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# $B(E2)$ Values in $^{12}$Be and Core Excitation

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**Abstract**

I have examined the $B(E2)$'s in $^{12}$Be connecting the first $2^+$ state to the first two $0^+$ states. I find that they can be understood in the simple model that has been successful for a variety of other observables in this nucleus, but only if the $2^+$ state has an excited-core component.

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# $B(E2)$ values in $^{12}$Be and core excitation

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I have examined the $B(E2)$'s in $^{12}$Be connecting the first $2^+$ state to the first two $0^+$ states. I find that they can be understood in the simple model that has been successful for a variety of other observables in this nucleus, but only if the $2^+$ state has an excited-core component.

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## I. INTRODUCTION

It is now fairly well established that the ground state (g.s.) and $2^+$ first-excited state of $^{12}$Be are dominated by two-neutron excitations into the $sd$ shell. This fact was first demonstrated [1] with the $^{10}$Be$(t,p)$ reaction, in which the g.s. is about five times as strong as the pure $p$-shell state would be, and the $2^+$ state is about 20 times the $p$-shell expectation. The analysis of the $(t,p)$ data and a calculation [2] of the $^{12}$Be-$^{12}$O energy difference determined the g.s. $p$-shell component to be about 32(3)%. A similar percentage was subsequently confirmed [3,4] in other independent processes.

For the $2^+$ state, the $(t,p)$ reaction demonstrated $sd$-shell dominance, but did not determine the $p$-shell admixture, but certainly it is less than about 0.3. Reanalysis [5] of data for the $^{14}$C$(p,t)$ reaction [6] suggested that the second $T = 2$ peak contained both the $2^+$ and the excited $0^+$ states, with the physical $2^+$ state containing about 19(9)% of the $p$-shell $2^+$ state. If all the cross section of the second peak had been attributable to the $2^+$ alone, this $p$-shell percentage would have been 24(8)%.

So, it seems reliably established that the g.s. has about 65%–70% $(sd)^2$ and the $2^+$ has 70%–90% $(sd)^2$. A simple shell-model calculation gave the $d^2/s^2$ ratio = 0.22/0.78 for the g.s. [5] and the $d^2/ds$ ratio = 0.13/0.87 for $2^+$ [7]. In our model, the excited $0^+$ at 2.24 MeV is just the orthogonal admixture of the two basis states: $(sd)^2$ and $p$-shell. These wave functions are listed in Table I.

Now, consider the $E2$ transitions connecting the $2^+$ to the two $0^+$ states. These two $B(E2)$'s (Table II) have recently been measured. For $0^+ \rightarrow 2^+$ they are 7.0(6) $e^2$ fm$^4$ for the excited $0^+$ state [8] and $40 \pm 11 \pm 4$ $e^2$ fm$^4$ for the g.s. [9]. The shell model of Ref. [8] predicts $B(E2;0_2 \rightarrow 2) = 25$ $e^2$ fm$^4$ with "standard" effective charges $e_p = 1.3e$, $e_n = 0.5e$. Reducing them to $e_p = 1.2e$ and $e_n = 0.2e$ still gives a large $B(E2) = 9.13$ $e^2$ fm$^4$ [8], but now comparable to the experimental value. With antisymmetrized molecular dynamics (AMD), the calculated $B(E2)$ underestimates the observed value by about a factor of three [8]. Using single particles in a deformed potential, with approximately equal mixing between $1/2^+$ and $1/2^-$ Nilsson orbitals, Hamamoto and Shimoura [10] obtain $B(E2; 0_2 \rightarrow 2) = 141$ $e_n^2$ fm$^4$. Thus, to fit the observed value of 7.0(6) $e^2$ fm$^4$ would require a neutron effective charge of $e_n = 0.22(1)e$, quite a small value.

Umaya *et al.* [11] performed mixed-shell shell-model calculations for some neutron-rich Be and C nuclei, using two different *psd* interactions SFO [12] and PSDMK [13]. For the wave functions of the first $0^+$ and $2^+$ states in $^{12}$Be, results for the two calculations were quite different, and neither was close to the wave functions now in common use. The SFO results were closer, but had 58.5% $p$ shell in the g.s. and only 4.4% in the $2^+$ state. However, predictions for the g.s. $B(E2)$ were much better: 34.4 $e^2$ fm$^4$ with SFO and 31.0 with PSDMK, close to the later experimental value of $40 \pm 11 \pm 4$ [9]. They did not give a calculated value for $0_2^+ \rightarrow 2^+$.

