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Keywords
optical materials, metals, scattering particles

Comments
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Cloaking and transparency for collections of particles with metamaterial and plasmonic covers

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OCIS codes: (160.4670) Optical materials; (160.3900) Metals; (290.5850) Scattering, particles.

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18. We understand that a material like the one considered here, with combined plasmonic properties and magnetic permeability higher than that of free space, may not be readily available in nature. However, here and in [2] we are mainly concerned in showing the fundamental theoretical possibilities of this cloaking technique. In the present simulations, therefore, we have employed a sample cover that may simultaneously cancel two multipolar orders (i.e., electric dipole and magnetic dipole moments), requiring electric and magnetic parameters different from those of the background. In different cases, or for different purposes, however, as shown in [1], [3], it may be enough to rely just on plasmonic materials with required permittivity (with no magnetic response, i.e., with relative permeability of unity), which may be available in different ranges of frequencies. Moreover, the material suggested here may be fabricated in some frequency range, in order to have required values of permittivity and permeability. Distinctly from other proposed techniques for metamaterial cloaking, the examples reported here rely on isotropic and homogeneous materials or metamaterials.
19. Heuristically, we may justify this down-shift in the cloaking frequency for the horizontal polarization, when the objects touch or are merged together, with the following considerations: in the horizontal polarization the electrical contact and the resulting current flow between the two objects may generate an electric dipole moment somehow larger than those of two separate objects. Therefore, for the same cover geometry and material, a slightly lower frequency provides a closer-to-zero permittivity for the material cover, or effectively a larger induced “opposite” dipole moment that may cancel the increase in the dipole moment scattered by the merged object. In reality, the dynamics is more complex than this simple picture, due to contributions from higher-order multipoles and interactions between the object and the cover, but the results in Fig. 1-2 appears to be consistent with this explanation. The cloaking frequency may be easily re-tuned to the desired value by slightly increasing the cover thickness or changing the plasma frequency of the cloaking material.

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1. Introduction

In a recent contribution [1], we have shown how it may be possible to cloak an isolated dielectric or conducting object by covering it with a suitably designed plasmonic material. An artificial or natural material operating at slightly above or below its plasma frequency, and therefore having the real part of its permittivity being low-positive or negative, has been shown theoretically to potentially cancel, under suitable conditions, the dipolar scattering from an object, thus dramatically reducing the visibility of the covered object to an outside observer.

In [2] we have further investigated this anomalous phenomenon, providing full-wave numerical simulations and animations for the case of an impenetrable isolated sphere of size comparable with the wavelength of operation covered by a metamaterial with suitable parameters. The total scattering cross section (SCS) of the object was shown to be lowered more than 99% with respect to the uncovered case, and the impinging wave, both in the case of plane wave excitation and a more complex form of excitation, was shown to be “re-routed” through the plasmonic cover without any substantial reflection or perturbation of its wave fronts. We have also shown in [2] how this cloaking phenomenon, being “non-resonant”, is considerably robust to frequency variations near the design frequency, to changes in the shape of the cloaked object and/or to variations of geometrical and electromagnetic design parameters.

As shown analytically [1], this cloaking phenomenon is indeed based on the fact that the multipolar radiation from a given object may be cancelled term by term (for the significant scattering orders) with a judicious design of the plasmonic cover, as was done in [2] for the electric and magnetic dipole radiation from the impenetrable sphere under consideration. This implies that even for an observer very close to the object, its presence becomes hardly detectable after the cover is used. In [3] this analysis was extended to cylindrical objects of small cross section with arbitrary shape, and the metamaterial cover was suitably designed at microwave frequencies with simple parallel-plate conducting implants in a host dielectric.

This has suggested unambiguously how this technique may be applied without technological difficulties to RF as well as to infrared and optical frequencies, where the required isotropic and homogeneous plasmonic materials may naturally exist [4].

In the recent literature, other techniques to provide cloaking have been proposed by using metamaterial covers [5-9]. These methods rely on a distinct way of electromagnetically isolating a given region of space from its surrounding. Up to now, these approaches depend on specific anisotropic and inhomogeneous material response that may be challenging in aspects such as 3D realization, polarization constraints, and narrow bandwidth. Despite these limitations, an experimental realization has been recently reported at microwave frequencies using metamaterials with resonant inclusions [6]. We speculate that our approach presented in [1-3] may be practically more robust and relatively easier to realize.

Up to now, however, we have mainly dealt with one single isolated spherical or cylindrical object of size smaller than the wavelength of operation, surrounded by a homogenous background medium. We may indeed speculate that, due to the low scattering of a cloaked object (even in its very near-field), collections of such cloaked particles may maintain an overall “invisibility” effect, since it is expected that the electromagnetic coupling among the elements would be negligible once the scattering from each one of them has been drastically reduced.

In the following, therefore, we verify this heuristic prediction with full-wave electromagnetic simulations conducted with the Finite Integration Technique (FIT) commercial software CST Microwave Studio™ [10], providing numerical results and animations for several cases of multiple impenetrable objects suitably covered and placed in the near field of each other. Furthermore, we study how this phenomenon may be influenced by a modification of the polarization of the field or of the direction of incidence of the impinging electromagnetic wave in systems that are non-symmetric with respect to the excitation.

