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AGENCY PROBLEMS IN CORPORATE FINANCE

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AGENCY PROBLEMS IN CORPORATE FINANCE

Abstract
I investigate: (i) Agency problems between debt and equity holders, and their impact on capital structure and investment policy; (ii) Agency problems between firm managers and capital providers.

The first chapter, "Investment and Financing under Reverse Asset Substitution", shows that banks place investment and borrowing restrictions on firms that are in lending relationships even when firms face no risk of bankruptcy, in order to continue extracting surplus from the firms over multiple periods. This agency problem is especially severe for firms that suffer from larger information asymmetries with the credit market. I use the term Reverse Asset Substitution (RAS) to express this partial transfer of control that benefits banks at the expense of equity holders of the firm. Using six different approaches, including triple difference in difference, Instrumental Variables, Propensity Score Matching and Endogenous Self-Selection, I provide empirical evidence consistent with the existence of RAS. I find that firms enjoying perfect competition in credit supply invest 2% more in PP&E than firms facing a monopoly in credit supply by banks. RAS reduces firm growth (11% lower PP&E) and leverage (24% lower).

In the chapter titled "Heterogeneity in Corporate Governance: Theory and Evidence", I propose that the amount of management autonomy in a firm is chosen as a best response to exogenous firmcharacteristics, such as output variance. Shareholders in firms with higher exogenous variance attempt to reduce the information disadvantage they face by reducing autonomy of management. In addition, I find that over time, this information gap has decreased in US capital markets, and since Sarbanes-Oxley the information asymmetry does not play a role in the choice of corporate governance mechanisms.

In the last chapter, "Effort, Risk and Walkaway under High Water Mark Style Contracts", Sugata Ray and I model a hedge fund style compensation contract in which management fees, incentive fees and a high water mark (HWM) provision drive a fund manager’s effort and risk choices as well as walkaway decisions by both the fund manager and the investor. Welfare results for the calibrated model show that a higher management fee and lower incentive fee (e.g. a 2.5/10 contract) lead to Pareto improvement.

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To my parents.
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Chapter 1

Investment and Financing under Reverse Asset Substitution

1.1 Introduction

Bank financing is the most important source of external financing for firms. Using US data on borrowing firms, lending banks and credit agreements, I show that banks place investment and borrowing restrictions on firms that are in lending relationships even when firms face no risk of bankruptcy. Banks restrict growth of the borrowing firms to continue extracting surplus over multiple periods. If banks do not share the upside of firms’ growth beyond avoidance of bankruptcy, they may restrict firm growth to benefit from a longer lending relationship. This phenomenon is more pronounced for firms that suffer from larger information asymmetries with the credit market and hence have limited alternatives to bank financing. When the firms grow, and cost of collection of information in proportion to loan size decreases, other lenders compete and lending relationships become transactional and relatively less profitable for all lenders.
I start by showing two prominent features of bank covenants that are undocumented and unexplained by previous literature. These features are illustrated in Figures 1.1 and 1.2. Figure 1.1 shows the distribution of various covenants imposed by banks on borrowing firms. The panel on the right shows that more covenants of certain types (Minimum Debt to Tangible Networth, Minimum Current Ratio, Minimum Quick Ratio) are placed on firms that are smaller, which is consistent with theory that smaller firms face higher chances of asset substitution and other agency conflicts. However, the figure to the left, shows that a different set of covenants (Maximum Capex, Max Debt to EBITDA, Min. Interest Coverage and Min. Fixed Charge Coverage) are imposed in firms that are in the middle range. Most of these firms are in no immediate danger of bankruptcy. This relatively higher weight of covenants on the firms in the middle range is at odds with the notion that banks are imposing covenants to reduce the possibility of asset substitution. Secondly, Figure 1.2 provides the incidence of Capital Expenditure covenants among firms based on their access to debt markets and shows Capital Expenditures are popular over and beyond other covenants in firms that are closer to public debt market access. This feature suggests that banks are restricting firms that are about to access external financing, not just the firms that are close to bankruptcy.

These two features, (i) higher incidence of covenant restricting investment on mid-size firms and (ii) higher incidence of Capital Expenditures on firms closer to public debt market access, cannot be explained by existing literature. Covenants dealing with possibility of repayment of loan in case of bankruptcy (where the limiting term is either current assets or tangible networth) are consistent with the view that banks impose covenants to protect themselves in case of bankruptcy, i.e. to mitigate asset substitution. However, Capital Expenditure covenants that restrict the dollar amount of capital expenditures by a firm, even if the firm is not close to bankruptcy, cannot be explained unless we accept that different covenants are imposed with different motives. All covenants are not imposed for the sole purpose of mitigating conventional
asset substitution. This paper provides this other motive of lenders when entering financing contracts with firms, as discussed below.

Firms have “soft” information (such as the ability of the entrepreneur, quality of the product, etc.) that cannot be credibly communicated to outsiders. Thus, lending banks have information about their borrowers that other prospective lenders do not have. Lending banks capitalize on the private information about borrowing firms and attempt to lock-in borrowing firms in profitable long-standing relationships. To do so, lending banks restrict investment decisions of firms so that the firms grow slowly. I refer to this transfer of wealth from a firm’s equity holders to debt holders (i.e. banks) as Reverse Asset Substitution (RAS). This is in contrast to conventional asset substitution where in expectation, there is a transfer of wealth from debt holders to equity holders. Asset substitution is the agency problem when equity holders find it attractive to undertake risky projects. This is because if the project is successful, equity holders get all the upside, whereas if it is unsuccessful, debt holders get all the downside. Debt holders impose investment and borrowing restrictions on the borrowing firm to mitigate the possibility of conventional asset substitution. In this paper, I show that the investment and borrowing restrictions on firms are imposed even when the borrowing firm is not prone to conventional asset substitution, thus allowing the lending bank to extract benefits of a lending relationship from the borrowing firm for a longer period. I argue that RAS has important quantitative implications, and leads to significant under-leverage in equilibrium, as borrowers trade-off financial flexibility against investment flexibility.

The questions answered in this work are the following: (i) When a firm uses bank financing, how are the firm’s investment decisions impacted by the bank’s objective to maximize its own profits? (ii) Is bankruptcy cost the only cost of bank debt or are there any agency costs as well? (iii) What are the quantitative effects of bank’s policies on a firm’s investment, leverage and equity value? To answer the research questions (i) I discuss implications of the hypotheses
proposed in this paper; (ii) using firm, bank and bank-loan data, I test the implications; and (iii) I estimate the impact of Reverse Asset Substitution on firm investment and leverage decisions.

The testable implications of the contract between the lender and the borrower when the lender enjoys an exclusive relationship with the borrower are as follows: (P1): Firms that have limited outside options for borrowing, find that incumbent creditors impose investment constraints even when the firm is not prone to conventional asset substitution. (P2): Firms have lower leverage ex-ante as they are apprehensive of their growth being stymied by Reverse Asset Substitution. As soon as the relationship between the bank and the firm ends, a firm increases its leverage. (P3): The investment constraints are imposed using covenants on firm characteristics. Thus certain firm characteristics (such as leverage, networth, current ratio) that are used by lenders to write debt covenants have significance in investment decisions. (P4): The investment of a firm that has limited access to external financing is more sensitive to cash flow innovations and less sensitive to innovations in Tobin's Q than the investment of a firm that has access to public debt markets.

I employ six different approaches to provide evidence for propositions (P1)-(P4) and also estimate the impact of RAS on investment and leverage of firms: (i) An interval study of firms taking loans, (ii) a natural experiment with a triple difference in difference approach, (iii) An IV approach to measure the influence of banks on firm investment decision, (iv) Propensity Score Method to estimate Average Treatment Effect, (v) a bank credit supply model to measure the impact of bank market power on investment using a difference in difference approach, (vi) a self-selection model based approach. All these methods independently corroborate propositions (P1)-(P4).

Using Method (i), I find that within a year after ending the credit relationship with a bank, a firm increases investment and increases leverage, supporting proposition (P2). Firms also respond differently to innovations in cash flow and Tobin's Q after they gain access to debt mar-
kets, supporting proposition (P4). Using Method (ii), I find evidence that firms that achieve a large increase in bargaining power against banks due to an improved ability to tap public debt markets, are the ones that invest the fastest, supporting proposition (P1) that banks limit growth of certain firms. Using Method (iii), I show that investment decisions of firms that cannot issue investment grade bonds are influenced by firm characteristics on which bank loan covenants are written. However, once the firm has credible outside options of financing, banks appear to have no additional influence on firm investment decisions. This supports propositions (P1) and (P3). I also differentiate firms based upon their financial health characteristics and empirically find that banks influence firm investment decisions in all groups, providing further evidence in support of proposition (P1). Thus, it is not the case that firms closer to bankruptcy are the only ones affected by creditors’ interests - a conventional asset substitution story. I find that firms far away from bankruptcy are also restricted in their investment opportunities by banks.

In Method (iv), I show that firms in perfect competition can invest 2% more in their PP&E annually than firms facing a monopoly in credit supply. This supports proposition (P1). By noting the coefficient of the interaction term (Cash Flow × Market Power of Local Banks), I infer that a firm borrowing from a bank that enjoys a lending monopoly invests 3% of PP&E more for the same unit increase in cash flow than a firm facing perfect competition among banks. Thus, internal cash flow becomes more important as lenders become more restrictive. Using Methods (v) & (vi), I find further empirical evidence in support of the implications of the model by using a Propensity Score Method and a generalized self-selection (Roy model) setup. I find that firms with no or limited access to the public debt markets, are negatively impacted by the RAS channel. Ceteris paribus, such firms lag in annual investment by 11% of their PP&E compared to firms that have access to public debt markets. Firms that have access to public debt markets show no influence by the RAS channel. This further supports proposition (P1).

This paper contributes to the literature on Relationship Banking. Mayer (1988) discusses
the general inability of firms and creditors to commit to a mutually beneficial course of action since firms cannot commit to sharing future rents with their creditors. Fischer (1990) models the problem posed by competitive credit markets and suggests that a firm can commit to sharing future profits if the lending bank has information monopoly. Rajan (1992) shows that even the bank’s information monopoly may be insufficient to bind the firm when competition comes from arm’s-length markets. I argue that banks may restrict growth of the firm by limiting lending or investment to extend the duration of information monopoly. The firm may commit to this arrangement explicitly by accepting investment or debt covenants, or implicitly by accepting lower amounts of financing from the bank in each period. The arrangement thus allows the firm to commit to sharing future rents with the bank, which is mutually beneficial and circumvents the problem pointed out in Mayer (1988).

Literature on Financial Intermediation has shown under various scenarios that relationship banking has its benefits and costs. Regarding benefits, Hoshi, Kashyap, and Scharfstein (1991), Petersen and Rajan (1994) and recently Bharath, Dahiya, Saunders, and Srinivasan (2009) show that availability of bank financing increases if banks have closer ties with the firms. Petersen and Rajan (1995) show that banks are more likely to finance credit constrained firms if credit markets are concentrated. On the other hand, Sharpe (1990) and Rajan (1992) show that banks that enjoy information monopoly may extract surplus from the borrowing firms. Hale and Santos (2009) show that firms are able to borrow at lower interest rates following their bond IPO, providing evidence that banks price their information monopoly. Rice and Strahan (2010) show that small firms borrow at interest rates 80 to 100 basis points lower if states are more open to branch expansion by banks. In this paper, I follow the aforementioned literature regarding costs. I study the investment and financing restrictions (that may be explicit as covenants) that a borrowing firm faces when it is in such a relationship. The bank does not want to lose this profitable exclusive relationship - an eventuality as the firm grows. Hence, the lending bank re-
stricts the growth of the borrowing firm to reduce the possibility of losing the relationship with the firm. The result is that a firm invests less and takes lower leverage even in the absence of the possibility of bankruptcy.

This paper contributes to the literature on Capital Structure, specifically the “Under-Leverage” puzzle. Graham (2000) and subsequently Graham (2003) find that if a typical firm levers up to the point at which the marginal tax benefits begin to decline, it could add 7.5% to firm value, after netting out the personal tax penalty. Given that firms can borrow cheaply and also get tax advantage on interest payments, why do firms not take more debt? What are the counterbalancing costs of debt - is it only the bankruptcy cost? The findings of Dichev and Skinner (2002) - that covenant restrictions are placed relatively tightly and lenders do not impose serious consequences on borrowing firms in case of violations - would further exacerbate the under-leverage puzzle. I argue that the agency problem between equity holders and debt-holders is a two-way street, and while conventional asset substitution is a possibility, over-restriction is also observed in practice and is having an impact on firms’ willingness to take bank loans. This leads to a significant reduction in firm leverage due to the cost of explicit or implicit restrictions on firm investment policy.

Recent literature has investigated the impact of specific features of financing contracts on firm investment. Nini, Smith, and Sufi (2009) find that 32% of the agreements between banks and public firms in their dataset contain an explicit restriction on the capital expenditures of the firms. Acharya and Subramanian (2009) show that when bankruptcy code is creditor friendly, excessive liquidations cause levered firms to shun innovation. Acharya, Amihud, and Litov (2010) show that stronger creditor rights in bankruptcy reduce corporate risk-taking by inducing firms to make diversifying acquisitions. Thus, creditor rights in bankruptcy changes the investment behavior of firms. Rauh and Sufi (2008) find that while high credit quality firms enjoy access to a variety of sources of discretionary flexible sources of finance, low credit quality firms
rely on tightly monitored secured bank debt for liquidity. In this paper, I find that firms with lower credit quality or with no access to public debt markets are more susceptible to Reverse Asset Substitution. Thus, in addition to the \textit{ex post} bankruptcy costs, the \textit{ex ante} restrictions that come with the loan increase the cost of bank loans for firms.

Section 1.2 discusses the concept of Reverse Asset Substitution and testable implications (P1)-(P4). Section 1.3 describes the data used for analysis. Section 1.4 provides empirical support for the implications discussed in Section 1.2 and also estimates the impact of RAS on firms’ investment and financing policy. Section ?? concludes.

### 1.2 Reverse Asset Substitution

This section illustrates RAS in practice as it may play out between firms and lending banks. It then discusses implications that can be empirically tested.

#### 1.2.1 The Concept of Reverse Asset Substitution (RAS)

The concept of RAS can be illustrated by a two period example. At date 0, an entrepreneur finds an investment opportunity, that has no possibility of capital loss. The project pays a certain return \( r > 0 \) when it is successful, and returns the whole investment in case of failure. This assumption is for exposition purposes - it helps remove the motivation of the bank of mitigating Asset Substitution completely, since even in case of failure of the project, the bank gets back the whole investment. Hence the bank should not limit firm investment de to fear of Asset Substitution. We next show that yet, the bank will choose to restrict firm growth: due to RAS.

The entrepreneur would like to borrow \( L \). There are two types of lenders, a bank (representing a syndicate of banks or the whole bank sector) and a public debt market. Public debt market lenders are arm's length lenders as they cannot monitor a small firm day to day due to relatively large costs of monitoring compared to the size of loan. One the other hand, if a bank has private
information about the lender then this allows the bank to give the entrepreneur a better rate in the first period. The entrepreneur in such a case goes to the bank to get the loan in the first period. The private information about the entrepreneur becomes public in the second period if the firm grows sufficiently. This will allow the public debt markets to compete in the second period with the bank to lend to the firm.

If the bank does not consider its own profits in future periods, then the bank should lend the maximum amount $L$ to the entrepreneur in the first period, and charge an interest rate that extracts all the surplus so that the entrepreneur is indifferent between borrowing or not. This however, means that in the next period, the entrepreneur can go to the public debt markets, where she can borrow at the competitive rate. The bank anticipates this possibility, and only lends $L^{RAS}$ to the entrepreneur, where $L^{RAS} < L$. By limiting lending and thus investment, the bank ensures that the entrepreneur will return to the bank in the next period, allowing the bank to maximize profits over multiple periods. The bank can now extract rents from the entrepreneur for two periods in place of one - a dominating strategy if discount rate for the bank is sufficiently low. The firm’s equity holders do not realize full growth potential in the first period, and suffer continued transfer of wealth in the second period. Thus debt holders have higher profits at the expense of equity holders. This transfer of assets from equity holders to debt holders is referred to as Reverse Asset Substitution (RAS) in this work.

Figure 1.3 serves as an illustration of RAS. $E$ represents equity of the entrepreneur in the firm at each stage. Superscript $s, f$ represent success or failure in each period. $q$ is the probability of success or failure of the project in each stage. The minimum amount of equity above which the entrepreneur switches to arm’s length debt financing is shown by the horizontal line. $E_{1}^{RAS}$ is the amount of equity the owner will have at the end of the first period if the project succeeds in the presence of RAS. The bank realizes that if the entrepreneur has equity $E_{1}^{S}$, then she will switch to public debt financing, and hence the bank limits the growth of the firm to $E_{1}^{RAS}$ even
in case that the bank faces no possibility of loss of capital.

1.2.2 Testable Implications

I discuss the testable implication that are consistent with Reverse Asset Substitution below.

Expensive Financing

We know that banks can monitor firms better than arm’s length lenders, specially those firms that have more “soft information”. Hence banks should be able to provide cheapest financing to such firms.

Even then, many firms diversify away from banks to other sources of financing. If a firm wants to diversify away from the bank so that it can escape monitoring, then the arm’s length investors of the public debt market should anticipate this motive of the firm, and charge a higher rate since they have lower ability to monitor. This will mean that the cost of financing will increase when a firm goes to public debt markets.

However if RAS is present, and a bank charges information monopoly rents and tries to keep the firm in an exclusive relationship with itself, then when the firm diversifies away, it should pay a lower cost of financing than before. This would mean that the firm’s investment rate and leverage will go up after it accesses external financing.

Borrowing or Investment Restrictions

An important feature of RAS is that banks place explicit or implicit borrowing and investment restrictions on firms. In this way, banks prevent firms from growing so that banks can extract rents in future periods as well. Thus, one testable implication of RAS is that firms that are closer to the point of getting a bond rating should find themselves facing more restrictions on investment and financing. Firms that have limited sources of external financing, should see stronger
restrictions when they are close to accessing new financing, than large firms that already have outside financing options.

**Sensitivity of Investment to Innovations**

Another testable prediction of investment restrictions is the sensitivity of investment to innovations in firm's improved prospects. If the firm finds that it has more cash flow than expected, existing literature suggests that a fraction of it will be reinvested. However, if there are restrictions on the firm that the firm is trying to escape, then its investment will be even more sensitive to innovations in cash flow.

A separate effect may take place when growth options of a firm improve. Such a firm should respond by increasing investment. However, if the firm is restricted from growing due to RAS, then the firm will show lower sensitivity to improvement in Tobin's $Q$ than an otherwise similar firm that has no or fewer restrictions to start with.

I summarize the testable implications below: (P1): Firms that have limited outside options for borrowing, find that incumbent creditors impose investment constraints even when the firm is not prone to conventional asset substitution. (P2): Firms have lower leverage ex-ante as they are apprehensive of their growth being stymied by Reverse Asset Substitution. As soon as the relationship between the bank and the firm ends, a firm increases its leverage. (P3): The investment constraints are imposed using covenants on firm characteristics. Thus certain firm characteristics (such as leverage, networth, current ratio) that are used by lenders to write debt covenants have significance in investment decisions. (P4): The investment of a firm that has limited access to external financing is more sensitive to cash flow innovations and less sensitive to innovations in Tobin's $Q$ than the investment of a firm that has access to public debt markets.

Section 1.3 will introduce the data that I will use to provide empirical support for the implications of propositions (P1)-(P4).
1.3 Data

In this section, I provide a summary of the four main data sources used in this paper. Standard and Poor's Compustat database merged with CRSP tapes is used to obtain firm specific variables. I use the Compustat sample of firms due to availability of firm characteristics needed for analysis. Reports of Condition and Income data (Call reports) for all banks regulated by the Federal Reserve System, Federal Deposit Insurance Corporation (FDIC), and the Comptroller of the Currency are collected from the website of Federal Reserve Bank of Chicago. The relationship between firms and banks is established using Loan Pricing Corporation's Dealscan database. The Summary of Deposits (SOD) obtained from FDIC contains deposit data for branches and offices of all FDIC-insured institutions.

All financial and utilities firms (i.e. firms with SIC code 6000 - 6999 and 4900 - 4999 respectively) are excluded from the database. The database is then merged with CRSP database with the help of historical links.

1.3.1 Dealscan

Loan information is obtained from Loan Pricing Corporation's (LPC) Dealscan database. The data consists of private loans made by bank (e.g., commercial and investment) and non-bank (e.g., insurance companies and pension funds) lenders to U.S. corporations during the period 1987-2006. Dealscan records loans that firms have taken and also lists the corresponding banks involved in the deal. The basic unit of observation in Dealscan is a loan, also referred to as a facility or a tranche. Loans are often grouped together into deals or packages. Most of the loans used in this study are senior secured claims, which have features common to commercial loans, as noted in Bradley and Roberts (2004). The data contain information on many aspects of the loan such as amount, promised yield, maturity, and information on restrictive covenants.

Compustat data is merged with Dealscan data by matching company names. Financing of
continuing operations sums up to more than 75.56% of the reasons given for taking a loan. Corporate events (such as mergers and acquisitions including leveraged buyouts) are only 4th, 5th and 6th in the sorted list summing up to 16.67%. Table I.1 reports the incidence of covenants in the database in decreasing order of popularity. 7.5% covenants of all covenants are on the dollar amount of capital expenditures by a firm, irrespective of the health of the firm, supporting proposition (P1) that creditors do not impose investment constraints for mitigating Asset Substitution only but also due to RAS. Interest Coverage\(^1\) and Fixed Charge Coverage\(^2\) covenants are 26.84% of all covenants. (Tangible) Networth covenants are 18.32% of all covenants. Covenants on the amount of debt over earnings (Maximum Debt to EBITDA and Max. Senior Debt to EBITDA) form another 18.95% of the covenants. Current Ratio\(^3\) and Quick Ratio\(^4\) covenants combine to form 6.17% of all covenants.

Dealscan data is also used to match lenders with Callreport data. Dealscan has 3335 unique banks which are sorted by frequency of their appearance in loans. One loan may have more than one bank due to syndication. The first 117 US banks are chosen that represent 30.52% (79238 out of 259600) unique appearances of banks in Dealscan loans. When manually matched with Callreport, this leads to 32 matches that in turn provide 23 unique bank holding companies.

---

1Interest Coverage Ratio is defined as EBIT divided by Interest payments. Interest Coverage is a measure of a company's ability to meet its interest payments.

2Indicates the number of times the interest (on bonds and long-term debt) and lease expenses can be covered by the indebted firm's earnings (revenue). Since a failure to meet interest payments would mean a default under the terms of a bond indenture, this ratio indicates the available margin of safety. Formula: \((\text{EBIT} + \text{lease expense}) / (\text{Interest} + \text{lease expense})\).

3An indicator of a company's ability to meet short-term debt obligations; the higher the ratio, the more liquid the company is. Current ratio is equal to current assets divided by current liabilities. If the current assets of a company are more than twice the current liabilities, then that company is generally considered to have good short-term financial strength.

4A measure of a company's liquidity and ability to meet its obligations. Quick ratio, often referred to as acid-test ratio, is obtained by subtracting inventories from current assets and then dividing by current liabilities. In general, a quick ratio of 1 or more is accepted by most creditors; however, quick ratios vary greatly from industry to industry.
1.3.2 Call Report

Call report data has been obtained from Chicago Federal Reserve Website. Data are available from 1976 to present in SAS XPORT format. The data used in this paper is from 1989 to 2006. I follow Kashyap and Stein (2000) to interpret the downloaded data and extract total assets, total loans extended and total Tier One capital as reported by the bank on the Report of Condition and Income Report. Variable descriptions are provided in Appendix A.3.

Table 1.2 presents summary statistics of bank characteristics for the sample of banks used in this paper. The individual bank level data as reported have been aggregated using bank holding level information to form total assets, securities market value, total loans, commercial loans, total capital, tier 1 capital, excess allowance and net risk weighted assets respectively. Details on the variables is in appendix A.3. All reported numbers in Table 1.2 are in percentage of total assets, except excess allowance which is in basis points. All loans form 62.57% of the total assets of a median bank’s portfolio. Commercial loans are 15.94% of the median bank’s portfolio in the sample. Securities held are 20.71% of the median bank’s assets. Tier I (core) Capital is 7.29% of total assets of the median bank. Net Risk Weighted Assets are 73.28% of the total assets. Excess Allowance is the extra capital the bank has posted above regulatory requirements, which in this case is 5.11 basis points of total assets.

1.3.3 Summary of Deposits

I obtain Summary of Deposits data from FDIC. The Summary of Deposits (SOD) contains deposit data for branches and offices of all FDIC-insured institutions. The Federal Deposit Insur-

Tier 1 Capital is composed of core capital, which consists primarily of common stock and disclosed reserves (or retained earnings), but may also include irredeemable non-cumulative preferred stock. Tier 1 and Tier 2 capital are defined in the Basel II capital accord. The Tier 1 capital ratio is the ratio of a bank’s core equity capital to its total risk-weighted assets.

Risk-weighted assets are the total of all assets held by the bank which are weighted for credit risk according to a formula determined by regulators who generally follow the Bank of International Settlements (BIS) guidelines. Assets like cash and coins usually have zero risk weight, while debentures might have a risk weight of 100%.
The Federal Deposit Insurance Corporation (FDIC) collects deposit balances for commercial and savings banks as of June 30 of each year. Figure 1.4 shows the deposits in banks by geography and metropolitan areas in United States.

I calculate the relative competition between banks using deposits in each geographical unit. The geographical unit is either the city or the Metropolitan Statistical Area (MSA) of the city when available. The MSAs are based on the 2000 Census. These areas correspond to the state / county / CBSA relationships as defined by the Census Bureau. The Herfindahl Index from Hirschman (1964) proxies the market power of banks in that geographical location.

1.3.4 Sample Selection

Unless otherwise stated, the results use the sample set that consists of all firm-years in Compustat-CRSP merged database merged with Dealscan and banks in Callreport as described above. Each record in the dataset has both firm and bank characteristics. The time period of the sample dataset is 1989 - 2006. In total, there are 19,156 firm-year observations.

Summary statistics of the firms are shown in Table 1.3. Firms in an active relationship can borrow more and invest more than when they are not in an active relationship. This is because firms are better off in a lending relationship with a bank than just using their internal cash flow for investment. This, however, is different from Reverse Asset Substitution. RAS happens when banks limit firm's growth to continue extracting benefits of private information for a longer period. RAS implies that firms with bank relationship are borrowing and investing less than otherwise comparable firms with access to public debt markets. I provide evidence that this is indeed the case in Section 1.4.

The Herfindahl index, also known as Herfindahl-Hirschman Index or HHI, is a measure of the size of firms in relation to the industry and an indicator of the amount of competition among them. It is defined as the sum of the squares of the market shares of all the firms. It can range from 0 to 1, moving from a huge number of very small firms to a single monopolistic producer. Increases in the Herfindahl index generally indicate a decrease in competition and an increase of market power and vice-versa.
1.4 Empirical Results

In this section, I test implications (P1)-(P4). I employ six different approaches in this section to provide evidence for the propositions (P1)-(P4) and also estimate the impact of RAS on investment and leverage: (i) An interval study of firms taking loans, (ii) A natural experiment with a triple difference in difference approach, (iii) An IV approach to measure the influence of banks on firm investment decision, (iv) Propensity Score Method to estimate Average Treatment Effect, (v) A bank credit supply model to measure the impact of bank market power on investment using a difference in difference approach, (vi) A self-selection model based approach. All these methods independently corroborate propositions (P1)-(P4).

1.4.1 An Interval Study of Firms taking Loans

In this section, I show that firms increase investment at a much faster rate after their bank lending relationships end. To do this, I follow an event study approach. I follow each firm individually, and predict the level of investment in each period using Tobin's Q and cash flow.

Table 1.4 reports the abnormal change in investment and leverage in the following four periods relative to the time when a loan relationship is active between a firm and a bank: (a) the year before the relationship, (b) in the first year of the loan relationship, (c) in the last year of the loan and (d) the year after the end of the loan contract. In the first row, the numbers quoted provide abnormal return compared to long term investment over capital rates. As can be seen, as soon as the facility ends, the firm increases investment by 5.99% of capital. As investment is on an average 20% of capital annually, as shown in Table 1.3, this represents approximately 30% increase in investments just after a lending relationship ends.

This is strong evidence in favor of the proposition (P1) that states that investment is curtailed in presence of creditors. However, once the creditors leave, the firm increases investment. This cannot be considered conventional asset substitution since creditors have already
left. This can only be considered that Reverse Asset Substitution - i.e. restriction of firm investment policy so as to curtail some legitimate capital expenditure - ends when an exclusive lending relationship ends.

1.4.2 A Natural Experiment

In this section, I show that firms that exhibit a large increase in bargaining power against banks due to an improved ability to tap public debt markets, are the ones that invest the fastest, supporting the proposition that banks limit growth of certain firms. The natural experiment analyzes the European Corporate Bond market which got a big positive liquidity shock after the introduction of Euro in 1999. This is the natural experiment: due to this exogenous change in public debt markets' liquidity, firms that previously had access only to bank loans now have the ability to go to public debt markets, improving their bargaining power against their banks.

I follow a triple difference in difference approach to see the impact of the exogenous change in credit supply on firms in Europe. The triple differences are: Investment in (i) year 1999 against year 1998, (ii) in France and Germany which adopted the Euro, compared to UK which did not adopt the Euro, (iii) between larger and smaller firms. Figure 1.6 (left panel) reports the volume of bond issuances by credit quality in the countries that adopted Euro as the common currency in 1999. Figure 1.6 (right panel) compares the growth in investment of firms by decile in Continental Europe to those in United Kingdom after introduction of Euro as the common currency in Continental Europe on Jan 1st, 1999. The firms are divided into deciles by sales, and the investment grows fastest in the smaller firms.

Figure 1.6 shows that when compared to their counterparts in United Kingdom, small firms in Continental Europe invest at an average 6% more of their PP&E in 1999 compared to 1998. This result is consistent with proposition (P1), because the firms that were previously restricted are growing the fastest, and those are the mid-size firms that were closer to accessing public
debt markets through growth. The smallest firms, that have no possibility to go to debt markets, do not grow faster as they do not gain as much bargaining power from the introduction of Euro in the first place.

1.4.3 RAS Depending on Access to Public Debt Markets

In this section, I establish that firms that have fewer outside financing options find that their investment decisions are influenced by banks more than firms that have better outside financing options. Evidence in support of the aforementioned claim is consistent with propositions (P1) and (P3). Faulkender and Petersen (2006) show that firms with access to public markets take more debt. In that case, there is more competition among creditors for providing capital to such firms that have a larger choice set of creditors, and the influence of competing creditors on firm investment policy should be lower.

