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Abstract
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Critical Properties of Spin-Glasses*

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The critical properties of the model of a spin-glass proposed by Edwards and Anderson are studied using the renormalization group. The critical exponents are calculated in 6 - \epsilon spatial dimensions. It is argued that a tricritical point can exist where the non-ordering field is the skewness of the distribution of J.

Although spin-glasses, such as dilute solutions of Mn in Cu, have been studied experimentally for many years, only recently have formulations been given in terms of a microscopic Hamiltonian. Even so, the spin-glass transition has not been successfully related to the usual picture of phase transitions as we shall do here. In Refs. 3-6 we consider the spin Hamiltonian, \mathcal{H}, given by

\[ \mathcal{H}/kT = -\sum_{\mathbf{r},\mathbf{r}' \neq \mathbf{r}} K(J(\mathbf{r}, \mathbf{r}')) \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{r}'), \]

(1)

where \mathbf{S}(\mathbf{r}) = S_1(\mathbf{r}), S_2(\mathbf{r}), \ldots, S_m(\mathbf{r}) is a classical m-component spin of unit magnitude at the lattice point \mathbf{r}, and \( K(J(\mathbf{r}, \mathbf{r}')) = J(\mathbf{r}, \mathbf{r}') / kT \), where \( J(\mathbf{r}, \mathbf{r}') \) is a random variable with a probability distribution \( P(\mathbf{r}, \mathbf{r}'; J) \), and \( J(\mathbf{r}, \mathbf{r}') \) is assumed to be a finite-ranged interaction. We treat a quenched random system where the average free energy is calculated as the average, denoted [.]_v, over all configurations of \( J(\mathbf{r}, \mathbf{r}') \):

\[ F = [J(\{J\})]_v. \]

(2)

According to mean-field theory one expects a ferromagnetic or antiferromagnetic state if [\( J(\mathbf{r}, \mathbf{r}') \)]_v is sufficiently large in magnitude. If [\( J(\mathbf{r}, \mathbf{r}') \)]_v is zero, Edwards and Anderson (EA) argue that there will still be a transition at a freezing temperature \( T_f \) to an ordered state characterized by a new order parameter,

\[ q(\mathbf{r}) = \langle \mathbf{S}(\mathbf{r}) \rangle_\{J\} - \langle \mathbf{S}(\mathbf{r}) \rangle_\{J\}^\epsilon, \]

(3)

where \( \langle \mathbf{S}(\mathbf{r}) \rangle_\{J\} \) is the thermal average of \( \mathbf{S}(\mathbf{r}) \) for a given configuration \{J\}. Note that q is by definition a positive quantity. This will be important in what follows. EA calculate the properties of this spin-glass phase transition using mean-field theory and a Gaussian random distribution of \( J \)'s centered about [\( J(\mathbf{r}, \mathbf{r}') \)]_v = 0. They find a continuous transition with an order-parameter exponent of \( \beta = 1 \) and a finite discontinuity in the slope of the specific heat, \( dC(T)/dT \), at \( T = T_f \), so that \( \alpha = -1 \). Similar results were found by other more detailed calculations. A straightforward generalization of the EA treatment to in-
clude an external field conjugate to the order parameter yields susceptibility and correlation-length exponents $\gamma=1$ and $\nu=\frac12$. These exponents should be valid for spatial dimensionality, $d$, greater than a critical value, $d_c$. The value of $d_c$ may be determined as the value of $d$ for which the scaling relation $2\beta+\gamma=d\nu$ is satisfied by the mean-field values of the exponents given above. Thus $d_c=6$ for the spin-glass problem. The same argument was used by Toulouse to correctly predict $d_c=6$ for the percolation problem and by other authors in connection with similar random systems. Deviations from mean-field theory occur for $d<d_c$ and are of order $d_c-d=\epsilon$ for $d=d_c$. In this paper we will use the renormalization group (RG) to analyze the spin-glass transition. As predicted above, this analysis will lead to an $\epsilon$ expansion in $6-\epsilon$ dimensions. We begin with a Hamiltonian of Eq. (1) with only nearest-neighbor interactions $K(\mathbf{r},\mathbf{r}+\mathbf{d})$. As is convenient in treating random systems, we evaluate the partition function of the system replicated $n$ times:

$$Z^{(n)} = \int d\{K\} P(\{K\}) \text{Tr}_s e^{\frac{1}{kT} \sum_{\alpha} \alpha} \exp(-\frac{\alpha^{(n)}}{kT}),$$

where

$$\frac{\alpha^{(n)}}{kT} = -\sum_{\alpha=1}^{n} \sum_{\alpha} \alpha \langle K(\mathbf{r}, \mathbf{r}+\mathbf{d}) \rangle \langle \mathbf{S}^\alpha(\mathbf{r}) \cdot \mathbf{S}^\alpha(\mathbf{r}+\mathbf{d}) \rangle$$

and where $\text{Tr}_s$ is a trace over all spin variables. The free energy of Eq. (2) is then given by

$$F = \lim_{n \to 0} (-kT/n) \ln Z^{(n)}.$$ 

The integral over $\{K\}$ in Eq. (4) can be carried out formally for independent interactions and the result is

$$Z^{(n)} = \text{Tr}_s e^{\frac{\alpha}{kT}},$$

where $C_k$ is the $k$th cumulant of the distribution $P(K) = [K_{av}]^k, C_2 = [K^2]_{av} - [K^2]_{av}^2$. For $C_2 \neq 0$, the model described above has been used to study the critical properties of dilute quenched random systems. We now consider the spin-glass in which $C_1 = 0$. For simplicity, we consider first the case of a Gaussian random distribution for which $C_2 = 0$ for all $k > 2$. In this case Eq. (6b) simplifies and $\mathcal{K}/kT$ becomes

$$Z^{(n)} = \text{Tr}_s e^{\frac{\alpha}{kT}},$$

where $\alpha = \sum_{i=1}^{n} \langle Q_{i} \alpha^\alpha \rangle, \alpha \neq \beta$.

To construct a field-theoretic formulation we need to express $Z^{(n)}$ in terms of the tensor $\mathbf{Q}$ which orders at the spin-glass transition. Accordingly, we introduce a probability distribution for $\mathbf{Q}$ via

$$P(\{\mathbf{Q}\}) = \text{Tr}_s \prod_{\mathbf{r}, \alpha, \beta} \delta(\mathbf{Q}_{\alpha, \beta}(\mathbf{r}) - \mathbf{S}^\alpha(\mathbf{r}) \cdot \mathbf{S}^\beta(\mathbf{r})) \exp\left[ C_2 \sum_{\alpha, \beta} \sum_{\mathbf{r}} \mathbf{S}^\alpha(\mathbf{r}) \cdot \mathbf{S}^\beta(\mathbf{r}) \right].$$

The trace over $\mathbf{S}$ in Eq. (9) could be performed explicitly. However, since we wish to develop a continuum theory, we observe that the following form for $P(\{\mathbf{Q}\})$ will reproduce the constraint that $Q_{ij}^{\alpha\beta}(\mathbf{r})$ be obtained from spins of unit length according to the definition below Eq. (7):

$$P(\{\mathbf{Q}\}) \propto e^{\alpha \text{Tr}(\mathbf{Q}^2) + \omega \text{Tr}(\mathbf{Q}^3) - b (\text{Tr}(\mathbf{Q}^2))^2},$$

where

$$\text{Tr}(\mathbf{Q}^2) = \sum_{\alpha, \beta, i, j} Q_{ij}^{\alpha\beta} Q_{ji}^{\beta\alpha},$$

etc. This equation does not include nonlocal effects present in Eq. (9). These will contribute to the $\text{Tr}(\mathbf{Q}^2)$ term in Eq. (11b). Following the approach used by Wilson and Kogut for the Ising model, we note that if $a$ and $b$ tend to infinity in the appropriate ratio, then the normalization of $\text{Tr}(\mathbf{Q}^2)$ is fixed. Furthermore, for fixed $\text{Tr}(\mathbf{Q}^2) = n(n-1)$, $\text{Tr}(\mathbf{Q}^3)$ is a maximum when $Q_{ij}^{\alpha\beta}$ is of the form $S_i^\alpha S_j^\beta$ with $\mathbf{S}^\alpha \cdot \mathbf{S}^\beta = 1$. Thus in the limit when $\omega, a, \text{ and } b$ become infinite, in the appropriate way Eq. (10) is valid. A similar scheme has been used by Priest and Lubensky for generalizations of the Potts model. Us-

