Phenomenology of Heterotic String Theory: With Emphasis on the B-L/EW Hierarchy

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Phenomenology of Heterotic String Theory: With Emphasis on the B-L/EW Hierarchy

Abstract
E8 x E8 heterotic string and M-theory, when appropriately compactified, can give rise to realistic, N = 1 supersymmetric particle physics. In particular, the exact matter spectrum of the MSSM is obtained by compactifying on Calabi-Yau manifolds admitting specific SU(4) vector bundles. These "heterotic standard models" have the SU(3)C x SU(2)L x U(1)Y gauge group of the standard model augmented

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PHENOMENOLOGY OF HETEROTIC STRING THEORY:
WITH EMPHASIS ON THE B-L/EW HIERARCHY

Michael Ambroso

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ABSTRACT

PHENOMENOLOGY OF HETEROTIC STRING THEORY: WITH EMPHASIS ON THE B-L/EW HIERARCHY

Michael Ambroso

Burt Ovrut, Advisor

$E_8 \times E_8$ heterotic string and M-theory, when appropriately compactified, can give rise to realistic, $N = 1$ supersymmetric particle physics. In particular, the exact matter spectrum of the MSSM is obtained by compactifying on Calabi-Yau manifolds admitting specific $SU(4)$ vector bundles. These “heterotic standard models” have the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group of the standard model augmented by an additional gauged $U(1)_{B-L}$. In this thesis, we report on the phenomenological viability of these compactifications. Through a series of increasingly sophisticated analyses, we consider a wide variety of phenomenological effects and compare them to present experimental bounds. We have found that, given the constraints considered, phenomenologically viable regions of parameter space exist and lead to interesting phenomenology, including exact predictions of new particle mass spectra.
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Chapter 1

Introduction

In this thesis, we report on the work that lead to the results presented in [2, 3, 4, 5]. This undertaking took place from the fall of 2006 to the spring of 2010 and was in collaboration with Burt Ovrut and, to a limited extent, Volker Braun. The aim of this effort was to assess the phenomenological viability of the Heterotic compactifications given in [21] and [22]. We believe this work to be significant as it provides a rare connection of String Theory and String motivated Field Theories to experimentation. Through a series of increasingly sophisticated analyses, we consider a wide variety of phenomenological effects and compare them to present experimental bounds. We have found that, given the constraints considered, phenomenologically viable regions of parameter space exist and lead to interesting phenomenology, including exact predictions of new particle mass spectra.

We outline the author’s contributions as well as provide additional supporting
details to the above projects. We begin by giving a brief background on the subject.

**Background**

Smooth compactifications of the weakly coupled [65] and strongly coupled [68, 96, 69] $E_8 \times E_8$ heterotic string have been studied for many years. When the compactification is on a Calabi-Yau threefold with a slope-stable, holomorphic vector bundle, the low energy four-dimensional effective theory is $N = 1$ supersymmetric. In recent years, such compactifications have been extended to complete intersection and elliptically fibered Calabi-Yau spaces admitting vector bundles constructed using monads [42, 75, 18, 7, 8], spectral covers [61, 43, 44, 45] and by extension of lower rank bundles [46, 47]. The formalism for explicitly computing the low energy spectrum in each of these cases has been developed, and presented in [19, 9], [48, 49] and [50, 23] respectively. Cohomological methods have been used to calculate the texture of Yukawa couplings and other parameters in these contexts [2, 24, 10]. Finally, the non-perturbative string instanton contributions to the superpotential have been computed [67, 89, 80, 32] and used to discuss moduli stability, supersymmetry breaking and the cosmological constant [25]. These methods underlie the theory of “brane universes” [81, 51] and new approaches to cosmology [77, 33].

In a series of papers, compactifications of the $E_8 \times E_8$ superstring have been constructed on elliptically fibered Calabi-Yau spaces with $\mathbb{Z}_3 \times \mathbb{Z}_3$ homotopy over a $dP_9$ surface [26]. These spaces admit a specific class of slope-stable, holomorphic
vector bundles with structure group $SU(4)$ that are constructed by extension and are equivariant under $\mathbb{Z}_3 \times \mathbb{Z}_3$ [27]. The non-trivial homotopy allows one to extend these bundles with flat $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines. Using the methods referenced above, the four-dimensional spectrum of these theories was computed. It is found to have precisely the matter content of the minimal supersymmetric standard model (MSSM), including three right-handed neutrino chiral supermultiplets, one per family. In addition there are a small number of Higgs-Higgs conjugate pairs, two in the model presented in [21] and one in the vacuum of [22]. These are termed “heterotic standard models”. They all contain a relatively small number of geometric and vector bundle moduli and each possesses an acceptable texture of Yukawa couplings. In Section 2, we will consider the minimal heterotic standard model [21] containing the matter spectrum of the MSSM, two pairs of Higgs-Higgs conjugate superfields as well as six geometric moduli and a small number of vector bundle moduli. In Sections 3 and 4, we consider the a similar model with, again, the exact MSSM matter spectrum, but with one pair of Higgs-Higgs conjugate superfields as well as three complex structure moduli, three Kahler moduli and thirteen vector bundle moduli.

The four-dimensional gauge group is obtained through the sequential breaking of $E_8$ by the $SU(4)$ structure group of the vector bundle and the $\mathbb{Z}_3 \times \mathbb{Z}_3$ of the Wilson lines. We find that

$$E_8 \xrightarrow{SU(4)} Spin(10) \xrightarrow{\mathbb{Z}_3 \times \mathbb{Z}_3} SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}.$$
Note that in addition to the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, there is an extra gauged $U(1)_{B-L}$. This arises from the fact that $Spin(10)$ has rank five and that the rank must be preserved when the group is further broken by any Abelian finite group, such as $\mathbb{Z}_3 \times \mathbb{Z}_3$. Since the standard model group has rank four, an extra $U(1)$ gauge factor must appear, in this case precisely $U(1)_{B-L}$.

We would like to emphasize that there is a direct relationship between having three right-handed neutrino chiral multiplets, one per family, and the appearance of the additional $U(1)_{B-L}$ gauge factor. The $SU(4)$ structure group in heterotic standard models is chosen precisely because the decomposition of the $248$ of $E_8$ with respect to it contains the $16$ representation of $Spin(10)$. It is well-known that each $16$ is composed of one family of quarks and leptons, including a right-handed neutrino. This fact makes choosing a vector bundle with an $SU(4)$ structure group a natural way to ensure that the spectrum contains three right-handed neutrinos.

However, the rank of $Spin(10)$ is five, one larger than the standard model gauge group. Hence, when $Spin(10)$ is broken to $SU(3)_C \times SU(2)_L \times U(1)_Y$ by the Abelian $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines, an additional $U(1)_{B-L}$ must appear.

The existence of the extra $U(1)_{B-L}$ gauge factor, far from being being extraneous or problematical, is precisely what is required to make a heterotic vacuum with $SU(4)$ structure group phenomenologically viable. The reason is the following. As is well-known, four-dimensional $N = 1$ supersymmetric theories generically contain two lepton number violating and one baryon number violating dimension four oper-
ators in the superpotential. The former, if too large, can create serious cosmological difficulties, such as in baryogenesis and primordial nucleosynthesis [93, 35, 36, 53], as well as coming into conflict with direct measurements of lepton violating decays [16]. The latter can produce extremely rapid proton decay, far in excess of the observed bound on its lifetime [16, 70]. To avoid these problems, it is traditional in low-energy $N = 1$ supersymmetric theories to impose a discrete “matter parity”, the supersymmetric version of “R-parity” [70, 92, 57, 56, 55, 79, 41]. This $Z_2$ finite symmetry disallows these dimension four operators from appearing in the superpotential, thus solving all the above problems. Remarkably, the $B$-$L$ MSSM theory naturally contains matter parity as a $Z_2$ subgroup of $U(1)_{B-L}$. As long as this subgroup is unbroken, or weakly broken, the theory will be phenomenologically and cosmologically viable. Importantly, however, since a gauged $B$-$L$ Abelian symmetry is not observed at low energy, it is essential that $U(1)_{B-L}$ be spontaneously broken above the electroweak scale.

In supersymmetric grand unified theories (GUTs), where the matter content can be chosen arbitrarily, this breaking can occur at a high scale [88, 99, 76, 78, 83]. This is accomplished by adding multiplets to the MSSM, neutral under the standard model group, for which $3(B-L)$ is an even, non-zero integer [84]. It is then arranged for at least one of these multiplets to get a large vacuum expectation value (VEV). This spontaneously breaks the gauged symmetry, giving a large mass to the $B$-$L$ vector boson, but, since the $3(B - L)$ charge is even, leaves $Z_2$ matter parity as a
discrete symmetry. Hence, although $U(1)_{B-L}$ is broken at a high scale, $\mathbb{Z}_2$ matter parity is an exact symmetry at lower scales. However, this mechanism cannot occur within the context of the exact MSSM spectrum in the heterotic standard models, since for all fields $3(B - L)$ is either $\pm 1$, $\pm 3$ or 0. It follows that when $U(1)_{B-L}$ is broken by a non-zero VEV, there is no residual $\mathbb{Z}_2$ symmetry. Therefore, in heterotic standard models $U(1)_{B-L}$ gauge symmetry must be broken at a low scale, an order of magnitude or two above the electroweak scale. But how can this be accomplished?

In the MSSM spectrum, the only scalar fields that carry a non-trivial $B-L$ charge, but transform trivially under the standard model gauge group, are the right-handed sneutrinos. Thus, $B-L$ symmetry must be broken by at least one of these scalars acquiring a non-vanishing VEV from radiative corrections. However, to sufficiently suppress large baryon and lepton number violation, this must occur just above the electroweak scale. To analyze this, one adds to the supersymmetric MSSM the soft supersymmetry violating operators that arise from various sources, such as gaugino condensation and the moduli vacuum state, during compactification. Whatever the source, these operators are of a specific form first worked out in [63] and discussed within the context of generalized scenarios in [71, 82, 38, 90, 31]. The initial values of the parameters are set by the details of the compactification, and are generically moduli dependent. At any lower scale, these parameters are determined by a complicated set of intertwined, non-linear renormalization group equations
(RGEs) [85, 86, 98, 72, 73, 74, 39]. It is by no means clear that a non-vanishing sneutrino VEV will necessarily develop. If it does, one must still show that a neutral Higgs field will get a non-zero VEV, thus breaking electroweak symmetry, at a scale an order of magnitude or two lower than the sneutrino VEV. Finally, it is of interest to know whether this result requires extremely fine-tuned parameters or is, more or less, a natural hierarchy.
Chapter 2

Two Higgs Pair Model: A Simplified Analysis

In this section, we reproduce the contribution by the author to the discussion presented in [2]. To give context, we briefly outline the content of the first part of [2] before beginning the present discussion.

The first few sections of [2] go as follows: in Section 2, we presented the explicit elliptically fibered Calabi-Yau threefold and $SU(4)$ holomorphic vector bundle of our two Higgs pair vacua. Using techniques introduced in [52, 28, 29, 30, 20], the spectrum was shown to be precisely that of the MSSM with the addition of a second Higgs-Higgs conjugate pair. Also computed was the number of geometric and vector bundle moduli; $h^{1,1}(X) = h^{2,1}(X) = 3$ and 13 respectively. The texture of the cubic Yukawa terms in the superpotential was calculated in Section 3. These terms were
shown to arise as the cubic product of the sheaf cohomology groups associated to matter and Higgs-Higgs conjugate superfields. The internal properties of these cohomologies under the \((p, q)\) and \([s, t]\) “stringy” symmetries induced by the two Leray sequences were tabulated and shown to lead to explicit selection rules for these couplings. The associated texture of the quark/lepton mass matrix was computed explicitly and found to naturally have one light and two heavy families. Importantly, we showed that the stringy symmetries allow the coupling of left and right chiral matter to the first Higgs pair but disallow a cubic coupling of matter to the second Higgs-Higgs conjugate superfields. Thus, classically, these two Higgs pair Heterotic Standard Models have no flavor-changing neutral currents. In Section 4, a similar calculation was carried out for the cubic terms in the superpotential involving a single vector bundle modulus with the Higgs-Higgs conjugate pairs. The \((p, q)\) and \([s, t]\) symmetries of the associated sheaf cohomologies again induce a texture on these couplings, allowing only 9 of the 13 vector bundle moduli to form such couplings and restricting the Higgs content as well. This has important consequences for the magnitude of the Higgs induced flavor-changing neutral currents.

The remainder of the paper comprises the author’s contribution. They are labeled as Sections 5 and 6 in [2]. It is here that we begin.

In Section 2.1, we give a discussion of the superpotential, including a heavy Kaluza-Klein superfield and its cubic coupling to two zero-mode fields. It is shown that tree level supergraphs involving the exchange of a Kaluz-Klein superfield can
generate the coupling of quark/lepton chiral matter to the second Higgs-Higgs conjugate pair, but only at dimension 4 in the superpotential. Hence, there is a natural suppression by a factor of \(1/M_c\), where \(M_c\) is the compactification scale. Similarly, such supergraphs generate suppressed dimension 4 terms in the superpotential coupling all 13 vector bundle moduli to all Higgs pairs. By requiring that these vacua have the correct scale of electroweak symmetry breaking, one can put an upper bound on the size of the vector bundle moduli vacuum expectation values and, hence, on the magnitude of the Yukawa couplings to the second Higgs-Higgs conjugate pair. Finally, in Section 2.2, we represent the physics of our two Higgs pair vacua in terms of a simplified model. This is essentially the non-supersymmetric standard model with the addition of a second Higgs doublet and a real scalar field representing the 4 vector bundle moduli disallowed from forming cubic couplings. The fact that chiral matter is prevented classically from coupling to the second Higgs pair is enforced in the toy model by a \(Z_2\) symmetry [64]. The scalar vacuum state closest to that of the standard model is found and the associated Higgs and fermion masses and eigenstates computed. Using these, we compute the interaction Langrangian for the Higgs mediated flavor-changing neutral currents, constraining the coefficients of these interactions to be those determined in Section 5 in the supersymmetric string vacua. These interactions are compared with the experimental upper bounds in several \(\Delta F = 2\) neutral meson processes [14, 15] and found to be generically well below these bounds. However, by choosing certain parameters
to be of order unity, and for a sufficiently light neutral Higgs scalar, the flavor-changing neutral current contributions to some meson processes can approach the upper bounds.

2.1 Discussion of the Superpotential

As shown in Sections 3 and 4 of [2], the perturbative holomorphic superpotential for zero-modes of the two Higgs-Higgs conjugate pair vacua presented is given, up to operators of dimension 4, by

\[ W_0 = W_{\text{Yukawa}} + W_{\mu}, \]  

where

\[ W_{\text{Yukawa}} = \lambda_{u,ij}^1 Q_i H_1 u_j + \lambda_{d,ij}^1 Q_i \bar{H}_1 d_j + \lambda_{\nu,ij}^1 L_i H_1 \nu_j + \lambda_{e,ij}^1 L_i \bar{H}_1 e_j \]  

with the restriction \( i = 1, j = 2, 3 \) or \( i = 2, 3, j = 1 \), and

\[ W_{\mu} = \hat{\lambda}_{10}^m \phi_m H_1 \bar{H}_2 + \hat{\lambda}_{21}^m \phi_m H_2 \bar{H}_1, \]  

where \( m = 1, \ldots, 9 \). Quadratic mass terms do not appear in \( W_0 \) since all fields in the perturbative low energy theory are strictly zero-modes of the Dirac operator. Furthermore, the cubic terms are restricted by the “stringy” \( (p,q) \) and \( [s,t] \) Leray selection rules. Specifically, non-vanishing Yukawa terms can only occur between the first family of quarks/leptons and the second and third quark/lepton families. In addition, only the first pair of Higgs-Higgs conjugate fields, \( H_1 \) and \( \bar{H}_1 \), can appear
in these non-vanishing Yukawa couplings. Similarly, non-zero cubic $\mu$-terms can only occur between a specific 9-dimensional subset of the 13 vector bundle moduli and the restricted pairs $H_1\bar{H}_2$ and $H_2\bar{H}_1$.

It is important to note, however, that only the zero-modes need have vanishing mass terms. Non-zero-modes, that is, the superfields corresponding to Kaluza-Klein states, do add quadratic terms to the superpotential$^1$. For example, let $H$ and $\bar{H}$ be two superfields corresponding to Kaluza-Klein modes with the same quantum numbers as $H_{1,2}$ and $\bar{H}_{1,2}$. These contribute a mass term

$$W_{\text{mass},KK} = M_c HH$$

(2.4)

to the superpotential, where $M_c$ is of the order of the Calabi-Yau compactification scale. Similarly, the $(p,q)$ and $[s,t]$ Leray selection rules only apply to the cubic product of the sheaf cohomologies associated with the zero-modes of the Dirac operator. It follows that there is no restraint, other than group theory, on cubic terms involving at least one Kaluza-Klein superfield. The terms of interest for this paper are

$$W_{\text{Yukawa},KK} = \tilde{\lambda}_{u,ij} Q_i H u_j + \tilde{\lambda}_{d,ij} Q_i \bar{H} d_j + \tilde{\lambda}_{\nu,ij} L_i \nu_j + \tilde{\lambda}_{e,ij} L_i \bar{e}_j$$

(2.5)

and

$$W_{\mu,KK} = \tilde{\lambda}_m^k \phi_m H \bar{H}_k + \tilde{\lambda}_k^m \phi_m H_k \bar{H}$$

(2.6)

where the sums over $i, j = 1, 2, 3$ as well as $m = 1, \ldots, 13$ and $k = 1, 2$ are unconstrained.

$^1$For a brief review of Kaluza-Klein modes, see Appendix A
Figure 2.1: Kaluza-Klein mode mediated supergraphs giving rise to $W_4$ and effective Yukawa couplings of quarks/leptons to the second Higgs pair.

The significance of this is that such interactions can quantum mechanically induce amplitudes which, at energy small compared to the compactification scale, appear as irreducible, holomorphic higher-dimensional contributions to the superpotential. Despite the fact that such terms depend on zero-modes only, they are not subject to $(p, q)$ and $[s, t]$ selection rules since they are not generated as a triple cohomology product. There are two classes of tree-level supergraphs that are of particular interest for this paper. The first of these is shown in Figure 1. An analysis of these graphs shows that for energy-momenta much less than the compactification scale, that is, $k^2 \ll M_c^2$, they induce quartic terms in the superpotential of the form

$$W_4 = \tilde{\lambda}_{u,ij} \tilde{\lambda}_2^m \frac{\phi_m}{M_c} Q_i H_2 u_j + \tilde{\lambda}_{d,ij} \tilde{\lambda}_2^m \frac{\phi_m}{M_c} Q_i \bar{H}_2 d_j$$

$$+ \tilde{\lambda}_{\nu,ij} \tilde{\lambda}_2^m \frac{\phi_m}{M_c} L_i H_2 \nu_j + \tilde{\lambda}_{e,ij} \tilde{\lambda}_2^m \frac{\phi_m}{M_c} L_i \bar{H}_2 e_j,$$  \hspace{1cm} (2.7)

where the sums over $m = 1, \ldots, 13$ and $i, j = 1, 2, 3$ are unrestricted. These terms
are of physical significance since, if at least one of the vector bundle moduli has
a non-vanishing vacuum expectation value $\langle \phi_m \rangle$, they yield cubic Yukawa terms
where quark/lepton superfields couple to the second Higgs pair, $H_2$ and $\tilde{H}_2$. The
induced Yukawa interactions are of the form

$$W_{4,\text{Yukawa}} = \lambda_{u,ij}^2 Q_i H_2 u_j + \lambda_{d,ij}^2 Q_i \tilde{H}_2 d_j + \lambda_{\nu,ij}^2 L_i H_2 \nu_j + \lambda_{e,ij}^2 L_i \tilde{H}_2 e_j,$$

(2.8)

where

$$\lambda_{u(\nu),ij}^2 = \tilde{\lambda}_{u(\nu),ij} \tilde{\lambda}_m^m \frac{\langle \phi_m \rangle}{M_c}, \quad \lambda_{d(e),ij}^2 = \tilde{\lambda}_{d(e),ij} \tilde{\lambda}_m^m \frac{\langle \phi_m \rangle}{M_c}. \quad (2.9)$$

Such couplings were disallowed classically by the $(p, q)$ and $[s, t]$ Leray selection
rules, as discussed above, but can be generated from the quartic terms in $W_4$ when
the vector bundle moduli have non-vanishing expectation values. It is important
to note, however, that since these Yukawa couplings to the second Higgs pair arise
from higher dimension operators, they are naturally suppressed by the factors

$$\tilde{\lambda}_m^m \frac{\langle \phi_m \rangle}{M_c} \ll 1, \quad \tilde{\lambda}_m^m \frac{\langle \phi_m \rangle}{M_c} \ll 1. \quad (2.10)$$

An estimate of the magnitudes of these factors will be presented below. Let us
assume, for example, that the cubic couplings of quarks/leptons to the Kaluza-Klein
Higgs pair $H, \tilde{H}$ are of the same order of magnitude as their Yukawa couplings to
$H_1, \tilde{H}_1$; that is, $\tilde{\lambda}_{u(\nu),ij} \sim \lambda_{u(\nu),ij}^1, \tilde{\lambda}_{d(e),ij} \sim \lambda_{d(e),ij}^1$. Then it follows from (2.10) that

$$\lambda_{u(\nu),ij}^2 \ll \lambda_{u(\nu),ij}^1, \quad \lambda_{d(e),ij}^2 \ll \lambda_{d(e),ij}^1. \quad (2.11)$$

Clearly this will remain true for a much wider range of assumptions as well, de-
pending on the magnitude of the suppression factors in (2.10). We conclude that
the Yukawa couplings of quarks/leptons to the second Higgs pair are \textit{naturally suppressed} relative to the Yukawa couplings to the first Higgs pair. The physical implications of this will be discussed in detail below. Before doing that, however, let us provide an estimate for the suppression factors in (2.10).

The second class of supergraphs of interest is shown in Figure 2. In the low energy-momentum limit, $k^2 \ll M_c^2$, these induce quartic terms in the superpotential of the form

$$W_4' = \tilde{\lambda}^m_n \phi_m \frac{\phi_n}{M_c} H_k \bar{H}_l,$$

(2.12)

where the sums over $m, n = 1, \ldots, 13$ and $k, l = 1, 2$ are unrestricted. These terms are physically significant since, if at least one of the vector bundle moduli has a non-vanishing vacuum expectation value $\langle \phi_m \rangle$, they induce Higgs $\mu$-terms of the form

$$W_{4,\mu} = \mu_{kl} H_k \bar{H}_l$$

(2.13)
with coefficients

$$
\mu_{kl} = \left( \tilde{\lambda}_m^m \langle \phi_m \rangle \right) \left( \tilde{\lambda}_n^n \langle \phi_n \rangle \right) M_c .
$$  \hfill (2.14)

On generic grounds, if this theory is to naturally have appropriate electroweak symmetry breaking, these $\mu$-coefficients must satisfy

$$
\mu_{kl} \lesssim M_{EW} ,
$$  \hfill (2.15)

where $M_{EW} \approx 10^2 GeV$. It follows from (2.14) that

$$
\tilde{\lambda}_m^m \langle \phi_m \rangle \lesssim \sqrt{ \frac{M_{EW}}{M_c} } \approx 10^{-7} .
$$  \hfill (2.16)

In the final term, we have chosen $M_c \approx 10^{16} GeV$. This is consistent with the inequalities (2.10) and gives a natural estimate for their magnitude. Note that if this bound is saturated, the natural suppression (2.11) of the Yukawa couplings to the second Higgs pair will remain true even if the $\tilde{\lambda}_{u(\nu),ij}$, $\tilde{\lambda}_{d(e),ij}$ coupling parameters in (2.9) are as large as $\tilde{\lambda}_{u(\nu),ij} \sim \tilde{\lambda}_{d(e),ij} \sim 1$. In this case, one would have

$$
\lambda_{u(\nu),ij}^2 \sim 10^{-7} , \quad \lambda_{d(e),ij}^2 \sim 10^{-7} ,
$$  \hfill (2.17)

a fact we will use in the next section.

Let us now return to the low-energy theory described strictly by the zero-modes of the Dirac operator. The Kaluza-Klein superfields “decouple” and, hence, we can ignore all interactions containing at least one of these heavy fields. It follows that the relevant superpotential for the low-energy theory is given by

$$
W = W_{\text{Yukawa}} + W_\mu + W_4 + W'_4 ,
$$  \hfill (2.18)
where $W_{\text{Yukawa}}$, $W_\mu$, $W_4$ and $W'_4$ are given in eqns. (2.2), (2.3), (2.7) and (2.12) respectively. In broad outline, the physics described by the superpotential $W$ in (2.18), relevant to the fact that there are two Higgs-Higgs conjugate pairs, is the following. First, note that since the coefficients of the Yukawa couplings to the second Higgs pair, $H_2$ and $\bar{H}_2$, are suppressed, it follows that the masses of quarks and leptons are predominantly generated by the vacuum expectation values of the first Higgs pair, $H_1$ and $\bar{H}_1$, as in the standard MSSM. Second, the masses of the $W^\pm$ and $Z$ vector bosons receive contributions from both pairs of Higgs-Higgs conjugate superfields through their respective kinetic energy terms. Despite this, the GIM mechanism continues to apply at tree level and, hence, $Z$ couples only to flavor preserving currents. Third, recall that in the single Higgs pair MSSM, all flavor-changing currents coupled to the neutral Higgs scalar boson vanish. This is no longer true, however, when the spectrum contains a second Higgs pair. In this case, one expects Higgs-induced flavor changing neutral currents coupled to as many as three neutral Higgs bosons. If the coefficients of the Yukawa couplings to $H_2$ and $\bar{H}_2$ were arbitrarily large, then these Higgs-induced neutral currents would violate current phenomenological bounds on a number of processes. However, the coefficients in $W_{4,\text{Yukawa}}$ in (2.8) are not arbitrarily large. Rather, as mentioned above, they are all naturally suppressed by the factors presented in (2.10) and estimated in (2.16). Hence, if these factors are sufficiently small the Higgs-induced flavor-changing neutral currents will be consistent with present experimental data.
Be that as it may, they may still be sufficiently large in some region of parameter space to become relevant as the precision of the relevant data is improved.

A complete analysis of these issues would require the computation of the perturbative Kahler potential, the non-perturbative contributions to both the Kahler potential and the superpotential, stabilization of all moduli, a complete exposition of supersymmetry breaking and the explicit computation of electroweak and $U(1)_{B-L}$ symmetry breaking. Although much of the theory required to accomplish this already exists, it is clearly a long term project that we will not begin to attempt in this paper. Rather, we will explore the relevant physics within the context of a toy model which contains most of the salient features of our two Higgs pair vacua.

To make this toy model as simple as possible, we close this section by noting from $W_\mu$ in (2.3) that any non-vanishing vacuum expectation values $\langle \phi_{\bar{m}} \rangle$, $\bar{m} = 1, \ldots, 9$ will induce $\mu$-terms of the form

$$W_\mu = \mu_{12} H_1 \bar{H}_2 + \mu_{21} H_2 \bar{H}_1 + \ldots,$$

(2.19)

where

$$\mu_{12} = \hat{\lambda}_{12} \langle \phi_{\bar{m}} \rangle, \quad \mu_{21} = \hat{\lambda}_{21} \langle \phi_{\bar{m}} \rangle.$$  

(2.20)

Exactly as in (2.15), these $\mu$-coefficients must satisfy

$$\mu_{12}, \mu_{21} \lesssim M_{EW}$$

(2.21)

and, hence,

$$\hat{\lambda}_{12} \frac{\langle \phi_{\bar{m}} \rangle}{M_c} \sim \hat{\lambda}_{21} \frac{\langle \phi_{\bar{m}} \rangle}{M_c} \lesssim \frac{M_{EW}}{M_c} \approx 10^{-14}.$$  

(2.22)
Assuming the parameters $\hat{\lambda}_{\bar{m}_{12}}$ and $\hat{\lambda}_{\bar{m}_{21}}$ are of order unity, or, at least, not extremely small, it follows from (2.16) that the contribution of the first $\bar{m} = 1, \ldots, 9$ moduli to the induced Yukawa couplings $\lambda_{u(\nu),ij}^2$ and $\lambda_{d(e),ij}^2$ in (2.9) can be ignored. Since in this remainder of this paper we are concerned only with possible Higgs-mediated flavor-changing neutral currents, it is reasonable to simply drop all terms in the superpotential (2.18) containing these nine moduli and only consider terms with the four moduli $\phi_{\hat{m}}$ with $\hat{m} = 10, \ldots, 13$. When constructing the toy model in the next section, we will base it on this truncated supersymmetric theory.

### 2.2 A Simplified Model

Much of the technical difficulty in analyzing our two Higgs pair string vacua comes from the $N = 1$ local supersymmetry. Great simplification is achieved, while retaining the relevant physics, by choosing our toy model to be non-supersymmetric. We will also, for simplicity, ignore the $U(1)_{B-L}$ gauge symmetry, since its inclusion would not alter our conclusions. That is, we take our gauge group to be the $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the standard model. Hence, after electroweak symmetry breaking our vector boson spectrum consists of three massive bosons, $W^\pm, Z$ and the massless photon $A$. 

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2.2.1 The Spectrum

We begin by including all of the matter fields of the standard model. That is, the spectrum contains three families of quark and lepton fermions, each family transforming as

\[ Q = (3, 2, 1), \quad u = (3, 1, 4), \quad d = (3, 1, -2) \] \hspace{1cm} (2.23)

and

\[ L = (1, 2, -3), \quad e = (1, 1, -6), \quad \nu = (1, 1, 0) \] \hspace{1cm} (2.24)

under \( SU(3)_C \times SU(2)_L \times U(1)_Y \). We have displayed the quantum number \( 3Y \) for convenience. Note from eqn. (2.24) that each family contains a right-handed neutrino.

To complete the standard model spectrum, we add a complex Higgs scalar boson which transforms as

\[ H_1 = (1, 2, 3) \] \hspace{1cm} (2.25)

under the gauge group. This naturally forms Yukawa terms with the “up” quark and neutrino singlets, whereas the “down” quark and lepton singlets couple to \( H_1^* \). This is unlike the supersymmetric case, where one must introduce an independent \( \tilde{H}_1 \) superfield.

So far, our toy model is exactly the standard model. However, to reflect the physics of our two Higgs pair string vacua, we now make several important additions to the spectrum. First, in analogy with the second Higgs-Higgs conjugate pair
$H_2, \bar{H}_2$, we introduce a second complex Higgs boson field $H_2$ (and, hence, $H_2^*$), transforming as

$$H_2 = (1, 2, 3). \quad (2.26)$$

Second, to play the role of the vector bundle moduli in the string vacua, we must add gauge singlet scalar fields to the spectrum. Recall that there are thirteen such moduli fields, which break into two types; nine that are allowed by the $(p, q)$ and $[s, t]$ selection rules to form cubic $\mu$-terms with the Higgs fields and four that are not. As discussed above, the moduli that form cubic $\mu$-terms give a sub-dominant contribution to the Yukawa couplings to the second Higgs pair and, for the purposes of this paper, can be ignored. Hence, we will not introduce them into our toy model. On the other hand, those moduli that are disallowed from forming cubic $\mu$-terms give the dominant contribution to these Yukawa couplings and must be part of the analysis. Therefore, we include them in the toy model. For simplicity, we add a single, real scalar field $\phi$ to the spectrum to represent this type of field. As do moduli, this transforms trivially as

$$\phi = (1, 1, 0) \quad (2.27)$$

under the gauge group. Choosing this field to be complex and/or adding more than one such field would greatly complicate the analysis without altering the conclusion.
2.2.2 Discrete Symmetry

If this model had no further restrictions, one would generically find, after electroweak symmetry breaking, flavor changing currents coupling with large coefficients to the neutral Higgs bosons. These Higgs mediated flavor-changing neutral currents would easily violate the experimental bounds on a large number of physical processes. As shown long ago [64], this problem can be naturally resolved in two ways. First, one can introduce a discrete symmetry which only allows Yukawa couplings of “up” quark and neutrino singlets to $H_1$ and “down” quark and lepton singlets to $H_2^*$. This is similar to having a single superfield pair $H_1, H_1^*$ in a supersymmetric model and is not analogous to the physics of our two Higgs pair vacua. For this reason, we follow the second method; that is, we introduce a discrete symmetry that allows all quarks/leptons to couple to either $H_1$ or $H_1^*$, but forbids any Yukawa couplings of quarks/leptons to $H_2$ and $H_2^*$ at the classical level. Note that this discrete symmetry is the field theory analogue of the “stringy” $(p,q)$ and $[s,t]$ Leray selection rules for cubic Yukawa couplings in our two Higgs pair vacua.

