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NOTE: At the time of publication, author Daniel Koditschek was affiliated with Yale University. Currently, he is a faculty member in the Department of Electrical and Systems Engineering at the University of Pennsylvania.
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Autonomous Mobile Robots
Controlled by Navigation Functions

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Abstract

This paper reviews the theory of navigation functions and the attendant use of natural control techniques with emphasis upon applications to mobile autonomous robots. Results to date will be discussed in the context of a larger program of research that seeks effective parametrizations of uncertainty in robot navigation problems. Constructive solutions to particular cases of mobile robot navigation problems with complete certainty are provided as well.

1 Introduction

This paper provides a tutorial sketch of recent mathematical results concerning the existence and construction of robot controllers for exact navigation in cluttered worlds. Honoring the stated theme of the present Workshop, particular emphasis is given the application of this theory to autonomous mobile robots.

1.1 Intended Scope of Application

Although the body of this paper has been kept relatively free of mathematical formulas, the abstract technical nature of the discussion may nevertheless obscure the ultimate goals of our work. At the risk of erring in the opposite extreme, it seems worth injecting a brief account of our larger ambitions. Thus, consider the following scenario.

Example 1.1.1 A Fantasy.

You tell the robot to enter your office and go to the window behind your desk without banging into the desk or chairs. The robot has never been in your office before, but it knows, for example, that an office is a topologically deformed solid ball, and, similarly, that a chair or a desk is a deformed pretzel, and so on. Moreover, it possesses a parametrized family of coordinate transformations that successively deform each model of Euclidean balls, cylinders, and pretzels into ever more detailed particular instances. The robot develops a plan of navigation in the model space, then as it begins to carry out the plan in the real world, it starts to adapt its parametric representation of the real room and chairs (as well as their relative location) in accordance with new sensory data. The plan is sufficiently conservative that no collisions occur along the way. The plan is sufficiently industrious that exploration of the real environment will continue until enough particular understanding of the geometric details is available to complete the task (if it is physically possible to do so).

This example must presently stand as fantasy because it presumes a parametrized representation of the world which is simultaneously effective with respect to the navigation problem; to the control implemention; and to the robot’s perceptual apparatus. This paper sketches a mathematical formalism we are developing that encompasses the first two notions of effectiveness. Constructive Recipes for solving very particular navigation problems for which perfect knowledge of the parameters is already known are given as well. Presently, we do not entirely understand how to formalize the third notion, and little more will be said about it here in consequence.

1.2 Representations of the Navigation Problem: a Manifold with Boundary

There are several levels of geometric and kinematic complexity that might be considered in the analysis of autonomous vehicle navigation. Here, we present a brief sketch of a hierarchy of problems. The intent is to distinguish the task domain over which our results may be considered immediately practicable as opposed to merely theoretically applicable. In all cases, the formal representation of the navigation task is a compact manifold with boundary whose interior corresponds to the freespace — the set of all robot placements that avoid intersections with the environment — and whose boundary components represent the obstacles. We have shown [12] that the topological properties of these bounded spaces define the invariant features of any navigation problem. That is to say two problems are identical if their representations are homeomorphic. Hence it is fitting to provide some intuition concerning the homeomorphism equivalence class of the various examples below.

The first example is uninteresting from any practical point of view and is included simply to afford a trivial setting in which to develop intuition in the sequel.

Example 1.2.1 Point Robot in a Line Segment.

The configuration space is some closed real interval, \(J = [a, b]\). It is evidently compact because it is closed and bounded. Its boundary is the set \(\partial J = \{a, b\}\). It is embedded in a Euclidean vector space, \(\mathbb{R}^n\), but is topologically distinct. Unlike the open interval, \((a, b)\), no change of coordinates can be used to identify the closed interval \([a, b]\) with \(\mathbb{R}^n\).

We now suppose a small robot is assigned to navigate in a relatively uncluttered room. The walls of the room and the obstacles may be approximated by circular disks without too much loss of accuracy. The robot’s physical extent will be ignored.

Example 1.2.2 Point Robot in a Sphere World.