The IV regression model for investment is as follows (suppressing time subscripts):

$$\begin{align*}
Y &= \gamma_0 + \gamma_1 Q + \gamma_2 CF + \gamma_3 Z + \eta \\
I/K &= \beta_0 + \beta_1 Q + \beta_2 CF + \beta_3 Y + \epsilon,
\end{align*}$$

(1.1) (1.2)

where $I/K$ represents investment in period $t$, scaled by net Property, Plant and Equipment (PP&E) $K$ at the end of the previous period, $Q$ represents Tobin's Q, and $CF$ represents cash flow also scaled by previous period's PP&E. $Y, Z$ represent a relevant set of firm and bank characteristics respectively in time period $t$, and $\epsilon$ and $\eta$ are error terms. I assume that bank's characteristics have no direct impact on the borrowing firm's investment decisions other than through the covenant channel. Two-step efficient generalized method of moments (GMM) is used for estimation. The standard errors are robust under heteroscedasticity. The variables $I, Q, CF$ among others are described in appendix A.3. Firm quarterly data and bank data are relevant for the
In equation 1.1 firm characteristics are instrumented using bank characteristics to address possible endogeneity of characteristics relevant for covenants with the firm borrowing from the bank in the first place. The bank's characteristics used to instrument firm characteristics are of the lending bank only. I take three different firm characteristics $Y$ to be instrumented: (a) Tangible Net Worth (scaled by Total Assets of the firm) (b) Current Ratio (Current Assets / Current Liabilities) (c) (Book) Leverage. The choice of these three characteristics is based on their wide use in practice. (See Chava and Roberts (2007) for details on various types of covenants in DealScan). Also, these three firm characteristics are relatively easier to quantify accurately using Compustat data. Tangible Net Worth scaled by Total Assets gives an indication of the recovery rate of debt in case of default. Existing literature has established tangibility as an important characteristic in the determination of leverage of firms (See Rajan and Zingales (1995) ). The ratio of Current Assets to Current Liabilities is an important measure of the short term solvency of a firm. A related measure is Quick Ratio, that excludes inventories from current assets. The intuition in both cases is to measure the ability of a firm to pay its short term liabilities out of its short term assets so that it can continue operations. The third characteristic used in the analysis is (Book) Leverage. Bank characteristics $Z$ that are used to instrument the firm characteristics are (a) Tier 1 Capital and (b) Total loans extended, both scaled by the total assets of the bank. Appendix A.3 provides details about the exact fields used from the databases.

I divide firms that have active relationships with banks on the basis of the credit rating of the firm. Following Faulkender and Petersen (2006), I use the presence of credit rating to establish whether a firm has access to public debt markets or not. If propositions (P1) and (P3) hold, then we should see that firms that have better access to external debt are less influenced by banks, and therefore investment is less sensitive to firm characteristics denoted by $Y$, for such firms.

The results are reported in Table 1.5. Panel A shows all unrated firms and panel B shows all
investment grade firms. In both cases, as expected, Tangible Networth and Current Ratio have a positive impact on investment, and leverage has a negative impact. This is because debtors can allow a firm with more tangible assets and more current assets more autonomy, all else being equal. Similarly, debtors allow firms with higher leverage lower autonomy. The three instrumented variables, Tangible Networth, Current Ratio and Leverage are statistically significant in Panel A. In Panel B, the same characteristics for a similar order of magnitude of observation set have no statistical significance. This implies that the instrumented characteristics do not influence the investment of firms in Panel B, i.e. the investment grade firms, implying that access to public debt markets reduces the ability of banks to influence firms' investment decisions. Thus, Table 1.6 shows that firms with limited access to public markets are more influenced by lenders (in this case banks) in their investment policy decision than their counterparts that have access to public debt markets.

1.4.4 Estimating Average Treatment Effect using Propensity Score

In this section, I estimate the average partial effects of access to debt markets on investment and leverage under conditional moment independence assumptions. I estimate the following model:

\[
\frac{I}{K} = \beta_0 + \beta_1 Q + \beta_2 CF + \beta_3 I(\text{Access}) + \beta_4 I(\text{Access})Q + \beta_5 I(\text{Access})CF + \epsilon
\]  

(1.3)

Here, I regress investment on an indicator variable \(I(\text{Access})\) that marks the time when a firm gets access to credit markets, after controlling for Tobin's \(Q\) and cash flow \(CF\) (scaled by previous period's PP&E). I also include interaction terms between firm characteristics (Tobin's \(Q\) and cash flow \(CF\)) and access to public debt markets: \(I(\text{Access})Q\) and \(I(\text{Access})CF\) respectively.

Tables 1.6 and 1.7 report the results of the analysis. Table 1.7 reports results for investment regression \((I/K)\) using annual observations. All numbers are in %. The standard errors are
robust under heteroscedasticity. Column (2) shows that firms that gain access to public debt markets invest more in net Property, Plant and Equipment each year. However this estimate has a selection bias, that I will correct for next. Columns (3) and (4) estimate the average treatment effect of getting access to public debt market financing on investment for firms that previously only had access to bank financing for credit. The endogeneity problem in column (2) is that firms that use public debt differ from firms that use bank financing over many firm characteristics, and hence any treatment effect measurement needs to control for this difference. To solve this problem, I follow the propensity score method suggested for non-random sampling (See Horvitz and Thompson (1952), Rosenbaum and Rubin (1983) and Wooldridge (2004)). To do this, I estimate probability of treatment $p(x)$ given the covariates $x$, which are the firm characteristics in this case:

$$p(x) = P(w = 1|x),$$

where $w$ is an indicator for treatment and $p(x)$ is the propensity score of accessing the public debt markets. Denis and Mihov (2003), examine the determinants of the source of new debt. Using their work, I use firm size (measured by log(assets)), Tobin's $Q$ and cash flow scaled by PP&E (to measure growth options), fixed asset ratio (Property, Plant and Equipment over Total Assets), Book Leverage (Debt / Total Assets), Profitability (Operating Income / Total Assets) to predict the source of debt financing using a probit model. Table 1.6 reports the estimation results of the probit regression. Column (4), that includes fixed asset ratio, profitability, book leverage, firm age, firm size, Tobin's $Q$ and cash flow as right hand side firm characteristics, is used for the estimation of the propensity score (propensity of having access to public debt market access). The predicted value $\hat{p}(x)$ is then used as a control in the regression:

$$I/K = \beta_0 + \beta_1 Q + \beta_2 CF + \beta_3 \hat{p}(x) + \beta_4 I(\text{Access}) + \sum_i \beta_i \text{Interaction Terms}_i + \epsilon. \quad (1.4)$$
Rosenbaum and Rubin (1983) and Wooldridge (2004) show that the coefficient on the access indicator variable and interaction variables are consistent for the average treatment effect, ATE.

Table 1.7 reports the results of the estimation of the average partial effects of access to debt markets on investment under conditional moment independence assumptions. I use annual observations for firms in DealScan Database that could be matched to Compustat. All numbers are in %. The standard errors are robust under heteroscedasticity. I obtain the historical ratings from S&P Credit Ratings database. In column (3), I find that after controlling for propensity score, Tobin's $Q$ and $CF$, and other interaction terms, firms with access to public debt markets invest 11% more PP&E per annum. This supports proposition (P1). Column (5) excludes the firms that have propensity scores above 90% or below 10%. This excludes firms that have no possibility of being in the other category (access or no access). The sample now contains firms that are closer in characteristics. This is confirmed by noting that the propensity score is not significant anymore in the estimation consistent with the premise that the selection bias has been controlled for. When looking at this set of firms that have a larger common support in the sample, I find that firms with access to debt markets grow faster by 11% of their PP&E per annum. Time and firm fixed effects are included for all columns. The reported standard errors are robust to heteroscedasticity and allow for firm level intra-cluster correlation of errors.

Table 1.7 also provides evidence in support of proposition (P4). As expected, investment is not as sensitive to innovations in cash flow when the firm has access to only bank debt, which comes with investment restrictions. I infer this by noting the coefficient of the interaction term $(Cash\ Flow) \times Access$. In column (6), one percentage increase in cash flow leads to 7% excess increase in investment for firms with limited access to public debt markets in column (4). Proposition (P4) also suggests that when the outside financing options of an unconstrained firm improve, its investment shows a higher sensitivity to Tobin's $Q$ than a firm that has more restrictions. This inference is corroborated by noting the coefficient of 0.6% for the interaction
term $Q \times \text{Access}$, giving support to proposition (P4).

Table 1.8 estimates the average partial effects of access to debt markets on leverage under conditional moment independence assumptions. The model estimated is as follows:

$$
\text{Leverage} = \beta_0 + \beta_1 \text{Profitability} + \beta_2 \text{Fixed Asset Ratio} + \beta_3 \text{Market to Book} \\
+ \beta_4 \text{Sales} + \beta_5 \hat{p}(x) + \beta_6 \mathbf{1}(\text{Access}) + \epsilon. \quad (1.5)
$$

In column (2), after controlling for propensity score, profitability, tangibility, size and growth options (which are commonly used explanatory variables for leverage) and interaction terms, I estimate that firms, when they get access to public debt markets, have an average of 35% more leverage. This supports proposition (P2). Faulkender and Petersen (2006) have also shown that firms with access to debt market take similar amount of additional leverage. Column (3) excludes the firms that have propensity scores above 90% or below 10%. This excludes firms that have no possibility of being in the other category (access or no access). Even then, access to public debt markets has an average treatment effect of 24% Time and firm fixed effects are included. The reported standard errors are robust to heteroscedasticity and allow for firm level intra-cluster correlation of errors.

### 1.4.5 Estimating RAS using Banks' Market Power

In this section, I estimate the differential impact of banks’ market power on firm investment. I estimate the following model:

$$
\frac{I}{K} = \beta_0 + \beta_1 Q + \beta_2 CF + \beta_3 \text{Market Power of Local Banks} \\
+ \beta_4 \text{Mkt. Power of Local Banks} \times Q + \beta_5 \text{Mkt. Power of Local Banks} \times CF + \epsilon \quad (1.6)
$$


Here, I regress investment on a variable Market Power of Local Banks that proxies the outside options of the borrowing firms, and the market power of each bank, after controlling for Tobin’s $Q$ and cash flow $CF$ (scaled by previous period’s PP&E). I also include interaction terms between firm characteristics (Tobin’s $Q$ and cash flow $CF$) and market power of banks: Market Power of Local Banks $\times Q$ and Market Power of Local Banks $\times CF$ respectively. The market power used in the above estimation is computed using the banks that are in the same Metropolitan Statistical Area (MSA) as the firm.

Table 1.9 reports results for investment regression $(I/K)$ using annual observations. All numbers are in %. The standard errors are robust under heteroscedasticity. Column (1) shows that as competition between banks decreases, firms grow their PP&E at a lower rate. Firms in perfect competition can invest 2% more in their PP&E annually, than firms facing a monopoly in credit supply. This is relatively 10% more investment since mean sample investment in PP&E is 20%. This supports proposition (P1). Column(2) shows that for only investment grade firms, competition between banks has no effect on investment. Thus, the effect is concentrated in firms that depend on bank financing since such firms have limited or no access to public debt markets.

Banks limit borrowing firms’ growth if they are apprehensive that the competing banks will steal their business once the firms grow. This means that a firm that faces a lending bank that has higher market power, is less constrained by its bank as the bank is less apprehensive of losing the firm’s business. On the other hand, growth in this case means larger loan sizes in the future and more profits for the bank. This is supported in data. By noting the coefficient of the interaction term (Cash Flow) $\times$ Market Power of Local Banks in columns (3) – (5), I infer that a firm in a bank monopoly invests 3% of PP&E more for the same unit increase in cash flow than a firm in perfect competition. Proposition (P4) also suggests that when a bank is less restrictive of a firm, which the bank can afford to be when competition among banks decreases
(Herfindahl Index increases), then the borrowing firm’s investment shows a lower sensitivity to improvement in Tobin’s $Q$ than a firm that has more restrictions to start with, and benefits more from outside options. This inference is corroborated by noting the negative coefficient 0.20% for the interaction term $Q \times$ Market Power of Local Banks in column (5), giving support to proposition (P4).

### 1.4.6 Estimation of RAS using a Self-Selection Model

So far, I have provided evidence in support of the implications of the propositions presented in section 1.2. Firms realize that if they go to the banks the information asymmetry with the bank will be less, but the bank will impose investment restrictions on the firm over and beyond bankruptcy costs due to RAS. The firm owners also know that this will not happen with public debt market, but entering that market is costly. I will now show that even though RAS cannot be directly observed, it can be identified off of firms’ choice of debt source.

Using a generalized Roy (1951) framework, I am able to estimate the sensitivity of investment to RAS controlling for Tobin’s $Q$ and cash flow. I will show that RAS does not have an influence in the group of firms that have outside financing options (access to public debt markets), while it has a strong negative influence on investment for firms that do not have outside financing options. This evidence supports propositions (P1) and (P4) and also estimates the importance of the Reverse Asset Substitution channel for investment. The selection model allows for endogeneity in the choice of financing (public debt or bank loan) exercised by a firm. The firm can graduate to a group with access to public debt markets by paying a cost based on its characteristics, similar to the education choice decisions in labor economics literature where the approach used in this section is usually applied, such as in Heckman and Navarro-Lozano (2003), Carneiro, Hansen, and Heckman (2003) and Cunha, Heckman, and Navarro (2005).

In my framework, a firm can choose to go to the public debt markets at a cost. If it does
not pay the cost, it stays in the group where firms have access only to banks, and faces stronger RAS. Let \( s = 0, 1 \) represent the two choices available to firms. In this case, firms are divided into those that have investment grade ratings and thus have good access to public debt markets \((s = 1)\) and firms that do not have any credit rating \((s = 0)\). Small firms often belong to the second group. Following Denis and Mihov (2003), I estimate the cost of getting access to the public debt markets using \(Q\), Leverage, Firm Size, Firm Age, Fixed Asset Ratio, Profitability. The investment decision depends on Tobin’s \(Q\) and cash flow. Variable \(\theta\) is used to denote RAS: the unobservable channel of influence on firm investment when the firm has access to only bank debt. In the estimation of the cost function, RAS also plays a role because if the lending bank exerts strong influence on the firm’s investment decision, then the firm finds it harder to switch to public debt markets. This is because due to RAS, the firm’s growth is impeded, and therefore the firm remains smaller, thus increasing costs of access to public debt markets. Therefore, RAS plays a role in the investment equation, as shown by implications (P1)-(P4) discussed in section 1.2.

For notational convenience, in this section, let \( I \) represent investment and \( C \) represent the cost of accessing public debt markets. \( CF, Q \) continue to represent cash flow and Tobin’s \( Q \). I assume that the error terms \( \varepsilon_0, \varepsilon_1 \) and \( \varepsilon_c \) are mutually independent of each other and that factor \( \theta \) is independent of the error terms, \( \varepsilon_c \). I denote the firm characteristics that affect investment \( I \) by \( X \) and that affect cost \( C \) of accessing public debt markets by \( Z \):

\[
X = [1, Q, CF] \\
Z = [Q, Leverage, Firm Size, Firm Age, Fixed Asset Ratio, Profitability].
\]

(1.7)

The system of equations to be estimated is as follows (suppressing time subscripts \( t \), but keep-
ing individual firm subscripts $i$ for clarity):

\[ I_s^i = \beta_{0,s} + \beta_{1,s} Q^i + \beta_{2,s} CF^i + \alpha_s \theta^i + \epsilon_s^i \quad s \in \{0, 1\} \quad (1.8) \]

\[ C^i = \gamma'z + \alpha_c \theta^i + \epsilon_c^i \quad (1.9) \]

where $s = 0, 1$ represents the two groups - group (0) has no access to public debt markets, and group (1) has access to public debt markets. $\beta_j, s$ for $j = 0, 1, 2$ represent constant, and the coefficients of Tobin's $Q$ and cash flow respectively for each group $s = 0, 1$. $\alpha_s$ represents coefficient for unobservable characteristic $\theta$ that represents RAS for each group $s$. $\gamma$ represents a vector of coefficients for firm characteristics that affect the ability of the firm to access public debt markets. The errors are assumed to follow normal distribution:

\[
\begin{bmatrix}
\epsilon_0^i \\
\epsilon_1^i \\
\epsilon_c^i
\end{bmatrix} \sim \mathcal{N}
\left(
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
\sigma^2_{\epsilon} & 0 & 0 \\
0 & \sigma^2_{\epsilon} & 0 \\
0 & 0 & \sigma^2_{c}
\end{bmatrix}
\right)
\]

The unobservable term $\theta$ that represents RAS is also assumed to be drawn from a normal distribution $\theta \sim \mathcal{N}(0, \sigma^2_{\theta})$.

More details of the identification and estimation strategy are in Appendix A.2. The results of maximizing the log likelihood function (Equation A.19 in Appendix A.2) are reported in Table 1.10. The dataset is at annual frequency and the same as described in section 1.3. Panel A and B tabulate the estimation results of equations 1.8 and 1.9. In total, 17 parameters are estimated using MLE. The reported values are mean and standard error. All values are in %.

Table 1.10 shows strong evidence directly in favor of RAS. $\alpha_0$ is the coefficient of RAS for firms that do not have access to public debt markets and $\alpha_1$ is the coefficient of RAS for firms that have access to public debt markets. I find that RAS has no impact on firms with access to
public debt markets, i.e. \( \alpha_1 \) is statistically indistinguishable from 0. The coefficient of \( \theta \) that represents RAS in Panel A first column is \( \alpha_0 = -17.4\% \) compared to \( \alpha_1 = 0 \) in second column, implying that the difference between the mean rates of growth of PP&E between firms that face RAS and those that do not is 17.4\% of PP&E annually. This is in comparison to a lower 11\% impact of RAS estimated by the propensity score method in Section 1.4.4 that does not allow for endogenous selection. The loading on RAS is thus quite large for firms that have difficulty switching from their bank with which they have lending relationship to public debt markets. This is strong evidence in support of proposition (P1). Table 1.10 also shows that firms that suffer more from RAS. By comparing coefficients of cash flow in Panel A, I infer that the firms with no access to public debt markets - show economically significant higher sensitivity to cash flow. When such firms get an innovation in internal cash flow, they are willing to invest a larger fraction of it so that they can escape their bank lending relationships. This is consistent with proposition (P4).

### 1.4.7 Evidence Against Other Hypotheses

In this section, I will provide evidence against competing hypotheses that could appear to provide partial explanations for the empirical evidence.

The first alternative hypothesis that I consider is that banks are not trying to maximize profits as stated by RAS, instead, they only restrict firms that make risky investment decisions. I use Altman Z-Score to proxy for the probability of bankruptcy of a firm.\(^8\) Table 1.11 report the results of the regression. As noted in the third row column (1) and (2), the impact of bank’s market power remains negative and similar in magnitude (1.7\%) even after this control, providing evidence against this alternate hypothesis. Thus, banks restrict firm growth even when the firm

---

\(^8\)Altman Z-Score is used to predict the probability that a firm will go into bankruptcy within two years. The Z-score is a linear combination of four or five common business ratios, weighted by coefficients. The coefficients were estimated by identifying a set of firms which had declared bankruptcy and then collecting a matched sample of firms which had survived, with matching by industry and approximate size (assets).
is not prone to asset substitution. Furthermore the coefficient on the interaction term between bank's market power and the proxy for riskiness of firm (Altman Z-Score × Banks’ Market Power) is not significant in the 5th row of column (2) suggesting that banks restrict growth of all borrowing firms irrespective of how risky the firms are. Therefore, this alternative hypothesis does not hold.

Another competing hypothesis is that banks are still lending at zero profits, but since they do not have the ability to get the loans off their books, they have to limit the risk they take and hence, they limit firm investments. Figure 1.5 shows the development of the Collateralized Debt Obligations (CDO) markets in United States. I include year 1998 and year 2003 as time dummies when the CDO market made significant strides. Columns (3)-(6) of Table 1.11 report the results. The impact of bank's market power (coefficients reported in the third row of respective columns) remains negative and similar in magnitude even after this control, providing evidence against this hypothesis.

A third argument against some implications of RAS could be made using the omitted variable bias argument: firms that go to the banks and the firms that go to the debt markets are inherently different based on characteristics that I have not controlled for in section 1.4.4. To address this concern, I use bank's concentration power in the geography of the firm to instrument firm's access to public debt markets. The intuition for the instrument is that higher bargaining power of banks makes public debt market access more attractive, and that this variation in banks’ market power by geography is as good as randomly assigned. The results of the first stage are reported in column (7) of Table 1.11. The second stage regression results reported in column (8) estimate the impact of public debt market access on firms’ ability to investment. The results are free of omitted variable bias as long as the omitted firm characteristics are orthogonal to local banks’ market power. The result is a large positive impact (18.1% of PP&E as reported in the last row of column (8)) of access to public debt markets on firms' investment,
consistent with earlier results in section 1.4.6 obtained using a completely different methodology. This corroboration of estimated impact of access to public debt markets provides robust evidence against any omitted variable bias argument. Therefore this alternative hypothesis also does not hold.

Section 1.4 tested the propositions presented in Section 1.2 that are consistent with the existence of RAS. It also estimated the impact of RAS on firm investment (reduction of PP&E growth by 11%) and leverage (reduction of 24%). It showed that firms with limited outside options in terms of financing, face restrictions on investment, even if bankruptcy is not probable. This supports the hypothesis that banks limit firm growth to extract benefits of information monopoly from lending relationships for a longer duration.

1.5 Conclusion

If a lender has private information about the borrower, then the lender can extract benefits from the borrower for a longer period if the borrower grows slowly. I use the term Reverse Asset Substitution (RAS) to express the agency problem where creditors benefit from slower growth of borrowing firms. This agency problem is more pronounced for firms that have larger information asymmetries with the credit market and the firms that do not have access to public debt markets. Equity holders take this agency problem along with potential bankruptcy costs of debt into account when choosing firm leverage. RAS can explain low leverage levels observed in practice.

In this paper, I present four testable propositions (P1)-(P4) that are consistent with Reverse Asset Substitution. I find empirical evidence in support of the propositions by following six different approaches including an IV approach using GMM, and a Roy Model based approach using MLE. Confirming the propositions, I find that firms with no or limited access to the public debt markets are negatively impacted by the RAS channel. Firms that have access to public debt
markets show no influence by the RAS channel. Using a completely different method (Average Treatment Effect with matching using Propensity Score Method) I find robust confirmation that RAS has a strong negative influence on investment (a reduction of 11% PP&E per year) and leverage (24% lower) for affected firms. I also find that firms in perfect credit supply competition can invest 2% more in their PP&E annually than firms facing a monopoly in credit supply by banks.

Agency problems between equity holders and debt-holders are a two way street, and while conventional asset substitution is a possibility, over restriction of investment by debt holders is also observed in practice and is having an impact on firms’ willingness to take bank loans.

If banks can also share the upside with firms, then the bank will not restrict the growth of the firm. Thus, if a bank holding company has a commercial bank arm that lends to firms when they are less successful, and also has an investment bank arm that will benefit from the firm growing and issuing public debt, then the agency problem discussed in this work will be less severe. Recent market developments of major investment banks changing to bank holding companies, may have this unexpected and welcome effect of improving credit supply to firms.
Table 1.1: Incidence of Covenants by Frequency

This table reports the incidence of covenants for firms in DealScan. The sample period is 1987-2006. Number of Deals counts each package where a certain covenant has been imposed. The frequency column gives the relative frequency of each covenant type in the whole sample.

<table>
<thead>
<tr>
<th>Covenant</th>
<th>Number of Deals</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Debt to EBITDA</td>
<td>6,817</td>
<td>15.81</td>
</tr>
<tr>
<td>Min. Interest Coverage</td>
<td>6,023</td>
<td>13.97</td>
</tr>
<tr>
<td>Min. Fixed Charge Coverage</td>
<td>5,590</td>
<td>12.97</td>
</tr>
<tr>
<td>Min. Tangible Net Worth</td>
<td>4,248</td>
<td>9.85</td>
</tr>
<tr>
<td>Min. Net Worth</td>
<td>3,650</td>
<td>8.47</td>
</tr>
<tr>
<td>Max. Capex</td>
<td>3,081</td>
<td>7.15</td>
</tr>
<tr>
<td>Max. Leverage ratio</td>
<td>2,801</td>
<td>6.5</td>
</tr>
<tr>
<td>Max. Debt to Tangible Net Worth</td>
<td>2,628</td>
<td>6.1</td>
</tr>
<tr>
<td>Min. Debt Service Coverage</td>
<td>2,303</td>
<td>5.34</td>
</tr>
<tr>
<td>Min. Current Ratio</td>
<td>2,004</td>
<td>4.65</td>
</tr>
<tr>
<td>Min. EBITDA</td>
<td>1,374</td>
<td>3.19</td>
</tr>
<tr>
<td>Max. Senior Debt to EBITDA</td>
<td>1,353</td>
<td>3.14</td>
</tr>
<tr>
<td>Min. Quick Ratio</td>
<td>656</td>
<td>1.52</td>
</tr>
<tr>
<td>Min. Cash Interest Coverage</td>
<td>281</td>
<td>0.65</td>
</tr>
<tr>
<td>Max. Debt to Equity</td>
<td>239</td>
<td>0.55</td>
</tr>
<tr>
<td>Max. Loan to Value</td>
<td>40</td>
<td>0.09</td>
</tr>
<tr>
<td>Max. Senior Leverage</td>
<td>21</td>
<td>0.05</td>
</tr>
</tbody>
</table>

43,109  100
Table 1.2: Summary Statistics of Banks in the sample

This table presents summary statistics for bank holding level data. The source of bank characteristics is Call Report data from Chicago Federal Reserve Website for the period 1989-2006. All reported numbers are in percentage of total assets, except excess allowance which is in basis points. The individual bank level data as reported have been aggregated using bank holding level information. Details on the variables is in appendix A.3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Loans</td>
<td>55.61</td>
<td>62.57</td>
<td>19.82</td>
</tr>
<tr>
<td>Commercial Loans</td>
<td>15.75</td>
<td>15.94</td>
<td>8.35</td>
</tr>
<tr>
<td>Securities Market Value</td>
<td>24.55</td>
<td>20.71</td>
<td>16.04</td>
</tr>
<tr>
<td>Tier 1 Capital</td>
<td>7.22</td>
<td>7.29</td>
<td>3.78</td>
</tr>
<tr>
<td>Excess Allowance</td>
<td>17.52</td>
<td>5.11</td>
<td>3.62</td>
</tr>
<tr>
<td>Net Risk Weighted Assets</td>
<td>69.11</td>
<td>73.28</td>
<td>21.23</td>
</tr>
<tr>
<td>Num of Observations</td>
<td>2,219</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1.3: Firm, Loan and Bank Sample Collection Comparison

This table presents summary statistics - means, [medians], and standard deviation - for four samples of firm-bank-year observations. The CRSP-Compustat sample consists of all firm-year observations from nonfinancial firms in the merged CRSP-Compustat database. The Dealscan sample consists of all firm-bank-year observations from nonfinancial firms in the merged CRSP-Compustat database for which a private loan was found in Dealscan. The time period of the sample dataset is 1989 - 2006. The during and before/after samples consist respectively of firm-bank-year observations when a lending relationship is active or inactive. Tangible Networth/Assets, ROA, Capital/Assets, Investment/Capital, Cash Flow, Leverage are quoted in percentage. All values are winsorized at 1%.

<table>
<thead>
<tr>
<th>Variable</th>
<th>CRSP-Compustat</th>
<th>DealScan</th>
<th>Relation Inactive</th>
<th>Relation Active</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Current Ratio</td>
<td>2.25</td>
<td>1.63</td>
<td>2.05</td>
<td>1.37</td>
</tr>
<tr>
<td>[1.80]</td>
<td>[1.71]</td>
<td></td>
<td>[1.77]</td>
<td></td>
</tr>
<tr>
<td>Net Worth</td>
<td>1224.49</td>
<td>4921.61</td>
<td>1247.03</td>
<td>5444.97</td>
</tr>
<tr>
<td>[168.08]</td>
<td>[201.61]</td>
<td></td>
<td>[170.93]</td>
<td></td>
</tr>
<tr>
<td>Tangible NW/Assets</td>
<td>31.22</td>
<td>29.54</td>
<td>27.99</td>
<td>28.47</td>
</tr>
<tr>
<td>[32.10]</td>
<td>[29.56]</td>
<td></td>
<td>[31.88]</td>
<td></td>
</tr>
<tr>
<td>log(Assets)</td>
<td>5.98</td>
<td>2.16</td>
<td>6.26</td>
<td>1.99</td>
</tr>
<tr>
<td>[5.92]</td>
<td>[6.20]</td>
<td></td>
<td>[5.98]</td>
<td></td>
</tr>
<tr>
<td>Market/Book</td>
<td>1.43</td>
<td>1.06</td>
<td>1.39</td>
<td>0.98</td>
</tr>
<tr>
<td>[1.10]</td>
<td>[1.09]</td>
<td></td>
<td>[1.09]</td>
<td></td>
</tr>
<tr>
<td>Tobin's Q</td>
<td>3.93</td>
<td>7.68</td>
<td>3.47</td>
<td>6.96</td>
</tr>
<tr>
<td>[1.59]</td>
<td>[1.47]</td>
<td></td>
<td>[1.47]</td>
<td></td>
</tr>
<tr>
<td>Capital/Assets</td>
<td>37.74</td>
<td>22.73</td>
<td>40.07</td>
<td>23.00</td>
</tr>
<tr>
<td>[34.00]</td>
<td>[36.32]</td>
<td></td>
<td>[36.02]</td>
<td></td>
</tr>
<tr>
<td>Investment/Capital</td>
<td>18.60</td>
<td>17.17</td>
<td>19.87</td>
<td>19.03</td>
</tr>
<tr>
<td>[13.94]</td>
<td>[14.96]</td>
<td></td>
<td>[14.59]</td>
<td></td>
</tr>
<tr>
<td>Book Leverage</td>
<td>19.89</td>
<td>16.92</td>
<td>22.60</td>
<td>17.97</td>
</tr>
<tr>
<td>[18.11]</td>
<td>[21.28]</td>
<td></td>
<td>[19.51]</td>
<td></td>
</tr>
<tr>
<td>Num of Obs.</td>
<td>269,844</td>
<td>19,156</td>
<td>14,445</td>
<td>4,711</td>
</tr>
</tbody>
</table>
Table 1.4: An event study approach to investments and returns

This table reports the change in investment and leverage in the following four period relative to the time when a loan relationship is active between a firm and a bank: (a) the year before the relationship ends, (b) in the first year of the loan relationship, (c) in the last year of the loan and (d) the year after the end of the loan contract. The CRSP-Compustat sample consists of all firm-year observations from nonfinancial firms in the merged CRSP-Compustat database. The Dealscan sample consists of all firm-year observations from nonfinancial firms in the merged CRSP-Compustat database for which a private loan was found in Dealscan. The time period of the sample dataset is 1989-2006. The reported values are mean and (standard error).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment/Capital</td>
<td>0.60</td>
<td>−6.65**</td>
<td>2.33</td>
<td>5.99**</td>
</tr>
<tr>
<td></td>
<td>(4.72)</td>
<td>(2.63)</td>
<td>(2.05)</td>
<td>(2.66)</td>
</tr>
<tr>
<td>Leverage</td>
<td>1.32**</td>
<td>1.37</td>
<td>−0.78</td>
<td>3.19**</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(1.67)</td>
<td>(3.85)</td>
<td>(1.32)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

*p < 0.10, **p < 0.05, ***p < 0.01
Table 1.5: Does the influence of banks depend on firm access to debt markets?