In addition, we investigate how these results are affected when the covers, or the objects themselves, are intersected or merged into one another, and we explore under which conditions similar results may be achieved when the total size of the system is larger than the wavelength of operation. We have indeed shown that such homogeneous metamaterial covers [1-2] may be designed to cloak up to two multipoles (i.e., electric and magnetic dipoles) of scattering. The scattering from objects larger than the wavelength are generally characterized by higher multipole orders of scattering. Here, however, we show how in principle this technique is still substantially working, even when the objects are merged together to form a system with transverse dimension larger than the wavelength.

As shown in [1-2], this phenomenon may rely on the polarizability of plasmonic materials with low-positive or negative electric permittivity. Such kind of materials are available in nature at THz, infrared (IR) and optical frequencies, since noble metals and polar dielectrics have low-loss regions near their plasma frequencies [4], which make them particularly suitable for the purpose of the present topic. At microwaves and radio frequencies, artificial structures and metamaterials [11] with plasmonic features may be synthesized, as it has been shown over the years [3, 12-13]. In particular, as mentioned above, in [3] we have applied such metamaterial designs to the transparency and cloaking problem. Similarly, the synthesis of metamaterials with the desired values of permittivities and low losses may be extended to infrared and optical frequencies by employing metallic artificial “molecules” in order to properly tailor the optical properties of materials. This problem is currently under investigation by our group.

2. Theoretical background

A given monochromatic $e^{-i\omega t}$ electromagnetic wave propagating in a background material with permittivity ε_0 and permeability μ_0 and impinging on a scatterer centered at the origin of a spherical coordinate system may always be expanded in terms of a superposition of orthogonal spherical harmonics [14], as:

$$\begin{aligned}\mathbf{E} &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n} a_{nm}\nabla\times\nabla\times\left(\mathbf{r}\psi_n^m\right)+i\omega\mu_0\sum_{n=1}^{\infty}\sum_{m=-n}^{n} b_{nm}\nabla\times\left(\mathbf{r}\psi_n^m\right)\\ \mathbf{H} &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n} b_{nm}\nabla\times\nabla\times\left(\mathbf{r}\psi_n^m\right)-i\omega\varepsilon_0\sum_{n=1}^{\infty}\sum_{m=-n}^{n} a_{nm}\nabla\times\left(\mathbf{r}\psi_n^m\right)\end{aligned}, \qquad (1)$$

where $\mathbf{r}$ is the radial vector and ψ_n^m are scalar spherical harmonics, which are the solutions of the Helmholtz equation in the spherical coordinate systems [14]. Each one of these terms corresponds to a TM^r or TE^r spherical wave, which represent, respectively, the electric or magnetic multipole of order (n,m) with amplitude a_{nm}, b_{nm}.

Owing to the orthogonality of such spherical waves and the linearity of Maxwell equations, the scattering problem may be separately solved for each one of them, and the field scattered by the object may therefore be represented by an analogous sum of electric and magnetic scattering multipoles [15]. In particular, depending on the size of the scatterer, only contributions up to a given order N are relevant, since for a regular scatterer the amplitude of the scattering coefficient of order (n,m) is of the order $o(k_0 a)^{2n+1}$ where a is the characteristic size of the scatterer and $k_0=\omega\sqrt{\varepsilon_0\mu_0}$ is the background wave number [16].

Provided that this size is sufficiently small (usually up to the order of half-wavelength for the characteristic size), only the scattering multipoles of order $n=1$ are significantly excited by the impinging wave and it is possible to drastically lower the amplitude of the total scattering cross section of the object by covering it with a plasmonic metamaterial with suitable values of its constitutive parameters at the frequency of interest [1-3]. This effect may be justified [1] by considering the fact that the $n=1$ electric multipole radiation is provided by the integral of the polarization vector $\mathbf{P}=(\varepsilon-\varepsilon_0)\mathbf{E}$ over the scattering volume [4], with ε being the local value of permittivity and $\mathbf{E}$ being the local electric field. A plasmonic material with low-positive or negative permittivity induces a local polarization vector 180 degrees out of phase with the local electric field, providing the possibility of canceling part or all of the in-phase contribution given by the scattering object. This heuristic explanation may be applied to any multipole order and, as found in [1], the exact analytical condition for total cancellation of a given multipolar order for a spherical object of radius a, permittivity ε, permeability μ and wave number $k=\omega\sqrt{\varepsilon\mu}$ is given by:

$$\begin{vmatrix} j_n(ka) & j_n(k_c a) & y_n(k_c a) & 0\\ \left[ka\, j_n(ka)\right]'/\varepsilon & \left[k_c a\, j_n(k_c a)\right]'/\varepsilon_c & \left[k_c a\, y_n(k_c a)\right]'/\varepsilon_c & 0\\ 0 & j_n(k_c a_c) & y_n(k_c a_c) & j_n(k_0 a_c)\\ 0 & \left[k_c a_c\, j_n(k_c a_c)\right]'/\varepsilon_c & \left[k_c a_c\, y_n(k_c a_c)\right]'/\varepsilon_c & \left[k_0 a_c\, j_n(k_0 a_c)\right]'/\varepsilon_0 \end{vmatrix}=0, \qquad (2)$$

with j_n, y_n being the spherical Bessel functions [17], ε_c and μ_c the cover permittivity and permeability, $k_c=\omega\sqrt{\varepsilon_c\mu_c}$ the corresponding wave number and a_c the outer radius of the spherical shell cover. This formula is independent of the azimuthal order m, and is valid for small as well as electrically large objects.