The configuration space, \(J\), is a compact connected subset of \(\mathbb{R}^3\) whose boundary is the disjoint union of circles — an outer circle represents the walls of the room and each circle inside represents a distinct obstacle. Again, this space is topologically distinct from \(\mathbb{R}^2\). Consider, instead, a two-dimensional sphere in \(\mathbb{R}^3\). Cut out a circle around the north pole and identify this boundary with the outer wall of the room. Cut out...
a circle in the remaining portion of the sphere for each interior obstacle. The resulting punctured sphere is topologically identical to \( J \).

If the room is more tightly cluttered but still entirely connected then the exact location of each obstacle becomes much more important and the geometric details of each boundary must be explicitly modeled. This situation might be modeled as follows.

**Example 1.2.3 Point Robot in a Blob World.**

The configuration space, \( J \), is a compact connected subset of \( \mathbb{R}^2 \) whose boundary is the disjoint union of deformed circles. For example, each obstacle might be represented as the zero level set of a proper (that is, every compact set in its range has a compact pre-image in its domain) scalar valued function on \( \mathbb{R}^2 \). This example is topologically indistinguishable from the previous one.

If, instead, the room is sufficiently cluttered that the physical size of the robot precludes free passage through apparently free regions then the configuration space differs from the workspace and must be computed by “growing the obstacles” and shrinking the robot to a point, for example, as in Lozano-Pérez and Wesley [13]. A plausible model in this case adds the possibility of separate components of the configuration space.

**Example 1.2.4 Spherical Robot in a Blob World.**

The configuration space may be disconnected. However each connected component is topologically identical to some punctured sphere world as defined in Example 1.2.2.

Finally, if the room is so cluttered that the robot can only move through narrow passages by adjusting its orientation then the configuration space differs in dimension from the workspace, and we arrive at what is arguably the first problem domain sufficiently complex to require the full power (and computational burden) of Schwartz and Shapiz’s [21], solution to the generalized “piano mover’s problem”.

**Example 1.2.5 Blob Robot in a Blob World.**

The configuration space is a solid torus with smaller solid tori removed from the interior. [4]. It is topologically distinct from any of the previous examples.

To preview the contents of this paper we offer a constructive solution to almost all problems up to the level of Example 1.2.3 assuming perfect information concerning the workspace obstacles taking the form of a scalar valued function (whose zero level represents the boundary) for each obstacle. Our constructions would be immediately applicable to the problem represented by Example 1.2.4 if some further processing resulted in an implicit representation of each of the configuration space boundary components. It is important to note, however, that the only way to determine whether or not the robot is presently in the same connected component of the configuration space as the desired point of destination is to actually run the algorithm: the robot arrives at the destination with probability one if a path exists. Our theoretical results guarantee the existence of solutions to all navigation problems including the generalized piano mover’s problem via the techniques presented here. However we have not presently attempted any constructive solution to the problem domain of Example 1.2.5.

1.3 The Link to Control Theory: Dissipative Mechanical Systems

This section provides a brief discussion of the robot control systems assumed by our theory. Given a configuration space, \( J \), the phase space, \( P \) models all possible velocities at any possible configuration.

For the sake of clarity, we will use the coordinates \( q \in J \) to denote configurations, and the coordinates, \( p = (p_1, p_2)^T \in P \) to denote phases: thus, \( p_1 \) is identified with \( q \) in all future formulae.

A *mechanical control system* is a second order system, as determined by the ordered pair, consisting of a configuration space and a choice of kinetic energy,

\[
\Sigma = (J, \kappa),
\]

defined by

\[
f_c(p, u) = f_c(p) + \begin{bmatrix} 0 \\ M^{-1} (p_1) u \end{bmatrix},
\]

where \( f_c \) is the Lagrangian vector field specified by the kinetic energy.

