4-1-1990

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Kinematics of Redundantly Actuated Closed Chains

VILAY KUMAR, MEMBER, IEEE, and JOHN F. GARDNER

Abstract—The instantaneous kinematics of a hybrid manipulation system, which combines the traditional serial chain geometry with parallelism in actuation, and the problem of coordination is discussed. The indeterminacy and singularities in the inverse kinematics and statics equations and measures of kinematic performance are analyzed. Finally, coordination algorithms that maintain an optimal force distribution between the actuators while avoiding or exploiting singularities are presented.

I. INTRODUCTION

Broadly speaking, there are two types of geometries for robot manipulators: serial chain and parallel chain linkages. However, robotic systems such as two cooperating arms, walking vehicles, and multifingered grippers consist of several actively controlled articulations (serial linkages), which act in parallel on an object/end-effector/ground. Unlike serial manipulators, they include one or more closed kinematic chains in their structure, and in addition, unlike completely parallel manipulators, there is more than one actuator in a particular chain, and the number of actuators typically exceeds the mobility typically. In this paper, such devices are called hybrid manipulators.

Examples of robots with completely in-parallel actuation and the kinematic and dynamic analysis of such parallel systems have been presented in [1], [2], [7], and [12]. The dualities that exist between serial and parallel geometries and between kinematics and statics have been pointed out in [17]. The control problem for such dynamically constrained systems has also been analyzed (see, e.g., [4], [5], [16], [20]).

Hybrid systems are characterized by redundantly-actuated closed chains. The actuator rates are uniquely determined by the specified trajectory, but the actuator forces are underdetermined. The redundancy in such systems is dual to the kinematic redundancy in the serial chain manipulators in which the number of actuators exceeds the dimension of the task space [11]. The presence of redundancy engenders a need for techniques that will resolve (or even exploit) the redundancy in the system.

Redundancy in manipulation systems with parallelism has been studied with reference to multifingered grippers [8], [15] and walking vehicles [3], [10], [14]. Since all these systems involve interaction between several actively controlled arms with a passive object [9], most reports have described attempts to optimize contact conditions [3], [8], [14]. These optimization efforts have largely ignored the performance of actuators at the joint level and have treated each articulation (leg/finger/arm) independently rather than considering the entire system. In research on multiple arm systems (see, e.g., [13], [16], [18], [20]), the force distribution problem has not been addressed directly. Instead, in most proposed control schemes, the force distribution is automatically determined by trajectory errors, which results in unconstraining large interaction forces, thus affecting the system performance adversely.

Even in studies in which this problem has been addressed, the load distribution has been a priori specified [13], [16]. This paper addresses the issue of coordination in robotic systems with redundantly actuated closed chains. First, an instantaneous kinematic analysis of hybrid devices is presented. The singularities in the kinematics and statics equations, which are characteristic of systems with closed chains, are analyzed. We next discuss the kinematic characterization of the actuators to improve the understanding of the nature of redundancy in hybrid systems. Coordination schemes, which avoid singularities and compute optimal force distributions, are derived from this characterization. Simulations are used to demonstrate the efficacy of the proposed schemes.

II. INSTANTANEOUS KINEMATIC ANALYSIS

The analysis in this paper is restricted to symmetric configurations (same number of links, joints, and actuators on each chain). It is assumed that the joints are frictionless and that inertial forces are negligible since the main objective of the paper is to gain an insight into the problem rather than work with a perfect model. Let the manipulation system be comprised of n parallel chains, each consisting of m serially connected links (and possessing m degrees of freedom), and let the task space of d-dimensional (where d \leq m). Let \( a \leq m \) be the number of actuators in each of the n chains. There are \( r = (n \times d) \) actively controlled joints. The special case \( a = 1 \) \((r = n)\) corresponds to a scheme of actuation that is completely parallel; an extensive survey of such mechanisms can be found in [7], whereas \( n = 1 \) (\( a \) must equal m in this case) denotes the standard serial arm.

