Coarsening of a Two-dimensional Foam on a Dome

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Abstract
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Coarsening of a two-dimensional foam on a dome

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In this paper we report on bubble growth rates and on the statistics of bubble topology for the coarsening of a dry foam contained in the narrow gap between two hemispheres. By contrast with coarsening in flat space, where six-sided bubbles neither grow nor shrink, we observe that six-sided bubbles grow with time at a rate that depends on their size. This result agrees with the modification to von Neumann’s law predicted by J. E. Avron and D. Levine [Phys. Rev. Lett. 69, 208 (1992)]. For bubbles with a different number of sides, except possibly seven, there is too much noise in the growth rate data to demonstrate a difference with coarsening in flat space. In terms of the statistics of bubble topology, we find fewer three-, four-, and five-sided bubbles, and more bubbles with six or more sides, in comparison with the stationary distribution for coarsening in flat space. We also find good general agreement with the Aboav-Weaire law for the average number of sides of the neighbors of an $n$-sided bubble.

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I. INTRODUCTION

Coarsening is a process in foams by which there is diffusion of gas across films such that some bubbles grow and other bubbles shrink. This progresses in such a way that the average bubble area increases over time [1]. Coarsening is not limited to foams and is also relevant in other systems involving domain growth [2,3]. For an ideal dry two-dimensional foam, von Neumann showed that the rate of change of area of a bubble in a two-dimensional foam is [4]

$$\frac{dA}{dt} = K_o(n - 6), \quad (1)$$

where $n$ is the number of sides of a bubble, and $K_o$ is a constant of proportionality. Remarkably, the shape of the bubble, its edge lengths, and its set of neighbors all do not matter.

In 1992 Avron and Levine [5] generalized von Neumann’s law to predict the rate of area change for bubbles coarsening on a curved surface. The essential ingredient is that the sum of turning angles around each bubble is no longer $2\pi$, as in flat space but, rather, depends on the integral of Gaussian curvature, $\kappa_G$, over the bubble area. This modifies the von Neumann law to

$$\frac{dA}{dt} = K_o \left[ (n - 6) + \frac{3}{\pi} \int \kappa_G dA \right]. \quad (2)$$

In the case of a surface of constant positive curvature, such as a dome of radius $R$, this reduces to

$$\frac{dA}{dt} = K_o \left[ (n - 6) + \frac{3A}{\pi R^2} \right]. \quad (3)$$

The rate of change of the bubble area thus depends on the number of sides and the area of the bubble.

There have been numerous theoretical and simulation studies of coarsening for foams in two-dimensional flat space [6–14]. But to date we are aware of only one simulation that includes the effect of substrate curvature [15]. There, the authors used a modified Potts model for two-dimensional foam coarsening on spheres, toroids, and pseudospheres. For spheres, they focus on how the area distribution and average area change over time and find that at late times the dynamics are dominated by the appearance of “singular bubbles” much larger than the average that quickly grow to cover the sphere.

While the coarsening of foams in two-dimensional flat space has been well measured [12,16–25], we are unaware of any experiments to test the modified law of Avron and Levine for foam in two-dimensional curved space. However, metallurgical grain growth on curved substrates has been reported. In Ref. [26], the results are said to be preliminary and no growth rate data are shown. In Ref. [27], the deviation from the coarsening rate for flat space is masked by noise, but statistical analysis is reported to demonstrate consistency with Eq. (2). In this paper we use a hemispheric cell to create a curved two-dimensional foam. We use image analysis to track individual bubbles and measure bubble dynamics such as coarsening rate. We also measure bubble statistics, such as the distribution of number of sides, and compare this to results from a flat cell. Our image quality and analysis methods are sufficient to demonstrate directly, for six-sided bubbles, that the growth rates are different from flat space and are consistent with Avron and Levine [5].

II. MATERIALS AND METHODS

To measure coarsening rates of two-dimensional foams on a curved surface, we constructed a cell from two hemispherical polycarbonate domes. The smaller dome has an outer diameter of 12.5 cm, and the larger dome has an inner diameter of 13.3 cm, creating a 4-mm gap. The smaller dome was glued to a flat acrylic plate. The larger dome was placed over the smaller dome and separated from the plate by an O-ring 0.25 in. in diameter. We were careful to ensure that the two domes were aligned concentrically. The upper dome was then screwed to the plate to create a sealed chamber of constant curvature.