**Example 1.2.1 (continued).**

The phase space over \( J = [a, b] \) is the closed vertical strip in \( \mathbb{R}^2 \), \( P = [a, b] \times \mathbb{R} \). Its boundary is formed by the two vertical lines through the endpoints of the configuration space.

\[
\partial P = \{ a \times \mathbb{R} \} \cup \{ b \times \mathbb{R} \}
\]

Suppose our one degree of freedom prismatic robot has mass \( M \). Then the kinetic energy is given by

\[
\kappa(q, v) = \frac{1}{2} M v^2,
\]

the Euler-Lagrange operator applied to \( \kappa \) yields a double integrator,

\[
p_1 = p_2, \\
p_2 = M^{-1} I_0,
\]

thus

\[
f_c(p) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} p.
\]

The control system (2) remains qualitatively the same as in the previous one degree of freedom case for Example 1.2.2 through Example 1.2.4 (it is now a pair of two forced double integrators) assuming that the two degrees of freedom can be simultaneously and independently actuated. We make this assumption throughout the sequel, thereby eliminating the more interesting nonholonomic case of a robotic cart with steering wheel treated in [1, §5.1] and [17, pp. 33-36]. For example, this model applies to Moravec’s cart Pluto, but not to Uranus [15].

If the input to (2) takes the form of a generalized PD controller,

\[
u = -\nabla \varphi (p_1) - G(p_2),
\]

where \( \varphi \) is a scalar valued map on the configuration space \( J \), and \( G \) has the property that \( p_2 G(p_2) \) is a positive definite function, then the resulting closed loop system is said to be a *dissipative mechanical system* [10, 11]. A century old result of Lord Kelvin [23] states that trajectories of the resulting closed loop system that start in a neighborhood of a local minimum of \( \varphi \) tend asymptotically toward that local minimum.

**Example 1.3.1 A Hook-Rayleigh System.**

The the sake of concreteness suppose that the configuration space, \( J \), in Example 1.2.1 is an asymmetric interval about the origin, \([−e, 1]\), and that the origin is the desired destination point. A Hook’s law spring potential, \( \frac{1}{2} q^2 K_1 q \) \((K_1 \text{ is positive}) \) encodes the “plan” since all trajectories of the negative gradient system,

\[
\dot{q} = -\nabla \varphi(q) = -K_1 q
\]

that start in \( J \) arrive at the origin, and no trajectories that start in \( J \) ever leave the set (crash into the boundary, \( \partial J \)).

To form the associated dissipative mechanical system, let us take a Rayleigh damping law \( G(p_2) = K_2 p_2 \) \((K_2 \text{ is positive as well}) \). Now apply the feedback law to the robot as dictated...
by the recipe in (3). The resulting closed loop is a "Hook-Rayleigh" dissipative mechanical system, \( \Delta_{HR} \), given by

\[
p = f_{\Delta_{HR}}(p) = \begin{bmatrix} 0 & 1 \\ -K_1 & -K_2 \end{bmatrix} p,
\]

a familiar second order linear time invariant system. All trajectories of this second order system asymptotically approach the origin of phase space, just as trajectories of the planning system approach the desired destination in configuration space. Formally, this may be proved by noting that the total energy for \( \Delta_{HR} \),

\[
\eta = \frac{1}{2} K_1 p_1^2 + \frac{1}{2} K_2 p_2^2,
\]

is non-increasing along trajectories.

The observation that total energy decreases in dissipative mecha-
nical systems is the basis for interest in artificial potential field
methods of robot control.

2 Robot Navigation with Perfect Information

Motivated by Lord Kelvin's assurance that dissipative mechanical systems end up at the local minima of the potential field, a great deal of interest in robotics has centered around the construction of artificial potential field to encode navigation problems. In this sec-
tion we will briefly review the general literature (for a less superficial
review, see [10]) and sketch the our own contributions in the case of
perfect information.

2.1 Previous Experience with Artificial Potentials

In his original 1978 paper with Le Maitre [6], Khatib assumed that
each obstacle is described as the zero level surface of a known scalar
valued analytic function, \( f(x, y, z) = 0 \), which he used to form a local
inverse square potential law. This construction goes to infinity as the
inverse square of \( f \) near the obstacle, and gets cut off at zero at some
positive level surface, \( f(x, y, z) = f_0 \), presumably "far enough" away
from the obstacle, as determined by the designer. A particle moving
according to Newton's Laws in such a potential field would clearly
never hit this obstacle. Khatib further observed that the sum of the
gradients is the gradient of the sum: thus adding up the potential
functions as

\[
\nabla = A \text{H and } B \text{R,}
\]

infinite value of the artificial potentials required to prevent trajec-
tories of the ultimate mechanically controlled system from crashing
through obstacle boundaries obviously could not be achieved in the
physical world and there were no clear guarantees as to when the
saturation torque levels of the robot's actuators would indeed suffice
to prevent collisions.

