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Using A Varifocal Mirror

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A New Range Finding Method Using A Varifocal Mirror

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ABSTRACT
A new range finding method is proposed in this paper which makes use of a varifocal mirror. The three-dimensional object space is first discretized into a sequence of spherical shells with a specially designed nonlinear vibrating varifocal mirror. These discrete spherical shell images are then recorded by a video camera. A deblurring algorithm is introduced in this paper which is used to remove the blurred components in the images. Different depth ranges can be obtained by controlling the vibration amplitude and the direct current component of the driving wave for the varifocal mirror. The depth accuracy is adjusted by varying the vibration period of the varifocal mirror. This range finding technique can be made real time by increasing the frame frequency of the camera.

1 Introduction
The range finding methods in the area of machine vision can be classified into three categories: active range finding methods, passive range finding methods, and their combinations.

In the categories of active range finding methods, the ultrasonic wave, the radio wave or the microwave, the visible light, the infrared wave, as well as other form of waves are used to detect the range between the wave source and the object. All these active range finding methods emit some kind of energy towarding the object, then receive the reflected energy so that the range can be measured according to the difference of the energy transmitting time and its transmitting speed. For example, the accurate radar range finding instruments and the laser range finders belong to the active range finding methods [1] [2] [3].

The passive range finding methods use the stereo vision parallax, the phase differences, the defocus disparities, the focus information, and as well as any other disparities coming from the objects which can be used to measure the range from the viewing point to the viewed object. All these passive range finding methods only receive the energy from the objects. For example, the stereo camera range finding method and the focus range finder are well known passive range finding methods [4] [5] [6]. In some other range finding methods, the active and passive range finding methods are combined together in order to obtain some special range finding results or to satisfy some special purposes [7].

In every range finding method mentioned above, there exist some advantages and some disadvantages. For instance, although the active range finding methods can obtain more accurate range data, but they dissipate a lot of energy, and even some emitting wave is very harmful to human being. In military applications, the emitting wave can be detected by the opponent. The passive range finding methods, such as stereo range finding techniques, are...
very attractive to the researchers, but there are various problems which are hard to overcome, such as the stereo matching errors and the measured range accuracy [8][9].

Facing these problems and the rapid arising requirements to three-dimensional machine vision, the researchers have to try to find the other new methods to finding range. Recently, Pietikainen and Harwood proposed a new method to find range known as the depth from three camera stereo [10]. After Pentland’s new sense for depth of field was published [11], Ma and Olsen announced their depth from zooming range finding method [12]. All these new methods of finding range make better progress in the field of three-dimensional machine vision. There will certainly be more and more new range finding methods to appear in the near future.

This paper proposes a new range finding method which makes use of the varifocal mirror. The concept of varifocal mirrors comes from a three-dimensional imaging technique which was first proposed by Muirhead in 1961 and the first three-dimensional imaging prototype was announced by Traub in 1967 [12]. Because the varifocal mirror 3D imaging technique has some attractive features which can almost display a pseudo autostereoscopic images, many researchers tried to use this method to record, transmit and display the three-dimensional pictures in broadcasting television system. Unfortunately as there exist some serious problems in varifocal mirror imaging techniques (flickering effects, ghost images, blurred pictures, etc.) which can not be solved so far, the varifocal mirror 3D imaging technique did not succeed [13][14][15].

However, because there are no displaying problems in three-dimensional machine vision, we can make use of the varifocal mirror as a powerful tool in range finding. First we discretize the three-dimensional object space into a sequence of the spherical shells which have the same origin. Then we do the volume scanning in this discretized 3D space and record these spherical shells separately with a specially designed varifocal mirror camera. Finally, we process these spherical images for creating a sequence of depth maps which can be used to find range. This new range finding method has some advantages:

1. It is a passive range finding method that only receive the reflecting light from the object.
2. There are no stereo matching and calibrating problems because only one camera is to be used in this method.
3. Through controlling the vibration frequency of the mirror, we can obtain the different depth range and different accuracy from coarse to fine.
4. By increasing the frame frequency of the video camera, we can increase the volume scanning frequency of the varifocal mirror. Therefore, we may use this method as a real time range finding method.

2 An Introduction to Varifocal Mirror 3D Imaging Technique

A brief introduction to Traub’s varifocal mirror three-dimensional imaging technique will be helpful for us to understand this range finding method.