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ing Eq. (10) and taking the continuum limit we obtain

\[
\mathcal{Z}^{(n)} = \int \mathcal{D}Q e^{-\mathcal{H}}
\]

\[
Z = \int d^d x \left[ \frac{1}{4} \text{Tr} \mathcal{Q}^2 + \frac{1}{4} \text{Tr} (\nabla \mathcal{Q})^2 - w \text{Tr} \mathcal{Q}^3 + u (\text{Tr} \mathcal{Q}^2)^2 + \sum_i v_i f_i(\mathcal{Q}) \right],
\]

where \( r = T - T^* \), and \( T^* = [(\partial J)_{\text{av}}]^{1/2} \) is the mean-field transition temperature. The \( f_i(\mathcal{Q}) \) are the fourth-order invariants other than \( (\text{Tr} \mathcal{Q}^2)^2 \) which are generated by repeated iterations of the RG. For the \( m = 1 \), Ising case \( f_1(\mathcal{Q}) = \text{Tr} \mathcal{Q}^4 \), \( f_2(\mathcal{Q}) = \sum_{\alpha \delta} (Q_{11} \alpha \delta)^2 \), and \( f_3(\mathcal{Q}) = \sum_{\alpha \delta} (Q_{11} \alpha \delta)^3 \).

For \( m \geq 2 \) there are many \( f_i \)'s. The Hamiltonian of Eq. (11b) may also be used when \( P(\mathcal{Q}) \) is non-Gaussian provided \( [J]_{\text{av}} = 0 \). In this case the initial values of the potentials will depend on the details of \( P(\mathcal{Q}) \).

The mean-field minima of Eq. (11b) with \( v_i = 0 \) can be located by setting \( Q_{ij}^{\alpha \delta} = m^{1/2} \delta_{ij} f_{\alpha \beta} \), where \( f_{\alpha \beta} \) is a symmetric off-diagonal tensor with unit entries. Then we have

\[
\frac{m^2 Z}{n(n-1)! kT} = \frac{1}{4} m^2 q^2 - w(n-2)q^3 + n(n-1)uq^4,
\]

where \( \Omega \) is the volume. The extrapolation of this mean-field Hamiltonian into the regime \( n < 1 \) is ambiguous, as may also be the case in Ref. 6. There is, however, no ambiguity for \( n > 1 \). We, therefore, calculate physical quantities such as susceptibilities or specific heats, etc., for \( 1 < n < 2 \) and analytically continue the results to \( n = 0 \). In so doing, no anomalies as a function of \( n \) are encountered. For instance, consider the calculation of the order parameter. Remembering that \( q \) must be positive, we see that Eq. (12) predicts a first-order transition whenever \( (n-2)w > 0 \) and a second-order transition when \( (n-2)w < 0 \). For \( w > 0 \) and \( n < 2 \) one has a second-order transition with \( q = mw/(6(n-2)w) \) and we believe this result can be extended to \( n = 0 \).

We now discuss the \( \epsilon \) expansion in \( 6 - \epsilon \) dimensions. The recursion relations are obtained in the standard way and in the notation of Ref. 12 are

\[
\nu' = b^{2-n} \left[ \nu - 36(n-2)m w^3 [A(0) - 2K_0 \ln b] \right],
\]

\[

\nu' = b^{\epsilon/2 + 3n/2} \left[ w + 36(n-3)m + 1] w^3 K_0 \ln b] \right],
\]

\[
\eta = 12(n-2)m w^3 K_0.
\]

These equations have a stable fixed point with \( (\nu^*)^2 = -\epsilon(36K_0/[n+2])^{-1} \) whenever \( n < 4 \)

\[
- (2/m). \] The exponents for \( n = 0 \) are listed in Table I.