2.2 Some New Concepts

Our work addresses and overcomes the limitations of artificial po-
tential fields described above. We discuss here the conceptual foun-
dations of this work along with concrete illustrations through simple
examples.

2.2.1 Global Mechanical Analog Computers

We proposed in 1985 [8] a systematic study of the properties of vec-
tor fields on a configuration space that make them effective planning
systems for mechanical systems on the phase space. Since regular
gradient systems are guaranteed to have a simple limit set consist-
ing of isolated extrema (in contrast to the usual complex limit sets
of more general classes of nonlinear vector fields) [11] they are a
natural choice for specifying asymptotic desired behaviors such as
"go there and stay". Moreover, the transients of a gradient flow are
readily shaped by adjusting the level curves of the associated cost
function, since trajectories run orthogonally to them [5]: any be-
behavioral properties that can be translated into the geometry of level
curves may be planned in this fashion. We have demonstrated how
the addition of several technical properties to the list of conditions on
gradient fields ensures that the generalized PD controller (3) will
cause the same global limit behavior in the second order mechanical
system (on a compact subset of phase space that includes the entire
configuration space) as in the first order gradient system [11]. The
conceptual advance involved in this contribution (which is otherwise
a perfectly unsurprising extension of Lord Kelvin's century old re-
sult on the dissipation of total energy) was a rigorous examination of
how to prevent finite escape through the boundaries of the configu-
ration space (arising, e.g., from robot joint limits, or obstacles in the
workplace) without requiring infinite actuator forces. The problem
is best summarized by examining the trivial one degree of freedom
Hook-Rayleigh system, Example 1.3.1.

Example 1.3.1 (continued).

The boundary of the phase space, \( \mathcal{P} \), of the Hook-Rayleigh
System, is the union of the two parallel lines

\[
\partial \mathcal{P} = ((-\epsilon) \times \mathbb{R}) \cup (\{1\} \times \mathbb{R}),
\]

running vertically through each boundary point of the configu-
ration space considered as a segment in the horizontal axis of
\( \mathcal{P} \). Unfortunately, however, \( f_{\Delta_{HR}} \) is directed away
from the interior of this infinite vertical strip of a phase space on the
upper half of the line through the point \([1,0]\) and the lower
half of the line through \([-\epsilon,0]\). Consequently, it may be ob-
served that the trajectory of \( f_{\Delta_{HR}} \) through every initial condi-
tion in a neighborhood of these open half line segments must
escape from \( \mathcal{P} \) in finite time. Thus, successful obstacle avoid-
ance properties of the gradient "planning system" on \( \mathcal{J} \) fail
to guarantee that the dissipative mechanical system will enjoy
the same properties.

In fact, we may once again use the total energy to determine the
extent of "safety" — those initial conditions of the dissipative me-
chanical system that are guaranteed to have collision free trajectories
— as follows.

Example 1.3.1 (continued).

The total energy function for \( \Delta_{HR} \) is

\[
\eta = \frac{1}{2} K_1 p_1^2 + \frac{1}{2} K_2 p_2^2.
\]

The energy level set \( \eta \geq 1 \) is a truncated ellipse just touching
(1,0) and tangent to the vertical line 1 x \( \mathbb{R} \) comprising the
right hand boundary of \( \mathcal{P} \). This ellipse is truncated on the
left hand side of the plane by the line segment contained in the
left hand boundary, \( -\epsilon \times \mathbb{R} \). Thus \( \mathcal{J} \) is bounded only

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partially by a total energy surface. Trajectories originating in this set, \( E^i \), cannot escape through the ellipsoidal portion of the boundary but, as we have seen, certainly can escape through the left hand truncating line.

On the other hand, the energy level set \( \eta = K/1^2 \) only touches the boundary of phase space on the zero velocity axis. All initial conditions within this set are "safe".

