4-4-2012

Properties of the Lowest $1/2^+, T = 3/2$ States in $A = 11$ Nuclei

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Properties of the Lowest $1/2^+, T = 3/2$ States in $A = 11$ Nuclei

Abstract
Analysis of energies and widths of the lowest $1/2^+, T = 3/2$ states in $A = 11$ nuclei suggests that the excitation energy in $^{11}$C should be about 200 keV below the energy in the literature, and the width should be 4 to 5 times the literature value. Properties of the state in $^{11}$B and $^{11}$N are in agreement with the present model.

Disciplines
Physical Sciences and Mathematics | Physics

Comments

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Abstract. Analysis of energies and widths of the lowest 1/2+ states in 11B and 11C[1] have been a puzzle for a long time. In a study of the low-lying levels of the A = 11 isospin quartet [2], we found that the 1/2− and 5/2− states behaved appropriately in the four nuclei, but there was a problem for the 1/2+ states. For the ground state (g.s.) of 11N, the various experimental determinations of its energy and width did not agree within the assigned uncertainties. And, for the known (11B) and supposed (11C) 1/2+ states, the experimental widths were only 1/3 (or less) of the values expected. Barker disagreed [3], arguing that these states could have lost width by mixing with T = 1/2 states—ignoring the fact that if these states lost width by mixing, then some nearby states would have acquired this missing width. No such states are known.

We have previously used a simple potential model [4] to compute energies of 0+, T = 2 states using a nuclear plus Coulomb potential to couple states in nuclei A = 1 to single nucleons to produce the T = 2 nuclei A. The model has worked reasonably well. The present situation for A = 11, 12 is similar to that for A = 15, 16 [5]. In the latter, the 0+, T = 2 state was not known in 15F [6], and the 1/2+, T = 3/2 state was unknown in 15O [7]. Also, the energy of the 1/2+ g.s. of 15F was poorly defined because of its large width [8,9]. For A = 11, 12 the 0+, T = 2 state in 12N has not been identified [1], and the 1/2+, T = 3/2 state in 11C is questionable. Similar to 15F, the g.s. of 11N [10,11] is too wide to provide a precise energy for it.

For A = 15, 16, we were able to use the known masses and relationships among the masses in our model to put constraints on the unknown energies and on the percentage of s² component in the 0+, T = 2 state (assumed equal for all five T = 2, A = 16 nuclei) [12]. That procedure also provided “best” values for the energy of the g.s. of 15F. Here, we have attempted to apply that technique to A = 11, 12.

II. 11B

The best evidence for the lowest T = 3/2 state in 11B comes from the 10Be(p,γ) reaction [13]. Those authors found a T = 3/2 state with J = 1/2+ or (3/2+) at an excitation energy of 12.55(3) MeV and with a width of 230(65) keV. This width persisted in the compilations for more than 30 years [1], until those data were refit [14] and it was found that the data required a broad peak in order to explain the cross section. And this width could come only from the 1/2+, T = 3/2 state. The resulting excitation energy and width were 12.61(5) MeV and 640(33) keV, respectively, rather than the 210(20) keV width listed in the compilations [1]. Alternative fits with various assumptions (e.g., energy-dependent width vs constant width, four states vs three) gave widths of about 730 and 700(100) keV. Barker later refit the (p,γ) data and provided a width “of order 600 keV” [15]. So, the 11B puzzle was solved, but the 11C problem remained.

III. 11C

Here, a state at Eγ = 12.16(4) MeV has been assigned T = 3/2 in several reactions [1], but J* has never been assigned. But, of the known states, it is the only candidate to be the required 1/2+ state. This state was observed in the reactions 11B(3He, t), 9Be(3He, n), and in 10B(p,p′) resonance inelastic scattering [16]. In the latter the width is all for decay to the 0+, T = 1 state of 10B. The (3He, t) data are especially compelling because they were compared to results of the inelastic reaction 11B(3He, 3He′) leading to the 11B state discussed above. All these reactions found a small width (as did the inelastic reaction for the 11B state).

Earlier [4] we found that the value of αR² (the s² component) in 10O(g.s.) needed to explain its Coulomb energy was 53(3)%.