For the \( i \)th serial chain, the end-effector rate \( \dot{\theta} \) is given by

\[
\dot{\theta} = J^\top \theta \quad \text{(1)}
\]

where the leading superscript \( i \) denotes the \( i \)th serial chain, \( J^i \) is the \((m \times 1)\) vector of joint rates, and \( J \) is the \((d \times m)\) Jacobian matrix for the \( i \)th chain. Similarly, using the principle of virtual work

\[
i_r = J^\top f \quad \text{(2)}
\]

where \( i_r \) is the \( m \times 1 \) vector of joint torques (or forces), and \( f \) is the vector of forces and moments exerted by the \( i \)th chain on the end effector (platform) or object. Obviously, \( m-a \) joint torques in \( i_r \) must equal zero. Equations (1) and (2) can be written for any of the \( n \) serial chains in the system.

Let us assume for the moment that the \( i \)th chain is in a nonsingular configuration, that is, \( m = d \), and the \( m \) joint freedoms in the \( i \)th chain are linearly independent. We can invert \( J \) in (1) to obtain the joint rates required to effect a desired velocity of the end effector in the task space. If this is done for all the \( n \) chains, we can write the inverse equations for the \( mn \) rates compactly:

\[
\begin{bmatrix}
\dot{\theta}_1 \\
\vdots \\
\dot{\theta}_n
\end{bmatrix} = \begin{bmatrix}
J_1 \\
\vdots \\
J_n
\end{bmatrix}^{-1} x.
\]

(3)

The rates for the \( r = (n \times a) \) actively controlled joints can be extracted from (3)

\[
\Theta_a = \Gamma x
\]

(4)

where \( \Theta_a \) is a \( r \times 1 \) vector of joint rates, where the subscript \( \dot{\theta} \) indicates that only the rates of the actively controlled joints are included in the vector. Each row of \( \Gamma \) (a \( r \times d \) Jacobian matrix) is a row in the \( mn \times d \) matrix of inverses in (3). Equation (4) represents the inverse rate kinematics equations for the system. Clearly, if \( r = d = na \) and \( \Gamma \) is nonsingular, the direct kinematics equations may be obtained by inverting (4):

\[
x = \Gamma^{-1} \Theta_a.
\]

(5)

Notice that in systems with parallelism, the inverse kinematics (3), (4) are simpler than the direct kinematics [12], [17]. Further, the
inverse kinematics require that each Jacobian matrix \((J)\) is nonsingular, but the direct kinematics, in addition to that, require that \(\Gamma\) be nonsingular.

Similar transformations from joint space to Cartesian (end-effector) space are possible in statics in the absence of friction and gravity. By using the principle of virtual work, from (4), we obtain \((\tau_j\) is the vector of joint torques for all the actively controlled joints) \[
\tau_j^f = \tau_j^T \Gamma^T = w^T \dot{x}
\]
or
\[
w = \Gamma^T \tau_j
\]

Once more, if \(R = na = d\) and \(\Gamma\) is invertible, the inverse problem can be solved.

A. Singularities

There are two types of special configurations that are encountered in control algorithms for such hybrid systems. The first type of singularity is caused by the joint freedoms in a single chain becoming linearly dependent, which results in that chain losing one or more degrees of freedom, thus constraining the end effector along one or more directions. This is the well-known kinematic singularity, which is characteristic of serial chain arms. However, even if none of the \(n\) serial chains are singular (that is, there is no kinematic singularity), it is possible for the \(d\) rows in the \(d \times d\) \(\Gamma\) matrix to become linearly dependent. In this case, the direct kinematics transformation (5) is not possible. In such a situation, a desired unique velocity of the end effector cannot always be specified. Alternatively, the manipulator is underconstrained, and there exist one or more wrenches that cannot be resisted. Therefore, alternatively, we can describe this singularity as a singularity in the “inverse statics” since it also prevents the determination of \(\tau\) for a given \(w\) in (7). In other words, this type of singularity is dual to the first kind and may even be called a static singularity as opposed to a kinematic singularity. We note that since we are only concerned about geometric singularities, computational singularities that arise from a particular mathematical modeling method are not an issue here.