2.1 Imaging Formula and Imaging Principle of A Varifocal Mirror

As it is well known that the image distance of an object in an optical system will be changed when the focal length of the optical system varies. Muirhead and Traub adopted this basic imaging principle in their three-dimensional imaging technique [12][13][14].

Let’s describe how their system works. First of all, we should know the imaging function of a varifocal mirror. We consider a circular mirror with a spherical surface whose circular radius and spherical curvature radius are denoted respectively by R and r (Ref. Figure 1). According to the geometrical theory of optical imaging [16], we can derive the imaging formula of the varifocal mirror:

\[ S_{f1} = \frac{(R^2 + \Delta^2)(-S_{f0} + \Delta)}{R^2 - 3\Delta^2 + 4\Delta S_{f0}} \]  

where \( S_{f1} \) denotes the image distance, \( S_{f0} \) denotes the object distance, \( \Delta \) is the vibration amplitude of the varifocal mirror, and here
Figure 1: Imaging Principle of the Varifocal Mirror

we assume that an approximation condition \( \Delta << R \) is satisfied. From the Equation(1) we can see that the image of the object in three-dimensional object space will be moving with the vibration of the varifocal mirror, so that a point in three-dimensional object space will be displayed as a three-dimensional line in image space, and a line in three-dimensional object space will be displayed as a three-dimensional plane. If we arrange a series of two-dimensional pictures which contain the different depth information on a fixed object distance in the separative time sequences and control the vibration of the mirror in some order, we are supposed to see the three-dimensional images through the varifocal mirror.

Figure 2 shows how an object in three-dimensional space can be recorded, transmitted and displayed using a varifocal mirror. This method was first proposed by King and Berry in 1970 [14]. \( S \) is an object in the three-dimensional object space, \( VM-1 \) is a varifocal mirror for recording the images of the objects, \( BS-1 \) is the beam splitter which creates the virtual images of the mirror around the object plane, \( L \) is a lens which has a large aperture and short focal length and so its depth of field will be so small that only the virtual images near the object plane of the lens can be clearly recorded on the video camera, and \( D \) is the image plane of both the lens and the video camera.

While the mirror vibrating with some frequency, the virtual images of the three-dimensional objects will be moving along the optical axis of the lens, and therefore, the video camera can record a series of pictures which contain the different depth information passing through its object plane, then all these recorded image signals are transmitted via some telecommunication means to the receiver.

At the receiver part, another varifocal mirror \( VM-2 \) which has the same specifications as the \( VM-1 \) may reconstruct the three-dimensional object images from the series of two-dimensional pictures displayed on the television screen. The observers will see the three-dimensional effect through the beam splitter \( BS-2 \).

2.2 The Problems of A Varifocal Mirror Imaging System

We have mentioned in Section 1 that the varifocal mirror imaging system has some serious problems. Now let us explain them in details. We assume that the frame frequency of the camera is \( f_T \), and vibration frequency of the varifocal mirror is \( f_V \). It is obvious to see
that the depth resolution of the varifocal mirror imaging system is:

\[
D_R = \frac{f_T}{f_V}
\] (2)

For a standard TV camera, the frame frequency \(f_T\) is 25HZ, if the vibration frequency \(f_V\) of the mirror is 10HZ, we can only obtain 2.5 depth resolution. For getting a reasonable stereoscopic effects, we are not able to raise the \(f_V\) although this frequency will lead to a serious flickering and for the compatible reason of a television system we also can not raise the frame frequency of the camera. That is why we almost can not get rid of the flickering problem for the system.

As the nonlinear magnification characteristics of the varifocal mirror, a point in the three-dimensional object space will appear to the viewer although the same viewer in the object space may not see this point at all and the result is that the objects behind the opaque objects will be displayed out, and an opaque object become transparent! That is the ghost image effects of the system. Because persistence of vision, the ghost image effect can not be overcome in varifocal mirror imaging system.

Because there is a depth field of the lens, not only the virtual images just on the object plane, but also the virtual images around the object plane may be recorded on to the video camera, although these images out of the object plane are blurred. As this result, the varifocal mirror 3D imaging system can not give us the clear 3D pictures.

All these disadvantages which are very hard to be overcome hindered the varifocal mirror three-dimensional imaging technique to be used in practice. Finally, after fifteen years passed, it is almost forgot by the world.

Because there are no task to dispay a three-dimensional image on some range finding purposes, we should recall the varifocal mirror imaging technique and introduce some features of it into the range finding technical field. The following sections will express our research work in this aspect.