After the analytical examples provided in [1], this technique has been successfully applied in [2] to cloak an isolated impenetrable sphere of size comparable with the operating wavelength, by using a plasmonic cover with $\varepsilon_c=0.1\varepsilon_0$ and $\mu_c=5.1\mu_0$. The reduction of the total scattering cross section of the system has been shown to be 99.2% at the design frequency. This result has been obtained by canceling the dominant scattering orders, which for the case at hand were the electric and magnetic dipoles. Physical insights and numerical full-wave animations showing this effect have been also provided in [2] in order to better understand the physics underlying this phenomenon. Also the robustness to variations in the design parameters, such as the shape of the object or frequency of operation have been extensively shown in [2], justified by the intrinsic non-resonant nature of this cloaking setup.

The question of whether the presence of multiple objects placed in close proximity of one another may affect these results and if it may worsen the performance of “invisibility” may be addressed by considering the generality of Eq. (1). The multipole expansion of the impinging field, and the subsequent condition (2) for making transparent a given multipolar order n, is indeed valid for any shape of the impinging wave front, even when it is not represented by a simple plane wave, as it was assumed in [1]. In order to show the independence of this transparency phenomenon on the form of excitation, in [2] the case in which the isolated sphere was excited by a short electric dipole placed in close proximity of its surface was also simulated. The results were analogous to the plane wave case, and they show drastic reduction of the scattered fields. Similarly, it may be speculated that the coupling between closely spaced objects, each of them covered by a proper shell to induce this transparency phenomenon, may be essentially cancelled, since the reduction or suppression of the multipolar scattering is valid in the near as well as in the far field. It should be realized in this context that the interaction among closely spaced particles is based on higher-order multipoles that, although decaying faster in the far-field with respect to the lower-order contributions, are dominant in the near-field of the object [12]. This interaction, however, can still be represented by the general form of excitation given in (1) for each one of the particles in the system, and the transparency phenomenon remains valid once Eq. (2) is satisfied for the significant scattering terms.

In the following, we present some numerical simulations confirming this prediction and showing how the presence of systems of multiple objects does not affect the results of the previous studies on this topic [1-3], which were obtained for single isolated particles. Moreover, we investigate under which limitations the same phenomenon may be obtained when the covered objects are merged together to form a single body with a physical cross section of multiple wavelengths. These results represent an important extension of the transparency effect we first presented in [1] in order to consider collections and systems of objects occupying volumes larger than the wavelength, and therefore typically characterized by several orders of multipolar scattering. The number of contributing multipoles N in the total scattering from electrically large systems indeed increases rapidly with the total occupied volume, and the simple technique presented in [1] may not in principle be as effective for notably reducing the total scattering cross section, but it may be rather effective just for canceling one or two of the dominant scattering orders. In the following sections, however, we show how it is indeed possible to obtain a very low scattering for such systems, if we employ the concepts of [1-3] but applied independently to the sub-wavelength sub-unit elements of the systems.

These results may open up the possibility of cloaking more complex objects than those analyzed in [1-3], by subdividing the system judiciously, and then covering each portion individually.

3. Dramatic reduction of the total scattering cross section

As a first example, we have analyzed the combination of two closely spaced impenetrable spherical particles each with the same geometry analyzed in [2], i.e., spheres of diameter $2a = 0.4\lambda_0$, with λ_0 being the background wavelength at the design frequency f_0, covered with a plasmonic shell with $\varepsilon_c = 0.1\varepsilon_0$, $\mu_c = 5.1\mu_0$ and outer shell radius $a_c = 1.09a$. As shown in [2], when a single isolated sphere is considered, this design allows reduction of the total scattering cross section of the system by 99.2% at the frequency $f_0 = \omega_0 / 2\pi$.

This case is of particular interest, since it represents the most difficult situation in which the transparency condition can be satisfied. In this case, the impinging wave indeed cannot penetrate through the volume of object, which is of a size comparable with the wavelength of excitation, and therefore the cover has to effectively reroute the energy around the impenetrable material in such a way that the outer space is not affected by the presence of the obstacle and of the cover. This phenomenon is clearly evident in the simulations reported in [2] and in the following for the case of systems of multiple objects. Note that this situation may correspond to a conducting sphere at microwave frequencies or to a metallic object with sufficiently negative permittivity at infrared and optical frequencies.