And, as noted above, our model assumes this component is the same in the five A = 12 nuclei. Here, we present our results for various values of this parameter. We use the symbol of a nucleus to represent the mass excess of that nucleus, and an asterisk to denote the lowest T = 3/2 state in a Tz = ±1/2 nucleus. We define ΔB and ΔC so that 11B* = 11B+ (Ref. [1]) + ΔB, 11C* = 11C+ (Ref. [1]) + ΔC. As noted above, refitting the 10Be(p,γ) data provided ΔB = 50(50) keV [14]—a small correction.

The 11B* and 11C* masses are needed as input to compute the energy of the lowest 0+, T = 2 state of 12C. If we require that the model fits this energy exactly within the uncertainties, we arrive at a constraint connecting ΔB, ΔC, and αR² represented in Fig. 1. We have temporarily suppressed the uncertainties in the figure, but we return to them shortly. First, we note that the small correction Δγ from the (p,γ) refit [14] (horizontal dashed lines) is consistent with a wide range of values of αR². Secondly, the required value of ΔC is negative; that is, the “best” excitation energy in 11C is below the one in the compilation [1]. For αR² in the previously mentioned
range, the value of $\Delta_C$ is about $-0.23$ MeV. The uncertainty in this value can perhaps be seen better in Fig. 2, where we replot this constraint differently. Here we plot $\Delta_C$ vs $\alpha^2$ for various values of $\Delta_B$. Recall that the earlier estimate for $\Delta_B$ is $0.1$ MeV [14]. The vertical line at $\alpha^2 = 0.53$ is from Ref. [4].

The result for $\Delta_C$ is negative, but with a disappointingly large uncertainty. Smaller uncertainties in the relevant excitation energies would be a great help.

IV. $^{11}$N

For $^{11}$N(g.s.), the experimental $p + ^{10}$C resonance energies [10,11,17–20] cover the range from 1.27 to 1.63 MeV, with widths ranging from 0.24(24) to 1.44(2) MeV. Theoretical resonance energies [4,21–23] span a similar range, but calculated widths are all about 0.8 MeV or larger. These are summarized in Table I and Figs. 3 and 4. In Ref. [21], the large uncertainty in the predicted width comes largely from the uncertainty in predicted energy. We also list three “averages” of the experimental results. The most recent $A = 11$ compilation [20] averaged results of the three experiments with the best resolution to get $E_p = 1.49(6)$ MeV, $\Gamma = 0.83(3)$ MeV. The mass evaluation [24] has an average of $E_p = 1.315(46)$ MeV. If we average all five experimental values, the results are $E_p = 1.41(10)$ MeV, $\Gamma = 0.78(11)$ MeV. Our predictions [4] were 1.35(7) and 0.87(10) MeV, respectively.

So far here, we have not made use of the isobaric multiplet mass equation (IMME). If we use the uncorrected energies for $A = 11$, we can compute the value of $d$—the coefficient of a possible cubic term in the IMME. For $A = 12$, $T = 2$, the result was $d = -8.4(17)$ keV. Thus, those masses do not require a nonzero value for $d$. With $d = 0$ in $A = 11$, $T = 3/2$, the masses obey a simple relation: $^{11}$N = $^{11}$Be $-3^{11}$B$^+ + 3^{11}$C$^*$. The mass tables [24] list a $^{11}$N mass

![FIG. 1. (Color online) Plot of energy correction in $^{11}$B vs the correction in $^{11}$C needed to fit the $0^+$, $T = 2$ energy in $^{12}$C for various values of $\alpha^2$, the $s^2$ fraction in the $0^+$ state. Horizontal dashed lines represent the 50(50) keV $^{11}$B correction from Ref. [14].](image1)

![FIG. 2. (Color online) Same information as in Fig. 1, but plot of the $^{11}$C correction vs $\alpha^2$, for various $^{11}$B corrections. Vertical line at $\alpha^2 = 0.53$ is from Ref. [4].](image2)

