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Two-neutron transfer in the "island of inversion"

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Cross sections for (p,t) and (t,p) reactions near neutron-shell closures depend sensitively on the amount of intruder configuration in the relevant states. For several nuclei in the "island of inversion," I present calculated cross section ratios for the first two 0^+ states as functions of the intruder-normal-state mixing.

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I. INTRODUCTION

Two-nucleon transfer reactions, especially (t,p) and (p,t), have long been useful tools for determining the major components of wave functions of simple states. The coherent nature of the process makes it particularly useful for uncovering destructive interference. The case of ^{18}O [1–3] is a good example. In a simple coexistence model [1], single-neutron transfer to and from ^{18}O led to a complete determination of the wave function for the ground state (g.s.) but allowed two different solutions for the wave functions of the second and third 0^+ states. Data for the ^{16}O(t,p) reaction provided a clear choice between the two [1]. This case has become a textbook example [2] of the procedure for determining the dominant features of wave functions from transfer reactions. It led to a larger application [3] of using (primarily) transfer and $E2$ strengths to obtain wave functions for all the low-lying positive-parity states of ^{18}O.

Data for the ^{12}C(t,p) reaction leading to the first three 0^+ states of ^{14}C provided an estimate [4] of 12% for the intensity of the core-excited $(sd)^2(1p)^{-4}$ component in ^{14}C(g.s.). In the ^{10}Be(t,p) reaction [5], the g.s. of ^{12}Be was five to seven times as strong as it would have been if it were a pure p-shell state. This result, together with a calculation of the ^{12}Be-^{12}O Coulomb energy difference [6], determined the ^{12}Be(g.s.) to have approximately 68% of the configuration ^{10}Be(g.s.) $\times$ $v(sd)^2$—a value later confirmed by other means [7,8].

Care must be exercised when applying this technique. In a theoretical paper [9] on ^{19}F/^{19}Ne, the authors adjusted their Nilsson-model parameters to reproduce the 3.91/4.03-MeV $3/2^+$ mirror states in these nuclei as $(sd)^3$ states, having (in their results) 70% of the $(3/2^+)^3$ Nilsson configuration. As evidence for this assignment, they cited the strength of the ^{19}Ne 4.03-MeV state in the ^{21}Ne(p,t) reaction [10]. However, this state has long been thought to have the dominant character of $(sd)^5(1p)^{-2}$ and/or $(sd)^7(1p)^{-4}$. Another paper [11] used the ^{21}Ne(p,t) reaction to prove the dominance of the former. The absence of any appreciable $(sd)^3$ component in ^{19}F(3.91) was demonstrated by its weakness in the ^{17}O(^{3}He,p) reaction [12] and by its very small α-particle spectroscopic factor [13].

Neutron-rich nuclei near $N = 20$ have been referred to as being in an "island of inversion." Many properties of these nuclei make it clear that neutron excitations into the fp shell must be included to understand the low-lying states. An important question has been whether these $(fp)^2$ excitations ever become dominant in the g.s. for $N \leqslant 20$. Many have claimed that, in ^{32}Mg, the energy of the first 2^+ state and its $B(E2)$ to the g.s. require both states to be dominated by fp-shell components. Others have disagreed. The history is summarized in Refs. [14,15] and in two more recent papers [16,17].

In the ^{30}Mg(t,p) reaction, in reverse kinematics, the excited 0^+ state was observed [14] at 1.058 MeV with a cross section that was 62(6)% of the g.s. Straightforward analysis of those data, in a simple two-state mixing model, demonstrated [15] that the g.s. was mostly an sd shell state and the excited state was mostly the $(fp)^2$ intruder. Resulting wave-function intensities in the g.s. were 0.81 and 0.19 [15].

Here, we use the same simple model to investigate the results to be expected in (p,t) and (t,p) reactions to the lowest two 0^+ states in these and other nearby nuclei.

II. THE MODEL

The model assumes two basis states dominate the structure of the g.s. and first-excited 0^+ state in these nuclei. For $N = 20$, the g.s. is written as g.s.$(20) = a + b(fp)^2\,(sd)^{-2}$, and the excited 0^+ is written as exc$(20) = -b + a(fp)^2(sd)^{-2}$ where all the action is in the neutrons.