Panel A reports results for investment regression (I/K using equations 1.1 and 1.2) during the facility for the unrated and below investment grade firms. Panel B reports investment regression results using data during the time when the bank has an active loan facility with the firm for the investment grade firms. Reported coefficients are for quarterly investments scaled up by 1000 over net PP&E. Two-step efficient generalized method of moments (GMM) is used for estimation. The standard errors are robust under heteroscedasticity. The reported values are mean and (standard error).

<table>
<thead>
<tr>
<th></th>
<th>Panel A</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Tobin's Q</td>
<td>3.33***</td>
<td>1.02***</td>
<td>1.76</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td>(1.04)</td>
<td>(0.27)</td>
<td>(6.72)</td>
<td>(3.31)</td>
</tr>
<tr>
<td>Cash Flow</td>
<td>3.48</td>
<td>1.87</td>
<td>2.29</td>
<td>3.40</td>
</tr>
<tr>
<td></td>
<td>(2.14)</td>
<td>(1.88)</td>
<td>(5.32)</td>
<td>(2.31)</td>
</tr>
<tr>
<td>Tangible NW/Assets</td>
<td>76.89***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(26.69)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Ratio</td>
<td>89.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(55.73)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td></td>
<td></td>
<td>−150.32***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(37.44)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>77.79***</td>
<td>28.35***</td>
<td>121.05</td>
<td>106.70***</td>
</tr>
<tr>
<td></td>
<td>(10.88)</td>
<td>(9.84)</td>
<td>(110.72)</td>
<td>(11.56)</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>13,362</td>
<td>11,964</td>
<td>11,968</td>
<td>12,124</td>
</tr>
<tr>
<td>Number of groups</td>
<td>2,799</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time Dummy?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

|                  | Panel B |          |          |          |
|                  | (1)     | (2)      | (3)      | (4)      |
| Tobin's Q        | 9.17*** | 6.78***  | 8.07***  | 8.57     |
|                  | (0.91)  | (2.23)   | (1.92)   | (7.53)   |
| Cash Flow        | 32.23***| 59.84*** | 59.55*** | 47.24*** |
|                  | (5.40)  | (18.68)  | (18.92)  | (12.41)  |
| Tangible NW/Assets| 81.98   |          |          |          |
|                  | (82.06) |          |          |          |
| Current Ratio    | 2.39    |          |          |          |
|                  | (8.52)  |          |          |          |
| Leverage         |         |          | −3.32    |          |
|                  |         |          | (346.69) |          |
| Constant         | 52.30***| 13.44    | 37.54*** | 42.36    |
|                  | (4.06)  | (27.31)  | (11.33)  | (88.40)  |
| Number of obs.   | 5,610   | 4,783    | 4,790    | 4,903    |
| Number of groups | 1,155   |          |          |          |
| Time Dummy?      | Yes     | Yes      | Yes      | Yes      |

Standard errors in parentheses

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 1.6: Probit Estimation of Access to Public Debt Markets

This table reports the loadings on various firm characteristics $x$ to estimate the probability $P(w = 1|x)$ of a firm having access to public debt markets. The sample set is annual observations for firms in DealScan Database that could be matched to Compustat. The time period of the sample dataset is 1989-2006. The reported values are mean and (standard error).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Asset Ratio</td>
<td>0.924***</td>
<td>0.515***</td>
<td>0.290***</td>
<td>0.206***</td>
</tr>
<tr>
<td></td>
<td>(0.0324)</td>
<td>(0.0346)</td>
<td>(0.0422)</td>
<td>(0.0473)</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.246**</td>
<td>0.866***</td>
<td>0.856***</td>
<td>0.951***</td>
</tr>
<tr>
<td></td>
<td>(0.0976)</td>
<td>(0.102)</td>
<td>(0.135)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>Book Leverage</td>
<td>1.678***</td>
<td>1.416***</td>
<td>1.400***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0447)</td>
<td>(0.0533)</td>
<td>(0.0536)</td>
<td></td>
</tr>
<tr>
<td>Firm Age</td>
<td>0.129***</td>
<td>0.115***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0168)</td>
<td>(0.0170)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm Size</td>
<td>0.616***</td>
<td>0.618***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00694)</td>
<td>(0.00695)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td></td>
<td>-0.00554***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00177)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Flow/$K$</td>
<td></td>
<td>-0.0292</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0396)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.635***</td>
<td>-0.952***</td>
<td>-5.145***</td>
<td>-5.061***</td>
</tr>
<tr>
<td></td>
<td>(0.0195)</td>
<td>(0.0219)</td>
<td>(0.0667)</td>
<td>(0.0696)</td>
</tr>
<tr>
<td>Observations</td>
<td>29,092</td>
<td>29,092</td>
<td>29,092</td>
<td>29,092</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 1.7: Effect on Investment of Access to Public Debt Markets as Treatment

This table reports results of the estimation of the average treatment effect of access to debt markets on investment \((I/K)\) using annual observations for firms in DealScan Database that could be matched to Compustat. I follow the propensity score method suggested for non-random sampling. To do this, I need to estimate probability of treatment \(p(x)\) given the covariates \(x\), which are the firm characteristics in this case \(p(x) = P(w = 1|x)\), where \(w\) is an indicator for treatment and \(p(x)\) is the propensity score of accessing the public debt markets. I use firm size (measured by log(assets)), Tobin's \(Q\) and cash flow scaled by PP&E (to measure growth options), fixed asset ratio (Property, Plant and Equipment over Total Assets), Book Leverage (Debt/Total Assets), Profitability (Operating Income/Total Assets) to predict the source of debt financing using a probit model. Table 1.6 reports the estimation results of the probit regression. Column (4), that includes fixed asset ratio, profitability, book leverage, firm age, firm size, Tobin's \(Q\) and cash flow as right hand side firm characteristics, is used for the estimation of the propensity score (propensity of having access to public debt market access). The predicted value \(\hat{p}(x)\) is then used as a control in the regression:

\[
I/K = \beta_0 + \beta_1 Q + \beta_2 CF + \beta_3 \hat{p}(x) + \sum_i \beta_i \text{Interaction Terms}_i + \epsilon
\]

All numbers are in %. The CRSP-Compustat sample consists of all firm-year observations from nonfinancial firms in the merged CRSP-Compustat database. The time period of the sample dataset is 1989 - 2006. Time and firm fixed effects are included. The standard errors are robust under heteroscedasticity and adjusted for clustering at firm level. The reported values are mean and (standard error).

<table>
<thead>
<tr>
<th></th>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow/(K)</td>
<td>5.575***</td>
<td>5.572***</td>
<td>4.490***</td>
<td>4.387***</td>
<td>4.482***</td>
<td>6.080***</td>
</tr>
<tr>
<td></td>
<td>(0.961)</td>
<td>(0.961)</td>
<td>(1.309)</td>
<td>(1.194)</td>
<td>(1.549)</td>
<td>(1.696)</td>
</tr>
<tr>
<td>(Q)</td>
<td>0.654***</td>
<td>0.653***</td>
<td>0.715***</td>
<td>0.661***</td>
<td>0.724***</td>
<td>0.636***</td>
</tr>
<tr>
<td></td>
<td>(0.0585)</td>
<td>(0.0584)</td>
<td>(0.0892)</td>
<td>(0.0842)</td>
<td>(0.106)</td>
<td>(0.0895)</td>
</tr>
<tr>
<td>Access to Debt Mkt</td>
<td>50.56*</td>
<td>10.70***</td>
<td>8.675***</td>
<td>10.80***</td>
<td>9.203***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(26.10)</td>
<td>(1.140)</td>
<td>(1.713)</td>
<td>(1.279)</td>
<td>(1.953)</td>
<td></td>
</tr>
<tr>
<td>Propensity Score</td>
<td>23.35***</td>
<td>22.47***</td>
<td>21.70***</td>
<td>21.87***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.296)</td>
<td>(4.410)</td>
<td>(4.875)</td>
<td>(4.816)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Q \times \text{Access to Debt Mkt})</td>
<td>0.472</td>
<td>0.589*</td>
<td>(0.289)</td>
<td>(0.324)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Flow/(K) \times \text{Access to Debt Mkt}</td>
<td>-0.867</td>
<td>-6.982**</td>
<td>(3.823)</td>
<td>(3.444)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>16.34***</td>
<td>6.134</td>
<td>9.177***</td>
<td>9.709***</td>
<td>7.888***</td>
<td>8.152***</td>
</tr>
<tr>
<td></td>
<td>(0.663)</td>
<td>(5.304)</td>
<td>(1.039)</td>
<td>(1.086)</td>
<td>(1.408)</td>
<td>(1.503)</td>
</tr>
<tr>
<td>Observations</td>
<td>15450</td>
<td>15450</td>
<td>9989</td>
<td>9989</td>
<td>6272</td>
<td>6272</td>
</tr>
<tr>
<td>Adjusted (R^2)</td>
<td>0.130</td>
<td>0.131</td>
<td>0.135</td>
<td>0.138</td>
<td>0.136</td>
<td>0.140</td>
</tr>
</tbody>
</table>

\(^* p < 0.10, ^{**} p < 0.05, ^{***} p < 0.01\)
Table 1.8: Effect on Leverage of Access to Public Debt Markets as Treatment

This table reports results for fixed effects book leverage regression using annual observations for firms in DealScan Database that could be matched to Compustat. I follow the propensity score method suggested for non-random sampling. Propensity score estimate (propensity of having access to public debt market access) from Table 1.6 Column (4), that includes fixed asset ratio, profitability, book leverage, firm age, firm size, Tobin's Q and cash flow as right hand side firm characteristics, is used for the second stage reported in this table. The model estimated is as follows:

\[
\text{Leverage} = \beta_0 + \beta_1 \text{Profitability} + \beta_2 \text{Fixed Asset Ratio} + \beta_3 Q \\
+ \beta_4 \text{Sales} + \beta_5 \hat{p}(x) + \beta_6 \text{I(Access)} + \epsilon.
\]

All numbers are in %. The CRSP-Compustat sample consists of all firm-year observations from non-financial and non-utilities firms in the merged CRSP-Compustat database. The time period of the sample dataset is 1989 - 2006. Time and firm fixed effects are included. The standard errors are robust under heteroscedasticity and adjusted for clustering at firm level. The reported values are mean and (standard error). In column (4) observations with extreme propensity scores (higher than 90% and less than 10%) have been dropped for comparing observations closer to common support.

<table>
<thead>
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<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profitability</td>
<td>-31.78***</td>
<td>-29.53***</td>
<td>-41.16***</td>
</tr>
<tr>
<td></td>
<td>(2.733)</td>
<td>(3.441)</td>
<td>(4.763)</td>
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<tr>
<td>Fixed Asset Ratio</td>
<td>18.48***</td>
<td>8.778***</td>
<td>-2.649</td>
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<tr>
<td></td>
<td>(2.515)</td>
<td>(2.871)</td>
<td>(3.315)</td>
</tr>
<tr>
<td>Q</td>
<td>0.00303</td>
<td>-0.0384</td>
<td>-0.152**</td>
</tr>
<tr>
<td></td>
<td>(0.0246)</td>
<td>(0.0284)</td>
<td>(0.0627)</td>
</tr>
<tr>
<td>Log Sales</td>
<td>3.167***</td>
<td>-8.114***</td>
<td>-22.78***</td>
</tr>
<tr>
<td></td>
<td>(0.539)</td>
<td>(1.006)</td>
<td>(1.443)</td>
</tr>
<tr>
<td>Propensity Score</td>
<td>97.21***</td>
<td>138.4***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.210)</td>
<td>(6.190)</td>
<td></td>
</tr>
<tr>
<td>Access to Debt Mkt</td>
<td>34.73***</td>
<td>24.06***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.266)</td>
<td>(1.407)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>3.823</td>
<td>40.06***</td>
<td>126.9***</td>
</tr>
<tr>
<td></td>
<td>(2.923)</td>
<td>(5.200)</td>
<td>(7.839)</td>
</tr>
<tr>
<td>Observations</td>
<td>15432</td>
<td>9989</td>
<td>6272</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.077</td>
<td>0.294</td>
<td>0.502</td>
</tr>
</tbody>
</table>

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
This table estimates the impact of banks' market power on firm investment. I estimate the following model:

\[
I/K = \beta_0 + \beta_1 Q + \beta_2 CF + \beta_3 \text{Market Power of Local Banks} \\
+ \beta_4 \text{Market Power of Local Banks} \times Q + \beta_5 \text{Market Power of Local Banks} \times CF + \epsilon
\]

The table reports results of regression of investment on Market Power of Local Banks that proxies the outside options of the borrowing firms, and the market power of each bank, after controlling for Tobin's \( Q \) and cash flow \( CF \) (scaled by previous period's PP&E).

Column (2) reports results for only firms that have investment grade access. I also include interaction terms between firm characteristics (Tobin's \( Q \) and cash flow \( CF \)) and market power of banks: Market Power of Local Banks \( \times Q \) and Market Power of Local Banks \( \times CF \) respectively. The market power used in the above estimation is computed using the banks that are in the same MSA as the firm. The data set annual observations for nonfinancial firms in DealScan Database that could be matched to Compustat. All numbers are in %.

The time period of the sample dataset is 1989 - 2006. The Herfindahl Index proxies the market power of banks in the geographical location of the respective firm. The standard errors are robust under heteroscedasticity and adjusted for clustering at firm level. The reported values are mean and (standard error).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow/K</td>
<td>6.848***</td>
<td>8.932***</td>
<td>5.540***</td>
<td>5.559***</td>
<td>5.595***</td>
</tr>
<tr>
<td></td>
<td>(0.472)</td>
<td>(1.912)</td>
<td>(0.677)</td>
<td>(0.671)</td>
<td>(0.671)</td>
</tr>
<tr>
<td>( Q )</td>
<td>0.694***</td>
<td>0.351***</td>
<td>0.775***</td>
<td>0.701***</td>
<td>0.694***</td>
</tr>
<tr>
<td></td>
<td>(0.0334)</td>
<td>(0.085)</td>
<td>(0.0479)</td>
<td>(0.0478)</td>
<td>(0.0481)</td>
</tr>
<tr>
<td>Banks' Market Power</td>
<td>-1.776***</td>
<td>-0.664</td>
<td>-2.371***</td>
<td>-1.891***</td>
<td>-7.106***</td>
</tr>
<tr>
<td></td>
<td>(0.555)</td>
<td>(0.654)</td>
<td>(0.702)</td>
<td>(0.697)</td>
<td>(3.490)</td>
</tr>
<tr>
<td>( Q \times \text{Banks' Market Power} )</td>
<td>-0.229**</td>
<td>-0.224**</td>
<td>-0.203**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0978)</td>
<td>(0.0970)</td>
<td>(0.0979)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Flow/K ( \times \text{Banks' Market Power} )</td>
<td>3.691***</td>
<td>3.467**</td>
<td>3.390**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.372)</td>
<td>(1.361)</td>
<td>(1.362)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm's Age ( \times \text{Banks' Market Power} )</td>
<td></td>
<td></td>
<td>-4.870***</td>
<td>-5.423***</td>
<td>1.660</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.371)</td>
<td>(0.519)</td>
<td>(1.089)</td>
</tr>
<tr>
<td>Constant</td>
<td>15.87***</td>
<td>8.906***</td>
<td>16.07***</td>
<td>29.99***</td>
<td>31.71***</td>
</tr>
<tr>
<td></td>
<td>(0.786)</td>
<td>(1.199)</td>
<td>(0.800)</td>
<td>(1.324)</td>
<td>(1.741)</td>
</tr>
<tr>
<td>Observations</td>
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<td>2,592</td>
<td>9,787</td>
<td>9,787</td>
<td>9,787</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>14.71</td>
<td>30.76</td>
<td>14.76</td>
<td>16.23</td>
<td>16.24</td>
</tr>
<tr>
<td>Time Dummy?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

\* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
This table reports results for Roy (Self-Selection with unobserved heterogeneity) model where firms are divided into two groups: (i) Firms with no access to public debt markets, and (ii) Firms that have access to public debt markets. A firm can choose to go to the public debt markets at a cost. If the firm does not pay the cost, it stays in the group where firms have access only to banks, and faces stronger RAS. The investment decision depends on Tobin's Q, Cash flow and an unobservable RAS. The results are reported in Panel A. Panel B estimates the cost of getting access to the public debt markets using Tobin's Q, Leverage, Firm Size, Firm Age, Fixed Asset Ratio, Profitability. \( I/K \) represents investment and \( C \) represents the cost of accessing public debt markets. \( CFQ \) continue to represent cash flow and Tobin's Q. Assume that the error terms \( \varepsilon_0, \varepsilon_1 \) and \( \varepsilon_c \) are distributed normally and mutually independent of each other. RAS represented by \( \theta \) is also assumed to be independent of the error terms. The system of equations to be estimated is as follows (suppressing time subscripts \( t \), but keeping individual firm subscripts \( i \) for clarity):

\[
I/K_i^s = \beta_{0,s} + \beta_{1,s}Q_i^s + \beta_{2,s}CF_i^s + \alpha_s \theta_i^s + \varepsilon_i^s, \quad s \in \{0, 1\}
\]

\[
C_i = \gamma'Z_i + \alpha_c \theta_i + \varepsilon_c^i
\]

where \( Z = [Q, \text{Leverage}, \text{Firm Size}, \text{Firm Age}, \text{Fixed Asset Ratio}, \text{Profitability}] \), and the unobservable term is drawn from a standard normal distribution. The reported values are mean and (standard error). All values in Panel A are in percentage.

### Panel A: Estimated Parameters

<table>
<thead>
<tr>
<th></th>
<th>Firm with no access</th>
<th>Firms with access</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tobin's Q</td>
<td>0.60***</td>
<td>0.68***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Cash Flow</td>
<td>5.21***</td>
<td>2.29***</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(8.29)</td>
</tr>
<tr>
<td>RAS</td>
<td>-17.41***</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Constant</td>
<td>12.52***</td>
<td>8.58**</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(4.04)</td>
</tr>
</tbody>
</table>

### Panel B: Estimation of Cost Function

<table>
<thead>
<tr>
<th></th>
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<th>Standard Error</th>
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</thead>
<tbody>
<tr>
<td>( \sigma_{\varepsilon} )</td>
<td>0.097***</td>
<td>0.00964</td>
</tr>
<tr>
<td>Tobin's Q</td>
<td>-0.39</td>
<td>2.46</td>
</tr>
<tr>
<td>Leverage</td>
<td>-1.03***</td>
<td>0.06</td>
</tr>
<tr>
<td>Firm Size</td>
<td>-3.39***</td>
<td>1.23</td>
</tr>
<tr>
<td>Firm Age</td>
<td>-2.46***</td>
<td>0.56</td>
</tr>
<tr>
<td>Fixed Asset Ratio</td>
<td>-2.15***</td>
<td>0.09</td>
</tr>
<tr>
<td>Profitability</td>
<td>1.17***</td>
<td>0.03</td>
</tr>
<tr>
<td>RAS</td>
<td>-0.17</td>
<td>0.24</td>
</tr>
<tr>
<td>Constant</td>
<td>0.96</td>
<td>1.29</td>
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</table>

Standard errors in parentheses, * \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Table 1.11: Evidence Against Competing Explanations of Reduction in Investment

This table provides evidence against explanations that compete with RAS. The table reports results for investment regression $I/K$ using annual observations for nonfinancial firms in DealScan Database that could be matched to Compustat. All numbers are in %. The Herfindahl Index proxies the market power of banks in the geographical location of the respective firm. The column marked “Access?” reports the results of the first stage regression where access to public debt markets is estimated using firm characteristics and banks’ market power. The reported values are mean and (Std. Error). Constant is not reported.

<table>
<thead>
<tr>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I/K$</td>
<td>4.960 ***</td>
<td>4.959 ***</td>
<td>5.048 ***</td>
<td>5.047 ***</td>
<td>4.833 ***</td>
<td>4.834 ***</td>
<td>0.0446 ***</td>
<td>5.537 ***</td>
</tr>
<tr>
<td></td>
<td>(0.969)</td>
<td>(0.970)</td>
<td>(0.982)</td>
<td>(0.983)</td>
<td>(0.971)</td>
<td>(0.971)</td>
<td>(0.0109)</td>
<td>(1.090)</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.522 ***</td>
<td>0.522 ***</td>
<td>0.539 ***</td>
<td>0.539 ***</td>
<td>0.540 ***</td>
<td>0.540 ***</td>
<td>0.000510</td>
<td>0.605 ***</td>
</tr>
<tr>
<td></td>
<td>(0.0502)</td>
<td>(0.0503)</td>
<td>(0.0515)</td>
<td>(0.0515)</td>
<td>(0.0514)</td>
<td>(0.0514)</td>
<td>(0.000519)</td>
<td>(0.0540)</td>
</tr>
<tr>
<td>Banks’ Market Power</td>
<td>-1.716 ***</td>
<td>-1.953 ***</td>
<td>-1.779 ***</td>
<td>-1.976 ***</td>
<td>-1.776 ***</td>
<td>-1.835 ***</td>
<td>-0.0805 ***</td>
<td>(0.0116)</td>
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<tr>
<td></td>
<td>(0.427)</td>
<td>(0.856)</td>
<td>(0.431)</td>
<td>(0.597)</td>
<td>(0.431)</td>
<td>(0.469)</td>
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<tr>
<td>Altman Z-Score</td>
<td>0.253 ***</td>
<td>0.233 ***</td>
<td>0.238 ***</td>
<td>0.238 ***</td>
<td>0.252 ***</td>
<td>0.252 ***</td>
<td>0.0563</td>
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<td></td>
<td>(0.0646)</td>
<td>(0.0902)</td>
<td>(0.0658)</td>
<td>(0.0658)</td>
<td>(0.0654)</td>
<td>(0.0654)</td>
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<tr>
<td>Altman Z-Score × Banks’ Market Power</td>
<td>0.0563</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.184)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Year &gt; 1998</td>
<td></td>
<td>-1.395 ***</td>
<td>-1.549 ***</td>
<td>(0.331)</td>
<td>(0.470)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Year &gt; 1998 × Banks’ Market Power</td>
<td></td>
<td>0.443</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>(0.850)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Year &gt; 2003</td>
<td></td>
<td>-1.515 ***</td>
<td>-1.641 ***</td>
<td>(0.401)</td>
<td>(0.572)</td>
<td></td>
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<tr>
<td>Year &gt; 2003 × Banks’ Market Power</td>
<td></td>
<td>0.364</td>
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<tr>
<td>Firm’s Age</td>
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<td></td>
<td></td>
<td>0.0783 ***</td>
<td>-6.285 ***</td>
<td>(0.00768)</td>
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<td>(0.0210)</td>
<td>13.82 ***</td>
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<td>Fixed Asset Ratio</td>
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<td></td>
<td>0.0447 ***</td>
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<td>(3.365)</td>
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<td></td>
<td>(0.0235)</td>
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<td>(3.058)</td>
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<td>Book Leverage</td>
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<td></td>
<td></td>
<td>0.515 ***</td>
<td>-11.38 ***</td>
<td>(0.0428)</td>
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<td></td>
<td>(0.0235)</td>
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<td>(6.119)</td>
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<td>Profitability</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0972 **</td>
<td>7.302 **</td>
<td>(0.0428)</td>
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<td></td>
<td></td>
<td>(0.0235)</td>
<td></td>
<td>(3.058)</td>
</tr>
<tr>
<td>Access to Public Debt Market</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.071</td>
<td></td>
<td>(0.062)</td>
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<tr>
<td>Observations</td>
<td>12318</td>
<td>12318</td>
<td>12318</td>
<td>12318</td>
<td>12318</td>
<td>12318</td>
<td>9992</td>
<td>9972</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.161</td>
<td>0.161</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
<td>0.071</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
This figure provides the distribution of covenants among firms divided into deciles based on their sales. The dataset is all firms in DealScan database. The figure to the right shows that more covenants of certain types (Minimum Debt to Tangible Networth, Minimum Current Ratio, Minimum Quick Ratio) are placed on firms that are smaller. However, the figure to the left, shows that more covenants of certain types (Maximum Capex, Max Debt to EBITDA, Min. Interest Coverage and Min. Fixed Charge Coverage) are imposed in firms that are in the middle range. The relative distribution of covenants of two different types cannot be explained if all covenants play the same role of just reducing asset substitution.
Figure 1.2: Timeline of Capital Expenditure Covenants on Firms relative to Public Debt Market Access

The figure provides the incidence of Maximum Capital Expenditure covenants among firms based on their access to debt markets. The dataset is all firms in DealScan database that eventually received a credit rating. The vertical axis shows the number of capital expenditure covenants scaled by the number of total covenants. The horizontal axis is time in years. Time 0 is when a firm gets access to public debt markets. The relative frequency of capital expenditure covenants are adjusted by total number of covenants in each bin, to see differential importance of Capital Expenditure covenants over other covenants.
Figure 1.3: Event Tree

This figure shows the possible events in the two-period setup presented in section 1.2. $E$ represents equity of the entrepreneur in the firm at each stage. Superscript $s, f$ represent success or failure in each period. $q$ is the probability of success or failure of the project in each stage. The minimum amount of equity above which the entrepreneur switches to arm's length debt financing is shown by the horizontal line. $E_{1}^{RAS}$ is the amount of equity the owner will have at the end of the first period if the project succeeds in the presence of RAS. The bank realizes that if the entrepreneur has equity $E_{1}$, then she will switch to public debt financing, and hence the bank limits the growth of the firm to $E_{1}^{RAS}$ even in case of success of the project in the first period.
Figure 1.4: Deposits in Banks by Geography & Metropolitan Areas in United States

This first figure provides the deposits in US Banks by state. The second figure shows the Metropolitan Statistical Areas in United States. The information is used to compute bank deposit concentration (Herfindahl Index) by geography, that proxies the market power of banks in the geographical location of the respective firm.
The figure shows the trends and jumps in securitization in United States. Specifically, it reports the volume of Collateralized Debt Obligations (CDOs) issued in United States each year. Various shades are used to mark individual types of CDOs.
Figure 1.6: Growth in Corporate Debt Market in the Euro Zone & Relative Growth Rates of Firms in the Euro Zone in 1999

The first figure reports the volume of issuance by credit quality in the countries that adopted Euro as the common currency in 1999. The second figure compares the growth in investment of firms in Continental Europe to those in United Kingdom after introduction of Euro as the common currency on Jan 1st, 1999. The European Corporate Bond market got a big positive liquidity shock after the introduction of Euro. It is a visual representation of a triple difference methodology, with nations (France, Germany) against nation (UK), year (1999) over year (1998) and large firms (right deciles) against small firms (left deciles).
Chapter 2

Heterogeneity in Corporate Governance: Theory and Evidence

2.1 Introduction

I argue that firm specific exogenous performance uncertainty plays an important role in the choice of managerial discretion allowed by investors. Ideally, insider management and outside investors can write an optimal contract so that the manager's actions are incentive compatible. In such a case, no intermediate interference by outside investors is necessary. However, the amount of ownership that management needs to have for incentive compatibility, is often not practical for large firms. I argue that corporate governance mechanisms help achieve second best control for the investor under such conditions.

Figure 2.1 suggests a heterogeneity in corporate governance levels adopted by equity-holders in 1500 largest US firms. This distribution is among the largest firms, where highly concentrated ownership is less frequent, hence blockholder heterogeneity (as in Burkart, Gromb, and Panunzi (1997)) cannot explain this heterogeneity. Since, these are all US firms as well, that are mostly
incorporated in Delaware, laws (as in La Porta, Lopez-de Silanes, Shleifer, and Vishny (2000)) do
not have a large effect as well. Hence, the heterogeneity in corporate governance among 1500
largest US firms is puzzling. Figure \ref{fig:2.2} presents another puzzling fact. It shows how the relative
vary over time. Gompers, Ishii, and Metrick (2003) show that in the 1990s, an abnormal return of 8.5% could have been earned by buying more democratic firms and selling firms that
practice near dictatorship. This is referred to as the governance portfolio. Core, Guay, and Rusticus (2006) and Yang (2005) find that the abnormal returns of the governance portfolio become
negative between 2000 and 2003. In any case, it can be concluded from above literature, that
corporate governance has an important effect on firm output. The figure also shows the puzz-
ling correlation that relative performance of growth and value firms have with the democracy
- dictatorship portfolio.

This paper answers the following questions: (i) Why do we need corporate governance mea-
sures if optimal contracts are possible? (ii) What features of optimal contracts between outside
investors and insiders are hard to implement? (iii) Can laws, blockholder heterogeneity, or man-
gerinal talent heterogeneity explain the heterogeneity in corporate governance? If not, what de-
termines the level of control that firm investors choose to give to the firm management? (iv) Did the Sarbanes-Oxley Act of 2002, also known as the Public Company Accounting Reform and
Investor Protection Act have an impact on investors’ choice of managerial discretion?

To answer these questions, I first introduce the optimal contract in a principal agent frame-
work, where the insider agent has more information about firm performance than the outsider
principal. There is heterogeneity among firms as firms face shocks that are not equal in vari-
ance - i.e. shocks are heteroscedastic over the distribution of firms. The agent also has a private
diversion technology. The shareholders own the company and have incomplete information
about the day to day workings, and the management get a certain wage to manage the assets


of the firm. There are no creditors. Shareholders can affect management quality by voting on
governance guidelines. On the other hand, management affects productivity through day to
day operating decisions. A higher autonomy vested in the management allows it to contribute
more to the firm, but also allows the management to divert more funds for private benefits.
Tighter governance control has the opposite effect. I show that for large enough firms, opti-
mal contracts become impractical, and corporate governance is the only answer. Investors use
corporate governance mechanisms to enforce second best control over insider management.
Corporate governance controls implemented by investors depend upon the firm specific un-
certainty, and hence we observe heterogeneity in managerial discretion. I find that autonomy
of management is positively correlated with size of the firm, but controlling for autonomy, there
is a negative correlation between variance and autonomy. I also find evidence indicating that
over time, the information asymmetry between shareholders and management has decreased.
Furthermore, since the implementation of Sarbanes-Oxley Act, the choice of corporate gover-
nance mechanisms are statistically independent of this information asymmetry.

This work contributes to the literature on corporate governance by making governance qual-
ity endogenous to the contract between equity-holders and management. I show that under
small enough adjustment cost, management decisions to take or reject risky projects depends
on the contract it faces, and shareholders respond to management behavior by choosing the
level of governance to maximize the value of their interests. When management chooses to take
new projects with higher asset variance, it is optimal for the shareholders to rein in such a man-
agement with higher shareholder rights and less autonomy - if allowed by adjustment costs. On
the other hand, if volatility of capital growth of the firm is low, it is optimal for the shareholders
to allow such a management more freedom, so as to encourage the latter to take projects with
higher asset variance. Thus, I argue that the governance policy and management investment
policy are jointly determined, compared to the common consensus in the literature that man-
agement responds to corporate governance chosen by shareholders but not vice-versa. Having an equilibrium of our form allows us to explain the corporate governance heterogeneity that exists in practice.