B. Redundancies

If any of the \(n\) chains has more than \(d\) joints, and \(m \geq d\), the inverse kinematics problem is underdetermined. It is possible to find more than one set of joint rates for a desired end-effector velocity, and this situation is called kinematic redundancy. Optimization techniques to resolve the kinematic redundancy can be found in the literature [6] and is beyond the scope of this study. In this paper, we will concentrate on systems in which \(m = d\). A situation that is dual to this occurs in hybrid systems when the problem of determination of the forces and moments exerted by each of the \(n\) chains of the end effector is underspecified. In this case, the problem is statically indeterminate—the situation of static redundancy is dual to the concept of kinematic redundancy. This problem of distributing the load between the \(n\) chains is called the force distribution problem [2], [9]-[11], [14].

III. KINEMATIC CHARACTERIZATION OF ACTUATORS

A. Partitioning of Actuators

If the rows of \(\Gamma\) are denoted by \(R_1, R_2, \ldots, R_n\), (7) can be rewritten as
\[
w = \Gamma^T \tau = [R_1^T \ldots R_n^T] [\tau_1 \ldots \tau_n]
\]
This represents an underdetermined set of equations when \(r > d\). In such a situation, it is possible to designate any \(d\) of the \(r\) available actuators as primary (the other \(r - d\) are called secondary) similar to the approach followed in [4] and [5]. In this framework, the primary actuator set would control the motion of the manipulated object, whereas the secondary actuators would cater to secondary objectives—one alternative is the control of interaction or constraint forces on the object [20]. In any event, the control inputs corresponding to the primary actuator set, in general, span the task space. From this point on, we ignore such secondary considerations and assume that joints with secondary actuators are free.

Clearly, there are \(C_d\) choices for the primary actuator set. The input vector of torques corresponding to a particular set that consists of actuators \(i_1, i_2, \ldots, i_d\) is denoted here as \(u_{i_1, i_2, \ldots, i_d}\). Each such \(r \times 1\) vector has \(d\) nonzero torques \((\tau_{i_1}, \tau_{i_2}, \ldots, \tau_{i_d})\), the other \(r - d\) torques are zero. These \(C_d\) vectors span the \(d\)-dimensional joint space, and any vector of inputs (torques) can be expressed as a linear combination of these vectors. Any of the \(C_d\) primary actuator sets can be used to control the motion and the input vector \(u_{i_1, i_2, \ldots, i_d}\) can be found by solving for the \(d\) unknowns \(\tau_{i_1}, \tau_{i_2}, \ldots, \tau_{i_d}\)
\[
w = \Gamma^T [\tau_{i_1, i_2, \ldots, i_d}], \quad \text{where} \quad \Gamma = [R_1^T R_2^T \ldots R_n^T]^T.
\]

B. Measures of Kinematic Performance

Several measures of kinematic performance have been sought for serial robot manipulators. The manipulability [19] and the condition number [15] are two well-known indices. Both measures of kinematic optimality may be adapted to meet our requirements.

For a given primary actuator set consisting of actuators \(i_1, i_2, \ldots, i_d\) (input vector \(u_{i_1, i_2, \ldots, i_d}\)), we can define
\[
\mu_{i_1, i_2, \ldots, i_d} = \sqrt{\det(\Gamma_{i_1, i_2, \ldots, i_d}^T \Gamma_{i_1, i_2, \ldots, i_d})}
\]
and
\[
r_{i_1, i_2, \ldots, i_d} = \frac{1}{c(\Gamma_{i_1, i_2, \ldots, i_d})}
\]
as measures of optimality of the primary actuator sets, where \(c(\cdot)\) is the condition number. Clearly, \(\mu_{i_1, i_2, \ldots, i_d}\) and \(r_{i_1, i_2, \ldots, i_d}\) are functions of the position of the object in task space as well as the choice of actuators \(i_1, i_2, \ldots, i_d\). It is important to note, however, that \(\mu_{i_1, i_2, \ldots, i_d}\) is the volume (except for a multiplicative constant) of the force ellipsoid given by
\[
||\mu_{i_1, i_2, \ldots, i_d}||^2 = w^T (\Gamma_{i_1, i_2, \ldots, i_d}^T \Gamma_{i_1, i_2, \ldots, i_d})^{-1} w = 1.
\]
A larger value of \(\mu_{i_1, i_2, \ldots, i_d}\) implies that for a given load, the input torques are smaller. Similarly, \(r_{i_1, i_2, \ldots, i_d}\) is a measure of the isotropy of the force ellipsoid.