This example suggests that the potential fields should be so constructed that some resulting total energy surface just touch the phase function must take its minimum uniquely on the desired destination fashion.

We are led to impose further conditions on the potential function to it is polar space comprises the notion of a navigation function that we introduced in order for its "lift" to the mechanical system (via feedback compensation) to effectively carry out the prescribed plan on a region of phase space that entirely contains the (zero velocity) configuration space comprises the notion of a navigation function that we introduced to the literature a year ago [20]. Roughly speaking, such a function must take its minimum uniquely on the desired destination set to ensure convergence from almost every initial state (formally, it is polar), and, moreover, must take its maximal value exactly and uniformly on the boundary of the configuration space (formally, it is admissible) to avoid trajectories which "crash through" out of the legal space as in Example 1.3.1, above. The following example provides simple instance of a navigation function on the one degree of freedom configuration space introduced in Example 1.2.1.

Example 2.2.1 An Admissible Potential.

Now reconsider the problem in Example 1.3.1. We would like to retain the Hook's Law spring, \( \varphi_A \) since it forces all trajectories to converge toward the desired position, \( 0 \in J \), and remain there. However, we would like to avoid crashing into the boundary points, \( B_J = [ -c, 1 ] \). These "bad" points may be readily represented as the zero level set of two additional appropriately chosen scalar valued functions, say

\[
\beta_L(q) \triangleq |q + 1|^2; \quad \beta_R(q) \triangleq |q - 1|^2.
\]

Dividing the "good" function by the product of the "bad" functions \( \varphi_A / \beta_L \beta_R \) results in a new scalar valued function that "encodes" our goals by assigning smallest (zero) cost to the good configuration and the largest (infinite) cost to the bad configurations. It is easy to show that \( \tilde{\varphi} \) has only one minimum. Moreover, both of the boundary configurations are assigned the same largest cost value. Unfortunately, \( \tilde{\varphi} \) is not bounded on \( J \), and must be rejected as physically unrealizable.

Now consider the map,

\[
\sigma(z) \triangleq \frac{2}{\lambda + z},
\]

which "squashes" the infinite half interval \([0, \infty)\) to the bounded interval \([0, 1]\) for all positive scales \( \lambda > 0 \). Composing \( \tilde{\varphi} \) with \( \sigma \) (taking \( \lambda = 1 \) in this case) results in a new cost function,

\[
\varphi_A \triangleq \sigma \circ \tilde{\varphi} = \frac{2 \varphi_A}{\beta_L \beta_R},
\]

that attains its lowest value (zero) on the good configuration, 0, and its highest value (one) on the bad configurations, \( B_J \). Since \( \sigma \) is monotonic, the second derivative of \( \varphi_A \) has the same sign as that of \( \tilde{\varphi} \) at any critical point, thus \( \varphi_A \) has only one minimum since \( \tilde{\varphi} \) enjoys that property: \( \varphi_A \) is a navigation function on \( J \).

The gradient system

\[
\dot{q} = -\text{grad} \varphi_A = \frac{-q \varphi_A (q + 1)(q - 1)}{(q - \varphi_A)(q - 1)^2(q + 1) + \varphi_A(q + 1)(q - 1)}
\]

has the same invariance and convergence properties as does \( \text{grad} \varphi_A \), and the computational cost is now more than an order of magnitude greater. However, the associated dissipative mechanical system, \( \Delta = A \),

\[
\ddot{q} = -\text{grad} \varphi_A (p_1) - K/1^2
\]

offers the significant improvement over \( \Delta = A \), that every initial zero velocity state, \((p_1, 0) \in P \) converge to the desired equilibrium state, \((\eta, 0) \in P \) with the guarantee of never crashing into the boundary lines \( \partial P \). Of course, this property holds true for many more initial conditions as well — namely, all of those in the energy set

\[
E^1 = \left\{ p \in P : \varphi_A (p_1) - \frac{1}{2} |p^2| \leq 1 \right\}.
\]

