6-1-2007

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Keywords
conformal antennas, cylindrical antennas, leaky waves, metamaterials

Disciplines
Electrical and Computer Engineering

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Theory and Simulations of a Conformal Omni-Directional Subwavelength Metamaterial Leaky-Wave Antenna

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Abstract—The detailed theory of a single subwavelength conformal radiator that exploits the resonant properties of thin cylindrical metamaterial shells supporting leaky waves is presented. It is shown and reviewed analytically and numerically how a circularly symmetric resonant leaky mode may be supported by a properly designed subwavelength homogenous cylindrical shell of low negative permittivity. Some physical insights are provided and numerical simulations with a feed point, considering also the finiteness of the antenna in the longitudinal direction, are presented and discussed. Moreover, possibilities and limitations of a practical realization of this setup are mentioned, considering in details the possible anisotropies in the metamaterials.

Index Terms—Conformal antennas, cylindrical antennas, leaky waves, metamaterials.

I. INTRODUCTION

In cellular and satellite applications using omnidirectional antennas with down-tilted directive beams, the need for new-generation compact conformal radiators with a low-profile structure has increased. One approach requires the use of multiple directive radiators tilted towards the ground to cover the cell of interest without interfering with the other neighboring cells. This requires a certain overall size of the radiating system, together with a corresponding complexity in its design and in the feeding technique.

The recent advances in development of metamaterials may provide novel techniques in antenna design that may overcome some of the inherent limitations of conventional materials commonly used in loading and building radiators. For example, the use of double-negative (DNG) [1] and single-negative (SNG) [2] metamaterials has been shown theoretically to be promising as antennas and resonators. DNG metamaterials are characterized by having negative values for the real parts of their effective permittivity and permeability in a given frequency range, whereas SNG materials have only one of the two constitutive parameters negative, distinguishing them into ε-negative (ENG) and μ-negative (MNG) materials. When these materials are coupled together and/or with standard dielectrics, the complementary interfaces separating them may support plasmonic resonances [2], which are useful in squeezing the resonant dimensions in several electromagnetic setups (see, e.g., [3]–[11]). In particular, applications to antenna setups have been envisioned in recent works, applying these resonances to patch antennas [7]–[9], short dipoles [10] and leaky-wave antennas [11].

Another class of metamaterials having low-positive or low-negative permittivity or permeability, called ε-near-zero (ENZ) and μ-near-zero (MNZ) materials, have been shown to support leaky-wave and polariton modes with directive radiation properties, with potential applications in different setups [11]–[13]. The possibility of manipulating the phase patterns of radiation fields with such low-index materials may be heuristically envisioned by resorting to the low variation of the phase distribution at the interface between an ENZ or MNZ material and a conventional medium (e.g., free space) [14]. This property may have interesting leaky-wave antenna applications (see, e.g., [12], [13] and references therein). Combining the salient features of low-index materials and the interface resonance of oppositely signed media has been shown theoretically to be useful in the design of planar leaky-wave antenna setups [11].

In the cylindrical and spherical geometry, subwavelength polaritonic resonances of different multipole orders of Mie type may be obtained by using these same families of metamaterials, as shown for scattering problems in [6].

Related to these anomalous properties of ENG and ENZ metamaterials, we have recently proposed a setup involving a low-permittivity ENG cylindrical shell supporting a resonant circularly-symmetric (i.e., azimuthally independent) leaky-wave [15]–[17]. This may provide the possibility for designing antennas with omnidirectional patterns in the azimuthal plane and a highly directive beam in the elevation angle, down-tilted towards the antenna’s cylindrical axis, without having its radiation pattern to overlap with that of a neighboring antenna.

Here we present and review the details of the theory behind our recently proposed setup and provide some full-wave numerical simulations and physical insights into the anomalous radi-
ation properties of such metamaterial shells. The issues on the possible realization of this antenna, its limitation in bandwidth and robustness to design changes are also discussed. Full-wave simulations using finite integration technique (FIT) commercial software CST Microwave Studio [18], considering the presence of a realistic feed and the finiteness of the antenna in the longitudinal direction, are also presented. Possible anisotropies in the metamaterials considered here, which may arise when a practical realization is foreseen, are also fully considered.