The aim here is to investigate whether these $B(E2)$'s can be understood within the framework of the same model that has been successful for several other properties of $^{12}$Be.

## II. THE MODEL

In the simplest model the wave functions are

$$\begin{aligned}
\text{g.s.} &= a\ ^{10}\text{Be}(p\text{-shell g.s.}) \times v(sd)_0^2 \\
&\quad + b\ ^{12}\text{Be}(p\text{-shell g.s.}), \\
2^+ &= A\ ^{10}\text{Be}(p\text{-shell g.s.}) \times v(sd)_2^2 \\
&\quad + B\ ^{12}\text{Be}(p\text{-shell } 2_1^+), \\
0^+_{\text{exc}} &= -b\ ^{10}\text{Be}(p\text{-shell g.s.}) \times v(sd)_0^2 \\
&\quad + a\ ^{12}\text{Be}(p\text{-shell g.s.}).
\end{aligned}$$

Later I add another term to the $2^+$ wave function: $C\ ^{10}\text{Be}(p\text{-shell } 2_1^+) \times v(sd)_0^2$.

Define $M^2(E2) = (2J_i + 1)B(E2; i \rightarrow f)$ so that the value of $M$ is independent of the direction of the transition. Then, in an obvious notation, for the two-component $2^+$ state, the physical $E2$ amplitudes are

$$\begin{aligned}
M(\text{exc}) &= -bA\, M(sd) + aB\, M(1p); \\
M(\text{g.s.}) &= aA\, M(sd) + bB\, M(1p),
\end{aligned}$$

where $M(sd)$ is the $E2$ amplitude connecting the $0_1^+$ and $2_1^+$ $(sd)^2$ states, and $M(1p)$ is that for the $p$-shell $0^+$ and $2^+$ states. In terms of shell-model amplitudes, we have

$M(E2) = e_pA_p + e_nA_n$, where $A_p$ and $A_n$ are bare shell-model $E2$ transition matrix elements. In the present space, $M(sd) = e_nA_n(sd)$, because we have no $sd$-shell protons; and $M(1p) = e_pA_p(1p)$, because the neutron $p$ shell is filled. The value of $M(1p)$ is about 5.88 $e$ fm$^2$ from a shell-model calculation [14] totally within the $p$ shell, using a proton effective charge of $e_p = 1.2e$ (the neutron shell is full; hence, no $n$ participation.) This value of $M(1p)$ corresponds

 

TABLE I. Energies and wave-function intensities in $^{12}$Be.

| $J^\pi$ | Space | State | $E_x$ (MeV) | $s^2$ | $d^2$ | $p$ shell | Reference |
|---|---|---|---|---|---|---|---|
| $0^+$ | $(sd)^2$ | $0^+_1$ | 0.20 | 0.78 | 0.22 | — | |
| | $(sd)^2$ | $0^+_2$ | 4.35 | 0.22 | 0.78 | — | |
| | $(sd)^2$ + $p$ shell | g.s. | | 0.53 | 0.15 | 0.32 | [2,5] |
| | | exc $0^+$ | | –0.25 | –0.07 | 0.68 | |
| | | | | $ds$ | $d^2$ | | |
| $2^+$ | $(sd)^2$ | $2^+_1$ | 3.63 | 0.87 | 0.13 | — | |
| | $(sd)^2$ | $2^+_2$ | 5.42 | 0.13 | 0.87 | — | |
| | $(sd)^2$ + $p$ shell | 2.11 MeV | | 0.71 | 0.10 | 0.19 | [7] |

to $B(E2; 2^+ \rightarrow 0^+) = 6.92\ e^2$ fm$^4$ for the $p$-shell $^{12}$Be. With typical $0^+_1$ and $2^+_1$ states within the $(sd)^2$ space, the $sd$-shell $E2$ matrix element is $M(sd) \sim 4.5\ e$ fm$^2$ for an effective charge of $e_n = 0.5e$. (With an effective charge of $e_n = 0.5e$ and the pure $sd$-shell wave functions of Table I, $M(sd)$ is 4.33 $e$ fm$^2$.) If the effective charge is $e_n = (Z/A)e$, we have $M(sd) \sim 3.3\ e$ fm$^2$. This is a pure neutron excitation. We would hope to be able to find a solution with $M(sd)$ between these two limits.