The solution we used to create our foam was a liquid consisting of 75% deionized water, 20% glycerin, and 5% deionized water.
Dawn Ultra Concentrated dishwashing liquid. This created a foam that was stable and generally lasted many days. The foam was prepared by putting 35 mL of solution into the chamber (this fills the dome to about 2 cm above the O-ring) and shaking it until a uniform opaque foam was created, with an average bubble size much less than the separation of the domes.

The scale bar is 1 cm.

FIG. 1. (Color online) Sample photograph of a two-dimensional foam, coarsening between nested polycarbonate hemispheres with a 4-mm gap. The plateau borders are defined correctly, we recognize that the image is an orthographic projection of the foam using the radius of the top dome and the plateau borders on the bottom dome are an orthographic projection of the foam using the radius of the bottom dome; both are combined into the same image. To undo this transformation, we take the inverse of the transformation twice, once using the radius for the top dome and once using the radius for the bottom dome. The resulting two images are thresholded and dilated. The images are then multiplied. This kills the plateau borders that do not correspond to the transformation. That is, the plateau borders from the bottom dome that were transformed using the radius of the top dome are killed, and vice versa. The result is a binary image with the correct latitudes and longitudes of the plateau borders on the dome.

After we have accounted for the fact that the plateau borders on both the inner and the outer domes are visible, we can consider the areas of the individual bubbles. The binary image with the correct latitudes and longitudes of the plateau borders has errors but does well for a region of interest in the center. This resulting image, however, does not preserve the areas of the cells. A simple projection that will preserve areas is the Lambert cylindrical equal area projection. This projection is defined by [28]

$$x = R \cos \varphi \sin \lambda, \quad y = R \sin \varphi,$$

where, as before, $\lambda$ is the longitude and $\varphi$ is the latitude. The $R$ used here is the average of the two domes used. This produces an image with cells that have distorted shapes but have the same areas as the actual cells on the curved surface. This allows us to track individual bubble areas over time. The result of this image analysis is shown in Fig. 2.

III. BUBBLE DYNAMICS

Using the method of finding the correct areas of individual bubbles described above, it is possible to track the areas of individual bubbles over time. The method of identifying the correct plateau borders and finding the correct areas sometimes has errors of failing to identify a plateau border or adding an
FIG. 3. (Color online) Area versus time for three six-sided bubbles, with initial area $A_0$, as labeled. Lines are a linear fit to the data, from which $A_0$ was found and subtracted from the data. Vertical error bars represent the perimeter times the pixel size divided by the square root of the number of points in the perimeter; this statistical uncertainty matches the scatter in the data points.

extra one, especially farther from the center, where distortion is greater. It was possible to find bubbles, especially near the center, that could be correctly identified for a significant length of time. The viewing region where bubbles can be tracked is greater. It was possible to find bubbles, especially near the extra one, especially farther from the center, where distortion corrections such as $K_0 t / R^2$ are much, much smaller than the linear term, so it is sufficient to fit to an ordinary line. The key feature in Fig. 3 is that the six-sided bubbles grow, and that the larger ones grow faster. This agrees with Avron and Levine and contrasts strongly with the case of a flat-sided cell, where six-sided bubbles neither grow nor shrink according to the usual von Neumann equation.

We now consider the form that the area-versus-time traces for the bubbles should take. Avron and Levine’s prediction for the coarsening of foam on a spherical substrate of radius $R$, Eq. (3), is a linear differential equation that can be solved analytically for area versus time:

$$A = A_0 + \frac{\pi R^2}{3} (n - 6) + \frac{3 A_0}{\pi R^2} \left(1 + \frac{3 K_0 t}{2 \pi R^2} + \ldots\right).$$

Here $A_0$ is the area of a bubble at time $0$, and Eq. (7) is the Taylor expansion in $(K_0 t / R^2)$. Note that the standard von Neumann result, $A = A_0 + K_o (n - 6) t$, is obtained in the limit $R \to \infty$.

Example data for the area versus time for three six-sided bubbles with different initial areas are plotted in Fig. 3. There the initial area of each bubble was subtracted off so that the traces are easily comparable. The lines are a linear fit to the data, giving a constant growth rate $dA/dt$ that is positive. It is possible to fit the data to the full exponential form of Eq. (6), but the additional terms in the expansion from Eq. (7) are much, much smaller than the linear term, so it is sufficient to fit to an ordinary line. The key feature in Fig. 3 is that the six-sided bubbles grow, and that the larger ones grow faster. This agrees with Avron and Levine and contrasts strongly with the case of a flat-sided cell, where six-sided bubbles neither grow nor shrink according to the usual von Neumann equation.