C. A Planar Dual Arm Manipulation System—An Example

As an example, we consider two planar robot manipulators, where each has two revolute joints. We model the gripped object as a small cylinder (the radius is small compared with the link lengths), and the interaction between the two arms is modeled as a revolute joint. In these circumstances, the system may be modeled as a closed five-bar chain (with five revolute joints) and four actuators, as is shown in Fig. 1. The mobility of the linkage is equal to two, and the task space is the 2D translational space. Since the number of control inputs (actuators) is four, the system is redundant.

Let \((x, y)\) be the coordinates of the object and \(\theta_i\) be the joint variables. Further, let \(c_i, s_i, c_{i,j}, s_{i,j}\) denote \(\cos \theta_i\), \(\sin \theta_i\),
cos(θ₁ + θ₂), and sin(θ₁ + θ₂), respectively. The rate kinematics equations are

\[ \dot{x} = [x, y]' = -J[\dot{\theta}_1, \dot{\theta}_2]' \]

(12)

where

\[ J = \begin{bmatrix}
-l_1 s_1 + l_2 s_{12} & -l_2 s_{12} \\
(l_1 c_1 + l_2 c_{12}) & l_2 c_{12}
\end{bmatrix} \]

and

\[ J = \begin{bmatrix}
-l_1 s_1 + l_2 s_{34} & -l_2 s_{34} \\
(l_1 c_1 + l_2 c_{34}) & l_2 c_{34}
\end{bmatrix} \]

Inverting the Jacobians analytically, we get expressions similar to (4) and (7):

\[ \Theta_x = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]' = \Gamma \dot{x} \]

\[ w = [w_x, w_y]' = \Gamma^T \tau = [R, R, R, R, R, \tau_1, \tau_2, \tau_3]^T \]

where

\[ \Gamma = \begin{bmatrix}
\frac{c_{12}}{s_1} & \frac{s_{12}}{s_1} \\
-\frac{1}{s_1} & -\frac{1}{s_1} & \frac{1}{s_1} & \frac{1}{s_1} & \frac{1}{s_1} \\
\frac{c_{34}}{s_4} & \frac{s_{34}}{s_4}
\end{bmatrix} \]

(13)

Clearly, there are 4C₂ (i.e., 6) choices for the primary actuator set. The six input vectors, denoted by \( u_{12}, u_{13}, u_{24}, u_{25}, u_{35}, \) and \( u_{34} \) span the 4D joint space, and any vector of torques can be expressed as a linear combination of these vectors. The input vector \( u_{ij} \) for the primary actuator set consisting of actuators \( i \) and \( j \) is required for a load \( \tau \) to be found by solving the above equations for the two unknowns \( \tau_1 \) and \( \tau_2 \).

For example, for \( u_{11} = [\tau_1, 0, \tau_2, 0]' \)

\[ w = \Gamma\tau \]

where

\[ \Gamma_{11} = \begin{bmatrix}
\frac{c_{12}}{s_1} & \frac{s_{12}}{s_1} \\
\frac{c_{34}}{s_4} & \frac{s_{34}}{s_4}
\end{bmatrix} \]

Even for the simple geometry shown in Fig. 1, the workspace is replete with singularities. These can be classified into six categories (cases A–F), all of which are shown in Fig. 2 and described in Table I.

If the primary actuator set consists of actuators from one of the two arms only (that is, the inputs \( u_{12}, u_{13} \), or \( u_{34} \), are considered), a special configuration or singularity occurs when an arm is completely extended or retracted (cases A and F in Fig. 2 and Table I); the determinant of \( J \) (or \( J_t \)) equals zero. If the system is considered to be a closed chain, that is, the dichotomy of two arms is abandoned, we are automatically faced with more special configurations. For example, for \( u_{34} \), equating the determinant of \( \Gamma_{11} \) to zero yields upon simplification the condition \( \sin(\theta_1 + \theta_2 - \theta_3 - \theta_4) = 0 \). This corresponds to the distal links of each manipulator being aligned, or more precisely, to axes of joints 2, 4, and 5 being coplanar. In this situation, a force exerted perpendicular to the plane of the axes cannot be resisted, and the object, therefore, is unrestrained. Similarly, the singularities for other actuator sets can be found. Unless a simple leader-follower type of a scheme is employed for coordination, all these singularities can be expected to affect the control of the system. A good coordination algorithm must avoid (or exploit) these singularities appropriately, as is shown in the next section.