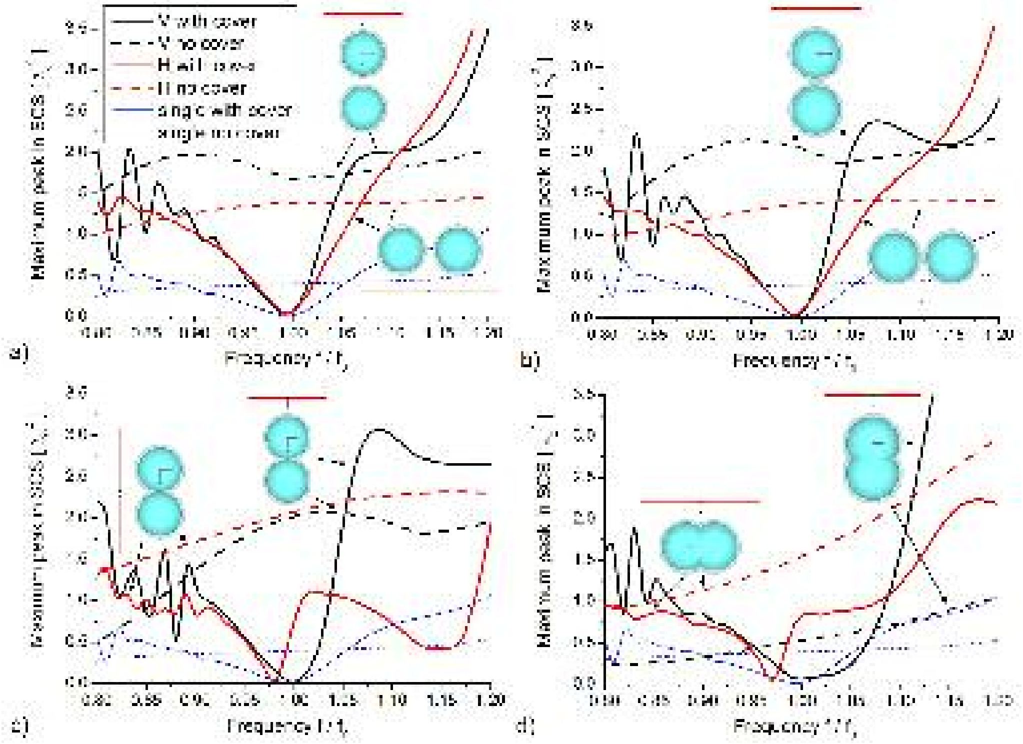

Fig. 1. Maximum peak in the scattering cross section pattern for a system of two spheres, each of them composed of an impenetrable core with diameter $2a = 0.4\lambda_0$ and (only for the solid lines) a cover shell with $\varepsilon_c = \varepsilon_0\left(1-\omega_p^2/\left[\omega(\omega-i\gamma)\right]\right)$, $\mu_c = 5.1\mu_0$, $a_c = 1.09a$, $\omega_p = 0.95\omega_0$, $\gamma = 0.016\omega_0$. The black lines correspond to vertical polarization of the impinging electric field, the red lines to horizontal polarization. (By 'vertical' and 'horizontal', we mean the electric field vector of the incident wave is 'perpendicular' and 'parallel' with the axis of the chain of spheres, respectively.) The dashed lines correspond to the uncovered cases. The four figures correspond to: (a) closely spaced spheres with a gap between the covers of $\lambda_0/25$; (b) touching covers; (c) touching cores and merged covers; (d) merged cores and covers. In all these figures, for sake of comparison, the lines relative to a single isolated sphere have also been plotted.

The results are reported in Fig. 1, which plots the maximum peak of the scattering cross section (normalized to the square wavelength) of the two-sphere system, as calculated by the FIT software CST Microwave Studio™ [10]. We note here how, as done in [2], the cover material for each one of the spheres has been assumed to have a reasonable plasma-like dispersion and material losses, following the Drude model $\varepsilon_c = \varepsilon_0\left(1-\dfrac{\omega_p^2}{\omega\left(\omega-i\gamma\right)}\right)$ with $\omega_p = 0.95\omega_0$ and $\gamma = 0.016\omega_0$, to achieve $\varepsilon\left(\omega_0\right) = \left(0.1+i\,0.015\right)\varepsilon_0$.

We recall here as an aside that the present phenomenon does not rely on any resonant phenomenon [1-3] and it is therefore weakly affected by the presence of ohmic losses in the materials or imperfections in the shape of the object, as extensively discussed in [3]. This is why the value of material losses considered here is not particularly relevant, nor the spherical shape of the particles. The material values and the frequency dispersion considered here are typical of realistic low-loss plasmonic materials or metamaterials available at optical, THz or microwave frequencies [18].

Since the available software does not compute the total scattering cross section of the object automatically, for sake of simplicity in the evaluation, in Fig. 1 we have plotted the maximum peak in the 3-D scattering cross section as a function of frequency in the range between $0.8 f_0$ and $1.2 f_0$. This is done for the case of covered (solid lines) and uncovered (dashed) objects, keeping the same distance between the centers of the two impenetrable spheres. The system was illuminated by a linearly polarized plane wave with electric field being orthogonal (vertical (V) polarization, black lines) or parallel (horizontal (H), red lines) to the segment connecting the centers of the two spheres. The four cases correspond to the different situations in which: (a) the two spheres are close to each other, with a gap between the two covers of $\lambda_0 / 25$ ($a/10$); (b) the two covers are touching; (c) the two impenetrable spheres are touching, and therefore the two covers are merged together; (d) the two impenetrable spheres are merged together in such a way that circumference of one sphere passes at the center of the other sphere (i.e., the joined impenetrable object has therefore dimensions $3a \times 2a \times 2a$). In the insets of each figure, the covered objects and the direction of the impinging waves are shown. For sake of comparisons, in all these figures we have also plotted the case of a single isolated particle with and without the cover (thin blue lines), consistent with the results presented in [2].

The figures convey plenty of information. First of all, it is evident how the covers work well in the case in which the two spheres are closely coupled, but still separate from each other (Fig. 1(a)), as heuristically expected from the discussion of the previous section. The reduction of the maximum peak of the scattering cross section is clearly evident at the design frequency and it works equally well for both polarizations. Also, the range of frequency over which a reasonable reduction of cross section is obtained through this phenomenon is relatively broad, again an indication of how the transparency achieved here is not a resonant phenomenon and therefore its dependence on frequency, even considering the material dispersion, is relatively less sensitive. It should be also underlined that the reduction of the total scattering cross section in this design is even more than what may be inferred from these plots, since the scattering radiation pattern is generally more directive when the covers are employed than in the uncovered case. This is due to the fact that the residual scattering is mainly due to the residual higher-order multipoles, producing a ‘faster’ spatial variation of the field. This phenomenon, already noticed in [2], ensures that the total scattering cross section reduction is even more evident than in these plots, since here just the peak of the scattering maxima at each frequency is shown. The results of Fig. 1(a) are analogous to those obtainable with a single isolated particle, as the thin blue lines show, confirming how the coupling between two covered objects does not affect significantly this transparency phenomenon. Even the bandwidth over which the cross section reduction is noticeable is analogous to that of the single sphere case.