We now measure the growth rate for all bubbles, as illustrated in Fig. 3, and we compare it to the expected relationship from Avron and Levine’s modification to von Neumann’s law. This is plotted in Fig. 4, where the $y$ axis is the coarsening rate, and the $x$ axis is the expected proportionality for a dome of constant curvature given by Eq. (3). Each point represents a single bubble. The line is a proportionality, with slope $K_0$, which is the only fitting parameter. The inset is a blowup showing all the six-sided bubbles and highlighting the three bubbles featured in Fig. 3. For six-sided bubbles the growth rates are all positive, except for one or two outliers. There is notable scatter, but the evident trend is that $dA/dt$ increases with bubble size.

Another way to compare growth rate data to Avron and Levine is to plot the coarsening rate against the area, as shown in Fig. 5. There each point represents a single bubble, color-coded by the number of sides. Horizontal lines are the expected relationship from the unmodified von Neumann’s law, as seen in Eq. (1), using the same constant of proportionality $K_o$, as found in Fig. 4. Solid lines are the expected relationship from the modified von Neumann’s law, as shown in Eq. (3), again using the same value of $K_o$. We see that the data are generally consistent with the predicted modification. This is most evident in the six-sided bubbles, which are virtually all growing. The rate of area change also appears to increase with area for $n = 7$ but is masked by noise for other values of $n$.

IV. BUBBLE DISTRIBUTIONS

With our system it was also possible to measure distributions such as $p(n)$, the probability that a bubble has $n$ sides, and $m(n)$, the average number of sides of the neighbors of an $n$-sided bubble. Unlike the case of the flat cell, we do not expect
to reach a scaling state where these statistical quantities remain constant over time. Because the growth rate of a bubble grows with its area, we expect, at long times, to have large bubbles grow rapidly to dominate the system, and this will cause bubble statistics and distributions to change with time. Our system is at much earlier times, where the modification to a bubble’s growth rate due to its area is small. This modification still should have some impact, however, and we do indeed find that our statistics deviate from the ordinary scaling state observed at much earlier times, where the modification to a bubble’s statistics and distributions to change with time. Our system is taken from Ref. [25]. Vertical error bars are from a fractional area of 1 over the square root of the number of bubbles with the specified side number.

From these same data we can measure \( m(n) \), the average number of sides of the neighbors of an \( n \)-sided bubble. We expect \( m(n) \) to be related to \( n \) by the Aboav-Weaire law, which predicts \( m(n) = 6 - a + [(6a - \mu_2)/n] \). In this equation \( a \) is

\[
\sum p(n)n(n - 6)^2 = \mu_2 = 1.30 \pm 0.05.
\]

This value is lower than measured for the flat cell, \( \mu_2 = 1.56 \pm 0.02 \) [25], indicating that the width of the distribution is narrower.
a fitting parameter which is usually found to be around 1. Our measurements for \(m(n)\) are shown in Fig. 7, along with the measurements of \(m(n)\) for a flat cell. Fits to the Aboav-Weaire law are also shown, using the relevant value for \(\mu_2\) in each case. We find \(a = 1.1 \pm 0.08\) for the flat cell [25] and \(a = 0.96 \pm 0.09\) for the dome. We see that in both cases there appears to be more curvature in the data than predicted. The data for the flat cell also seem to fit the form better than for the dome.

V. CONCLUSION

In this experiment we were able to measure both bubble statistics and bubble dynamics of a foam on a curved two-dimensional surface of radius \(R\). For bubble statistics we find that bubbles with few sides are under-represented compared to a two-dimensional foam in flat space. We also find that the Aboav-Weaire law generally holds, though not quite as well for the dome as for the flat cell. For bubble growth rates, in general, it is difficult to observe the effect of the term added to von Neumann’s law by Avron and Levine to account for substrate curvature. Our maximum bubble size is around \(A_{\text{max}} = 3.5\, \text{cm}^2\), compared to \(R^2 = 41.6\, \text{cm}^2\); therefore, for all our bubbles \(3A/(\pi R^2) \ll |n-6|\) holds, except for \(n = 6\). This is why all the data in Fig. 4 lie at \(x\) values near \((n-6)\). Even if we managed to get a single bubble of 20 \(\text{cm}^2\) to completely fill our viewing area, the rate of area change would be \(dA/dt = K_0(n-6) + 0.46\), so that a seven-sided bubble would have less than a 50% increase in growth rate. For these reasons, the clearest signal we see of curvature effects is that six-sided bubbles grow systematically and do so more rapidly for larger bubbles. The coarsening data as a whole are consistent with Avron and Levine’s modification to von Neumann’s law to account for coarsening on a curved surface.

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