IV. COORDINATION OF REDUNDANT ACTUATORS

We now address the problem of specifying torque set points for the controller and resolving the redundancy in (8) in an effective manner. It is clear from the discussion in the previous section that negotiating the myriad singularities is one of the key issues. However, if one considers the major task of the system to be that of resisting an applied load, it makes sense to take advantage of some of the singularities. In particular, the singularities that belong to the first class (kinematic singularities) are potentially attractive since they represent configurations in which externally applied loads are not reacted by the actuators at all. If we formulate a control scheme that actually favors those sets of actuators that are at or near kinematic singularities and avoid those that are at or near "static" singularities, we can achieve greater load (for example, lifting) capacity with less joint deflections, greater accuracy, and superior performance.

The simplest coordination scheme involves switching from one set of primary actuators to another in order to constantly command the optimal primary actuator set while the secondary actuator set is idle. However, this would result in a discontinuity in the torques at the instants when the actuator sets are switched. To circumvent this, we propose a coordination scheme based on different weighted averages.
of the $C_p$ possible input vectors that enables a continuous switching between different actuator sets.

For a given primary actuator set consisting of actuators $i_1, i_2, \ldots, i_d$, the input vector $u_{i_1, i_2, \ldots, i_d}$ can be computed unless \( \Gamma_{i_1, i_2, \ldots, i_d} \) is singular. If $\Gamma_{i_1, i_2, \ldots, i_d}$ is an index that is used as a measure of performance of $u_{i_1, i_2, \ldots, i_d}$, an appropriate vector of torques can be computed quite simply:

$$
\tau = \frac{\sum_{i_1, i_2, \ldots, i_d} \rho_{i_1, i_2, \ldots, i_d} u_{i_1, i_2, \ldots, i_d}}{\sum_{i_1, i_2, \ldots, i_d} \rho_{i_1, i_2, \ldots, i_d}}.
$$

The use of either $\mu_{i_1, i_2, \ldots, i_d}$ or $r_{i_1, i_2, \ldots, i_d}$ as a weighting factor makes it possible to accommodate singularities with ease since in either case, the weighting factor vanishes when the input vector corresponding to a singular $\Gamma_{i_1, i_2, \ldots, i_d}$ matrix. Using $\mu_{i_1, i_2, \ldots, i_d}$ results in a larger force ellipsoid for the system and therefore smaller torques. This automatically favors kinematic singularities (see cases A and F in Fig. 2 and Table I) but avoids "static" singularities (cases B, C, D, and E in Fig. 2 and Table I). The use of $r_{i_1, i_2, \ldots, i_d}$ on the other hand would increase the isotropy of the force ellipsoid but would also (indirectly) avoid "static" singularities that cause the force ellipsoid to flatten out along one of its principal axes.

A. Example—A Simulation of the Dual-Arm Manipulation System

We illustrate the application of (14) using the indexes $\mu_{i_1, i_2, \ldots, i_d}$ and $r_{i_1, i_2, \ldots, i_d}$ with simulations on a quasistatic model of the system in the example considered earlier (see Figs. 3 and 4). We consider the two trajectories shown in Figure 3(a) and 4(a) with a constant load, $W$, of $-i-j$ units as examples.

The horizontal trajectory of Fig. 3(a) passes through a kinematic singularity (case A) at $x = 0$. Fig. 3(b) and (c) shows the torques required to counteract the applied load in a master-slave type control scheme. Fig. 3(b), actuators 1 and 2 (input vector, $u_{12}$) and Fig. 3(c),
Fig. 4. Simulation results (Example 2). The trajectory is a vertical path 
\( x = 0.5, \quad -0.5 \leq y \leq 1.5 \) with a constant load of \( \frac{y}{J} \) applied 
on the object (including the weight). \( s \) is a parameter that varies from 0 
to 1 along the trajectory, and \( u_{ij1} \) and \( u_{ij2} \) denote \( t_i \) and \( t_j \) 
respectively, for the primary actuator set consisting of actuators \( i \) and \( j \).