There are several discrete symmetries that can be imposed on our toy model to implement the “decoupling” of $H_2$ from quark/leptons. The simplest of these is a $Z_2$ symmetry defined as follows. Constrain the Lagrangian to be invariant under the action

$$(\bar{Q}, \bar{L}) \longrightarrow (\bar{Q}, \bar{L}), \quad (u,d,\nu,e) \longrightarrow (u,d,\nu,e)$$

(2.28)
and

\[ H_1 \rightarrow H_1, \quad H_2 \rightarrow -H_2, \quad \phi \rightarrow -\phi. \]  \hspace{1cm} (2.29)

Then, up to operators of dimension 4 in the fields, the Lagrangian is restricted to be of the form

\[ \mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{potential}}, \]  \hspace{1cm} (2.30)

where \( \mathcal{L}_{\text{kinetic}} \) is the canonically normalized gauged kinetic energy for all of the fields,

\[ \mathcal{L}_{\text{Yukawa}} = \lambda_{u,ij} \bar{Q}_i H_1^{*} u_j + \lambda_{d,ij} \bar{Q}_i H_1 d_j + \lambda_{\nu,ij} \bar{\nu}_i H_1^{*} \nu_j + \lambda_{e,ij} \bar{L}_i H_1^{*} e_j + h.c \]  \hspace{1cm} (2.31)

with \( i, j = 1, 2, 3 \) unrestrained and \( \mathcal{L}_{\text{potential}} = -V \) with

\[ V = V_F + V_D + \mathcal{V} \]  \hspace{1cm} (2.32)

such that

\[ V_F = \lambda_1 (H_1^* \cdot H_2)(H_2^* \cdot H_1) + \lambda_2 ((H_1^* \cdot H_2)(H_1^* \cdot H_2) + (H_2^* \cdot H_1)(H_2^* \cdot H_1)) \]  \hspace{1cm} (2.33)

\[ V_D = \lambda_3 |H_1|^4 + \lambda_4 |H_2|^4 + \lambda_5 |H_1|^2 |H_2|^2 \]  \hspace{1cm} (2.34)

and

\[ \mathcal{V} = -\mu_1^2 |H_1|^2 - \mu_2^2 |H_2|^2 - \frac{\mu_\phi^2}{2} \phi^2 + \rho_3 \phi (H_1^* \cdot H_2 + H_2^* \cdot H_1) \]

\[ + \phi^2 (\gamma_1 |H_1|^2 + \gamma_2 |H_2|^2) + \rho_4 \phi^4. \]  \hspace{1cm} (2.35)

Note that we have, for simplicity, taken \( \lambda_2 \) and \( \rho_3 \) to be real. For \( V \) to be hermitian, all other coefficients in (2.33), (2.34) and (2.35) must be real. Finally, to ensure vacuum stability we choose all coupling parameters to be positive.
In addition to the Yukawa couplings to $H_2$ being disallowed, the potentials $V_F$ and $V_D$ are also consistent with the potential energy of our two Higgs pair string vacuum. Specifically, the $F$-term contribution to the potential generated from the classical superpotential $W_\mu$ in (2.3), disregarding the terms with $\phi_m$ and setting $\bar{H}_1$, $\bar{H}_2$ to be $H_1^*$, $H_2^*$ respectively for the reasons discussed previously, contains precisely the same terms as in $V_F$. They differ only in that their coefficients are related in the supersymmetric case, whereas $\lambda_1$, $\lambda_2$ in $V_F$ can be completely independent. Similarly, the $D$-term contribution to the supersymmetric potential, again setting $\bar{H}_1$, $\bar{H}_2$ to be $H_1^*$, $H_2^*$, contains the same terms as in $V_D$, albeit with constrained coefficients. The coefficients $\lambda_3$, $\lambda_4$, $\lambda_5$ in $V_D$ can be independent.

There are several other important, but more subtle, features of our two Higgs pair string vacua that are captured in the remaining term $V$ of the potential. First, recall that in these string vacua quadratic mass terms do not appear for the Higgs fields since they are zero modes of the Dirac operator. However, supersymmetry breaking and radiative corrections are expected to induce non-vanishing vacuum expectation values for these fields. This symmetry breaking is modeled in our $\mathbb{Z}_2$ toy theory by the appearance of such mass terms in $V$ with negative sign. To be consistent with electroweak breaking, we will choose parameters $\mu_1$, $\mu_2$ and $\lambda_i$, $i = 1, \ldots, 5$ so that

$$\langle H_1 \rangle \sim \langle H_2 \rangle \approx M_{EW}$$

(2.36)

Second, moduli fields must have a vanishing perturbative potential in string theory.
However, non-perturbative effects and supersymmetry breaking are expected to induce a moduli potential leading to stable, non-zero moduli expectation values. This is modeled in our toy theory by the the pure $\phi^2$ and $\phi^4$ terms in $V$. Since $\phi$ represents moduli with potentially large expectation values, we will choose parameters $\mu_\phi$ and $\rho_4$ so that

$$\langle \phi \rangle \lesssim M_c \,. \tag{2.37}$$

Finally, note that the $Z_2$ symmetry allows mixed cubic and quartic $\phi$-$H$ couplings in $V$. Such cubic terms cannot arise from a cubic superpotential. Quartic terms might occur, but are disallowed by the $(p, q)$ and $[s, t]$ selection rules of our string vacua. However, both terms can be expected to arise in the string potential energy after supersymmetry breaking, radiative corrections and non-perturbative effects are taken into account. To ensure that these terms are consistent with electroweak symmetry breaking (2.36) and the large modulus expectation value (2.37), one must choose coefficients $\rho_3$ and $\gamma_1, \gamma_2$ to satisfy

$$\rho_3 \sim \left( \frac{M_{EW}}{M_c} \right) M_{EW} \,, \quad \gamma_1, \gamma_2 \sim \left( \frac{M_{EW}}{M_c} \right)^2 \,. \tag{2.38}$$

From the point of view of the toy model with $Z_2$ discrete symmetry, this is fine-tuning of the coefficients. However, it is a natural requirement if we want our toy model to reflect the appropriate electroweak symmetry breaking in the two Higgs pair string vacua.

Of course, there is an infinite set of operators that are of order dimension five and higher in the fields that are consistent with the $Z_2$ discrete symmetry. Here,
we will be interested only in the dimension five operators
\[ \mathcal{L}_5 = \hat{\lambda}_{u,ij} \frac{\phi}{M_c} \bar{Q}_i H_1^* u_j + \hat{\lambda}_{d,ij} \frac{\phi}{M_c} \bar{Q}_i H_2 d_j + \hat{\lambda}_{\nu,ij} \frac{\phi}{M_c} \bar{L}_i H_2^* \nu_j + \hat{\lambda}_{e,ij} \frac{\phi}{M_c} \bar{L}_i H_2 e_j + h.c \] (2.39)
related to flavor-changing neutral currents. Note that a non-vanishing vacuum expectation value \( \langle \phi \rangle \neq 0 \) will induce Yukawa couplings of the quarks/leptons to the the second Higgs doublet \( H_2 \) of the form
\[ \mathcal{L}_{5,Yukawa} = \lambda^2_{u,ij} \bar{Q}_i H_2^* u_j + \lambda^2_{d,ij} \bar{Q}_i H_2 d_j + \lambda^2_{\nu,ij} \bar{L}_i H_2^* \nu_j + \lambda^2_{e,ij} \bar{L}_i H_2 e_j + h.c \] , (2.40)
where
\[ \lambda^2_{u(\nu),ij} = \hat{\lambda}_{u(\nu),ij} \frac{\langle \phi \rangle}{M_c} , \quad \lambda^2_{d(e),ij} = \hat{\lambda}_{d(e),ij} \frac{\langle \phi \rangle}{M_c} . \] (2.41)
Since one expects \( \frac{\langle \phi \rangle}{M_c} < 1 \), the Yukawa couplings to the second Higgs \( H_2 \) are naturally smaller that the couplings to \( H_1 \). To be consistent with the two Higgs pair string vacua, it follows from (2.11) that we should choose
\[ \lambda^2_{u(\nu),ij} \ll \lambda^1_{u(\nu),ij} , \quad \lambda^2_{d(e),ij} \ll \lambda^1_{d(e),ij} . \] (2.42)
More specifically, from (2.9), (2.16) and the associated discussion one might expect
\[ 10^{-7} \lambda^1_{u(\nu),ij} \lesssim \lambda^2_{u(\nu),ij} \lesssim 10^{-7} , \quad 10^{-7} \lambda^1_{d(e),ij} \lesssim \lambda^2_{d(e),ij} \lesssim 10^{-7} . \] (2.43)

2.2.3 The Vacuum State

To find the vacuum of this theory, one has to find the local minima of the potential \( V \). To do this, define the component fields of the two Higgs doublets by
\[ H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 + i h_2 \\ h_3 + i h_4 \end{pmatrix} , \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_5 + i h_6 \\ h_7 + i h_8 \end{pmatrix} \] (2.44)
It turns out that for a generic choice of coefficients there are several local minima. For simplicity of the analysis, we choose the one most closely related to the standard model vacuum. The analytic expressions for the vacuum expectation values, as well as the scalar mass eigenvalues and eigenstates, greatly simplify if we take all coefficients $\lambda_i$, $i = 1, \ldots, 5$ to have the identical value $\lambda$. With this simplification, this local minimum is specified by

$$\langle h_3 \rangle = \frac{\mu_1}{\sqrt{\lambda}}, \quad \langle h_8 \rangle = \frac{\mu_2}{\sqrt{\lambda}}, \quad \langle \phi \rangle = \frac{\mu\phi}{2\sqrt{\mu_4}}$$

(2.45)

with all other expectation values vanishing. This vacuum clearly spontaneously breaks $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$. Note that both Higgs doublets contribute to the mass matrix of the vector bosons. Despite this, as mentioned above, the GIM mechanism continues to apply at tree level and all $Z$ mediated flavor-changing currents vanish.

The scalar mass matrix is easily evaluated and diagonalized in this vacuum. Expanding around the vacuum expectation values in (2.45) and writing $h_3 = \langle h_3 \rangle + \bar{h}_3$, $h_8 = \langle h_8 \rangle + \bar{h}_8$ and $\phi = \langle \phi \rangle + \bar{\phi}$, we find that the square of the mass eigenvalues and the associated eigenstates are given respectively by

$$
\begin{align*}
M^2_{h'_1} &= 0, & M^2_{h'_2} &= 0, \\
M^2_{h'_3} &= 4\mu^2_1, & M^2_{h'_4} &= 0, \\
M^2_{h'_5} &= 4(\mu^2_1 + \mu^2_2), & M^2_{h'_6} &= \mu^2_1 + \mu^2_2, \\
M^2_{h'_7} &= \mu^2_1 + \mu^2_2, & M^2_{h'_8} &= 4\mu^2_2, \\
M^2_{\phi'} &= 2\mu^2_\phi
\end{align*}
$$

(2.46)
and

\[ h'_1 = -\bar{\mu}_1 h_4 + \bar{\mu}_2 h_7, \quad h'_2 = \bar{\mu}_1 h_1 - \bar{\mu}_2 h_6, \]
\[ h'_3 = \bar{h}_3, \quad h'_4 = \bar{\mu}_1 h_2 + \bar{\mu}_2 h_5, \]
\[ h'_5 = \bar{\mu}_2 h_4 + \bar{\mu}_1 h_7, \quad h'_6 = -\bar{\mu}_2 h_1 - \bar{\mu}_1 h_6, \]
\[ h'_7 = -\bar{\mu}_2 h_2 + \bar{\mu}_1 h_5, \quad h'_8 = \bar{h}_8, \]
\[ \phi' = \bar{\phi} \]

where

\[ \bar{\mu}_i = \frac{\mu_i}{\sqrt{\mu^2_1 + \mu^2_2}} \quad i = 1, 2 . \]  

(2.48)

Clearly \( h'_1, h'_2 \) and \( h'_4 \), which can be rotated into the charged eigenstates

\[ G^0 = h'_1, \quad G^\pm = \frac{1}{\sqrt{2}}(h'_2 \pm ih'_4) , \]

(2.49)

are the Goldstone bosons. Since in the unitary gauge they will be absorbed into the longitudinal components of the \( Z \) and \( W^\pm \) vector bosons, we will henceforth ignore these fields. The remaining Higgs scalars we group into charge eigenstates as

\[ \mathcal{H}^0_1 = h'_3, \quad \mathcal{H}^0_2 = h'_5, \quad \mathcal{H}^0_3 = h'_8 \]

(2.50)

and

\[ \mathcal{H}^\pm = \frac{1}{\sqrt{2}}(h'_6 \pm ih'_7) , \]

(2.51)

with masses

\[ M^2_{\mathcal{H}^0_1} = 4\mu^2_1, \quad M^2_{\mathcal{H}^0_2} = 4(\mu^2_1 + \mu^2_2), \quad M^2_{\mathcal{H}^0_3} = 4\mu^2_2 \]

(2.52)

and

\[ M^2_{\mathcal{H}^\pm} = \mu^2_1 + \mu^2_2 \]

(2.53)
respectively. Since we are interested in flavor-changing neutral currents, we will ignore \( H^\pm \) and consider the currents coupling to \( H_1^0, H_2^0 \) and \( H_3^0 \) only. The charge neutral field \( \phi' \) does mediate a flavor-changing neutral current. However, it will naturally be suppressed by the factor \( \frac{\langle H_2 \rangle}{M_c} \) and, hence, is negligible.

### 2.2.4 Flavor-Changing Neutral Currents

Having determined the vacuum state, we can expand the two Yukawa terms given in (2.31) and (2.40) to find the fermion mass matrices and the Higgs induced flavor-changing neutral interactions. For simplicity, we will always assume \( \lambda_{u(\nu),ij}^{1,2} \) and \( \lambda_{d(e),ij}^{1,2} \) are real and symmetric. First, consider the fermion mass matrices. For up-quarks, one finds

\[
(L_{\text{Yukawa}} + L_{5,\text{Yukawa}})|_{\text{up-mass}} = \bar{U}_i \left( \frac{\lambda_{u,ij}^1}{\sqrt{2}} \langle h_3 \rangle - i \frac{\lambda_{u,ij}^2}{\sqrt{2}} \langle h_8 \rangle \right) u_j + h.c.
\]

This can always be written in terms of a diagonal mass matrix and its eigenstates. For example, the first term becomes

\[
\bar{U}_i \left( \frac{\lambda_{u,ij}^1}{\sqrt{2}} \langle h_3 \rangle - i \frac{\lambda_{u,ij}^2}{\sqrt{2}} \langle h_8 \rangle \right) u_j = \bar{\tilde{U}}_i M_{u,ij}^{\text{diag}} \tilde{u}_j ,
\]

which allows us to re-express

\[
\frac{\lambda_{u,ij}^1}{\sqrt{2}} \bar{U}_i u_j = \bar{\tilde{U}}_i M_{u,ij}^{\text{diag}} \tilde{u}_j + i \frac{\lambda_{u,ij}^2}{\sqrt{2}} \bar{\tilde{U}}_i \tilde{u}_j .
\]

Note that, in the last term, we have replaced \( \bar{U}_i, u_j \) by the eigenstates \( \bar{\tilde{U}}_i, \tilde{u}_j \). This is valid to leading order since it follows from (2.36) and (2.42) that

\[
\lambda_{u,ij}^2 \langle h_8 \rangle \ll \lambda_{u,ij}^1 \langle h_3 \rangle .
\]
Similar expressions hold for the hermitian conjugate terms, down-quarks and the \( \nu, e \)-leptons.

One can now evaluate the flavor-changing neutral interactions. For up-quarks, we find that

\[
\left( \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,\text{Yukawa}} \right)_{\text{up-neutral}} = \frac{\lambda^2_{u,ij}}{\sqrt{2}} \bar{U}_i (i \langle h_8 \rangle (\bar{h}_3 - i \bar{h}_4) + (h_7 - i h_8)) \bar{u}_j + \text{h.c.},
\]

where we have used expression (2.56) and dropped the flavor-diagonal \( M_{u,ij}^{\text{diag}} \) term.

From (2.45), (2.47) and (2.50), one can write (2.58) in terms of the neutral Higgs eigenstates. The result is

\[
\left( \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,\text{Yukawa}} \right)_{\text{up-neutral}} = \frac{\lambda^2_{u,ij}}{\sqrt{2}} \bar{U}_i (i \tilde{\mu}_2 \mathcal{H}_1^0 + \frac{1}{\mu_1} \mathcal{H}_2^0 - i \mathcal{H}_3^0) \bar{u}_j + \text{h.c.}.
\]  

(2.59)

Written in terms of the Dirac spinors

\[ q_{u,i} = \bar{U}_i \oplus \tilde{u}_i, \]

(2.60)

this becomes

\[
\left( \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,\text{Yukawa}} \right)_{\text{up-neutral}} = \frac{\lambda^2_{u,ij}}{\sqrt{2}} (- i \tilde{\mu}_2 \bar{q}_{u,i} \gamma^5 q_{u,j}) \mathcal{H}_1^0 + \frac{1}{\mu_1} (\bar{q}_{u,i} q_{u,j}) \mathcal{H}_2^0 + i (\bar{q}_{u,i} \gamma^5 q_{u,j}) \mathcal{H}_3^0.
\]  

(2.61)

Similar expressions hold for the down-quarks and \( \nu, e \)-leptons. Putting everything together, we find that the flavor-changing neutral interactions are given by

\[
\left( \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,\text{Yukawa}} \right)_{\text{neutral}} = J^1 \mathcal{H}_1^0 + J^2 \mathcal{H}_2^0 + J^3 \mathcal{H}_3^0,
\]

(2.62)

where

\[
J^1 = -i \frac{\lambda^2_{u(\nu),ij}}{\sqrt{2}} \frac{\tilde{\mu}_2}{\mu_1} (\bar{q}_{u(\nu),i} \gamma^5 q_{u(\nu),j}) + i \frac{\lambda^2_{d(e),ij}}{\sqrt{2}} \frac{\tilde{\mu}_2}{\mu_1} (\bar{q}_{d(e),i} \gamma^5 q_{d(e),j}).
\]  

(2.63)
\[ J^2 = \frac{\lambda_{u(\nu),ij}^2}{\sqrt{2}} \frac{1}{\mu_1} (\bar{q}_{u(\nu),i} q_{u(\nu),j}) + \frac{\lambda_{d(e),ij}^2}{\sqrt{2}} \frac{1}{\mu_1} (\bar{q}_{d(e),i} d_{d(e),j}) , \]  

\[ J^3 = i \frac{\lambda_{u(\nu),ij}^2}{\sqrt{2}} (\bar{q}_{u(\nu),i} \gamma^5 q_{u(\nu),j}) - i \frac{\lambda_{d(e),ij}^2}{\sqrt{2}} (\bar{q}_{d(e),i} \gamma^5 d_{d(e),j}) \]  

(2.64)

(2.65)

Note that these flavor-changing currents all vanish as \( \lambda_{u(\nu),ij}, \lambda_{d(e),ij} \to 0 \), as they must.

### 2.2.5 Phenomenology

The most stringent bounds on Higgs mediated flavor changing neutral currents arise from the experimental data on the mass splitting of neutral pseudoscalar \( F^0 - \bar{F}^0 \) meson eigenstates. Theoretically, the mass difference \( \Delta M_F \) is given by

\[ M_F \Delta M_F = |\langle F^0 | \mathcal{L}_{\text{eff}} | F^0 \rangle| , \]  

(2.66)

where \( \mathcal{L}_{\text{eff}} \) is the low energy \( \Delta F = 2 \) effective Lagrangian arising from a variety of processes [14, 15]. First, there is a well-known contribution from the standard model part of our simplified theory. In addition, we have terms rising from the flavor-changing neutral Higgs vertices in (2.62)-(2.64). These lead to the tree-level graphs shown in Figure 3 which, at low energy, give extra contributions to the mass splitting. Using the results of [14], we find that the Higgs mediated flavor changing neutral currents lead to an additional contribution to the mass splitting given by

\[ M_F \Delta M_F^{\text{FCNC}} = \frac{B_F}{8} \left( \lambda_{(u,d),ij}^2 \right)^2 \left[ (\pm) \{ \left( \frac{\mu_2}{\mu_1} \right)^2 - \frac{1}{\mu_1^2} \} P_{ij}^F + \frac{1}{\mu_1^2} S_{ij}^F \right] , \]  

(2.67)
\[ q_{(u),i}^{(a)} \quad \bar{q}_{(d),i}^{(a)} \quad \gamma^0 \quad H_1^0, H_3^0 \quad q_{(u),i}^{(b)} \quad \bar{q}_{(d),i}^{(b)} \quad \gamma^5 \quad H_2^0 \]

(a) \hspace{2cm} (b)

Figure 2.3: Feynman diagrams of the tree level contributions to neutral meson mixing mediated by Higgs bosons. Note that graphs (a) and (b) involve pseudoscalar and scalar interactions respectively.

\[
P_{ij}^F = -\frac{f_F^2 M_F^2}{6} \left(1 + \frac{11 M_F^2}{(m_i + m_j)^2}\right), \quad S_{ij}^F = \frac{f_F^2 M_F^2}{6} \left(1 + \frac{M_F^2}{(m_i + m_j)^2}\right)
\]

are associated with the pseudoscalar and scalar interaction graphs, Figure 3(a) and Figure 3(b), respectively. Here \(f_F\) is the pseudoscalar decay constant, \(M_F\) is the leading order meson mass, \(m_i\) is the mass of the \(i\)-th constituent quark and \(B_F\) is the \(B\)-parameter of the vacuum insertion approximation defined in [14]. The label \((u, d)\) tells one to choose the \(\lambda\) coefficient associated with the up-quark or down-quark content of the meson \(F\) and the indices \(i, j\), where \(i \neq j\), indicate which two families compose \(F\). In this paper, we simplify the analysis by considering two natural limits of (2.67), each consistent with all previous assumptions. The first
limit is to take $\mu_2 = \mu_1 \approx M_{EW}$. Expression (2.67) then simplifies to

$$M_F \Delta M_F^{FCNC(I)} = \frac{B_F}{8} \left( \lambda^2_{(u,d),ij} \right)^2 \frac{1}{M^2_{EW}} S^F_{ij}. \quad (2.69)$$

As a second limit, let us assume that $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, the $\mu_1$ contribution is sub-dominant and (2.67) becomes

$$M_F \Delta M_F^{FCNC(II)} = \mp \frac{B_F}{8} \left( \lambda^2_{(u,d),ij} \right)^2 \frac{1}{\mu^2_2} \tilde{S}^F_{ij}, \quad (2.70)$$

which can be written as

$$\Delta M_F^{FCNC(II)} = \mp \Delta M_F^{FCNC(I)} \left( \frac{M^2_{EW}}{\mu^2_2} \right) \left( \frac{\tilde{S}^F_{ij}}{S^F_{ij}} \right). \quad (2.71)$$

It follows from (2.68) that, in general, $\frac{\tilde{S}^F_{ij}}{S^F_{ij}} \sim 10$ and from our assumption that $\frac{M^2_{EW}}{\mu^2_2} \gg 1$. Hence,

$$|\Delta M_F^{FCNC(II)}| \gg \Delta M_F^{FCNC(I)}. \quad (2.72)$$

We will analyze the implications of both limits. Before proceeding, recall from (2.43) that a natural range for the Yukawa coefficients $\lambda^2_{(u,d),ij}$ is

$$10^{-7} \lambda^1_{(u,d),ij} \lesssim \lambda^2_{(u,d),ij} \lesssim 10^{-7}. \quad (2.73)$$

There are various ways to estimate the flavor non-diagonal coefficients $\lambda^1_{(u,d),ij}$, $i \neq j$. Here, we will simply assume each is of the same order of magnitude as the largest diagonal Yukawa coupling of the $u$ or $d$ type corresponding to the $i$ and $j$ families. Other commonly used estimates simply strengthen our conclusions.

In this paper, we will consider the $F^0$ mesons $K^0 = \bar{s}d$, $B^0_d = \bar{b}d$ and $D^0 = \bar{c}u$, since their mass mixings with their conjugates are the best measured. The values
Table 2.1: Table of data pertinent to the calculation of $\Delta M_F$. The data in the first two columns have dimensions $GeV^4$, those in column three are dimensionless while the entries in the last two columns are in $GeV$.

<table>
<thead>
<tr>
<th>$F^0$</th>
<th>$\mathcal{P}^F$</th>
<th>$S^F$</th>
<th>$B_F$</th>
<th>$\Delta M_F^{SM}$</th>
<th>$\Delta M_F^{Exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0$</td>
<td>-27.5</td>
<td>2.5</td>
<td>0.75</td>
<td>$1.4 - 4.6 \times 10^{-15}$</td>
<td>$3.51 \times 10^{-15}$</td>
</tr>
<tr>
<td>$B_d^0$</td>
<td>-2.65</td>
<td>0.37</td>
<td>1</td>
<td>$10^{-13} - 10^{-12}$</td>
<td>$3.26 \times 10^{-13}$</td>
</tr>
<tr>
<td>$D^0$</td>
<td>-0.52</td>
<td>0.068</td>
<td>1</td>
<td>$10^{-17} - 10^{-16}$</td>
<td>$&lt; 1.32 \times 10^{-13}$</td>
</tr>
</tbody>
</table>

\[
\Delta M_K^{FCNC(I)} \approx 4.72 \times 10^{-5} (\lambda_{d,12}^2)^2 GeV \quad (2.74)
\]

Assuming that $\lambda_{d,12}^1 \sim \lambda_s^1 \sim 10^{-4}$, the range (2.73) becomes

\[
10^{-11} \lesssim \lambda_{d,12}^2 \lesssim 10^{-7} \quad (2.75)
\]

and, hence,

\[
4.72 \times 10^{-27} GeV \lesssim \Delta M_K^{FCNC(I)} \lesssim 4.72 \times 10^{-19} GeV \quad (2.76)
\]

This sits comfortably below the upper bound

\[
\Delta M_K^{FCNC} \lesssim 10^{-15} GeV \quad (2.77)
\]
obtained using the $K^0$ entries in the last two columns of Table 1. Next, consider the second limit where $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, we know from (2.72) that this choice of parameters will come closer to saturating the upper bound. Using (2.71) and Table 1 we find that

$$|\Delta M_{K}^{FCNC(I)}| = \Delta M_{K}^{FCNC(I)} \left( \frac{1.1 \times 10^5 GeV^2}{\mu_2^2} \right)$$  \hspace{1cm} (2.78)

If, for example, we take

$$\mu_2 \approx 7 GeV,$$  \hspace{1cm} (2.79)

corresponding to an $H_3^0$ mass of 14 GeV, then it follows from (2.76) and (2.78) that

$$10^{-23} GeV \lesssim |\Delta M_{K}^{FCNC(I)}| \lesssim 10^{-15} GeV.$$  \hspace{1cm} (2.80)

The choice of $\mu_2$ in (2.79) is purely illustrative, chosen so that the Higgs mediated flavor changing currents can induce $K^0$ mixing of the same order as the experimental data. A more detailed study of our theory would be required to determine if a neutral Higgs boson can be this light relative to the electroweak scale. Of course, if the mass of $H_3^0$ is larger, its contribution to neutral meson mixing would rapidly decrease. We conclude that if $\lambda_{d,12}^2$ saturates its upper bound of $10^{-7}$ and the neutral Higgs $H_3^0$ is sufficiently light, then the contribution of the Higgs mediated flavor-changing neutral currents can play a measurable role in $K^0 - \bar{K}^0$ mixing.

Next, let us discuss $B^0_d - \bar{B}^0_d$ mixing. In the limit that $\mu_2 = \mu_1 \approx M_{EW}$, it follows from (2.69), Table 1 and $M_{B_d} = 5.28 GeV$ that

$$\Delta M_{B_d}^{FCNC(I)} \approx 0.876 \times 10^{-6} \lambda_{d,13}^2 GeV.$$  \hspace{1cm} (2.81)
Assuming that $\lambda_{d,13}^1 \sim \lambda_b^1 \sim 10^{-2}$, the range (2.73) becomes

$$10^{-9} \lesssim \lambda_{d,13}^2 \lesssim 10^{-7}$$

(2.82)

and, hence,

$$0.876 \times 10^{-24} \text{GeV} \lesssim \Delta M^{FCNC(I)}_{B_d} \lesssim 0.876 \times 10^{-20} \text{GeV} .$$

(2.83)

This contribution is well below the upper bound of

$$\Delta M^{FCNC}_{B_d} \lesssim 10^{-13} \text{GeV}$$

(2.84)

obtained using the $B_d^0$ entries in the last two columns of Table 1. Next, consider the second limit where $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, we know from (2.72) that this choice of parameters will come closer to saturating the upper bound. Using (2.71) and Table 1 we find that

$$|\Delta M^{FCNC(II)}_{B_d}| = \Delta M^{FCNC(I)}_{B_d} \left( \frac{7.16 \times 10^4 \text{GeV}^2}{\mu_2^2} \right)$$

(2.85)

If we take, for example,

$$\mu_2 \approx 7 \text{GeV} ,$$

(2.86)

thus saturating the upper bound in the $K^0$ case, then it follows from (2.83) and (2.85) that

$$1.28 \times 10^{-21} \text{GeV} \lesssim |\Delta M^{FCNC(II)}_{B_d}| \lesssim 1.28 \times 10^{-17} \text{GeV} .$$

(2.87)

We conclude that even if $\lambda_{d,13}^2$ saturates its upper bound of $10^{-7}$ and the neutral Higgs $H_3^0$ is sufficiently light to saturate the upper bound in the $K^0$ case, the
contribution of the Higgs mediated flavor-changing neutral currents to $B_d^0 - \bar{B}_d^0$ mixing remains well below the presently measured upper bound.

Finally, consider the $D^0 - \bar{D}_0$ case. If we assume that $\lambda^1_{u,12} \sim \lambda^1_c \sim 5 \times 10^{-3}$, the range (2.73) becomes

$$5 \times 10^{-10} \lesssim \lambda^2_{u,12} \lesssim 10^{-7}.$$  \hspace{1cm} (2.88)

It follows from this, (2.69), Table 1 and $M_{D^0} = 1.86 GeV$ that in the limit that $\mu_1 = \mu_2 \approx M_{EW}$

$$1.14 \times 10^{-25} GeV \lesssim \Delta M_{D}^{FCNC(I)} \lesssim 4.56 \times 10^{-21} GeV ,$$ \hspace{1cm} (2.89)

well below the upper bound of

$$\Delta M_{D}^{FCNC} \lesssim 10^{-13} GeV$$ \hspace{1cm} (2.90)

obtained using the $D^0$ entries in the last two columns of Table 1. Finally, consider the second limit where $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, using (2.71), Table 1, (2.89) and $\mu_2 \approx 7 GeV$, we obtain

$$1.77 \times 10^{-22} GeV \lesssim |\Delta M_{D}^{FCNC(II)}| \lesssim 7.11 \times 10^{-18} GeV .$$ \hspace{1cm} (2.91)

We conclude that even if $\lambda^2_{u,12}$ saturates its upper bound of $10^{-7}$ and the neutral Higgs $H_3^0$ is sufficiently light to saturate the upper bound in the $K^0$ case, the contribution of the Higgs mediated flavor-changing neutral currents to $D^0 - \bar{D}^0$ mixing remains well below the presently measured upper bound.
Chapter 3

Single Higgs Pair Model: Quasi Analytic Analysis

Here we present the results of a renormalization group analysis of the minimal heterotic standard model with a reasonable set of assumptions about the initial soft supersymmetry breaking parameters. These assumptions are consistent with basic requirements of phenomenology, such as suppressed flavor changing neutral currents, but are further constrained so as to allow a quasi-analytic solution of the RGEs. We found that $B$-$L$ symmetry is indeed spontaneously broken by a radiatively induced VEV of at least one right-handed sneutrino. Electroweak symmetry is then radiatively broken by a Higgs VEV at a lower scale, with the $B$-$L$/electroweak hierarchy of $\mathcal{O}(10)$ to $\mathcal{O}(10^2)$. The purpose of this section is to present the detailed renormalization group calculations leading to those conclusions. These include both
analytic, quasi-analytic and purely numerical solutions of the relevant equations. Specifically, we will do the following. In Section 3.1, the chiral fields of the $U(1)_{B-L}$ extended MSSM are presented and their supersymmetric interactions, via the superpotential and $D$-terms, are discussed. The complete set of soft supersymmetry breaking operators in this context are then introduced. Section 3.2 is devoted to presenting and solving the RGEs associated with the spontaneous breaking of the $U(1)_{B-L}$ gauge symmetry. This analysis involves the gauge parameters, gaugino and slepton masses and both the $B$-$L$ and $Y$ Fayet-Iliopoulos (FI) parameters [58]. Using these results, it is shown that $U(1)_{B-L}$ is indeed radiatively broken by a non-zero right-handed sneutrino VEV, and the details of this vacuum are presented. In Section 3.3, the analysis is extended to include both up and down Higgs and squark masses, as well as the $\mu$ and $B$ parameters. It is then shown that at the $B$-$L$ scale, electroweak symmetry, as well as color and charge, remain unbroken. All RGEs are then scaled down several orders of magnitude. We demonstrate that a Higgs VEV now develops which spontaneously breaks electroweak symmetry, without breaking color or charge. The $B$-$L$ and Higgs VEVs are presented and all squark, slepton and Higgs masses are calculated in this vacuum. The results are a detailed function of the initial right-handed sneutrino and Higgs mass parameters, $m_\nu(0)$ and $m_H(0)$ respectively, as well as inverse powers of $\tan\beta$. The relationship between $m_\nu(0)$ and $m_H(0)$ is also presented. Finally, in Section 3.4, we analyze the resultant $B$-$L$/electroweak hierarchy and show that it is of order $10^2$. The complete
spectrum of squark and slepton masses, written in terms of the $B-L$ boson mass, is then evaluated. Our analysis depends on a numerical solution for the Higgs mass parameter, $m_H(t)^2$. This is discussed and presented in Appendix B. In Appendix C, we numerically calculate the relationship between $m_\nu(0)$ and $m_H(0)$ used in the text. Finally, in Appendix D we verify that our $B-L$/electroweak breaking vacuum satisfies the standard constraint and minimization equations presented, for example, in [39, 6, 87].