For $N = 20 + n$, we have g.s.$(20 + n) = a(fp)^n + b(fp)^{n+2}(sd)^{-2}$, and for $N = 20 - m$, we have g.s.$(20 - m) = a(sd)^{-m} + b(fp)^2(sd)^{-m-2}$. In all cases, the excited 0^+ state is taken to be the orthogonal linear combination. I use direct-reaction phases throughout so that a positive relative sign in the wave function corresponds to constructive interference in $2n$ transfer. The connection between shell-model and direct-reaction phases is addressed in Ref. [15].

I deal only with cross section ratios, both calculated and experimental. As before, I define $R^2 = \sigma(fp)/\sigma(sd)$, where $\sigma(fp)$ is the calculated cross section for the (t,p) reaction leading from an empty shell to $(fp)^2$, and $\sigma(sd)$ is for ^{30}Mg(g.s.) to ^{32}Mg(g.s.) within the sd shell. Similarly, I define $T^2 = \sigma[(sd)^{-4} \to (sd)^{-2}]/\sigma[(sd)^{-2} \to \text{closed shell}]$ and $R'^2/R^2 = \sigma[(fp)^2 \to (fp)^4]/\sigma(\text{empty shell} \to (fp)^2]$. As mentioned in Ref. [15], if one theoretical cross section in the ratio is known, then the ratio can usually be computed from the summed experimental cross section to the two 0^+ states. Of course, if detailed wave functions are known, then the ratios can always be calculated. For (t,p) experimental cross sections, I define $r^2 = \sigma(\text{exc})/\sigma(\text{g.s.})$. For (p,t), I use r'^2. Whenever an ambiguity is possible, I have a subscript for the nucleus A. For r and r', the subscript will always refer to

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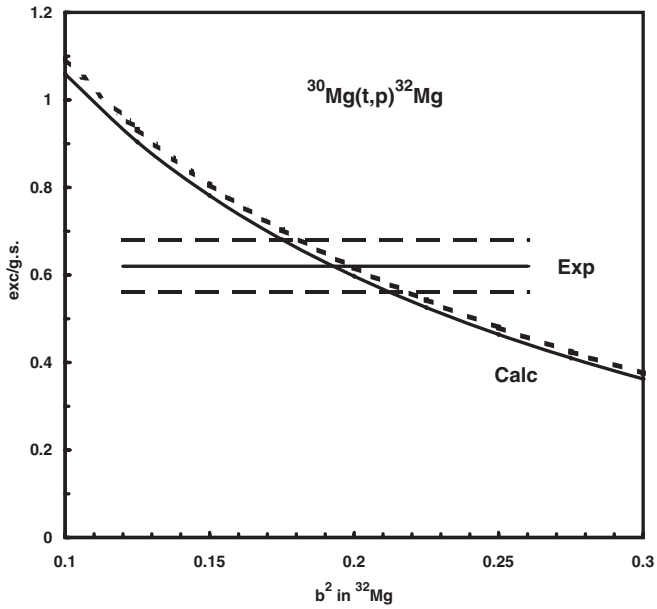

FIG. 1. Ratio σ(exc)/σ(g.s.) of cross sections for the reaction ^{30}Mg(t,p) ^{32}Mg. Curved lines are for pure (solid) and mixed (dashed) ^{30}Mg(g.s.). (See text.) Horizontal solid and dashed lines are the experimental value and the $\pm 1\sigma$ limits, respectively [14]. The abscissa is the intensity of the $(fp)^2$ intruder configuration in ^{32}Mg(g.s.).

the final nucleus. If I define $x = b/a$, then, for example, for the reaction ^{32}Mg(t,p) ^{34}Mg, we have

$$r_{34} = [x_{32}R' - x_{34}(R + x_{32})]/[(R + x_{32}) + x_{32}x_{34}R'],$$

and for ^{32}Mg(p,t) ^{30}Mg, we have

$$r'_{30} = [x_{32}T - x_{30}(1 + x_{32}R)]/[(1 + x_{32}R) + x_{30}x_{32}T].$$

For even-even nuclei, the g.s.-to-g.s. $2n$ transfer amplitude is always constructive—all the terms will have the same sign. This feature was treated by Yoshida [18] years ago, and it was discussed at some length recently [15]. Therefore, a, b, and R are all non-negative. Because the transfer to the excited state involves destructive interference, r and r' can have either sign.