The question immediately arises whether such desirable features may be achieved in general. For example, it is hopeless to attempt global asymptotic stability of a single destination point via a smooth memoryless time invariant controller (in all but uninterestingly simple problems like the example above) for fundamental topological reasons [9]. Are there similar topological obstructions to a navigation function? Fortunately, we were able to modify a construction introduced by Smale three decades ago in his proof of the Poincaré Homeomorphism Conjecture [22] in order to demonstrate the fact (surprising to us) that smooth navigation functions exist on any compact connected smooth manifold with boundary [12]. Thus, in any problem involving motion of a mechanical system through a cluttered space (with perfect information and no requirement of physical contact) if the problem may be solved at all, we are guaranteed that it may be solved by a navigation function. There remains the engineering problem of how to construct such functions.

2.2.3 Changes of Coordinates

The importance of coordinate changes and their invariants is by now a well known theme in control theory. Roughly speaking, these notions formalize the manner in which two apparently different problems are actually the same. Their most familiar instance is undoubtedly encountered in the category of linear maps on linear vector spaces whose invariants (under changes of basis) determine closed loop stability. Of course, many other instances may be found in the control literature and, more recently, the utility of coordinate changes in robotics applications has been proposed independently by Brockett [3] as well.

The relevant invariant in navigation problems is the topology of the underlying configuration space [9]. In this regard, the significant virtue of the navigation function is that its desirable properties are invariant under diffeomorphism [12]. Thus, instead of building a navigation function for each particular problem, we are encouraged to devise "model problems", construct the appropriate model navigation functions, and then "deform" them into the particular details of a specified problem. This notion pervades the remainder of the paper.

Example 2.2.2 Admissibility of a Hook's Law Spring by Change of Coordinates.

Any closed real interval is an affine coordinate change away from any other. For example, the configuration space, \( J = [-c, 1] \), of Example 1.3.1 may be identified with a "model
fying via coordinate changes, the more complicated the construction problem introduced in Example 1.2.2. While the construction of Section 2.3.2, solve the navigation problem introduced in Example 1.2.3 and Example 1.2.4 (assuming further processing results in a description of the configuration space). Thus, the composition function, \( \phi_M(q) \equiv (\varphi \circ h_q)(q) = \frac{1}{2} K_1((2q - (1 + r))^2) \), is admissible on \( \mathcal{J} \), since the "height" of any configuration \( q \in \mathcal{J} \) is determined by the "height" assigned to its identified image value \( h_q \in \mathcal{D}' \). In particular, since \( \mathcal{J}' \) is identified with \( \mathcal{D}' \), the admissibility property of \( \phi_M \) is preserved.

Of course, the more features of the problem we insist on identifying via coordinate changes, the more complicated the construction of the new coordinate system becomes. If we merely desire the identification of the interval's end points then the composition of a linear scaling and affine translation will do as the previous example shows. Stronger identifications require less rigid transformation classes and more care.

**Example 2.2.2** (continued).

Since \( h_1 \) takes the midpoint of \( \mathcal{J} \) to the origin of \( \mathcal{D}' \), \( \phi_M \) has a minimum at the midpoint of \( \mathcal{J} \). We require, however, that our potential field have a minimum at the origin of \( \mathcal{J} \), hence, we must construct a diffeomorphism taking the endpoints and the origin of the \( \mathcal{J} \) to the endpoints and origin, respectively, of \( \mathcal{D}' \).

Define an "analytic switch",

\[ \sigma_i(q) = \frac{\varphi_{hi}(q)}{\varphi_M + \lambda h_i h_q} \]

which uses the "squashing" function introduced in Example 2.2.1 to vanish at the origin and attain unity at the boundary of \( \mathcal{J} \). The function

\[ h(q) = \sigma_i h_1 + (1 - \sigma_i) g \]

takes the boundary of \( \mathcal{J} \) to the boundary of the model, \( \mathcal{D}' \), and also takes the origin to the origin, since \( h \) is the convex combination of two monotone increasing functions it is not hard to see graphically, and may be readily shown algebraically that \( h \) is itself monotone increasing for sufficiently large values of \( r \). Thus, \( h < 0 \) and preserves the boundary and origin. We have already argued that admisibility is preserved by \( h \). Moreover, since \( 0 \) is the unique minimum of \( \varphi_M + \lambda h_i h_q \), it is intuitively clear that \( \varphi_M h \) has its unique minimum at zero on \( \mathcal{J} \). A little more thought will quickly convince the reader that \( \varphi_M h \) is a navigation function for the origin of \( \mathcal{J} \).