Figure 3: A Discrete 3D Object Space

3 Discretizing, Recording the 3D Objects

We break up a continuous three-dimensional object space into a discrete three-dimensional object space under a spherical coordinates and record these discrete spherical shells separately with a varifocal mirror and a video camera. For recording a linear changing distance, we must use a nonlinear driving wave to drive the varifocal mirror.

3.1 Discretizing the 3D Object Space

The later analysis will show that the three-dimensional object space is broken up into a discrete three-dimensional object space under a spherical coordinates is very convenient to derive some useful results. Let us put the optical center of a varifocal mirror on to the origin of a spherical coordinate and let the optical axis of the varifocal mirror coincide with the x axis. Figure 3 depicts the discrete situation. Assuming the area that we are interested is within:

\[
\begin{align*}
  r_{\text{min}} & \leq r_i & \leq r_{\text{max}} \\
  \phi_{\text{min}} & \leq \phi_j & \leq \phi_{\text{max}} \\
  \theta_{\text{min}} & \leq \theta_k & \leq \theta_{\text{max}}
\end{align*}
\] (3)

\[
i = 1, 2, \ldots N. \\
\phi = 1, 2, \ldots M.
\]
Therefore, given every \( r_i \), we obtain a corresponding spherical shell \( S_i \). Through controlling the values of \( r_{\text{min}}, r_{\text{max}}, \phi_{\text{min}}, \phi_{\text{max}}, \theta_{\text{min}}, \theta_{\text{max}} \), we can obtain the different range area. Adjusting the number of \( N \), we can control the depth resolution, or in other words, the range accuracy. We may perform this work through setting the definite amplitude, the direct current component of driving wave and the vibration period of the varifocal mirror.

### 3.2 Nonlinear Driving Wave for A Varifocal Mirror

From the introduction to the varifocal mirror three-dimensional imaging technique we have known that the video camera object plane is fixed. In order to obtain a sequence of spherical shell images which are separated by the equal intervals, we must control the moving of the virtual images in a nonlinear state. According to the geometrical theory of imaging, we can obtain the imaging function for a spherical reflecting mirror under its geometrical symmetry condition (Ref. Figure 4) as follows:

\[
x_i = \frac{r_m}{2x_p - r_m} x_p
\]

\[
y_i = \frac{-r_m}{2x_p - r_m} y_p
\]

where \( x_p, y_p \) is an object point on some discrete spherical shell, \( x_i, y_i \) is the image point of the object point on the image plane, and \( r_m \) is the radius of the varifocal mirror. We first square Equation (4) on both sides and add them together, then substitute \( x_i^2 + y_i^2 \) and \( x_p^2 + y_p^2 \) with \( R_p \) and \( R_i \) respectively. Introducing the approximation condition \( r_m \gg x_p \), we derive another imaging formula:

\[
R_i = \frac{r_m}{2R_p - r_m} R_p
\]

Let us suppose the farthest spherical shell \( S_N \) is moving towards the origin of the varifocal mirror with a constant speed \( V_0 \) and for obtaining the linear depth varies we should keep the \( R_i \) unchanged during the motion of the spherical. So we can differentiate Equation (5) and set its derivative to be zero. Solving this equation, we obtain the relation between the linear moving of the spherical shell and the nonlinear vibration of the varifocal mirror:

\[
r_m(t) = \frac{2R_i (v_0 t + R_{\text{max}})}{R_i + (v_0 t + R_{\text{max}})}
\]

\[
v_0 = \frac{R_{\text{min}} - R_{\text{max}}}{T}
\]

If we keep the radius of the mirror vibration to follow Equation (6), we can record a sequence of spherical images with an equal depth interval.

A varifocal mirror can be made up in several ways and a practical method makes use of a loudspeaker on which a silver vibration film is fixed with a flange plate. The film is driven by the air within the paper basin and the paper basin is vibrated with the driving current via the speaker coil. A analysis shows that the vibration form of the film will follow the driving current wave form and if the amplitude of the mirror is not so large the film surface is almost ideally spherical. Finding the relation between \( r_m(t) \) and the driving wave will be very useful in practice. Let us see Figure 5, where \( r_m(t) \) is the radius of the vibrating silvering film, \( R_m \) is the radius of the speaker and \( \Delta(t) \) is the vibration amplitude of the silver film. The relation between \( \Delta(t) \) and \( r_m(t) \) can be expressed as follows:

\[
\Delta(t) = -r_m(t) + \sqrt{r_m(t)^2 - R_m^2}
\]
Expanding the last part of Equation (7) with a power series and omitting the higher order terms of $R_m/r_m$, we obtain the formula of the driving wave form for the varifocal mirror:

$$
\Delta(t) = -\left(\frac{R_m}{2}\right)\left(\frac{1}{v_0 t + R_{max}} + \frac{1}{R_{max}} \right)
$$

$$
v_0 = \frac{R_{min} - R_{max}}{T}
$$

Figure 6 represents the driving wave form which can be easily performed by a general function generator. From Equation (8), Equation (2) and Figure 6, we obviously see that the different maximum depth ranges can be controlled by adjusting the vibrating amplitude of the varifocal mirror, and the various minimum depth ranges may be obtained through setting the different direct current component of the driving wave. For getting more accurate range data, we can decrease the vibration period of the varifocal mirror so that we can record more multiple spherical shells during one period and through increasing the frame frequency of the video camera we even can use this range finding method as a real time range finding technique. Figure 7 represents a practical structure and specifications of a range finding varifocal mirror camera.

### 4 Removing the Blurred Images

As the influence of depth field of the video camera, not only the virtual images on the object plane, but also the virtual images around the object plane will be recorded on to the video camera. To obtain a clear sequence of the discrete spherical shell images, all blurred components must be removed. There are two kinds of techniques which can be used in this task: digital deblurring method and analog filtering method. The former leads to an ideal deblurring result and the latter results in real time processing speed in spite of losing some low frequency components of the images.
4.1 The P.S.F. of the Varifocal Mirror Camera

An optical image can be expressed as the three-dimensional convolution of a 3D object with its 3D point spread function (p.s.f.), and also a blurred image can be expressed as the result of the convolution of a 3D object with its defocus p.s.f. To remove the blurred components of the images we must know the p.s.f. with respect to the blurred images. Hopking and Stokseth discovered the properties of a defocus optical system dealing with the coherent light [17] [18]. From their research work, we can derive the infocus p.s.f. and the defocus p.s.f. under the white light condition. In the former case, the p.s.f. can be expressed as follows:

\[
I(0, \nu) = I_0 \int_{\nu0nm}^{700nm} \frac{2J_1(\nu)}{\nu} d\lambda \quad (9)
\]

\[
\nu = \left( \frac{2\pi}{\lambda} \right) \left( \frac{a_c}{f_c} \right) r_c
\]

\[
I_0 = \left( \frac{\pi a_c^2 A_o}{\lambda f_c^2} \right)^2
\]

where \(a_c\) is the radius of the video camera lens aperture, \(r_c\) is its radius variable, \(\lambda\) is the wavelength of light, \(A_o\) is the incident light intensity from the objects, and \(f_c\) is the video camera image distance. Figure 8 depicts the distribution of the intensity in the exact focal plane. Because the Equation(9) contains the first order Bessel function whose computation is very expensive, we have to find a simple function to approximate it. As is known to all, the Gaussian function has the similar distribution as Figure 8 and it can be proved that for any infocus point spread function in white light condition, we can find a Gaussian function with a definite variance \(\sigma\), which can approximate the distribution of Equation(9). Figure 9 represents this approximation. In the defocus condition, we can derive the expression of p.s.f. as follows:

\[
I(\mu, \nu) = I_0 \int_{\nu0nm}^{700nm} \frac{2}{\mu} (U1^2 + U2^2) d\lambda
\]

\[
I_0 = \left( \frac{\pi a_c^2 A_o}{\lambda f_c^2} \right)^2
\]

\[
U_m(\mu, \nu) = \sum_{s=0}^{\infty} (-1)^s \left( \frac{\mu}{\nu} \right)^{m+2s} J_{m+2s}(\nu)
\]
where $U_n(\mu, \nu)$ is the Lommel function and $x$ is defocus deviation. Although the distribution of $I(\mu, \nu)$ is so complicated that we even cannot use it in practice, we also can find a Gaussian function with a definite variance $\sigma$ as an approximate expression of Equation (10), as long as the defocus deviation $x$ is not too large [16]. Figure 10 depicts the p.s.f. intensity distribution at different defocus. It is obvious that a definite Gaussian may coincide with the defocus point spread function. If we encounter some defocus images with a large deviation, we may use other more accurate approximate functions to Equation (10) which can be derived from Ref. [19] [20], but the computation will be more expensive.