This work contributes to the contracting literature, by suggesting that even if optimal contracts may be possible, it may not always be practical. Under perfect information, Holmstrom and Milgrom (1987) and more recently Ou-Yang (2005) have shown that the optimal contract is composed of the reservation wage of the management, its effort cost and a share of the risk in the price movements of the stock. Principal agent problems with incomplete information are generally posed between an entrepreneur (who also owns the firm) and creditors. Hart and Moore (1998) have shown that debt contract is optimal in such a setup. Dewatripont, Legros, and Matthews (2003) show that under limited liability and investor's earnings having a monotonic relation with firm profits, debt and convertible debt maximize the entrepreneur's incentives to exert effort. DeMarzo and Sannikov (2006) also have a principal that suffers from an information disadvantage compared to the agent. In their solution they provide management with enough equity stake to dissuade the agent from private diversion of firm funds. This paper addresses the case when the agent cannot be given such an equity stake, due to practical reasons - such as when the firm is large.

Section 2.2 presents the principal agent model, and solves for the optimal autonomy and optimal corporate governance policy. Section 2.3 presents data to support the model and discusses the results. Section 2.4 concludes.

2.2 Model

I start by showing the main tradeoff in section 2.2.1. Section 2.2.2 presents the dynamic optimal contract.
2.2.1 Framework

This is a principal (P) agent (A) model with incomplete information. The next three paragraphs describe the actors in the play:

The Agent

The agent chooses the amount of effort \( \eta(t) \) she puts in the firm and the amount of output \( \bar{d}(K,M,t) \) she diverts for private consumption. \( M \) is a measure of the autonomy given to the agent by the principal, i.e. as the value of \( M \) increases, the oversight faced by the agent decreases. I use the governance index of Gompers, Ishii, and Metrick (2003), as an instrument for \( M \).

Assumption 2.2.1 The agent incurs a cumulative cost of \( \int_0^T c(\eta, t) \, dt \) associated with managing the firm from time 0 to \( T \). The cost rate is given by a convex function:

\[
c(\eta, t) = \frac{1}{2} \kappa(t) \eta^2(t)
\]

where \( \kappa(t) \) is a deterministic function of time \( t \) and where the marginal cost rate increases with the level of effort, thus guaranteeing a solution to the manager's maximization problem. \( \kappa(t) \) may serve as a proxy for a manager's skill, that is, the higher a manager's skill, the lower the value of \( k(t) \).

Furthermore, I make an assumption about the diversion function of the agent:

Assumption 2.2.2 The amount of output the agent can divert for personal consumption non-verifiably is also a convex function of the form:

\[
\bar{d}(M,K,t) = \frac{1}{2} \zeta K(t) M(t)^2,
\]
where $\zeta$ is a proxy for the propensity of the agent to divert capital.\footnote{As we shall see later, the agent may not divert the maximum amount allowed, since in an incomplete information setting, she might want to smooth output. Smoothing output reduces the information the principal can infer from observing the output of the firm regarding the characteristics of the agent. But, more on that will follow later!}

The agent has a negative exponential utility, and has a risk aversion of $R_A$. $\bar{W}_A$ represents the certainty equivalent wealth of the agent required for him to run the firm. At time $T$, the agent receives a salary $S(T)$ from the principal as compensation for their expended effort $\eta$ in running the firm. The salary and the diversion $d(K, M, t)$ of output for private consumption are the only two sources of income for the agent. The salary is, of course, observable by the principal, but the diversion is not.

Given the principal’s contract $S(T)$, the agent expends effort to maximize her own expected utility:

$$
\sup_{\eta(t), d(M, K, t)} \mathbb{E}_0 \left( -\frac{1}{R_A} \exp \left\{ -R_A \left[ S(T) + \int_0^T d(K, M, t) - c(\eta, t) dt \right] \right\} \right),
$$

(2.3)

where $\{ \eta(t), d(M, K, t) \} \equiv \{ \eta(t), d(M, K, t) : 0 \leq t \leq T \} \in \Xi_{0,T}$, with $\Xi_{0,T} \in \mathcal{F}^2$ representing the set of measurable processes on $[0, T]$ adapted to the agent’s information. Since $\kappa(t)$ in equation 2.1 is deterministic function of time, $\bar{\kappa}(t) \equiv e^{-rt}\kappa(t)$ and hence I omit the discount factor for agent’s cost function in equation 2.3 as in Ou-Yang (2005).

### The Firm

The firm has a stochastic Cobb Douglas production function of the form:

$$
dY(t) = K(t)^a M(t)^\eta(t) dt + \sigma(Y)dw,
$$

(2.4)
tonomy granted to the agent by the principal. Higher autonomy allows the agent to work more effectively, thus increasing output. Labor is conspicuous by its absence, which I assume to be completely inelastic in our setting.

**The Principal**

The principal pays the agent $S(t)$. I treat the shareholders or the principal as a homogeneous group as this paper addresses the dynamics between the principal i.e. shareholders and the agent i.e. the management.

The principal does not observe: (i) shocks to productivity $d w$ (even though I assume $\sigma(Y)$ is known by the principal, this assumption can be relaxed, but does not help gain any intuition in our work), and (ii) diversion $\bar{d}$ by the agent. The principal observes (i) Cost of effort $\kappa(t)$ for agent, (ii) Risk aversion $R_A$ of the agent, and (iii) Required reservation wage $\bar{W}_A$ of the agent.

The principal, much as the agent, has a negative exponential utility function with a constant risk-aversion coefficient $R_P$. The principal owns the capital of the firm. The principal decides on the level of autonomy $M$ she would give to the agent. She increases or decreases $M$ using control up $U$ or down $D$ respectively. Thus, by choosing controls $U, D$ and the wage contract with the agent $S(t)$, the principal attempts to maximize expected utility over her terminal wealth:

$$\max_{U,D,S(t)} \mathbb{E}_0 \left( -\frac{1}{R_P} \exp \left\{ -R_P \left[ K(T) - S(T) - D(T) \right] \right\} \right),$$

subject to agent's participation and incentive compatibility constraints as well as the investor's budget constraints that will be stated shortly.

I assume the principal cannot solve her problems and attain risk neutrality by trading away shares or diversifying the portfolio. This is a reasonable assumption for large blockholders.
2.2.2 The Problem

Armed with the knowledge of the economy, I pose the complete problem. The problem of the principal is to maximize her expected terminal wealth by choosing optimal wage contract for the agent $S$ and optimal management control $M$ while she suffers from incomplete information on agent’s behavior and firm output. Formally:

$$\max_{M,S} \mathbb{E}_0 \left( -\frac{1}{R_P} \exp \left\{ -R_P \left[ K(T)-S(T)-D(T) \right] \right\} \right)$$

subject to the incentive compatibility (IC) and participation constraint (PC) of the agent. The principal above has a negative exponential utility with risk aversion $R_P$. She only cares about her capital $K$ at time $T$ net of agent’s wage $S(T)$ and cumulative (and properly discounted) diversion $D(T)$ i.e. $K(T)-S(T)-D(T)$.

The capital of the firm incurs a cost of capital $r$ and depreciates at rate $\delta$. The agent’s diversion of capital for private consumption $\tilde{d}(M, K, t)$ is also funded from the capital of the firm. Along with the drift term, the control exercised by principal $U, D$ also changes management autonomy.

Since the certainty equivalent wealth required by the agent is $W_A$ and the wage contract is $S(t)$, the participation constraint (PC) for the agent, at equilibrium effort $\eta^*(t)$ and equilibrium management oversight $M^*(t)$ is:

$$\max_{\eta, d} \mathbb{E}_0 \left( -\frac{1}{R_A} \exp \left\{ -R_A \left[ S(T) + \int_0^T \tilde{d}(M, K, t)-c(\eta^*, t)d t \right] \right\} \right) \geq -\frac{1}{R_A} \exp(-R_A W_A).$$ (2.6)

This constraint allows the agent to get a minimum level of expected utility that is equal to her outside option of working elsewhere. $c(\eta, t)$ includes the cost of effort and the private diversion of capital by the agent.
The incentive compatibility (IC) constraint of the agent is that given \( S(t) \), the agent’s equilibrium effort \( \eta^*(t) \) satisfies the agent’s dynamic maximization problem:

\[
\eta(t)^* = \arg \max_{\eta(t)} \mathbb{E}_0 \left( -\frac{1}{R_A} \exp \left\{ -R_A \left[ S(T) + \int_0^T \tilde{d}(M, K, t) - c(\eta^*, t) dt \right] \right\} \right) \quad (2.7)
\]

To solve the problem I take the following approach: I solve for the optimal contract under no adjustment cost of changing agent’s autonomy \( M \). I then introduce adjustment costs and solve for optimal control where agent tries to maximize private benefits of diversion from \( \tilde{d} \) and principal tries to minimize it.²

**Optimal Contract**

As the principal’s maximization problem is subject to agent’s PC and IC constraint, I will first impose these constraints on the contract form ².8 and then I can solve the unconstrained problem.


**Assumption 2.2.3** The compensation to the agent is given by:

\[
S(T) = q(T, K(T)) + \int_0^T g(t, K(t)) dt + \int_0^T h(t, K(t)) dK(t), \quad (2.8)
\]

where functions \( q(T, K(T)) \in \mathcal{R} \) and \( g(t, K(t)), h(t, K(t)) \) are continuous on \([0, T] \times \mathcal{R}\). Thus \( q(T, K(T)), g(\cdot) \) and \( h(\cdot) \) are measurable and adapted to principal’s information set \( \mathcal{F}_t = \{ K_s : s = t \} \).

²I am working on solving for the evolution of the wage contract as the principal updates the beliefs about the characteristics of the agent from her behavior as follow up research.
Before getting an expression for the agent’s equilibrium compensation, I also need the following definition.

**Definition 2.2.1** Given the agent’s compensation $S(t)$ in equation 2.8, the value function process $V_A(t, K(t))$ for the agent’s maximization problem is given by:

$$
\sup_{\eta, \tilde{d}} \mathbb{E}_t \left[ - \frac{1}{R_A} \exp \left( - R_A \left\{ q(T, K) + \int_t^T \left[ g(u, K) + \tilde{d}(M, K, u) - c(\eta, u) \right] du + \int_t^T h(u, K) dK \right\} \right) \right],
$$

(2.9)

where the agent’s information set is given by $\mathcal{F}(t) = \{ K(s), s \leq t \}$. $V(t, K(t))$ is assumed to be continuously differentiable in $t$ and twice continuously differentiable in $K \in \mathcal{R}$.

The Bellman equation for the agent’s maximization problem is given by:

$$
0 = \sup_{\eta, \tilde{d}} - R_A V(t, K) \left[ g(t, K) + h(t, K) \mu(K) + \tilde{d}(M, K, t) - c(\eta, t) - \frac{1}{2} R_A \sigma(K)^2 h^2(t, K) \right]
+ V_t + V_K \left[ \mu(K) - R_A \sigma(K)^2 h(t, K) \right] + \frac{1}{2} V_{KK} \sigma(K)^2,
$$

(2.10)

where $\mu(K), \sigma(K)$ are defined as below from the law of motion of capital in equation 2.6

$$
\mu(K) \equiv K(t)^\alpha M(t)^{\beta(t)} - (r + \delta)K(t) - \tilde{d}
$$

$$
\sigma(K) \equiv \sigma(Y)
$$

(2.11)

The derivation of agent’s equilibrium contract is given in appendix B.2. I state the equilibrium contract here as a lemma.

**Lemma 2.2.2** The agent’s equilibrium compensation is given by:

$$
S(T) = \bar{W}_A + \int_0^T \left[ c(\eta, t) - \tilde{d}(M, K, t) + \frac{R_A}{2} \sigma(K)^2 \bar{h}^2 \right] dt + \int_0^T \sigma(K) \bar{h} dW(t),
$$

(2.12)
where \( \bar{h} \equiv h(t, K(t)) - \frac{V(T)K}{RAV} \). Agent’s certainty equivalent wealth \( \bar{W}_A \) and cost of effort \( c(\eta, t) \) have already been introduced in section 2.2.1.

Thus, the principal pays the agent a reservation wage, reimburse the agent’s effort costs and compensates the agent with the third term for the risk borne by the agent in the fourth term that is stochastic.

Now, I will incorporate the agent’s IC constraint, into the principal’s maximization problem. To do so, I need to impose the first-order condition (FOC) of the agent’s Bellman equation 2.10 on the principal’s problem.

From the FOC by differentiating on \( \eta \), I get:

\[
0 = \frac{\partial}{\partial \eta} \left[ g(t, K) + \bar{h} \mu(K) - c(\eta, t) - \frac{1}{2} RA \sigma^2(K) h^2(t, K) \right] \\
\bar{h} = \frac{\kappa(t) \eta(t)}{K(t)^a M(t)^{b(t)}} \log M(t),
\]

(2.13)

where \( \eta(t) \) and \( h \) shall be determined from the principal’s maximization problem. The agent’s PC and IC will now be satisfied if we substitute \( h \) derived above into the expression for agent’s compensation 2.2.2.

Now, I can solve the principal’s maximization problem for optimal \( \eta \) without any additional constraints. We can then construct the optimal contract in terms of variables such as capital which are observed by both the principal and the agent. The proof of the next theorem which solves for the optimal contract and the optimal effort is in appendix.

**Theorem 2.2.3** The optimal effort level \( \eta^* \), the optimal management \( M \) and the optimal contract
and: 

\[ S(t) = W_A + \frac{1}{2} \int_0^T \left[ c(\eta^*, M(t), \tilde{d}) + \frac{R_A}{2} \tilde{h}^2 \sigma^2(K) d t - \tilde{h} \mu(K) \right] d t + \int_0^T \tilde{h} d K(t), \] (2.14)

where, \( \mu(K), \sigma(K) \) are defined in equation 2.11 and \( \tilde{h} \) is shown in equation 2.13.

The above relations might not be so enlightening, and in fact, a bit disheartening, as I do not get closed form solutions.

However, consider this simplification obtained from the system of equations 2.14:

\[ \frac{1 - \eta \log M}{1 + \eta \log M} \left( \frac{M \log M}{\eta} \right) = \frac{\partial}{\partial \eta} \left( \mu(K) - c(\eta, M, \tilde{d}) e^{-\int_0^T \mu(K) d t} \right) \] (2.15)

that shows the relative tradeoff between \( \eta \) and \( M \). At the margin, the net increase in capital growth obtained by increasing \( \eta \) after paying the agent’s effort cost, and the relative increase in capital growth by increasing the management capital \( M \) after paying the increased amount the agent can divert have a ratio given by the left hand side of equation 2.15.

2.2.3 Optimal Policy under Adjustment Cost

I now introduce adjustment costs and solve for optimal control where agent tries to maximize private benefits of diversion from \( \tilde{d} \) and principal tries to minimize it. I rewrite the maximiza-
tion problem of the principal in continuous time, adding a proportional cost \( \lambda M \) for changing management:

\[
\begin{align*}
\max_{U,D} & \quad \mathbb{E}_0 \left\{ \frac{1}{R_P} \exp \left\{ -R_P \left[ K(T) - D(T) - S(T) \right] \right\} \right\} \\
\text{s.t.} & \quad d K(t) = d Y(t) - ((r + \delta)K(t) - \theta)d t - (1 + \lambda_+)Md U_t - (1 - \lambda_-)Md D_t \\
& \quad d M_t = d U_t - d D_t,
\end{align*}
\]

(2.16)

where \( K(T) \) represents the firm capital at the terminal time \( T \), \( D(T) \) represents the cumulative diversion done till time \( T \) and \( S(T) \) is the contracted wage given to agent \( A \). The principal has constant absolute risk averse risk preferences with \( R_P > 0 \) as the coefficient of absolute risk aversion. The principal uses controls \( U, D \) (up/down) to change autonomy \( M \) and maximize terminal capital \( K(T) \) of the firm which she consumes at time \( T \) after suffering diversion and paying the contracted wage to the agent. The charges for a change in management autonomy are \( \lambda_- M \) if the management autonomy is reduced and \( \lambda_+ M \) if the autonomy is increased.

The principal’s position in firm capital \( K(t) \) and management autonomy \( M(t) \) are constrained to lie in the closed region:

\[
\mathcal{S} = \left\{ (x, y) \in \mathbb{R}^2 : x - (1 + \lambda_+)Md U_t - (1 - \lambda_-)Md D_t \geq 0 \text{ and } y \in \mathbb{R} \right\}
\]

(2.17)

The solution for optimal policy is in the tradition of Davis and Norman (1990). A policy for governance is a couplet \( \{D(t), U(t)\} \) of adapted processes that represent the cumulative decrease and increase of management autonomy. \( \{D(t)\} \) and \( \{U(t)\} \) are right-continuous and non-decreasing with \( D(0) = U(0) = 0 \). As shown in problem 2.16 the principal’s holdings \( (K(t), M(t)) \) starting with a position \( (x, y) \in \mathcal{S} \) evolve in the following way in response to a given policy
\( (U, D): \)

\[
    d K_t = d Y_t - (r + \delta) K_t dt - d(M, t) dt - (1 + \lambda_+) M d U_t - (1 - \lambda_-) M d D_t, \quad K_0 = x
\]

An admissible policy is a policy \((D, U)\) for which \(\tau = \infty\) a.s. or equivalently, \(P[(K_t, M_t) \in \mathcal{F} \forall t \geq 0] = 1\). I denote by \(\mathcal{U}\) the set of admissible policies.

The principal's objective is to maximize over \(\mathcal{U}\) the utility:

\[
    J_{x,y}(U, D) = \mathbb{E}_{x,y} \left( -\frac{1}{R_P} \exp \left( -R_P \left[ K(T) - D(T) - W_A \right] \right) \right),
\]

where \(x, y\) represent the initial positions in \(K, M\).

**Theorem 2.2.4** The optimal growth of capital \(d K(t)\), the optimal policy to change management \(U, D\) is given by:

\[
    d K^* = -\frac{1}{R_P} \log \left( \frac{J_K}{R_P} \right) \tag{2.18}
\]

\[
    l = \begin{cases} 
        \theta & \phi_M \geq (1 + \lambda_-) e^{-\int_0^T \mu_K dt} \\
        0 & \phi_M < (1 + \lambda_-) e^{-\int_0^T \mu_K dt}
    \end{cases}
\]

\[
    u = \begin{cases} 
        \theta & \phi_A \geq (1 - \lambda_+) e^{-\int_0^T \mu_K dt} \\
        0 & \phi_A < (1 - \lambda_+) e^{-\int_0^T \mu_K dt}
    \end{cases}
\]

where the principal's value function is given by:

\[
    J(t, K_t, M_t) = -\frac{1}{R_P} \exp[-R_P(f_1(t)K_t + f_2(t) + \phi(M))] \\
    f_1 = e^{-\int_0^T \mu_K dt}
\]

When the marginal of \(\phi\), which is part of the value function, with respect to \(M\) crosses a
threshold that depends on rate of growth of capital and transaction cost $\lambda$, the principal needs to increase or decrease management autonomy to keep it inside the threshold. In other words, we have a bang-bang equilibrium.

**Optimal Bounds for Optimal Policy**

So far, I have not talked about the nature of the transaction cost. This is where the incomplete monitoring nature of the game plays a role. Kalai and Lehrer (1993) show that when playing a Harsanyi-Nash equilibrium of an infinitely repeated game of incomplete information about opponents' payoff matrices, players will eventually play a Nash equilibrium of the real game, as if they had complete information. However, this takes time. Agents hired by the principal do not run the firm for an infinite time.

The result is the following sub-optimal choices which will happen on the equilibrium path until principal has learnt enough of the agent:

- **Type I error or False Positive:** In this case, the growth of capital of the firm is low due to unfavorable shock. However, the principal decides to reduce autonomy as she blames the agent for it. This results in lower ability of the agent to perform and is thus a cost which I represent by $\lambda_-$.

- **Type II error or False Negative:** In this case, the growth of capital of the firm is high due to favorable shock. However, the principal decides to increase autonomy as she credits the agent for it. This results in higher ability of the agent to steal and is thus a cost which I represent by $\lambda_+$.

As everything else is observable, the probability of making the errors mentioned above depends on the unobservable diversion principal thinks the agent is making and the unobservable pro-
ductivity shocks:

\[ \lambda_- \equiv P(FP) = P\left(-\int_{0}^{T} \sigma dW > \int_{0}^{T} \tilde{\mu}_K - \frac{dK}{dK} dK < \frac{dK}{dK}\right) \]

\[ \lambda_+ \equiv P(FN) = P\left(\int_{0}^{T} \sigma dW > \int_{T}^{0} \frac{dK}{dK} - \frac{\bar{\mu}_K}{dK} dK > \frac{\bar{d}K}{dK}\right), \]  

(2.19)

where \( \tau \) represents the time when principal chooses to exercise control \( M \). The next lemma shows the probability of the errors dependent upon the time \( T \) the agent has been at the helm of the firm.

**Lemma 2.2.5** The probabilities of type I and type II errors are:

\[ P(FP) = \Phi\left(\log\left(\frac{K}{K}\right) + \frac{1}{2} \sigma^2(T-t)\right) \]

\[ P(FN) = \Phi\left(\log\left(\frac{K}{K}\right) + \frac{1}{2} \sigma^2(T-t)\right) \]

(2.20)

This is an intuitive result. The probability of making wrong control decisions depends on the variance \( \sigma^2 \) of the output and also on the acceptable rates of growth of capital \( \frac{K}{K} \). Furthermore, the longer the principal takes to decide i.e. \( T \), the lower the probability of an error.\(^4\) Let us assume for now that the principal observes the capital growth at constant intervals, such as in annual shareholder meetings. In such a case, as \( \sigma^2 \) increases, to hold the rate of making errors constant, the bounds \( \frac{K}{K} \) of the bang-bang equilibrium need to widen.

\(^4\) However, waiting infinitely is not optimal, since then the agent will have deviated an enormous amount of capital before getting reigned in. Abel, Eberly and Panageas (2007) address the question of optimal inattention with costly information access for an investor. A similar question arises here, due to incomplete information. Two paths exist for us: Choosing optimal bounds on capital growth \( \frac{K}{K} \) or choosing optimal \( T \). I presently hold \( T \) constant and focus on capital growth.
Filtering Problem

In this problem I have \((d, K)\) as a two-dimensional partially observable random process where true diversion by management \(d = (d_t, \mathcal{F}_t), 0 \leq t \leq T\), is the unobservable component, and \(K = (K_t, \mathcal{F}_t)\) is the observable component. Shareholders in our problem will face the optimal filtering problem for a partial observable process \((d, K)\) which will require the construction of an optimal mean square estimate of some \(\mathcal{F}_t\) measurable function \(\hat{d}_t\) of \((d, K)\) on the basis of observation results \(K_s, s < t\).

Assumption 2.2.4 I need a few assumptions to characterize \(\pi_t(\hat{d})\):

- \(M \hat{d}_t^2 < \infty\). In such a case, the optimal estimate is the a posteriori mean \(\pi_t(\hat{d}) = M(\hat{d}_t|\mathcal{F}_t^K)\).
- The process \(\hat{d}, K), t \leq T\), can be represented as follows:
  \[
  \hat{d}_t = \hat{d}_0 + \int_0^t D_s K_s ds + x_t, \quad (2.21)
  \]
  where \(X = (x_t, \mathcal{F}_t, t \leq T)\) is a martingale, and \(D = (D_t, \mathcal{F}_t), t \leq T\), is a random process with \(\int_0^T |D_t| ds < \infty\) almost surely.
- The observable process \(K = (K_t, \mathcal{F}_t)\) is an Itô process:
  \[
  K_t = K_0 + \int_0^t \Gamma_s(\omega) ds + \int_0^t B_s(K) dw_s, \quad (2.22)
  \]
  with \(\Gamma \equiv A_t K_t M_t^0 - (r + \delta)K_t - D_t K_t\) and \(B_s(K) d \tilde{w}_s \equiv \sigma_t(K) d w_s - x_t\).

Now, I am ready to use Theorem 8.1 from Lipster and Shiryae (2001) to write the optimal nonlinear filtering equation for our problem:
Lemma 2.2.6  Under correct assumptions, I have for each $t, 0 \leq t \leq T,$

$$\pi_t(\hat{d}_t) = \pi_0(\hat{d}) + \int_0^t \pi_s(D) ds + \int_0^t \left\{ \pi_s \left( \frac{d < x, \tilde{w} >}{dt} \right) + \left[ \pi_s(\hat{d}\Gamma) - \pi_s(\hat{d})\pi_s(\Gamma) \right] B_s^{-1}(K) \right\} d \tilde{w}_s,$$

(2.23)

where

$$\tilde{w}_s = \int_0^t \frac{dK_t - \pi_s(\Gamma) ds}{B_s(K)}$$

Shareholders update their optimal estimate $\pi_t(\hat{d}_t)$ of diversion using the equation above.

But management knows that too. Hence, the problem faced by management is:

$$\max_{\{x, \{D\}} d - \pi(\hat{d}),$$

(2.24)

i.e. management tries to maximize diversion while minimizing estimate of diversion at each instant.

To see the importance of optimal governance, let us first discuss the solution to the unconstrained problem posed above. I obtain a corner solution in such a case as the third and fourth term which are essentially covariance terms can be minimized by taking maximum feasible and opposing value of $x$ for every value of $\tilde{w}$.

The management will thus attempt to create a pessimistic picture of the firm - whenever there are profits, management will divert it to show minimum growth required for maintaining employment at the firm. Naturally, the question arises, what is the lower bound of performance the management can get away with? This is where corporate governance control are needed. The bounds provided in section 2.2.3 provide the minimum performance required by shareholders under the optimal policy outlined in section 2.2.3.
Optimal diversion by agent in equilibrium

Let us assume that the production shocks in equation 2.4 are i.i.d. with mean 0. Furthermore, if the agent has no advantage of information on the principal, the agent’s diversion policy will be independent of the shocks. However, let us now assume that the shock is composed of two factors:

\[ \tilde{w} = \tilde{v} + \tilde{u}, \] (2.25)

where \( \tilde{v} \) is observed by the agent before the shock is realized, giving her private information about production. This is a reasonable assumption as insiders have more information about a firm’s fortunes than the market. Kyle (1985) now provides inspiration as I am looking at a setting where an insider, who knows \( \tilde{v} \) beforehand, is trying to maximize payoff in presence of noise created by \( \tilde{u} \).

So far, in the game, the principal has chosen the contract and optimal policy. Now, the agent chooses the quantity of capital \( \tilde{d} \) she diverts, after observing \( \tilde{v} \), i.e. \( \tilde{d} = V(\tilde{v}) \), where \( V(\cdot) \) is a measurable function of \( \tilde{v} \). In the next step, the principal observes \( \tilde{v} + \tilde{u} \) jointly, and tries to make the best estimate \( \hat{d} \) of \( \tilde{d} \), i.e. \( \hat{d} = D(\tilde{v} + \tilde{u}) \), where \( D(\cdot) \) is a measurable function of \( \tilde{v} + \tilde{u} \).

The profits of the agent, denoted \( \tilde{\pi} \) are given by \( \tilde{\pi}(V, D) = \tilde{d}(V, D) - \hat{d}(V, D) \).

**Definition 2.2.7** An equilibrium is defined as a pair defined by \( V, D \) such that the agent maximizes cumulative diversion while minimizing the knowledge of the diversion process that the principal infers by observing net capital growth.

\[
\sup_{V, D} \int_0^T (\tilde{d} - \hat{d}) dt \quad (2.26)
\]

**Theorem 2.2.8** There exists a unique equilibrium in which actual diversion by the agent \( \tilde{d} \) and
diversion expected by the principal \( \hat{d} \) are linear functions. The equilibrium \( \hat{d} \) and \( \tilde{d} \) are given by:

\[
\hat{d} = \beta \tilde{v}, \quad \hat{d}(\tilde{d} + \tilde{u}) = \lambda(\tilde{d} + \tilde{u})
\] (2.27)

The above lemma postulates that a fraction of output proportional to the variance of the firm will be diverted by the management.

### 2.2.4 Optimal Leverage given Optimal Control by shareholders

As diversion by management can increase in case of positive shocks, management has an incentive in choosing higher leverage. The downside is that higher leverage may mean that in case of bad shocks, the firm might go bankrupt if the firm capital deteriorates.

In other words, a firm with capital \( K \) and choice of leverage \( L \), can see its capital deteriorate to \( \underline{K}(L) \) with a probability \( p(L) \) and increase to \( \overline{K}(L) \) with probability \( 1 - p(L) \), such that:

\[
\underline{K}(L) \leq K \leq \overline{K}(L)
\]

However, if firm capital falls below \( \underline{K}(L) \), then the shareholders of the firm will remove the management. Thus, management will always take on leverage, such that if a bad shock hits, and it is going to be removed, then the firm might as well go bankrupt. Debtholders, know that, and in equilibrium will not provide any leverage that in any state with measure allows firm capital to be lower than the performance bound allowed by the shareholders:

\[
\underline{K}(L) = K
\] (2.28)

The lower performance bound chosen by shareholders is a function of autonomy granted, thus giving us a relation between autonomy and leverage. Firms with higher autonomy, should show
higher leverage, and that is what we observe in practice.

In this section, I first introduce the optimal contract in a principal agent framework, where the insider agent has more information about firm performance than the outsider principal. There is heterogeneity among firms as firms face shocks that are not equal in variance - i.e. shocks are heteroscedastic over the distribution of firms. The agent also has a private diversion technology. The shareholders own the company and have incomplete information about the day to day workings, and the management get a certain wage to manage the assets of the firm. There are no creditors. Shareholders can affect management quality by voting on governance guidelines. On the other hand, management affects productivity through day to day operating decisions. A higher autonomy vested in the management allows it to contribute more to the firm, but also allows the management to divert more funds for private benefits. Tighter governance control has the opposite effect. I show that investors use corporate governance mechanisms to enforce second best control over insider management. Corporate governance controls implemented by investors depend upon the firm specific uncertainty, and hence we observe heterogeneity in managerial discretion.

2.3 Evidence

In this section we test the implications of the theory presented in section 3.2. Section 2.3.1 describes the main data used. Section 2.3.2 describes the reduced form that is used for estimation of the importance of various determinants of corporate governance. As a robustness check, section 2.3.3 includes block holders as suggested in Burkart, Gromb, and Panunzi (1997) as a determinant of corporate governance. The importance of firm characteristics remains robust to this treatment.
2.3.1 Data

The data for corporate governance is obtained from Investor Responsibility Research Center (IRRC) corporate governance data from Wharton Research Data Services (WRDS). The IRRC Governance database (also known as IRRC Takeover Defense database) provides information on takeover defense and other corporate governance provisions for major US firms. It also includes state takeover laws. IRRC indicates that they collect the data from 10-K, 10-Q, annual reports, and other documents filed with the SEC. This is supplemented with data obtained from other public sources. The IRRC Governance database in WRDS also includes the Gompers, Ishii and Metrick Governance Index (Gindex). I use this index as an instrument for corporate governance. Explanations and definitions of the variables in the dataset and details on the construction of Gindex are provided in Gompers, Ishii, and Metrick (2003). I match the Investor Responsibility Research Center (IRRC) corporate governance data to the Center for Research in Security Prices (CRSP) and Standard and Poors Compustat database. Our sample includes an average of 1,700 firms per year from September 1990 to December 2006.