3.1 The $N = 1$ Supersymmetric Theory

We will consider an $N = 1$ supersymmetric theory with gauge group

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$ (3.1)

and the associated vector superfields. The gauge parameters are denoted by $g_3$, $g_2$, $g_Y$ and $g_{B-L}$ respectively. The matter spectrum consists of three families of quark and lepton chiral superfields, each family with a right-handed neutrino. They transform under the gauge group in the standard manner as

$$Q_i = (3, 2, 1/3, 1/3), \quad u_i = (\bar{3}, 1, -4/3, -1/3), \quad d_i = (\bar{3}, 1, 2/3, -1/3)$$ (3.2)

for the left and right-handed quarks and

$$L_i = (1, 2, -1, -1), \quad \nu_i = (1, 1, 0, 1), \quad e_i = (1, 1, 2, 1)$$ (3.3)
for the left and right-handed leptons, where $i = 1, 2, 3$. In addition, the spectrum has one pair of Higgs-Higgs conjugate chiral superfields transforming as

$$H = (1, 2, 1, 0), \quad \bar{H} = (1, 2, -1, 0).$$

(3.4)

When necessary, the left-handed $SU(2)_L$ doublets will be written as

$$Q_i = (U_i, D_i), \quad L_i = (N_i, E_i), \quad H = (H^+, H^0), \quad \bar{H} = (\bar{H}^0, \bar{H}^-).$$

(3.5)

There are no other fields in the spectrum.

The supersymmetric potential energy is given by the usual sum over the modulus squared of the $F$ and $D$-terms. In principle, the $F$-terms are determined from the most general superpotential invariant under the gauge group,

$$W = \mu H\bar{H} + \sum_{i,j=1}^{3} (\lambda_{u,ij}Q_i Hu_j + \lambda_{d,ij}Q_i \bar{H}d_j + \lambda_{\nu,ij}L_i H\nu_j + \lambda_{e,ij}L_i \bar{H}e_j)$$

(3.6)

Note that an innocuous mixing term of the form $L_i H$, as well as the dangerous lepton and baryon number violating interactions

$$L_i L_j e_k, \quad L_i Q_j d_k, \quad u_i d_j d_k$$

(3.7)

which generically would lead, for example, to rapid nucleon decay, are disallowed by the $U(1)_{B-L}$ gauge symmetry. To simplify the upcoming calculations, we will assume that we are in a mass-diagonal basis where

$$\lambda_{u,ij} = \lambda_{d,ij} = \lambda_{\nu,ij} = \lambda_{e,ij} = 0, \quad i \neq j.$$

(3.8)

Note that once these off-diagonal couplings vanish just below the compactification scale, they will do so at all lower energy-momenta. We will denote the diagonal Yukawa couplings by $\lambda_{ii} = \lambda_i, \quad i = 1, 2, 3$. 41
Next, observe that a constant, field-independent $\mu$ parameter cannot arise in a supersymmetric string vacuum since the Higgs fields are zero modes. However, the $H\bar{H}$ bilinear can have higher-dimensional couplings to moduli through both holomorphic and non-holomorphic interactions in the superpotential and Kahler potential respectively. When moduli acquire VEVs due to non-perturbative effects, these can induce non-vanishing supersymmetric contributions to $\mu$. A non-zero $\mu$ can also be generated by gaugino condensation in the hidden sector. Why this induced $\mu$-term should be small enough to be consistent with electroweak symmetry breaking is a difficult, model dependent problem. In this paper, we will not discuss this “$\mu$-problem”, but simply assume that the $\mu$ parameter is at, or below, the electroweak scale. In fact, so as to emphasize the $B$-$L$/electroweak hierarchy and simplify the calculation, we will take $\mu$, while non-zero, to be substantially smaller than the electroweak scale, making its effect sub-dominant. This can be implemented consistently throughout the entire scaling regime. The exact meaning of “sub-dominant” is quantified in Appendix D, where we also present the upper bound on $\mu$ and, hence, the Higgsino mass in our approximation scheme.

The $SU(3)_C$ and $SU(2)_L$ $D$-terms are of the standard form. We present the $U(1)_Y$ and $U(1)_{B-L}$ $D$-terms,

$$D_Y = \xi_Y + g_Y \phi_A^I (Y/2)_{AB} \phi_B$$  \hspace{1cm} (3.9)

and

$$D_{B-L} = \xi_{B-L} + g_{B-L} \phi_A^I (Y_{B-L})_{AB} \phi_B$$  \hspace{1cm} (3.10)
where the index $A$ runs over all scalar fields $\phi_A$, to set the notation for the hypercharge and $B-L$ charge generators and to remind the reader that each of these $D$-terms potentially has a Fayet-Iliopoulos additive constant. However, as with the $\mu$ parameter, constant field-independent FI terms cannot occur in string vacua since the low energy fields are zero modes. Field-dependent FI terms can occur in some contexts, see for example [11]. However, since both the hypercharge and $B-L$ gauge symmetries are anomaly free, such field-dependent FI terms are not generated in the supersymmetric effective theory. We include them in (3.9),(3.10) since they can, in principle, arise at a lower scale from radiative corrections once supersymmetry is softly broken [72]. Be that as it may, if calculations are done in the $D$-eliminated formalism, which we use in this paper, these FI parameters can be consistently absorbed into the definition of the soft scalar masses and their beta functions. Hence, we will no longer consider them.

In addition to the supersymmetric potential, the Lagrangian density also contains explicit “soft” supersymmetry violating terms. These arise from the spontaneous breaking of supersymmetry in a hidden sector that has been integrated out of the theory. This breaking can occur in either $F$-terms, $D$-terms or both in the hidden sector. In this paper, for simplicity, we will restrict our discussion to soft supersymmetry breaking terms arising exclusively from $F$-terms. The form of these terms is well-known and, in the present context, given by [63, 71, 82, 90, 87]

$$V_{soft} = V_{2s} + V_{3s} + V_{2f}, \quad (3.11)$$
where $V_{2s}$ are scalar mass terms

$$V_{2s} = \sum_{i=1}^{3} (m_{Q_i}^2|Q_i|^2 + m_{u_i}^2|u_i|^2 + m_{d_i}^2|d_i|^2$$

$$+ m_{L_i}^2|L_i|^2 + m_{\nu_i}^2|\nu_i|^2 + m_{e_i}^2|e_i|^2$$

$$+ m_{H_i}^2|H|^2 + m_{\tilde{H}}^2|\tilde{H}|^2) - (B H \tilde{H} + hc),$$

(3.12)

$V_{3s}$ are scalar cubic couplings

$$V_{3s} = \sum_{i=1}^{3} (A_{u_i} Q_i H u_i + A_{d_i} Q_i \tilde{H} b_i + A_{\nu_i} L_i H \tilde{\nu}_i + A_{e_i} L_i \tilde{H} e_i + hc)$$

(3.13)

and $V_{2f}$ contains the gaugino mass terms

$$V_{2f} = \frac{1}{2} M_3 \lambda_3 \lambda_3 + \frac{1}{2} M_2 \lambda_2 \lambda_2 + \frac{1}{2} M_Y \lambda_Y \lambda_Y + \frac{1}{2} M_{B-L} \lambda_{B-L} \lambda_{B-L} + hc.$$  

(3.14)

As above, to simplify the calculation we assume the parameters in (3.12) and (3.13) are flavor-diagonal. This is consistent since once the off-diagonal parameters vanish just below the compactification scale, they will do so at all lower energy-momenta.

### 3.2 The Renormalization Group and $B-L$

In this section, we discuss the spontaneous breakdown of the gauged $B-L$ symmetry.

The parameters in our theory all scale with energy-momentum, each obeying the associated renormalization group equation (RGE). In this section, we will solve those equations required in the analysis of $B-L$ breaking to the one-loop level.
**Gauge Parameters:**

We begin by considering the RG running of the gauge coupling parameters. Since our low energy theory arises from an $SO(10)$ compactification of heterotic string theory broken to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ by Wilson lines, it is conventional to redefine the hypercharge and $B-L$ gauge parameters as

$$g_1 = \sqrt{\frac{5}{3}} g_Y, \quad g_4 = \sqrt{\frac{4}{3}} g_{B-L}.$$  

With these redefinitions, and defining $t = \ln(\mu / M_u)$, the four running gauge parameters $g_a(t), a = 1, \ldots, 4$ all unify to a value $g(0)$ at a scale $M_u$. Precision measurements set [6, 12, 62]

$$g(0) \simeq 0.726, \quad M_u \simeq 3 \times 10^{16} \text{GeV}.$$  

Note, $g(0)$ is simply obtained from [6] and the equations there in. For specificity, we will use these values in our analysis, ignoring as sub-dominant the defocussing effects of possible string thresholds and non-universal soft breaking parameters. The RGEs are given by

$$\frac{dg_a}{dt} = \frac{1}{16\pi^2} \beta_a, \quad a = 1, \ldots, 4.$$  

1There are two related, but not identical, notions of “scale” that occur in this paper. The first is the value of the RG parameter $\mu$ or, equivalently, the associated energy-momentum where the running couplings are evaluated. The second is the magnitude of a VEV or, equivalently, the mass of the associated vector boson. This scale is set by various mass and coupling parameters evaluated at a given $\mu$. Which scale is being referred to should be clear from context.
where

\[ \beta_a = b_ag_a^3, \quad \bar{b} = \left( \frac{33}{5}, 1, -3, 12 \right). \] (3.18)

These can be integrated directly to yield

\[ g_a(t)^2 = \frac{g(0)^2}{1 - \frac{g(0)^2 b_a}{8\pi^2}}, \quad a = 1, \ldots, 4. \] (3.19)

In this section, we are interested in scaling all parameters from the unification mass \( M_u \simeq 3 \times 10^{16}\text{GeV} \) to the \( B-L \) scale \( \mu_{B-L} \simeq 10^4\text{GeV} \); that is,

\[ t_{B-L} \simeq -28.7 \leq t \leq 0 = t_u. \] (3.20)

The RG scaling to lower energies will be carried out in subsequent sections\(^2\) In this range, we find from (3.18), (3.19) that the infrared free parameters \( g_1, g_2 \) and \( g_4 \) decrease for small energy-momentum as

\[ .441 \leq \frac{g_1(t)^2}{g(0)^2} \leq 1, \quad .839 \leq \frac{g_2(t)^2}{g(0)^2} \leq 1, \quad .303 \leq \frac{g_4(t)^2}{g(0)^2} \leq 1 \] (3.21)

whereas the asymptotically free coupling \( g_3 \) grows as

\[ 2.36 \leq \frac{g_3(t)^2}{g(0)^2} \leq 1. \] (3.22)

\(^2\)We seek a solution to the RGEs which sequentially breaks the gauge symmetry; first \( B-L \) followed by \( SU(2)_L \times U(1)_Y \). By \( \mu_{B-L} \), we mean a value of energy-momentum at which one expects \( B-L \), but not electroweak symmetry, to be broken. Although not strictly necessary, it is natural to take the RG scale \( \mu_{B-L} \) to be in the range where, on phenomenological grounds, \( B-L \) symmetry breaking should occur. Order \( 10^4 \text{ GeV} \) is a convenient choice.
Gaugino Masses:

Denoting $M_Y = M_1$ and $M_{B-L} = M_4$, the RGEs for the gaugino masses in (3.14) are [86]

$$\frac{dM_a}{dt} = \frac{1}{8\pi^2} b_a g_a^2 M_a, \quad a = 1, \ldots, 4$$  \hspace{1cm} (3.23)

where the $b_a$ coefficients are given in (3.18). This is immediately solved to give

$$M_a(t) = \frac{M_a(0)}{1 - \frac{g(0)^2 b_a t}{8\pi^2}}, \quad a = 1, \ldots, 4.$$  \hspace{1cm} (3.24)

A priori, there is no constraint on the initial values $M_a(0)$. In the scaling range (3.20), it follows from (3.18), (3.24) that

$$0.441 \leq \frac{M_1(t)}{M_1(0)} \leq 1, \quad 0.839 \leq \frac{M_2(t)}{M_2(0)} \leq 1, \quad 0.303 \leq \frac{M_4(t)}{M_4(0)} \leq 1$$  \hspace{1cm} (3.25)

and

$$2.36 \geq \frac{M_3(t)}{M_3(0)} \geq 1.$$  \hspace{1cm} (3.26)

As will be seen shortly, the quantity we will be most interested in is $g_a(t)^2 |M_a(t)|^2$. Using (3.19) and (3.24), this combination runs under the RG as

$$g_a(t)^2 |M_a(t)|^2 = \frac{g(0)^2 |M_a(0)|^2}{(1 - \frac{g(0)^2 b_a t}{8\pi^2})^3}.$$  \hspace{1cm} (3.27)

Note that even if one assumes that the gaugino masses are “unified” at $t = 0$, making any ratio $\frac{g_a(t)^2 |M_a(0)|^2}{g_a(0)^2 |M_a(0)|^2}$ unity, it is clear that the gluino mass contributions will quickly grow to dominate. For example, at the electroweak scale the ratio of the gluino to the $SU(2)_L$ gaugino terms is 25.6. In this paper, so as to simplify the calculation and allow for a quasi-analytic solution, we will not assume unified
gaugino masses, instead taking $|M_1(0)|^2, |M_2(0)|^2, |M_4(0)|^2 \ll |M_3(0)|^2$. It then follows from (3.27) that

$$g_1^2 |M_1|^2, g_2^2 |M_2|^2, g_4^2 |M_4|^2 \ll g_3^2 |M_3|^2$$  \hspace{1cm} (3.28)

over the entire scaling regime. Recall that “non-unified” gaugino masses easily occur in string vacua, while unification requires additional “minimal” criteria [39, 87]. These are not generically satisfied in our MSSM theory.

**Slepton Masses:**

The RGEs for the slepton mass parameters $m_{L_i}$, $m_{e_i}$ and $m_{\nu_i}$ are given by [86]

$$16\pi^2 \frac{d m_{L_i}^2}{dt} = 2(m_{L_i}^2 + m_{H_i}^2 + m_{\nu_i}^2)|\lambda_{\nu_i}|^2 + 2(m_{L_i}^2 + m_{H_i}^2 + m_{\nu_i}^2)|\lambda_{e_i}|^2$$

$$+ 2|A_{\nu_i}|^2 + 2|A_{e_i}|^2 - 2g_1^2 |M_1|^2 - 6g_2^2 |M_2|^2 - \frac{3}{2}g_3^2 |M_4|^2$$

$$- \frac{3}{5}g_1^2 S - \frac{3}{4}g_4^2 S'$$,  \hspace{1cm} (3.29)

$$16\pi^2 \frac{d m_{e_i}^2}{dt} = 4(m_{L_i}^2 + m_{H_i}^2 + m_{e_i}^2)|\lambda_{e_i}|^2 + 4|A_{e_i}|^2$$

$$- \frac{24}{5} g_1^2 |M_1|^2 - \frac{3}{2} g_2^2 |M_4|^2 + \frac{6}{5} g_3^2 S + \frac{3}{4} g_4^2 S'$$,  \hspace{1cm} (3.30)

$$16\pi^2 \frac{d m_{\nu_i}^2}{dt} = 4(m_{L_i}^2 + m_{H_i}^2 + m_{\nu_i}^2)|\lambda_{\nu_i}|^2 + 4|A_{\nu_i}|^2$$

$$- \frac{3}{2} g_1^2 |M_1|^2 + \frac{3}{4} g_4^2 S'$$  \hspace{1cm} (3.31)
where

\[ S = m_H^2 - m_H^2 + \sum_{i=1}^{3} (m_{Q_i}^2 - 2m_{u_i}^2 + m_{d_i}^2 - m_{L_i}^2 + m_{e_i}^2) \]

\[ = Tr(\frac{Y}{2}m^2), \quad (3.32) \]

\[ S' = \sum_{i=1}^{3} (2m_{Q_i}^2 - m_{u_i}^2 - m_{d_i}^2 - 2m_{L_i}^2 + m_{e_i}^2 + m_{\nu_i}^2) \]

\[ = Tr(Y_{B-L}m^2). \quad (3.33) \]

A full numerical solution of these equations will be presented elsewhere. Here, we give an approximate solution based on the following observations. First, note that the initial conditions for the \( A \)-coefficients in equation (3.13) are, ignoring phenomenological constraints for the time-being, completely arbitrary. However, it is conventional [87] to let

\[ A_{u_i} = \lambda_{u_i} \tilde{A}_{u_i}, \quad A_{d_i} = \lambda_{d_i} \tilde{A}_{d_i}, \quad A_{\nu_i} = \lambda_{\nu_i} \tilde{A}_{\nu_i}, \quad A_{e_i} = \lambda_{e_i} \tilde{A}_{e_i} \quad (3.34) \]

where the dimensionful \( \tilde{A} \)-parameters satisfy

\[ \tilde{A}_{u_i} \sim \mathcal{O}(m_{u_i}), \quad \tilde{A}_{d_i} \sim \mathcal{O}(m_{d_i}), \quad \tilde{A}_{\nu_i} \sim \mathcal{O}(m_{\nu_i}), \quad \tilde{A}_{e_i} \sim \mathcal{O}(m_{e_i}). \quad (3.35) \]

This is not a requirement in the “non-minimal” string vacua that we are discussing. Be that as it may, for simplicity of presentation we will assume (3.34) and (3.35) for the remainder of this paper. Having done this, it follows that every term on the right hand side of equations (3.29), (3.30) and (3.31), with the exception of the terms involving the gaugino masses, has the form of either \(|\lambda|^2m^2\) or \(g^2m^2\). Our second observation is that the Yukawa couplings appearing in (3.29), (3.30) and
(3.31) satisfy

$$|\lambda_{\nu_1}| < |\lambda_{\nu_2}| < |\lambda_{\nu_3}| \simeq 10^{-9} \ll g_a, \quad |\lambda_{e_1}| < |\lambda_{e_2}| < |\lambda_{e_3}| \simeq 10^{-2} \ll g_a$$  \(3.36\)

throughout the scaling range (3.20) for \(a = 1, \ldots, 4\). Using (3.34), (3.35) and (3.36), it follows that one can approximate the slepton mass RGEs as

$$16\pi^2 \frac{dm_i^2}{dt} \simeq -2g_1^2|M_1|^2 - 6g_2^2|M_2|^2 - \frac{3}{2}g_4^2|M_4|^2 - \frac{3}{5}g_1^2S - \frac{3}{4}g_1^2S',$$

(3.37)

$$16\pi^2 \frac{dm_i^2}{dt} \simeq -\frac{24}{5}g_1^2|M_1|^2 - \frac{3}{2}g_4^2|M_4|^2 + \frac{6}{5}g_1^2S + \frac{3}{4}g_4^2S',$$

(3.38)

$$16\pi^2 \frac{dm_i^2}{dt} \simeq -\frac{3}{2}g_4^2|M_4|^2 + \frac{3}{4}g_4^2S'.$$  \(3.39\)

Third, recall that the initial gaugino masses \(M_a(0), a = 1, \ldots, 4\) are chosen so that (3.28) is satisfied, but are otherwise arbitrary. Henceforth, we further restrict them so that

$$g_1^2|M_1|^2, g_2^2|M_2|^2, g_4^2|M_4|^2 \ll g_4^2S'$$  \(3.40\)

over the entire scaling range (3.20). Fourth, we make a specific choice for the scalar masses at the unification scale \(M_u\). These are taken to be

$$m_H(0)^2 = m_{\tilde{H}}(0)^2, \quad m_{Q_i}(0)^2 = m_{u_j}(0)^2 = m_{d_k}(0)^2$$  \(3.41\)

and

$$m_{L_i}(0)^2 = m_{e_j}(0)^2 \neq m_{\nu_k}(0)^2$$  \(3.42\)

for all \(i, j, k = 1, 2, 3\). Note that the sneutrino masses are different than those of the remaining sleptons. \textit{This asymmetry is one ingredient in breaking} \(U(1)_{B-L}\) at
an appropriate scale. Other than that, this choice is taken so as to simplify the RGEs as much as possible and to allow a quasi-analytic solution. We point out that soft scalar masses need not be “universal” in string theories, since they are not generically “minimal”. We emphasize that a B-L/electroweak hierarchy is possible for a much wider range of initial parameters.

Finally, let us consider the $g_4^2S$ and $g_4^2S'$ terms. Note from (3.32) and (3.33) that if one chooses the initial scalar masses to satisfy (3.41) and (3.42) then

$$S(0) = 0, \quad S'(0) = \sum_{i=1}^{3} (-m_{L_i}(0)^2 + m_{\nu_i}(0)^2) \neq 0.$$  \hspace{1cm} (3.43)

Additionally, we will take $S' > 0$ with the scale set by the initial sneutrino masses. It then follows from (3.43) that over the scaling range (3.20)

$$g_4^2S \ll g_4^2S'.$$  \hspace{1cm} (3.44)

In fact, we can go one step further. Decompose $S'$ in (3.33) as

$$S' = S'_0 + S'_1,$$  \hspace{1cm} (3.45)

where

$$S'_0 = \sum_{i=1}^{3} (2m_{Q_i}^2 - m_{u_i}^2 - m_{d_i}^2 - m_{L_i}^2 + m_{e_i}^2), \quad S'_1 = \sum_{i=1}^{3} (-m_{L_i}^2 + m_{\nu_i}^2).$$  \hspace{1cm} (3.46)

Then we see from (3.41) and (3.42) that

$$S'_0(0) = 0, \quad S'_1(0) = \sum_{i=1}^{3} (-m_{L_i}(0)^2 + m_{\nu_i}(0)^2) \neq 0$$  \hspace{1cm} (3.47)

and, hence, over the scaling range (3.20)

$$g_4^2S'_0 \ll g_4^2S'_1.$$  \hspace{1cm} (3.48)
We conclude from (3.40), (3.44) and (3.48) that a further approximation to the
slepton mass RGEs is given by
\[
16\pi^2 \frac{d m^2_{\tilde{L}_i}}{dt} \simeq -\frac{3}{4} g^2_i S'_1, \tag{3.49}
\]
\[
16\pi^2 \frac{d m^2_{\tilde{e}_i}}{dt} \simeq \frac{3}{4} g^2_i S'_1, \tag{3.50}
\]
\[
16\pi^2 \frac{d m^2_{\nu_i}}{dt} \simeq \frac{3}{4} g^2_i S'_1. \tag{3.51}
\]

From (3.46), (3.49) and (3.51) one finds a RGE for \( S'_1 \) given by
\[
\frac{d S'_1}{dt} = \frac{9}{32\pi^2} g^2_i S'_1. \tag{3.52}
\]

Using (3.19), this is easily solved to give
\[
S'_1(t) = \frac{S'_1(0)}{(1 - \frac{g(0)^2 b_4 t}{8\pi^2})^{\frac{9}{4} b_4}}. \tag{3.53}
\]

It follows from (3.16) and (3.18) that in the scaling range (3.20)
\[
.800 \leq \frac{S'_1(t)}{S'_1(0)} \leq 1. \tag{3.54}
\]

It is now straightforward to solve (3.49), (3.50) and (3.51) for \( m^2_{\tilde{L}_i}, m^2_{\tilde{e}_i} \) and \( m^2_{\nu_i} \),
respectively. From (3.19), (3.50) and (3.53) one finds
\[
m^2_{\tilde{L}_i}(t) = m^2_{\tilde{L}_i}(0)^2 \frac{1}{6} (1 - (1 - \frac{g(0)^2 b_4 t}{8\pi^2})^{-9/4 b_4}) S'_1(0) \tag{3.55}
\]
and
\[
m^2_{\tilde{e}_i}(t) = m^2_{\tilde{e}_i}(0)^2 - \frac{1}{6} (1 - (1 - \frac{g(0)^2 b_4 t}{8\pi^2})^{-9/4 b_4}) S'_1(0) \tag{3.56}
\]
\[
m^2_{\nu_i}(t) = m^2_{\nu_i}(0)^2 - \frac{1}{6} (1 - (1 - \frac{g(0)^2 b_4 t}{8\pi^2})^{-9/4 b_4}) S'_1(0). \tag{3.57}
\]
These equations are almost identical with the exception of the sign of the second term. This is positive for $m_{L_i}^2$, whereas it is negative in the expressions for $m_{e_i}^2$ and $m_{\nu_i}^2$. This sign has important physical consequences. Using (3.16) and (3.18), the mass parameter parameter for $m_{L_i}^2$ increases from its initial value at $t = 0$ to

$$m_{L_i}(t_{B-L})^2 = m_{L_i}(0)^2 + (3.35 \times 10^{-2})S_1'(0)$$

(3.58)

at $t_{B-L}$. Thus it always remains positive. On the other hand, the mass parameters for $m_{e_i}^2$ and $m_{\nu_i}^2$ start at their initial values at $t = 0$ but decrease to

$$m_{e_i}(t_{B-L})^2 = m_{e_i}(0)^2 - (3.35 \times 10^{-2})S_1'(0),$$

(3.59)

$$m_{\nu_i}(t_{B-L})^2 = m_{\nu_i}(0)^2 - (3.35 \times 10^{-2})S_1'(0)$$

(3.60)

at $t_{B-L}$. If the initial masses are sufficiently small and $S_1'(0)$ is sufficiently large, then one or more of these squared masses can become negative signaling possible symmetry breaking.

**Spontaneous $B-L$ Breaking:**

Given the RGE solutions in the previous subsections, one can now study the scalar field potential at any scale in the range (3.20) and, in particular, search for local minima. Here, we limit the discussion to the slepton fields. Higgs fields and squarks will be discussed in the next subsection, where the consistency of this two-step procedure will be demonstrated.

Let us begin by considering the quadratic mass terms near the origin of field
space. The relevant part of the scalar potential is

\[ V = V_{2s} + \frac{1}{2} D_{B-L}^2 \] (3.61)

where \( V_{2s} \) and \( D_{B-L} \) are given in (3.12) and (3.10) respectively. Expanding this using the \( B-L \) quantum numbers listed in (3.4), the quadratic terms are given at any scale \( t \) by

\[ V = \cdots + \sum_{i=1}^{3} (m_{L_i}^2 |L_i|^2 + m_{e_i}^2 |e_i|^2 + m_{\nu_i}^2 |\nu_i|^2) + \cdots \] (3.62)

The reader should recall that the FI terms have been absorbed into the soft mass parameters. All \( \beta \) functions in the D-eliminated formalism are written in terms of these redefined masses. As the theory is scaled from \( t = 0 \) toward \( t_{B-L} \), the \( m_{L_i}^2 \), \( m_{e_i}^2 \) and \( m_{\nu_i}^2 \) parameters scale as in (3.58),(3.59) and (3.60) respectively.

The first requirement for spontaneous \( B-L \) breaking is that at least one of the slepton squared masses becomes negative at \( t_{B-L} \). Clearly, this cannot happen for \( m_{L_i}(t_{B-L})^2 \), which is always positive. However, if the initial squared masses are sufficiently small and \( S'_1(0) \) sufficiently large, both \( m_{e_i}(t_{B-L})^2 \) and \( m_{\nu_i}(t_{B-L})^2 \) can become negative. Since the \( e_i \) fields are electrically charged, we do not want them to get a vacuum expectation value and, hence, we want \( m_{e_i}(t_{B-L})^2 \) to be positive. On the other hand, the \( \nu_i \) fields are neutral in all quantum numbers except \( B-L \). Hence, if they get a nonzero VEV this will spontaneously break \( B-L \) at \( t_{B-L} \), but leave the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge symmetry unbroken. We now show that for a wide range of initial parameters this is indeed possible. For simplicity, let us
choose the initial right-handed slepton masses to be

\[ m_{\nu_1}(0) = m_{\nu_2}(0) = C m_\nu(0), \quad m_{\nu_3}(0) = m_\nu(0) \]  \hspace{1cm} (3.63)

and

\[ m_{e_1}(0) = m_{e_2}(0) = m_{e_3}(0) = A m_\nu(0) \]  \hspace{1cm} (3.64)

for some dimensionless constants \( C \) and \( A \) to be determined. Using (3.42), (3.43) and (3.63), (3.64) we see that \( S'_1(0) \) is parameterized by

\[ S'_1(0) = (1 + 2C^2 - 3A^2)m_\nu(0)^2 . \]  \hspace{1cm} (3.65)

Let us first consider \( \nu_3 \). It follows from (3.60), (3.63) and (3.65) that

\[ m_{\nu_3}(t_{B-L})^2 = (1 - (3.35 \times 10^{-2})(1 + 2C^2 - 3A^2))m_\nu(0)^2 . \]  \hspace{1cm} (3.66)

For specificity, we will henceforth take

\[ (3.35 \times 10^{-2})(1 + 2C^2 - 3A^2) = 5 . \]  \hspace{1cm} (3.67)

This choice yields the simple result that

\[ m_{\nu_3}(t_{B-L})^2 = -4m_\nu(0)^2 , \]  \hspace{1cm} (3.68)

suggesting a non-zero VEV in the \( \nu_3 \) direction. Now consider \( \nu_1, \nu_2 \). In this case, we see from (3.60), (3.63), (3.65) and (3.67) that

\[ m_{\nu_1,2}(t_{B-L})^2 = (C^2 - 5)m_\nu(0)^2 . \]  \hspace{1cm} (3.69)

Now let us examine the \( e_{1,2,3} \) fields. Using (3.59), (3.64), (3.65) and (3.67) one finds

\[ m_{e_i}(t_{B-L})^2 = (A^2 - 5)m_\nu(0)^2 \]  \hspace{1cm} (3.70)
for \( i = 1, 2, 3 \). Since all \( m_{e_i}(t_{B-L})^2 \) must be positive, the coefficient \( A \) must satisfy \( A^2 - 5 > 0 \). Again, for specificity we will choose

\[
A = \sqrt{6} ,
\]

which yields the simple result that

\[
m_{e_i}(t_{B-L})^2 = m_{e_i}(0)^2
\]

for \( i = 1, 2, 3 \). Putting \( A = \sqrt{6} \) into expression (3.67) gives

\[
C = 9.12 .
\]

Hence, we see from (3.69) that both \( m_{\nu_{1,2}}(t_{B-L})^2 \) are positive and given by

\[
m_{\nu_{1,2}}(t_{B-L})^2 = 78.2 m_{\nu}(0)^2 .
\]

We conclude from (3.72) and (3.74) that, near the origin of field space, there are positive quadratic mass terms in the \( e_{1,2,3} \) and \( \nu_{1,2} \) field directions. Finally, let us consider the \( L_i, i = 1, 2, 3 \) masses. Using (3.42), (3.58), (3.64), (3.65), (3.67) and (3.71), we find that

\[
m_{L_{1,2,3}}(t_{B-L})^2 = 11 m_{\nu}(0)^2 ,
\]

that is, a positive quadratic mass term in each if the \( L_i \) field directions at the origin. We note for future reference that using (3.71) and (3.73), equation (3.65) becomes

\[
S'_1(0) = 149 m_{\nu}(0)^2 .
\]

Given these mass terms, as well as \( g_4 \) in (3.21), one can minimize the complete potential to determine the vacuum at \( t_{B-L} \). The part of this potential relevant
to finding a local minimum in the slepton fields is \( V \) in (3.61). We note that, in principle, the \( |\lambda_{\nu_3}|^2 |L_3|^2 |\nu_3|^2 \) term in the \( |\partial_H W|^2 \) contribution to the potential, where \( W \) is given in (3.6), could play a role in selecting the vacuum. However, since \( \lambda_{\nu_3} \ll g_4 \), this term can be safely ignored. Expanding out the contributions to \( V \) and collecting terms, the important interactions are

\[
V = m_{\nu_3}^2 |\nu_3|^2 + \frac{3}{4} g_4^2 |\nu_3|^4 + (m_{\nu_{1,2}}^2 + \frac{3}{4} g_4^2 |\nu_3|^2) |\nu_{1,2}|^2 \\
+ \sum_{i=1}^{3} ((m_{e_i}^2 + \frac{3}{4} g_4^2 |\nu_3|^2) |e_i|^2 + (m_{L_i}^2 - \frac{3}{4} g_4^2 |\nu_3|^2) |L_i|^2) + \ldots \tag{3.77}
\]

Recalling from (3.68) that \( m_{\nu_3} (t_{B-L})^2 = -4m_{\nu}(0)^2 \), the first two terms in (3.77) can be minimized by the non-zero VEV

\[
\langle \nu_3 \rangle = \frac{2 m_{\nu}(0)}{\sqrt{\frac{3}{4} g_4 (t_{B-L})}}. \tag{3.78}
\]

Is this point a local minimum of the complete slepton potential? To determine this, consider potential (3.77) near this VEV. Clearly, the first derivatives of all fields vanish at this point. Now determine the masses in all slepton field directions evaluated at (3.78). It follows from (3.77) that

\[
V = \langle m_{\nu_3}^2 \rangle |\delta \nu_3|^2 + \langle m_{\nu_{1,2}}^2 \rangle |\nu_{1,2}|^2 + \sum_{i=1}^{3} ((\langle m_{e_i}^2 \rangle |e_i|^2 + \langle m_{L_i}^2 \rangle |L_i|^2) + \ldots, \tag{3.79}
\]

where

\[
\langle m_{\nu_3}^2 \rangle = -2 m_{\nu_3}^2, \\
\langle m_{\nu_{1,2}}^2 \rangle = m_{\nu_{1,2}}^2 + \frac{3}{4} g_4^2 \langle \nu_3 \rangle^2, \\
\langle m_{e_i}^2 \rangle = m_{e_i}^2 + \frac{3}{4} g_4^2 \langle \nu_3 \rangle^2, \\
\langle m_{L_i}^2 \rangle = m_{L_i}^2 - \frac{3}{4} g_4^2 \langle \nu_3 \rangle^2. \tag{3.80}
\]
Using (3.68), (3.72), (3.74), (3.75) and (3.78) we find that at \( t_{B-L} \)

\[
\langle m_{\nu_3}^2 \rangle = 8 \, m_\nu(0)^2, \quad \langle m_{\nu_{1,2}}^2 \rangle = 82.2 \, m_\nu(0)^2,
\]

\[
\langle m_{e_i}^2 \rangle = 5 \, m_\nu(0)^2, \quad \langle m_{L_i}^2 \rangle = 7 \, m_\nu(0)^2.
\] (3.81)

Since all of these masses are positive, we conclude that the vacuum specified by

\[
\langle \nu_{1,2} \rangle = 0, \quad \langle \nu_3 \rangle = \frac{2 \, m_\nu(0)}{\sqrt{3} g_4(t_{B-L})}
\] (3.82)

and

\[
\langle e_i \rangle = \langle L_i \rangle = 0 \quad i = 1, 2, 3
\] (3.83)

is indeed a local minimum of the slepton potential \( V \). This vacuum spontaneously breaks the gauged \( B - L \) symmetry while preserving the remaining \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge group. The massless Goldstone boson is “eaten” by the \( B-L \) vector boson giving it a mass

\[
M_{A_{B-L}} = \sqrt{2} \, g_{B-L}(t_{B-L}) \, \langle \nu_3 \rangle.
\] (3.84)

Using (3.15) and (3.82), this becomes\(^3 \)

\[
M_{A_{B-L}} = 2\sqrt{2} \, m_\nu(0).
\] (3.85)

We now have to include the Higgs fields and squarks, analyze their masses at \( t_{B-L} \) around vacuum (3.82), (3.83) and show that this is a local minimum in the

\(^3\)Note from (3.68),(3.82) and (3.84) that the sneutrino expectation value and the mass of the \( B-L \) gauge boson are determined from a specific combination of mass and gauge parameters evaluated at \( t_{B-L} \). Hence, they depend on the initial values of these parameters as well as the beta functions determining the RG running. They are not identical to \( \mu_{B-L} \).
complete scalar field space. To do this, one must discuss the Higgs and squark RGEs, to which we now turn.