Moving the impenetrable particles closer, as in Fig. 1(b), until the two covers touch each other (no gap between them) does not affect this discussion and the results are still very similar. Again, the coupling between the two covered spheres indeed remains negligible. When the covers merge together (Fig. 1(c)) and the two impenetrable objects touch, in effect creating a single impenetrable object with twice as large transverse cross section, however, the geometry is modified substantially. Interestingly enough, if the vertical polarization is not significantly affected by this change, in the horizontal polarization the cover produces the maximum reduction of scattering at a slightly lower frequency. Clearly, the horizontal polarization is the most affected by the geometrical variation induced in Fig. 1(c), since in this case the polarizing electric field “sees” an impenetrable object twice as large. It is still surprising to see how even in this case the plasmonic cover manages to drastically reduce the scattering cross section of the object. We underline here that the design of the covers has *not* been modified, and it is the same employed in [2], which is found by using Eq. (2) to cancel the electric and magnetic dipole radiation from just the original isolated particle. A numerical optimization of the cover thickness may be easily conducted to bring back the minimum of scattering in the horizontal polarization to f_0, and possibly reduce the scattering even further. It is clear, however, that the two minima in the two polarizations have slightly different positions in this coupled geometry -- an effect of the strong modification of the whole geometry [19]. Therefore, although a single cover that can produce a low scattering cross section at any angle and for any incident wave may not be easily achievable for this "extreme" merged geometry, such plasmonic covers may still grant a significant reduction of scattering for a broad range of frequencies, angles and polarization, even for this non-optimized design.

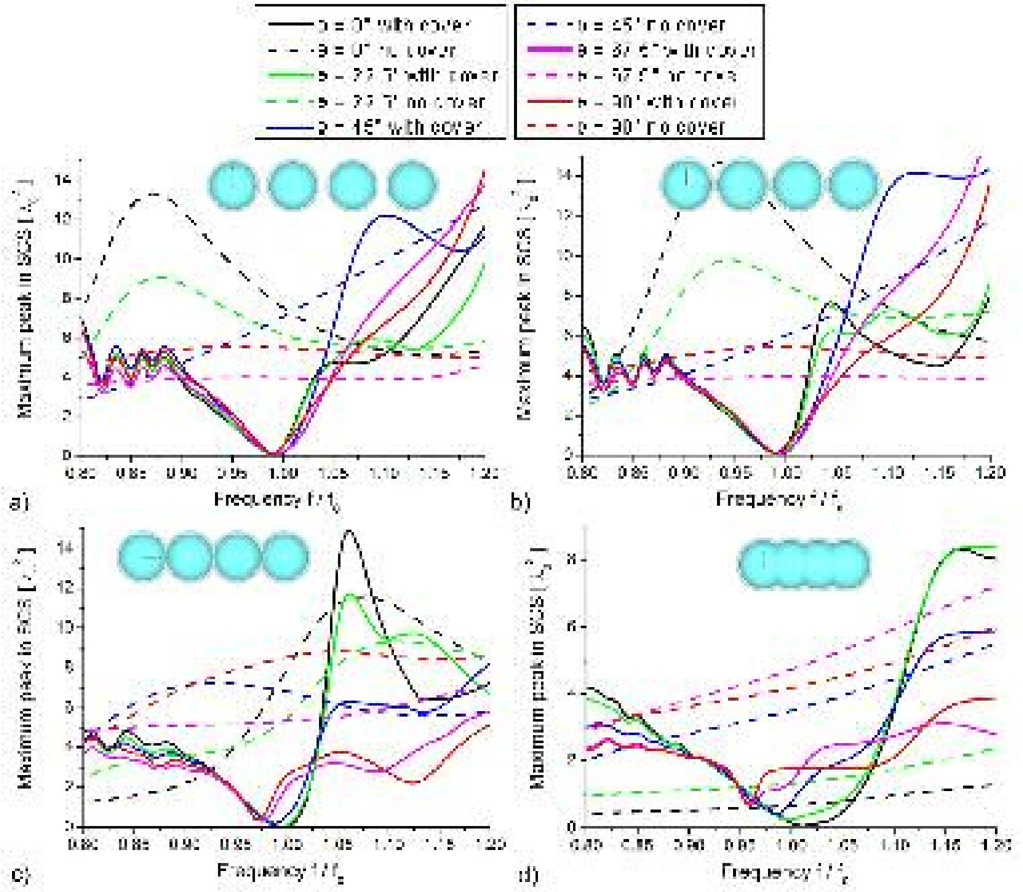

Fig. 2. Maximum peak in the scattering cross section pattern for a system of four spheres, analogous to Fig. 1. Different colors correspond to various angles of incidence of the impinging wave (with the electric field in the plane of incidence). The four figures correspond to different spacing between the spheres, analogously to Fig. 1.

In Fig. 1(d) this concept is stressed even more and the two impenetrable objects become really a single object, having merged together two half spheres into one. The minima in the scattering cross section in this last case are slightly larger, even though still the 'visibility' of the object is drastically lowered around f_0. Also the position of the two minima in the two polarizations is distinct, due to the strong anisotropy of the geometry in this case. Again, here no optimization of the cover has been performed, and we expect even better results once a numerical optimization is performed.