An interesting possibility exists if \( P(\mathcal{Q}) \) is not symmetric in \( J \). If \( [J^3]_{\text{av}} \) is nonvanishing, there will be an extra term in the Hamiltonian proportional to \( -[J^3]_{\text{av}} \text{Tr} \mathcal{Q}^3 \). Thus, if the distribution is sufficiently skewed to the antiferromagnetic side, the sign of \( w \) in Eq. (11) can change. In this case, there would be a first-order transition in mean-field theory for \( n < 2 \). Thus the point \( w = 0 \) would correspond to a tricritical point with the skewness of the distribution acting as the non-ordering field. The tricritical exponents can be determined from the recursion relations in \( 4 - \epsilon \) dimensions with \( w = 0 \). To first order in \( \epsilon \) they are of the form

\[
u' = b^{\epsilon} \left[ u - 128u^2 K_0 \ln b - \sum_i A_i u v_i - \sum B_i v_i v_i \right],
\]

\[
v_i' = b^{\epsilon} \left[ v_i - 128u v_i K_0 \ln b - \sum_i C_i v_i v_i \right],
\]

\[
r' = b^{\epsilon} \left[ r - 32ru K_0 \ln b + D u \right],
\]

where \( A_i, B_i, C_i, \) and \( D \) are constants of order \( \ln b \). Thus, at the "Heisenberg" fixed point (HFP) with \( u^* = \epsilon/(128K_0) \), \( v_i^* = 0 \) for all \( i \), one has \( \lambda^* \) exponent \( \lambda^- = -\epsilon, \lambda(v) = -2\epsilon, \) for all \( i \), so that the HFP is stable. The other fixed points are assumed to be unstable or unphysical. Also, to lowest order in \( \epsilon \) near the HFP, one has

\[
u' = b^{\epsilon/2} w [1 - 96K_0 \ln b],
\]

from which one can determine the crossover exponent \( \varphi^K \) defined so that the free energy depends on the variable \( w \) as \( w/\varphi^K \). The tricritical ex-

<p>| Table I. Values of exponents correct to first order in ( \epsilon ). Other exponents are obtained by ( \alpha = 2 - d \nu, \beta = 1/2(d + 1 - 2\nu) ). |
|---------------------------------|---------------------------------|---------------------------------|</p>
<table>
<thead>
<tr>
<th>Exponent</th>
<th>Critical point at ( \nu = 12/7 )</th>
<th>Critical point at ( \nu = 12/7 )</th>
</tr>
</thead>
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<td>( \nu )</td>
<td>( 1/2 + 5 m \epsilon/12(2m-1) )</td>
<td>( 1/2 - \epsilon/16 )</td>
</tr>
<tr>
<td>( \eta )</td>
<td>( -m \epsilon/8(2m-1) )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( \varphi^K )</td>
<td>( 4 - 1/4 \epsilon )</td>
<td>( 4 - 1/4 \epsilon )</td>
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Temperature Dependence of Electric Field Gradients in Noncubic Metals*

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Temperature dependence of the nuclear quadrupole frequency, $\nu_Q$, of noncubic metals has been studied theoretically. It is shown that the electronic contribution to the field gradient is largely responsible for the observed $T^{3/2}$ behavior. The conclusions are general and apply to all noncubic metals.

The study of electric field gradients, $eq$, in metals is of great importance since it not only provides a detailed knowledge of the electronic wave functions in the occupied Fermi volume, but can also yield valuable information regarding the nuclear quadrupole moment, $Q$. Experiments using a variety of probes, such as nuclear magnetic resonance, time-differential perturbed angular correlation, and Mössbauer effect, have been performed on both pure noncubic metals and alloys to study the distribution of $eq$. In several systems the sign of the nuclear quadrupole coupling, $\nu_Q = e^2qQ/\hbar$, has also been determined.

Recently the temperature dependence of $\nu_Q$ of several metals, such as Cd, Zn, In, Sb, and Ga, has been studied experimentally. An analysis of these results reveals the interesting feature that $\nu_Q$ generally decreases as $T^{3/2}$ for all these metals, namely,

$$\nu_Q \propto \nu_Q^0 (1 - \alpha T^{3/2}),$$

where $\nu_Q^0$ is the value of the nuclear quadrupole frequency at $T = 0$ K and $\alpha$ is a constant. Since the electronic structures of all these metals are very different from each other, this “universal” form of the temperature dependence suggests that