It should be mentioned that radiators with similar properties have been proposed in the past exploiting cylindrical rods with periodic corrugations (see, e.g., [19]–[22]). However, the structure proposed here has the advantage of subwavelength cross section, thanks to the properties of metamaterials, and of its complete symmetry in the azimuthal plane, ensuring a totally omnidirectional pattern, thus potentially overcoming the limitations of the other setups proposed in literature for these purposes.

II. LEAKY-WAVE DISPERSION

The geometry we have recently proposed [15]–[17] is depicted in Fig. 1, embedded in a suitable cylindrical system of coordinates (ρ, φ, z). It consists of a cylindrical metamaterial shell with inner radius a_{in}, outer radius a_{out} > a_{in}, permittivity ε and permeability μ_{0}, embedded in air with the same permeability and with permittivity ε_{0}. The system is excited by a monochromatic φ-st time dependence. We are interested in finding the possible conditions on the parameters of this geometry for supporting φ-independent leaky waves traveling along the z axis in the subwavelength case of a_{out} ≪ \min(λ_0, λ), where λ and λ_0 are, respectively, the wavelength in the metamaterial and in free space. This may result in a leaky-wave antenna with sub-wavelength cross section supporting directive beams pointing towards a given desired direction in the elevation plane (i.e., towards a given angle from the ŵ axis), but omnidirectional in the azimuthal plane.

As we mentioned in [17], the leaky-wave field distribution in such a geometry corresponds to a natural mode of this open waveguide with e^{-jβz} longitudinal variation and with no dependence on the φ coordinate. The analysis of the natural modes supported by ENG (plasma) cylinders or cylindrical shells has been carried out in different papers (see, e.g., [23]–[26]), although mainly focused on surface wave propagation. An analogous analysis is applied here for the supported leaky-waves. In the TM^p polarization the electric and magnetic field distributions in this structure may be written as in (1), shown at the bottom of the page, where J_n(·) and Y_n(·) are respectively the Bessel and Neumann functions of the n-th order [27], H_n(·) = J_n(·) - jY_n(·) is the Hankel function of the second kind, k_t = \sqrt{\omega^2μ_0ε - β^2}, k_0 = \sqrt{\omega^2μ_0ε_0 - β^2} are the transverse wave numbers in each material. For leaky-waves, which are the modes of interest here, β is a complex quantity even when ohmic losses are not considered, since its imaginary part takes into account the radiation losses of the mode that couple energy with free space. An expression analogous to (1), may be written for the TE^p case using duality and by substituting the ratio (ε/ε_{0}) with unity.

The complex coefficients c_{n} for the problem at hand may be determined by matching the boundary conditions at the two interfaces, implying the dispersion relation (2), shown at the bottom of the page, for the TM^p natural modes supported by the cylindrical shell of Fig. 1.

Highly directive leaky-waves radiating at the angle \theta = \cos^{-1}(\text{Re}[β/ω\sqrt{ε_{0}μ_{0}}]) from the normal to the cylinder axis are supported by this shell provided that (2) admits complex solutions for β with Re[β] < ω\sqrt{ε_{0}μ_{0}} and Im[β] ≪ ω\sqrt{ε_{0}μ_{0}}. These conditions for the solutions of (2) are hardly met by using standard materials with ε > ε_{0}, since the relevant natural modes of a standard dielectric shell are represented by surface waves with β > ω\sqrt{ε_{0}μ_{0}}. The analysis of leaky and complex modes traveling along cylindrical dielectric rods has been carried out in [28], [29], but it shows narrow frequency
windows of leaky-wave operation, with limited directivity and strict requirements on the transverse thickness of the rods, which cannot be made too thin with respect to the wavelength. A proper periodic corrugation of cylindrical dielectric rods supporting surface modes has been proposed in [19], [20] in order to perturb the surface guided modes into leaky-waves, but this configuration requires the surface mode to be properly bounded around the dielectric shell, so that the perturbation takes effect. Again this requires its transverse cross section to be sufficiently thick to bound energy near its surface.