To study further this phenomenon in presence of multiple objects, we have explored the case of four aligned spheres, as reported in Fig. 2. Since here the asymmetry of the system is even more pronounced, in each plot we have considered five different excitations by a plane wave, by varying the angle of incidence θ, which is the angle between the incident wave vector and the axis along which the spheres are aligned, and considering the electric field as being polarized in the plane formed by these two lines (for which the scattering is larger, consistent with the previous discussion). The angle θ is varied between 0°, corresponding to vertical polarization (where the electric field is perpendicular to the axis along which the spheres are aligned), to 90°, which corresponds to horizontal polarization (when it is parallel with this axis).

Again, in Fig. 2(a) and 2(b) where the covered spheres are not yet intersecting, the coupling among each element is negligible and, despite the relevant dimensions of the whole scattering system (around two free-space wavelengths), the results confirm a dramatically reduced scattering around f_0 for all angles of incidence. In Fig. 2(c), when the covers merge together and the four impenetrable spheres are all touching, similar with the previous results, the vertical polarization and the case $\theta = 22.5°$ are again surprisingly weakly affected by the presence of an impenetrable body with size of about two wavelengths and the cover design still works quite well. Increasing the angle of incidence the situation is slightly deteriorated and the minimum of scattering is lowered with frequency, with the worst case again being represented by the horizontal polarization. Even in this situation, however, the presence of the plasmonic cover allows a drastic reduction of the scattering. These results are even more interestingly confirmed in the case of joined spheres (Fig. 2(d)), even though, due to the size of the impenetrable body, all the minima are now of higher value and, as expected, a small residual scattering is visible even for the vertical polarization. We stress again how the plots report only the scattering maxima and that the covers have not been numerically optimized to obtain the minimum total scattering cross section independent of the polarization angle. These results indeed open up the realistic possibility of cloaking large collections of scatterers by synthesizing plasmonic covers that cancel the scattering from sub-wavelength portions of them.

4. Numerical simulations and animations

In order to gain further insights into this phenomenon, here we reproduce some results of our full-wave simulations in the four-sphere scenario presented in the previous section. This may show how the "tunneling" of wave through the system takes place and provide some further clarification on the cloaking of such collections of objects whose total size is larger than the wavelength of operation.

Figure 3 shows the phase of the total magnetic field distribution for the case of four closely spaced spheres, as in Fig. 2(a), excited by a plane wave in the case of $\theta = 0°$ angle of incidence (Fig. 3(a)-(b)), oblique angle of incidence with $\theta = 45°$ (Fig. 3(c)-(d)) and normally incident wave ($\theta = 90°$) (Fig. 3(e)-(f)). The corresponding movies, posted online, refer to the time-domain animations of the magnetic field distribution for all the six cases. The phase distribution and animations compare the covered case (Fig. 3(a),(c),(e)) with the uncovered case (Fig. 3(b),(d),(f)) and show the striking "tunneling" of the incident energy through the system when the covers are employed. In all these cases, the electric field is in the plane of incidence.

It is evident how the covers "restore" the plane wave phase fronts, making the spheres hardly detectable for an external observer, even if it were placed in the small gaps between the spheres. The reduction of scattering due to the use of the plasmonic covers is striking in these plots, and it can be seen how the lines of equi-amplitudes and equi-phases are rapidly bent in the thin plasmonic shell to match the boundary conditions on the impenetrable surface of the core sphere while maintaining the original phase fronts just outside the cover. We have obtained similar performance for the other configurations of closer spheres (not reported here for sake of brevity), showing a restoration of the wave fronts even when the objects are touching or intersecting each other. As predicted by the results of the previous section, in the cases in which the covered spheres intersect or merge together, better performance and complete tunneling are still obtained under vertical polarization, whereas the horizontal polarization starts to show some residual (small) perturbation of the impinging field and possibility of some scattering from the object.