2.3.2 Testing the Model

The model described above posits that a fraction of the variance of the output will be diverted by the management. This will still leave the remaining fraction of the volatility drag as part of the growth rate of the firm.

I regress Governance Index which instruments management autonomy on assets growth, and variance in assets. The equation used is as follows:

\[
\log M_t = \log \left[ \text{constant} + \nu \left( \frac{K_{t+1}}{K_t} \right) + \theta \sigma^2 K_t \right] - (\alpha - 1) \log K_t,
\]

which is rewritten from the law of motion of capital.
Equation 2.27 shows that in equilibrium when shareholders suffer from incomplete information about the output of the firm, management diverts capital in proportion to variance of assets. A higher variance, in our model, implies higher diversion, when the shareholders have incomplete information about the firm. To reign in the management, the shareholders reduce the autonomy so that despite of higher variation, less diversion happens. Hence, to support our hypothesis that shareholders choose management depending on the process of capital growth, I expect to find a negative coefficient to variance in the above regression.

\( \theta \) represents the coefficient of variance in the regressions below. I find \( \theta \) is negative and significant before and inclusive of 2000 (1980-2000). I see a progressive increase in the value of \( \theta \) over the years (1980-1990 and 1990-2000). In fact, in recent times, \( \theta \) is not differentiable from 0 at 95% level and the point estimate is positive (2000-2004). One possible explanation for this is the information disadvantage of shareholders has decreased, and hence they do not feel the need to control the autonomy of the firm to avoid diversion. This can be an evidence in favor of Sarbanes-Oxley (2002), or parts of it. Laws that encourage disclosure to increase transparency can help shareholders feel more confident to provide the firm larger autonomy, which, in our model, leads to higher returns through equation 2.4. However, even before the law was enacted, as per our model, US Capital Markets were becoming more transparent progressively.

The coefficient of growth \( \nu \) is consistently negative - showing that firms with higher growth are more democratic. As firms grow faster, shareholders are less pressed to give the management autonomy to increase growth rate - since that would mean larger bounds on diversion on the flip side. \( \alpha \), the exponent of capital in our model, is consistently less than 1 - which shows that as firm size \( K_t \) increases, firms become more autocratic. This is consistent with the literature on management entrenchment and empire building.\(^6\)

\(^5\)under the assumption that the composition of firms in the cross section has remained the same
\(^6\)Representative literature includes seminal work by Shleifer and Vishny (1989) and Scharfstein and Stein (2000)
2.3.3 Relation between Blockholders and Autonomy

As an alternate specification, let production function of the firm be:

\[ dY_t = K_t^{\alpha} M_t^{\eta} B_t^{\beta} dt + \sigma Y dw, \]  

(2.30)

where \( B \) represents the percentage of shares held by outside blockholders - another determinant of control shareholders have on management. I regress Governance Index now on assets growth, and variance in assets and percentage shares held by outside blockholders. The equation used is as follows:

\[ \log M_t = \log \left[ \text{constant} + \nu \left( \frac{K_{t+1}}{K_t} \right) + \theta \sigma^2 K_t \right] - (\alpha - 1) \log K_t + \beta \log B_t. \]  

(2.31)

Table 2.5 shows the results of the regression for various time periods. As in Table 2.1, \( \theta \) represents the coefficient of variance in the regressions below. I again find \( \theta \) is negative and significant before and inclusive of 2000 (1980-2000) - as our model predicts. The progressive increase in the point estimate of \( \theta \) over the years (1980-1990 and 1990-2000) is seen here as well. In the period since 2000, \( \theta \) is not differentiable from 0 at 95% level in this alternative model specification as well. This evidence gives further credence to our hypothesis that information disadvantage of shareholders has decreased, and they feel less pressed to control the autonomy of the firm to avoid diversion. Introduction of shares held by blockholders does not change the coefficient of growth \( \nu \). It is consistently negative in this model as well - showing that firms with higher growth are more democratic. Similar to table 2.1, \( \alpha \), the exponent of capital, is consistently less than 1 - which shows that as firm size \( K_t \) increases, firms become more autocratic.
2.3.4 Further Robustness Checks

I sort of 1500 largest firms for which I have governance data into 4 bins according to governance index and sort each bin further into 5 quintiles according to size. Then I tabulate the mean assets, mean growth rate and variance of the median firm for each of the 20 bins thus created. Table 2.3 and 2.4 shows the results for periods of 1980-2000 and 2000-2004 respectively.

Figure 2.5 shows the same results visually. It is clear on inspection that as management autonomy increases, variance decreases in both periods. However, it the gradient of descent is much steeper in the earlier period of 1980-2000: this endorses the results of the regressions about the significant negative $\theta$ values before 2000 and the almost 0 value of $\theta$ after 2000. The trends in growth are descending but much less so, as was predicted by the small values of $\nu$ in the regression. Finally, the positive correlation between management autonomy and firm size is shown in the rightmost subplot.

2.3.5 Relation between Leverage and Autonomy

Firms with higher autonomy, should show higher leverage. This is because management will always take on leverage, such that if a bad shock hits, and it is going to be removed, then as far as the manager is concerned, the firm might as well go bankrupt. Debt holders, know that, and in equilibrium will not provide any leverage that in any state with measure allows firm capital to be lower than the performance bound allowed by the shareholders: Lower bound on capital by debt holders = Lower bound on firm capital by shareholders. The lower performance bound chosen by shareholders is a function of autonomy granted, thus giving us a relation between autonomy and leverage. Firms with higher autonomy, should show higher leverage, and that is what is observed in the table below. I regress firm leverage (Long term Debt/Assets) on G Index
and moments of Tobin’s Q:

\[
\text{Leverage} = \alpha_1 \text{G Index} + \alpha_2 \text{Variance in Firm Q} + \alpha_3 Q + \alpha_4 \text{Mean growth in Q} + \alpha_5 \text{Time Dummy}
\]

(2.32)

Table 2.6 reports the results.

I find that as autonomy increases, debt holders also let the firm take on large debt. I also observe the known fact that growth firms that have larger Tobin’s Q take on less debt - this is because such firms have less collateralizable assets. In other words, it is due to equity being cheaper in comparison to debt.

2.3.6 Does autonomy change with firm characteristics? : A Causality Test

I have proposed in this paper that shareholders choose autonomy as a response to firm characteristics. Thus, when Tobin’s Q changes enough, if shareholders are really responding to firm characteristics, then management autonomy should change as well.

In table 2.7, The unrestricted model tests whether the G-Index values of last two years and Tobin’s Q of last two years Granger causes present G-Index. The restricted model tests causality against only G-Index values of last two years. The results test compare the errors from the two models and show that changes in Q causes changes in autonomy. The null is not rejected. This supports our theory that investors choose autonomy as a response to change in firm characteristics.

2.3.7 Dividend Smoothing

The accounting literature has built up a strong case that managers engage in income smoothing. This work has contributes to that literature. In Fudenberg, Drew and Tirole, Jean (1995), income smoothing arises due to the following features. First, the manager enjoys a private benefit from
running the firm - which they call incumbency rent. Second, the firm cannot commit itself to a long-term incentive contract. The third key feature is that recent income observations are more informative than older ones about the future prospects. In Trueman and Titman (1988) the managers act to defend the shareholders' interests rather than their own. These papers also specify a preference for smoother income on the part of the external capital market. In Lambert (1984), risk-averse managers without access to capital markets want to smooth the firm's reported income in order to provide themselves with insurance. Except Fudenberg, Drew and Tirole, Jean (1995), the other papers do not consider optimal contracts, and it is not clear whether income smoothing could be eliminated by a change in the managerial compensation scheme.

Our model also generates income smoothing. I have shown that with higher variance, management autonomy is reduced in our model. Higher management autonomy allows higher private diversion. Now, if shareholders do not know variance \( \sigma^2_Y \) of the firm, it is in the interest of the manager to have a lower observable variance \( \hat{\sigma}^2_Y \), so that shareholders respond with higher autonomy to the management. Of course, in equilibrium, shareholders will converge to the right value. However, firm characteristics are ever changing and management tenures are short, hence the updating of beliefs regarding variance may not be fast enough - thus yielding more than nominal transitory gains for management while shareholders learn.

In this section, I have shown that firm specific exogenous performance uncertainty plays an important role in the choice of managerial discretion allowed by investors. Equityholders in firms with higher exogenous variance attempt to reduce the information disadvantage they face by reducing autonomy of management. Thus, in practice, we observe a range of governance control that is negatively correlated to the variance of firm output.


2.4 Conclusion

I show that for large enough firms, optimal contracts become impractical, and corporate governance is the only answer. Investors use corporate governance mechanisms to enforce second best control over insider management. Corporate governance controls implemented by investors depend upon the firm specific uncertainty, and hence we observe heterogeneity in managerial discretion. I find that autonomy of management is positively correlated with size of the firm, but controlling for autonomy, we see a negative correlation between variance and autonomy. I find evidence indicating that over time, the information asymmetry between shareholders and management has decreased. Furthermore, since the implementation of Sarbanes-Oxley Act, the choice of corporate governance mechanisms are statistically independent of this information asymmetry.

Management decisions to take or reject risky projects depends on the contract it faces, and shareholders respond to management behavior by choosing the level of governance to maximize the value of their interests. When management chooses to take new projects with higher asset variance, it is optimal for the shareholders to rein in such a management with higher shareholder rights and less autonomy - if allowed by adjustment costs. On the other hand, if volatility of capital growth of the firm is low, it is optimal for the shareholders to allow such a management more freedom, so as to encourage the latter to take projects with higher asset variance. This work argues that governance policy and management investment policy are jointly determined, compared to the common consensus in the literature that management responds to corporate governance chosen by shareholders but not vice-versa. Having an equilibrium as described here provides an explanation of the corporate governance heterogeneity that exists in practice.
The figure shows us the heterogeneity in corporate governance as measured by G-Index that exists in practice. We argue that the governance policy and management investment policy are jointly determined, after taking exogenous firm variance into account. This is in contrast to the common consensus in the literature that management responds to corporate governance chosen by shareholders but not vice-versa. Having an equilibrium of our form allows us to explain the corporate governance heterogeneity observed above.
Figure 2.2: Relative Performance of Growth/Value Firms and Democracy/Dictatorship Firms

This figures shows the relative performance of growth firms against value firms and democracy firms compared to dictatorship firms.
Figure 2.3: Tradeoff between higher autonomy and higher effort

The figure shows the relative trade-off between agent’s talent $Q$ and autonomy $M$. At the margin, the net increase in capital growth obtained by increasing $Q$ after paying the agent's effort cost, and the relative increase in capital growth by increasing the management capital $M$ after paying the increased amount the agent can divert have a ratio.
Figure 2.4: Optimal Control Policy and Region of Inaction

This figure shows that when the marginal of $\phi$, which is part of the value function of the principal, with respect to autonomy $M$ crosses a threshold that depends on rate of growth of capital and transaction cost $\lambda$, the principal needs to increase or decrease autonomy of the management to stay inside the threshold. In other words, we have a bang-bang equilibrium.
Figure 2.5: Size, Growth and Variance of the Median Firm sorted by Governance Index: The top figure is pre 2000 and the Bottom is post 2000

e sort 1500 largest firms for which we have governance data into 4 bins according to governance index and sort each bin further into 5 quintiles according to size. Then we tabulate the mean assets, mean growth rate and variance of the median firm for each of the 20 bins thus created. The top figure is pre 2000 and the Bottom is post 2000. It is clear on inspection that as management autonomy increases, variance decreases in both periods. However, the gradient of descent is much steeper in the earlier period of 1990-2000: this endorses the results of the regressions about the significant negative $\theta$ (coefficient of variance) values before 2000 and the almost 0 value after 2000. Notice, the largest firms do not obey this negative relationship between variance and autonomy. Our story is based upon incomplete information about the firm - and we know that the largest firms face the most scrutiny by media and analysts. Hence, the fact that they do not obey the relationship, in fact, supports our theory even more. The trends in growth are descending but much less so, as was predicted by the small values of $\nu$ in the regression (coefficient of asset growth). Finally, the positive correlation between management autonomy and firm size is shown in the rightmost subplot.
Table 2.1: Regression of Management Autonomy on Firm Characteristics

I use Non Linear Least Squares using Maximum Likelihood as the estimation procedure. Governance Index instruments management autonomy on the left hand and assets growth, and variance in assets on the right. The equation used is as follows:

\[ \log M(t) = \log \left[ \text{constant} + \nu \left( \frac{K(t+1)}{K(t)} \right) + \theta \sigma(K)^2 \right] - (\alpha - 1) \log K(t), \]

which is rewritten from the law of motion of capital. In equilibrium when shareholders suffer from incomplete information about the output of the firm, management diverts capital in proportion to variance of assets. A higher variance, in our model, implies higher diversion, when the shareholders have incomplete information about the firm. To reign in the management, the shareholders reduce the autonomy so that inspite of higher variation, less diversion happens. In panel A, we show supporting evidence for the hypothesis that shareholders choose management depending on the process of capital growth. we expect to find a negative coefficient to variance in the above regression. \( \theta \) represents the coefficient of variance in the table I. We find \( \theta \) is negative and significant before and inclusive of 2000 (1990-2000). We see a progressive increase in the value of \( \theta \) over the years. In fact, in recent times, \( \theta \) is not differentiable from 0 at 95% level and the point estimate is positive (2000-2006). One possible explanation for this is the information disadvantage of shareholders has decreased, and hence they do not feel the need to control the autonomy of the firm to avoid diversion. This can be an evidence in favor of Sarbanes-Oxley (2002), or parts of it. Laws that encourage disclosure to increase transparency can help shareholders feel more confident to provide the firm larger autonomy, which, in our model, leads to higher returns. However, even before the law was enacted, as per our model, US Capital Markets were becoming more transparent progressively. The coefficient of growth \( \nu \) is consistently negative - showing that firms with higher growth are more democratic. As firms grow faster, shareholders are less pressured to give the management autonomy to increase growth rate - since that would mean larger bounds on diversion on the flip side. \( \alpha \), the exponent of capital in our model, is consistently less than 1 - which shows that as firm size \( K(t) \) increases, firms become more autocratic. This is consistent with the literature on management entrenchment and empire building.

<table>
<thead>
<tr>
<th>Time periods</th>
<th>( \alpha )</th>
<th>( \nu )</th>
<th>( \theta )</th>
<th>Pr &gt; F Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990 – 2006</td>
<td>0.9686</td>
<td>-0.0005</td>
<td>-0.2816</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.00123)</td>
<td>(0.000258)</td>
<td>(0.0387)</td>
<td></td>
</tr>
<tr>
<td>1990 – 2000</td>
<td>0.9638</td>
<td>-0.0005</td>
<td>-0.6886</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.00171)</td>
<td>(0.000216)</td>
<td>(0.0530)</td>
<td></td>
</tr>
<tr>
<td>2000 – 2006</td>
<td>0.9737</td>
<td>-0.0011</td>
<td>0.1636</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.00209)</td>
<td>(0.000471)</td>
<td>(0.1124)</td>
<td></td>
</tr>
</tbody>
</table>
I regress variance in firm's Tobin's Q scaled by book value of the firm on governance index, leverage and first two moments of Tobin's Q (scaled again by book value of the firm) on the right. The equation used is as follows:

\[
\text{Variance in Firm's Q} = \alpha_1 \text{G Index} + \alpha_2 \text{Leverage} + \alpha_3 \text{Mean Q} \times \text{Book Value} \\
+ \alpha_4 \text{Mean Growth in Q} \times \text{Book Value} + \alpha_5 \text{Time Dummy},
\]

The results again show a negative relation between observed variance in Tobin's Q and governance in the earlier period, and a similar reversal in trend in last few years as observed in Panel A. As explained before, I consider it evidence in favor of my hypothesis that information disadvantage of shareholders has decreased, and hence they do not feel the need to control the autonomy of the firm to avoid diversion. In fact, they allow more autonomy to enjoy higher productivity of management which comes with more autonomy. Thus, the results are robust to the choice of performance measure - Tobin's Q, Firm Assets or other measures.

<table>
<thead>
<tr>
<th>Time periods</th>
<th>G Index</th>
<th>Leverage</th>
<th>Mean Q × Book Value</th>
<th>Mean Growth of Q × Book Value</th>
<th>Time Dummy</th>
<th>R-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980 – 2006</td>
<td>69.51</td>
<td>755.83</td>
<td>0.7411</td>
<td>128.38</td>
<td>3.93</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>(18.80)</td>
<td>(260.09)</td>
<td>(0.0014)</td>
<td>(7.17)</td>
<td>(9.31)</td>
<td></td>
</tr>
<tr>
<td>2000 – 2006</td>
<td>81.96</td>
<td>737.46</td>
<td>0.2018</td>
<td>465.7</td>
<td>8.79</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>(18.78)</td>
<td>(231.03)</td>
<td>(0.0008)</td>
<td>(2.39)</td>
<td>(25.17)</td>
<td></td>
</tr>
<tr>
<td>1990 – 2000</td>
<td>112.71</td>
<td>1056.91</td>
<td>0.6192</td>
<td>60.10</td>
<td>33.53</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(20.44)</td>
<td>(299.13)</td>
<td>(0.0022)</td>
<td>(4.21)</td>
<td>(18.55)</td>
<td></td>
</tr>
</tbody>
</table>
We sort 1500 largest firms for which we have governance data into 4 bins according to governance index and sort each bin further into 5 quintiles according to size. Then we tabulate the mean assets, mean growth rate and variance of the median firm for each of the 20 bins thus created. The top row in each cell is asset variance, middle row is asset growth and bottom row is mean assets of the median firm. This table shows the results for periods of 1990-2000 and 2000-2006. It is clear that as management autonomy increases, variance decreases in both periods. However, the gradient of descent is much steeper in the earlier period of 1990-2000: this endorses the results of the regressions about the significant negative $\theta$ (coefficient of variance) values before 2000 and the almost 0 value after 2000. The trends in growth are descending but much less so, as was predicted by the small values of $\nu$ in the regression (coefficient of asset growth). Finally, the positive correlation between management autonomy and firm size is shown in the third number in each cell - suggesting management entrenchment.

<table>
<thead>
<tr>
<th>Governance Index</th>
<th>0 – 5</th>
<th>5 – 9</th>
<th>10 – 13</th>
<th>14 –</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom 20%</td>
<td>-2.4362</td>
<td>-2.4267</td>
<td>-2.6206</td>
<td>-3.2948</td>
</tr>
<tr>
<td></td>
<td>0.0109</td>
<td>-0.0031</td>
<td>0.0054</td>
<td>0.0164</td>
</tr>
<tr>
<td></td>
<td>4.4503</td>
<td>4.8173</td>
<td>5.1151</td>
<td>5.3384</td>
</tr>
<tr>
<td>20 – 40%</td>
<td>-1.6411</td>
<td>-2.1619</td>
<td>-2.2013</td>
<td>-1.5640</td>
</tr>
<tr>
<td></td>
<td>0.1450</td>
<td>0.0676</td>
<td>0.0877</td>
<td>0.0979</td>
</tr>
<tr>
<td></td>
<td>5.1048</td>
<td>5.4919</td>
<td>5.9425</td>
<td>6.1026</td>
</tr>
<tr>
<td>40 – 60%</td>
<td>-1.6686</td>
<td>-1.7908</td>
<td>-2.0913</td>
<td>-2.2375</td>
</tr>
<tr>
<td></td>
<td>0.1469</td>
<td>0.1160</td>
<td>0.1183</td>
<td>0.0816</td>
</tr>
<tr>
<td></td>
<td>5.6585</td>
<td>6.0488</td>
<td>6.8170</td>
<td>7.3223</td>
</tr>
<tr>
<td>60 – 80%</td>
<td>-2.2948</td>
<td>-1.9256</td>
<td>-2.3765</td>
<td>-2.2386</td>
</tr>
<tr>
<td></td>
<td>0.1261</td>
<td>0.1536</td>
<td>0.1308</td>
<td>0.1928</td>
</tr>
<tr>
<td></td>
<td>6.7898</td>
<td>6.9606</td>
<td>7.8828</td>
<td>7.7408</td>
</tr>
<tr>
<td>Top 20%</td>
<td>-1.7973</td>
<td>-2.0913</td>
<td>-2.0444</td>
<td>-1.8321</td>
</tr>
<tr>
<td></td>
<td>0.3425</td>
<td>0.2080</td>
<td>0.1651</td>
<td>0.5225</td>
</tr>
<tr>
<td></td>
<td>8.2637</td>
<td>8.6567</td>
<td>8.8859</td>
<td>8.9969</td>
</tr>
</tbody>
</table>
Table 2.4: Relation between Firm Assets Variance, Growth and Median and Governance Index (Post 2000)
Top row in each cell is Asset Variance, Middle row is Asset Growth and Bottom row is Mean assets of the Median firm

<table>
<thead>
<tr>
<th>Asset Percentile</th>
<th>0 – 5</th>
<th>5 – 9</th>
<th>10 – 13</th>
<th>14 –</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom 20%</td>
<td>-2.9104</td>
<td>-2.9744</td>
<td>-3.2059</td>
<td>-4.6560</td>
</tr>
<tr>
<td></td>
<td>-0.0974</td>
<td>-0.1323</td>
<td>-0.0645</td>
<td>-0.0662</td>
</tr>
<tr>
<td></td>
<td>5.5031</td>
<td>5.5481</td>
<td>5.8804</td>
<td>6.1493</td>
</tr>
<tr>
<td>20 – 40%</td>
<td>-3.4290</td>
<td>-3.0446</td>
<td>-3.5671</td>
<td>-2.8385</td>
</tr>
<tr>
<td></td>
<td>-0.0107</td>
<td>-0.0246</td>
<td>0.0211</td>
<td>0.1029</td>
</tr>
<tr>
<td></td>
<td>6.4551</td>
<td>6.4127</td>
<td>7.0422</td>
<td>7.3560</td>
</tr>
<tr>
<td>40 – 60%</td>
<td>-3.1279</td>
<td>-3.1946</td>
<td>-3.6780</td>
<td>-3.2500</td>
</tr>
<tr>
<td></td>
<td>0.0241</td>
<td>0.0745</td>
<td>0.0900</td>
<td>0.2031</td>
</tr>
<tr>
<td></td>
<td>7.0482</td>
<td>7.1293</td>
<td>7.8822</td>
<td>8.1964</td>
</tr>
<tr>
<td></td>
<td>0.1617</td>
<td>0.1533</td>
<td>0.1753</td>
<td>0.3753</td>
</tr>
<tr>
<td></td>
<td>7.8166</td>
<td>8.0121</td>
<td>8.8251</td>
<td>8.7676</td>
</tr>
<tr>
<td>Top 20%</td>
<td>-3.2819</td>
<td>-3.3592</td>
<td>-3.8812</td>
<td>-3.0484</td>
</tr>
<tr>
<td></td>
<td>1.6314</td>
<td>0.4047</td>
<td>0.4604</td>
<td>0.9035</td>
</tr>
</tbody>
</table>
Table 2.5: Relation between Firm Characteristics, Outside Block holders and Corporate Governance

As an alternate specification, we let production function of the firm be:

\[ dY(t) = K(t)^{\alpha}M^\nu B^\theta d t + \sigma(Y)d w, \]

where B represents the percentage of shares held by outside block holders - another determinant of control shareholders have on management, raised to a constant \( \beta \). We regress Governance Index now on assets growth, and variance in assets and percentage shares held by outside block holders. The equation used is as follows:

\[ \log M(t) = \log \left[ \text{constant} + \nu \left( \frac{K(t + 1)}{K(t)} + \theta \sigma(K)^2 \right) - (\alpha - 1) \log K(t) + \beta \log B_t. \]

This table shows the results of the regression for various time periods. As in panel A of Table I, \( \theta \) represents the coefficient of variance in the regressions below. We again find \( \theta \) is negative and significant before and inclusive of 2000 (1990-2000) - as our model predicts. The increase in the point estimate of \( \theta \) over the years is seen here as well. In the period since 2000, \( \theta \) is not differentiable from 0 at 95% level in this alternative model specification as well. This evidence gives further credence to our hypothesis that information disadvantage of shareholders has decreased, and they feel less pressed to control the autonomy of the firm to avoid diversion. Introduction of shares held by blockholders does not change the coefficient of growth \( \nu \). It is consistently negative in this model as well - showing that firms with higher growth are more democratic. \( \alpha \), the exponent of capital, is consistently less than 1 - which shows that as firm size \( K(t) \) increases, firms become more autocratic.

<table>
<thead>
<tr>
<th>Time periods</th>
<th>( \alpha )</th>
<th>( \nu )</th>
<th>( \theta )</th>
<th>( \beta )</th>
<th>Pr &gt; F Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990 – 2006</td>
<td>0.9726</td>
<td>-0.0005</td>
<td>-0.4466</td>
<td>0.00200</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.00145)</td>
<td>(0.000281)</td>
<td>(0.0461)</td>
<td>0.000181</td>
<td></td>
</tr>
<tr>
<td>1990 – 2000</td>
<td>0.9672</td>
<td>-0.0005</td>
<td>-0.8736</td>
<td>0.00186</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.00190)</td>
<td>(0.000233)</td>
<td>(0.0587)</td>
<td>(0.000231)</td>
<td></td>
</tr>
<tr>
<td>2000 – 2004</td>
<td>0.9795</td>
<td>-0.0010</td>
<td>-0.3056</td>
<td>0.00291</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.00275)</td>
<td>(0.000659)</td>
<td>(0.1886)</td>
<td>(0.000351)</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.6: Relation between Leverage and Corporate Governance

Management will always take on leverage, such that if a bad shock hits, and it is going to be removed, then as far as the manager is concerned, the firm might as well go bankrupt. Debt holders, know that, and in equilibrium will not provide any leverage that in any state with measure allows firm capital to be lower than the performance bound allowed by the shareholders: Lower bound on capital by debt holders = Lower bound on firm capital by shareholders. The lower performance bound chosen by shareholders is a function of autonomy granted, thus giving us a relation between autonomy and leverage. Firms with higher autonomy, should show higher leverage, and that is what we observe in the table below. We regress firm leverage (Long term Debt/Assets) on G Index and moments of Tobin’s Q.

\[
\text{Leverage} = \alpha_1 \text{G Index} + \alpha_2 \text{Variance in Firm Q} + \alpha_3 \text{Q} + \alpha_4 \text{Mean growth in Q} + \alpha_5 \text{Time Dummy}
\]

we find that as autonomy increases, debt holders also let the firm take on large debt. We also observe the known fact that growth firms that have larger Tobin’s Q, take on less debt - this is because such firms have less collateralizable assets. In other words, it is due to equity being cheaper in comparison to debt.

<table>
<thead>
<tr>
<th>Time periods</th>
<th>G Index</th>
<th>Variance in Q</th>
<th>Q</th>
<th>Mean Growth of Q</th>
<th>Time Dummy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990 - 2006</td>
<td>0.0031</td>
<td>-0.0074</td>
<td>-0.0056</td>
<td>-0.0084</td>
<td>-0.00003</td>
</tr>
<tr>
<td></td>
<td>(0.00041)</td>
<td>(0.00061)</td>
<td>(0.00049)</td>
<td>(0.0053)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>2000 - 2006</td>
<td>0.0046</td>
<td>-0.0040</td>
<td>-0.0082</td>
<td>-0.0068</td>
<td>-0.0019</td>
</tr>
<tr>
<td></td>
<td>(0.00076)</td>
<td>(0.0027)</td>
<td>(0.0015)</td>
<td>(0.0098)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>1990 - 2000</td>
<td>0.0034</td>
<td>-0.0031</td>
<td>-0.0060</td>
<td>-0.0062</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>(0.00054)</td>
<td>(0.00076)</td>
<td>(0.00066)</td>
<td>(0.0021)</td>
<td>(0.00049)</td>
</tr>
</tbody>
</table>
Table 2.7: Granger Test: Does change in Firm characteristics cause change in Autonomy?

We have proposed in this paper that shareholders choose autonomy as a response to firm characteristics. Thus, when Tobin's Q changes enough, if shareholders are really responding to firm characteristics, then management autonomy should change as well. The unrestricted model tests whether the G-Index values of last two years and Tobin's Q of last two years Granger causes present G-Index. The restricted model tests causality against only G-Index values of last two years. The results test compare the errors from the two models and show that changes in Q causes changes in autonomy. The Granger test shows that the null hypothesis that states that changes in Q cause changes in Autonomy is not rejected. This supports our theory that investors choose autonomy as a response to change in firm characteristics.

<table>
<thead>
<tr>
<th>Time periods</th>
<th>F-Test</th>
<th>Chi Test</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990 – 2006</td>
<td>18682.20</td>
<td>37369.47</td>
<td>0</td>
</tr>
<tr>
<td>1990 – 2000</td>
<td>5934.39</td>
<td>11871.66</td>
<td>0</td>
</tr>
<tr>
<td>2000 – 2006</td>
<td>450.21</td>
<td>901.25</td>
<td>0</td>
</tr>
</tbody>
</table>
Chapter 3

Effort, Risk and Walkaway Under High Water Mark Contracts

With Sugata Ray

3.1 Introduction

As more and more funds found themselves under their high water marks (HWMs) though the course of the recent economic crisis, investors and the financial press increased their scrutiny of the impact of a distant high water mark (HWM) on fund manager incentives. Most hedge funds have a management fee (generally 2%) and a performance fee (generally 20%) with a high water mark that stipulates the performance fee is paid only on “new” profits. In other words, if a fund with a HWM stipulation has made a loss, it must recoup the loss before earning the 20% performance fee. This leads to a distortion of incentives when funds are below their HWMs. In particular, fund managers far below their HWMs may increase portfolio risk and lose focus on alpha generation, which in turn may lead to investors pulling out of such funds. With over two trillion dollars under management in hedge funds and much of it still under their respective HWMs, both fund managers and investors are currently grappling with this very problem.

The central role of the high water mark feature in driving fund managers’ and investors’
behavior is readily apparent in the following anecdote. Following the beginnings of the credit-related market correction in August 2007, a well known hedge fund at a bulge bracket bank, which had lost a lot of money and was reportedly more than 30% below its high water mark, offered a ‘sale’ on new fund inflows. Fees charged on new money would consist solely of a 10% incentive on any profits generated, and the management fee would be waived. That is, the fund adopted a “0/10” structure rather than the standard “2/20.” The implications of such a move were manifold. Original monies in the fund were sufficiently far away from their high water mark that fund employees, discouraged about prospects of ever hitting the high water mark (and earning substantial bonuses), considered leaving and joining other funds. As a result, original investors in the fund questioned whether the fund would be able to maintain its team through the crisis and considered withdrawing funds. Investors might also have been concerned that managers would take inordinate risks in a desperate attempt to generate returns above the high water mark. The infusion of new capital, fresh and untethered to the old high water mark, alleviated these concerns. Employees would stand to earn incentive fees (and thus bonuses) on this new money immediately, although the fees would accrue at the rate of 10% rather than the full 20%. The original investors were reassured that the fund would be able to maintain its team and did not withdraw their funds.