Metamaterials may provide alternative ways for designing subwavelength shells supporting these leaky modes, even without the need of any periodic corrugation along the cylinder. Similar to what was found in the planar geometry [11], the use of low index materials, together with a properly designed complementary interface, may indeed confine in a transverse subwavelength cross section the desired leaky modes. As we briefly reported in [17], by imposing that $\varepsilon \ll \varepsilon_0$ and that $a_{\text{out}} \ll \text{min}(\lambda_0, \lambda)$, and after algebraic manipulations, (2) may be conveniently simplified into the following design formula:

$$\text{Re}[\beta]^2 \approx \frac{-2\varepsilon/\varepsilon_0}{a_{\text{in}}^2 \ln(a_{\text{out}}/a_{\text{in}})}$$

(3)

involving the real part of the complex guided wave number and the geometrical and electrical parameters of the shell. In this subwavelength limit, the imaginary part of the guided wave number $\beta$ may be shown to tend to zero, as long as a leaky-mode can be supported (and (3) can be consequently satisfied by the geometry considered here). This suggests the possibility of supporting highly-directive leaky-modes in a subwavelength transverse cross section, confirming analytically the previous heuristic discussion. We note that, although vanishingly small, an imaginary part of $\beta$ is present, as required to justify radiation losses.

We reiterate that (3) implies the use of a low-permittivity ENG material (with $\varepsilon < 0$), which explains the anomalous resonance in such a subwavelength shell in terms of the interface resonance phenomenon between materials with oppositely signed constitutive parameters [2]. The electric permittivity here plays an important role because of the chosen $\text{TM}^p$ polarization characterized by a non-zero radial electric field, which becomes dominant inside the metamaterial shell as shown in the following. Analogous results may be obtained with an MNZ shell in the dual polarization. Since we limit our analysis to ENZ materials, however, under the subwavelength assumption $a_{\text{out}} \ll \text{min}(\lambda_0, \lambda)$, (3) ensures the presence of only $\text{TM}^p$ leaky modes supported by the structure of Fig. 1.

We note how the smaller the outer radius of such a shell (i.e., $a_{\text{out}}$) is, the more $[\beta]$ should be close to zero in order to maintain the constraint $\text{Re}[\beta] \ll \omega\sqrt{\varepsilon_0\mu_0}$ in (3). This implies that a too thin cross section for the metamaterial shell would dramatically enhance the radial component of the electric field inside the metamaterial to match the continuity of the electric displacement at the interfaces between the shell and free space. This results in increase of the reactive fields around the feed and of the sensitivity to the unavoidable ohmic losses. In other words, a trade off between robustness of the design and thickness of the subwavelength shell has to be found in conducting a proper subwavelength design, as we discuss in the next section.

As a final note, it may be pointed out that, as in the planar case [11], (3) implies that for the radiation to be close to the normal (i.e., with $\beta \simeq 0$) values of permittivity for the metamaterial shell should be closer and closer to zero, making the setup more sensitive, with a narrower bandwidth and little robustness. The broadside radiation represents a kind of “quasi-static” cut-off for this setup in terms of the required values of permittivity, even though in the ideal case there is not a lower limit on how close such a shell may radiate in the broadside, provided that a material with sufficiently low permittivity may be manufactured. Larger angles of radiation indeed provide a better design robustness, consistently with what we have found in the planar geometry [11].

The subwavelength highly confined (in the transverse plane) $\text{TM}^p$ leaky modes satisfying (3) are backward modes, i.e., their phase velocity travels antiparallel to the net energy flow due to the negative permittivity of the material and to the fact that they concentrate most of the power inside the material shell, again similar to the analogous results found in the planar geometry [11] and to the surface waves guided by plasma cylinders [23]. Here, we do not discuss further details of such anomalous power flow, since this is not of direct interest in the present paper.