3.3 The RGEs for Higgs Fields and Squarks

In this section, we present and analyze the RGEs for the up and down Higgs fields, the left and right squarks, as well as the associated $\mu$ and $B$ parameters. Using these results, we compute the Higgs and squark masses at $t_{B-L}$ and show that they are all positive at vacuum (3.82), (3.83), as desired.
“Up” Higgs and Squark Masses:

The RGEs for the “up” Higgs and squark mass parameters \( m_H, m_Q, \) and \( m_u \) are given by [86]

\[
16\pi^2 \frac{d m_H^2}{d t} = \sum_{i=1}^{3} (6(m_Q^2 + m_H^2 + m_u^2)|\lambda_i|^2 + 2(m_L^2 + m_H^2 + m_u^2)|\lambda_i|^2 \\
+ 6|A_{u_i}|^2 + 2|A_{u_i}|^2) - \frac{6}{5} g_1^2|M_1|^2 - 6g_2^2|M_2|^2 \\
+ \frac{3}{5} g_1^2S, 
\]

\[
16\pi^2 \frac{d m_Q^2}{d t} = 2(m_Q^2 + m_H^2 + m_u^2)|\lambda_i|^2 + 2(m_Q^2 + m_H^2 + m_d^2)|\lambda_d|^2 \\
+ 2|A_{u_i}|^2 + 2|A_{d_i}|^2 - \frac{12}{5} g_1^2|M_1|^2 - 6g_2^2|M_2|^2 - \frac{32}{3} g_3^2|M_3|^2 \\
- \frac{1}{6} g_4^2|M_4|^2 + \frac{1}{5} g_1^2S + \frac{1}{4} g_3^2S', 
\]

\[
16\pi^2 \frac{d m_u^2}{d t} = 4(m_Q^2 + m_H^2 + m_u^2)|\lambda_i|^2 + 4|A_{u_i}|^2 \\
- \frac{32}{15} g_1^2|M_1|^2 - \frac{32}{3} g_3^2|M_3|^2 - \frac{1}{6} g_4^2|M_4|^2 \\
- \frac{4}{5} g_1^2S - \frac{1}{4} g_3^2S', 
\]

where \( S \) and \( S' \) are given in (3.32) and (3.33) respectively.

A full numerical solution of these equations will be presented elsewhere. Here, we give an approximate solution based on the following observations. First, as discussed in the previous section we will assume that the \( A \)-coefficients satisfy (3.34) and (3.35). Having done this, it follows that every term on the right hand side of equations (3.86), (3.87) and (3.88), with the exception of the terms involving the gaugino masses, has the form of either \( |\lambda|^2 m^2 \) or \( g^2 m^2 \). Our second observation is
that the Yukawa couplings appearing in (3.86), (3.87) and (3.88) satisfy

\[ |\lambda_{\nu_1}| < |\lambda_{\nu_2}| < |\lambda_{\nu_3}| \simeq 10^{-9} \ll g_a, \]
\[ |\lambda_{d_1}| < |\lambda_{d_2}| < |\lambda_{d_3}| \simeq 5 \times 10^{-2} \ll g_a, \]  
\[ (3.89) \]

and

\[ |\lambda_{u_1}| < |\lambda_{u_2}| \simeq 10^{-2} \ll g_a, \quad |\lambda_{u_3}| \simeq 1 \]  
\[ (3.90) \]

throughout the scaling range (3.20) for \( a = 1, \ldots, 4 \). We see from (3.90) that \( |\lambda_{u_3}| \) is of \( \mathcal{O}(1) \) and, hence, terms containing it cannot be dropped from the RGEs. As in the previous section, we will continue to assume that the initial gaugino masses \( M_a(0), a = 1, 2, 4 \) are chosen sufficiently small that the inequalities in (3.40) remain satisfied. However, we will not make a similar assumption about \( M_3(0) \), allowing it, for the time being, to be arbitrarily large and not sub-leading to \( S'_1 \). Since the initial scalar masses satisfy (3.41) and (3.42), the inequalities (3.44) and (3.48) are still satisfied. Finally, we will assume that terms with \( g_2^2S \) are small compared to any term proportional to \( |\lambda_{u_3}|^2 \). Using all these inputs and assumptions, it follows that one can approximate the squark mass RGEs as

\[ 16\pi^2 \frac{dm_{Q_3}^2}{dt} \simeq 2 (m_{Q_3}^2 + m_H^2 + m_{u_3}^2) |\lambda_{u_3}|^2 + 2 |\lambda_{u_3}|^2 |\tilde{A}_{u_3}|^2 + \frac{32}{3} g_3^2 |M_3|^2 + \frac{1}{4} g_4^2 S'_1, \]
\[ (3.91) \]

\[ 16\pi^2 \frac{dm_{u_3}^2}{dt} \simeq 4 (m_{Q_3}^2 + m_H^2 + m_{u_3}^2) |\lambda_{u_3}|^2 + 4 |\lambda_{u_3}|^2 |\tilde{A}_{u_3}|^2 - \frac{32}{3} g_3^2 |M_3|^2 - \frac{1}{4} g_4^2 S'_1. \]
\[ (3.92) \]
and

\[
16\pi^2 \frac{dm^2_{Q_{3,4}}}{dt} \simeq -\frac{32}{3} g_3^2 |M_3|^2 + \frac{1}{4} g_4^2 S'_1, \quad (3.93)
\]

\[
16\pi^2 \frac{dm^2_{u_{3,4}}}{dt} \simeq -\frac{32}{3} g_3^2 |M_3|^2 - \frac{1}{4} g_4^2 S'_1. \quad (3.94)
\]

Let us begin by analyzing equations (3.91) and (3.92). Note that (3.91) and (3.92) can be written as

\[
16\pi^2 \frac{dm^2_{Q_3}}{dt} \simeq 16\pi^2 \frac{d(\frac{1}{3}m^2_H)}{dt} - \frac{32}{3} g_3^2 |M_3|^2 + \frac{1}{4} g_4^2 S'_1 + C_{Q_3}, \quad (3.95)
\]

\[
16\pi^2 \frac{dm^2_{u_3}}{dt} \simeq 16\pi^2 \frac{d(\frac{2}{3}m^2_H)}{dt} - \frac{32}{3} g_3^2 |M_3|^2 - \frac{1}{4} g_4^2 S'_1 + C_{u_3} \quad (3.96)
\]

respectively. These are easily solved to give

\[
m^2_{Q_3} \simeq \frac{1}{3} m^2_H - \frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 + \frac{1}{64\pi^2} \int_0^t g_3^2 S'_1 + C_{Q_3}, \quad (3.97)
\]

\[
m^2_{u_3} \simeq \frac{2}{3} m^2_H - \frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 - \frac{1}{64\pi^2} \int_0^t g_3^2 S'_1 + C_{u_3} \quad (3.98)
\]

where

\[
-\frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 = -\frac{8}{3b_3} \left( \frac{1}{(1 - \frac{g(0)^2 b_3 t}{8\pi^2})^2} - 1 \right) |M_3(0)|^2, \quad (3.99)
\]

\[
-\frac{1}{64\pi^2} \int_0^t g_4^2 S'_1 = -\frac{1}{18} \left( \frac{1}{(1 - \frac{g(0)^2 b_4 t}{8\pi^2})^2} - 1 \right) S'_1(0) \quad (3.100)
\]

are evaluated using (3.19), (3.24) and (3.53). The integration constants are

\[
C_{Q_3} = m_{Q_3}(0)^2 - \frac{1}{3} m_H(0)^2, \quad C_{u_3} = m_{Q_3}(0)^2 - \frac{2}{3} m_H(0)^2 \quad (3.101)
\]

where we have used our assumption (3.41) that the initial squark masses are degenerate. Note that since \(b_3 = -3\) and \(t \leq 0\), expression (3.99) is always non-negative. Similarly, since \(b_4 = 12\), it follows that (3.100) is also non-negative.
Now consider the $m_H^2$ equation (3.86). If we insert (3.97) and (3.98) into (3.86), we get an expression for $m_H^2$ without the the squark mass squared terms. It is important to note that in doing so, we would gain a term in the beta function that depends on the gluino mass squared. If we then apply (3.40) and the discussion above (3.91) (where we took the gluino mass squared term to be of the same order as $S'_1$ in our approximation), we get

$$\frac{dm_H^2}{dt} \simeq \frac{3}{8\pi^2} |\lambda_{u_3}|^2 (2m_H^2 - \frac{4}{3\pi^2} \int_0^t g_3^2 |M_3|^2 + m_C^2 + |\tilde{A}_{u_3}|^2) \quad (3.102)$$

with $m_C^2$ defined by

$$m_C^2 = C_{Q_3} + C_{u_3} = 2m_{Q_3}(0)^2 - m_H(0)^2. \quad (3.103)$$

Henceforth, to simplify our discussion, we assume

$$m_{Q_3}(0)^2 = \frac{m_H(0)^2}{2}, \quad (3.104)$$

which sets

$$m_C^2 = 0. \quad (3.105)$$

Furthermore, we will choose the initial value $\tilde{A}_{u_3}(0)$ so that

$$|\tilde{A}_{u_3}|^2 \ll 2m_H^2 - \frac{4}{3\pi^2} \int_0^t g_3^2 |M_3|^2 \quad (3.106)$$

over the entire scaling range. As a final simplification, we note that $\lambda_{u_3}$ scales by only a few percent over the scaling range. Hence, we approximate it as a constant with its phenomenological value of

$$\lambda_{u_3} \simeq 1. \quad (3.107)$$
Using (3.105), (3.106) and (3.107), RGE equation (3.102) simplifies to

\[ \frac{dm_H^2}{dt} \simeq \frac{3}{8\pi^2} (2m_H^2 - \frac{4}{3\pi^2} \int_0^t g_3^2 |M_3|^2) \]. (3.108)

In this paper, we will solve equation (3.108) subject to the constraint that \( m_H^2 \), which is positive at \( t = 0 \), remain positive over the entire scaling range. Then, RGE (3.108) is equivalent to an integral equation for \( m_H^2 \) given by

\[ m_H^2 \simeq m_H(0)^2 e^{-\frac{3}{8\pi^2} \int_0^t (1 + |\frac{\frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2}{m_H^2}|} \). (3.109)\]

Note that (3.104) implies

\[ C_{Q3} = \frac{1}{6} m_H(0)^2, \quad C_{u3} = -\frac{1}{6} m_H(0)^2. \] (3.110)

It follows that the \( m_{Q3}^2 \) and \( m_{u3}^2 \) equations in (3.97) and (3.98) become

\[ m_{Q3}^2 \simeq \frac{1}{3} m_H^2 - \frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 + \frac{1}{64\pi^2} \int_0^t g_4^2 S_1' + \frac{1}{6} m_H(0)^2, \] (3.111)

\[ m_{u3}^2 \simeq \frac{2}{3} m_H^2 - \frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 - \frac{1}{64\pi^2} \int_0^t g_4^2 S_1' - \frac{1}{6} m_H(0)^2 \] (3.112)

respectively. Finally, let us consider RGEs (3.93) and (3.94). These are easily solved to give

\[ m_{Q_{1,2}}^2 \simeq -\frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 + \frac{1}{64\pi^2} \int_0^t g_4^2 S_1' + \frac{m_H(0)^2}{2}, \] (3.113)

\[ m_{u_{1,2}}^2 \simeq -\frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 - \frac{1}{64\pi^2} \int_0^t g_4^2 S_1' + \frac{m_H(0)^2}{2} \] (3.114)

where the \( g_3^2 |M_3|^2 \) and \( g_4^2 S_1' \) integrals are given in (3.99) and (3.100) respectively and we have used assumption (3.41) that the initial squark masses are degenerate.
“Down” Higgs and Squark Masses:

The RGEs for the “down” Higgs and squark mass parameters $m_R$ and $m_{d_i}$ are given by [86]

\[
16\pi^2 \frac{dm_R^2}{dt} = \sum_{i=1}^{3} (6(m_Q^2 + m^2_R + m_{d_i}^2)|\lambda_{d_i}|^2 + 2(m_L^2 + m_R^2 + m_{e_i}^2)|\lambda_{e_i}|^2 + 6|A_{d_i}|^2 + 2|A_{e_i}|^2 - \frac{6}{5} g_1^2 |M_1|^2 - 6g_2^2 |M_2|^2 \tag{3.115}
\]

\[
-\frac{3}{5} g_1^2 S,
\]

\[
16\pi^2 \frac{dm^2_{d_i}}{dt} = 4(m_Q^2 + m_R^2 + m_{d_i}^2)|\lambda_{d_i}|^2 + 4|A_{d_i}|^2
- \frac{8}{15} g_1^2 |M_1|^2 - \frac{32}{3} g_3^2 |M_3|^2 - \frac{1}{6} g_4^2 |M_4|^2
\]

\[
+ \frac{2}{5} g_1^2 S - \frac{1}{4} g_2^2 S' \tag{3.116}
\]

where $S$ and $S'$ are given in (3.32) and (3.33) respectively. First consider the $m_{d_i}$ equation. Using the assumptions listed in (3.34), (3.35), (3.40), (3.48) and (3.89), equation (3.116) can be approximated by

\[
16\pi^2 \frac{dm^2_{d_i}}{dt} \simeq - \frac{32}{3} g_3^2 |M_3|^2 - \frac{1}{4} g_4^2 S'_{1}. \tag{3.117}
\]

This can be immediately integrated to give

\[
m_{d_i}^2 \simeq - \frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 - \frac{1}{64\pi^2} \int_0^t g_3^2 S'_{1} + \frac{m_R(0)^2}{2}, \tag{3.118}
\]

where the first and second terms on the right hand side are evaluated in (3.99) and (3.100) respectively, and we have used assumptions (3.41) and (3.104). Note that these are both non-negative and, hence, $m^2_{d_i}, i = 1, 2, 3$ are all non-negative throughout the entire scaling range (3.20). Now consider the $m_R$ equation (3.115).
All of the terms on the right hand side of this expression are small compared to 
\( g_3^2 |M_3|^2 \) and \( g_4^2 S_1' \). In addition, these terms are multiplied by \(|\lambda_d|^2\), which is small. Hence, to the order we are working

\[
\frac{d m_H^2}{dt} \simeq 0 \quad (3.119)
\]

and

\[
m_H^2 \simeq m_H(0)^2, \quad (3.120)
\]

where we have used (3.41).

**The \( \mu \) Parameter:**

The \( \mu \) parameter enters the potential for the Higgs supermultiplets \( H, \bar{H} \) through the superpotential \( W \) in (3.6). The RGE is given by [86]

\[
16\pi^2 \frac{d\mu}{dt} = \mu \left( \sum_{i=1}^{3} (3|\lambda_{u_i}|^2 + 3|\lambda_{d_i}|^2 + |\lambda_{v_i}|^2 + |\lambda_{e_i}|^2) - \frac{3}{5} g_1^2 - 3g_2^2 \right). \quad (3.121)
\]

This is easily integrated to

\[
\mu = \mu(0) e^{-\frac{1}{16\pi^2} \int_t^0 \left( \sum_{i=1}^{3} (3|\lambda_{u_i}|^2 + 3|\lambda_{d_i}|^2 + |\lambda_{v_i}|^2 + |\lambda_{e_i}|^2) - \frac{3}{5} g_1^2 - 3g_2^2 \right)}. \quad (3.122)
\]

As discussed earlier, a constant field-independent \( \mu(0) \) parameter cannot arise in a supersymmetric string vacuum since the Higgs fields are zero modes. However, a \( \mu \)-term can arise from higher-dimensional couplings to moduli. Although typically chosen to be of electroweak order, in this paper, to emphasize the \( B-L/\text{electroweak} \) hierarchy and simplify the calculation, we will choose \( \mu(0) \) to be substantially

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smaller, making its effects sub-dominant. It is clear from (3.122) that \( \mu \) runs slowly over the scaling range and, thus, once chosen to be sub-dominant at the unification scale it remains so throughout the scaling range. The exact meaning of the term “sub-dominant” will be given in Appendix D.

The \( B \) Parameter:

The \( B \) parameter enters the potential for the Higgs scalars \( H, \bar{H} \) through the soft supersymmetry breaking term \( V_{2s} \) in (3.12). The RGE for \( B \) is given by [86]

\[
16\pi^2 \frac{dB}{dt} = B \left( \sum_{i=1}^{3} (3|\lambda_{ui}|^2 + 3|\lambda_{di}|^2 + |\lambda_{ui}|^2 + |\lambda_{vi}|^2) - \frac{3}{5} g_1^2 - 3g_2^2 \right) + \mu \left( \sum_{i=1}^{3} (6A_{ui}\lambda_{ui}^* + A_{di}\lambda_{di}^* + 2A_{ei}\lambda_{ei}^* + 2A_{vi}\lambda_{vi}^*) \right) + 6g_2^2 M_2 + \frac{6}{5} g_1^2 M_1 \right) .
\]

(3.123)

Recalling the discussion in the previous subsection, we take the term proportional to \( \mu \) to be sub-leading. Equation (3.123) is then solved by

\[
B = B(0)e^{-\frac{1}{16\pi^2} \int_0^t (\sum_{i=1}^{3} (3|\lambda_{ui}|^2 + 3|\lambda_{di}|^2 + |\lambda_{ui}|^2 + |\lambda_{vi}|^2) - \frac{3}{5} g_1^2 - 3g_2^2)} .
\]

(3.124)

Here, unlike the constant, field-independent \( \mu(0) \) parameter, the dimension two \( B(0) \) parameter arises from supersymmetry breaking and need not vanish. Using (3.36), (3.89) and (3.90), the RGE for \( B \) can be approximated by

\[
B \simeq B(0)e^{-\frac{3}{16\pi^2} \int_0^t |\lambda_{ui}|^2} .
\]

(3.125)
This parameter will make its appearance in the analysis of the Higgs potential below. The assumption that the term proportional to $\mu$ is sub-leading is easily checked using (3.106) and the relative sizes of $B$ and $\mu$ presented in Appendix D.

**Higgs and Squark Masses at the $B$-$L$ Breaking Vacuum:**

At the end of the previous section we showed that, when restricted to slepton scalars only, the potential energy has a local minimum given in (3.82) and (3.83). Clearly, however, to understand the stability of this minimum it is essential to extend this analysis to the entire field space; that is, to include all Higgs and squark scalars and as well as the sleptons. Given the RGE solutions in the previous subsections, one can now study the full scalar potential for any scale in the range (3.20).

Let us begin by considering the quadratic mass terms, not at the origin of field space but, rather, at the slepton minimum given in (3.82) and (3.83). The relevant part of the scalar potential is still (3.61). However, this is now evaluated for all scalar fields near the slepton VEVs. Note that, in principle, the $|\lambda_{\nu_3}|^2|H|^2|\nu_3|^2$ term in the $|\partial L_3 W|^2$ contribution to the potential, where $W$ is given in (3.6), could play a role in selecting the vacuum. However, since $\lambda_{\nu_3} \ll g_4$, this term can safely be ignored. Similarly, under the assumptions (3.34) and (3.35), one can neglect the $A_{\nu_3} L_3 H \nu_3 + h.c$ term in $V_{3s}$. Expanding out $V$ using the $B$-$L$ quantum numbers listed in (3.4), the quadratic terms at any scale $t$ are

$$V = \cdots + V_{m_{\text{slepton}}}^2 + V_{m_{\text{Higgs}}}^2 + V_{m_{\text{squarks}}}^2 + \cdots,$$

(3.126)
where $V_{m_{\text{slepton}}}$ is given in (3.79), (3.80). The Higgs contribution to the quadratic potential is

$$V_{m_{\text{Higgs}}} = m_H^2 |H|^2 + m_{\tilde{H}}^2 |\tilde{H}|^2 - B(H\tilde{H} + h.c) \quad (3.127)$$

where $m_H^2$, $m_{\tilde{H}}^2$ and $B$ are given in (3.109), (3.120) and (3.125) respectively. Note from (3.109), (3.120) that for $t \ll 0$ the quantity $|m_{\tilde{H}}^2 - m_H^2|$ becomes non-zero and large. Henceforth, we will assume that in this range of $t$ the coefficient $B$ is such that

$$4 \left( \frac{B}{m_{\tilde{H}}^2 - m_H^2} \right)^2 \ll 1 \quad (3.128)$$

This is easily arranged by adjusting $B(0)$. For $t \ll 0$, the Higgs mass matrix (3.127) can be diagonalized to

$$V_{m_{\text{Higgs}}} = m_{H'}^2 |H'|^2 + m_{\tilde{H}}^2 |\tilde{H}'|^2 \quad (3.129)$$

where

$$m_{H'}^2 \simeq m_H^2 - m_{\tilde{H}}^2 \left( \frac{B}{m_{\tilde{H}}^2 - m_H^2} \right)^2, \quad m_{\tilde{H}}^2 \simeq m_H^2 - m_{\tilde{H}}^2 \left( \frac{B}{m_{\tilde{H}}^2 - m_H^2} \right)^2 \quad (3.130)$$

and

$$H' \simeq H + \left( \frac{-B}{m_{\tilde{H}}^2 - m_H^2} \right) \tilde{H}^*, \quad \tilde{H}' \simeq \left( \frac{B}{m_{\tilde{H}}^2 - m_H^2} \right) H^* + \tilde{H} \quad (3.131)$$

Here, $H^{*A} = g^{AC} \epsilon_{CD} \tilde{H}^{*B}$ and $H^{*A} = g^{AC} \epsilon_{CD} H^{*B}$ have the same $SU(2)_L \times U(1)_Y$ transformations as $H$ and $\tilde{H}$ respectively. In component fields

$$\tilde{H}^* = (\tilde{H}^{*-}, -\tilde{H}^{0*}), \quad H^* = (H^{0*}, -H^{*+}) \quad (3.132)$$
It follows from (3.109), (3.120), (3.128) and (3.130) that anywhere in the range \( t \ll 0 \), and specifically at \( t_{B-L} \),

\[
m_{H}^{2} \simeq m_{H}^{2} = m_{H}(0)^{2} > 0 . \tag{3.133}
\]

Importantly, however, we see from (3.109), (3.120) that as \( t \) becomes more negative \( m_{H}^{2} \) can approach, become equal to and finally become smaller than \( m_{H}^{2} \left( B/(m_{H}^{2} - m_{H}^{2}) \right)^{2} \). This corresponds to \( m_{H}^{2} \) being positive, zero and negative respectively. The vanishing of \( m_{H}^{2} \) signals the onset of electroweak symmetry breaking. Clearly, to evaluate \( m_{H}^{2} \) at \( t_{B-L} \) we must solve equation (3.108) or, equivalently, (3.109) for the running of \( m_{H}^{2} \). The solution of this equation will depend on two arbitrary parameters, \( m_{H}(0)^{2} \) and, using (3.24), \( M_{3}(0) \). It follows from the exponential form of (3.109) and the fact that integral (3.99) is non-negative that increasing \( |M_{3}(0)| \) decreases \( m_{H}(t)^{2} \) for any fixed value of \( t \), and vice versa. Hence, specifying \( |M_{3}(0)| \) is equivalent to specifying the value of \( m_{H}(t)^{2} \) at some fixed \( t \). For reasons discussed in detail below, the physics is most transparent if we choose this to be the electroweak scale corresponding to \( t_{EW} \simeq -33.3 \). Specifically, we take

\[
m_{H}(t_{EW})^{2} = \frac{m_{H}(0)^{2}}{T'^{2}} , \tag{3.134}
\]

where

\[
T'^{2} = \frac{T^{2}}{1 - \Delta^{2}} . \tag{3.135}
\]

\( T \) is defined by

\[
T^{2} \equiv \left( \frac{B}{m_{H}^{2} - m_{H}^{2}} \right)^{-2} \gtrsim 40 , \tag{3.136}
\]
thus explicitly satisfying constraint (3.128). \( \Delta \) is a parameter with the range \( 0 < \Delta^2 < 1 \). The upper bound follows from our requirement that \( m_H^2 \) be positive over the entire physical scaling range and, hence, positive at \( t_{EW} \). Note from (3.130), (3.133), (3.134), (3.135) and (3.136) that

\[
m_H(t_{EW})^2 = \frac{-\Delta^2}{T^2} m_H(0)^2 .
\]

(3.137)

It follows that electroweak symmetry will be broken at \( t_{EW} \) only if \( \Delta^2 \) is strictly positive. Hence, the lower bound on this parameter.

The role of (3.134) in electroweak breaking will be thoroughly discussed in the next subsections. Its relationship to \( |M_3(0)| \) and, hence, to the solution of (3.108) for \( m_H(t)^2 \) will be derived in Appendix B. Here, we will simply state the result that specifying this value at \( t_{EW} \) is equivalent to choosing

\[
|M_3(0)|^2 = 0.0352(1 - \frac{11.5}{T'^2})m_H(0)^2 .
\]

(3.138)

Using this expression for \( |M_3(0)| \), we can solve (3.108) numerically for any fixed value of \( T' \). The numerical results can then be fit to a smooth curve. We find that the numerical data is well-represented over the entire scaling regime by

\[
m_H(t)^2 = \left(1 - \frac{1}{T'^2}\right)\left(\frac{t_{EW} - b}{t_{EW}}\right)\left(\frac{t}{t - b}\right)m_H(0)^2 ,
\]

(3.139)

where \( b \) is a function of \( T' \) of the form

\[
b(T') = 19.9\left(1 - \frac{0.186}{T' - 3.69}\right).
\]

(3.140)

Note that at \( t = 0 \) and \( t = t_{EW} \), \( m_H^2 \) is given by \( m_H(0)^2 \) and \( m_H(0)^2/T'^2 \) respectively, as it must be. One can now use expression (3.139) to evaluate \( m_H^2 \) and, hence,
$m^2_H$ at $t_{B-L}$ for any $T$ and parameter $\Delta^2$. It suffices here to give the lower bound. To do this, first note that for $\Delta^2 = 1$ the $T$ dependence drops out of (3.139) and (3.140). Furthermore, it follows from (3.137) that this corresponds to the largest negative $H^T$ squared mass at $t_{EW}$ and, hence, a $T$-dependent lower bound on $m^2_{H'}$ at $t_{B-L}$. Taking $\Delta^2 = 1$, we find

$$m_H(t_{B-L})^2 = 0.0565 \, m_H(0)^2$$

(3.141)

and, hence,

$$m_{H'}(t_{B-L})^2 = 0.0565(1 - \frac{17.7}{T^2}) \, m_H(0)^2 .$$

(3.142)

Since, from (3.136), $T^2 \gtrsim 40$, this expression is always positive. We conclude that at $t_{B-L}$, for any $T$ and $0 < \Delta^2 < 1$,

$$m^2_{H'} > 0.0565(1 - \frac{17.7}{T^2}) \, m_H(0)^2 > 0 .$$

(3.143)

Finally, note that evaluated at the slepton vacuum (3.82), (3.83) the diagonalized quadratic Higgs potential remains

$$V_{m_{H'}} = \langle m^2_{H'} \rangle |H'|^2 + \langle m^2_{H'} \rangle |\bar{H}'|^2 ,$$

(3.144)

with

$$\langle m^2_{H'} \rangle = m^2_{H'}, \quad \langle m^2_{\bar{H}'} \rangle = m^2_{\bar{H}'} .$$

(3.145)

It then follows from (3.143) and (3.133) respectively that at $t_{B-L}$

$$\langle m^2_{H'} \rangle > 0.0565(1 - \frac{17.7}{T^2}) \, m_H(0)^2, \quad \langle m^2_{\bar{H}'} \rangle = m_H(0)^2$$

(3.146)
and that they are both positive.

The squark contribution to the quadratic potential is

\[ V_{m_{\text{squark}}^2} = \sum_{i=1}^{3} (\langle m_{Q_i}^2 \rangle |Q_i|^2 + \langle m_{u_i}^2 \rangle |u_i|^2 + \langle m_{d_i}^2 \rangle |d_i|^2) , \]  

(3.147)

with

\[ \langle m_{Q_i}^2 \rangle = m_{Q_i}^2 + \frac{1}{4} g_4^2 \langle \nu_3 \rangle^2 \]  

(3.148)

and

\[ \langle m_{u_i}^2 \rangle = m_{u_i}^2 - \frac{1}{4} g_4^2 \langle \nu_3 \rangle^2 , \]

\[ \langle m_{d_i}^2 \rangle = m_{d_i}^2 - \frac{1}{4} g_4^2 \langle \nu_3 \rangle^2 . \]  

(3.149)

Here \( m_{Q_i}^2 \) are given in (3.111), (3.113), \( m_{u_i}^2 \) are given in (3.112), (3.114) and \( m_{d_i}^2 \) are given in (3.118).

Using this result, one can now evaluate the squark masses in (3.148) and (3.149). Let us begin by computing the second term on the right hand side of these equations.