The force distributions in (d) and (e) are derived from (14) with 
\( p_i \) and \( p_j \) as weighting factors, respectively.

In the second example, the trajectory is a vertical path (see Fig. 4(a)), which passes through three singularities (cases B, C, and D).
The so-called "static" singularities can be clearly seen in Fig. 4(b) 
from the time histories of \( u_{13} \) and \( u_{14} \) (cases B and C respectively).
Fig. 4(c) once again suggests that a simple switching algorithm would 
ensure a condition number less than 3.0 \( (r_{ij} > 0.33) \) through the 
trajectory. The force distribution resulting from \( p_i \) as a weighting 
factor is shown in Fig. 4(d). This time, the torques are discontinuous 
in the slope (as opposed to the torques in Fig. 3(f)). This is a 
consequence of the singularities. However, if the weighting factor is 
made a higher power of \( \mu_{ij} \), for example, with \( p_i = \mu_{ij}^r \), we obtain 
a force distribution in which the maximum torque is approximately 
the same, but the torques are much smoother, as is shown in Fig. 4(f) (compare with Fig. 3(f)).

In conclusion, the use of weighting factors enables efficient utilization 
of all the actuators and allows a "closed chain" approach to the 
problem as opposed to a "master-slave" approach. The weighting 
factors circumvent singularities in the statics equations, whereas the 
kinematic singularities are exploited to minimize motor torques. This
approach results in an even distribution of forces, as is evidenced by
Fig. (3(a) and (f) and Fig. 4(c) and (f).

B. Remarks
1. Dynamic Loads and Inertial Forces: The assumptions regarding
the quasi-static nature of the problem are realistic in some applications,
as was mentioned earlier. However, if inertial forces become signifi-
cant, they must be incorporated in the vector \( \mathbf{w} \). Now, \( \mathbf{w} \) varies with time, but this does not preclude the application of
the ideas presented in this paper. The only difference is that the
"static" singularities are not as important at high speeds since the
inertia of the system will carry it through such singularities. It may
be speculated that control problems will not be as severe. Neverthe-
less, at low speeds, even if the inertia of the system is significant,
these singularities (belonging to the second kind) are an important
consideration.

2. Computational Load: The computational complexity of the sug-
gested scheme increases with the dimension of the task space. This is
particularly so since \( \tau_{ij} \) requires the computation of eigenvalues
of a matrix and is unsuitable for on-line computation. However, the
use of \( \mu_{ij} \) is well suited to real-time operation, especially since
the input vectors \( \xi_{ij} \) can be computed in parallel. Even in 3D geometry, the complexity
in the computation of the weighting factors is similar to the com-
plexity in the computation of the determinant of a Jacobian matrix
in an industrial robot, which is not at all expensive when efficient
analitical expressions for the determinant are used.

In fact, (14) can be further simplified if \( \mu_{ij} = \mu_{i'j'} \)

\[
\frac{1}{\sum_{i<j} \mu_{ij}} \sum_{i<j} \mu_{ij} \sum_{i<j} \mu_{ij} = \sum_{i<j} \mu_{ij} \sum_{i<j} \mu_{ij} \]

where \( \mu \) represents the adjoint of the matrix. Now, no inverses
have to be computed to find \( \xi_{ij} \), and there is never any scope
division by a small number.

V. Concluding Remarks

The kinematics of a hybrid manipulation system, which combines
the traditional serial chain geometry with parallelism in actuation,
is discussed. These systems have two key attributes: a closed chain
structure and redundancy in actuation. In this paper, the structural
characteristics and the kinematic performance of such systems are
studied. The special configurations or singularities, which are clas-
sified into two dual categories, and are dual to one another, are
studied. A coordination algorithm which automatically avoids unde-
sirable singularities while favoring preferred singularities is devel-
oped. Simulation results demonstrate the utility and practicality of
the scheme.

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