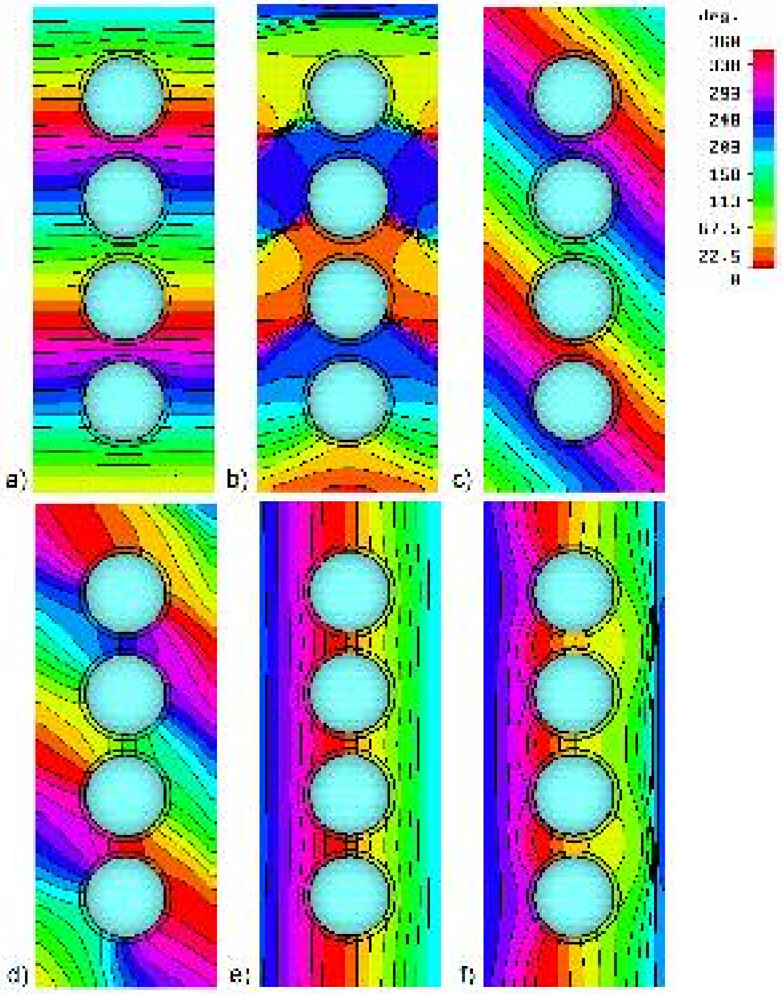

Fig. 3. Phase of the total magnetic field distribution in the E plane for the case of four aligned spheres as in Fig. 2a with: (a) angle of incidence $\theta = 0°$, covered case (movie, 1.55 MB); (b) angle of incidence $\theta = 0°$, uncovered case (movie, 1.19 MB); (c) oblique incidence $\theta = 45°$, covered (movie, 2.12 MB); (d) oblique incidence $\theta = 45°$, uncovered (movie, 1.64 MB); (e) angle of incidence $\theta = 90°$, covered (movie, 1.74 MB); (f) angle of incidence $\theta = 90°$, uncovered (movie, 1.20 MB). The movies show the time-domain animations relative to the same field distribution to which each one of the panels refers.

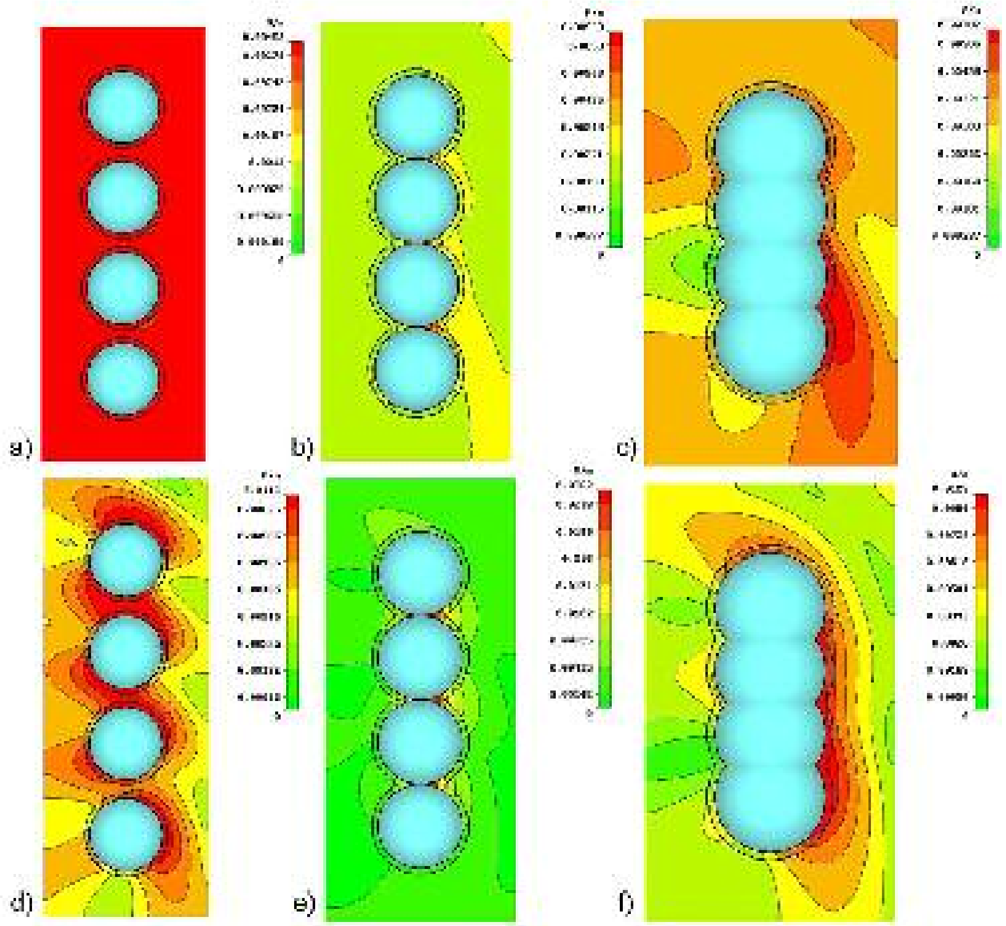

Fig. 4. Amplitude of the total magnetic field distribution in the E plane for the case of four aligned spheres: (a) with a gap of $\lambda_0/25$ between neighboring covers, covered case; (b) with touching core spheres and intersecting covers; (c) with merged core spheres; (d)-(f) are the same as (a)-(c) but removing the cover materials. A TM plane wave impinges at $\theta = 45°$ to excite the system.