Using (3.16), (3.21), (3.76) and (3.78), we find that at \( t_{B-L} \)

\[ \frac{1}{4} g_4^2 \langle \nu_3 \rangle^2 = \frac{4}{3} m_{\nu}(0)^2 . \]  

(3.150)

First consider \( \langle m_{Q_{1,2}}^2 \rangle \). Recall from (3.113) that

\[ m_{Q_{1,2}}^2 \approx -\frac{2}{3\pi^2} \int_0^t g_5^2 |M_3|^2 + \frac{1}{64\pi^2} \int_0^t g_4^2 S_1 + \frac{m_H(0)^2}{2} . \]  

(3.151)

For any value of \( t \), the first term can be evaluated using (3.99), (3.138) and the
second with (3.76), (3.100). We find that at $t_{B-L}$

$$-\frac{2}{3\pi^2}\int_0^{t_{B-L}} g_\beta^2|\mathcal{M}_3|^2 = .143\left(1 - \frac{11.5}{T^2}\right)m_H(0)^2,$$

$$\frac{1}{64\pi^2}\int_0^{t_{B-L}} g_4^2 \mathcal{S}_1' = -1.66\ m_\nu(0)^2$$

and, hence,

$$m_{Q_{1,2}}^2 \simeq .643\left(1 - \frac{2.57}{T^2}\right)m_H(0)^2 - 1.66\ m_\nu(0)^2 .$$

Using this and (3.150) in (3.148) gives

$$\langle m_{Q_{1,2}}^2 \rangle = .643\left(1 - \frac{2.57}{T^2}\right)m_H(0)^2 - 336\ m_\nu(0)^2 .$$

To further evaluate this squared mass and to determine whether or not it is positive, one must give an explicit relationship between the parameters $m_\nu(0)^2$ and $m_H(0)^2$. This will be discussed in detail in Appendix C. Here, we will simply use the result.

By demanding that all squark mass squares remain minimally positive at any scale $t_{EW} \leq t \leq 0$, we find that

$$m_\nu(0)^2 = .864\left(1 - \frac{2.25}{T^2}\right)m_H(0)^2 .$$

Using this to eliminate the $m_H(0)^2$ parameter, expression (3.155) becomes

$$\langle m_{Q_{1,2}}^2 \rangle = .408\left(1 - \frac{.583}{T^2}\right)m_\nu(0)^2 .$$

Note that this expression is positive for all parameters $T^2 \gtrsim 40$ and $0 < \Delta^2 < 1$.

Also, for sake of comparison to the sneutrino VEV, we will write all masses at $t_{B-L}$ in terms of $m_\nu(0)^2$. The form of $m_{u_{1,2}}^2$ and $m_{d_i}^2$ for $i = 1, 2, 3$ given in (3.114) and
These mass squares are positive for all \( T^2 \gtrsim 40 \) and \( 0 < \Delta^2 < 1 \).

Now consider \( \langle m^2_{Q_3} \rangle \). Recall from (3.111) that
\[
m^2_{Q_3} \simeq \frac{1}{3} m^2_H - \frac{2}{3\pi^2} \int_0^t g^2_3 |M_3|^2 + \frac{1}{64\pi^2} \int_0^t g^2_3 S'_1 + \frac{1}{6} m_H(0)^2 .
\] (3.159)
The second and third terms are given in (3.152) and (3.153) respectively. Unlike the masses evaluated above, however, \( m^2_{Q_3} \) also depends on the mass parameter \( m^2_H \).

This can be evaluated using (3.139), (3.140) for any value of \( t \). At \( t_{B-L} \) we find
\[
m^2_H \simeq 0.0565(1 - \frac{0.101}{T'} + 16.3\frac{T'}{T'^2}) m_H(0)^2 .
\] (3.160)
Using this together with (3.152) and (3.153), \( m^2_{Q_3} \) at \( t = t_{B-L} \) is given by
\[
m^2_{Q_3} \simeq -1.66 m_\nu(0)^2 + 0.328(1 - \frac{0.00629}{T'} - 4.09\frac{T'}{T'^2}) m_H(0)^2 ,
\] (3.161)
which, using (3.156), becomes
\[
m^2_{Q_3} \simeq -1.29(1 + \frac{0.00185}{T'} + 0.545\frac{T'}{T'^2}) m_\nu(0)^2 .
\] (3.162)

Similarly, \( m^2_{u_3} \) given by (3.112) is found to be
\[
m^2_{u_3} \simeq 1.68(1 - \frac{0.00284}{T'} - 0.691\frac{T'}{T'^2}) m_\nu(0)^2
\] (3.163)
at \( t = t_{B-L} \). Using these results and (3.150), we find from (3.148),(3.149) that
\[
\langle m^2_{Q_3} \rangle \simeq 0.0435(1 - \frac{0.0549}{T'} - 16.2\frac{T'}{T'^2}) m_\nu(0)^2 ,
\]
\[
\langle m^2_{u_3} \rangle \simeq 0.353(1 - \frac{0.0136}{T'} - 3.30\frac{T'}{T'^2}) m_\nu(0)^2 .
\] (3.164)
Note that they are both positive for all $T^2 \gtrsim 40$ and $0 < \Delta^2 < 1$. Putting everything together, the squark masses, expressed in terms of $m_{\nu}(0)^2$, are given in (3.157), (3.158) and (3.164) and are all positive.

From (3.81), (3.133), (3.143), (3.157), (3.158) and (3.164) we conclude that at $t = t_{B-L}$ the vacuum specified by

$$\langle \nu_{1,2} \rangle = 0, \quad \langle \nu_3 \rangle = \frac{2 m_{\nu}(0)}{\sqrt{\frac{3}{4}} g_1}, \quad \langle e_i \rangle = \langle L_i \rangle = 0$$

(3.165)

$$\langle Q_i \rangle = \langle u_i \rangle = \langle d_i \rangle = 0$$

(3.166)

and

$$\langle H' \rangle = \langle \bar{H}' \rangle = 0$$

(3.167)

is a local minimum of the potential energy. It has positive mass squares in every field direction including $H'$ and $\bar{H}'$, signaling that, at $t = t_{B-L}$, electroweak symmetry has not yet been spontaneously broken.

**Electroweak Symmetry Breaking:**

To explore the breaking of electroweak symmetry, one must add to $V_{m_{\text{Higgs}}^2}$ in (3.127) all other relevant interactions involving the Higgs fields. It follows from previous discussions that the relevant part of the Higgs potential is

$$V = V_{m_{\text{Higgs}}^2} + \frac{1}{2} D_Y^2 + \frac{1}{2} \sum_{a=1}^{3} D_{SU(2)L,a}^2$$

(3.168)

where, written in the mass diagonal fields $H'$ and $\bar{H}'$ defined in (3.131),

$$D_Y = \sqrt{\frac{3}{5}} \frac{g_1}{2} (|H'|^2 - |\bar{H}'|^2) ,$$

(3.169)
\[ D_{SU(2)_L} = \frac{g^2}{2}(H'^\dagger \sigma_a H' + \bar{H}'\sigma_a \bar{H}') \] (3.170)

and we have dropped terms of \( O(T^{-1}) \). The parameters in this potential should now be evaluated, not at \( t_{B-L} \), but, rather, at the electroweak scale \( \mu_{EW} \simeq 10^2 \text{GeV} \) which corresponds to a \( t_{EW} \simeq -33.3 \). The gauge couplings are easily evaluated there using (3.18) and (3.19) and found to be

\[
\frac{g_1^2(t_{EW})}{g(0)^2} = .405, \quad \frac{g_2^2(t_{EW})}{g(0)^2} = .818 .
\] (3.171)

Also, it follows from (3.120) and (3.137) that

\[
m_{H'}(t_{EW})^2 \simeq m_H(0)^2, \quad m_{H'}(t_{EW})^2 = \frac{-\Delta^2}{T^2} m_H(0)^2
\] (3.172)

where \( T^2 \gtrsim 40 \) and \( 0 < \Delta^2 < 1 \).

We can now proceed to minimize potential (3.168). Writing

\[ H' = (H'^+, H'^0), \quad \bar{H}' = (\bar{H}'^0, \bar{H}'^-) \],

potential (3.168) becomes

\[
V = -\frac{\Delta^2}{T^2} m_H(0)^2 (|H'^0|^2 + |H'^+|^2) + m_H(0)^2 (|\bar{H}'^0|^2 + |\bar{H}'^-|^2) \\
+ \frac{1}{8} \left( \frac{3}{5} \right) g_1^2 (|H'^0|^2 + |H'^+|^2 - |\bar{H}'^0|^2 - |\bar{H}'^-|^2)^2 \\
+ \frac{1}{8} g_2^2 (|H'^0|^2 + |H'^+|^2 - |\bar{H}'^0|^2 - |\bar{H}'^-|^2)^2 \\
+ 4 |H'^+ \bar{H}'^0 + H'^0 \bar{H}'^-|^2 ) .
\] (3.174)

This is easily minimized to give

\[
\langle \langle H'^0 \rangle \rangle = \frac{2\Delta m_H(0)}{T \sqrt{\frac{3}{5} g_1(t_{EW})^2 + g_2(t_{EW})^2}}, \quad \langle \langle H'^+ \rangle \rangle = 0 ,
\] (3.175)

77
\[ \langle \langle \tilde{H}^0 \rangle \rangle = \langle \langle \tilde{H}'^- \rangle \rangle = 0, \]  
\[ \langle \langle \bar{H}^0 \rangle \rangle = \langle \langle \bar{H}'^- \rangle \rangle = 0, \]  
(3.176)

where the double bracket \( \langle \langle \rangle \rangle \) indicates the vacuum at \( t_{EW} \). It is straightforward to compute the squared masses of the radial fluctuation \( \delta H^0 \) and complex \( \bar{H}^0, \bar{H}'^- \) fields at this vacuum. We find that

\[
V = \langle \langle m_{H^0}^2 \rangle \rangle |\delta H^0|^2 + \langle \langle m_{\bar{H}^0}^2 \rangle \rangle |\bar{H}^0|^2 + \langle \langle m_{\bar{H}'^-}^2 \rangle \rangle |\bar{H}'^-|^2 + \ldots
\]
(3.177)

where

\[
\langle \langle m_{H^0}^2 \rangle \rangle = 4 \Delta^2 m_H(0)^2
\]
(3.178)

and

\[
\langle \langle m_{\bar{H}^0}^2 \rangle \rangle = (1 - \frac{\Delta^2}{T^2}) m_H(0)^2, \quad \langle \langle m_{\bar{H}'^-}^2 \rangle \rangle = (1 - \frac{\Delta^2}{T^2} \frac{g_1^2 - g_2^2}{g_1^2 + g_2^2}) m_H(0)^2.
\]
(3.179)

Note that the Higgs mass squares are positive for all \( T^2 \equiv 40 \) and \( 0 < \Delta^2 < 1 \).

Evaluated at this minimum, the phase of \( H^0 \) and \( H'^-, \bar{H}'^-\) are Goldstone bosons which are “eaten” by the Higgs mechanism to give mass to the \( Z \) and \( W^\pm \) vector bosons. For example, the \( Z \) mass is given by

\[
M_Z = \frac{\sqrt{2} \Delta m_H(0)}{T} \simeq 91 GeV.
\]
(3.180)

Although the mass eigenstate basis \( H', \bar{H}' \) is the most natural for analyzing this vacuum, it is of some interest to express it in terms of the original \( H \) and \( \bar{H}' \) fields.

Using (3.5), (3.131) and (3.136), we find

\[
\langle \langle H^+ \rangle \rangle = \langle \langle \bar{H}^- \rangle \rangle = 0
\]
(3.181)
and, to leading order, that

\[ \langle \langle H^0 \rangle \rangle = \frac{2\Delta m_H(0)}{T \sqrt{\frac{2}{3} g_1(t_{EW})^2 + g_2(t_{EW})^2}}, \quad \langle \langle \bar{H}^0 \rangle \rangle = \frac{1}{T \langle \langle H^0 \rangle \rangle}. \]  

(3.182)

Note that the condition \( \langle \langle \bar{H}^0 \rangle \rangle = 0 \) in (3.176) does not imply the vanishing of \( \langle \langle H^0 \rangle \rangle \). Rather, \( \langle \langle \bar{H}^0 \rangle \rangle \) is non-zero and related to \( \langle \langle H^0 \rangle \rangle \) through the ratio

\[ \frac{\langle \langle H^0 \rangle \rangle}{\langle \langle \bar{H}^0 \rangle \rangle} \equiv \tan \beta = T + \mathcal{O}(T^{-1}). \]

(3.183)

We have indicated the \( \mathcal{O}(T^{-1}) \) contribution to emphasize that although \( \tan \beta = T \) to leading order, this relationship breaks down at higher order in \( T^{-1} \). We refer the reader to Appendix D for a more detailed discussion of this point.

We conclude that electroweak symmetry is spontaneously broken at scale \( t_{EW} \) by the non-vanishing \( H^0 \) vacuum expectation value in (3.175). This vacuum has a non-vanishing value of \( \tan \beta \) which, using the assumption for \( T^2 \) given in (3.136), satisfies

\[ \tan \beta \approx 6.32. \]  

(3.184)

As far as the Higgs fields are concerned, the vacuum specified in (3.175) and (3.176) is a stable local minimum. As a check on our result, choose \( \mu^2 \) of order \( \mathcal{O}(T^{-4}) \) or smaller, that is, non-vanishing but sub-dominant in all equations. Then (3.133),(3.134),(3.135) and (3.136) satisfy the constraint equations, given, for example, in [87], for the Higgs potential to be bounded below and have a negative squared mass at the origin. Furthermore, to the order in \( T^{-1} \) we are working, (3.180) and (3.182) for the Higgs vacuum satisfy the minimization conditions in [87]. This is
shown in detail in Appendix C.

**Slepton and Squark Masses at the EW Breaking Vacuum:**

Clearly, to understand the complete stability of this minimum, it is essential to extend this analysis to the entire field space; that is, to include all slepton and squark scalars as well as the Higgs fields. The relevant part of the potential energy is

\[
V = V_{2s} + \frac{1}{2} D_{B-L}^2 + \frac{1}{2} D_Y^2 + \frac{1}{2} \sum_{a=1}^{3} D_{SU(2)_{L_a}}^2 ,
\]

(3.185)

where \(V_{2s}, D_{B-L}, D_Y\) are given in (3.12), (3.10), (3.9) respectively and \(D_{SU(2)_{L_a}}\) is the extension of (3.170) to include all slepton and squark doublets. Note that, in principle, the superpotential (3.6) can also contribute to the squark/slepton potential energy. However, it follows from (3.36) and (3.89),(3.90) that such terms will be negligibly small compared to those in (3.185) with the notable exception of the third family up-squarks. We will take this contribution into account when it arises below. Now expand \(V\) to quadratic order in the fields using the quantum numbers listed in (3.2), (3.3) and (3.4). We find that, evaluated at the slepton and Higgs VEVs in (3.82), (3.83) and (3.175), (3.176) respectively,

\[
V = \cdots + V_{m_{Higgs}^2} + V_{m_{sleptons}^2} + V_{m_{squarks}^2} + \cdots ,
\]

(3.186)

where \(V_{m_{Higgs}^2}\) is given in (3.177), (3.178) and (3.179).
The slepton contribution to the quadratic potential is

\[
V_{m_{\text{sleptons}}}^2 = \langle (m_{\nu_3}^2) | \delta \nu_3 |^2 + \langle (m_{\nu_1,2}^2) | \nu_1,2 |^2 + \sum_{i=1}^3 \langle (m_{\nu_i}^2) | e_i |^2 \\
+ \langle (m_{N_i}^2) | N_i |^2 + \langle (m_{E_i}^2) | E_i |^2 \rangle + \ldots ,
\]

(3.187)

where

\[
\langle (m_{\nu_3}^2) \rangle = \langle m_{\nu_3}^2 \rangle, \quad \langle (m_{\nu_1,2}^2) \rangle = \langle m_{\nu_1,2}^2 \rangle, \\
\langle (m_{e_i}^2) \rangle = \langle m_{e_i}^2 \rangle + \frac{3}{2} \left( \frac{3}{5} g_1^2 \langle (H^0) \rangle^2 \right), \\
\langle (m_{N_i}^2) \rangle = \langle m_{L_i}^2 \rangle - \frac{1}{4} \left( \frac{3}{5} g_1^2 + g_2^2 \right) \langle (H^0) \rangle^2, \\
\langle (m_{E_i}^2) \rangle = \langle m_{L_i}^2 \rangle - \frac{1}{4} \left( \frac{3}{5} g_1^2 - g_2^2 \right) \langle (H^0) \rangle^2.
\]

(3.188)

The squared masses \( \langle m_{\nu_3}^2 \rangle \), \( \langle m_{\nu_1,2}^2 \rangle \), \( \langle m_{e_i}^2 \rangle \) and \( \langle m_{N_i}^2 \rangle \) were defined in (3.80) and evaluated at \( t_{B-L} \) in (3.81). Now, however, these values must be corrected by scaling down to \( t_{EW} \). Using (3.16) and (3.18), the slepton masses defined in (3.55)-(3.57) can be evaluated at \( t_{EW} \). Note that the parameters \( A, C \) and, hence, \( S'_1(0) \) given in (3.71),(3.73) and (3.76) remain the same. We find that

\[
m_{\nu_3}^2 = -4.38 m_\nu(0)^2, \quad m_{\nu_1,2}^2 = 77.8 m_\nu(0)^2, \\
m_{e_i}^2 = 0.625 m_\nu(0)^2, \quad m_{L_i} = 11.4 m_\nu(0)^2.
\]

(3.189)

Using these values, slepton potential (3.77) has a local minimum at

\[
\langle \langle \nu_3 \rangle \rangle = (1.05) \frac{2 m_\nu(0)}{\sqrt{\frac{3}{2} g_4(t_{EW})}}, \quad \langle \langle \nu_{1,2} \rangle \rangle = 0 \\
\langle \langle e_i \rangle \rangle = \langle \langle L_i \rangle \rangle = 0 \quad i = 1, 2, 3
\]

(3.190)

(3.191)
We can now evaluate the squared slepton masses in (3.80) at $t_{EW}$. We find

$$
\langle m_{\nu_3}^2 \rangle = 8.75 m_\nu(0)^2; \quad \langle m_{\nu_{1,2}}^2 \rangle = 82.2 m_\nu(0)^2; \\
\langle m_{\tau}^2 \rangle = 5 m_\nu(0)^2; \quad \langle m_{e_i}^2 \rangle = 7 m_\nu(0)^2. \quad (3.192)
$$

Inserting these results into (3.188) and using (3.175) gives

$$
\langle \langle m_{\nu_3}^2 \rangle \rangle = 8.75 m_\nu(0)^2; \quad \langle \langle m_{\nu_{1,2}}^2 \rangle \rangle = 82.2 m_\nu(0)^2; \\
\langle \langle m_{\tau}^2 \rangle \rangle = 5 m_\nu(0)^2 + 2 \left( \frac{3}{5} g_1^2 + g_2^2 \right) \frac{\Delta^2 m_H(0)^2}{T^2}. \quad (3.193) \\
\langle \langle m_{N_i}^2 \rangle \rangle = 7 m_\nu(0)^2 - \frac{\Delta^2 m_H(0)^2}{T^2}, \\
\langle \langle m_{E_i}^2 \rangle \rangle = 7 m_\nu(0)^2 + \left( \frac{-3}{5} g_1^2 + g_2^2 \right) \frac{\Delta^2 m_H(0)^2}{T^2}. \quad (3.194)
$$

Finally, using expression (3.156) which relates $m_\nu(0)^2$ to $m_H(0)^2$ and (3.171), these squared masses become

$$
\langle \langle m_{\nu_3}^2 \rangle \rangle = 8.75 m_\nu(0)^2; \quad \langle \langle m_{\nu_{1,2}}^2 \rangle \rangle = 82.2 m_\nu(0)^2; \\
\langle \langle m_{\tau}^2 \rangle \rangle = 5 m_\nu(0)^2 (1 + \Delta^2 \frac{0.0791}{T^2}) \simeq 5 m_\nu(0)^2, \quad (3.194) \\
\langle \langle m_{N_i}^2 \rangle \rangle = 7 m_\nu(0)^2 (1 - \Delta^2 \frac{0.123}{T^2}) \simeq 7 m_\nu(0)^2, \\
\langle \langle m_{E_i}^2 \rangle \rangle = 7 m_\nu(0)^2 (1 + \Delta^2 \frac{0.0669}{T^2}) \simeq 7 m_\nu(0)^2. 
$$

Note that the slepton squared masses are positive for all $T^2 \gtrsim 40$ and $0 < \Delta^2 < 1$.

Similarly, the squark contribution to the quadratic potential is

$$
V_{m_{squark}} = \sum_{i=1}^{3} \left( \langle \langle m_{U_i}^2 \rangle \rangle |U_i|^2 + \langle \langle m_{D_i}^2 \rangle \rangle |D_i|^2 + \langle \langle m_{u_i}^2 \rangle \rangle |u_i|^2 + \langle \langle m_{d_i}^2 \rangle \rangle |d_i|^2 \right) \quad (3.195)
$$
with
\[
\langle\langle m^2_{U_3} \rangle\rangle = \langle m^2_{Q_i} \rangle + \left(\frac{\frac{1}{2} g_1^2 - g_2^2}{4} + |\lambda_{u_1}|^2 \delta_{i3}\right) \langle\langle H^0 \rangle\rangle^2,
\]
\[
\langle\langle m^2_{D_3} \rangle\rangle = \langle m^2_{Q_i} \rangle + \left(\frac{\frac{1}{2} g_1^2 + g_2^2}{4}\right) \langle\langle H^0 \rangle\rangle^2 \tag{3.196}
\]
and
\[
\langle\langle m^2_{U_i} \rangle\rangle = \langle m^2_{U_i} \rangle - \left(\frac{1}{5} g_1^2 - |\lambda_{u_1}|^2 \delta_{i3}\right) \langle\langle H^0 \rangle\rangle^2,
\]
\[
\langle\langle m^2_{d_i} \rangle\rangle = \langle m^2_{d_i} \rangle + \frac{1}{10} g_1^2 \langle\langle H^0 \rangle\rangle^2. \tag{3.197}
\]

Note that in the expressions for \(\langle\langle m^2_{U_3} \rangle\rangle\) and \(\langle\langle m^2_{u_3} \rangle\rangle\) we have included the non-negligible superpotential contribution. The squared masses \(\langle m^2_{Q_i} \rangle\), \(\langle m^2_{u_i} \rangle\) and \(\langle m^2_{d_i} \rangle\) were defined in (3.148), (3.149) and evaluated at \(t_{B-L}\) in (3.164). Now, however, these values must be corrected by scaling down to \(t_{EW}\). First, we must compute \(g_4\) at \(t_{EW}\). Using (3.16) and (3.19) we find
\[
\frac{g_4^2(t_{EW})}{g(0)^2} = .272. \tag{3.198}
\]
One can now evaluate the second term on the right hand side of (3.148) and (3.149) using (3.190). The result is
\[
\frac{1}{4} g_4^2 \langle \nu_3 \rangle = 1.46 m_{\nu}(0)^2. \tag{3.199}
\]
The squark masses defined in (3.111),(3.112),(3.113),(3.114) and (3.118) can be evaluated at \(t_{EW}\) using (3.95),(3.96),(3.93),(3.94),(3.102), (3.118) and (3.156). Adding
these to (3.199), expressions (3.148), (3.149) become

\[
\begin{align*}
\langle m_{Q_3}^2 \rangle &= 0.132 \left(1 - \frac{14.7}{T^2}\right) m_H(0)^2, \\
\langle m_{Q_{1,2}}^2 \rangle &= 0.465 \left(1 - \frac{4.87}{T^2}\right) m_H(0)^2, \\
\langle m_{u_3}^2 \rangle &= 0.374 \left(1 - \frac{7.72}{T^2}\right) m_H(0)^2, \\
\langle m_{u_{1,2}}^2 \rangle &= 1.041 \left(1 - \frac{3.42}{T^2}\right) m_H(0)^2, \\
\langle m_{d_i}^2 \rangle &= 1.041 \left(1 - \frac{3.42}{T^2}\right) m_H(0)^2.
\end{align*}
\]

(3.200)

Inserting (3.200) and (3.175) into (3.196) and (3.197), we find

\[
\begin{align*}
\langle\langle m_{U_3}^2 \rangle \rangle &= 0.132 \left(1 - \frac{14.7}{T^2}\right) m_H(0)^2 + \left(\frac{\frac{1}{3} g_1^2 - g_2^2 + 4|\lambda_{us}|^2}{\frac{3}{5} g_1^2 + g_2^2}\right) \frac{\Delta^2 m_H(0)^2}{T^2}, \\
\langle\langle m_{D_3}^2 \rangle \rangle &= 0.132 \left(1 - \frac{14.7}{T^2}\right) m_H(0)^2 + \left(\frac{\frac{1}{3} g_1^2 + g_2^2}{\frac{3}{5} g_1^2 + g_2^2}\right) \frac{\Delta^2 m_H(0)^2}{T^2}, \\
\langle\langle m_{U_{1,2}}^2 \rangle \rangle &= 0.465 \left(1 - \frac{4.87}{T^2}\right) m_H(0)^2 + \left(\frac{\frac{1}{3} g_1^2 - g_2^2}{\frac{3}{5} g_1^2 + g_2^2}\right) \frac{\Delta^2 m_H(0)^2}{T^2}, \\
\langle\langle m_{D_{1,2}}^2 \rangle \rangle &= 0.465 \left(1 - \frac{4.87}{T^2}\right) m_H(0)^2 + \left(\frac{\frac{1}{3} g_1^2 + g_2^2}{\frac{3}{5} g_1^2 + g_2^2}\right) \frac{\Delta^2 m_H(0)^2}{T^2}, \\
\langle\langle m_{u_3}^2 \rangle \rangle &= 0.374 \left(1 - \frac{7.72}{T^2}\right) m_H(0)^2 - \left(\frac{\frac{4}{5} g_1^2 - 4|\lambda_{us}|^2}{\frac{3}{5} g_1^2 + g_2^2}\right) \frac{\Delta^2 m_H(0)^2}{T^2}, \\
\langle\langle m_{u_{1,2}}^2 \rangle \rangle &= 1.041 \left(1 - \frac{3.42}{T^2}\right) m_H(0)^2 - \left(\frac{\frac{2}{5} g_1^2}{\frac{3}{5} g_1^2 + g_2^2}\right) \frac{\Delta^2 m_H(0)^2}{T^2}, \\
\langle\langle m_{d_i}^2 \rangle \rangle &= 1.041 \left(1 - \frac{3.42}{T^2}\right) m_H(0)^2 + \left(\frac{\frac{2}{5} g_1^2}{\frac{3}{5} g_1^2 + g_2^2}\right) \frac{\Delta^2 m_H(0)^2}{T^2}.
\end{align*}
\]
Finally, using expression (3.171) and taking $|\lambda_u|^2 = 1$, these squared masses become

$$\langle\langle m_{U_3}^2 \rangle\rangle = 0.132m_H(0)^2(1 - \frac{14.7 - 63.5\Delta^2}{T^2}) \simeq 0.132m_H(0)^2,$$
$$\langle\langle m_{D_3}^2 \rangle\rangle = 0.132m_H(0)^2(1 - \frac{14.7 - 21.1\Delta^2}{T^2}) \simeq 0.132m_H(0)^2,$$
$$\langle\langle m_{U_{1,2}}^2 \rangle\rangle = 0.465m_H(0)^2(1 - \frac{4.87 - 3.37\Delta^2}{T^2}) \simeq 0.465m_H(0)^2,$$
$$\langle\langle m_{D_{1,2}}^2 \rangle\rangle = 0.465m_H(0)^2(1 - \frac{4.87 - 6.69\Delta^2}{T^2}) \simeq 0.465m_H(0)^2, \hspace{1cm}(3.202)$$
$$\langle\langle m_{U_3}^2 \rangle\rangle = 0.374m_H(0)^2(1 - \frac{7.72 - 26.0\Delta^2}{T^2}) \simeq 0.374m_H(0)^2,$$
$$\langle\langle m_{U_{1,2}}^2 \rangle\rangle = 1.041m_H(0)^2(1 - \frac{3.42 - 3.12\Delta^2}{T^2}) \simeq 1.041m_H(0)^2,$$
$$\langle\langle m_{d_i}^2 \rangle\rangle = 1.041m_H(0)^2(1 - \frac{3.42 - 3.56\Delta^2}{T^2}) \simeq 1.041m_H(0)^2.$$

Note that the squark mass squares are positive for all $T^2 \gtrsim 40$ and $0 < \Delta^2 < 1$. The superpotential contributions to both third family up-squark masses can become significant when $T^2$ and $\Delta^2$ are simultaneously in the lower and upper part of their range respectively. Be that as it may, for typical values of these parameters such contributions are negligible. Therefore, for simplicity, we ignore them when giving the leading order estimate of the squared masses in (3.202).

We conclude from (3.178), (3.179), (3.194) and (3.202) that

$$\langle\langle \nu_{1,2} \rangle\rangle = 0, \hspace{0.5cm} \langle\langle \nu_3 \rangle\rangle = (1.05)\frac{2m_{\nu}(0)}{\sqrt{\frac{3}{2}g_4(t_{EW})}},$$
$$\langle\langle e_i \rangle\rangle = \langle\langle L_i \rangle\rangle = 0 \hspace{0.5cm} i = 1, 2, 3 \hspace{1cm}(3.203)$$
$$\langle\langle H^0 \rangle\rangle = \frac{2\Delta m_H(0)}{T\sqrt{\frac{3}{2}g_1(t_{EW})^2 + g_2(t_{EW})^2}},$$
$$\langle\langle H^+ \rangle\rangle = \langle\langle H^- \rangle\rangle = \langle\langle H^0 \rangle\rangle = 0.$$
is a stable local minimum of the potential energy at scale $t_{EW}$.

### 3.4 The $B$-$L$/Electroweak Hierarchy

The vacuum state (3.203) spontaneously breaks both $B$-$L$ and electroweak symmetry, and exhibits a distinct hierarchy between the two. Color and electric charge are left unbroken. Using (3.156), we see that the $B$-$L$/electroweak hierarchy, expressed as the ratio of the third right-handed sneutrino and Higgs vacuum expectation values, is

$$\frac{\langle\langle \nu_3 \rangle\rangle}{\langle\langle H^0 \rangle\rangle} \approx (0.976) \frac{\sqrt{\frac{3}{5} g_1^2 + g_2^2} \tan \beta}{\sqrt{\frac{3}{4}} g_4} \frac{\Delta}{\Delta},$$

(3.204)

where the gauge parameters are computed at $t_{EW}$. We have dropped the term proportional to $2.25(1 - \Delta^2)/\tan^2 \beta$ in (3.156) since it is always much less than unity in our parameter regime where $\tan \beta \gtrsim 6.32$ and $0 < \Delta^2 < 1$. Note that for fixed $\tan \beta$ the ratio of VEVs in (3.204) can be made arbitrarily large by fine-tuning $\Delta \to 0$. Conversely, by fine-tuning $\Delta \to 1$ this ratio approaches $2.22 \tan \beta$, where we used (3.171) and (3.198). A more natural value for $\Delta$ would lie in the middle of the range $0 \leq \Delta^2 \leq 1$. For specificity, let us take $\Delta = \frac{1}{\sqrt{2}}$. In this case, the ratio (3.204) evaluated in the region $6.32 \leq \tan \beta \leq 40$ is found to be

$$19.9 \leq \frac{\langle\langle \nu_3 \rangle\rangle}{\langle\langle H^0 \rangle\rangle} \leq 126.$$

(3.205)

A second measure of the $B$-$L$/electroweak hierarchy is given by the ratio of the $B$-$L$ vector boson mass to the mass of the $Z$ boson. It follows from (3.85),(3.156),(3.180)
and (3.190) that

$$\frac{M_{AB-L}}{M_Z} \simeq (1.95) \frac{\tan \beta}{\Delta}. \quad (3.206)$$

Note that if we take $\Delta \to 0$ this mass ratio becomes arbitrarily large, whereas if $\Delta \to 1$, the upper bound in our approximation, then $\frac{M_{AB-L}}{M_Z}$ is $1.95 \tan \beta$. Again, using the more natural value $\Delta = \frac{1}{\sqrt{2}}$ and evaluating this mass ratio in the range $6.32 \leq \tan \beta \leq 40$, one finds

$$17.5 \leq \frac{M_{AB-L}}{M_Z} \leq 110. \quad (3.207)$$

We conclude from (3.204) and (3.206) that for typical values of $\Delta$, the vacuum (3.203) exhibits a $B$-$L$/electroweak hierarchy of $\mathcal{O}(10)$ to $\mathcal{O}(10^2)$ over the physically interesting range $6.32 \leq \tan \beta \leq 40$.