III. SPECIFIC REACTIONS

A. ^{30}Mg(t,p) ^{32}Mg

First, I review the situation for ^{30}Mg(t,p) for which the data were published in Ref. [14] and the two-state analysis in Ref. [15]. Of all the reactions considered here in the island of inversion, this is the only one whose experimental results are known. The solid curve in Fig. 1 was calculated by assuming the ^{30}Mg(g.s.) was a pure sd shell state. The ordinate is the calculated exc/g.s. cross section ratio, which uses the value of $R^2 = 4.3$ extracted [15] from the summed measured cross sections to the two 0^+ states. The abscissa is b^2, the intensity of the $(fp)^2$ intruder component in ^{32}Mg(g.s.). The horizontal solid line is the measured ratio, and the dashed horizontal lines are the $\pm 1\sigma$ limits on that ratio. (The dashed curve is discussed later, below.) Note that the calculation and experiment intersect at $b^2 = 0.19$. This is the same calculation that gave $b^2 = 0.19$ in Ref. [15]. The uncertainty on the experimental ratio corresponds to an uncertainty in b^2 of about $\Delta(b^2) = 0.02$.

In ^{30}Mg, the energy of the excited 0^+ state is known, and the $E0$ transition strength connecting it to the g.s. has been measured [19]. That analysis provided an estimate of $b^2 = 0.0319(76)$ for the intensity of the intruder $(fp)^2$ component in ^{30}Mg(g.s.). The dashed curve in Fig. 1 was computed by using this mixing in ^{30}Mg. Because this mixing is so small, the mixed and pure results for this reaction are very similar.

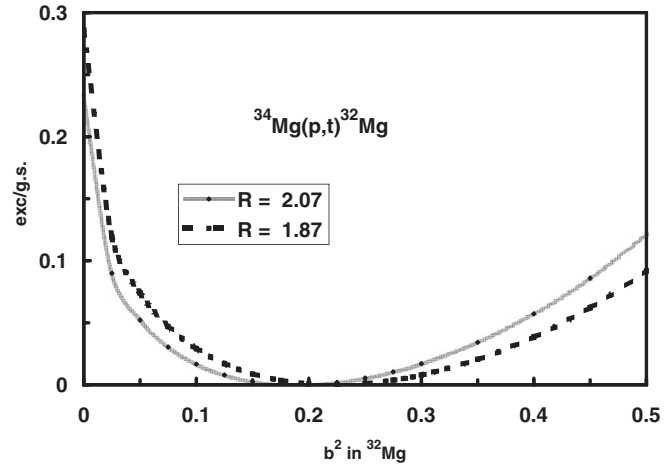

FIG. 2. Plot of σ(exc)/σ(g.s.) for the reaction ^{34}Mg(p,t) vs the assumed intensity of the $(fp)^2$ intruder configuration in ^{32}Mg(g.s.). Solid line has $R^2 = 4.3$, dashed line has 3.5. Both are for pure $(fp)^2$ ^{34}Mg(g.s.). A value of $b^2 = 0.19$ was estimated [15] from (t,p).

B. ^{34}Mg(p,t) ^{32}Mg

We first assume that the ^{34}Mg(g.s.) is predominantly $(fp)^2$ in character. Then, we have $r' = (1 - xR)/(R + x)$, where $x = b/a$ in ^{32}Mg and R^2 is defined above. Reference [15] found $R^2 \sim 4.3$ and $b^2 \sim 0.19$. In Fig. 2, I plot the predicted cross section ratio vs b^2 for two values of $R^2 = 4.3$ and 3.5. We note that, for any value of b^2 near the estimate of Ref. [15], the ratio is extremely small. Any neglected small components in the wave functions will increase the g.s. cross section because all components will add constructively for the g.s. But, because the small result for the excited-state cross section arises from destructive interference, such small components will have a larger effect on the excited state—probably leading to a partial filling in of the minimum in the ratio. Nevertheless, the prediction is that the excited 0^+ state of ^{32}Mg will be quite weak in the ^{34}Mg(p,t) reaction if there is no core excitation in ^{34}Mg(g.s.).