We focus this study on this intricate set of risk, effort and walkaway decisions driven by the high water mark (HWM) feature of the hedge fund contract. Modeling a simple one period principal-agent relationship governed by such a contract, we derive the empirical predictions in terms of risk choices, effort choices and walkaway behavior of the parties. Extending the model to infinite periods, we highlight the disciplining role of investor walkaway on manager behavior. We calibrate parameters in the infinite period model using observed hedge fund return data and

1 Although the fund staved off immediate closure and the cash inflows from the sale led to a positive bounce, the hedge fund eventually lost money over the rest of the year as the credit crisis worsened, ending the year with a 38% loss.

90
use the calibrated model to posit welfare enhancing modifications to the current ‘2/20’ contract.

Although the model itself is fairly complex due to the optionality embedded in the hedge fund contract, the empirical implications of the model resonate with the intuition that the contract functions as if the fund manager simply has a call option on the returns of the fund. The strike price of this option is the HWM and the return required to hit the HWM (RR) is a measure of the moneyness of this option (the higher RR, the further the option is out of the money). In particular, we predict differences in both fund manager and investor behavior depending on how close the manager’s option is to being in the money. We expect the following: (i) The lower the return required to hit the HWM (RR), the more likely the fund manager is to expend costly effort, and the higher the subsequent period returns will be. (ii) The manager of a fund far below from the HWM is likely to increase the risk of his portfolio in the hopes of breaking the HWM barrier again and earning performance fee based bonuses. (iii) Once a fund falls to a certain point below the HWM, the danger of walkaway increases, as the principal may worry that the agent is no longer expending an appropriate level of effort and is taking too much risk. These empirical predictions are supported by empirical findings from Ray (2009).

We calibrate the free parameters of the model to observed hedge fund return data. Once calibrated, moments generated from simulating an economy of model hedge funds an investors closely match actual observed moments. Using the calibrated model, we examine various other permutations of a “x/y%” management/incentive fee contract in search of a contract that provides better incentives for the fund manager without raising the expected fees paid by the investor. We find that substituting an increased management fee in lieu of incentive fees (e.g., a “2.5/10” contract rather than the standard “2/20” contract) leads to improved outcomes for both the investor and the fund manager.

For funds near the HWM, this is achieved through improved risk sharing. In addition to improved risk sharing, for funds further away from the HWM, the increased continuation value of
such a contract, combined with the enhanced ability to maintain a costly alpha-generating team through periods of poor returns, leads to more investor-friendly behavior by the fund manager and a correspondingly lower incidence of walkaway by the investor.

We also find that a constantly renegotiated HWM improves manager welfare at the expense of investor welfare. Such a renegotiation is akin to the manager always having an at the money option and although it mitigates some of the incentive problems in the original contract, it generates a fresh set of perverse incentives for fund managers.

While we admit that the welfare analysis is largely a thought experiment, as changing the contract form will likely change other calibrated parameters that were held constant in the welfare analysis, the results do highlight potential implications of changing the contract terms as well as the effects of renegotiation. These implications suggest numerous practical applications: (1) Portfolio allocation decisions by investors and fund-of-funds would find direct use for these findings in optimizing their portfolios. In particular, investors should carefully consider how far they should let their funds drop below the HWM before renegotiating or walking away entirely. (2) Additionally, the welfare analysis suggests higher management fees and lower incentive fees (e.g., the “2.5/10” contract above) may lead to improved outcomes for both investor and manager. (3) These findings also have implications for hedge fund marketing strategies. The anecdote above, where the fund had the ‘0/10’ sale to mitigate many of these concerns, exemplifies this. (4) A clearer level of disclosure about a fund’s high water mark(s), might also generate value for current and potential investors alike.

Extant literature related to this study includes Goetzmann, Ingersoll, and Ross (2003), a seminal paper in this field which examines the unique, high watermark (HWM) structure of the hedge fund compensation contract and computes the alpha-generation potential necessary to justify paying a fund manager according to such a contract. Hodder and Jackwerth (2007) conducts a theoretical study of the impact of the HMW in a finite time horizon setting. Panageas
and Westerfield (2008) presents and solves a theoretical model where an exogenously set fund closure point (bankruptcy) leads to fund managers moderating risk taking behavior despite the optionality in the contract. Kerr Christoffersen and Musto (2008) considers the impact of the HWM in a two period model with Bayesian learning and show how varying levels of performance, in conjunction with the HWM, affect fund flows, fund closure and alpha generation by the hedge fund. Metrick and Yasuda (2007) looks at the empirical differences between the “2/20” fee structures of hedge funds, venture capital firms and private equity shops.

Our paper contributes to this body of research through two extensions. The first contribution is to theory; our model is the first to formalize endogenous investor walkaway in the context of a hedge fund HWM contract. This feature has been explicitly suggested in Panageas and Westerfield (2008). Using this extension, we are able to posit clear empirical predictions of the impact of the HWM on walkaway. We are also able to calibrate our model to match hedge fund walkaway rates, among other moments, and obtain economically meaningful model parameters, such as the manager’s risk aversion and the fraction of investors who monitor their investments. The second contribution is to the quantification of welfare implications of changing the contract. Using the calibrated model, we calculate investor expected returns and fund manager expected utility under different contract forms. This also allows us to search for Pareto improving contracts in our calibrated model and examine policy functions under such contracts to gain intuition on what, specifically, drives the Pareto improvement.

The rest of this paper is divided into four sections. Section 3.2 describes the model and presents the solutions to the model (the solutions to the model include optimal effort and risk choices by the manager and optimal walkaway decisions for the investor). Section 3.3 calibrates the model to observed empirical data. Section 3.4 uses the calibrated model to examine the welfare implications of changing the hedge fund manager compensation contract and section 3.5 concludes.
3.2 Model

We model a risk averse hedge fund manager (the agent) and a risk neutral principal in a hedge fund contract, with management fees, incentive fees, and a high water mark driving manager actions and investment decisions. We begin by examining a one period model under such a setup and use this to gain insight into the manager’s decision making. We then extend the model to infinite periods and incorporate the investor’s walkaway decision (Section 3.2.3).

3.2.1 One period model without continuation value

In this single period model, we have a hedge fund manager who has been given a sum of money, \( v_0 \), by the principal at time \( t_0 \). The manager will be compensated at \( t_1 \) according to the returns to the fund. The fund starts at the high water mark and all returns will be subject to the incentive fee over the \( t_0 \rightarrow t_1 \) period.

\[
w_1 = k v_0 + s v_0 \max(\tilde{r}, 0),
\]

where

\[
\tilde{r} = a + \sigma \tilde{\epsilon}
\]

\[
\tilde{\epsilon} \sim U(-1, 1)
\]

\( w_1 \) denotes the compensation of the hedge fund manager at \( t_1 \). Thus, in a standard 2/20 contract, \( k = 2\% \) and \( s = 20\% \). The \( \tilde{r} \) is the return before fees in the next period, \( a \) and \( \sigma \) are effort and risk choices made by the manager. Effort is costly to produce, and risk can be increased costlessly. The manager has CRRA utility and strives to maximize his utility from \( t_1 \) wealth. The shock is uniform over the \([-1, 1]\) interval. The manager’s problem can be represented as follows:

\[
V_m = \max_{a, \sigma} \mathbb{E} \beta \left[ u(k v_0 + s v_0 \max(0, \tilde{r}) - \frac{v_0}{2} c_a \sigma^2) \right]
\]
where

\[ u(x) = \frac{x^{1-\gamma}}{1-\gamma} \]  

(3.3)

\( V_m \) is the value of the contract to the agent, which he seeks to maximize. \( u(x) \) represents the agent’s utility function and \( \beta \) and \( \gamma \) are the agent’s discount rate and coefficient of risk aversion, respectively. \( \frac{v_0 c_a}{2} a^2 \) is the term that represents the cost of effort that is paid at \( t_1 \). We assume a quadratic cost of effort, scaled by \( v_0 \). This is similar to the form used in Hodder and Jackwerth (2007) and is consistent with decreasing returns to scale with respect to increased effort expenditure. Although effort can be interpreted traditionally as how hard the manager works, a more apt interpretation views it as hiring costly alpha-generating talent from a common portfolio manager pool. \( c_a \) is a parameter representing the cost of effort in this market for alpha-generating talent.

We solve this simple model and show that effort, \( a \), has an interior solution (given a finite and fixed \( \sigma \)). \( \sigma \) will not have an interior solution - it is either 0% or infinity, depending on the choice of \( a \), the distribution of shocks, \( \varepsilon \), \( \gamma \) and other contract parameters.

### 3.2.2 One period model with continuation value and investor walkaway

We can extend this one period model to capture investor walkaway and some of the effects of the HWM by treating this as a hedge fund that has been in existence for some time. Thus, the fund may potentially be below the HWM. The distance of the fund from the HWM is characterized by the return required to hit the HWM (RR), denoted by \( r r_0 \).

\[ 1 + r r_0 = \max(h_0/v_0,1), \]  

(3.4)

\( r r_0 \) is the return required to hit the HWM at \( t_0 \). \( v_0 \) is the value of the fund at \( t_0 \) and \( h_0 \) is the
HWM at \( t_0 \). If the fund is at the HWM, \( h_0 = v_0 \) and the return required to hit the HWM is zero \((rr_0 = 0)\). Additionally, the model allows the investor to withdraw the investment, \( v_0 \), at time \( t_0 \), before the manager can make his effort and risk choices. The investor is risk neutral and will withdraw the investment if the expected value of the fund after fees in the second period is lower than the value of the fund currently. If the investor decides to continue, then the manager will make effort and risk choices, the return will be realized, and the contract will terminate at \( t_1 \). To capture the continuation value of the contract to the manager, the investor will also have the opportunity to withdraw funds in \( t_1 \). If the investor withdraws the funds, the manager will receive his outside option, \( v_y \). If the investor remains invested, the manager will receive a lump sum value, \( v_c \), representing the continuation value of the contract. We will assume that the continuation value of the contract is higher than the manager’s outside option \((v_c > v_y)\).

A timeline for this extension is as follows, and a graphical representation of the timeline can be found in figure 3.1

- At time \( t_0 \):
  - Assets under management (AUM) are \( v_0 \) and the fund high watermark is at \( h_0 \).
  - \( rr_0 \) represents the return required to reach the last watermark (RR). (as calculated in equation 3.4)
  - The investor remains invested in the firm if she anticipates that the effort and variance chosen by the manager will lead to positive expected returns after fees are deducted.
  - However, if she chooses to walk away from the fund, the contract is terminated, the investor receives the fund value \( v_0 \), and the agent receives his outside option.
  - If the investor chooses to continue with the portfolio manager at \( t_0 \), then the manager chooses effort, \( a \), and variance, \( \sigma \). (See section 3.2.2).
At time $t_1$:

- The management fee is paid to the fund manager based on the previous period's AUM ($k v_0$).
- The rate of return, $\hat{r}_{01}$, is realized. If the old HWM is exceeded, or $\hat{r}_{01} > r r_0$, then an incentive fee of $s \times v_0(\hat{r}_{01} - r r_0)$ is paid to the fund manager.
- The return required to hit the HWM (RR) at $t_1$, $r r_1$, is computed, and the investor makes a decision based on $r r_1$ whether to continue investing in the fund going forward. There is a walkaway threshold, $r r_1^*$, above which the investor will walk away at time $t_1$. $r r_1^*$ is set exogenously in the one period model, although this will become endogenous in the infinite period model.
- If the investor does not walk away, then the manager receives a lump sum $v_c$, which represents the continuation value of the contract - a stylized way to capture future incentive and management fees. If walkaway occurs, the manager will receive his outside option, $v_y$. We will assume that $v_c > v_y$. In the infinite period model, $v_c$ will be replaced by a function of RR, allowing us to construct a Bellman equation for the manager's problem.
- The contract then terminates at time $t_1$.

We solve for the manager's effort and risk decisions at time $t_0$, depending on what $rr_0$ is, such that the manager maximizes expected utility from incentive fees, management fees, and potential contract continuation. Given the manager's actions, we can determine the investor's expected returns after fees as a function of $rr_0$ and the region of $rr_0$ for which this expected return is negative. In this region, the investor will walk away in time $t_0$, before the manager makes the decision.
Portfolio Manager’s Problem

The portfolio manager will act at $t_0$ to maximize his expected utility at $t_1$. $V_m$ is the fund manager’s expected utility at $t_0$:

$$V_m = \max_{a, \sigma} \beta \mathbb{E} \left[ u(k v_0 + s v_0 \max(0, \bar{r}_{01} - r r_0) - \frac{v_0}{2} c_a a^2) + p(\bar{r}_{01} > r_{01}^*) \times \pi \right], \quad (3.5)$$

where

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

$$1 + r_{01}^* = \frac{1 + r r_0}{1 + r r_1^*}$$

$$\bar{r}_{01} = 1 + a + \sigma \bar{e}$$

$$\pi = u(v_c) - u(v_y)$$

$$\bar{e} \sim U(-1,1)$$

where $\bar{r}_{01}$ represents the rate of return from time $t_0$ to $t_1$. The utility function remains the same as before, with $\gamma$ representing the coefficient of risk aversion for the CRRA manager and $\beta$ representing his discount rate. The cost of effort also remains the same as before, $\frac{v_0}{2} c_a a^2$. The difference now is that the fund may not be at the HWM, and thus, the incentive fee is paid only on returns above the HWM ($s v_0 \max(0, \bar{r}_{01} - r r_0)$). The shock retains the uniform distribution, $\epsilon \sim U(-1,1)$. $p(\bar{r}_{01} > r_{01}^*)$ is the probability that the return over the period leads to a RR in the second period that is low enough to ensure the continuation of the fund, and $\pi$ is the increased utility to the manager from receiving this continuation value instead of the outside option. The manager will choose effort $a$ and risk $\sigma$ for the next period to maximize his expected utility:

$$\{a^*, \sigma^*\} = \arg \max_{a, \sigma} \beta \mathbb{E} \left[ u(k v_0 + s v_0 \max(0, \bar{r}_{01} - r r_0) - \frac{v_0}{2} c_a a^2) + p(\bar{r}_{01} > r_{01}^*) \times \pi \right] \quad (3.6)$$
**Investor's problem**

The investor's walkaway decision at $t_0$ depends on the returns after fees that she can expect given $rr_0$, the RR at time $t_0$, and the manager's optimal effort and risk choices for this $rr_0$. The value of the fund after fees at $t_1$ can be represented as follows:

$$
E[v_1] = E[v_0\tilde{r}_{01} - k v_0 - s v_0 \max(0, \tilde{r}_{01} - r_{r0})]|\tilde{r}_{01} = 1 + a^* + \sigma^* \tilde{e}],
$$

(3.7)

where $v_0, v_1$ are fund values at time $t_0, t_1$, respectively, and $\tilde{r}_{01}$ is the return given the manager's anticipated choice of effort, $a^*$, and variance, $\sigma^{*2}$, as given in equation 3.6. Fees paid to the manager in the above equation are simply the management and incentive fees from the contract $(k v_0 + s v_0 \max(0, \tilde{r}_{01} - r_{r0}))$. As the investor is risk neutral, if $E[v_1|rr_0] \leq v_0$, the investor will walk away at time $t_0$. $E[v_1]$ is decreasing in $rr_0$ and the minimum RR at which the investor walks away is denoted by $rr_0^*$.

At time $t_1$, the investor walks away if return required, $rr_1$, is greater than an assumed and fixed $rr_1^*$ (akin to $rr_0^*$ walkway point discussed above, but fixed by assumption in the one period model). This leads to a $t_1$ walkaway point in terms of the realized return for the period:

$$1 + r_{01}^* = \frac{1 + rr_0}{1 + r_{r1}^*}$$

(3.8)

If $r_{01} < r_{01}^*$, the investor will walk away at $t_1$. In the infinite period model, this walkaway point will be determined endogenously using the Bellman equation.

**Optimal Effort, Risk and Walkaway in the one period case**

Although closed form solutions to the manager's optimal effort and risk choices, $a^*$ and $\sigma^*$, exist, it is easier to glean intuition from graphical representations of these solutions which are presented in Figure 3.2. We can see that optimal effort generally decreases with RR: optimal effort remains at its maximum, 5%, until $RR = 1.8\%$, then falls to about 2% and then slowly
decreases to near-zero levels at \( RR = 10\% \). Optimal risk is at the minimum when \( RR < 1.8\% \).
Above this, risk spikes to its maximum value, 10\%, and remains there for \( RR \geq 1.8\% \).

Given this set of optimal effort and risk choices, we solve for the fund manager’s value from the contract (given no investor walkaway at \( t_0 \)) as a function of RR. This is shown in figure 3.3. This graph has three lines. The “Payoff from Present Period” is the utility derived from the net of the management fees, incentive fees and cost of effort \( (u(k v_0 + s v_0 \max(0, \tilde{r}_0 - r_0) - \frac{v_0}{2} c_a a^2)) \) from equation 3.5. The “Payoff from Continuation” is the expected utility from receiving the continuation value over the outside option \( (\rho(\tilde{r}_0 > r_0^*) \times \pi) \) from equation 3.5. Finally, the “Total Payoff” is the sum of these and reflects the total expected utility to the manager from this contract, given the RR. Note that given the power utility function, the sum of the utility from the total payoff is not the sum of the utilities from the individual components. We see that the manager’s utility decreases monotonically with RR, and the contract continues with certainty until \( RR = 1.8\% \). Beyond that, the probability of continuation in the second period decreases linearly with RR.

Finally, using the optimal effort and risk policies above, we can determine the expected return after fees, given fund continuation. This is shown as a function of RR in figure 3.4. We see that this increases from 1\% to its peak at 2.25\% as the fund goes from 5\% above the HWM to 1.8\% below it. Then it precipitously falls to -0.6\% and slowly decreases with RR, finally settling at about -2\% for \( RR > 10\% \). The precipitous fall coincides with the switch in optimal risk policy from minimum risk to maximum risk. A rational investor would thus withdraw the funds at \( t_0 \) if \( RR > 1.8\% \). We also note the initial increasing expected return after fees (over the domain \(-5\% \leq RR \leq 1.8\% \)) is due to the reduced incidence of fees. The further the fund is below the HWM, the less it will have to pay in terms of incentive fees, as these fees are only charged on returns over the HWM.
3.2.3 Infinite Period Model and Solution

To extend this model to a more realistic infinite period contract, we convert equation 3.5 into a Bellman equation. We retain the timing of the decisions made by the players from section 3.2.2. Specifically, during each period, the decisions are made as follows:

1. The investor, who has funds with the manager indexed to a HWM, decides whether to remain invested with the manager or whether to withdraw funds.
2. If the investor remains invested, the manager chooses how much effort to expend and how much risk to take.
3. The return is realized as per the shock and the effort and risk choices.
4. Management and incentive fees, if any, are paid to the manager.
5. Any returns to the fund are disbursed, and if the fund loses money, the investor replenishes the fund to $v_0$ with new money entering indexed to the old HWM.
6. A new HWM is established if the old one is exceeded and the investor contemplates the walk-away decision all over again.

Once we replace the lump sum continuation value in the second period in equation 3.5 with a function of the RR to represent the continuation value of the contract, the agent’s value function becomes the Bellman equation shown in section 3.2.3 below. We assume (in step 5 above) that the investor will maintain assets of the fund at $v_0$. That is, she will withdraw funds above $v_0$ if the fund made a profit and will replenish the fund to $v_0$ if it made a loss. We will further assume that the contract allows her to replenish the fund with monies indexed to her old watermark, although this is not a central assumption. We can appeal to an optimal fund size for.

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2 Allowing the investor to replenish funds such that the funds invested have a blended HWM also produces qualitatively similar results.
alpha-generation to justify this assumption. An S-shape alpha-generation function (with initial economies of scale followed by decreasing returns to scale) will provide some justification for this optimal $v_0$ fund size. Additionally, a number of hedge fund investors, such as pension funds, fund of funds, and high net worth individuals, often have portfolio allocation targets, which comport with maintaining the investment in the fund at a given size. These assumptions greatly increase tractability and allow us to isolate the impact of the watermark on the incentives of the fund manager, rather than reflecting the manager's incentive to grow the fund to increase fee revenue. While flow of funds constitutes an important part of the asset management literature (see Kerr Christoffersen and Musto (2008) and Berk and Green (2004)), the natural convexity of the ‘2/20’ contract allows us to discern the impact of incentives on the control variables without considering fund flows.

**Portfolio Manager’s Problem**

The portfolio manager’s value function can be represented as the Bellman equation below:

$$
V_m(rr) = \max_{a, \sigma} \beta \mathbb{E} \left[ u(k v_0 + s v_0 \max(0, \tilde{r} - r r) - \frac{v_0}{2} c_a a^2) + V_n(r r') \right]
$$

(3.9)

where

$$
u(x) = \frac{x^{1-\gamma}}{1-\gamma}
$$

$$
r r' = \max(0, \frac{r r - r_{pf}}{1 + r_{pf}})
$$

$$\tilde{r} = a + \sigma \tilde{\epsilon}
$$

$$r_{pf} = \tilde{r} - k - s \max(0, \tilde{r} - r r)
$$

$$\tilde{\epsilon} \sim N(0, 1)
$$

and
\[ V_n(rr') = \begin{cases} 
V_m(rr') & \text{for } W(rr') = 0 \\
\alpha V_y + (1-\alpha)V_{nw,a}(rr') & \text{for } W(rr') = 1 
\end{cases} \quad (3.10) \]

The payoff in the next period from incentive fees, management fees and cost of effort remains the same as in the one period model. Note that all payouts (including the cost of effort paid by the manager) occur at the end of the current period. This allows us to avoid a savings decision for the manager. There is an active literature on other specifications for agent consumption and savings (see Brown, Goetzmann, and Ibbotson (1999) and Brown, Goetzmann, and Park (2001) among others), however, in our paper, the manager will consume all income from a given period. He will not have a savings decision, nor is he able to invest in his own fund. \( W(rr') = 1 \) and \( W(rr') = 0 \) represent the walkaway point, in terms the RR, being breached or not being breached in the second period, respectively. This function is described further in section 3.2.3 below. \( V_y \) and \( V_{nw,a} \) are the value functions for a manager receiving his outside option, and a manager with an investor who does not monitor, respectively. Given the specification of the cost of effort and the utility function, together with the replenishing of the fund (step 5 in the timing above), the fund size, \( v_0 \), is not a state variable and is normalized to 1 in the solution and calibration sections. \( \alpha \) is the fraction of investors who monitor their investments. We discuss this fraction in further detail in section 3.2.3 below. Setting \( \alpha = 1 \) would represent a world in which all investors monitored, and the manager would always receive the outside option if the walkaway threshold were to be breached. \( V_y \) and \( V_{nw,a}(rr) \) can be represented as follows:

\[ V_y = \sum_{i=0}^{\infty} \beta^i u(y v_0) \quad (3.11) \]

and

\[ V_{nw,a}(rr) = \max_{a,\sigma} \beta \mathbb{E} \left[ k v_0 + s v_0 \max(0, \bar{r} - rr) - \frac{v_0}{2} c_a a^2 + V_{nw,a}(rr') \right] \quad (3.12) \]
If the manager learns that the investor monitors (this happens once the walkaway threshold is breached, with probability $\alpha$), his choices are no longer relevant, as the fund would have been liquidated; the manager will take up his outside option and earn $y v_0$ into perpetuity. This can be interpreted as taking up a position at an actively managed mutual fund with the same AUM as the hedge fund at the time of closure and making a fixed expense ratio, $y$, on these investments forever. However, if the manager learns that the investor does not monitor (this happens once the walkaway threshold is breached, with probability $1 - \alpha$), he will correctly determine that the investor will never walk away and will act as if this contract will never be terminated. Note that the continuation value ($V_{nw}(rr')$) in the no walkaway Bellman equation remains of the same form, regardless of how far the fund falls from the HWM. This is in contrast to the case for the manager who does not yet know if the investor monitors, and the continuation value is of a different form ($V_{n}(rr')$ rather than $V_{m}(rr)$ in equation 3.9).

Before solving the manager's problem, we consider the investor's walkaway decision in order to obtain $W(rr')$ above.

**Monitoring Investor's Problem**

We modify the investor's walkaway decision from equation 3.7 in the one period model. We assume that only a fraction of investors, $\alpha$, monitor their investments. The investors who do not monitor will never walk away. This section discusses the monitoring investor's walkaway decision.

The “monitoring” parameter, $\alpha$, represents the fraction of investors who constantly monitor their investments. Each fund either has an investor who constantly monitors the fund at zero cost, or has an investor who has a prohibitively high cost of monitoring the fund and does not monitor. $\alpha$ is the fraction of total investors who costlessly monitor the fund. If the investor does

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3The purpose of $\alpha$ becomes clear in the calibration section as we try to match the dispersion of fund observations by distance from the HWM. For now, setting $\alpha = 1$ would represent a world where all investors monitor and the remaining analysis would go through.
not monitor, she will never walk away. Initially, the manager does not know what type of investor he works for; the manager will act such that there is $\alpha$ probability that the investor monitors and $1 - \alpha$ probability that the investor will not monitor and will never walkaway. This concept of the attentive/inattentive actor is not new in the literature. Both the mutual fund literature and mortgage literature have extensively studied this phenomenon (see Berk and Tonks (2007), Christoffersen and Musto (2002), Carhart, Carpenter, Lynch, and Musto (2002) and Schwartz and Torous (1989)). While it might be argued that investors in hedge funds are different and are much less likely to leave their funds unattended, we note that there is a large variation in the type of hedge fund investors. A large fund of hedge funds would be unlikely to leave its funds unmonitored indefinitely, but retired high net worth individuals and pension funds might well adopt this “set it and forget it” stance seen among non-monitoring investors.

The monitoring investor is risk neutral and seeks to maximize expected funds by deciding when/if to withdraw funds from this alternative investment and to deposit them into an account bearing no interest. In order to posit an equilibrium, we assume the investor believes that if she does not walk away when she should, the manager will behave as if the investor is of the unmonitoring type and will never walk away (i.e. the manager will determine $a^*$ and $\sigma^*$ by solving $V_{nw}(rr)$ from equation 3.12). This is an off the equilibrium path belief. In equilibrium, the monitoring investor will always walk away when the walkaway condition is met.

Even with these beliefs, there may still be multiple equilibria for the investor's walkaway threshold. We take a number of steps to identify one such equilibrium. We posit a walkaway rule consistent with the off the equilibrium path beliefs discussed above, and we verify that this rule, combined with the manager's actions under that walkaway policy, constitutes a stable equilibrium. We propose the following walkaway policy: The investor will walk away based on

\footnote{Although we do not consider reinvestment into another fund as an admissible strategy in our model, if such a reinvestment were to be accompanied by an upfront sales fee that expropriated all investor surplus from entering a contract at the HWM, this would be equivalent to having a zero-interest bearing deposit account as an alternative.}
the outcome of returns and fees over the next $N$ periods. More specifically, the investor will walk away if expected returns from the next $N$ periods are negative, regardless of interim walkaway, given the RR and that the manager’s investment decisions are as in the no walkaway case (i.e. given $a^*$ and $\sigma^*$ from the solution to $V_{nw}(rr)$ for each given $rr$ from equation 3.12). Thus, the walkaway rule can be expressed as:

$$W(rr) = \begin{cases} 
1 & \text{for } E_{nw}[\tilde{r}_{pf, t_0 \rightarrow t_n}|rr] < 0 \\
0 & \text{otherwise} \\
\forall \ n \in [1, 2, 3...N] 
\end{cases}$$

(3.13)

Where $E_{nw}[\tilde{r}_{pf, t_0 \rightarrow t_n}|rr]$ represents the expected return after fees under the belief that the manager will behave as if the investor never monitors over the next $n$ periods. Such a rule can be interpreted as assuming a bounded rational investor who is willing to analyze the next $N$ periods to make the right decision. We now have to verify that this rule is consistent with the behavior of a value maximizing investor. It is easy to show that the investor should never walk away when $W(rr) = 0$ because (1) expected returns will always be higher when the manager is uncertain if the investor is monitoring than when the manager knows for sure the investor is not monitoring (this is shown for the calibrated model in section 3.2.3 below) and (2) expected returns for the investor will be positive (over some period from $t_1$ to $t_N$) under the belief that the manager acts as if the investor is not monitoring. (1) and (2) together imply that walking away when $W(rr) = 0$ would mean walking away from positive expected returns and depositing the funds in a zero interest bearing account, which is suboptimal. We also to verify that continuing the contract when $W(rr) = 1$ is suboptimal in terms of returns over the next $N$ periods. This is trivially true given the walkaway condition in equation 3.13. Note that while this is a walkaway threshold consistent with the investor’s beliefs, we do not identify it as a unique walkaway threshold. It is possible that a different walkaway threshold may be a feasible policy function in equilibrium as well. This walkaway policy does, however, provide us a clear path to solving the problem:
We solve for the manager's optimal policy in a world where he believes the investor does not monitor. Using these policies, we determine the walkaway policy for the investor from equation 3.13. Using the walkaway policy, we solve for the manager's policy functions using the Bellman equation, equation 3.9.

**Optimal effort, risk and walkaway in the infinite period case**

We solve for the optimal effort and risk policy functions for the manager in the no walkaway case using value function iteration. The functions are shown in figure 3.5. These simply represent how a manager would behave if he knew the investor would never walk away, and they are the solutions to the maximization problem in equation 3.12. Using these functions, we can determine the regions of walkaway as per the walkaway condition above (equation 3.13). The walkaway decision is shown in figure 3.6 and is the function presented in equation 3.13. Under the model, only $\alpha$ fraction of the investors monitor their funds. The only way for the manager to find out if the investor is of the monitoring type is if the fund ever reaches a point where the return required is higher than the walkaway threshold shown in figure 3.6. At that point, the value function is resolved into either $V_y$ or $V_{wa}(rr)$. Once we add in the walkaway policy for the monitoring investor, we can model the manager as choosing $\alpha$ and $\sigma$ as per the Bellman equation above (equation 3.9) with investor walkaway policy in figure 3.6, conditional on the monitoring investor not walking away. Using value function iteration, we solve for the manager’s optimal policy and value function. The policy functions are shown in figure 3.5. Combining the optimal policy functions for the manager and the investor, we determine the value function for the manager, which is shown in figure 3.7. Note the steep dropoff to the right of the walkaway point indicating the potential loss of the contract. The expected moments for investors’ returns under the solved model are shown in figure 3.6. We can see that expected return next period decreases and expected return variance increases with RR.