To finish our analysis, we would like to emphasize that once the $\frac{M_{AB-L}}{M_Z}$ ratio is fixed, that is, once the $B$-$L$ vector boson mass is measured, the spectrum of squarks and sleptons is completely determined. To see this, first recall that at $t_{EW}$

$$M_{AB-L} = \sqrt{2} g_{B-L}(t_{EW}) \langle \langle \nu_3 \rangle \rangle. \quad (3.208)$$

It then follows from (3.15), (3.190) and (3.156) that

$$m_H(0) = 0.362 \, M_{AB-L}. \quad (3.209)$$

Here and henceforth we drop all $1/T'^2$ terms since they are much less than unity in our parameter regime. It then follows from (3.194) that the slepton masses, in
the order of decreasing mass, are

\[ \langle \langle m_{\nu_1,2} \rangle \rangle = 5.26 \, M_{A_{B-L}}, \]
\[ \langle \langle m_{\nu_3} \rangle \rangle = 1.72 \, M_{A_{B-L}}, \quad (3.210) \]
\[ \langle \langle m_N \rangle \rangle = \langle \langle m_{E_i} \rangle \rangle = 1.54 \, M_{A_{B-L}}, \]
\[ \langle \langle m_{e_i} \rangle \rangle = 1.30 \, M_{A_{B-L}}. \]

Similarly, we see from (3.202) that the squark masses, in descending order, are given by

\[ \langle \langle m_{u_{1,2}} \rangle \rangle = \langle \langle m_{d_i} \rangle \rangle = 0.614 \, M_{A_{B-L}}, \]
\[ \langle \langle m_{U_{1,2}} \rangle \rangle = \langle \langle m_{D_{1,2}} \rangle \rangle = 0.410 \, M_{A_{B-L}}, \quad (3.211) \]
\[ \langle \langle m_{u_3} \rangle \rangle = \langle \langle m_{d_3} \rangle \rangle = 0.369 \, M_{A_{B-L}}, \]
\[ \langle \langle m_{U_3} \rangle \rangle = \langle \langle m_{D_3} \rangle \rangle = 0.219 \, M_{A_{B-L}}. \]

Note that the sleptons masses are on the order of \( M_{A_{B-L}} \) and each is heavier than any squark. The squark masses are lighter, being on the order of about 20%-60% of \( M_{A_{B-L}} \), with the third family left-handed up and down squarks being the lightest. We conclude that the radiative \( B-L \)/electroweak hierarchy also leads to a computable hierarchy in the squark/slepton masses.
3.5 Conclusions

There are vacua of heterotic string theory whose four-dimensional effective theories have exactly the spectrum of the MSSM with three right-handed neutrino chiral multiplets. These theories have, in addition to the standard model gauge group, a gauged $U(1)_{B-L}$ symmetry which must be spontaneously broken not too far above the electroweak scale. In this paper, it is shown that for a specific range of initial parameters the $U(1)_{B-L}$ group is indeed broken by radiative corrections with a phenomenologically acceptable $B$-$L$/electroweak hierarchy.

Let us review the reasons for the existence and magnitude of the $B$-$L$/electroweak hierarchy. First, initial conditions (3.41),(3.42) are chosen so as to set $S$ and $S'_0$, that is, the parameters not containing the right-handed sneutrino masses, to zero, thus minimizing their role in the RG scaling. On the other hand, the second part of initial condition (3.42) gives emphasis to the right-handed sneutrinos by not requiring their masses be degenerate with the $L_i$ and $e_i$ soft masses. This, along with (3.63),(3.64) enables the $S'_1$ parameter (3.65) not only to be non-vanishing but, in addition, to be large enough to dominate all contributions to the RGEs with the exception of the gluino mass terms. This drives $m_{\nu_3}^2$ negative and initiates $B$-$L$ breaking at scale $m_\nu(0)$. The coefficients $C$ and $A$ in (3.63) and (3.64) respectively are chosen to ensure that $m_{\nu_1,2}^2$ and $m_{e_i}^2$, $m_{L_i}^2$, $i = 1,2,3$ remain positive under the RG scaling. With this choice of soft parameters, the $B$-$L$ breaking vacuum at energies of order a $10$ TeV is a local minimum in all scalar field directions, including
the Higgs fields. We point out that the $C$ coefficient can be generalized so as to allow two and three sneutrino mass squares to become negative, leading to a more complicated $B$-$L$ breaking vacuum.

Second, $B$ and $M_3$ (hence, $m_H(t_{EW})$) are chosen to satisfy constraints (3.134), (3.135) and (3.136) respectively at electroweak energies, with $0 < \Delta^2 < 1$. This ensures electroweak breaking for positive $m^2_H$ at a scale proportional to $\Delta m_H(0)/\mathcal{T}$. The values for $\mathcal{T}$ assumed in (3.136) imply that the non-vanishing VEV is largely in the $H^0$ direction, allowing one to identify $\mathcal{T}$, to leading order, with $\tan \beta$. Hence, in this paper $\tan \beta$ can take any value in excess of 6.32, a physically interesting range. In the calculation of the electroweak breaking vacuum we have made two further assumptions. The first is that the $\mu$ parameter at the electroweak scale satisfy $\mu \ll M_Z$ and, hence, be sub-dominant throughout the RG scaling. Second, we treat the soft $A$-parameters in the usual way, assuming that each is proportional to the associated Yukawa coupling times a supersymmetry breaking mass parameter. This makes all $A$ coefficients negligible, with the exception of $A_{u_3}$. However, it follows from (3.106) that in the region of parameter space we are working in $A_{u_3}$ can also be neglected. With these inputs, at the order of 100 GeV the potential exhibits an electroweak breaking vacuum which is a stable local minimum. The scale of the Higgs VEVs is set by $m_H(0)$.

Third, equation (3.156) ensures that squark/slepton squared masses are positive at all scales. By giving the relationship between $m_{\nu}(0)$ and $m_H(0)$, it allows one to
compare the $B$-$L$ and electroweak VEVs at any scale. We find that at low energy the electroweak breaking is smaller than the $B$-$L$ scale, with the $B$-$L$/electroweak hierarchy proportional to $\tan \beta / \Delta$. For $\tan \beta$ in the allowed range and typical values of $\Delta$, this hierarchy is consistent with present experimental data.

Finally, many of the assumptions for the initial conditions were chosen so as to obtain a \textit{quasi-analytic} solution of the RGEs. These indicate that a physically acceptable $B$-$L$/electroweak radiative hierarchy is indeed achievable. However, we want to emphasize that by explicitly solving the RGEs numerically one can explore this hierarchy over a wide range of initial parameter space. We have partially carried this out with two results: 1) a numerical calculation using the above initial parameters reproduces the results of our quasi-analytic approach and 2) we find a physically acceptable radiative hierarchy can be achieved over a much wider range of initial parameters. These results will be reported elsewhere.
Chapter 4

Single Higgs Pair: Numerical Analysis

In the previous section, we performed a quasi-analytic analysis of the single Higgs pair model presented in Section 3.1. We demonstrated that for a specific set of initial parameters, a series of phenomenologically interesting effects can be found, including a $B$-$L$/EW hierarchy. This was shown for a limited range of initial parameters that allowed for considerable simplifications in the corresponding renormalization group equations, thus enabling a quasi-analytic solution. We now seek in this section to generalize these results by using numerical solving methods to explore a much wider region in the initial parameter space.

We begin in Sections 4.1 and 4.2 by doing an in-depth analysis of each of the running parameters defined in (3.11) - (3.14), which comprise the entire list of
running parameters in the theory. In Section 4.3, the exact MSSM spectrum and the associated $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ quantum numbers are presented, along with the associated superpotential, $D$-terms and soft supersymmetry breaking quadratic and cubic terms. Section 4.4 is devoted to extending the ideas in [3, 4] relevant to ensuring the spontaneous breaking of $U(1)_{B-L}$ through right-handed sneutrino VEVs, as well as specifying some physically less interesting initial parameters. The number of initial parameters is reduced to four, related to the squarks, right-handed sneutrinos, the $\mu$ parameter and $\tan \beta$. Three phenomenological constraints are then presented. The first is two inequalities that ensure radiative breaking of electroweak symmetry through up- and down-Higgs VEVs. Second, we give the constraints required to make the $B$-$L$/electroweak vacua local minima of the potential energy. Third, the lower bounds on the masses of all superpartners, as well as the Higgs fields, are presented. The calculations in this paper will satisfy all three constraints. Our main numerical results are presented in Section 4.5. This section is broken into three subsections and a brief summary. The three subsections reflect the fact that all $B$-$L$ MSSM vacua can be categorized by the sign of the left-handed squark and right-handed slepton squared masses; that is, 1) all $m^2 > 0$, 2) all masses positive except $m^2_{Q_i} < 0$ and 3) all masses positive except $m^2_{\tilde{e}_i} < 0$. In each case, we find the complete region of parameter space for which one obtains a realistic theory, compute the $B$-$L$/electroweak hierarchy over each acceptable region and, at some representative points, explicitly compute the...
sparticle and Higgs mass spectrum. We verify that all phenomenological constraints are indeed satisfied at these points.

Our results are predictive, since many low-energy phenomena arise from the radiative breaking of a right-handed sneutrino. Perhaps the most striking aspect of this is that the non-vanishing sneutrino VEV “grows back” the previously disallowed lepton number violating dimension four terms in the superpotential, each with an explicitly calculable coefficient. Following [34, 95], we confront our results with various cosmological constraints, such as baryon asymmetry and primordial nucleosynthesis. We find that they are all easily satisfied in the $B$-$L$ MSSM theory. Furthermore, we show that the our theory is consistent with gravitino dark matter and rapidly decaying standard model sparticles. Another important aspect of breaking $B$-$L$ symmetry with a right-handed sneutrino is that the previously disallowed baryon violating dimension four operator does not grow back from the dimension four superpotential. It can only reappear from higher dimensional operators with calculable, and naturally suppressed, coefficients. Putting in our calculated results, we find that proton decay through dimension four operators is sufficiently suppressed to satisfy all bounds on the proton lifetime. These lepton and baryon number violating results are presented in Section 4.6.

Finally, we want to point out that the $B$-$L$ MSSM theory, with spontaneous breaking of $U(1)_{B-L}$ through right-handed sneutrinos, was presented from a “bottom up” point of view in [17, 59, 54]. These authors discussed various phenomeno-
logical predictions and applied similar ideas to other low-energy theories [60].

4.1 Dimensionless quantities

Gauge Couplings

The first parameters we wish to examine are the gauge couplings in the theory. We begin by observing the experimental values for these couplings. It is important to note that since these couplings run over the scaling range, we must pick a scale at which to report them; we will use $M_Z$ as that scale. Also, there is a mild dependence on exactly what is meant by coupling constant and how they are calculated. We will follow the standard convention of reporting the value derived by the modified minimal subtraction method.

In [12], we see they used the values $\alpha_1^{-1}(M_Z) = 58.98 \pm 0.04$, $\alpha_2^{-1}(M_Z) = 29.57 \pm 0.03$ and $\alpha_3^{-1}(M_Z) = 8.40 \pm 0.14$ for the gauge coupling. To confirm these values of the gauge couplings we refer to the current review from the Particle Data Group [6], where the measured value of the strong coupling is reported to be

\[ \alpha_s^{-1}(M_Z) = 8.503 \Rightarrow g_3(M_Z) = 1.215 \ . \quad (4.1) \]

Next, we consider the $SU(2)$ coupling. There are several ways to derive its value from data given. We choose to use the Fermi coupling constant ($G_F$) and the mass of the W bosons ($M_W$) and find

\[ \alpha_2^{-1}(M_Z) = \frac{\sqrt{2}}{\pi} G_F M_W^2 = \frac{1}{29.55} \Rightarrow g_2(M_Z) = 0.6530 \ . \quad (4.2) \]
Figure 4.1: This plot shows the running of the gauge couplings $g_1$ (red), $g_2$ (yellow), $g_3$ (green) and $g_4$ (blue) and their subsequent unification at $2 \times 10^{16}$ GeV. For this plot, $t = \ln(\mu/(2.2 \times 10^{16}))$.

Lastly, we have the $U(1)_Y$ coupling. We simply use the term $\sin \theta_W(M_Z)$ which relates $g_2$ to $g_1$. Using the above value for $g_2$ we find

$$g_1(M_Z) = \frac{3}{5} g_Y(M_Z) = \frac{3}{5} g_2(M_Z) \sqrt{\frac{\sin^2 \theta_W(M_Z)}{1 - \sin^2 \theta_W(M_Z)}} = 0.46202.$$ (4.3)

We now use the well known RGE equations:

$$\frac{dg_1}{dt} = \frac{1}{16\pi^2} \frac{33}{5} g_1^3, \quad \frac{dg_2}{dt} = \frac{1}{16\pi^2} g_2^3, \quad \frac{dg_3}{dt} = \frac{-3}{16\pi^2} g_3^3.$$ (4.4)

where we have taken $t = \ln(\mu/M_g)$ and $M_g$ is the GUT scale. Using the above values, we find gauge coupling unification at roughly $2.19 \times 10^{16}$ at a value of $g_{unification} = 0.7235$. Note, that this analysis did not take into account threshold corrections. Using these, which depend on the gauge group and matter content, a wider range of values can be obtained.

We now expand the MSSM to include a $U(1)_{B-L}$ and thus a new coupling constant $g_{B-L} = \sqrt{\frac{3}{4}} g_4$. Though it is straight forward to derive the RGE equation
Figure 4.2: This plot shows the running of the gauge couplings $\alpha_{-1}$ (red), $\alpha_{-2}$ (yellow), $\alpha_{-3}$ (green) and $\alpha_{-4}$ (blue) versus the Log of the energy scale $\mu$. As shown in the previous plot, these couplings unify at $2.2 \times 10^{16}$ GeV.

for this coupling, there clearly does not exist a measured value for it at any scale, thus it is a free parameter. If we set $g_4$ to unify with the others at $M_G$ then we find at $M_Z$ then we get the value $g_4 = 0.3795$.

Yukawa Couplings

We begin by recalling the value of the Standard Model Yukawa couplings at the scale $M_Z$. These can be obtained simply from knowing the masses of each of the fermions in the SM as well as the value of the Higgs VEV. In the SM, the Higgs, denoted as $H$, can obtain a VEV, $\langle H \rangle$. In general we find the mass of a fermion to be

$$m_i = \lambda_i \langle H \rangle. \quad (4.5)$$
Using this equation and letting $\langle H \rangle = \frac{246}{\sqrt{2}} \text{GeV}$, we find the list of Yukawa values displayed in Figure 4.3. Note, we define a convention here of including the $\sqrt{2}$ in the definition of the Higgs VEV. We will maintain this convention throughout.

For the MSSM, there are two different Higgs doublets that give mass to fermions, so the situation is more complicated. One couples to the Up type fermions, the other, the Down type. The ratio of the VEVs of these Higgs bosons is commonly referred to as $\tan \beta$. For a $\tan \beta$ of 1 the values of the Yukawa couplings would be twice the size reported for the SM in Figure 4.3 but the ratio of the coupling would remain the same. As $\tan \beta$ increases, the value of the Yukawa couplings for the Down type fermions would increase and those for the Up type fermions would decrease.

As will be shown below, in each of the RGE equations for all the free parameters in the MSSM, any Up or Down type fermion always appears in sums with the others families of it’s type. In other words, the charm Yukawa coupling always appears with the top and up couplings, and likewise for the down couplings. However, the ups do not always appear with the downs, and vice versa, in the same equation. Since the value of $\tan \beta$ does not affect the ratio of the size of the various Up type couplings with each other, and same for the Downs, we find that it would be a reasonable approximation to take the 1st and 2nd family Yukawa couplings to be subleading to the 3rd family and thus drop them in all RGE equations. Doing so we obtain the following RGE equations for the Yukawa couplings of the third family.
<table>
<thead>
<tr>
<th>Yukawa Coupling</th>
<th>SM value at $M_Z$</th>
<th>MSSM at $M_Z$ and $tan\beta = 10$</th>
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</thead>
<tbody>
<tr>
<td>$\lambda_t$</td>
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<td>1.08</td>
</tr>
<tr>
<td>$\lambda_b$</td>
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<td>0.26</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>$7.3 \times 10^{-3}$</td>
<td>$8.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>$6.0 \times 10^{-4}$</td>
<td>$6.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\lambda_d$</td>
<td>$2.9 \times 10^{-5}$</td>
<td>$3.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\lambda_u$</td>
<td>$1.1 \times 10^{-5}$</td>
<td>$1.3 \times 10^{-4}$</td>
</tr>
<tr>
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<td>$1.0 \times 10^{-2}$</td>
<td>0.11</td>
</tr>
<tr>
<td>$\lambda_\mu$</td>
<td>$6.0 \times 10^{-4}$</td>
<td>$6.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\lambda_e$</td>
<td>$2.9 \times 10^{-6}$</td>
<td>$3.2 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Figure 4.3: This table shows values of various Yukawa couplings in the Standard Model given at the scale of $M_Z$ as well as values for the couplings in the MSSM for a $tan\beta$ value of 10.
Figure 4.4: This plot shows the running of the $\log_{10}$ of the inverse Yukawa couplings $\lambda^{-1}_t$(red), $\lambda^{-1}_b$(yellow), $\lambda^{-1}_\tau$(green) and $\lambda^{-1}_\nu$(blue) versus $t$. This plot demonstrates how slowly they run over the entire scaling range.

quarks and leptons,

$$\begin{align*}
\frac{d\lambda_t}{dt} &= \frac{1}{16\pi^2} \lambda_t (6\lambda_t^2 + \lambda_b^2 + \lambda_\nu^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 - \frac{1}{12} g_4^2) \\
\frac{d\lambda_b}{dt} &= \frac{1}{16\pi^2} \lambda_b (6\lambda_b^2 + \lambda_t^2 + \lambda_\tau^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 - \frac{1}{12} g_4^2) \\
\frac{d\lambda_\tau}{dt} &= \frac{1}{16\pi^2} \lambda_\tau (4\lambda_\tau^2 + 3\lambda_b^2 - 3g_2^2 - \frac{9}{5} g_1^2 - \frac{3}{4} g_4^2) \\
\frac{d\lambda_\nu}{dt} &= \frac{1}{16\pi^2} \lambda_\nu (4\lambda_\nu^2 + 3\lambda_t^2 - 3g_2^2 - \frac{9}{5} g_1^2 - \frac{3}{4} g_4^2).
\end{align*}$$

(4.6)

Note that these can not be solved explicitly, but can be quickly solved numerically with no need of any further approximation. However, as noted above, the exact values of these couplings depend on the value of $\tan \beta$. We note however, that for most of the range of $\tan \beta$ considered, the Yukawa couplings will not change by more than thirty percent, some as few as just a few percent. For the sake of illustration, we present in Figure 4.4 the running for the case of $\tan \beta = 10$ (who’s values at $M_Z$ are also given in Figure 4.3). We can quickly observe that the running of these couplings is extremely slow over the entire scaling range.
4.2 Dimensionful Couplings

The $\mu$ Parameter

Next we consider the parameter $\mu$. We report the RGE equation where we have dropped the first and second family Yukawa couplings.

$$\frac{d\mu}{dt} = \frac{1}{16\pi^2} \mu (3\lambda_t^2 + 3\lambda_b^2 + \lambda^2 + \lambda^2 - 3g_2^2 - \frac{3}{5}g_1^2) \quad (4.7)$$

This equation is fairly straightforward to solve numerically. It is a first order equation and thus requires a boundary condition to obtain a specific solution. Since there is no experimental value for this parameter, we leave it free. We observe briefly that over the entire scaling range, the $\lambda_t$ term will dominate by more than an order of magnitude and will itself change slowly while it is running. Thus we expect the running of $\mu$ to be fairly consistent over the enter scaling range and to approximately follow the much simpler equation

$$\frac{d\log(\mu)}{dt} = \frac{3}{16\pi^2} \quad (4.8)$$

Given a value for $\tan\beta$ and an initial value $\mu_0$, we can exactly solve (4.7) using numerical methods. In Figure 4.5, we see that the above estimation is a good one for extreme values of $\tan\beta$ where $\lambda_t$ or $\lambda_b$ dominate.
Figure 4.5: We see the running of $\mu/\mu_0$ for several values of $\tan\beta$ as well as the estimated running in equation (4.8). In this plot, $\tan\beta = 7$ is red, $\tan\beta = 10$ is yellow-green, $\tan\beta = 40$ is green and the estimation is blue. We can see for extreme values of $\tan\beta$, (4.8) is a good estimate. For this plot, $t = \text{Ln}(\mu/(2.2 \times 10^{16}))$.

Gaugino Masses

The gaugino mass RGE equations are entirely trivial to solve. They have the same form as the gauge couplings. We reproduce these equations here:

$$\begin{align*}
\frac{dM_1}{dt} &= \frac{1}{8\pi^2} \frac{33}{5} g_1^2 M_1 \\
\frac{dM_2}{dt} &= \frac{1}{8\pi^2} g_2^2 M_2 \\
\frac{dM_3}{dt} &= \frac{1}{8\pi^2} (-3) g_3^2 M_3 \\
\frac{dM_4}{dt} &= \frac{1}{8\pi^2} 12 g_4^2 M_4 . \quad (4.9)
\end{align*}$$

Again, we do not have an experimental value to fit these parameters to, so they are free. We illustrate an example of the running for these mass couplings in Figure 4.6. Notice, due to the fact that we do not know their values at any scale, we simply plot them modulo some scale $M_0$. 

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Figure 4.6: This plot shows the running of the gaugino mass couplings $M_1$(red), $M_2$(yellow), $M_3$(green) and $M_4$(blue) divided by a scale $M_0$ for the case of $M_1$, $M_2$ and $M_4$ and an example scale $1.5M_0$ for $M_3$. This shows possibility for partial unification of some of the gaugino masses at the GUT scale. For this plot, $t = \ln(\mu/(2.2 \times 10^{16}))$.

Trilinear Scalar term A

Here we reproduce the RGE equations for the trilinear couplings:

$$16\pi^2 \frac{dA_t}{dt} = A_t \left( -\frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 - \frac{1}{12}g_4^2 + \lambda_b^2 + 8\lambda_t^2 + \lambda_r^2 \right)$$

$$+ \lambda_t \left( \frac{26}{15}M_1g_1^2 + 6M_2g_2^2 + \frac{32}{3}M_3g_3^2 + \frac{1}{6}M_4g_4^2 + 2A_b\lambda_b \right)$$

$$+ 10A_t\lambda_t + 2A_r\lambda_r \right) \quad (4.10)$$

$$16\pi^2 \frac{dA_b}{dt} = A_b \left( -\frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 - \frac{1}{12}g_4^2 + 8\lambda_b^2 + \lambda_t^2 + \lambda_r^2 \right)$$

$$+ \lambda_b \left( \frac{14}{15}M_1g_1^2 + 6M_2g_2^2 + \frac{32}{3}M_3g_3^2 + \frac{1}{6}M_4g_4^2 + 10A_b\lambda_b \right)$$

$$+ 2A_t\lambda_t + 2A_r\lambda_r \right)$$

$$16\pi^2 \frac{dA_r}{dt} = A_r \left( -\frac{9}{5}g_1^2 - 3g_2^2 - \frac{3}{4}g_4^2 + 3\lambda_b^2 + 6\lambda_r^2 \right)$$

$$+ \lambda_r \left( \frac{18}{5}M_1g_1^2 + 6M_2g_2^2 + \frac{3}{2}M_4g_4^2 + 6A_b\lambda_b + 6A_r\lambda_r \right)$$

$$16\pi^2 \frac{dA_\nu}{dt} = A_\nu \left( -\frac{3}{5}g_1^2 - 3g_2^2 - \frac{3}{4}g_4^2 + 3\lambda_t^2 + 6\lambda_\nu^2 \right)$$

$$+ \lambda_\nu \left( \frac{6}{5}M_1g_1^2 + 6M_2g_2^2 + \frac{3}{2}M_4g_4^2 + 6A_t\lambda_t + 6A_\nu\lambda_\nu \right) .$$
Figure 4.7: In this plot, we show $A_t/A_0$ over the whole scaling range of $t$ for several different values of $\tan\beta$ and for $A_0 = M_0$ and $M_3(0) = 1.5 M_0$. The values of $\tan\beta$ given are 7 (red), 10 (yellow), and 40 (green). We observe the weak dependence of $A_t$ on $\tan\beta$. For this plot, $t = \ln(\mu / (2.2 \times 10^{16}))$.

We have dropped the first and second family Yukawa couplings as explained in previous sections. The running of the above $A$ couplings is not trivial and very numerically intensive.

To make these equations more tractable for our numerical solving software, we set the $\lambda_i'$s to be constants. Doing this, we can easily solve the above equations numerically for some initial values of $M_i'$s and $A_i'$s as well as $\tan\beta$. To understand the initial values of $A_i$, we will make the assumption derived from supergravity theories in a unified theory. When a flat Kahler potential is assumed, we find that you can write $A_i = \lambda \tilde{A}_i$. Where the values of $\tilde{A}_i$'s unite at the GUT scale. In our theory, this is not a necessary constraint, but we find it to be a helpful starting point in our analysis. So we will take $A_i = \lambda A_0$.

Now we wish to understand more clearly the dependence of $A_t$ on the various
Figure 4.8: In these plots, we show $A_t/A_0$ and $A_t/M_0$ over the whole scaling range of $t$ for several different values of the ratio and for fixed $M_3(0) = 1.5M_0$. For these examples, we take the values of $\tan \beta = 10$. The values of the ratio shown are 100 (green), 10 (yellow), 1 (red), 0.1 (light blue), 0.01 (blue). For these plots, $t = \frac{\text{ln}(\mu)}{2.2 \times 10^{16}}$.

initial conditions. First we examine its dependence on $\tan \beta$. We show in Figure 4.7 that the $\tan \beta$ dependence is very weak so we will ignore it in further analysis of $A_t$.

In this plot we took $A_0 = M_0$ and $M_3(0) = 1.5M_0$ which is the splitting we used in Figure 4.6. We also find this trend holds for a wide range of ratios of $A_0 / M_0$ which we will now explore.

Next we wish to explore how $A_t$ depends on the ratio of $A_0$ to $M_0$. The initial conditions of Figure 4.6, we can examine a variety of ratios of $A_0 / M_0$. In plots presented in Figure 4.8, we can see that for a ratio less than 1, the $M$ terms dominate and the running of $A_t$ is strongly dominated by the value of $M_0$. For a high ratio, the opposite is the case and $A_0$ dominates. We can see that the most “tame” case is when the ratio is close to 1. Note, in these figures, we give a plot of both $A_t/A_0$ and $A_t/M_0$. 

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Figure 4.9: In this plot, we show $B_t/B_0$ for the minimal case of $A_0 = M_0 = \mu_0 = B_0$ and $\tan \beta = 10$. For this plot, $t = \ln(\mu/(2.2 \times 10^{16}))$.

in (a) and $A_t/M_0$ in (b). We do this to gain intuition as to how changing the size of $A_0$ and $M_0$ affect the running. Plot (a) is the case of leaving $A_0$ fixed and adjusting the size of $M_0$ relative to it, while (b) is the opposite.

The Bilinear term B

There exists a parameter in this theory that has mass dimension 2. This is the B parameter. The RGE for this coefficient is

$$\frac{dB}{dt} = \frac{1}{16\pi^2} \left( B(3\lambda_t^2 + 3\lambda_b^2 + \lambda_\nu^2 + \lambda_r^2 - 3g_2^2 - \frac{3}{5}g_1^2) 
+ \mu(6A_t\lambda_t + 6A_b\lambda_b + 2A_r\lambda_r + 2A_\nu\lambda_\nu + 6g_2^2M_2 + \frac{6}{5}g_1^2M_1) \right) \quad (4.11)$$

where again, we have dropped the first and second family Yukawa couplings. We can see that this is a fairly involved RGE equation. As in the case of the $A$ terms, we will take the Yukawa couplings to be constants relative to $t$. This still leaves for a somewhat challenging equation as it depends on the still free $\tan \beta$ as well
as the initial conditions of the $A$’s, $M$’s, $\mu_0$ and $B$ itself. For the minimal case of $A_0 = M_0 = \mu_0 = B_0$ and $\tan \beta = 10$, we get the running shown in Figure 4.9.

### 4.3 The $N = 1$ Supersymmetric Theory

We will now again explore the model presented in Section 3.1, however, this time using the tools of computer generated numerical approximations. For ease of reference, we will briefly repeat the discussion of the theory presented in Section 3.1. In addition, we will highlight several generalizations in the assumptions presented previously. Recall that, in the last section, many of the assumptions made were to simplify the RGE equations presented to allow for a quasi-analytic solution. We are now free to generalize to more universal assumptions.

Consider an $N = 1$ supersymmetric theory with gauge group

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$  \hspace{1cm} (4.12)

and the associated vector superfields. The gauge parameters are denoted by $g_3$, $g_2$, $g_Y$ and $g_{B-L}$ respectively. The matter spectrum consists of three families of quark and lepton chiral superfields, each family with a right-handed neutrino. They transform under the gauge group in the standard manner as

$$Q_i = (3, 2, 1/3, 1/3), \quad u_i = (\bar{3}, 1, -4/3, -1/3), \quad d_i = (\bar{3}, 1, 2/3, -1/3)$$  \hspace{1cm} (4.13)

for the left and right-handed quarks and

$$L_i = (1, 2, -1, -1), \quad \nu_i = (1, 1, 0, 1), \quad e_i = (1, 1, 2, 1)$$  \hspace{1cm} (4.14)

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for the left and right-handed leptons, where \( i = 1, 2, 3 \). In addition, the spectrum has one pair of Higgs-Higgs conjugate chiral superfields transforming as

\[
H = (1, 2, 1, 0), \quad \bar{H} = (1, 2, -1, 0).
\]  

(4.15)

When necessary, the left-handed \( SU(2)_L \) doublets will be written as

\[
Q_i = (U_i, D_i), \quad L_i = (N_i, E_i), \quad H = (H^+, H^0), \quad \bar{H} = (\bar{H}^0, \bar{H}^-).
\]  

(4.16)

There are no other fields in the spectrum.

The supersymmetric potential energy is given by the usual sum over the modulus squared of the \( F \) and \( D \)-terms. In principle, the \( F \)-terms are determined from the most general superpotential invariant under the gauge group,

\[
W = \mu H \bar{H} + \sum_{i,j=1}^{3} \left( \lambda_{u,ij} Q_i H u_j + \lambda_{d,ij} Q_i \bar{H} d_j + \lambda_{\nu,ij} L_i H \nu_j + \lambda_{e,ij} L_i \bar{H} e_j \right)
\]  

(4.17)

Note that the quadratic mixing term of the form \( L_i H \), as well as the dangerous lepton and baryon number violating interactions

\[
L_i L_j e_k, \quad L_i Q_j d_k, \quad u_i d_j d_k
\]  

(4.18)

which generically would lead, for example, to rapid nucleon decay, are disallowed by the \( U(1)_{B-L} \) gauge symmetry. To simplify the upcoming calculations, we will assume that we are in a mass-diagonal basis where

\[
\lambda_{u,ij} = \lambda_{d,ij} = \lambda_{\nu,ij} = \lambda_{e,ij} = 0, \quad i \neq j.
\]  

(4.19)

Note that once these off-diagonal couplings vanish just below the compactification scale, they will do so at all lower energy-momenta. We will denote the diagonal
Yukawa couplings by $\lambda_{ii} = \lambda_i$, $i = 1, 2, 3$. Next, observe that a constant, field-independent $\mu$ parameter cannot arise in a supersymmetric string vacuum since the Higgs fields are zero modes. However, the $H\bar{H}$ bilinear can have higher-dimensional couplings to moduli through both holomorphic and non-holomorphic interactions in the superpotential and Kahler potential respectively. When moduli acquire VEVs due to non-perturbative effects, these can induce non-vanishing supersymmetric contributions to $\mu$. A non-zero $\mu$ can also be generated by gaugino condensation in the hidden sector. Why this induced $\mu$-term should be small enough to be consistent with electroweak symmetry breaking is a difficult, model dependent problem. In this paper, we will not discuss this “$\mu$-problem”. Instead, we will consider the $\mu$ parameter as an input to our analysis and consider a range of possible values.

The $SU(3)_C$ and $SU(2)_L$ $D$-terms are of the standard form. We present the $U(1)_Y$ and $U(1)_{B-L}$ $D$-terms,

$$D_Y = \xi_Y + g_Y \phi_A^\dagger (Y/2)_{AB} \phi_B$$

and

$$D_{B-L} = \xi_{B-L} + g_{B-L} \phi_A^\dagger (Y_{B-L})_{AB} \phi_B$$

where the index $A$ runs over all scalar fields $\phi_A$, to set the notation for the hypercharge and $B-L$ charge generators and to remind the reader that each of these $D$-terms potentially has a Fayet-Iliopoulos (FI) additive constant. However, as with the $\mu$ parameter, constant field-independent FI terms cannot occur in string vacua since the low energy fields are zero modes. Field-dependent FI terms can occur...
in some contexts, see for example [11]. However, since both the hypercharge and $B\!-\!L$ gauge symmetries are anomaly free, such field-dependent FI terms are not generated in the supersymmetric effective theory. We include them in (4.20),(4.21) since they can, in principle, arise at a lower scale from radiative corrections once supersymmetry is softly broken [72]. Be that as it may, if calculations are done in the $D$-eliminated formalism, which we use in this paper, these FI parameters can be consistently absorbed into the definition of the soft scalar masses and their beta functions. Hence, we will no longer consider them.