With core excitation in ^{34}Mg(g.s.), the calculated ratio will be larger because then, three reaction amplitudes can contribute to (p,t): $v(fp)^4$ to $v(fp)^2$, $v(fp)^2$ to the empty shell, and ^{32}Mg(g.s.) to ^{30}Mg(g.s.) within the sd shell. Ratios of these reaction amplitudes are defined in Sec. II above. The expected ratio is then,

$$r'_{32} = [(1 + x_{34}R') - x_{32}R]/[R + x_{34}(1 + x_{34}R')],$$

and the cross section ratio is r'^2. I have used $x_{32} = 0.48$ (from $b^2 = 0.19$) and $R'/R = 1$ and 1.29 as likely outer limits.

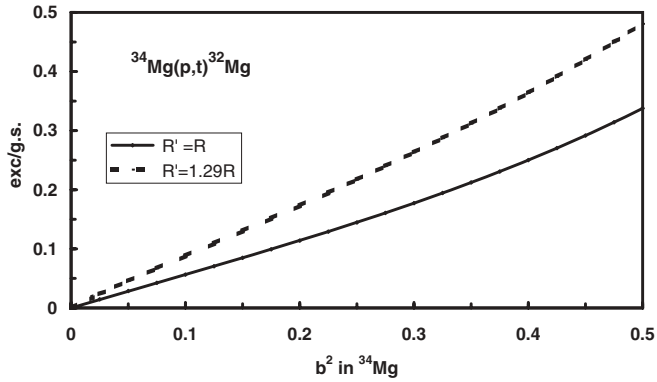

FIG. 3. Ratio for the same reaction as in Fig. 2 but now vs the intensity of intruder $(fp)^4(sd)^{-2}$ configuration in ^{34}Mg(g.s.) by assuming $b^2 = 0.19$ in ^{32}Mg. Two different curves are described in the text.

I arrived at these limits by inspection of the microscopic occupancies of the two lowest fp-shell Nilsson orbitals for positive deformation. For any reasonably small core excitation in ^{34}Mg(g.s.), from Fig. 3, we see that the cross section ratio is small. The predicted (p,t) ratio increases monotonically with an increase in the core-excitation component in ^{34}Mg(g.s.). A measurement of this ratio should then provide an estimate of this core excitation.

C. ^{32}Mg(t,p) ^{34}Mg

With the same definitions, the expression for r_{34} is given in Sec. II above. These results are plotted in Fig. 4. Here, the predicted cross section ratio is extremely small for a wide range of b^2 from about 0.05 to about 0.35 in ^{34}Mg. From consideration of the $B(E2)$, I expect the relevant region to be within these limits. So, this reaction would not be a good way to discover the excited 0^+ state in ^{34}Mg [unlike ^{30}Mg (t,p) ^{32}Mg].

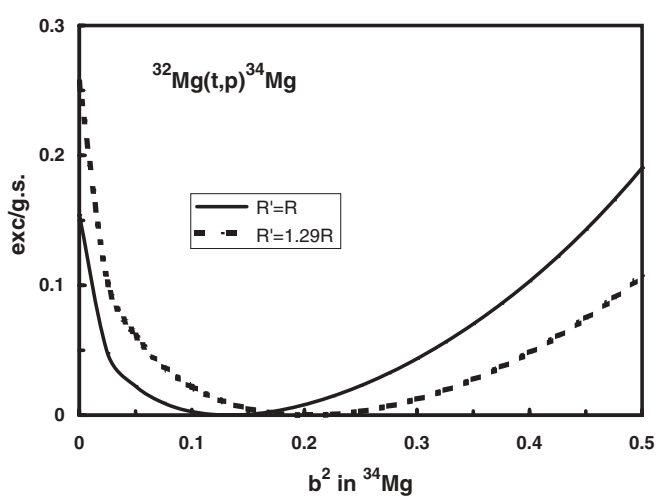

FIG. 4. Results for the reaction ^{32}Mg(t,p) ^{34}Mg, using $b^2 = 0.19$ for ^{32}Mg(g.s.), plotted vs b^2 in ^{34}Mg(g.s.). Two curves are described in the text.