### III. Radiation Characteristics

Fig. 2 presents the theoretical findings for the radiation patterns in the elevation plane for a subwavelength cylindrical shell with $\varepsilon = -10^{-7}\varepsilon_0$, $a_{\text{out}} = 1.83\ \text{mm}$, $a_{\text{in}} = 1.5\ \text{mm} = \lambda_0/100$ at the operating frequency $f_0 = 2\ \text{GHz}$, designed according to (3) in order to radiate at an angle of 60° from the broadside direction at the central frequency $f_0$. In this idealized example, no losses have been considered in the involved materials and the patterns have been calculated by applying the reciprocity theorem, similarly to how we derived the radiation features in the planar geometry for analogous leaky-wave antennas [11], [12]. The feed in this example is represented by an infinitesimal electric dipole located at the origin and directed along the $z$ axis. In the elevation plane the calculated directivity at the central frequency $f_0$ is 69.15 dB. Due to the symmetry of excitation and idealized infinite length of the antenna in this case, the plot shows four peaks in the elevation plane since the feed excites two leaky-waves flowing along the positive and
the negative $z$ direction and radiating in the forward- and backward-wave manner. In the realistic scenario of a finite cylinder, where two of these beams are undesired, they may be suppressed by feeding the shell at one end and properly terminating the antenna on the other end to avoid reflections. We show this possibility in a realistic design of the antenna in the next section, where the length of the antenna is chosen to be sufficiently long in order to radiate away most of the energy in the traveling mode, avoiding significant reflections at its termination. The patterns in the azimuthal plane (not reported here, they are available in [17]) are totally symmetric and uniform, due to the symmetry of the structure and of the excitation.

The variation of the radiation pattern of Fig. 2 with frequency is evaluated considering a typical Drude model dispersion for the material permittivity, i.e., $\varepsilon = \varepsilon_0(1 - \omega_p^2/\omega^2)$. It is interesting to note that the sensitivity to the material dispersion is very high in this example, since the permittivity is varying with frequency in this plot, and a small variation of the design parameters noticeably affects the radiation pattern in this very subwavelength design. This is due to the reduced dimensions of the antenna that, as noticed in the previous paragraph, influences the robustness of the design and increases the $Q$ factor of the leaky-mode resonance, consistent with the fundamental Chu limitation for subwavelength radiators consisting of passive materials [31]. The overall bandwidth performance of these subwavelength radiators are indeed limited when passive metamaterials with negative parameters are considered in the design, since they necessarily exhibit temporal dispersion, and thus their parameters vary with frequency, in accordance with the Kramers-Kronig relations [30]. This implies that relation (3) involves an implicit frequency dispersion when the required variation of $\varepsilon$ with frequency is considered, as done in Fig. 2, unless the use of active materials is envisioned. This is consistent with the properties of other subwavelength resonant geometries employing metamaterials [3]–[12].

As a second example, we have conducted a less “extreme” design in order to get a reasonable tolerance over a variation of the design parameters. In this second case, the shell has the following geometry: $\varepsilon = -10^{-2}\varepsilon_0$, $a_{\text{out}} = 17.8$ mm, $a_{\text{in}} = 15$ mm $= \lambda_0/10$ at the same working frequency $f = 2$ GHz. Fig. 3 shows the variation of the radiation pattern of this structure varying the frequency of operation, with a shell permittivity again following a lossless Drude model. In this case at the central frequency the antenna directivity in the elevation plane is reduced to 17.46 dB. It is noticeable how an increase in the shell radius (maintaining still its subwavelength features) improves the performance of the antenna in terms of pattern bandwidth.

Fig. 4 shows how the directivity of the radiation pattern in this second example is affected when losses are considered in the metamaterial, following the conventional lossy Drude model $\varepsilon = \varepsilon_0(1 - \omega_p^2/\omega^2)\left[\varepsilon - j\omega\cdot\mu\right]$. The sensitivity to material losses is reasonable, as the figure shows, and a relatively high directivity may be achieved with realistic material losses included in the model. It is worth mentioning in this context that the values of permittivity in the analysis are close to the plasma frequency...
of the shell metamaterials. This means that at the operating frequencies the materials are relatively far from their resonances and, consequently, material losses may be expected to be relatively low. If we compare these results with our others reported in [17] on the robustness to material losses of the example of Fig. 2, we notice that the example of Fig. 3 here provides a more robust design to the presence of the losses. This is again due to the high Q-factor of the subwavelength resonance in the first example, for which directivity and radiation efficiency are rapidly worsened by the presence of realistic losses in the metamaterial considered here.

In order to provide further insights into the physical behavior of the leaky-wave resonance responsible for these anomalous radiation properties, we report in Fig. 5 the variation of the real and imaginary parts of guided wave number $\beta$ with the shell permittivity [Fig. 5(a)], ratio of radii $a_{\text{out}}/a_{\text{in}}$ [Fig. 5(b)] and frequency of operation [Fig. 5(c)] for the example of Fig. 3.

