Intruder–normal-state mixing in $^{30}\text{Mg}$

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Intruder–normal-state mixing in $^{30}$Mg

Abstract
In $^{30}$Mg, existing data for $B(E2)$ strengths connecting the ground and excited 0+ states to the first 2+ state have been used, together with earlier shell-model predictions of normal and intruder $E2$ strengths, to estimate the intruder-normal state mixing in the 0+ and 2+ states. Resulting mixing is small, as expected, and for the ground state my value of 0.11(7) has a larger uncertainty, but is in quantitative agreement with the estimate of 0.0319(76) obtained earlier from the measured $E0$ strength connecting the 0+ states.

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I. INTRODUCTION

In neutron-rich nuclei near neutron number $N = 20$, the structure of the low-lying states is changing rapidly with changing neutron number. This region of nuclei has been called the “island of inversion.” For many of these nuclei, intruder neutron excitations into the $fp$ shell are important in order to reproduce the properties of the low-lying states. But, the extent to which such intruder configurations mix into the ground states (g.s.) of the even-$A$ nuclei is still a matter of some debate. The lowering of the $(sd)(fp)$ shell gap has been attributed to deformation and/or pairing. Many calculations agree that $^{34}$Mg, which must have at least two neutrons in the $fp$ shell, has a deformed g.s., while $^{30}$Mg is probably spherical but β soft. In between these two is $^{32}$Mg, about which there is a large difference of opinion. Many groups have claimed that the low energy of the first 2$^+$ state and the large $B(E2)$ connecting it to the g.s. require the g.s. and first 2$^+$ state to be dominated by the $(fp)^2$ intruder configuration. However, Ref. [1] found that in both $^{30,32}$Mg, the 2$^+$ energy and the $B(E2)$ could be understood with spherical states. Also, information from the recent $^{30}$Mg($t,p$) experiment [2] contradicts the conventional explanation. This reaction, in reverse kinematics, was used to locate the excited 0$^+$ state at $E_x = 1.058$ MeV [2]. Straightforward analysis [3] of the cross-section ratio for the two 0$^+$ states, in a two-state model, demonstrated that the g.s. of $^{32}$Mg is predominantly sd shell and the excited 0$^+$ state has most of the $(fp)^2$ intruder configuration. The observed exc/g.s. ratio was much too large for the g.s. to be mostly the intruder. Analysis of those data obtained a value of 19(2)$\%$ for the intruder admixture in the g.s. [3]. The same model was reasonably successful in accounting for the g.s. to 2$^+$ $B(E2)$ in this nucleus. Analysis with this g.s. wave function demonstrated [4] that the $B(E2)$ could be understood with a 2$^+$ state that was also largely sd shell. It remains to be seen whether mixed-shell shell-model calculations can reproduce this behavior.

I turn now to $^{30}$Mg, where the mixing is expected to be small. Both shell-model (sm) [5–9] and Hartree-Fock-Bogoliubov (HFB) [10–12] calculations suggest that its g.s. is almost pure sd shell. This view is supported by a measurement [13] of the $E0$ strength connecting the g.s. and excited 0$^+$ state at 1.789 MeV. A simple model of this $E0$ strength resulted in an estimate of the mixing intensity, $b^2 = 0.0319(76)$ [13]. In $^{30}$Mg, the $B(E2)$’s are known for both 0$^+$ states to the first 2$^+$ state at 1.481 MeV [14,15]. These are listed in Table I, along with shell-model predictions [7] for the normal sd-shell transition and for the intruder $(fp)^2$ one. Courrier et al. [7], performed sm calculations for $^{30}$Mg totally within the sd shell and for two nucleons in the $fp$ shell. They predicted energies and $B(E2)$ values for the unmixed states. The yrast experimental $B(E2)$ is only slightly larger than the sd sm value (also listed in Table I), and significantly smaller than in $^{32}$Mg. Here, I investigate whether these two experimental $B(E2)$’s [14,15] can be understood in a simple two-state mixing model, and used to obtain another estimate of the intruder-normal state mixing.