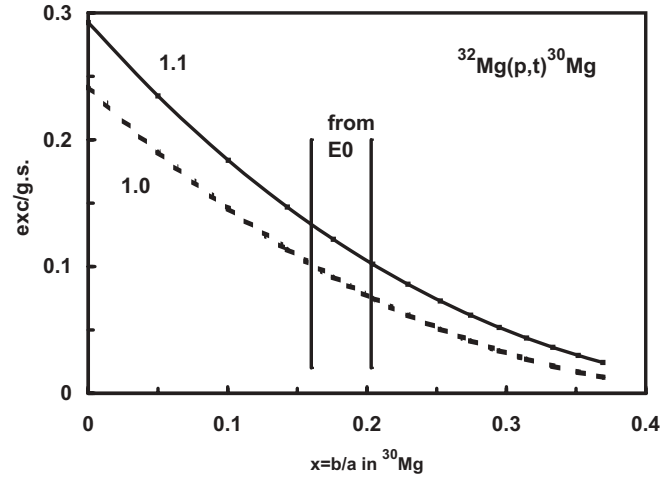

FIG. 5. Results for the reaction ^{32}Mg(p,t) ^{30}Mg, using $b^2 = 0.19$ in ^{32}Mg(g.s.), plotted vs $x = b/a$ in ^{30}Mg(g.s.). Solid vertical lines are at the limits on x from the $E0$ measurement [17]. The two curves result from two different values of T (see text).

D. ^{32}Mg (p,t) ^{30}Mg and ^{28}Mg(t,p) ^{30}Mg

Here, too, the excited 0^+ state is known at 1.789 MeV. The expression for r'_{30} in (p,t) is given in Sec. II above. The predicted ratio depends on the value of T defined above. For spherical sd-shell wave functions, I expect T to be close to, but perhaps slightly larger than, unity. It is easily calculated with shell-model wave functions, but here, we are interested in general trends. So, I have performed the calculations for $T = 1.0$ and 1.1.

The predicted cross section ratio is plotted vs x_{30} in Fig. 5 where I used the b^2, a^2 values of 0.19, 0.81 previously determined for ^{32}Mg. Here, the abscissa is x, rather than b^2 because the expected value of b^2 is so small in ^{30}Mg. From a measurement of the $E0$ strength between the two 0^+ states, Ref. [19] estimated $b^2 = 0.0319(76)$, and hence, $x = 0.182(22)$. Here, we see that the excited state is again

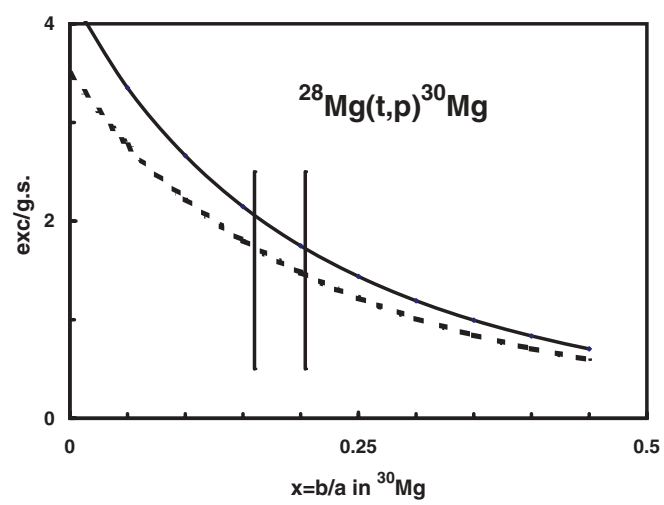

FIG. 6. Results for the ^{28}Mg(t,p) ^{30}Mg reaction vs $x = b/a$ in ^{30}Mg(g.s.), assuming a pure sd shell ^{28}Mg(g.s.). Solid vertical lines are as in Fig. 5. Note large predicted ratio.