## III. CALCULATIONS

Oddly enough, the $B(E2)$ from the excited $0^+$ state is very close to the value it would have (Table III) if the $2^+$ state were pure $^{10}$Be($p$-shell g.s.) $\times$ $\nu(sd)^2_2$, that is, if $B = 0$ in the $2^+$ wave function. However, if we ignore the presence of any $p$-shell amplitude in the $2^+$, it is obvious that the ratio of $B(E2)$'s connecting the $0^+$ states to the $2^+$ is independent of the $d^2/s^2$ ratio in the $0^+$ state and independent of the $ds/d^2$ ratio in the $2^+$. Also, the ratio is independent of the value of the $2^+ \rightarrow 0^+$ $(sd)^2$ $E2$ matrix element [and, of course, independent of $M(1p)$]. Furthermore, the ratio of $B(E2)$'s is just equal to the $(sd)^2$ intensity ratio in the two $0^+$ states. The measured ratio of 0.175(52) is significantly smaller than the ratio of 0.32/0.68 expected. Also, the sum of the two experimental $B(E2)$'s is too large for the assumption of no $p$-shell component in the $2^+$ state.

I use the procedure outlined below. Throughout I adopt the $0^+$ values of $a^2 = 0.68$, $b^2 = 0.32$ from Refs. [2] and [5]. The equations for $M$(exc) and $M$(g.s.) given above can be rewritten as

$$AM(sd) = aM(\text{g.s.}) - bM(\text{exc}) \text{ and}$$
$$BM(1p) = aM(\text{exc}) + bM(\text{g.s.}).$$

From the experimental $B(E2) = 7.0(6)\ e^2$ fm$^4$, we have $M(\text{exc}) = \pm\ 2.65(11)\ e$ fm$^2$ (Table II). Clearly, the negative square root must be chosen for $M$(exc). With $a$, $b$ known, results are $BM(1p) = 1.39(53)\ e$ fm$^2$ and $AM(sd) = 6.71(98)\ e$ fm$^2$, listed in Table IV. Recall that we are using $M(1p) = 5.88\ e$ fm$^2$. Thus, $B = 0.236(90)$, $B^2 = 0.056(42)$. Then, with $A$, $B$ related via $A^2 + B^2 = 1$, we can compute $M(sd) = 6.9(10)\ e$ fm$^2$. Note that, even for such small values of $B^2$, the value of $M(sd)$ needed becomes unreasonably large. This is not surprising, because this experimental $B(E2)$ is very close to the result for $B = 0$. Thus, even though we obtain an approximate fit to both $B(E2)$'s with only two terms in the $2^+$ wave function, the results are unsatisfactory: The values of $B^2$ are very small, and the required value of $M(sd)$ is larger than seems reasonable. In the two-component $2^+$, the sum of the two $B(E2)$'s is $\Sigma\ B(E2) = A^2\ M^2(sd) + B^2\ M^2(1p)$. Here, too, the experimental value is larger than the calculated one for any reasonable range of $M(sd)$.

So, what is wrong with this simple model, and why is it unable to account for these two $B(E2)$'s? I investigate next the possible presence of a component of the $^{12}$Be $2^+$ state that has a $^{10}$Be excited core, specifically $^{10}$Be($2^+$) $\times$ $\nu(sd)^2_0$. We know this term is present at some level, but it has been ignored before because it has no direct one-step route in the $(t,p)$ reaction. With this new term for the $2^+$, we have

$$2^+ = A\ ^{10}\text{Be}(p\text{-shell g.s.}) \times \nu(sd)^2_2 + B\ ^{12}\text{Be}(p\text{-shell } 2^+_1) + C\ ^{10}\text{Be}(p\text{-shell } 2^+_1) \times \nu(sd)^2_0.$$