It is evident how varying each one of these parameters away from the design values of Fig. 3 scans the radiation angle from broadside (for which $\beta = 0$) to endfire (with $\beta \approx k_0$). The imaginary part of $\beta$, albeit sufficiently small to guarantee a high directivity, varies as well with the beam angle, reaching zero at the two cut-off limits for which radiation is no longer present (here the material losses have indeed been neglected and $\text{Im}[\beta]$ is only related to radiation losses). We note that for $\varepsilon < -0.035 e_0$ or $\omega < 0.988 \omega^*$ the antenna enters a surface wave regime and $\text{Im}[\beta] = 0$.

To conclude this section, Fig. 6 depicts the excited leaky-wave field distribution at $f = 2$ GHz. $\eta = \sqrt{\mu_0/\varepsilon}$ is the characteristic impedance in the metamaterial.

![Fig. 5. Variation of the complex wave number $\beta$ for the example of Fig. 4 varying: (a) the shell permittivity $\varepsilon$, (b) the ratio of radii $a_{\text{out}}/a_{\text{in}}$, (c) the frequency of operation.](image)

![Fig. 6. Electric and magnetic near-field distribution for the antenna of Fig. 3 at $f = 2$ GHz. $\eta = \sqrt{\mu_0/\varepsilon}$ is the characteristic impedance in the metamaterial.](image)
by taking into account the presence of a realistic feed, of material loss and dispersion.

IV. NUMERICAL SIMULATIONS

We have simulated the setup of Fig. 1 using finite-integration technique commercial software CST Microwave Studio [18]. In particular, we have designed an ideal metamaterial shell with Drude permittivity and with geometry consistent with the antenna of Fig. 3. Material losses have been added to the setup and the antenna has been designed to have a finite size. Moreover, we have fully considered in the simulations the presence of a realistic feed in the form of a simple coaxial cable. The following numerical results aim to confirm the theoretical predictions of the previous section taking into account some realistic aspects of the design, i.e., small perturbation due to numerical noise and discretization in the design geometrical and electromagnetic parameters, finite length of the antenna and the presence of a feed.

In this first set of simulations the length of the cylindrical shell has been chosen to be $L = 75$ cm, a length which ensures the leaky mode to have decayed enough due to radiation losses before reaching the end of the antenna, following the results reported in Fig. 5 for $\text{Im}[\beta]$. In particular, for the case at hand, at the operating frequency $f_0 (2)$ yields a complex $\beta = (0.5 + i0.04) k_0$, which implies that the selected length $L$ is sufficient to let the leaky mode radiate over 95% of its total input power before reaching the antenna termination.

Due to the symmetry of the leaky-wave field, we have fed the antenna with a coaxial cable with characteristic impedance of 50 $\Omega$, inner radius of 2 mm and outer radius of 5 mm, placed at the center of the hollow core, as depicted in Fig. 7(a). The near-zone electric field distribution on a cross section of the antenna far from the feed confirms the excitation of the expected leaky-wave field, as predicted in the previous section, with a dominant radial component of the electric field inside the metamaterial, as Fig. 7(b) shows.

The specific linear polarization of the electric field inside the metamaterial may, in principle, allow a design of the inclusions composing the metamaterial shell. We speculate that wire inclusions directed along the radius of the cylinder, when designed properly, may be sufficient for inducing a low-negative effective permittivity in artificial materials [32], [33] for the radial component of the electric field. We discuss these aspects in more detail in the following section. The subwavelength thickness of the shell may introduce some difficulties in a practical realization of such inclusions, and we are currently exploring some of these issues and concepts.

From these full-wave simulations, the antenna central resonance is found to be at $f = 1.975$ GHz, very close to the frequency design from our analytical approach, confirming that the antenna is sufficiently robust to small perturbations in its design parameters caused by the numerical discretization of the problem. The input impedance at the port has not been optimized in this setup, and still represents a practical challenge to be addressed. We are currently working on a proper optimization of the feeding technique to reduce the mismatch, but these simulations do indeed already show promising directions, and implied a realistic possibility of exciting properly this anomalous resonant leaky-mode. A thorough optimization of the feed for reducing the mismatch at the input port and increasing the antenna efficiency, however, is beyond the scope of the present paper, since it should also fully consider the metamaterial realization, i.e., the proper design of the inclusions realizing the anomalous electromagnetic behavior of the thin ENZ shell.