II. THE MODEL

I define wave functions

$$\Psi(\text{g.s.}) = a\Phi_{0N} + b\Phi_{0I}.$$  
$$\Psi(2^+) = A\Phi_{2N} + B\Phi_{2I}.$$  

The two 0$^+$ states are obviously orthogonal. Normalization requires $a^2 + b^2 = 1$, and $A^2 + B^2 = 1$. Here, $\Phi_{0N}$ and $\Phi_{2N}$ are, respectively, the wave functions of the g.s. and first 2$^+$ state of $^{30}$Mg from a shell-model calculation totally within the sd shell. The intruder states, labeled I, are more complicated. They consist of two $fp$-shell nucleons coupled to a complete set of sd-shell $A = 28$ states, subject only to the total wave function having good $J^\pi$ and isospin [16]. It appears that the intruder g.s. contains components with $J = 2$ for both the core and the $fp$-shell pair—and presumably also $J = 4$ and 6. The 2$^+$ intruder state could then presumably contain terms all the way from $0 \times 2 \rightarrow 8 \times 6$, where the first factor refers to $J$ of the core and the second one to $J$ of the $fp$-shell pair. I have seen no indications of the likely magnitudes of these various terms. And the $fp$-shell nucleons were not restricted to be neutrons, although it turned out that they were mostly neutrons. The number quoted for the 0$^+$ intruder in $^{32}$Mg is 1.95 $fp$-shell neutrons and 0.05 protons [17].

Luckily, we do not need the detailed wave functions because the $B(E2)$’s connecting normal states and connecting intruder states are given [7], together with the statement that the $B(E2)$ transitions between $N$ and $I$ vanish. We define $B(E2; i \rightarrow f) = M^2/(2J_i + 1)$, so that if $J_i = 0$, $B(E2) = M^2$. 

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TABLE I. Relevant $B(E2)$ values ($e^2\text{fm}^4$) in $^{30}\text{Mg}$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Transition</th>
<th>$B(E2)$</th>
<th>Ref.</th>
<th>$B(E2)$ Sum</th>
<th>Amp. ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. g.s. $\rightarrow 2^+_1$</td>
<td>295(26)</td>
<td>15</td>
<td>348(27)</td>
<td>2.36(17)</td>
<td></td>
</tr>
<tr>
<td>$0^+_{\text{exc}} \rightarrow 2^+_1$</td>
<td>53(6)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calc. 0 $\rightarrow 2$, $N$</td>
<td>265</td>
<td>7</td>
<td>825</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>Calc. 0 $\rightarrow 2$, $I$</td>
<td>560</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Then we have

\[ M(\text{g.s.}) = a A M_N + b B M_I, \quad M(\text{exc}) = -a A M_N + a B M_I \]

and the two terms will be constructive for the g.s.

Here I take the unmixed $E2$ amplitudes from the shell-model calculations. With the normal and intruder $B(E2)$'s from Ref. [7] (listed in Table I), the $M$'s are $M_N = 16.3$ and $M_I = 23.7$, both in $\text{e}^2\text{fm}^4$.

III. ANALYSIS AND RESULTS

I first attempt a fit with the model given above for the $2^+$ state and the two $0^+$ states, with $b$ and $B$ allowed to vary. With only two unknowns (the $0^+$ and $2^+$ mixing) and two known experimental $B(E2)$ values, it is a simple matter to obtain a unique solution for the mixing from the $E2$ data. Actually, it is easier to fit the $B(E2)$ sum and the ratio of the experimental $M$'s, because we have

\[ \sum_{\text{g.s.}} = B(E2; \text{g.s.} \rightarrow 2^+_1) + B(E2; 0^+_{\text{exc}} \rightarrow 2^+_1) = A^2 M_N^2 + B^2 M_I^2 = 348(27) \text{e}^2\text{fm}^4. \]

This relation gives $B^2$ directly (using $A^2 + B^2 = 1$): $B^2 = 0.281(92)$.

Then, defining $r = M(\text{g.s.})/M(\text{exc})$, we have $r = (1 + xyR)/(yR - x)$, where $x = b/a$, $y = B/A$, and $R = M_I/M_N$. The experimental value of $r = 2.36(17)$, combined with the value of $y$ computed from the value of $B^2$ above, leads to $b^2 = 0.109(16)$. This value of $b^2$ is significantly larger than the estimate [13] of $0.0319(76)$ from the $E0$ analysis. For any given value of $B^2$, the allowed range of $b^2$ is quite small. But, for values computed for $B^2$ within its 1$\sigma$ range, $b^2$ can be significantly different. This behavior is demonstrated in Fig. 1, where I have plotted $\Delta^2$ vs $b^2$ for three values of $B^2$—its central value and $\pm 1\sigma$. The definition of $\Delta^2$ is

\[ \Delta^2 = [(M(\text{g.s. exp}) - M(\text{g.s. calc})/\Delta M(\text{g.s.}))^2 + [(M(\text{exc exp}) - M(\text{exc calc})/\Delta M(\text{exc}))^2. \]

The allowed range of $b^2$ is thus from 0.04 to 0.19, with “best” fit at $b^2 = 0.11$. This range certainly overlaps the range of $b^2$ from the $E0$ analysis, but the uncertainty is disappointingly large (even though the upper limit of $b^2$ is still reasonably small). Results are listed in Table II.