Then the $E2$ amplitudes are

$$M(\text{g.s.}) = aAM(sd) + b[BM(1p) + CM(^{10}\text{Be})];$$
$$M(\text{exc}) = -bAM(sd) + a[BM(1p) + CM(^{10}\text{Be})],$$

where $M^2(^{10}\text{Be}) = 5B(E2; 2^+ \rightarrow 0^+)$ in $^{10}$Be. The new term could contribute significantly here, because the $^{10}$Be $B(E2)$ is large, about 9.2 $e^2$ fm$^4$ [15]. I expect $C^2$ to be small, certainly <0.3 from results of the $(t,p)$ reaction. With this third component in the $2^+$ state, a fit to the excited $0^+$ $B(E2)$ is underdetermined. An extra complication here is that the $C$ term also contributes (but only slightly) to the $^{14}$C$(p,t)$ reaction

TABLE II. Experimental $B(E2; 0^+ \rightarrow 2^+)$ in $^{12}$Be.

| Transition | $B(E2)$ ($e^2$ fm$^4$) | Reference | $M(E2)$ ($e$ fm$^2$) |
|---|---|---|---|
| $0_2 \rightarrow 2_1$ | 7.0(6) | [8] | $\pm$2.65(11) |
| g.s. $\rightarrow 2_1$ | 40(12) | [9] | 6.32(92) |

TABLE III. Calculated $B(E2; 0_2 \rightarrow 2_1)$ with pure $(sd)^2$ $2^+$.

| $e_n$ | $M(sd)$ ($e$ fm$^2$) | $B(E2)$ ($e^2$ fm$^4$) |
|---|---|---|
| $0.5e$ | 4.5 | 6.48 |
| $(Z/A)e$ | 3.3 | 3.43 |

TABLE IV. Results for two-component $2^+$ state. Units of $M$ are $e$ fm$^2$.

| $M$(exc) | $M$(g.s.) | $AM(sd)$ | $BM(1p)$ | $B$[a] | $M(sd)$ |
|---|---|---|---|---|---|
| $\pm 2.65(11)$ | 6.32(92) | 6.71(98) | 1.39(53) | 0.236(90) | 6.9(10) |

[a]Computed from value of $BM(1p)$ in fourth column, using $M(1p) = 5.88\ e$ fm$^2$ (see text).

through core excitation [16] in $^{14}$C(g.s.). So I withhold the $(p,t)$ constraint until later.

A set of approximate solutions have $B \sim C$, close to the parameter contour that would maximize the sum of the two $B(E2)$'s. I mentioned above that, in the simplest model, this sum is larger experimentally than expected for reasonable values of $M(sd)$. Results of one such search are displayed in Fig. 1, where I plot vs $A^2$ the value of $M(sd)$ that fits the excited $0^+$ $B(E2)$ together with the calculated and experimental g.s. $M(E2)$. Note that the calculated and experimental values agree to better than $1\sigma$ for a range of $A^2$ values just below $A^2 = 0.67$. The horizontal lines crossing the $M(sd)$ curve are at $M(sd) =$ 4.5 and 3.3 $e$ fm$^2$, the values corresponding to $e_n = 0.5e$ and $(Z/A)e$, respectively. Agreement with $M$(g.s.) can be had for most of the $M(sd)$ range between these two limits. If we now impose a constraint from $(t,p)$ that $A^2 = 0.70(7)$, our solution can be limited to the upper end of the allowed range in Fig. 1. For $A^2 = 0.66$, $B^2 = 0.15$, $C^2 = 0.19$, we have $M(sd) =$ 3.61(25) $e$ fm$^2$. Furthermore, if I use these values of $B$, $C$ to estimate the $2^+$ cross section in $^{14}$C$(p,t)$, I find that the $2^+$ wave function contains 18% of the $p$-shell $2^+$ cross section. These results are summarized in Table V. These results are reasonably close to the estimates of Ref. 7 (in Table I), but the $2^+$ $p$-shell component is now about 15%, rather than 19(9)%, and the $sd$-shell intensity of 85% (instead of 81%) has an excited-core component.