In addition to the supersymmetric potential, the Lagrangian density also contains explicit “soft” supersymmetry violating terms. These arise from the spontaneous breaking of supersymmetry in a hidden sector that has been integrated out of the theory. This breaking can occur in either $F$-terms, $D$-terms or both in the hidden sector. In this paper, for simplicity, we will restrict our discussion to soft supersymmetry breaking terms arising exclusively from $F$-terms. The form of these terms is well-known and, in the present context, given by [63, 71, 82, 90, 87]

\[ V_{\text{soft}} = V_{2s} + V_{3s} + V_{2f}, \]  

(4.22)

where $V_{2s}$ are scalar mass terms

\[ V_{2s} = \sum_{i=1}^{3} (m_{Q_i}^2 |Q_i|^2 + m_{u_i}^2 |u_i|^2 + m_{d_i}^2 |d_i|^2 + m_{L_i}^2 |L_i|^2 + m_{\nu_i}^2 |\nu_i|^2 \]

\[ + m_{e_i}^2 |e_i|^2) + m_H^2 |H|^2 + m_{\tilde{H}}^2 |\tilde{H}|^2 - (BH\tilde{H} + h.c), \]  

(4.23)
$V_{3s}$ are scalar cubic couplings

$$V_{3s} = \sum_{i=1}^{3} (A_u Q_i H u_i + A_d Q_i \bar{H} d_i + A_{\nu} L_i H \nu_i + A_{e} L_i \bar{H} e_i + \text{hc})$$  \hfill (4.24)

and $V_{2f}$ contains the gaugino mass terms

$$V_{2f} = \frac{1}{2} M_3 \lambda_3 \lambda_3 + \frac{1}{2} M_2 \lambda_2 \lambda_2 + \frac{1}{2} M_Y \lambda_Y \lambda_Y + \frac{1}{2} M_{B-L} \lambda_{B-L} \lambda_{B-L} + \text{hc}.$$

As above, to simplify the calculation we assume the parameters in (4.23) and (4.24) are flavor-diagonal. This is consistent since once the off-diagonal parameters vanish just below the compactification scale, they will do so at all lower energy-momenta. Finally, note that lepton and baryon violating scalar cubic terms of the form (4.18) are disallowed in $V_{3s}$ by the $U(1)_{B-L}$ gauge symmetry.

### 4.4 Initial Parameter Space

The four-dimensional effective theory described in the previous section arises at an initial energy-momentum just below the compactification scale given by the inverse Calabi-Yau radius. In order to carry out a detailed renormalization group analysis, we must specify this initial energy-momentum precisely. We will do this as follows.

#### 4.4.1 Gauge Coupling Parameters

It is well known that precision measurements [12, 6, 62] carried out at the electroweak scale indicate that the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings, $g_3, g_2$
and \( g_1 = \sqrt{\frac{5}{3}} g_Y \) respectively, unify to

\[
g(0) \simeq 0.726 \tag{4.26}
\]

at scale

\[
M_u \simeq 3 \times 10^{16}\text{GeV} \ . \tag{4.27}
\]

For simplicity, so that we can ignore a discussion of threshold effects, we will assume that the initial energy momentum for our effective theory is precisely the unification scale \( M_u \). In addition, since the \( SU(4) \) vector bundle breaks \( E_8 \) to \( SO(10) \), we will take the \( U(1)_{B-L} \) gauge coupling \( g_4 = \sqrt{\frac{1}{3}} g_{B-L} \) to unify with the three other couplings at \( M_u \).

Having fixed the initial energy-momentum as \( M_u \), one must now specify the initial values of all parameters in the effective theory at this scale. In principle, string theory would predict these parameters as functions of the moduli VEVs. In this paper, however, we will be content with simply choosing the initial parameters subject to the dictates of simplicity, the “universality” of some parameters observed in minimal supergravity and simple string compactifications [39, 87] and the necessity to break \( U(1)_{B-L} \) through a VEV of at least one right-handed sneutrino. Having chosen all the initial parameters, their values at any lower scale, specified by

\[
t = \ln\left( \frac{\mu}{M_u} \right) , \tag{4.28}
\]

are determined by the associated renormalization group equations (RGEs). These
are discussed in detail in several reviews, see, for example [85, 86, 98, 72, 73, 74, 39],
and were generalized to include the $U(1)_{B-L}$ symmetry in our previous papers [3, 4].

In this paper, all calculations will be carried out at the one-loop level.

The initial unified gauge coupling is given in (4.26). We now turn to specifying
the initial values for all other parameters in our effective low-energy theory. We
begin with the dimensionful parameters.

4.4.2 Gaugino Mass Parameters

Consider the soft supersymmetry breaking gaugino mass parameters that appear
in $V_{2f}$ in (4.25). Following standard notation, we henceforth denote $M_Y = M_1$ and
$M_{B-L} = M_4$. We now make the assumption that at the compactification scale the
gaugino masses unify, that is,

$$|M_1(0)| = |M_2(0)| = |M_3(0)| = |M_4(0)|.$$  (4.29)

Such universal gaugino masses naturally occur in minimal supergravity [71, 38, 31]
and simple string theories [82, 90]. Here, we choose (4.29) for reasons of simplicity.
4.4.3 Higgs, Squark and Slepton Masses

The RGEs for the soft supersymmetry breaking Higgs, squark and slepton masses all contain a term proportional to $g_1^2 S$ where

$$ S = Tr \left( \frac{Y}{2} m^2 \right) $$

$$ = m_H^2 - m_H^2 + \sum_{i=1}^{3} (m_{Q_i}^2 - 2m_{u_i}^2 + m_{d_i}^2 - m_{L_i}^2 + m_{e_i}^2). $$

It greatly simplifies the boundary conditions of these RGEs to choose the initial soft breaking masses so that $S(0) = 0$. A natural way to achieve this is to impose a separate unification of the Higgs masses, squark masses and the left doublet/down right singlet slepton masses. That is, we henceforth choose

$$ m_{H}(0)^2 = m_{H}(0)^2, \quad m_{Q_i}(0)^2 = m_{u_i}(0)^2 = m_{d_i}(0)^2 $$

and

$$ m_{L_i}(0)^2 = m_{e_j}(0)^2 $$

for all $i, j, k = 1, 2, 3$. In addition to the hypercharge induced $g_1^2 S$ term, the gauged $U(1)_{B-L}$ symmetry of our effective theory introduces a new term into the RGEs for the squarks and slepton soft supersymmetry breaking masses. This term is of the form $g_1^2 S'$ where

$$ S' = Tr(Y_{B-L} m^2) = S'_0 + S'_1 $$

and

$$ S'_0 = \sum_{i=1}^{3} (2m_{Q_i}^2 - m_{u_i}^2 - m_{d_i}^2 - m_{L_i}^2 + m_{e_i}^2), \quad S'_1 = \sum_{i=1}^{3} (-m_{L_i}^2 + m_{e_i}). $$
It follows from (4.31) and (4.32) that $S'_0(0) = 0$. Note, however, that unlike $S$ and $S'_0$, the $S'_1$ term depends on the soft supersymmetry breaking right-handed sneutrino masses. We choose the initial values of these parameters not to be degenerate with the other slepton masses, that is,

$$m_{L_i}(0)^2 = m_{e_j}(0)^2 \neq m_{\nu_k}(0)^2$$

(4.35)

for all $i, j, k = 1, 2, 3$. It follows that

$$S'_1(0) = \sum_{i=1}^{3} (-m_{L_i}(0)^2 + m_{\nu_i}(0)^2) \neq 0 .$$

(4.36)

This asymmetry is an important ingredient in generating radiative breaking of the $U(1)_{B-L}$ symmetry. We point out that soft scalar masses need not be “universal” in string theories, since they are not generically “minimal”.

### 4.4.4 The A and B Parameters

Now consider the soft supersymmetry breaking up/down $A_i$ and $B$ parameters in equations (4.24) and (4.23) respectively. As already stated, we take the $A_i$ coefficients to be flavor diagonal. In addition, it is conventional [87] to let

$$A_{ui} = \lambda_{ui} \tilde{A}_{ui}, \quad A_{di} = \lambda_{di} \tilde{A}_{di}, \quad A_{\nu_i} = \lambda_{\nu_i} \tilde{A}_{\nu_i}, \quad A_{e_i} = \lambda_{e_i} \tilde{A}_{e_i}$$

(4.37)

for $i = 1, 2, 3$, where $\lambda_i$ are the Yukawa couplings and the dimensionful $\tilde{A}_i$ parameters are chosen to be of order the supersymmetry breaking scale. This is not a requirement in the “non-minimal” string vacua that we are discussing. Be that as
it may, for simplicity of presentation we will assume (4.37) for the remainder of this paper. The input Yukawa parameters will be discussed below. In this paper, we will, for simplicity, assume the $\tilde{A}_i$ parameters unify at the scale $M_u$. That is,

$$\tilde{A}_{ui}(0) = \tilde{A}_{di}(0) = \tilde{A}_{\nu k}(0) = \tilde{A}_{\tau l}(0)$$

(4.38)

for all $i, j, k, l = 1, 2, 3$.

The initial value of the soft breaking B parameter, $B(0)$, is taken to be arbitrary. However, in our analysis B will be treated differently than the other dimensionful parameters. As will be shown below, rather than choosing the value of the B parameter, we will instead input $\tan \beta$ and the supersymmetry breaking scale. This will dynamically fix the value of B for any given set of initial conditions.

### 4.4.5 The $\mu$ Parameter

The supersymmetric $\mu$ parameter has a fundamentally different origin than the soft supersymmetry breaking dimensionful couplings discussed above. In this paper, we will simply allow its initial value $\mu(0)$ to be arbitrary. As in conventional radiative breaking scenarios, to be compatible with electroweak symmetry breaking we expect it to be of $\mathcal{O}(100)\text{GeV}$. However, we make no attempt to solve this “$\mu$-problem”.

Having discussed the initial values for the dimensionful parameters, we now consider the dimensionless parameters in our effective theory.
4.4.6 Tan$\beta$ and the Yukawa Couplings

As with any MSSM-like model, our low energy theory requires two Higgs chiral supermultiplets, $H$ and $\bar{H}$, whose VEVs $\langle H \rangle$ and $\langle \bar{H} \rangle$ break electroweak symmetry and give mass to the $W^\pm$ and $Z$ vector bosons. The experimentally measured vector boson masses put a constraint on these VEVs. In terms of the $Z$ mass, this is

$$\langle H \rangle^2 + \langle \bar{H} \rangle^2 = \frac{2M_Z^2}{g_Y^2 + g_2^2} \simeq \left( \frac{246}{\sqrt{2}} \text{GeV} \right)^2.$$  (4.39)

Hence, giving one Higgs VEV completely determines the other. It is conventional to re-express the remaining Higgs VEV in terms of the ratio

$$\tan \beta = \frac{\langle H \rangle}{\langle \bar{H} \rangle}.$$  (4.40)

If the value of tan$\beta$ is given, one can easily find both Higgs VEVs using (4.39) and (4.40). The result is

$$\langle H \rangle = \left( \frac{246}{\sqrt{2}} \text{GeV} \right) \frac{\tan \beta}{\sqrt{1 + \tan^2 \beta}}, \quad \langle \bar{H} \rangle = \left( \frac{246}{\sqrt{2}} \text{GeV} \right) \frac{1}{\sqrt{1 + \tan^2 \beta}}.$$  (4.41)

In this paper, we will take tan$\beta$ as an input parameter.

In addition to the vector bosons, the Higgs VEVs $\langle H \rangle$ and $\langle \bar{H} \rangle$ give mass to the up and down quarks/leptons respectively. As with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings, the Yukawa couplings are highly constrained by experiment. Given a value of tan$\beta$ and, hence, $\langle H \rangle$ and $\langle \bar{H} \rangle$, the known masses of the quarks/leptons completely determine the Yukawa couplings at the electroweak scale. However, unlike the gauge couplings, the Yukawa coupling do not unify at $M_u$. Rather, when
run up to the unification scale using their RGEs, the initial values of the Yukawa couplings are a set of $\tan\beta$ dependent numbers with no particular relationship. Therefore, in this paper, rather than specifying the initial Yukawa couplings at scale $M_u$, we will instead input a value of $\tan\beta$ and use the associated Higgs VEVs and the measured quark/lepton masses to calculate all Yukawa parameters at the electroweak scale. These will then be run back to the unification scale and stored in our program. When required, the initial Yukawa parameters can then be input into any other RGE and scaled down along with the other relevant parameters.

It is important to note from (4.41) that as $\tan\beta$ is decreased, the up Higgs VEV $\langle H \rangle$ must get smaller. This then necessitates taking larger values for the up Yukawa couplings to be consistent with the measured masses. For $\langle H \rangle$ sufficiently small, the top quark Yukawa coupling will become much larger than unity and the theory becomes non-perturbative. This puts a bound on how small $\langle H \rangle$ can be and, hence, a lower bound on $\tan\beta$. Similarly, increasing $\tan\beta$ requires the down Higgs VEV $\langle \bar{H} \rangle$ to decrease. For $\langle \bar{H} \rangle$ sufficiently small, the bottom quark Yukawa coupling will become much larger than unity and the theory non-perturbative. This puts a bound on how small $\langle \bar{H} \rangle$ can be and, hence, an upper bound on $\tan\beta$. These bounds on $\tan\beta$ are typically estimated [6, 94] to be

$$4 \lesssim \tan\beta \lesssim 50.$$ (4.42)

When inputting $\tan\beta$ in this paper, we will always restrict it to be within these bounds.
4.4.7 Parameterizing the Initial Conditions

Recall that, with the exception of the $\mu$ parameter, all of the dimensional coefficients discussed above occur in soft supersymmetry breaking interactions. If we denote by $\mathcal{M}$ a mass characterizing the scale of supersymmetry breaking, then each of the above coefficients can be written in the form

$$c_i(t)\mathcal{M},$$

where $c_i(t)$ is dimensionless. This parameterization emphasizes that the soft dimensionful coefficients share a common supersymmetry breaking scale. The initial coefficients, $c_i(0)$, are arbitrary. However, naturalness would dictate that they not to be too much larger, or smaller, than unity. The exception to this is the parameter $\mu$. This arises in the supersymmetric quadratic Higgs term and is, a priori, unrelated to the scale $\mathcal{M}$. However, it can always be written in the form (4.43). In this case, however, one does not expect the associated coefficient to be of order unity. Be that as it may, the “$\mu$-problem” specifies that appropriate radiative electroweak breaking will require $\mu$ to be close to the scale $\mathcal{M}$.

Specifically, this parameterization of the dimensionful parameters allows us to write the initial value for the gaugino masses as

$$|M_1(0)| = |M_2(0)| = |M_3(0)| = |M_4(0)| = c_M(0)\mathcal{M},$$

as well as

$$m_H(0) = m_{\tilde{H}}(0) = c_H(0)\mathcal{M}$$
for the initial Higgs parameters. Similarly, the initial squark and doublet/down singlet slepton masses are

\[ m_{Q_i}(0) = m_{u_j}(0) = m_{d_k}(0) = c_q(0)\mathcal{M} \]

and

\[ m_{L_i}(0) = m_{e_j}(0) = c_e(0)\mathcal{M} \]

respectively. However, for the reasons discussed below, we will allow the initial right-handed sneutrino masses to have the texture

\[ m_{\nu_1}(0) = m_{\nu_2}(0) = c_{\nu_{1,2}}(0)\mathcal{M} , \quad m_{\nu_3}(0) = c_{\nu_3}(0)\mathcal{M} . \]

Finally, we write

\[ \tilde{A}_{u_i}(0) = \tilde{A}_{d_j}(0) = \tilde{A}_{e_k}(0) = \tilde{A}_{\nu_l}(0) = c_{\tilde{A}}(0)\mathcal{M} \]

and

\[ \mu(0) = c_\mu(0)\mathcal{M} , \quad B(0) = c_B^2(0)\mathcal{M}^2 \]

for the initial dimension-one \( \tilde{A} , \mu \) parameters and the dimension-two \( B \) parameter respectively. That is, there is a total of nine dimensionless \( c_i(0) \) parameters arising from the dimensionful parameters in our effective theory. However, this number can be reduced as follows.

First, note that all mass parameters scale with the same factor \( \mathcal{M} \). Hence, one can always redefine \( \mathcal{M} \) so as to absorb one of these coefficients. Without loss of generality, we can choose this to be the Higgs parameter. That is, set

\[ c_{H}(0) = 1 . \]
Second, in minimal supergravity and simple superstring vacua, the unified initial $\tilde{A}$ and gaugino mass parameters are numbers of order unity times the supersymmetry breaking scale $\mathcal{M}$. We will assume this in our calculation as well. For simplicity, choose

$$c_A(0) = 1 \, . \quad (4.52)$$

The initial value for $c_M$ is more subtle to determine. We have done an extensive numerical analysis of phenomenologically acceptable initial conditions allowing $c_M(0)$ to vary freely. The result is a bound given by $0.1 < c_M(0) < 1.2$. In this paper, for simplicity of presentation, we fix this initial parameter to a value in the middle of this range given by

$$c_M(0) = 0.6 \, . \quad (4.53)$$

Finally, we will also specify the coefficients $c_e(0)$ and $c_{\nu_1,2}(0)$ as follows.

In a previous paper [4], we presented a quasi-analytic solution to the RGEs in the $B-L$ MSSM theory subject to certain initial conditions on the parameters. To obtain an analytic solution, the initial parameters chosen were considerably more constrained than they are in this paper. Be that as it may, the generalized parameter space discussed here contains these initial conditions as a small subset. Specifically, we showed that at the $B-L$ scale $M_{B-L} \simeq 10^4 GeV$ the right-handed down slepton and right-handed sneutrino soft mass parameters are given by

$$m_{e_i}(t_{B-L})^2 = m_{e_i}(0)^2 - (3.35 \times 10^{-2})\mathcal{S}'_1(0) \, , \quad (4.54)$$

$$m_{\nu_i}(t_{B-L})^2 = m_{\nu_i}(0)^2 - (3.35 \times 10^{-2})\mathcal{S}'_1(0) \quad (4.55)$$
for \( i = 1, 2, 3 \) where, using (4.36), (4.47) and (4.48), one can write

\[
S'_1(0) = (1 + 2C^2 - 3A^2)m_{\nu_3}(0)^2
\]  

(4.56)

with

\[
C = \frac{c_{\nu_1,2}(0)}{c_{\nu_3}(0)} , \quad A = \frac{c_\nu(0)}{c_{\nu_3}(0)} .
\]  

(4.57)

For specificity, let us choose

\[
(3.35 \times 10^{-2})(1 + 2C^2 - 3A^2) = 5.
\]  

(4.58)

Then one obtains the simple result that

\[
m_{\nu_3}(t_{B-L})^2 = -4m_{\nu_3}(0)^2 ,
\]  

(4.59)

leading to a non-zero VEV in the \( \nu_3 \) direction. In this way, we guarantee radiative \( U(1)_{B-L} \) breaking in the theory. Similarly, using (4.58) we find that

\[
m_{\nu_{1,2}}(t_{B-L})^2 = (C^2 - 5)m_{\nu_3}(0)^2
\]  

(4.60)

and

\[
m_{e_i}(t_{B-L})^2 = (A^2 - 5)m_{\nu_3}(0)^2
\]  

(4.61)

for \( i = 1, 2, 3 \). The simplest vacuum structure occurs when all \( m_{e_i}(t_{B-L})^2 \) are positive. For this to be the case, the coefficient \( A \) must satisfy \( A^2 - 5 > 0 \). Again, for specificity we will choose

\[
A = \sqrt{6} ,
\]  

(4.62)

which yields the simple result that

\[
m_{e_i}(t_{B-L})^2 = m_{\nu_3}(0)^2
\]  

(4.63)
for $i = 1, 2, 3$. Putting $A = \sqrt{6}$ into expression (4.58) gives

$$C = 9.12 . \quad (4.64)$$

It then follows from (4.60) that both $m_{\nu_{1,2}}(t_{B-L})^2$ are positive and given by

$$m_{\nu_{1,2}}(t_{B-L})^2 = 78.2 \ m_{\nu_3}(0)^2 . \quad (4.65)$$

We conclude that the choice of the $A$ and $C$ parameters given in (4.62) and (4.64) respectively leads to a vacuum that has positive soft squared masses and, hence, vanishing VEVs for all sleptons with the exception of the third family right-handed sneutrino. This acquires a non-zero VEV which radiatively breaks $U(1)_{B-L}$ symmetry. It is clear that these choices for $A$ and $C$ are far from unique, and that a wide range of values would still lead to a vacuum with appropriate $U(1)_{B-L}$ symmetry breaking. Be that as it may, we find it convenient to continue to use (4.62) and (4.64) in the present paper. It then follows from (4.57) that we will choose

$$c_e(0) = \sqrt{6} \ c_{\nu_3}(0) , \quad c_{\nu_{1,2}}(0) = 9.12 \ c_{\nu_3}(0) . \quad (4.66)$$

The constraints given in (4.51), (4.52), (4.53) and (4.66) reduce the number of free parameters down to six— four $c_i$ parameters as well as $\mathcal{M}$ and $\tan \beta$. There are, however, important phenomenological constraints on these parameters, to which we now turn.
4.4.8 Phenomenological Constraints

It is well-known [87] that for an MSSM-like theory with two Higgs doublets, $H$ and $\bar{H}$, to have a stable vacuum solution that breaks electroweak symmetry, the parameters of the theory have to satisfy two constraints at the electroweak scale $M_{EW} \simeq 10^2 GeV$. These are

$$B^2 > (|\mu|^2 + m_H^2)(|\mu|^2 + m_H^2),$$

which ensures that one linear combination of $H$ and $\bar{H}$ has a negative squared mass, thus enabling a non-zero Higgs VEV to form, and

$$2B < 2|\mu|^2 + m_H^2 + m_{H}^2,$$

which guarantees that the quadratic part of the potential energy is positive along the $D$-flat directions and, hence, that the potential energy is bounded from below. Once these conditions are satisfied, the theory has a stable Higgs vacuum specified by the two minimization equations. Their solutions can be put in the form

$$\sin(2\beta) = \frac{2B}{m_H^2 + m_{\bar{H}}^2 + 2|\mu|^2}$$

and

$$M_Z^2 = \frac{|m_H^2 - m_{\bar{H}}^2|}{\sqrt{1 - \sin^2(2\beta)}} - m_H^2 - m_{\bar{H}}^2 - 2|\mu|^2$$

with the parameters evaluated at the electroweak scale.

In many analyses of electroweak breaking, all the soft masses and the $\mu$ parameter are given as input with $\tan \beta$ and $M_Z$ generated as solutions of (4.69) and (4.70).
However, as discussed above, it is convenient in this paper to take \( \tan \beta \) as an input parameter. It follows that equation (4.69) should be viewed as yet another constraint on the soft breaking parameters. Specifically, we will use (4.69) to solve for \( B \) as a function of \( \tan \beta, m_{H}^{2}, m_{\bar{H}}^{2} \) and \( \mu \) at the electroweak scale. This is possible since the RGEs for \( m_{H}^{2}, m_{\bar{H}}^{2} \) and \( \mu \) \cite{86} and, hence, the value of these parameters at the electroweak scale do not depend implicitly on \( B \). Written in terms of the notation introduced in the previous section, it follows that

\[
c_{B}^2 = \frac{\sin(2\beta)}{2}(c_{H}^2 + c_{\bar{H}}^2 + 2|c_{\mu}|^2).
\]

(4.71)

We can then scale this parameter back up to the \( M_u \) to determine the initial value \( B(0) \).

Similarly, we can input the experimental value of \( M_Z \) into (4.70) and use this to put a further constraint on the initial parameters. In terms of the above parameterization, (4.70) can be re-written as

\[
M_Z^2 = \left( \frac{|c_{\bar{H}}^2 - c_{H}^2|}{\sqrt{1 - \sin^2(2\beta)}} - c_{\bar{H}}^2 - c_{H}^2 - 2|c_{\mu}|^2 \right) M^2.
\]

(4.72)

From this equation we see that, given the initial values of \( c_i \) and \( \tan \beta \), one can use the experimentally derived value for \( M_Z \) to fix \( M \) and, thus, the soft breaking scale. Note that for fixed values of \( c_H, c_{\bar{H}} \) and \( \tan \beta \), mass \( M \) is a minimum as \( c_{\mu} \to 0 \) and becomes arbitrarily large as

\[
|c_{\mu}|^2 \to \frac{1}{2} \left( \frac{|c_{\bar{H}}^2 - c_{H}^2|}{\sqrt{1 - \sin^2(2\beta)}} - c_{\bar{H}}^2 - c_{H}^2 \right).
\]

(4.73)
It follows that the value of the supersymmetry breaking parameter is not particularly restricted by constraint (4.72). Be that as it may, obtaining its minimum value and, in particular, a large value requires fine-tuning $c_\mu$ to zero and (4.73) respectively. Without fine-tuning, the typical value for $\mathcal{M}$ is set by the $Z$-mass and, for the initial parameters in this paper, found to be of order a few hundred GeV up to order 10 TeV.

Applying constraints (4.71) and (4.72) to fix the values of $c_B(0)$ and $\mathcal{M}$ respectively, we are now left with four free parameters. They are

$$c_q(0), \ c_{\nu_3}(0), \ c_\mu(0), \ \tan \beta .$$ (4.74)

In the remainder of this paper, we analyze the vacuum state and mass spectrum of the $B$-$L$ MSSM theory over this four-dimensional initial parameter space. This is accomplished by numerically solving all RGEs for a given choice of initial conditions, scaling down from $M_u$ to $M_{EW}$. In doing this, however, we will impose several important phenomenological constraints—rejecting the initial parameters if the results fail to satisfy these constraints and accepting them if they are satisfied. In this way, one can map out the allowed region of the four-dimensional initial parameter space.

The phenomenological constraints we impose are the following.

- To ensure that a stable electroweak breaking vacuum can develop at low energy-momenta, we impose the constraint that inequalities (4.67) and (4.68)
be satisfied. This should be understood as a consistency check on our assumption, implicit in using $\tan \beta$ and the experimental value of $M_Z$ as input parameters, that a stable electroweak breaking vacuum described by (4.69) and (4.70) exists. In terms of the parameterization introduced in Subsection 3.7, these constraints are

\[ c_B^4 > (|c_\mu|^2 + c_H^2)(|c_\mu|^2 + c_{\bar{H}}^2) \]  

and

\[ 2c_B^2 < c_H^2 + c_{\bar{H}}^2 + 2|c_\mu|^2 \]  

respectively.

- As discussed above, condition (4.66) ensures that a vacuum expectation value develops in the third right-handed sneutrino. To guarantee that this is a stable local minimum, we impose the constraint that the effective squared masses of all squarks and sleptons evaluated at the $B-L$ breaking VEV $\langle \nu_3 \rangle$, for example,

\[ \langle m_{Q_i}^2 \rangle = m_{Q_i}^2 + \frac{1}{4} g_4^2 \langle \nu_3 \rangle^2, \quad \langle m_{L_i}^2 \rangle = m_{L_i}^2 - \frac{3}{4} g_4^2 \langle \nu_3 \rangle^2 \]  

are positive over the entire scaling range. It follows that color and charge symmetry are never spontaneously broken. Note that imposing the positivity of the effective masses does not necessarily restrict the soft squared masses to be positive. For example, the positivity of $\langle m_{Q_i}^2 \rangle$ does not require that $m_{Q_i}^2$ be positive. On the other hand, $m_{L_i}^2$ must be positive to ensure that $\langle m_{L_i}^2 \rangle$ is.
<table>
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<tr>
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<td>$Z'$ Boson</td>
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Table 4.1: Experimental lower bounds on the Higgs fields and sparticles in the MSSM. The $Z'$ mass is for an additional $U(1)$ gauge boson arising from spontaneously broken $SO(10)$.

This allows us to classify the $B$-$L$ MSSM vacua in terms of the signs of the soft squared masses at the electroweak scale. This will be discussed in detail later in the paper.

- An important phenomenological constraint is that our results be consistent with the observed bounds on the masses of the Higgs fields, Higgsinos and all squarks, sleptons and gauginos. These are given in the Particle Data Group review [6] and reproduced in Table 4.1. Note that these bounds serve as guidelines rather than strict bounds, since we are working with a model that
is somewhat different than the MSSM. The eigenstates of the various fields in the $B$-$L$ MSSM involve considerable mixing of the fields induced by the $\nu_3$ and $H, \bar{H}$ VEVs. This presents somewhat of a challenge in our analysis. Details of this diagonalization process, as well as a discussion of the role of the spontaneously broken $B$-$L$ gauge symmetry, are presented in Appendix E. In this paper, we compute the mass eigenvalues for the Higgs fields and all sparticles and compare the results to the values in Table 4.1. We disallow all initial conditions that violate these bounds.

4.5 Numerical Analysis

We now turn to the numerical analysis of the low-energy vacua associated with the four initial parameters given in (4.74). Even though the number of these parameters has been reduced to four, a systematic study of this space is still labor intensive. Happily, there is a natural splitting into two two-dimensional spaces. To see this, note that one of the physical properties we are most concerned with is the hierarchy between the $B$-$L$ and electroweak breaking. This hierarchy can be described in several ways [4]. Here, we will define the hierarchy as the ratio of the mass of the $U(1)_{B-L}$ gauge boson, given by

$$M_{A_{B-L}} = \sqrt{\frac{3}{2}} g_4 \langle \nu_3 \rangle, \quad \langle \nu_3 \rangle = \frac{|m_{\nu_3}|}{\sqrt{\frac{3}{4} g_4}}$$

(4.78)
evaluated at the electroweak scale, and the Z-boson mass given in (4.70). Written in terms of the parameterization introduced in Subsection 4.4.7, the hierarchy becomes

$$\frac{M_{A_{B-L}}}{M_Z} = \frac{\sqrt{2}|c_{\nu_3}|}{\left(\frac{c_H^2 - c_H^2}{\sqrt{1 - \sin^2(2\beta)}} - c_H^2 - c_H^2 - 2|c_H|^2\right)^{1/2}}. \quad (4.79)$$

The factor of $\mathcal{M}$ occurs in both the numerator and the denominator and, hence, cancels out of this expression. Of the five parameters in (4.79), only $c_{\nu_3}$, $c_\mu$ and $\tan \beta$ have arbitrary initial conditions. Noting that all $c_i$ coefficients, even when evaluated at the electroweak scale, are essentially of order unity, we see that the most influential factors in the size of the hierarchy are $c_\mu$ and $\tan \beta$. This is because for fixed $\tan \beta$ one can drive the denominator in (4.79) to zero, and, hence, the hierarchy to be arbitrarily large, by fine-tuning $c_\mu$. For this reason, we will examine the two-dimensional $c_\mu(0)$-$\tan \beta$ plane for different values of $c_q(0)$ and $c_{\nu_3}(0)$. This naturally splits the four-dimensional space of initial values into two two-dimensional surfaces, greatly simplifying the analysis.

### 4.5.1 All $m^2 > 0$

**Phenomenologically Allowed Regions and the Mass Spectrum:**

We first present our analysis subject to the following additional condition.

- With the exception of $m_{\nu_3}^2$, all squark and slepton soft squared masses are constrained to be positive over the entire scaling range.
To illustrate the procedure, pick an arbitrary point

\[ c_q(0) = 0.75, \quad c_{\nu_3}(0) = 0.75 \]  

(4.80)

in the \(c_q(0)-c_{\nu_3}(0)\) plane. For these initial values, we scan over the \(c_\mu(0)\)-tan \(\beta\) plane, first imposing the positive squark/slepton squared mass condition and then analyzing each point relative to the constraints discussed in the previous section. The results are shown in Figure 4.10. The positive squared mass condition is satisfied everywhere in the depicted region.

Figure 4.10(a) shows the regions where electroweak symmetry is and is not radiatively broken, indicated in yellow and white respectively. The yellow region is defined as the locus of points where both inequalities (4.75) and (4.76) are satisfied, whereas in any white region either one or both of these inequalities is violated. Before analyzing the individual areas, let us recall the consequences of each inequality. As discussed in Subsection 3.8, (4.75) guarantees that one linear combination of Higgs fields has a negative squared mass. In this case, satisfying inequality (4.76) implies a stable electroweak breaking vacuum. If, however, (4.76) is violated, the potential energy is not bounded from below and no stable vacuum state exits. On the other hand, violating inequality (4.75) indicates that the origin of Higgs space is either a local minimum or a local maximum of the potential energy, depending on whether or not (4.76) is satisfied.