The features of radiation patterns are also essentially confirmed by our full-wave simulations, as the gain patterns in Fig. 8 show. Although, as already noticed, the feed is not matched with the leaky mode of the antenna and therefore the efficiency at the input port is not high, a sufficiently high gain and a directive beam is found using the full-wave simulations, which is consistent with the predictions in the previous section. The beam scanning with frequency is also consistent with the analytical results, and it is found to vary almost linearly with frequency over the whole range of visible angles (the trend is not reported here for sake of brevity).

Fig. 9 shows the simulation of the same antenna but with a shorter length, i.e., $L = 25$ cm. In this case, the length of the antenna is not sufficient to radiate most part of the input power traveling as a leaky mode along the structure. In particular, for the case at hand the leaky-mode reaches the end of the antenna still carrying 35% of the impinging power, which is then reflected at the antenna termination. This is evident in
the near-field Poynting vector distribution. This results in a loss of directivity and the presence of some back radiation lobes in the corresponding patterns reported in Fig. 9.

It is worth noting how the complete symmetry of the feed and the antenna, with no requirement for inhomogeneity, corrugation or complex feeding networks in the whole antenna setup, allows obtaining a totally symmetrical and omni-directional beam in the azimuthal plane, as the full-wave simulations confirm. This is independent of the beam direction in the elevation plane. The scanning of the radiation angle with frequency and the effects of the antenna overall length on the directivity of the setup, predicted by these full-wave simulations, are expected effects common to any leaky-wave antenna and therefore are not further discussed in the present paper.

As a final note, we mention that the backward-wave nature of the leaky-mode excited along this cylindrical structure may result in a relevant advantage for practical setups. This antenna, in fact, in a vertical position may be excited directly from its bottom end in order to avoid any interference between the radiated field and the transmission line feed.

V. REALIZATION CHALLENGES

In this section, we briefly discuss some of the challenges to be faced in practical realization of the antenna discussed in the previous sections. The main aspects involve the metamaterial realization in the subwavelength cross section considered in these examples. Indeed we note that the main limitation of the previous analysis deals with the idealization of the involved metamaterials.

In order to address this issue, several techniques may be suggested for realization of the setup presented in this manuscript. A first solution may be the one of utilizing naturally available materials. While plasmonic materials are naturally available at infrared or optical frequencies, as noble metals, polar dielectrics and some semiconductors [34], [35], natural gaseous plasmas are available at microwave frequencies. The first scenario might be to scale up these concepts for the realization of nano leaky-wave optical antennas, by considering thin shells of plasmonic optical materials (e.g., silver, which has its plasma frequency in the ultraviolet regime and in the visible regime it follows a Drude model consistent with the one adopted in the previous calculations [34]). The second scenario may be the use of natural plasma in the form of free electron gas contained in a suitable transparent cylindrical shell, with the desired number density to scale the plasma frequency to the desired value. These suggestions may provide some proof-of-the-principle methods for experimental realization of the concepts presented here.
The realization of the required ENZ shell may also follow the direction of “engineered” materials: metamaterials may be realized with the required electromagnetic features by properly selecting shape, geometry and number density of small resonant electric inclusions in a host material. The techniques proposed in [32], [33], [37], [38] may be viable ways towards such realization of ENZ-ENG metamaterials with the desired properties. Since the region occupied by the metamaterial shell is electrically small in the transverse section, such realizations may be challenging, particularly when the required electromagnetic response needs to be isotropic in the three directions. Current attempt to bring down the size of each inclusion in ENG metamaterials to a fraction of wavelength has proven successful [37], [38] and several groups are currently working on further reducing the required dimensions of metamaterial inclusions.