<table>
<thead>
<tr>
<th>Label</th>
<th>Method</th>
<th>$E_p$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expt.</td>
<td>1 $p + ^{10}$C elastic</td>
<td>1.30(4)</td>
<td>0.99(0.10)</td>
<td>[17]</td>
</tr>
<tr>
<td></td>
<td>2 $p + ^{10}$C elastic</td>
<td>1.27(0.18)</td>
<td>1.44(2)</td>
<td>[10]</td>
</tr>
<tr>
<td></td>
<td>3 $^{10}$B($^{14}$N,$^{13}$B)</td>
<td>1.63(5)</td>
<td>0.4(1)</td>
<td>[18]</td>
</tr>
<tr>
<td></td>
<td>4 $^{14}$N($^3$He,$^4$He)</td>
<td>1.31(5)</td>
<td>0.2(24)</td>
<td>[19]</td>
</tr>
<tr>
<td></td>
<td>5 $p + ^{10}$C elastic</td>
<td>1.54(2)</td>
<td>0.83(3)</td>
<td>[11]</td>
</tr>
<tr>
<td>Ave.</td>
<td>7 Compilation</td>
<td>1.49(6)</td>
<td>0.83(3)</td>
<td>[20]</td>
</tr>
<tr>
<td></td>
<td>8 Mass evaluation</td>
<td>1.315(46)</td>
<td></td>
<td>[24]</td>
</tr>
<tr>
<td></td>
<td>9 Present</td>
<td>1.41(10)</td>
<td>0.78(11)</td>
<td>Present</td>
</tr>
<tr>
<td>Calc.</td>
<td>11 Mirror of $^{11}$Be</td>
<td>1.35(7)</td>
<td>0.87(10)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>12 Mirror of $^{11}$Be</td>
<td>1.60(22)</td>
<td>1.58(0.75)</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>13 Mirror of $^{11}$Be</td>
<td>1.2</td>
<td>1.1</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>14 Mirror of $^{11}$Be</td>
<td>1.34</td>
<td>1.47</td>
<td>23</td>
</tr>
</tbody>
</table>
These two proposed energy corrections for $^{11}\text{C}$ are summarized in Table II. The corrections are $\Delta C = -0.27(10)$ MeV for $^{11}\text{Be}, ^{12}\text{B}$, $^{12}\text{C}$, and $^{12}\text{O}$, and $\Delta C = -0.13(7)$ MeV for $^{11}\text{Be}, ^{11}\text{B}, ^{11}\text{N}$. A weighted average gives $\Delta C = -0.18(6)$ MeV. These corrections are consistent with the other analysis presented in Sec. III above.

V. DISCUSSION

The spectroscopic factors for the four $1/2^+$, $T = 3/2$ states are listed in Table III. For all but $^{11}\text{Be}$, these are obtained from the expression $C^2 S = \Gamma_{\exp}/\Gamma_{\text{sp}}$, where $C^2$ is $1/3$, $2/3$, and 1 for $^{11}\text{B}, ^{11}\text{C}$, and $^{11}\text{N}$, respectively. Estimates of $\Gamma_{\text{sp}}$ are listed in the table. They were calculated using a Woods-Saxon potential (plus Coulomb), with $r_0 = 1.25$ fm and $a = 0.65$ fm. The depths were adjusted to reproduce the observed energies. The difficulty with $^{11}\text{C}$ is apparent. The spectroscopic factor derived from its width is only about 20% of $S$ for the other three nuclei—and the $S$’s should all be equal. As pointed out above, if the $^{11}\text{C}$ state loses width (spectroscopic strength) by mixing with $T = 1/2$ states, then one or more of them should exhibit this strength, and none do. We recall that the $1/2^+$, $T = 3/2$ state in $^{15}\text{O}$ has also never been identified. An early candidate turned out to have $T = 1/2$, as demonstrated by its large width for a decay that would be forbidden for a $T = 3/2$ state.

The problem in $^{11}\text{C}$ is not with the $sp$ widths. Barker [3] used a potential model to compute $\Gamma_{sp}$ for states at the experimental energies. His values (last column of Table III) are similar to ours. For $^{11}\text{C}$, his $sp$ width is actually 12% larger than ours. We thus expect a $1/2^+$, $T = 3/2$ state near 12 MeV in $^{11}\text{C}$, with a width of about 1.2 MeV and $C^2 S_{1/2} \approx 0.50$.