Figure 4 studies the amplitude of the total magnetic field distributions in the E plane for oblique incidence ($\theta = 45°$) for the three cases of: closely spaced spheres (Fig. 4(a),(d)), analogous to Fig. 3(c)-(d); touching core spheres (Fig. 4(b),(e)); merged objects (Fig. 4(c),(f)). It compares the covered cases (Fig. 4(a)-(c)) with the uncovered cases (Fig. 4(d)-(f)) and shows once again how the plasmonic covers are very effective in drastically reducing the scattering from such collections of objects. As expected from the previous results, the case of non-intersecting spheres works quite well, analogously to the isolated case analyzed in [2], showing a uniform field distribution right outside the covers as provided just by the impinging plane wave (almost no scattering from the system, Fig. 4(a)). The reader may compare this result with Fig. 4(d) in which the covers have been removed and the oblique plane wave induces a strong scattering from the collection of particles. Similar results are obtained even when the impenetrable spheres are touching and the covers intersect (Fig. 4(b),(e)). Once again, the scattered field is much reduced by the presence of the plasmonic materials. In the last case of merged spheres (Fig. 4(c),(f)), although the scattering is sensibly reduced, still there is a residual scattering from the object, due to the strong modification of the geometry and under this oblique incidence the wave tunneling is not complete. Although the system observability has been sensibly reduced, better performance is obtained in the vertical polarization (not reported here), consistently with the previous results.

Figure 5 shows the real part of the time-averaged Poynting vector (power flow) distribution for the same examples of Fig. 4. These plots clearly show how the power is ‘rerouted’ by the plasmonic covers in order to leave unmodified the distribution of the impinging plane wave without perturbation due to the scattering from the objects. The $\theta = 45°$ power flow is restored even behind the object, particularly in the first two examples. These results confirm the previous predictions. Similar plots have been obtained also for the other angles of incidence (not reported here).

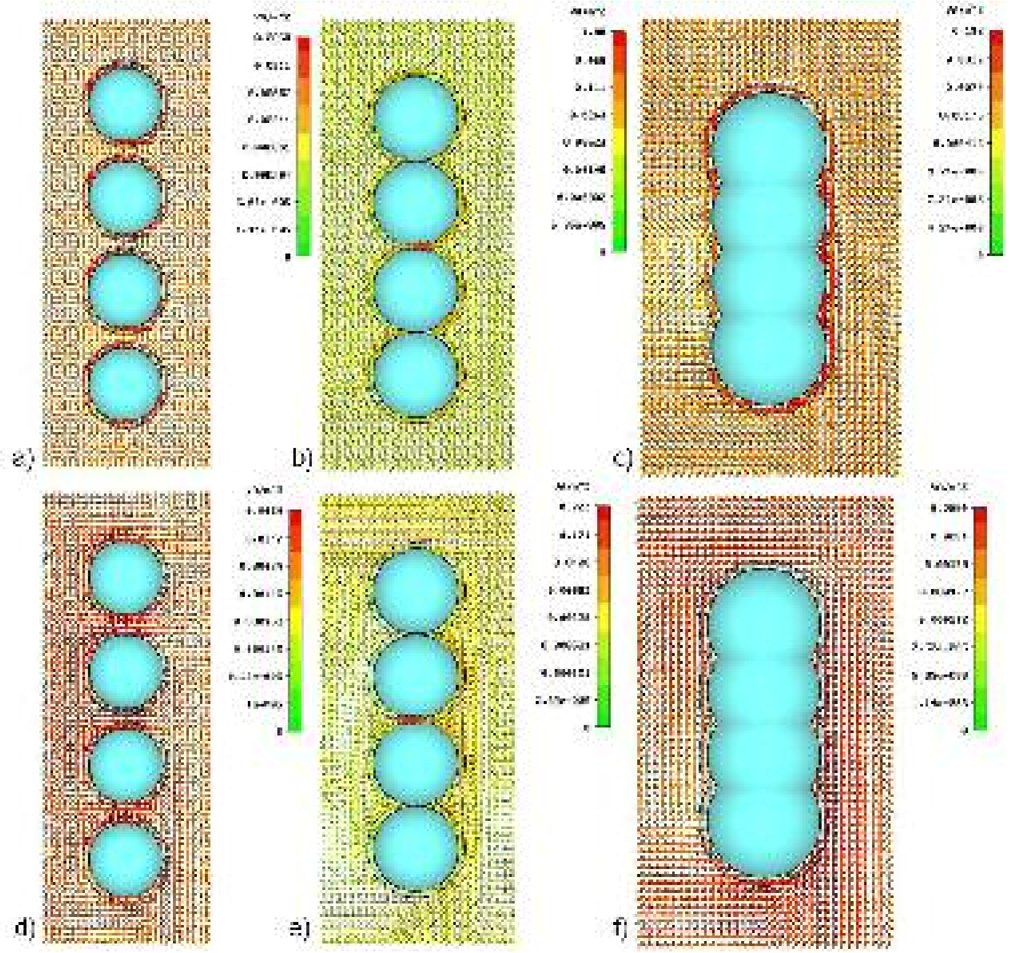

Fig. 5. Real part of the time-averaged Poynting vector (power flow) distribution in the E plane for the same cases as in Fig. 4.

5. Conclusions

In this paper we have extended our idea of employing plasmonic covers for cloaking and reducing the total scattering cross sections of dielectric and conducting objects to the case in which collections of particles and even joined particles form a system larger than the wavelength of excitation. This provides further possibility towards more realistic scenarios for inducing transparency for objects. The cases analyzed here refer to impenetrable objects, which are the more challenging cases for cloaking. Analogous results may be obtained for dielectric systems as well, following the results presented in [1].

Acknowledgments

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