We could ask what value of $B$ is required if we use the value of $b^2 = 0.0319(76)$ from the $E0$. Results are plotted in Fig. 2. For the entire range of $B^2$ from 0 to about 0.3, the computed $B(E2)$ for the g.s. is just slightly more than $1\sigma$ below the experimental value. However, the calculated value for the excited state varies rapidly with $B^2$, so that only a narrow range of $B^2$ is allowed. Thus, using $b^2$ from the $E0$ analysis, the resulting value of $B^2$ is 0.180(14), somewhat smaller than the value required by fitting the sum, but consistent with it.

There is another way to estimate the g.s. mixing. Given $0^+$ mixing coefficients $a$ and $b$, and an energy separation of $E$, the matrix element responsible for the mixing is $V = abE$. If I take $V = 0.415 \text{MeV}$ [4] from the $0^+$ mixing in $^{32}\text{Mg}$ (probably not correct, but perhaps a reasonable approximation), the equation can be used to determine $b^2$. The result is 0.057 (with an uncertainty that is difficult to estimate), reasonably close to the other two estimates. All three values of $b^2$ are listed in Table III.

<table>
<thead>
<tr>
<th>Model</th>
<th>g.s.</th>
<th>$2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit sum and ratio</td>
<td>0.109(16)</td>
<td>0.281(92)</td>
</tr>
<tr>
<td>Use $b^2$ from $E0$</td>
<td>0.0319(76)</td>
<td>0.180(14)</td>
</tr>
</tbody>
</table>

$^a$For the full range of $B^2$, $b^2$ is 0.11(7).

$^b$Held fixed at the value from Ref. [13].

![FIG. 2. Experimental and calculated $M(E2)$'s in $^{30}\text{Mg}$ connecting the g.s. (top) and excited $0^+$ (bottom) to the first $2^+$ state. Calculations use $b^2 = 0.0319(76)$ from the $E0$ analysis of Ref. [13]. Dashed lines represent the uncertainty in $b^2$.](024305-2)
TABLE III. Intruder intensity in 30Mg (g.s.) from various sources.

<table>
<thead>
<tr>
<th>Source</th>
<th>b²</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E0</td>
<td>0.0319(76)</td>
<td>13</td>
</tr>
<tr>
<td>E2's</td>
<td>0.11(7)</td>
<td>Present</td>
</tr>
<tr>
<td>g.s.—0⁺ₑexc energy</td>
<td>0.057</td>
<td>Present</td>
</tr>
</tbody>
</table>

Thus, we have three independent values of the intruder component mixing into the g.s. of 30Mg, and they all agree.

IV. THE SECOND 2⁺ STATE

The two-component picture of the first 2⁺ state can be used to make some estimates of the properties of the second 2⁺ state, which is currently unknown, but could be the state at 2.465 MeV [14,18]. Assuming the second 2⁺ state to be the orthogonal linear combination to the first one, viz.,

$$\Psi(2^+_2) = -B\Phi_{2N} + B\Phi_{2J},$$

and using the g.s. and 0⁺ₑexc wave functions from above, we can estimate the E2 strength to the two 0⁺ states. Results are B(E2; g.s. → 2^+_2) = 2 to 8 e²fm⁴, with a large uncertainty, and B(E2; 0⁺ₑexc → 2^+_2) = 475 or 500 e²fm⁴. Thus, within this model, the second 2⁺ state would have a very small B(E2) to the g.s., but a very large one to the excited 0⁺ state. Note that, in this simple model, the first two 2⁺ states preserve the summed E2 strength of the normal and intruder 0 → 2 transitions. It might be worthwhile to look for a g.s. branch from the 2.465-MeV state, which decays primarily to the first 2⁺ at 1.481 MeV—a transition that is probably mostly M1 [14].

Even with the very small g.s. B(E2), a g.s. decay branch from the 2.465-MeV state would still be favored over a branch to the excited 0⁺ state by a factor of 2 to 10, because of the E² factor.

V. SUMMARY

In 30Mg, the B(E2)’s connecting the first 2⁺ state to the g.s. and excited 0⁺ state have been previously measured. I have used a simple model, employing two-state mixing for the 0⁺ states and the first 2⁺ state, and B(E2)’s from a shell-model calculation, to estimate the mixing. The g.s. mixing is small, as expected, and is consistent with an earlier estimate from analysis of the E0 strength in a similar two-state model. I suggest that the second 2⁺ state should have a very weak g.s. branch.