Finally, we confirm that both the attentive investor's behavior and the manager's actions
constitute a Nash equilibrium. The investor will never walk away in regions to the left of the walkaway boundary in figure 3.6 as it is unprofitable to do so. To right of the boundary, she will always walk away as she believes that the manager will revert to a no-walkaway effort and risk policy if she does not. She will end up paying more in fees than she can hope to recoup in expected returns according to her off the equilibrium path beliefs, and thus, she will walk away. The manager will also not deviate, as every control decision taken by the manager optimizes the Bellman equation shown in equation 3.9 and deviation can only lower the manager's expected utility. Given that no unilateral deviation on the part of either party can improve their outcomes, these policy functions constitute a Bayesian Nash equilibrium.

3.3 Model Calibration

Using the empirically observed moments from Ray (2009), we calibrate a number of free parameters in our model. In particular, we are interested in the agent’s risk aversion ($\gamma$), the cost of effort ($c_a$), the fraction of investors that monitor their funds ($\alpha$), and the time horizon of the monitoring investor, $N$. Additionally, we also determine the impact of the outside option, $y$, and the discount factor for the agent, $\beta$. We also calibrate the domain of choice variables considered (the range of potential effort choices, $a$, and potential risk choices, $\sigma$). Using these parameters, we attempt to match observed data moments, including the population average after fee returns, the variance of the average after fee returns, walkaway rates, the relationship between investor returns and return required to hit the HWM, and the dispersion of fund-period observations in terms of how far funds are from the HWM. The summary statistics we try to match are reproduced from Ray (2009) and presented in Table 3.1.

Table 3.2 shows the moments generated by simulating the model under various sets of parameters alongside the actual empirical moments. We simulate 20,000 funds running for between 1 and 200 periods, each starting at the HWM. The fund starting points are distributed uni-
formly over the interval to simulate the staggered starting points of hedge funds in the CISDM dataset. The funds all have a 1.5/17.5 fee structure, mirroring the average management and incentive fees seen in the CISDM dataset.

The first column in the table shows moments obtained from Ray (2009), and the second column shows our base parameters. The observed sample moments can be matched quite closely by the simulation. In particular, the average returns and return standard deviation are slightly lower (0.86% vs. 0.95% and 5.78% vs. 5.99%). The walkaway rate predicted by the model (0.74%) is also slightly lower than that observed in the data (0.87%). The coefficients describing the relationship between expected returns, return variance and distance from the HWM also match directionally and in magnitude. $\delta_1$ suggests the model predicts that a fund 10% below the HWM will underperform a fund at the HWM by 1.9% in the next period. Multivariate regression analysis in Ray (2009) shows that in observed actual data, a fund 10% below the HWM will underperform a fund at the HWM by 2.8% over the following six months. Similarly, the model predicts that the standard deviation of monthly returns for a fund 10% below the HWM will be 9.4% higher than the standard deviation of a fund at the HWM. Multivariate regressions in Ray (2009) suggest that the standard deviation of monthly returns using 6 months of data are 1.6% higher for funds requiring a 10% return to hit the HWM compared to funds at the HWM. We face a tradeoff between using more periods to obtain a more accurate standard deviation of returns and diluting the impact of the HWM on risk in the next period, or using fewer periods and introducing noise into the measure of standard deviation. We use six months of returns to estimate the standard deviation of returns. We are not concerned that the sensitivity of risk to RR predicted by the model is higher than the observed sensitivity because risk calculation for the observed sensitivity relies on returns further into the future, which are less likely to be affected by the RR in the current period. We analytically compute investor-observed moments (average returns and return standard deviation), and how they vary depending on the return required to
hit the HWM. These moments are displayed in figure 3.6 and are used to determine the regression coefficients above. Finally, we note that the dispersion of the simulated observations by the distance required to hit the HWM also comports with observed data, except for a higher concentration in the $RR > 100\%$ category at the expense of lower concentration in $10\% < RR \leq 100\%$ category.

The third column shows an increase in the cost of effort $c_a$ to 0.85. It principally shows that due to the endogeneity of the walkaway, $c_a$ (and many other parameters) affects almost all the moments. As the cost of effort increases, expected returns decrease, and the contract has less value to the manager. Thus, the manager is more willing to gamble because losing the less valuable contract and receiving the outside option are less distressing now than when the cost of effort was lower. This naturally leads to increased risk and walkaway incidence. In general, the Jacobian of the parameter-moment relationship has very few zero elements, reflecting generally that parameters affect all moments due to the endogeneity of walkaway and choice variables. Additionally, to confirm that the parameters in the model can be uniquely identified, we note that this Jacobian matrix, evaluated at the base parameter set, has full rank.

The fourth column shows the impact of setting $\alpha = 1$. In a world where all investors monitor, we note that the dispersion of observations will never have any fund periods with $RR > 10\%$. The manager's discount rate ($\beta$) and outside option ($y$) are left unchanged through the results presented in the table. The outside option can be thought of as the manager leaving the hedge funds to go manage an actively managed mutual fund with similar assets under management. We set the expense ratio of the fund at 1.5%, although reducing this has little effect on the results, save to reduce the variance of the simulated returns. $\beta$ also has a limited impact, and the higher $\beta$ is, the less risk the manager takes. Additionally, at very high levels of $\beta$ ($\beta > 0.99$), the value function convergence slows, and obtaining meaningful solutions numerically becomes a time-intensive process. The parameters restricting policy function choice variables ($a \in [0\%,2\%]$ and
\( \sigma \in [3\%, 11\%]) \) primarily allow us to control the overall level of risk and excess return. While it is possible to introduce additional parameters in the cost of effort function or a Sharpe ratio to achieve similar results, we opt for a more transparent approach of simply limiting the range of \( a \) and \( \sigma \). We note that Hodder and Jackwerth (2007) similarly restrict their alpha-generation choices. We also note that in addition to risk aversion and the threat of walkaway, hedge funds typically have risk management divisions that often prohibit egregious risk taking.

Despite our ability to match a number of the moments quite closely with the model, we note that our model, like all models, does not capture the reality of these contracts in its entirety. For example, we do not capture Bayesian learning, by which the investor learns more about the skill of a manager through the return process (see Kerr Christoffersen and Musto (2008) for a model of this feature and its interaction with the HWM). We also do not capture renegotiation or new fund flows. The anecdote cited in the introduction, in which a fund below its HWM had a sale, shows both of these at work. In particular, fees can be renegotiated downwards to stave off fund closure, and new money introduced into a fund below the HWM will also prevent fund closure. Finally, we also note that reporting and survival biases will likely affect a number of the moments we seek to match. In particular, given the general positive bias of fund reporting, average investor returns are likely to be lower than observed, and walkaway rates are likely to be higher. These, together with the continued evolution of the hedge fund space, suggest that the calibration exercise does the best with the data we have and bears revisiting as new data become available and regulations surrounding the reporting of returns are codified.

### 3.4 Welfare Analysis

Using the calibrated model, we can analyze welfare considerations from modifying the specifications of the contract. Rather than adopting a completely free contract form, we maintain the basic HWM structure, where there is a fixed management fee and an incentive fee based on per-
formance relative to a historical HWM. Thus the return required to hit the HWM, $r r'$, follows the same laws of motion from equation 3.9. We vary the management fee (the 2% in 2/20 contracts) and the linear incentive fee (the 20% in 2/20 contracts). The initial metrics for measurement of welfare are the expected return for the investor after fees at the inception of the contract ($E[r_p]$) evaluated at $rr = 0$ and the value function for the agent evaluated at $rr = 0$. Since these two metrics cannot be added together, we look for Pareto improvements on the existing 2/20 contract, rather than finding a contract that maximizes “societal” welfare.

Optimal contract literature has benefitted from strong theoretical research - some of the representative papers include, but are not limited to, Holmstrom and Milgrom (1987), Jewitt (1988), Laffont and Tirole (1988), Kuhn (1994), Ou-Yang (2003), Demarzo and Sannikov (2004), Cadenillas, Cvitanic, and Zapatero (2005) and Sannikov (2006) among others. These papers provide optimal contracts for agents, such as portfolio managers, often focusing on proving that a linear (or other) contract form is optimal. This paper does not consider the optimality of a “x/y”, HWM contract, but focuses instead on identifying exact specifications that lead to Pareto improvement over the standard 2/20 contract under the calibrated model.

The methodology used for this analysis is as follows:

- We assume different contract forms, as given by different management and incentive fee specifications, while retaining the HWM structure of the contract
- Using the calibrated model from section 3.2.3 and section 3.3 we solve for the optimal effort and risk policy for the manager and the corresponding walkaway threshold for the investor for each contract specification
- This obtains the value function for the hedge fund manager and the expected return after fees for the investor in the next period across the domain of RR
- We compare the proposed welfare metrics across the different contract specifications. In par-
ticular, we compare the expected return for the investor after fees at the inception of the contract \( E[r_{pf}] \) evaluated at \( rr = 0 \) from equation 3.9 and the value function for the agent evaluated at \( rr = 0 \) and search for Pareto improvements over the base (2/20) case.

The results are shown in table 3.3. The columns show varying management fee levels from 1.5% to 3%, and the rows show varying incentive fee levels from 2.5% to 40%. We see that, in general, lowering the incentive fee and increasing the management fee leads to Pareto improvement for both the fund manager and the investor. In particular, we focus on a 2.5/10 contract and see that the investor expects to make a 1.33% return from a fund at the HWM compared to a 1.15% return from a 2/20 contract fund at the HWM. The manager, too, is better off by 75% (this simply means \( \frac{v_m(0|2.5/10) - v_m(0|2/20)}{v_m(0|2/20)} = 75\% \)), where \( v_m \) is the manager’s value function from equation 3.9, the Bellman equation.

The policy functions for the manager under these 2 contracts sheds light on this interesting finding. Figure 3.8 illustrates the policy functions under these two cases. Although there is no difference in manager policy at the HWM, as the manager is on his “best” behavior in both cases, the Pareto improvement comes from improved risk sharing. However, in the regions with \( RR > 0 \), we see that the manager exerts in more effort and takes lower risk for the 2.5/10 contract compared to the 2/20 contract. These management policy functions also serve to increase the walkaway threshold for the investor. These differences in the policy functions suggest that the higher continuation value of contracts and the enhanced ability of high-management-fee funds to retain costly alpha generating talent through periods of poor returns also improve manager and investor outcomes across the entire spectrum of RR. In figure 3.9 we graph the expected return after fees for the investor and the value function for the manager over RR for these two cases and find that for both investor and the manager, the 2.5/10 contract outcomes dominate the 2/20 contract outcomes across the RR spectrum.

This also comports with analysis from Ray (2009) comparing funds across the AUM and fee...
structure spectrum, which finds that large funds and funds with higher fee structures have a positive relationship between RR and expected returns after fees, further supporting the hypothesis that these funds are able to retain their costly alpha generating teams through periods of poor returns.

3.4.1 Renegotiation

Recently, a number of funds have been renegotiating their HWMs. This can be done explicitly through simply lowering the old HWM, or implicitly through increased new fund inflows from discounting (such as the case mentioned in the introduction). Using the calibrated model, we examine the effect of such renegotiation on the manager choices and investor welfare.

There are a number of ways to implement renegotiation. We choose a simple, transparent renegotiation where the HWM is simply reset each time the fund falls below it through constant renegotiation. Under such a set up, the investor never walks away and the $r_r$ state variable is always reset to 0. We implement such a set up using a 2/20 contract.

While such a contract does not suffer from the adverse effects when $r_r$ is high, the investor's expected return after fees when $r_r = 0$ (which is all the time, due to the constant renegotiation), is 0.56% per month, compared to 1.15% when $r_r = 0$ in the 2/20 contract that is not renegotiated (this is obtained from table 3.3, 2% management fee, 20% incentive fee cell). The manager is also 19.4% better off with renegotiation compared to a manager at the HWM holding a similar 2/20 contract without the prospect of renegotiation. Renegotiation removes the threat of falling below the HWM along with potential loss of incentive fees and investors funds and leads to reduced effort and higher risk choices by the manager.

Comparing the 2/20 contract that is never renegotiated (see Figure 3.9) with one that is constantly renegotiated, as long as $r_r \leq 16.5\%$ (which is also the walkaway point for the monitoring investor), the contract without renegotiation outperforms for the investor.

In reality, renegotiation often occurs only when a fund has fallen significantly below the
HWM. Additionally, renegotiation is often accompanied with fee reductions and other concessions by the fund manager. These can certainly be instituted in the calibrated model, but the negative effects of the renegotiation for the investor will be based on the same intuition described above, although they may be mitigated by savings in fees.

3.5 Conclusion

While some funds have largely recovered from the recent economic crisis, many others still find themselves far below their HWMs. The funds’ losses have irked investors and many funds have offered lowered fees as a concession to retain assets under management. A number of funds have also attempted renegotiating the HWM with their investors to better align fund management and investor incentives: some have succeeded and are back on track to earning incentive fees; some others have failed and are slowly chipping away at their old losses in the hope of once again making their 20% incentive fees.

All of these contracting decisions have direct impacts on fund manager incentives and affect management risk decisions, as well as the ability of management to retain costly investment talent. These decisions also affect investor portfolio allocation decisions: many disillusioned investors on the verge of withdrawing their funds have been persuaded to stay invested through a combination of fee reductions, HWM renegotiations and increased investment transparency.

We model the effects of the HWM fee structure on manager and investor behavior and calibrate the model to empirical data. We use the calibrated model to examine the impact of renegotiation and fee changes on investor and manager incentives, decisions and welfare. In addition to re-iterating the known incentive hazards associated with being below the HWM, the study suggests that increasing the management fee and decreasing the incentive fee (e.g., a 2.5/10 contract) may lead to Pareto improvement through improved risk sharing, moderated risk-taking by fund management and fund talent retention through periods of poor returns. The
study also highlights the theoretical time inconsistency problem of renegotiation, which, while optimal ex post, once a fund is below the HWM, leads to a poorer management behavior ex ante.

Overarching all the issues related to HWMs highlighted in this study, questions about the optimality of the prevailing management/incentive fee with a HWM contract remain. Even as the worst of the recent crisis recedes, we see changes to the standard hedge fund contract emerging (e.g. lower fees, increased transparency, implementation of clawbacks). While our calibrated model provides a starting point with which we can examine the welfare implications of these and other contract forms in a partial equilibrium setting, the elusive search for the optimal contract remains just that, elusive.
Figure 3.1: One period model timeline

The figure illustrates the timing of the decisions and the realization of shocks.

State: v, h

Contract begins

Yes: principal gets back $v_0$
No: principal remains invested

Agent gets $V$

Contract ends

Yes: principal gets back $v_1$
No: principal remains invested

Agent gets $V_c$

Walkaway?

Yes

Control: $a, \sigma$

No

$T_0$

$T_1$

$T_0$

$T_1$

Walkaway?
Figure 3.2: Optimal effort and risk under a one period model with continuation value

The graphs present the joint solution for optimal effort and optimal variance as a function of RR. These policy functions are under the one period setup with continuation value described in section 3.2.2. The parameters used for these solutions are $k = 0.02, s = 0.2, r r_1^* = 5\%, c_a = 3, \pi = u(0.2) - u(0.05), \gamma = 3.5, \epsilon \sim U[-1, 1]$. 

[Graphs showing the joint solution for optimal effort and optimal variance as a function of RR.]
Manager's value from the hedge fund contract as a function of RR under the one period setup with continuation value described in section 3.2.2. The parameters used for these solutions are $k = 0.02, s = 0.2, r_r^* = 5\%, c_a = 3, \pi = u(0.2) - u(0.05), \gamma = 3.5, \epsilon \sim U[-1, 1]$. These values assume no investor walkaway in period $t_0$. 

Figure 3.3: Manager's value function and components under a one period model with continuation value
Investor’s expected return after fees as a function of RR under the one period setup with continuation value described in section 3.2.2. The parameters used for these solutions are \( k = 0.02, s = 0.2, r_1^* = 5\%, c_a = 3, \pi = u(0.2) - u(0.05), \gamma = 3.5, \epsilon \sim U[-1, 1] \). These values assume no investor walkaway in period \( t_0 \). A horizontal reference line of \( v_1 = 1 \) represents the investor’s outside option if she chooses to walk away at \( t_0 \).
Figure 3.5: Optimal Effort and Risk Choices for Manager

This figure presents the joint solutions for optimal effort and optimal variance in the infinite period model described in section 3.2.3. The parameters used are the base parameter set presented in table 3.2 and discussed in section 3.3.
Figure 3.6: Walkaway decision and return moments as observed by Investor

This figure presents the walkaway decision taken by the investor and investor observed moments in the infinite period model described in section 3.2.3. The parameters used are the base parameter set presented in table 3.2 and discussed in section 3.3.

Optimal Walkaway Point

Investor Observed Moments
Figure 3.7: Value function for manager

The figure presents the value function for the manager in the infinite period model described in section 3.2.3. The parameters used are the base parameter set presented in table 3.2 and discussed in section 3.3.
This figure shows the manager's policy functions for the base 2/20 contract and those for a contract with increased management fees and decreased incentive fees (2.5/10). The top graph shows optimal effort exerted as a function of RR and the bottom graph shows optimal risk taken as a function of RR.
Figure 3.9: Manager's value function and investor's expected return after fees under 2.5/10 vs. 2/20

This figure shows the manager's value functions and investor's expected return after fees for the base 2/20 contract and the same graphs for a contract with increased management fees and decreased incentive fees (2.5/10). The top graph shows the manager's value function across the RR spectrum and the bottom graph shows expected returns after fees as a function of RR.
Table 3.1: Summary Statistics

The table reports the summary statistics of the database of hedge fund returns used and is reproduced from Ray (2009). Panel A reports the characteristics of the funds and Panel B reports the return and walkaway characteristics of the funds.

Panel A: Fund Types and Fee Structures

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incentive Fee (%)</td>
<td>17.23</td>
<td>6.45</td>
<td>8143</td>
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<tr>
<td>Management Fee (%)</td>
<td>1.51</td>
<td>0.91</td>
<td>8304</td>
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<tr>
<td>Sales Fee (%)</td>
<td>0.56</td>
<td>2.11</td>
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<tr>
<td>Active</td>
<td>0.51</td>
<td>0.5</td>
<td>8752</td>
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<tr>
<td>Gate Percent (%)</td>
<td>12.7</td>
<td>15.92</td>
<td>175</td>
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<tr>
<td>HWM Used?</td>
<td>0.96</td>
<td>0.19</td>
<td>872</td>
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<tr>
<td>CPO Flag</td>
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<td>0.33</td>
<td>8752</td>
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<td>CTA Flag</td>
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<tr>
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<tr>
<td>Index Flag</td>
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Panel B: Return and Walkaway Characteristics

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<th>Variable</th>
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<td>Walkaway</td>
<td>0.007</td>
<td>0.083</td>
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</tr>
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Table 3.2: Model Calibration

The table shows model generated simulated moments alongside moments observed in empirical data as described in Ray (2009). The rows display the parameters used, population moments, regression results and observation dispersion across return required. The first column shows actual moments observed in the data, the second column shows results for base parameters, the third column shows results for a higher cost of effort and the fourth column shows results where all principals constantly monitor their investments. The regression coefficients are those obtained from the following two regressions.

\[ r = \alpha + \delta_1 RR + \delta' controls + \epsilon \]

and

\[ \sigma = \alpha + \delta_2 RR + \delta' controls + \epsilon \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Observed</th>
<th>Base</th>
<th>( \gamma = 0 )</th>
<th>( \alpha = 1 )</th>
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<td>( c_a )</td>
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<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>( \alpha )</td>
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<td>0.85</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( \gamma )</td>
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<td>0.02</td>
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<td></td>
</tr>
<tr>
<td>( \beta )</td>
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<td>0.95</td>
<td>0.95</td>
<td></td>
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<tr>
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<td>0.064</td>
<td>0.064</td>
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<tr>
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<th>Base</th>
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<th>( \alpha = 1 )</th>
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<td>Walkaway Rate (%)</td>
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<td>0.00</td>
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<thead>
<tr>
<th>Coefficients</th>
<th>( \beta_1 ) (0% ( \leq RR &lt; 10% ))</th>
<th>( \beta_2 ) (0% ( \leq RR &lt; 10% ))</th>
<th>( \beta_1 ) (RR ( \geq 10% ))</th>
<th>( \beta_2 ) (RR ( \geq 10% ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-5.8</td>
<td>+25.7</td>
<td>+7.5</td>
<td>-11.4</td>
</tr>
<tr>
<td></td>
<td>+16.3</td>
<td>+192.6</td>
<td>+30.1</td>
<td>83.5</td>
</tr>
<tr>
<td></td>
<td>-0.0</td>
<td>+0.3</td>
<td>+7.5</td>
<td>-11.4</td>
</tr>
<tr>
<td></td>
<td>+0.0</td>
<td>+16.5</td>
<td>+30.1</td>
<td>83.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Observation Dispersion (%)</th>
<th>RR = 0%</th>
<th>0% ( &lt; RR \leq 10% )</th>
<th>10%( &lt; RR \leq 100% )</th>
<th>100% ( &lt; RR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR = 0%</td>
<td>40.5</td>
<td>62.5</td>
<td>66.5</td>
<td>66.2</td>
</tr>
<tr>
<td>0% ( &lt; RR \leq 10% )</td>
<td>26.7</td>
<td>33.5</td>
<td>24.7</td>
<td>33.8</td>
</tr>
<tr>
<td>10% ( &lt; RR \leq 100% )</td>
<td>30.1</td>
<td>1.1</td>
<td>5.2</td>
<td>0.0</td>
</tr>
<tr>
<td>100% ( &lt; RR )</td>
<td>2.7</td>
<td>2.9</td>
<td>3.6</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table 3.3: Welfare Analysis

This table shows model expected returns for the investor and for the hedge fund manager for different contracts. The columns vary management fee from 1.5% to 3.0%. The rows vary the incentive fees from 2.5% to 40.0%. The body of the table presents the expected return after fees (ERAF) for the investor in the next period for a fund at the HWM and the change in utility vis-a-vis the standard 2/20 contract for the fund manager managing a fund at the HWM ($\Delta v_m(.)$ is computed as $\frac{(v_m(0|x/y)-v_m(0|2/20))}{v_m(0|2/20)}$). For example, for a 2.5/10 contract, the investor expects a 1.33% return in the following period and the manager is happier by 74.94% compared to the 2/20 contract.

<table>
<thead>
<tr>
<th>Incentive Fee</th>
<th>Data</th>
<th>$r_{pf}$</th>
<th>1.50%</th>
<th>2.00%</th>
<th>2.50%</th>
<th>3.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50%</td>
<td>$r_{pf}$</td>
<td>0.76%</td>
<td>1.54%</td>
<td>1.50%</td>
<td>1.46%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-563.02%</td>
<td>-115.96%</td>
<td>17.88%</td>
<td>57.69%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.00%</td>
<td>$r_{pf}$</td>
<td>0.88%</td>
<td>1.48%</td>
<td>1.44%</td>
<td>1.40%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-524.75%</td>
<td>-27.78%</td>
<td>71.10%</td>
<td>88.61%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.50%</td>
<td>$r_{pf}$</td>
<td>1.08%</td>
<td>1.43%</td>
<td>1.39%</td>
<td>1.34%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-512.99%</td>
<td>-11.58%</td>
<td>73.13%</td>
<td>91.51%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.00%</td>
<td>$r_{pf}$</td>
<td>1.03%</td>
<td>1.37%</td>
<td>1.33%</td>
<td>1.29%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-505.96%</td>
<td>-12.85%</td>
<td>74.94%</td>
<td>91.16%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.50%</td>
<td>$r_{pf}$</td>
<td>0.98%</td>
<td>1.32%</td>
<td>1.27%</td>
<td>1.23%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-501.18%</td>
<td>-11.58%</td>
<td>73.13%</td>
<td>91.51%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.00%</td>
<td>$r_{pf}$</td>
<td>0.93%</td>
<td>1.26%</td>
<td>1.22%</td>
<td>1.18%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-497.80%</td>
<td>-0.06%</td>
<td>74.86%</td>
<td>92.14%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.50%</td>
<td>$r_{pf}$</td>
<td>0.88%</td>
<td>1.20%</td>
<td>1.16%</td>
<td>1.12%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-489.12%</td>
<td>0.11%</td>
<td>75.13%</td>
<td>92.24%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.00%</td>
<td>$r_{pf}$</td>
<td>0.83%</td>
<td>1.15%</td>
<td>1.10%</td>
<td>1.06%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-493.28%</td>
<td>0.00%</td>
<td>75.33%</td>
<td>93.31%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.50%</td>
<td>$r_{pf}$</td>
<td>0.78%</td>
<td>1.09%</td>
<td>1.05%</td>
<td>1.01%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-491.64%</td>
<td>0.60%</td>
<td>75.50%</td>
<td>91.71%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25.00%</td>
<td>$r_{pf}$</td>
<td>0.73%</td>
<td>1.03%</td>
<td>0.99%</td>
<td>0.95%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-490.24%</td>
<td>1.12%</td>
<td>74.06%</td>
<td>92.42%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27.50%</td>
<td>$r_{pf}$</td>
<td>0.68%</td>
<td>0.98%</td>
<td>0.93%</td>
<td>0.89%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-489.06%</td>
<td>0.48%</td>
<td>74.82%</td>
<td>91.80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.00%</td>
<td>$r_{pf}$</td>
<td>0.63%</td>
<td>0.92%</td>
<td>0.88%</td>
<td>0.84%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-487.98%</td>
<td>-1.95%</td>
<td>72.56%</td>
<td>90.63%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32.50%</td>
<td>$r_{pf}$</td>
<td>0.58%</td>
<td>0.86%</td>
<td>0.82%</td>
<td>0.78%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-487.08%</td>
<td>-1.61%</td>
<td>72.65%</td>
<td>90.19%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.00%</td>
<td>$r_{pf}$</td>
<td>0.53%</td>
<td>0.81%</td>
<td>0.76%</td>
<td>0.72%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-486.24%</td>
<td>-3.25%</td>
<td>72.72%</td>
<td>88.84%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37.50%</td>
<td>$r_{pf}$</td>
<td>0.48%</td>
<td>0.75%</td>
<td>0.71%</td>
<td>0.67%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-485.54%</td>
<td>-8.26%</td>
<td>71.69%</td>
<td>88.87%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.00%</td>
<td>$r_{pf}$</td>
<td>0.43%</td>
<td>0.69%</td>
<td>0.65%</td>
<td>0.61%</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_m(.)$</td>
<td>-508.08%</td>
<td>-8.04%</td>
<td>68.43%</td>
<td>86.88%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix A

Supplement to Investment and Financing under Reverse Asset Substitution

A.1 Identification of Selection (Roy) Model

I want to identify the relative loadings of $\alpha_s$ for $s = \{0, 1\}$ on the term that represents RAS in equation 1.8. If I find that firms with no access to the public debt markets have negative loading on RAS variable, and if I find that firms with access to public debt markets do not load significantly on RAS term, then that will be strong evidence in support of implications (P1) and (P2).

I need two normalizations on factor loadings to identify the model, hence normalizing $\sigma^2 = 1$, and $\sigma_c^2 = 1$ suffices. These normalizations do not change the relative weights of $a_0, a_1$, which is what is the goal of the exercise.

For shorthand, let us define:

\begin{align}
U_0 & = a_0 \theta + \epsilon_0 \\
U_1 & = a_1 \theta + \epsilon_1 \\
U_c & = a_c \theta + \epsilon_c 
\end{align}

(A.1)

The additional investment $D$ that the borrowing firm can make if the firm has access to public
debt markets is:

\[ D = \mathbb{E}[(I_1 - I_0) - C I | \mathcal{I}] \]

\[ = X(\beta_1 - \beta_0) - Z\gamma + \mathbb{E}((U_1 - U_0) - U_c | \mathcal{I}), \quad (A.2) \]

where \( \mathcal{I} \) represents the information set available to the borrowing firm making the choice between public debt markets and continued relationship with the lending bank. Using the definitions from equation (A.1) I get:

\[ D = X(\beta_1 - \beta_0) - Z\gamma - [(a_0 - a_1) + a_c] \theta - \epsilon_c, \quad (A.3) \]

Please note that the above decomposition assumes that agents know the \( \beta, \gamma, \) and \( a \). For the sake of clarity in derivation, I now define two terms that represent the costs and benefits in equation (A.3):

\[ \tilde{\mu}_I(X, Z) \equiv X(\beta_1 - \beta_0) - Z\gamma \quad (A.4) \]

\[ \varphi \equiv [(a_0 - a_1) + a_c] \theta + \epsilon_c \quad (A.5) \]

The above definition implies that \( \varphi \sim N(0, \sigma^2\varphi) \) where

\[ \sigma^2\varphi = [(a_0 - a_1) + a_c]^2 + 1 \quad (A.6) \]

Furthermore, we know that \( \text{Cov}(\epsilon_s, \varphi) = 0 \) for \( s = 0, 1 \) and

\[ \text{Cov}(\theta_1, \varphi) = [(a_0 - a_1) + a_c] \quad (A.7) \]

The decision rule of whether to pay the cost and access the public debt markets or to continue in a relationship with the bank and suffer RAS, depends on whether \( \tilde{\mu}_I(X, Z) \) is less than or more than can now be rewritten as:

\[ S = 1 \iff \varphi \leq \tilde{\mu}_I(X, Z), \quad S = 0 \iff \varphi > \tilde{\mu}_I(X, Z), \quad (A.8) \]
Thus I can calculate the expectation of firms accessing public debt markets:

\[
\mathbb{E}(S|X, Z) = P(S = 1|X, Z) = P(\varphi \leq \tilde{\mu}_I(X, Z)) = \Phi\left(\frac{X_1(\beta_1 - \beta_0)}{\sigma_\varphi} - Z \cdot \frac{\gamma}{\sigma_\varphi}\right) \tag{A.9}
\]

We can show that mean observed investment satisfies:

\[
\begin{align*}
\mathbb{E}(I_0|X, Z, \varphi > \tilde{\mu}_I(X, Z)) &= X_0 \beta_0 + \frac{\alpha_0 \left(\alpha_0 - \alpha_1\right) + \alpha_c}{\sigma_\varphi} 
\frac{\phi\left(\frac{\tilde{\mu}_I(X, Z)}{\sigma_\varphi}\right)}{1 - \Phi\left(\frac{\tilde{\mu}_I(X, Z)}{\sigma_\varphi}\right)} 
\tag{A.10} \\
\mathbb{E}(I_1|X, Z, \varphi \leq \tilde{\mu}_I(X, Z)) &= X_1 \beta_1 + \frac{\alpha_1 \left(\alpha_0 - \alpha_1\right) + \alpha_c}{\sigma_\varphi} 
\frac{\phi\left(\frac{\tilde{\mu}_I(X, Z)}{\sigma_\varphi}\right)}{\Phi\left(\frac{\tilde{\mu}_I(X, Z)}{\sigma_\varphi}\right)} 
\tag{A.11}
\end{align*}
\]

I now apply Heckman (1976) two-step procedure. From equation (A.9), I obtain consistent estimator of \(\frac{\gamma}{\sigma_\varphi}\) and \(\frac{\beta_1 - \beta_0}{\sigma_\varphi}\). From equation (A.10) and (A.11), I obtain consistent estimator of \(\beta_s\) for \(s = 0, 1\). Now, to identify \(\alpha_0, \alpha_1\), I use the variance and covariance relationships:

\[
\begin{align*}
\text{Cov}(I_0, D|X, Z) &= \alpha_0 \left(\alpha_0 - \alpha_1\right) + \alpha_c \\
\text{Cov}(I_1, D|X, Z) &= \alpha_1 \left(\alpha_0 - \alpha_1\right) + \alpha_c \tag{A.12}
\end{align*}
\]

The scaled covariance gives us a relation between the loadings on RAS in the two groups:

\[
\frac{\text{Cov}(I_1, D|X, Z)}{\text{Cov}(I_0, D|X, Z)} = \frac{\alpha_1}{\alpha_0} \tag{A.13}
\]

The other moment condition can be obtained from the variance equations:

\[
\begin{align*}
\text{Var}(I_0, D|X, Z) &= \alpha_0^2 + \sigma_\epsilon^2 \\
\text{Var}(I_1, D|X, Z) &= \alpha_1^2 + \sigma_\epsilon^2 \tag{A.14}
\end{align*}
\]

The above equations provide the following moment condition:

\[
\text{Var}(I_1, D|X, Z) - \text{Var}(I_0, D|X, Z) = \alpha_1^2 - \alpha_0^2 \tag{A.15}
\]

Using moment conditions (A.13) and (A.15), I identify \(\alpha_0, \alpha_1\). Furthermore, once I have \(\alpha_0, \alpha_1, \) I
also identify $\sigma_e$ using the set of variance equations A.14.