4.2 Deblurring Algorithm

As we described above, any optical imaging can be expressed by the convolution of a 3D object with the definite point spread functions. Referring Figure 3, 4, and Equation (5), let $p_n(r_i, \phi_j, \theta_k)$ denote a spherical image which is just positioned at the object plane of the video camera and let $h_n(r_i, \phi_j, \theta_k)$ denote the point spread function with respect to the $p_n(r_i, \phi_j, \theta_k)$, therefore, the image $g_n(y_j, z_k)$ on the video camera can be represented as follows:

$$g_n(y_j, z_k) = \sum_{i=1}^{N} h_n(r_i, \phi_j, \theta_k) \otimes p_n(r_i, \phi_j, \theta_k)$$

$$j = 1, 2, \cdots M.$$

$$k = 1, 2, \cdots P. \tag{11}$$

As the influence of depth field of the video camera, not only the spherical image on focal plane of the video camera, but also the spherical images around the focal plane will be recorded on to the video camera. Therefore, the image on the video camera at any sampled time during vibration of the varifocal mirror is a summation of responses to all spherical images. Because the vibration amplitude, the vibration frequency and the frame frequency of the video camera are all known factors, we can obtain the defocus p.s.f. in three-dimensional image space at any time sequence. Considering the blurred effect, the image on the video camera can be written out:

$$g_n(y_j, z_k) = \sum_{i=1}^{N} h_n(r_i, \phi_j, \theta_k) \otimes p_i(r_i, \phi_j, \theta_k)$$

$$j = 1, 2, \cdots M.$$

$$k = 1, 2, \cdots P. \tag{12}$$

Applying the Fourier transform to both sides of Equation (12), and introducing the convolution theorem, we can obtain the convenient multiplication form of Equation (12) in frequency domain:

$$G_n(\mu, \nu) = \sum_{i=1}^{N} H_n(\mu, \nu, \xi) P_i(\mu, \nu, \xi) \tag{13}$$

In Equation (13), $P_i(\mu, \nu, \xi)$ is the Fourier transform of the every spherical image at the image distance $r_i$ and do the inverse Fourier transform of $P_i(\mu, \nu, \xi)$ we can obtain the clear spherical image itself. For solving the $N$ equations we should have $N$ equations which contain $N \times N$ known $H_n(\mu, \nu, \xi)$ and $N$ known $G_n(\mu, \nu)$. As we have described in preceding sections, we can record $N$ spherical images during one period of vibration time of the varifocal mirror and the number of $N$ can be defined by the factors of the varifocal mirror camera according to the different requirement and we also can find the definite Gaussian as
the approximate functions of all infocus and defocus p.s.f. We substitute all point spread functions in Equations(11)(12)(13) with the Gaussian functions, therefore, we get the matrix equations bellow:

\[
\begin{align*}
G_1 &= H_{11}P_1 + H_{12}P_2 + \cdots + H_{1N}P_N \\
G_2 &= H_{21}P_1 + H_{22}P_2 + \cdots + H_{2N}P_N \\
&\vdots \\
G_N &= H_{N1}P_1 + H_{N2}P_2 + \cdots + H_{NN}P_N \\
\end{align*}
\]

(14)

From the Equation(14), we can solve the \( N \ P_i(\mu, \nu, \xi) \), and from the inverse transform of \( P_i(\mu, \nu, \xi) \), we can obtain all deblurred spherical images: \( p_i(r, \phi, \theta, \xi) \) and where \( i = 1, 2, \cdots N; \ j = 1, 2, \cdots M; \ k = 1, 2, \cdots P \).

4.3 Deblurring Method with Analog Filter

Performing the deblurring algorithm mentioned above can give out a sequence of clear images, but it will spend a lot of computing time. We can make use of the analog filtering techniques to remove these blurred images in real time speed, but some low frequency component in images will be lost.

5 Conclusion

This paper has proposed a range finding method which makes use of a specially designed varifocal mirror camera. The three-dimensional object space is discretized into a sequence of discrete spherical shells which have the definite depth range and depth interval. These discrete spherical shells are recorded respectively on to a video camera and then the blurred components are removed through a digital or an analog filtering technique. The recorded depth range and depth accuracy can be controlled by setting the parameters of the varifocal mirror system. Through increasing the frame frequency of the video camera, this range finding method can work in real time speed.

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