Comparing Example 2.2.1 with Example 2.2.2 it is not clear that the second construction is any simpler. However, as the detailed geometric features of the actual problem become increasingly complicated, the general techniques of Section 2.3.1 fail: spurious minima cannot be avoided by the simple multiplication and division procedures adopted there. Thus, we are led to generalize the ideas of Example 2.2.2, above, in order to increase the "range of geometric expressiveness" of our methodology.

### 2.3 Some New Tools

In this section we present some constructive results. It will be observed that the construction of Section 2.3.1 together with the recipe of (3) provides an immediate feedback controller for the type of navigation problem introduced in Example 1.2.2. These, together with the construction of Section 2.3.2, solve the navigation problem introduced in Example 1.2.3 and Example 1.2.4 (assuming further processing results in a description of the configuration space).
Our constructive results to date encompass the class of “star worlds.” A star shaped set is a diffeomorph of a Euclidean n-disk, $D^n$ possessed of a distinguished interior center point from which all rays intersect its boundary in a unique point. A star world is a compact connected subset of $E^n$ whose boundary is the disjoint union of a finite number of star shaped set boundaries. We now suppose the availability of an implicit representation for each boundary formation required in the construction to follow is detailed in the component, chief reference for this work [19].

To the center and minimum “radius” (the minimal distance from the given star world and the derived model sphere world geometrical” steps.

Thus, if $\phi$ is a navigation function on $M$, and a positive constant $\lambda$, then

$$h_\lambda : F \rightarrow M,$$

is an analytic diffeomorphism.

This family of transformations, mapping any star world onto the corresponding sphere world, induces navigation functions on a much larger class than the original sphere worlds, thus advancing our program of research toward the goal of developing “geometric expressiveness” rich enough for navigation amidst real world obstacles. A paper presently in preparation describes how the construction presented here may be extended to handle arbitrarily close approximations to any situation of the kind encountered in Example 1.2.4.

3 Conclusion

This paper reviews our work to date in exact robot navigation assuming perfect information. We provide explicit recipes for constructing control laws guaranteed to bring a mobile robot to a desired destination without hitting any obstacles (assuming a path exists). The classes of environments for which a recipe is provided here include all of the sample hierarchy introduced in Section 1.2 up to (arbitrarily close approximations to) the sphere robot in the blob world of Example 1.2.4. We have already shown that every navigation problem is amenable to solution by our methods, thus constructions are worth attempting in more complicated cases, for instance the blob robot in a blob world of Example 1.2.5.

Ultimately, we see the most exciting use of the results presented here in a planned series of extensions to the case of partial uncertainty as exemplified by the fantasy application of Section 1.1. Our reliance upon generalized PD controllers (3) represents not so much a fixed faith in their performance but rather the choice of a simple vehicle for eliciting those properties of task descriptions that are simultaneously effective with respect to controller design as well. The task-encoding formalism presented here has the advantage of reflecting any uncertainty (i.e., imperfectly known configuration space boundaries) into the ultimate closed loop dynamics via the lifted configuration space vector field. Thus parameterized models of task uncertainty immediately generate clearly posed parameter adaptation problems.

One difficulty in pursuing this program of research is that any interesting parametrization of task uncertainty will not result in a linear-in-parameters adaptation problem and we will be forced to abandon the traditional tools of linear adaptive systems theory [16] in favor of radically new adaptive laws. Preliminary results of this nature will be presented shortly [7] indicating how to successfully adapt the power parameter, $k$, required by the navigation function in Theorem 1 while still avoiding collisions through the configuration space boundaries.

Yet an even more fundamental problem concerns the nature of the parameterized family itself: the parametrization should be “effective” with respect to the robot’s perceptual apparatus. Lurking at the heart of this still fuzzy notion is a theory of continuous geometric reasoning that would translate new sensory data into updated parameter values in a rational manner.
References