### A.2 Likelihood Estimation for Selection Model

I calculate likelihood by constructing the likelihood conditional on the unobservable $\theta$ and then integrating $\theta$ out. First, I assume that the factor $\theta$ is observable. In this case, the data would be $(I, S, X, \theta)$ where

$$Y = (1 - S)I_0 + SI_1,$$  \hspace{1cm} (A.16)

and let $(I^i, s^i, X^i, \theta^i)$ denote the data from firm $i$. The likelihood is:

$$L = \prod_{i=1}^I \int [f(y^i_0 | s^i = 0, \theta^i)\mathbb{P}(s^i = 0|\theta^i)f(\theta^i)]^{1-s^i} [f(y^i_1 | s^i = 1, \theta^i)\mathbb{P}(s^i = 1|\theta^i)f(\theta^i)]^{s^i}d\theta^i$$ \hspace{1cm} (A.18)

where

$$f(\theta^i) = (2\pi)^{-1}(\sigma_1^2 \sigma_2^2)^{-\frac{1}{2}} \exp(-\frac{1}{2\sigma_1^2}(\theta^i)^2)$$

$$f(y^i_0 | s^i = 0, \theta^i) = (2\pi)^{-\frac{1}{2}}(\sigma_2^2)^{-\frac{1}{2}} \exp\{-\frac{1}{2\sigma_2^2}(I^i_0 - X^i \beta_0 - \alpha_0 \theta^i)^2\}$$

$$f(y^i_1 | s^i = 1, \theta^i) = (2\pi)^{-\frac{1}{2}}(\sigma_2^2)^{-\frac{1}{2}} \exp\{-\frac{1}{2\sigma_2^2}(I^i_1 - X^i \beta_1 - \alpha_1 \theta^i)^2\}$$

$$\mathbb{P}(s^i = 1|\theta^i) = \Phi(\tilde{\mu}_i(X^i, Z^i) - [(\alpha_0 - \alpha_1) + \alpha_c] \theta^i)$$ \hspace{1cm} (A.17)

Now I recognize the fact that we do not observe $\theta$, hence we can not write the likelihood conditional on $\theta$. I integrate it out:

$$L = \prod_{i=1}^I \int [f(y^i_0 | s^i = 0, \theta^i)\mathbb{P}(s^i = 0|\theta^i)f(\theta^i)]^{1-s^i} [f(y^i_1 | s^i = 1, \theta^i)\mathbb{P}(s^i = 1|\theta^i)f(\theta^i)]^{s^i}d\theta^i$$ \hspace{1cm} (A.18)

The log-likelihood can thus be written as:

$$l = \sum_{i=1}^I \ln \int [f(y^i_0 | s^i = 0, \theta^i)\mathbb{P}(s^i = 0|\theta^i)f(\theta^i)]^{1-s^i} [f(y^i_1 | s^i = 1, \theta^i)\mathbb{P}(s^i = 1|\theta^i)f(\theta^i)]^{s^i}d\theta^i$$ \hspace{1cm} (A.19)
A.3 Data Description

A.3.1 Bank Variables

- **Total Assets of the Bank**: RCFD2170 from Commercial Bank Data.
- **Total Loans extended**: RCFD2122 from Commercial Bank Data.
- **Loan Ratio**: Total loans as a fraction of total assets of the bank.
- **Tier 1 Capital of the Bank**: RCFD8274 from Commercial Bank Data.
- **Capital Ratio**: Total tier 1 capital as a fraction of total assets of the bank.
- **Bank Holding Company ID**: RSSD9348 (See Structure and Geographical Variables).

A.3.2 Firm Variables (Annual)

Annual data items from the CRSP/Compustat Merged database, are first listed, followed by the calculations underlying the constructed variables. The construction of the variables follows Eberly, Rebelo, and Vincent (2008), and hence this part of the appendix also follows the same.

- **I, Investment**: expenditures on property, plant, and equipment, data 30

- **CF, CashFlow**: Income before extraordinary items + depreciation and amortization + minor adjustments, calculated as follows (from the Compustat manual): Income Before Extraordinary Items, 123 + Depreciation and Amortization, 125 + Extraordinary Items and Discontinued Operations, 124 + Deferred Taxes, 126 + Equity in Net Loss (Earnings), 106 + Sale of Property, Plant, and Equipment and Sale of Investments.Loss(Gain), 213 + Funds from Operations - Other, 217 + Accounts Receivable - Decrease (Increase), 302 + Inventory - Decrease (Increase), 303 + Accounts Payable and Accrued Liabilities - Increase (Decrease), 304 + Income Taxes - Accrued - Increase (Decrease), 305 + Assets and Liabilities - Other (Net Change), 307
• **Inventories**: total inventories (end of period), data 330

• **Debt**: Long-term debt (end of period), data 9

• **PPE**: Book value of capital: property, plant, and equipment, - data 182: PPE - Beginning Balance - check if it is still reported after 1997; - data 187: PPE - Ending Balance (Schedule V); - data 184: PPE - Retirements (Schedule V) - not reported after 1997; - data 185: PPE - Other Changes (Schedule V) - not reported after 1997.

• **Price of capital**: implicit price deflator for nonresidential investment, Economic Report of the President, Table B-3, various years.

• **Market Value of Equity**: Closing stock price times number of common shares outstanding (end of period) plus redemption value of preferred stock (end of period) = \( \text{prc} \times \text{shrout}/1000 + \text{data56} \), where,
  - prc: closing stock price ;
  - shrout: Common shares outstanding ;
  - data 56: Preferred Stock - Redemption Value.

• **L**: Useful life of capital goods: by two-digit industry, the useful life of capital goods is calculated as \( L_j \equiv \frac{1}{N_j} \sum_{i \in j} \frac{PPE_{i,t-1} + DEPR_{i,t-1} + I_{i,t}}{DEPR_{i,t}} \), where \( N_j \) is the number of firms, \( i \), in industry \( j \). Using the double-declining balance method, the implied depreciation rate for industry \( j \), \( \delta_j \), is \( 2/L_j \).

• **K, replacement value of capital stock**: Using the method of Salinger and Summers (1983) the replacement value of the capital stock is constructed by firm from its book value using the recursion: \( K_{i,t} = \left( K_{i,t-1} \frac{P_{K,t}}{K_{K,t-1}} + I_{i,t} \right)(1 - \delta_j) \), where the recursion is initialized using the book value of capital.

• **Q, Tobin’s Q**: \( [(\text{market value of equity})_{t-1} + (\text{debt})_{t-1} - (\text{inventories})_{t-1}] / K_t \).
Appendix B

Supplement to Heterogeneity in Corporate Governance: Theory and Evidence

B.1 A Simple Static Model

The firm with capital $K$ pays an adjustment cost $\phi(I/K)$ on its investments $I$. Thus, the total cost per unit capital of investment is:

$$c(i) \equiv \frac{I}{K} + \phi(I/K)$$

The firm faces a depreciation rate of $\delta > 0$ on its capital. Therefore, the law of motion of capital is:

$$dK_t = (I_t - \delta K_t)dt, \quad t \geq 0$$

The cash flow process for the firm is as follows:

$$dY_t = K_t(dA_t - c(i_t)dt)$$

where $A_t$ is the production technology of the firm.

The agent’s efforts $a_t$ and the autonomy $M_t$ given to the agent affects the firm’s production technology as below:

$$dA_t = a_t M_t \mu dt + \sigma dZ_t, \quad t \geq 0$$

The agent enjoys private benefits at the rate $\lambda(M_t - a_t)\mu dt$.

Suppose that the performance $q$ of a firm is equal to effort $a$ times management autonomy $m$ with return to scale $\gamma$ minus money $d$ that the agent can divert without possible verification.
by the principal plus noise $\epsilon$:

$$q = am^\gamma - d + \epsilon,$$

where $\epsilon$ is normally distributed with zero mean and variance $\sigma_\epsilon^2$. The principal is assumed to be risk neutral. The agent has constant absolute risk averse (CARA) risk preferences represented by the following negative exponential utility function:

$$u(w, a) = -e^{-\eta[w + d - \psi(a)]},$$

where $w$ is the monetary compensation and $\eta > 0$ is the agent’s coefficient of absolute risk aversion. Cost of effort $\psi(a)$ is assumed to be quadratic: $\psi(a) = \frac{1}{2}ca^2$. Suppose that the principal and agent can write only linear contracts of form:

$$\hat{w} = t + sq,$$

where $t$ is the fixed compensation level and $s$ is the variable performance related compensation of compensation and $\hat{w}$ is the contracted wage. The amount $d$ that the agent can divert is proportional to autonomy $m$ and private information $u$ that the agent has about the output of the firm in the next period. The shock $\epsilon$ (say at time $t$) experienced by the firm is composed of two parts: $u$ that is observed by the agent before the period starts (at time $t - 1$), and a part $v$ which is unknown to both the principal and an agent.

$$\epsilon = u + v,$$

The agent chooses to divert a constant fraction $\theta$ of observed shock. But she does so only when the shock observed is positive, i.e. $u > 0$. This means the real wage the agent is getting is:

$$w = \hat{w} + d = t + sq + \theta 1(u > 0)$$

The principal maximizes the output of the firm after paying the wage of the agent, subject to the agent’s IC and IR constraints. The principal’s problem is then to solve:

$$\max_{a, m, s, t} \mathbb{E}(q - w)$$

s.t.  

$$a \in \arg\max_a \mathbb{E}\left[-e^{-\eta[w - \frac{1}{2}ca^2]}ight]$$

$$e^{-\eta \mu m^\mu} \leq \mathbb{E}\left[-e^{-\eta[w - \psi(a)]}\right]$$

The first constraint above is the IC constraint of the agent - agent chooses her effort to optimize

---

1This constraint is not required in the general case, but helps convey our point. In later sections, we will consider a larger contract space.

2This assumption can also be generalized. However, we will not do so in this section. The full model presented later does not make this assumption.
her total wage. The IR constraint, follows, where \( \bar{w} \) gives the reservation wage of the agent. As in Gabaix and Landier (2007) and Edmans, Gabaix and Landier (2007), we make the assumption that talent of CEOs vary. The principal first solves the optimal autonomy problem and then picks an agent from a pool with the correct talent level to handle optimal autonomy. Thus, autonomy and talent, have one to one correspondence to each other and as talent increases, autonomy increases as well and vice-versa. The reservation wage of such an agent, hence, also is proportional to autonomy \( m \) raised to a factor \( \mu \) that represents talent.

The utility function of the agent can be rewritten as:

\[
u(a, d) = -e^{-\eta t + s(a m^\gamma - d + \epsilon) + \frac{1}{2} c a^2}
= -e^{-\eta t + s a m^\gamma + d(1-s) - \frac{1}{2} c a^2 + s \epsilon}
= -e^{-\eta t + s a m^\gamma + \frac{1}{2} c a^2 + f(\eta, m^2, \sigma_u^2, \chi(1-s) - \frac{1}{2} s^2 \sigma_e^2)}
\equiv -e^{-\eta \hat{w}(a, m)}, \tag{B.7}
\]

where \( \hat{w}(a, m) \) is the certainty equivalent compensation of the agent. The certainty equivalent wage consists of her expected compensation and option value of private diversion net of cost of effort and risk premium. \( f(\eta, m^2, \sigma_u^2) \) represents the option value of private diversion. Risk premium \( \frac{1}{2} s^2 \sigma_e^2 \) is increasing in coefficient of risk aversion \( \eta \), share of firm output \( s \) and variance of firm output \( \sigma_e^2 \). The option value increases in agent autonomy \( m \), risk aversion \( \eta \) and variance of private information about firm output \( \sigma_u^2 \). However, the option value is multiplied by \( 1-s \) as the agent denies herself a fraction \( s \) of the output of the firm if she diverts before reporting firm performance.

The optimization problem of the agent is to get the most certainty equivalent wage by choosing effort \( a \):

\[
\begin{align*}
\arg\max_a \hat{w}(a, m) & = t + s a m^\gamma - \frac{1}{2} c a^2 + f(\eta, m^2, \sigma_u^2, \chi(1-s) - \frac{1}{2} s^2 \sigma_e^2) \\
sm^\gamma - ca & = 0 \\
a^* = \frac{sm^\gamma}{c}, \tag{B.8}
\end{align*}
\]

i.e. the optimal effort by the agent increases in fraction of output she receives, the autonomy she enjoys and decreases in the cost of effort she has to exert. We can now obtain the maximum certainty equivalent payoff for the agent in terms of her effort \( a^* \):

\[
\hat{w}(a^*, m) = t + \frac{1}{2} \frac{s^2 m^2}{c^2} + f(\eta, m^2, \sigma_u^2, \chi(1-s)(1-s) - \frac{1}{2} s^2 \sigma_e^2) \tag{B.9}
\]

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Given the optimal level of effort $a^*$ by the agent, principal solves the following problem:

$$\max_{m,t,s} \mathbb{E}[q-w] = \mathbb{E}[q-t-sq]$$

$$= \mathbb{E}[-t+(1-s)q]$$

$$= \mathbb{E}[-t+(1-s)(am-d+\epsilon)]$$

$$= -t+(1-s)\left(\frac{sm^{2y}}{c} - g(\eta,m^2,\sigma_u^2)\right) \quad \text{(B.10)}$$

where the agent's optimal effort enters as a constraint:

$$t + \frac{1}{2} s^2 m^{2y} \gamma c + f(\eta,m^2,\sigma_u^2)(1-s) - \frac{\eta}{2} s^2 \sigma^2 = \bar{w} m^\mu \quad \text{(B.11)}$$

$\mathbb{E}[d] = g(\eta,m^2,\sigma_u^2)$ represents private diversion that the principal expects the agent to make. \textit{This may not be the real amount diverted.} The dynamic game of interest is to figure out the optimal policy of diversion of the agent, given that she knows that the principal is updating his expectation about agent's diversion from output reported. The difference between $f(.)$ and $g(.)$ will decide the autonomy chosen by the principal, as we shall see later in equation \text{(B.14)}.

The Lagrangian in this case is:

$$\mathcal{L} = -t+(1-s)\left(\frac{sm^{2y}}{c} - g(\eta,m^2,\sigma_u^2)\right) + \lambda\left(t + \frac{1}{2} s^2 m^{2y} \gamma c + f(\eta,m^2,\sigma_u^2)(1-s) - \frac{\eta}{2} s^2 \sigma^2 - \bar{w} m^\mu\right)$$

on differentiating w.r.t. $t$ we get:

$$\lambda = 1$$

Differentiating w.r.t. $s$, we get:

$$\frac{\partial \mathcal{L}}{\partial s} = -s m^{2y} \gamma c + m^{2y} \sigma^2 - \eta s \sigma^2$$

This yields optimal fraction $s^*$ in terms of variance and autonomy:

$$s^* = \frac{m^{2y}}{m^{2y} + \eta c \sigma^2}, \quad \text{(B.13)}$$

i.e. the fraction of the performance to be given to the agent increases with the autonomy enjoyed by the agent, and decreases with variance of output, risk aversion of agent and cost of exerting effort by the agent. As agent autonomy increases, fraction of firm performance does not increase, then agent's incentive to divert will increase, and hence the result is intuitive.
Differentiating the lagrangian w.r.t. autonomy $m$, we get:

$$\frac{\partial \mathcal{L}}{\partial m} = \frac{\partial}{\partial m} \left[ (1-s) \left( \frac{s m^{2\gamma}}{c} - g(\eta, m^2, \sigma_u^2) \right) + \frac{1}{2} s^2 m^{2\gamma} + f(\eta, m^2, \sigma_u^2) (1-s) - \bar{w} m^\mu \right]$$

$$= \frac{\partial}{\partial m} \left[ (1-s) \left( \frac{s m^{2\gamma}}{c} - g(\eta, m^2, \sigma_u^2) + f(\eta, m^2, \sigma_u^2) \right) + \frac{1}{2} s^2 m^{2\gamma} - \bar{w} m^\mu \right]$$

$$= (1-s) \left( \frac{2s m^{2\gamma-1}}{c} - g'(\eta, m^2, \sigma_u^2) + f'(\eta, m^2, \sigma_u^2) \right) + \frac{\gamma s m^{2\gamma-1}}{c} - \mu \bar{w} m^{\mu-1}$$

Equating the right hand side to zero, we get:

$$[(1-s)c f_m(\eta, m^2, \sigma_u^2) - g_m(\eta, m^2, \sigma_u^2)] + [(2-s)\gamma s m^{2\gamma-1} - \mu \bar{w} m^{\mu-1}] = 0 \quad (B.14)$$

Taking the constant return to scale case on autonomy i.e. $\gamma = 1, \mu = 1$ and assuming principal expects no diversion i.e. $g(\eta, m^2, \sigma_u^2) = 0$ we have:

$$(1-s)c f_m(\eta, m^2, \sigma_u^2) = \mu \bar{w} - (2-s)\gamma s m \quad (B.15)$$

Option value increases in $\sigma_u^2$. Assuming that private information $u$ is a (stationary) fraction of total information $c$, we have shown that there is a negative relation between observed output variance and autonomy.

Section 2.2.1 provides a dynamic model and shows that the negative relation exists there as well.

### B.2 Proof of Lemma 2.2.2


By Ito’s Lemma, $d V(t, K(t), Y(t), M(t)) = d V(t, X(t))$ is given by:

$$d V(t, X(t)) = \left[ V_t(X(t), t) + V_X(X(t), t)^T \mu(X(t)) + \frac{1}{2} tr [V_{XX}(X(t), t) \sigma(K) \sigma(K)^T] \right] d t$$

$$+ V_X(X(t), t)^T \sigma(K) d w(t) \quad (B.16)$$

Rewriting agent’s bellman condition 2.10 we have:

$$V_t + V_X \mu(X(t)) + \frac{1}{2} tr [V_{XX}(X(t), t) \sigma(K) \sigma(K)^T] = R_t V(t, K) [g(t, X) + h(t, X) \mu(X) - c(t, \eta, M, d)]$$

$$- \frac{1}{2} R_t \sigma(X) [h(t, X) h^T(t, X) \sigma(X)^T(X)] + V_{X} \sigma(X) [h(t, X) \sigma(Y) \sigma(Y)^T] \quad (B.17)$$
Substitute for the drift term in equation B.16 with the above equation, we get:

\[ d V(t, X(t)) = \left[ R_A V(t, K) \left[ g(t, K) + h(t, K) \mu(X) - c(t, \eta(t), M(t), \bar{d}) \right] - \frac{1}{2} R_A \sigma(X) h(t, X) h^T(t, X) \sigma(X) \right] + V_S R_A \sigma(Y) h(t, X) \sigma^T(Y) d t + V_X \sigma(Y) d w \]  

(B.18)

We now define \( W(t) \) such that \( R_A W(t) = - \log(-R_A V(T, X(t))) \), where \( W_T = q(T, K_T) \) as defined in equation 2.8.

Differentiating \( W(t) \), and using the expression of \( d V(t, X(t)) \), we get:

\[ R_A d W(t) = - \frac{d V}{V} + \frac{1}{2} \left( \frac{d V}{V} \right)^2 \]

\[ = - R_A \left[ g + h \mu(X) - c - \frac{1}{2} R_A \bar{h} \sigma(X) \sigma^T(X) \bar{h} \right] d t + R_A (\bar{h} - h) \sigma(Y) d w, \]  

(B.19)

where \( \bar{h} \equiv h - \frac{V_k}{R_A V} \).

The last expression can be rearranged as:

\[ d W(t) + g(.). d t + h(\mu(X) d t + \sigma(Y) d w) = c d t + \frac{R_A}{2} \bar{h} \sigma(Y) \sigma^T(Y) \bar{h} d t + \bar{h} \sigma(Y) d w \]  

(B.20)

which is the same as:

\[ S(t) = \bar{W}_M + \int_0^T c(t, \eta(t), M(t), \bar{d}) d t + \frac{R_A}{2} \int_0^T \bar{h} \sigma(Y) \sigma^T(Y) \bar{h} d t + \int_0^T \bar{h} \sigma(Y) d w(t) \]  

(B.21)

B.3 Proof of Theorem 2.2.3

The investor's value function is given by:

\[ J(t, K, M) = \sup_{\eta} \mathbb{E}_0 \left[ - \frac{1}{R_p} e^{-R_p(d W_t - d S_t)} \right], \]

where \( K \) and \( M \) are the initial capital and autonomy values. Thus, we get the principal's Bellman equation:

\[ \sup_{\eta} J \left[ R_p c + \frac{R_A R_p}{2} \bar{h}^2 \sigma^2_K \right] + R_p^2 \bar{h}^2 \sigma^2_K + J_t + J_W \left[ r W + \mu + R_p \bar{h} \sigma^2_K \right] + \frac{1}{2} J_{WW} \sigma^2_K + J_K \left[ \mu_K + R_p \sigma^2_K \bar{h} \right] + \frac{1}{2} J_{KK} \sigma^2_K + J_{WK} \sigma^2_K = 0. \]  

(B.22)
Conjecture that the principal’s value function is given by:

\[ f(t, K_t, M_t) = -\frac{1}{R_p} \exp[-R_p(f_1(t)K_t + f_2(t))], \]

where \( f_1(t), f_2(t) \) are continuous deterministic functions of time with boundary conditions \( f_1(\infty) = 1, f_2(\infty) = 0 \).

The Bellman equation then becomes:

\[
\sup_{\eta} \left[ c + \frac{R_A + R_p}{2} \sigma^2_K \dot{\eta}^2 - \dot{f}_1(t)K - \dot{f}_2(t) - f_1(t) \left( \mu_K + R_p \dot{h} \sigma^2_K \right) + \frac{R_p}{2} f_1^2(t) \sigma^2_K \right] = 0
\]

To satisfy the above Bellman equation, we must have the following condition (by taking the coefficient of \( W \) from above):

\[ \dot{f}_1(t) + \mu_K f_1(t) = 0 \quad \text{or} \quad f_1 = e^{-\int_t^{\infty} \mu_K dt} \]

The FOC with respect to \( \eta \) now reduces to:

\[
\kappa \eta + \frac{(R_A + R_p)}{\log M} \sigma^2_K \dot{\eta} \left( \frac{\partial}{\partial \eta} \kappa(t) \right) + \kappa(t) \eta \eta \left( \frac{\partial}{\partial \eta} f_1(t) \right) \left( K^\alpha M^\eta + R_p \frac{\kappa(t) \eta}{K^\alpha M^\eta \log M} \sigma^2_K \right) = 0
\]

which yields an implicit solution for \( \eta^* \):

\[
\kappa \eta^* + \frac{(R_A + R_p)}{\log M} \sigma^2_K \dot{\eta} \left( \frac{\partial}{\partial \eta} \kappa(t) \right) + \kappa(t) \eta \eta \left( \frac{\partial}{\partial \eta} f_1(t) \right) \left( K^\alpha M^\eta + R_p \frac{\kappa(t) \eta}{K^\alpha M^\eta \log M} \sigma^2_K \right) = 0
\]

The FOC with respect to \( M \) reduces to:

\[
-d(t) + (R_A + R_p) \sigma^2_K \dot{\eta} \left( \frac{1}{\log M} \frac{\partial}{\partial M} \right) - \frac{\partial}{\partial M} e^{-\int_t^{\infty} \mu_K dt} \left( K^\alpha M^\eta + R_p \frac{\kappa(t) \eta}{K^\alpha M^\eta \log M} \sigma^2_K \right) = 0
\]

which yields:

\[
-\zeta(t)M - \frac{(R_A + R_p) \sigma^2_K \dot{\eta} \left( \frac{1}{\log M} \frac{\partial}{\partial M} \right)}{\left( \log M \right)^2 K^\alpha M^{\eta+1}} - e^{-\int_t^{\infty} \mu_K dt} \left( \eta K^\alpha M^{\eta-1} - \frac{R_p \kappa(t) \eta \left( \frac{1}{\log M} \frac{\partial}{\partial M} \right) \eta}{\left( \log M \right)^2 K^\alpha M^{\eta+1}} \right) + \eta K^\alpha M^{\eta-1} e^{-\int_t^{\infty} \mu_K dt} \left( K^\alpha M^\eta + R_p \frac{\kappa(t) \eta}{K^\alpha M^\eta \log M} \sigma^2_K \right) = 0
\]
B.4 Proof of Theorem 2.2.4

The principal’s value function is given by:
\[
J(t, K, M) = \sup_{d, u} \mathbb{E}_0 \left[ -\frac{1}{R_p} e^{-R_p(dW_t - dS_t)} \right],
\]
where \( K \) and \( M \) are the initial capital and autonomy values. As in Davis and Norman (1990), to easily characterize optimal policies, let us consider a restricted class of policies in which \( D \) and \( U \) are constrained to be absolutely continuous with bounded derivatives, i.e.
\[
D_t = \int_0^t d_s d_s, \quad U_t = \int_0^t u_s d_s, \quad 0 \leq l_s, u_s \leq \theta,
\]
(B.23)

Thus, we get the principal’s Bellman equation:
\[
\sup_\eta J(t, K, M) = -\frac{1}{R_p} \exp\left[-R_p(f_1(t)K + f_2(t) + \phi(M))\right],
\]
where \( f_1(t), f_2(t) \) are continuous deterministic functions of time with boundary conditions \( f_1(\infty) = 1, f_2(\infty) = 0 \).

The Bellman equation then becomes:
\[
\sup_\eta \left[ c + \frac{R_A + R_p}{2} \bar{h}^2 \sigma_k^2 - f_1(t)K - f_2(t) + f_1(t) \left( \mu_K + R_p \bar{h} \sigma_k^2 \right) + \frac{R_p}{2} f_1^2(t) \sigma_k^2 \right. \\
\left. - \phi_M \left( rM + R_p \bar{h} \sigma_k^2 \right) + \frac{R_p}{2} \phi_MM \sigma_k^2 + \frac{R_p}{2} f_3(t) \phi_M^2 \sigma_k^2 + \frac{R_p}{2} f_1(t) \phi_M \sigma_k^2 \\
+ \left[ \phi_M - (1 + \lambda_-) f_1(t) \right] I + \left[ f_1(t)(1 - \lambda_+) - \phi_M \right] u = 0 \right] = 0
\]

To satisfy the above Bellman equation, we must have the following condition (by taking the coefficient of \( K \) from above):
\[
f_1(t) + \mu_K f_1(t) = 0 \quad \text{or} \quad f_1 = e^{-\int_0^t \mu_K dt}
\]

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Taking the coefficients of controls $l, u$:

$$e^{-\int_{t}^{t+dt} \mu_K dt} (1 - \lambda) \leq \phi_M \leq (1 + \lambda) e^{-\int_{t}^{t+dt} \mu_K dt}$$

### B.5 Proof of Theorem 2.2.8

We give a quick intuition first:

$$\hat{d} = \mu + \lambda y, \quad \hat{\tilde{d}} = \alpha + \beta v$$

Given the rules above, diversion can be written as:

$$(\tilde{d} - \hat{d}(\tilde{d} + \hat{u}))(\sigma^2) = (v - \mu - \lambda \sigma^2)\sigma^2$$

We thus have from FOC:

$$1/\beta = 2\lambda, \quad \alpha = -\mu \beta$$

In equilibrium,

$$\hat{d} = \mathbb{E}(\tilde{d}|\alpha + \beta \tilde{v} + \tilde{u} = y),$$

which yields:

$$\lambda = \frac{\beta \text{var}(v)}{\beta^2 \text{var}(v) + \text{var}(u)}, \quad \mu = 0$$

Thus optimal diversion in equilibrium is proportional to the variance of output of the firm.

Now for the full proof. Substituting the laws of motion of capital and optimal contract, the agent's objective function becomes:

$$\sup_{V,D} \mathbb{E}_0 - \frac{1}{R_A} \exp \left\{-R_A \left[ W_M - \hat{d} + \tilde{d} + \frac{R_A}{2} \tilde{h}^2 \sigma^2_K d t - \tilde{h} \mu_K \right] d t + \tilde{h} \left( K_i^{\alpha} M^\eta + (r + \delta) K_i - \hat{d} \right) d t + \sigma_y d w \right\}$$

which can be written as:

$$\sup_{V,D} \mathbb{E}_0 - \frac{1}{R_A} \exp \left\{-R_A \left[ (g(K, M, \eta)) + \tilde{d}(1 - \tilde{h}) - \hat{d} \right] d t + \sigma_y d w \right\},$$

where $g(\cdot)$ collects all the terms which do not depend on the choice of $\tilde{d}$ in the present period. Taking out $g(K, M, \eta)$ from the exponential so that $\exp g(\cdot) \equiv G(\cdot)$, we have:

$$\sup_{V,D} \mathbb{E}_0 - \frac{G(K, M, \eta)}{R_A} \exp \left\{-R_A \left[ \tilde{d}(1 - \tilde{h}) - \hat{d} - \frac{1}{2} \tilde{h}^2 \sigma^2_U \right] d t \right\},$$

As this is a negative exponential utility function, the optimal policy $V$ for the principal given agent's policy $D$ is to have policy:

$$\tilde{d}(1 - \tilde{h}) - \hat{d} - \frac{1}{2} \tilde{h}^2 \sigma^2_U = 0.$$
i.e.:

\[ \tilde{d} = \frac{1}{1 - h}[\bar{d} + \frac{\bar{h}^2}{2}\sigma^2_{\bar{U}}] \]

In equilibrium, if \( \bar{d} = \bar{v} \), then:

\[ \bar{d} = \frac{1}{1 - h}[\bar{v} + \frac{\bar{h}^2}{2}\sigma^2_{\bar{U}}] \]  \hspace{1cm} (B.25)
Bibliography