As mentioned in the previous sections, the polarization of the electric field in the metamaterial shell may provide a hint for the proper realization and orientation of the inclusions. Since the electric field is inside the metamaterial shell is predominantly radial (although a small longitudinal component of electric field is indeed present and predicted by (1) for the polarization of interest), properly-designed radially directed compact resonant wires (e.g., space-filling compact resonators [39]) might be a suitable choice for realizing the required metamaterial response. They would provide, however, an intrinsically anisotropic metamaterial response, with a uniaxial permittivity tensor that may be modeled in the cylindrical reference system of Fig. 1 as \( \varepsilon = \varepsilon_r \hat{r} \hat{r} + \varepsilon_z \hat{z} \hat{z} \). In this case, \( \varepsilon_r \) would be mainly affected by the presence of radially directed short resonant wires in the shell region, whereas the other two tensor components would show a value close to the background permittivity value.

Such a problem, which may model more realistically the realization of a metamaterial shell with a short-wire medium, may be solved analytically, analogously to what was done in Section II, yielding the following expression for the different field components, shown in (4) at the bottom of the page, where \( k_{zt} = \sqrt{\varepsilon_0 k_0^2 \varepsilon_z - \beta^2} \), \( k_{tr} = \sqrt{\varepsilon_0 k_0^2 \varepsilon_r - \beta^2} \), consistent with (1).

Under the same assumptions of the previous section, namely \( |\varepsilon_r| \ll \varepsilon_0 \) and \( a_{\text{out}} \ll \lambda_0 \), it is possible to generalize (3) to this anisotropic scenario, obtaining the interesting formula

\[
\text{Re}[\beta]^2 \approx \frac{-4\varepsilon_r}{a_{\text{in}}^2 \ln(4 + 2\varepsilon_0 \ln(a_{\text{out}}/a_{\text{in}}))}. \tag{5}
\]

\[
\begin{align*}
H_{TM} & = 
\begin{cases}
  j\omega \varepsilon_0 \beta^{-1} c_0^2 H_1(k_0 \rho) e^{-j \beta z} \hat{\phi} & \rho < a_{\text{in}} \\
  j\omega \varepsilon_r \beta^{-1} c_0^2 \left[ c_0^2 M J_1(k_0 \rho) + c_0^2 M Y_1(k_0 \rho) \right] e^{-j \beta z} \hat{\phi} & a_{\text{in}} \leq \rho < a_{\text{out}} \\
  \frac{j \omega \varepsilon_0 \beta^{-1} c_0^2 H_1^{(2)}(k_0 \rho)}{k_{zt}} e^{-j \beta z} \hat{\phi} & \rho \geq a_{\text{out}}
\end{cases} \\
E_{TM} & = 
\begin{cases}
  j \frac{\varepsilon_0}{k_{tr}} \left[ c_0^2 M J_1(k_0 \rho) + c_0^2 M Y_1(k_0 \rho) \right] e^{-j \beta z} \hat{\rho} + \beta^{-1} k_0 H_0^{(1)}(k_0 \rho) e^{-j \beta z} \hat{\rho} & \rho < a_{\text{in}} \\
  \frac{j \kappa_{tr} c_0^2 M J_1(k_0 \rho) + c_0^2 M Y_1(k_0 \rho)}{k_{zt}} e^{-j \beta z} \hat{\rho} + \beta^{-1} k_{zt} \hat{\rho} & a_{\text{in}} \leq \rho < a_{\text{out}} \\
  j \frac{\varepsilon_0}{k_{tr}} \left[ c_0^2 M H_1^{(2)}(k_0 \rho) + c_0^2 M H_1^{(2)}(k_0 \rho) \right] e^{-j \beta z} \hat{\rho} & \rho \geq a_{\text{out}}
\end{cases}
\end{align*}
\]
with inclusions oriented radially, would slightly worsen the radiation properties of the antenna, consistently with Fig. 10 and (5).

These results are fairly encouraging, and allow to forecast the possibility of realizing such antennas following different venues and paths with current and future technologies.

VI. CONCLUSION

The detailed theory of the idea of an omni-directional cylindrical leaky-wave radiator utilizing metamaterial thin shells fed at the bottom end with a simple coaxial cable has been presented here. The metamaterials with low negative permittivity are shown to improve the directivity of the setup and allow a significant reduction of the overall cross section of the antennas. Full-wave numerical simulations with FIT commercial software confirm the results and allow an optimistic forecast on the future practical realization of the proposed setup. Issues related to practical realization of the antenna and possible anisotropy of the involved metamaterials have been also considered and discussed.

REFERENCES


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