We have given considerable thought to finding a reaction to make these states in $^{11}\text{C}$ (and $^{15}\text{O}$). The $(p,t)$ reaction does not work, because the targets do not contain the $2\Delta_{1/2}$ nucleon that is the main feature of these states. The $(^3\text{He},t)$ reaction populates both $T = 1/2$ and 3/2 states, as does $(^3\text{He},n)$. Finding a state at roughly the expected energy that preferentially decays to the 0$^+$, $T = 1$ state of $^{10}\text{B}$ in the $^{10}\text{B}(p,p')$ reaction was encouraging, but the width reported there is also too small by about a factor of five. In the $(^3\text{He},n)$ reaction the background (both real and from $T = 1/2$ states) is a serious problem. This reaction does have the advantage that cross-section ratios for different $T = 3/2$ final states should be approximately the same in $(^3\text{He},n)$ and $(t,p)$ on the same target and under similar kinematic conditions. Thus, for a $^{10}\text{Be}$ target, we expect the ratio $\sigma(1/2^+)/\sigma(1/2^-)$ in $(^3\text{He},n)$ to be roughly equal to the same ratio in $(t,p)$. In the latter, the ratio at the peak angle and the ratio of angle-integrated cross sections were both about 0.22. The best candidate might be $^{10}\text{C}(d,p)$ in reverse kinematics. In that reaction, $C^2 S_n$ would be about 0.25.

There is one last possibility to consider—could interference between overlapping $T = 1/2$ and $T = 3/2$ states cause a broad negative dip? If so, the void between 11.44 and 12.16 MeV in $^{11}\text{C}$ could actually be the negative profile of the $1/2^+$.
TABLE III. Widths (in MeV) and spectroscopic factors for $1/2^+$, $T = 3/2$ states in $A = 11$ nuclei.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>$E_x$ (MeV)</th>
<th>$\Gamma$</th>
<th>$\Gamma_{sp}$</th>
<th>$S$</th>
<th>$\Gamma_{sp}$ (Ref. [3])</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}\text{Be}$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{11}\text{B}$</td>
<td>12.61(5)$^a$</td>
<td>0.640(33)$^a$</td>
<td>$\sim$2.4</td>
<td>0.80$^b$</td>
<td>2.31</td>
</tr>
<tr>
<td>$^{11}\text{C}$</td>
<td>12.16(4)$^b$</td>
<td>0.27(5)$^b$</td>
<td>$\sim$2.4</td>
<td>0.16</td>
<td>2.69</td>
</tr>
<tr>
<td>$^{11}\text{N}_0$</td>
<td>0$^c$</td>
<td>0.83(3)$^d$</td>
<td>$\sim$1.3</td>
<td>$\sim$0.64</td>
<td>1.42</td>
</tr>
</tbody>
</table>

$^a$Includes the correction from Ref. [14].
$^b$Ref. [1].
$^cE_p = 1.32$ to 1.49 MeV (averages in Table I).
$^d$Average in Table I.

$^e$As averaged in Ref. [4].

$T = 3/2$ state. But, would this interference be about the same in, say, ($^3\text{He},t$) and ($p,p'$)?

VI. CONCLUSION

We have noted here that the global averages of the energy and width of $^{11}\text{N}(g.s.)$ are consistent with the calculations. We conclude that the small correction found earlier [14] for the energy of the $1/2^+$, $T = 3/2$ state in $^{11}\text{B}$ is consistent with the current analysis, and that the previous problem [2] with the width in $^{11}\text{B}$ has been solved [14]. A larger, negative energy correction [180(60) keV] is needed for $^{11}\text{C}$. That finding presents a problem, because in $^{11}\text{C}$ there is nothing between 11.44 and 12.16 MeV. The width in $^{11}\text{C}$ should be 4 to 5 times the currently accepted value. From inspection of the relevant spectra, it is difficult to see how the $1/2^+$ width could be several times the estimate in the compilation [1], especially if the energy is shifted lower. Of course, inspection of the spectra also provided a small width for $^{11}\text{B}^*$, which we now know was a factor of three too small. It would be very useful to find a way to settle this width question in $^{11}\text{C}$.

ACKNOWLEDGMENT

I acknowledge many interesting discussions with Rubby Sherr.