For a given value of $A$, the predictions are a very slow function of $B$ and $C$, as long as we have $A^2 + B^2 + C^2 = 1$. So, we have not really independently determined $B$ and $C$, but some combination of them. To get the $B(E2)$ sum large enough we need $C$ slightly greater than $B$.

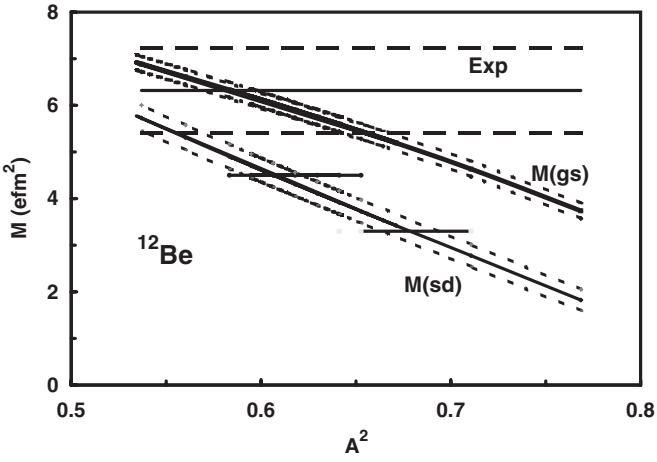


FIG. 1. The experimental value of $M$(g.s.) = 6.32(92) $e$ fm$^2$; plotted vs $A^2$ is the value of $M(sd)$ that fits the $0^+_{\rm exc} \rightarrow 2^+$ $B(E2)$, and the value of $M$(g.s.) computed with this $M(sd)$, but now for a three-component $2^+$ state. Horizontal lines crossing $M(sd)$ are at 3.3 and 4.5 $e$ fm$^2$ (see text).

## IV. POSSIBLE REFINEMENTS

Some minor components that are known to exist in the wave functions have been omitted in the current treatment. These include

(i) $(sd)^2$ configurations involving the $d_{3/2}$ orbital, abbreviated as $d$' here,
(ii) the configuration $^{10}$Be$(2^+) \times (sd)^2_2$, which we call $2 \times 2$, and
(iii) configurations with four $sd$-shell neutrons coupled to states in $^8$Be, which we call $4\ \hbar\omega$.

Inclusion of the $d$' components would increase the $(sd)^2$ $(t,p)$ cross section by a few percent, because of constructive coherence. Neither of the other two components has a direct one-step route in the $(t,p)$ reaction. Their inclusion would thus slightly decrease the $(t,p)$ predictions, through renormalization. Inclusion of all three components should leave the $(t,p)$ cross sections basically unchanged.

Each of the three components, if included, would lead to a renormalization of the quantity called $M(sd)$ above, thereby widening its allowed range. We do not use a calculated value for this quantity, but treat it as a free parameter, within certain limits. Because the resulting fitted value was within the previous allowed range, it would naturally also be within a wider allowed range.

We conclude that omission of these three components in our simple model has no appreciable effect on the outcome. Of course, even the simplest $(0+2)\ \hbar\omega$ shell-model calculation would include the first two. It is likely that such calculations should be expanded to include $4\ \hbar\omega$. We remarked in the Introduction that no shell-model calculations have been successful in explaining both the $B(E2)$'s that are the subject of the present analysis.

## V. SUMMARY

I have calculated the $B(E2)$'s connecting the first $2^+$ state of $^{12}$Be with the first two $0^+$ states, using previous wave functions for the latter and both two-and three- component models for the former. With two components in the $2^+$ state,

TABLE V. Wave-function intensities for three-component $2^+$ state.

| Component | Intensity |
|---|---|
| $^{10}$Be($p$-shell g.s.) $\times$ $\nu(sd)^2_2$ | 0.66 |
| $^{12}$Be($p$-shell $2^+_1$) | 0.15 |
| $^{10}$Be($p$-shell $2^+_1$) $\times$ $\nu(sd)^2_0$ | 0.19 |

the model requires the $sd$-shell $E2$ amplitude to be larger than is reasonable. With three components, the calculations are underdetermined, but agreement can be achieved for a range of the two small components, with about 66% of the dominant one. These are listed in Table V.

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