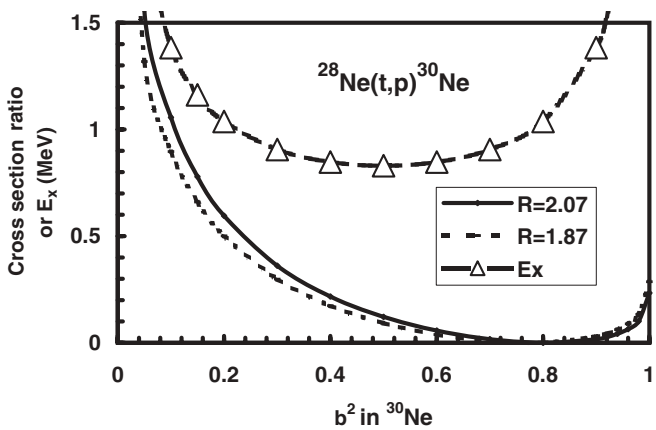

FIG. 7. Results for the reaction ^{28}Ne(t,p) ^{30}Ne plotted vs $(fp)^2$ intensity in ^{30}Ne(g.s.) for full range of b^2, assuming ^{28}Ne(g.s.) is a pure *sd* shell. Open triangles represent the expected excited 0^+ energy if the mixing matrix element in ^{30}Ne is the same as in ^{32}Mg.

predicted to be weak. Similar arguments apply here as above for slight changes expected from neglected small components in the wave function. We return to this point in Sec. IV below.

However, the situation is quite different for the reaction ^{28}Mg(t,p). The expected σ(exc)/σ(g.s.) ratio for this reaction is plotted in Fig. 6 and is seen to be very large. In fact, of all the ratios presented, this is the only one with an expectation greater than unity. Here, I have assumed that ^{28}Mg(g.s.) is well described totally within the *sd* shell. Again, neglected small components will change the predictions slightly, increasing the g.s. cross section because of constructive interference and either increasing or decreasing the excited-state cross section. The ratio to be found experimentally will probably be slightly smaller than the ratio plotted here.

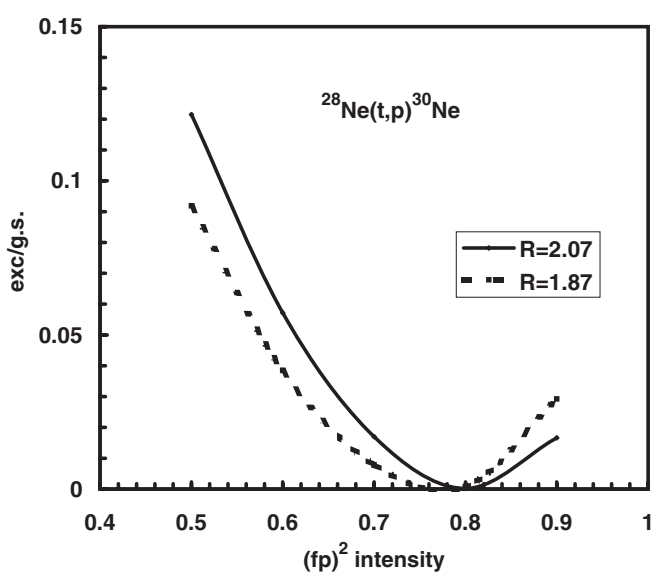

FIG. 8. As Fig. 7 but for a portion of the range. Note small predicted ratio.

E. ^{28}Ne(t,p) ^{30}Ne

Many groups suggest that ^{30}Ne(g.s.) has more $(fp)^2$ admixture than does ^{32}Mg(g.s.). If that is true, we might expect that to be apparent in the (t,p) reaction. With similar definitions for the two 0^+ states in ^{30}Ne, we get the ratio plotted in Figs. 7 and 8. Here, the abscissa is b^2 in ^{30}Ne. The excited 0^+ state is not known in ^{30}Ne. If the mixing matrix element between the two basis 0^+ states is the same in ^{30}Ne and ^{32}Mg, the excitation energy of the excited 0^+ state should depend on b^2 as indicated in Fig. 7. Figure 8 is an expanded version of a portion of Fig. 7.

IV. ROBUSTNESS OF THE PREDICTIONS

For all the calculations presented here, only the two dominant components of the two lowest 0^+ states are considered. Smaller components in the wave functions have been neglected. Their inclusion could change the results somewhat. Because all components in the g.s. will produce constructive interference in the $2n$ transfer, inclusion of the small components could slightly increase the predicted g.s. cross section. However, inclusion of these components would also slightly decrease the contributions of the two dominant components (through overall normalization). For the excited 0^+ state, for which the two major components interfere destructively, the effect could be larger and of either sign. If the predicted excited-state cross section is very small, any additional amplitudes are likely to increase it, whereas, if it is large, they will probably decrease it. For a quantitative estimate, I compare the simple two-component predictions with a calculation in which an excluded component is added in.

For definiteness, I consider that the "other" component mixes with the normal one at the 10% level, so that the ratio of other/normal intensities is the same (10%) in both the g.s. and $0^{+\prime}$. Most of the easily envisioned *other* components have no direct one-step cross section. These include, for example, excitations among the protons or a 2^+ core coupled to $v(fp)^2_2$. I consider two cases in which the cross section from a pure other configuration is 1% and 10% of the simple prediction. Results are listed in Table I. These are appropriate for the two processes ^{30}Mg $\rightarrow$ ^{32}Mg and ^{34}Mg $\rightarrow$ ^{32}Mg. We see that, when the *other* cross section is very small, the g.s. loses a bit because of overall normalization, but even a 10% cross section for *other* leads to an increase for the g.s. The effect on the excited 0^+ state depends on whether the new reaction amplitude is destructive or constructive with respect to the two-component one. When the simple exc/g.s. ratio is relatively large, as in ^{30}Mg(t,p) ^{32}Mg, the addition of this other configuration makes only a small change in the computed ratio. But, when the simple prediction is for a small ratio, as in ^{34}Mg (p,t) ^{32}Mg, the other amplitude can increase the exc/g.s. ratio by up to about 30% or can decrease it by as much as 40%, depending on relative signs. But, it is clear that none of these subtleties could turn a ratio of about 0.60 into 0.05 or vice versa. I conclude that the general flavor of the predictions is stable with respect to such considerations.

TABLE I. Relative cross sections for various assumptions regarding a hypothetical neglected component.

Reaction	Percentage of others	Relative cross section			
		Other	g.s.	Excited state	
				Constr.	Destr.
^{30}Mg(t,p) ^{32}Mg	0		1.0	0.60	
	10	0.01	0.98	0.61	0.57
	10	0.10	1.09	0.66	0.52
^{34}Mg(p,t) ^{32}Mg	0		1.0	0.05	
	10	0.01	0.98	0.055	0.043
	10	0.10	1.09	0.070	0.032

V. SUMMARY

Within the so-called island of inversion, an excited 0^+ state is known in ^{30,32}Mg. Others remain to be discovered. I have estimated cross section ratios $\sigma(\text{exc})/\sigma(\text{g.s.})$ for (p,t) and (t,p) reactions on several nuclei here. As noted earlier, in ^{30}Mg(t,p) ^{32}Mg, the measured ratio of 0.62(6) requires about 19(2)% of the $(fp)^2$ intruder in the ^{32}Mg(g.s.). At anywhere near this amount of mixing, the ratio expected in ^{34}Mg (p,t) is extremely small if ^{34}Mg(g.s.) is pure $(fp)^2$. An admixture of $(fp)^4(sd)^{-2}$ causes the expected ratio to increase monotonically with the magnitude of the admixture. Even with such an admixture in ^{34}Mg, the predicted ratio in ^{32}Mg(t,p) is quite small—leading to the conclusion that this reaction is not a good candidate for locating the excited 0^+ state.

From the $E0$ measurement [19] in ^{30}Mg, the intruder admixture is thought to be quite small—0.0319(76). In the vicinity of that value, the ratio predicted in ^{32}Mg(p,t) is about 0.1. Such a small admixture has a negligible effect on the ^{30}Mg(t,p) prediction. By far, the largest cross section ratio predicted here is for the reaction ^{28}Mg(t,p) where a ratio between 1 and 2 is expected. This result arises because ^{28}Mg is assumed to be a pure sd shell, and ^{30}Mg has only very little mixing with the excited state being the intruder. (Transfer into the fp shell is significantly stronger than into the sd shell.)

Expected results for the ^{28}Ne(t,p) ^{30}Ne reaction are presented for the entire range of intruder-normal-state mixing. If the g.s. of ^{30}Ne has more than 50% of the $(fp)^2$ intruder configuration, the predicted ratio is less than 0.1. I also present the expected excitation energy of the excited 0^+ state in ^{30}Ne if the mixing matrix element is the same as in ^{32}Mg.

All these predictions are based on a model in which the first two 0^+ states are orthogonal linear combinations of two simple basis states. A simple calculation in Sec. IV suggests that the addition of a weak neglected state into both, even at the level of 10%, does not cause a major change in the predictions.

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