Let us now discuss the individual regions. Anywhere in the yellow region both (4.75) and (4.76) are satisfied, leading to a stable electroweak breaking vacuum.
Figure 4.10: The $c_\mu(0)$-$\tan \beta$ plane corresponding to the point $c_\mu(0) = 0.75, c_\nu(0) = 0.75$. The yellow and white regions of (a) indicate where electroweak symmetry is and is not broken respectively. The individual regions satisfying the present experimental bounds for squarks and sleptons, gauginos and Higgs fields are shown in (b),(c) and (d), while their intersection is presented in (e). The dark brown area of (e) is the phenomenologically allowed region where electroweak symmetry is broken and all experimental mass bounds are satisfied. We present our predictions for the sparticle and Higgs masses at point (P).
Note that there are two separated areas where electroweak breaking does not occur. Our analysis shows that at any point in the upper white region it is the first inequality (4.75) that is violated, while (4.76) continues to be satisfied. This indicates a stable vacuum, but with vanishing Higgs VEVs. The transition between the yellow and upper white regions is defined by saturating inequality (4.75), that is,

$$c_B^4 = (|c_\mu|^2 + c_H^2)(|c_\mu|^2 + c_H^2).$$  \hspace{1cm} (4.81)

It follows from this and expression (4.71) that the boundary between these regions corresponds to the vanishing of $M_Z^2$ in (4.72), that is,

$$\frac{|c_B^2 - c_H^2|}{\sqrt{1 - \sin^2(2\beta)}} - c_H^2 - c_H^2 - 2|c_\mu|^2 = 0,$$  \hspace{1cm} (4.82)

plotted as a function of $\tan \beta$ and $c_\mu(0)$. Below this boundary $M_Z^2$ is positive, indicating electroweak symmetry breaking vacua. At and above this line, however, $M_Z^2$ vanishes, implying that electroweak symmetry is unbroken. Similarly, the lower right white region shown in Figure 4.10(a) also violates constraint (4.75) while satisfying (4.76). Hence, the above analysis applies here as well. For completeness, we point out that, beyond the boundaries shown in Figure 4.10(a), there is a transition of this lower right region to an area where both inequalities (4.75) and (4.76) are violated. In this regime, there are no stable vacua.

Figures 4.10(b),(c) and (d) indicate where our calculated masses of the squarks, sleptons, Higgs and gauginos respectively exceed the experimental lower bounds presented in Table 4.1. Finally, Figure 4.10(e) superimposes all of these with the
area of electroweak symmetry breaking, the dark brown region representing their intersection. Any point in this region has broken electroweak symmetry and a mass spectrum satisfying all experimental bounds. As an example, consider the point (P) indicated in this region. Our calculated values for the squark, slepton, Higgs and gaugino masses are presented in Table 4.2. Note that, as stated, their values all exceed the experimental bounds.

The above analysis was carried out for the arbitrarily chosen point (4.80) in the $c_q(0)$-$c_{\nu_3}(0)$ plane. We emphasize that although this point has a non-vanishing region in the $c_\mu(0)$-$\tan \beta$ plane satisfying all phenomenological bounds, this need not be the case for other points. To explore this, we now scan over the entire $c_q(0)$-$c_{\nu_3}(0)$ plane. At each point, we analyze the associated $c_\mu(0)$-$\tan \beta$ plane and see if an allowed region exists. The results are shown in Figure 4.11. The white region indicates points whose corresponding $c_\mu(0)$-$\tan \beta$ plane contains no locus of electroweak symmetry breaking. The yellow area represents points whose $c_\mu(0)$-$\tan \beta$ plane has a region where electroweak symmetry is broken. Finally, each point in the blue area has a phenomenologically allowed region in its corresponding $c_\mu(0)$-$\tan \beta$ plane satisfying the squark/slepton positive squared mass condition. Point (4.80) analyzed above is indicated by (A) in the diagram. It is of interest to see how the results change as we move to different phenomenologically allowed points in the $c_q(0)$-$c_{\nu_3}(0)$ plane. For example, consider point (B) shown in Figure 4.11.
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<td>(Z')</td>
<td>(A_{B-L}, \tilde{A}_{B-L})</td>
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Table 4.2: The predicted spectrum at point (P) in Figure 4.10(e). The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for right-handed fields. The considerable mixing between the third family left- and right-handed scalar fields is incorporated into these results.
Figure 4.11: A plot of the $c_q(0)$-$c_{\nu}(0)$ plane showing physically relevant areas. The yellow and white indicate points whose corresponding $c_\mu(0)$-tan$\beta$ plane does and does not contain a region of electroweak symmetry breaking respectively. Within the yellow area, the blue shading contains all points whose $c_\mu(0)$-tan$\beta$ plane has a non-vanishing region satisfying all experimental sparticle and Higgs bounds and for which all soft susy breaking masses remain positive over the entire scaling range. (A) and (B) indicate the two points analyzed in detail in the text.
This has the values

\[ c_q(0) = 1.4, \quad c_\nu(0) = 1.2. \tag{4.83} \]

For this point, the regions of the \( c_\mu(0) \)-tan\( \beta \) plane corresponding to the different constraints, as well as their intersection, are shown in Figure 4.12. The positive squared mass condition is satisfied everywhere in the depicted regime.

In the yellow region both (4.75) and (4.76) are satisfied, leading to stable electroweak breaking vacua. There are two separated areas where electroweak breaking does not occur. As occurred for point (A), anywhere in the upper white region the first inequality (4.75) is violated, while (4.76) continues to be satisfied. This indicates stable vacua, but with \textit{vanishing} Higgs VEVs. As discussed above, the boundary between the yellow and upper white regions corresponds to the vanishing of \( M_Z^2 \) in (4.72).

The regions where the squarks/sleptons, gauginos and Higgs exceed their experimental lower bounds are depicted in the indicated colors. Any point in the intersection area, shown in dark brown, has broken electroweak symmetry and a mass spectrum satisfying all experimental bounds. As an example, consider the point (Q) indicated in this region. Our calculated values for the squark, slepton, Higgs and gaugino masses are presented in Table 4.3. Note that, as stated, their values all exceed the experimental bounds.
Figure 4.12: The $c_\mu(0)$-$\tan \beta$ plane corresponding to the point $c_q(0) = 1.4, c_{\nu_3}(0) = 1.2$. The yellow and white regions indicate where electroweak symmetry is and is not broken respectively. The individual regions satisfying the present experimental bounds for squarks and sleptons, gauginos and Higgs fields are shown in the indicated colors. The dark brown area is their mutual intersection where electroweak symmetry is broken and all experimental mass bounds are satisfied. We present our predictions for the sparticle and Higgs masses at point (Q).
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<td>1314, 509</td>
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</table>

Table 4.3: The predicted spectrum at point (Q) in Figure 4.12. The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for right-handed fields. The mixing between the third family left- and right-handed scalar fields is incorporated.
The $B-L$/Electroweak Hierarchy:

We have determined the subspace of the $c_q(0)-c_{\nu_3}(0)$ plane for which each point has a region in the corresponding $c_{\mu}(0)$-$\tan \beta$ plane satisfying 1) the positive squark/slepton squared mass condition with 2) broken electroweak symmetry and 3) phenomenologically acceptable squark, slepton, Higgs and gaugino masses. Given such a point in the $c_q(0)-c_{\nu_3}(0)$ plane and choosing a point in the acceptable region in the $c_{\mu}(0)$-$\tan \beta$ plane, we now analyze the following question: What is the $B-L$/electroweak hierarchy for these initial values?

An expression for the $B-L$/electroweak hierarchy in terms of the $c_i$ coefficients and $\tan \beta$ was given in (4.80). We repeat it here for convenience.

$$M_{\delta^{B-L}/Z} = \frac{\sqrt{2}|c_{\nu_3}|}{\left(\sqrt{\frac{c^2_{\mu}-c^2_{H}}{1-\sin^2(2\beta)}} - c^2_{H} - 2|c_{\mu}|^2\right)^{1/2}}.$$  \hspace{1cm} (4.84)

For the specific point chosen in the initial $c_q(0)$, $c_{\nu_3}(0)$, $c_{\mu}(0)$, $\tan \beta$ parameter space, one can scale all quantities down to the electroweak scale and evaluate the hierarchy using (4.84). As a concrete example, consider point (A) in the $c_q(0)$-$c_{\nu_3}(0)$ plane of Figure 4.11. The corresponding regions of the $c_{\mu}(0)$-$\tan \beta$ plane were superimposed in Figure 4.10(e) and are presented again in Figure 4.13(a). The allowed region is the dark brown area. For (A) given in (4.81), the $B-L$/electroweak hierarchy is evaluated for each point in this allowed region and plotted in Figure 4.13(b). We find that the hierarchy takes values of 6.30-6.36 along the lower boundary of the allowed region. Note that below this boundary at least one of the gaugino or Higgs
Figure 4.13: Plot (a) shows the $c_\mu(0)$-tan $\beta$ plane corresponding to point (A) in Figure 4.11 with the phenomenologically allowed region indicated in dark brown. The mass spectrum at (P) was presented in Table 4.2. A plot of the hierarchy $M_{B-L}/M_Z$ over the allowed region is given in (b). Graph (c) shows the hierarchy as a function of $c_\mu(0)$ along the tan $\beta = 18$ line passing through (P).
masses violates their experimental bound. Hence, the lower values of the hierarchy are determined from the experimental data. On the other hand, as one approaches the boundary with the upper white region, the hierarchy becomes infinitely large. To understand this, recall from (4.82) that this boundary is determined by the vanishing of $M^2_Z$ in (4.73), that is,

$$\frac{|c_H^2 - c_H^2|}{\sqrt{1 - \sin^2(2\beta)}} - c_H^2 - c_H^2 - 2|c_\mu|^2 = 0.$$  (4.85)

Hence, at any point on this boundary the denominator in (4.84) vanishes and

$$\frac{M_{A_{B-L}}}{M_Z} \to \infty.$$  (4.86)

It follows that within the phenomenologically acceptable region, any value of the B-L hierarchy in the range $6.30 \lesssim M_{A_{B-L}}/M_Z < \infty$ can be attained.

Another way to analyze the data is to pick a specific point in the allowed region and to compute (4.84) as a function of $c_\mu(0)$ along the fixed tan $\beta$ line passing through it. For concreteness, choose the point (P) for which we calculated the mass spectrum in Table 4.2. This is shown in Figure 4.13(a) along with the dotted line $\tan \beta = 18$ intersecting it. The B-L/electroweak hierarchy along this line is plotted in Figure 4.13(c). Note that this begins at $M_{A_{B-L}}/M_Z = 6.35$ at the experimentally determined lower boundary, rises slowly to $M_{A_{B-L}}/M_Z \sim 20$ across most of the region, and then rapidly diverges to infinity as one approaches the upper boundary. Approaching both the lower and, especially, the upper boundary requires fine-tuning of $c_\mu(0)$. For “typical” values of $c_\mu(0)$, the hierarchy is naturally in the
\begin{equation}
10 \lesssim \frac{M_{A_{B-L}}}{M_Z} \lesssim 20.
\end{equation}

As a second example, consider point (B) in the $c_q(0)-c_{\nu_3}(0)$ plane of Figure 4.11. The corresponding regions of the $c_{\mu}(0)$-tan $\beta$ plane were superimposed in Figure 4.12 and presented again in Figure 4.14(a). The allowed region is the dark brown area. For (B) given in (4.84), the $B$-$L$/electroweak hierarchy is evaluated for each point in this allowed region and plotted in Figure 4.14(b). We find that the hierarchy takes values of 10.00-10.21 along the lower boundary of the allowed region, below which at least one of the gaugino or Higgs masses violates their experimental bound. Again, as one approaches the boundary with the upper white region, the hierarchy becomes infinitely large. It follows that within the phenomenologically acceptable region any value of the $B$-$L$ hierarchy in the range $10 \lesssim M_{A_{B-L}}/M_Z < \infty$ can be attained.

Another way to analyze the data is to pick a specific point in the allowed region and to compute (4.84) as a function of $c_{\mu}(0)$ along the fixed tan $\beta$ line passing through it. For concreteness, choose the point (Q) for which we calculated the mass spectrum in Table 4.3. This is shown in Figure 4.14(a) along with the dotted line tan $\beta = 12$ intersecting it. The $B$-$L$/electroweak hierarchy along this line is plotted in Figure 4.14(c). Note that this begins at $M_{A_{B-L}}/M_Z = 10.15$ at the experimentally determined lower boundary, rises slowly to $M_{A_{B-L}}/M_Z \sim 30$ across most of the region, and then rapidly diverges to infinity as one approaches the upper
Figure 4.14: Plot (a) shows the $c_\mu(0)$-tan$\beta$ plane corresponding to point (B) in Figure 4.11 with the phenomenologically allowed region indicated in dark brown. The mass spectrum at (Q) was presented in Table 4.3. A plot of the hierarchy $M_{B-L}/M_Z$ over the allowed region is given in (b). Graph (c) shows the hierarchy as a function of $c_\mu(0)$ along the tan$\beta = 12$ line passing through (Q).
boundary. For “typical” values of $c_\mu(0)$ not fine-tuned near either boundary, the hierarchy is *naturally* in the range

\[ 15 \lesssim \frac{M_{A_{B-L}}}{M_Z} \lesssim 30 . \]  \hspace{1cm} (4.88)

### 4.5.2 $m_{Q_3}^2 < 0$

**The Potential Energy for $m_{\nu_3}^2 < 0$ and $m_{Q_3}^2 < 0$:***

For the choice of parameters in (4.66), all sleptons have positive soft squared masses with the exception of the third family right-handed sneutrino, for which $m_{\nu_3}^2 < 0$. As noted in Subsection 3.8, imposing positivity on the effective masses of the left-handed squarks at the $B$-$L$ breaking VEV $\langle \nu_3 \rangle$, that is,

\[ \langle m_{Q_i}^2 \rangle = m_{Q_i}^2 + \frac{1}{4} g_3^2 \langle \nu_3 \rangle^2 > 0 , \]  \hspace{1cm} (4.89)

does *not* require that $m_{Q_i}^2$ be positive. In general, one or more of these soft squared masses can be negative. Despite our assumption in (4.31),(4.46) that the initial squark masses are universal, the effect of the large third family up-Yukawa coupling in the RGEs is to break this degeneracy, driving $m_{Q_3}^2$ negative more quickly than the first and second family squark masses. Therefore, for simplicity, we explore the possibility that only the third family left-handed squark soft mass becomes negative, $m_{Q_3}^2 < 0$, as it is scaled down to electroweak energy-momenta.

The electroweak phase transition breaks the left-handed $SU(2)_L$ doublet $Q_3$ into its up- and down- quark components $U_3$ and $D_3$ respectively. The leading
order contribution of the Higgs VEVs to their mass splits the degeneracy between these two fields, destabilizing the potential most strongly in the $D_3$ direction. For this reason, the relevant Lagrangian for analyzing this vacuum can be restricted to

$$\mathcal{L} = |D_{\nu_3\mu}\nu_3|^2 - \frac{1}{4} F_{B-L\mu\nu}F_{B-L}^{\mu\nu} + |D_{D_3\mu}D_3|^2 - \frac{1}{4} F_{Y\mu\nu}F_Y^{\mu\nu} - \frac{1}{4} F_{SU(2)\mu\nu}F_{SU(2)}^{\mu\nu} - \frac{1}{4} F_{SU(3)\mu\nu}F_{SU(3)}^{\mu\nu} - V(\nu_3, D_3)$$

(4.90)

where

$$D_{\nu_3\mu} = \partial_\mu - ig_{B-L}A_{B-L\mu},$$

(4.91)

$$D_{D_3\mu} = \partial_\mu - \frac{ig_{B-L}}{3} A_{B-L\mu} - \frac{ig_Y}{6} A_{Y\mu} - ig_2 A_{SU(2)\mu} - ig_3 A_{SU(3)\mu}$$

and

$$V(\nu_3, D_3) = m_{\nu_3}^2 |\nu_3|^2 + m_{D_3}^2 |D_3|^2 + \frac{g_{B-L}^2}{2} (|\nu_3|^2 + \frac{1}{3} |D_3|^2)^2$$

$$+ \frac{1}{2} \left( \frac{g_Y^2}{36} + \frac{g_2^2}{4} + \frac{g_3^2}{3} \right) |D_3|^4.$$  

(4.92)

The first two terms in the potential are the soft supersymmetry breaking masses in (4.23), while the remaining terms are supersymmetric and arise from $D_{B-L}$, $D_Y$ in (4.21), (4.20) and $D_{SU(2)}$, $D_{SU(3)}$ respectively. Using $\lambda_{d_3} \simeq 5 \times 10^{-2}$, a hierarchy with $\langle H^0 \rangle \ll \langle \nu_3 \rangle$ and assuming $|m_{D_3}|$ is of order $|m_{\nu_3}|$, terms proportional to the Higgs VEVs are small and are ignored in (4.92). For simplicity, we henceforth drop the small $\frac{g_{B-L}^2}{9} + \frac{g_Y^2}{36}$ piece of the $D$-term contribution.

If both $m_{\nu_3}^2 < 0$, $m_{D_3}^2 < 0$ at the electroweak scale, then the potential is unstable at the origin of field space and has two other local extrema at

$$\langle \nu_3 \rangle^2 = -\frac{m_{\nu_3}^2}{g_{B-L}^2}, \quad \langle D_3 \rangle = 0,$$

(4.93)
and

\[ \langle \nu_3 \rangle = 0, \quad \langle D_3 \rangle^2 = -\frac{m_{D_3}^2}{g_2^2/4 + g_3^2/3} \quad (4.94) \]

respectively. Using these, potential (4.92) can be rewritten as

\[
V(\nu_3, D_3) = \frac{g_{B-L}^2}{2} (|\nu_3|^2 - \langle \nu_3 \rangle^2)^2 + \frac{g_{B-L}^2}{3} |\nu_3|^2 |D_3|^2 \\
+ \frac{g_2^2/4 + g_3^2/3}{2} (|D_3|^2 - \langle D_3 \rangle^2)^2. \quad (4.95)
\]

Let us analyze these two extrema. Both have positive masses in their radial directions. At the sneutrino vacuum (4.93), the mass squared in the \( D_3 \) direction is given by

\[
m_{D_3}(\nu_3) = \frac{g_{B-L}^2}{3} \langle \nu_3 \rangle^2 - (\frac{g_2^2}{4} + \frac{g_3^2}{3}) \langle D_3 \rangle^2 = \frac{|m_{\nu_3}|^2}{3} - |m_{D_3}|^2, \quad (4.96)
\]

whereas at the \( D_3 \) vacuum (4.94), the mass squared in the \( \nu_3 \) direction is

\[
m_{\nu_3}(D_3) = \frac{g_{B-L}^2}{3} \langle D_3 \rangle^2 - g_{B-L}^2 \langle \nu_3 \rangle^2 = |m_{D_3}|^2 (\frac{g_{B-L}^2}{3g_2^2/4 + g_3^2}) - |m_{\nu_3}|^2. \quad (4.97)
\]

Note that either (4.96) or (4.97) can be positive, but not both. To be consistent with the hierarchy solution, we want (4.93) to be a stable minimum. Hence, we demand \( m_{D_3}(\nu_3) > 0 \) or, equivalently, that

\[
|m_{\nu_3}|^2 > 3|m_{D_3}|^2. \quad (4.98)
\]

We will impose (4.98) as an additional condition for the remainder of this subsection.

It then follows from (4.97) that \( m_{\nu_3}(D_3) < 0 \) and, hence, the \( D_3 \) extremum (4.94) is a saddle point. As a consistency check, note that \( V(\nu_3) < V(D_3) \) if and only if

\[
g_{B-L}^2 \langle \nu_3 \rangle^4 > (\frac{g_2^2}{4} + \frac{g_3^2}{3}) \langle D_3 \rangle^4 \quad (4.99)
\]
or, equivalently,

\[ |m_{\nu_3}|^2 > |m_{D_3}|^2 \left( \frac{g_{B-L}^2}{3g_2^2/4 + g_3^2} \right)^{1/2}. \]  

(4.100)

This follows immediately from constraint (4.98).

Finally, note that the potential descends monotonically along a path \( C \) from the saddle point at (4.94) to the absolute minimum at (4.95). Solving the \( \frac{\partial V}{\partial D_3} = 0 \) equation, this curve is found to be

\[ |D_3|^2 = \left( \left(\langle D_3 \rangle \right)^2 - |\nu_3|^2 \left( \frac{g_{B-L}^2}{3g_2^2/4 + g_3^2} \right) \right)^{1/2}. \]  

(4.101)

Note that it begins at \( \langle D_3 \rangle \) for \( \nu_3 = 0 \) and continues until it tangentially intersects the \( D_3 = 0 \) axis at \( |\nu_{30}| = \sqrt{3} \left( \frac{|m_{D_3}|}{|\nu_{30}|} \right) \langle \nu_3 \rangle \). From here, the path continues down this axis to the stable minimum at (4.93). We conclude that at the electroweak scale the absolute minimum of potential (4.92) occurs at the sneutrino vacuum given in (4.93).

**Phenomenologically Allowed Regions and the Mass Spectrum:**

In this subsection, we analyze our results subject to the following additional conditions.

- The **third family left-handed down-squark soft mass squared will be constrained to be negative**, that is, \( m_{D_3}^2 < 0 \). All other squark and slepton soft squared masses are positive over the entire scaling range, with the exception of \( m_{\nu_3}^2 \).

- To ensure that the \( B-L \) breaking VEV is the absolute minimum, we impose
condition (4.98),

\[ |m_{\nu_3}|^2 > 3|m_{D_3}|^2 , \]  

(4.102)

at the electroweak scale.

We will refer to these two conditions collectively as the \( m_{D_3}^2 < 0 \) mass condition.

As discussed in the previous subsection, we proceed by scanning over the entire \( c_q(0)-c_{\nu_3}(0) \) plane, at each point analyzing the associated \( c_\mu(0) - \tan \beta \) plane to see if an allowed region exists. The results are shown in Figure 4.15. As in Figure 4.11, the white region indicates points whose corresponding \( c_\mu(0)-\tan \beta \) plane contains no locus of electroweak symmetry breaking, whereas the yellow area represents points whose \( c_\mu(0)-\tan \beta \) plane has a region where electroweak symmetry is broken. Finally, each point in the red area has a phenomenologically allowed region in its corresponding \( c_\mu(0)-\tan \beta \) plane satisfying the \( m_{D_3}^2 < 0 \) mass condition. Note that this is distinct from the blue region in Figure 4.11, where all squark/slepton mass squares are positive. Let us analyze the properties of an arbitrary point in the red area. For example, consider point (C) shown in Figure 4.15. This has the values

\[ c_q(0) = 1.0 \ , \ c_{\nu}(0) = 1.1 \ . \]  

(4.103)

For this point, the regions of the \( c_\mu(0)-\tan \beta \) plane corresponding to the different constraints, as well as their intersection, are shown in Figure 4.16. The \( m_{D_3}^2 < 0 \) mass condition is satisfied everywhere in the depicted regime.

In the yellow region both (4.75) and (4.76) are satisfied, leading to stable electroweak breaking vacua. There are two separated areas where electroweak breaking
Figure 4.15: A plot of the $c_q(0)-c_{\nu}(0)$ plane showing physically relevant areas. The yellow and white indicate points whose corresponding $c_q(0)$-tan$\beta$ plane does and does not contain a region of electroweak symmetry breaking respectively. Within the yellow area, the red shading contains all points whose $c_q(0)$-tan$\beta$ plane has a non-vanishing region satisfying all experimental sparticle and Higgs bounds and for which $m_{D_3}^2 < 0$. (C) indicate the point analyzed in detail in the text.

This indicates stable vacua, but with vanishing Higgs VEVs. It follows that the boundary between the yellow and white regions corresponds to $M_Z^2$ in (4.72) becoming zero. The regions where the squarks/sleptons, gauginos and Higgs exceed their experimental lower bounds are depicted in the indicated colors. Any point in the intersection area, shown in dark brown, has broken electroweak symmetry and an acceptable mass spectrum. As an example, consider the point (R) indicated in this region. Our calculated values for the squark, slepton, Higgs and gaugino masses are presented in Table 4.4.
Figure 4.16: The $c_{\mu}(0)$-tan $\beta$ plane corresponding to the point $c_{q}(0) = 1.0, c_{\nu}(0) = 1.1$. The yellow and white regions indicate where electroweak symmetry is and is not broken respectively. The individual regions satisfying the present experimental bounds for squarks and sleptons, gauginos and Higgs fields are shown in the indicated colors. The dark brown area is their mutual intersection where electroweak symmetry is broken and all experimental mass bounds are satisfied. We present our predictions for the sparticle and Higgs masses at point (R).
The B-L/Electroweak Hierarchy:

We have determined the subspace of the $c_q(0)-c_{\nu_3}(0)$ plane for which each point has a region in the corresponding $c_\mu(0)$-$\tan \beta$ plane satisfying 1) the $m_{D_3}^2 < 0$ mass condition with 2) broken electroweak symmetry and 3) phenomenologically acceptable squark, slepton, Higgs and gaugino masses. Given such a point in the $c_q(0)$-$c_{\nu_3}(0)$ plane and choosing a point in the acceptable region in the $c_\mu(0)$-$\tan \beta$ plane, we now analyze the B-L/electroweak hierarchy for these initial values.

An expression for this hierarchy in terms of the $c_i$ coefficients and $\tan \beta$ was given in (4.84). For the specific point chosen in the initial $c_q(0), c_{\nu_3}(0), c_\mu(0), \tan \beta$ parameter space, one can scale all quantities down to the electroweak scale and use this expression to evaluate the hierarchy. As a concrete example, consider point (C) in the $c_q(0)$-$c_{\nu_3}(0)$ plane of Figure 4.15. The corresponding regions of the $c_\mu(0)$-$\tan \beta$ plane were superimposed in Figure 4.16 and are presented again in Figure 4.17(a). The allowed region is the dark brown area. For (C) given in (4.103), the B-L/electroweak hierarchy is evaluated for each point in this allowed region and plotted in Figure 4.17(b). We find that the hierarchy takes values of 8.99-9.06 along the lower boundary of the allowed region. Note that below this boundary at least one of the gaugino or Higgs masses violates their experimental bound. Hence, the lower values of the hierarchy are determined from the experimental data.


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<th>Symbol</th>
<th>Mass [GeV]</th>
<th>Particle</th>
<th>Symbol</th>
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<tr>
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<td>$Z'$</td>
<td>(A_{B-L}, \tilde{A}_{B-L})</td>
<td>1199, 398</td>
</tr>
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Table 4.4: The predicted spectrum at point (R) in Figure 4.16. The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for right-handed fields. The mixing between the third family left- and right-handed scalar fields is incorporated.
the other hand, as one approaches the boundary with the upper white region, the hierarchy becomes infinitely large. As discussed in the previous subsection, this is explained by the vanishing of $M_Z^2$ in (4.72). Hence, at any point on this boundary the denominator in (4.84) vanishes and $M_{A_{B-L}}/M_Z \rightarrow \infty$. It follows that within the phenomenologically acceptable region, any value of the $B$-$L$ hierarchy in the range $8.99 \lesssim M_{A_{B-L}}/M_Z < \infty$ can be attained.

Another way to analyze the data is to pick a specific point in the allowed region and to compute (4.84) as a function of $c_{\mu}(0)$ along the fixed $\tan \beta$ line passing through it. For concreteness, choose the point (R) for which we calculated the mass spectrum in Table 4.4. This is shown in Figure 4.17(a) along with the dotted line $\tan \beta = 14$ intersecting it. The $B$-$L$/electroweak hierarchy along this line is plotted in Figure 4.17(c). Note that this begins at $M_{A_{B-L}}/M_Z = 9.0$ at the experimentally determined lower boundary, rises slowly to $M_{A_{B-L}}/M_Z \sim 40$ across most of the region, and then rapidly diverges to infinity as one approaches the upper boundary. Approaching both the lower and, especially, the upper boundary requires fine-tuning of $c_{\mu}(0)$. For “typical” values of $c_{\mu}(0)$, the hierarchy is naturally in the range

$$15 \lesssim \frac{M_{A_{B-L}}}{M_Z} \lesssim 40 \ . \quad (4.104)$$

“Mixed” $m^2 > 0$ and $m^2_{D_3} < 0$ Mass Conditions:

It is of interest to superimpose the blue region in Figure 4.11, satisfying the $m^2 > 0$ mass condition, with the red region of Figure 4.15, defined by the $m^2_{D_3} < 0$ con-
Figure 4.17: Plot (a) shows the $c_{\mu}(0)$-tan$\beta$ plane corresponding to point (C) in Figure 4.15 with the phenomenologically allowed region indicated in dark brown. The mass spectrum at (R) was presented in Table 4.4. A plot of the hierarchy $M_{B-L}/M_{Z}$ over the allowed region is given in (b). Graph (c) shows the hierarchy as a function of $c_{\mu}(0)$ along the tan$\beta = 14$ line passing through (R).
Figure 4.18: A plot of the $c_q(0)$-$c_{\nu_3}(0)$ plane showing both the blue and red regions presented in Figures 4.11 and 4.15 respectively. They have a non-vanishing intersection, indicated in purple. Any point in this overlap has an allowed region in the $c_\mu(0)$-$\tan\beta$ plane that is divided into two areas—one with $m^2 > 0$ and the second with $m^2_{D_3} < 0$. (D) indicates a point in this overlap region analyzed in detail in the text.

This is shown in Figure 4.18. Note that there is a non-vanishing intersection between these two areas. This is comprised of points in the $c_q(0)$-$c_{\nu_3}(0)$ plane whose phenomenologically allowed regions in the corresponding $c_\mu(0)$-$\tan\beta$ plane are each divided into two regimes—one satisfying the $m^2 > 0$ mass condition and the other the $m^2_{D_3} < 0$ constraint. As a specific example, consider the point (D) shown in Figure 4.18. This has the values

$$
c_q(0) = 1.0, \quad c_\nu(0) = 0.9.
$$

(4.105)

For this point, the areas of the $c_\mu(0)$-$\tan\beta$ plane corresponding to the different constraints, as well as their intersection, are shown in Figure 4.19. The regions where the squarks/sleptons, gauginos and Higgs exceed their experimental lower bounds are depicted in the indicated colors. Any point in the intersection area,
Figure 4.19: The $c_p(0)$-tan $\beta$ plane corresponding to the point $c_q(0) = 1.0, c_{\nu_3}(0) = 0.9$. The yellow and white regions indicate where electroweak symmetry is and is not broken respectively. The individual regions satisfying the present experimental bounds for squarks and sleptons, gauginos and Higgs fields are shown in the indicated colors. The dark brown area is their mutual intersection where electroweak symmetry is broken and all experimental mass bounds are satisfied. We present our predictions for the sparticle and Higgs masses at point (S). The dotted line passing to the right of (S) separates the $m^2 > 0$ region, to the left of this line, from the area where $m^2_{D_3} < 0$, to the right.
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<td>3005, 1273</td>
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Table 4.5: The predicted spectrum at point (S) in Figure 4.19. The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for right-handed fields. The mixing between the third family left- and right-handed scalar fields is incorporated.
shown in dark brown, has broken electroweak symmetry and an acceptable mass spectrum. Importantly, however, note the dotted line dividing this plane. We find that the $m^2 > 0$ mass condition is satisfied everywhere to the left of this line, whereas the $m^2_{D_3} < 0$ constraint holds at all points to the right—consistent with (D) being a point in the intersection of the blue and red regions. The dotted line is vertical since, to leading order the $D_3$ mass squared, although a function of $\tan \beta$, is independent of $c_\mu(0)$. The sparticle and Higgs mass spectrum for point (S) in the allowed region is presented in Table 4.5.

4.5.3 $m^2_{e_3} < 0$

The Initial Conditions and Potential Energy for $m^2_{\nu_3} < 0$ and $m^2_{e_3} < 0$:

As discussed in Subsection 3.8, to guarantee that the $B$-$L$ vacuum is a stable local minimum, we impose the constraint that the effective squared masses of all squarks and sleptons evaluated at $\langle \nu_3 \rangle$ are positive over the entire scaling range. Similarly to the left-handed squark mass condition (4.88), imposing positivity on the effective right-handed down slepton masses at the $B$-$L$ breaking VEV $\langle \nu_3 \rangle$, that is,

$$\langle m^2_{e_i} \rangle = m^2_{e_i} + \frac{3}{4} g_4^2 \langle \nu_3 \rangle^2 > 0,$$

(4.106)

does not require that $m^2_{e_i}$ be positive. In general, one or more of these soft squared masses can be negative. Recall from (4.32),(4.47) that we have assumed that all left-handed and right-handed down sleptons have a universal initial mass. This is similar to the initial condition on squark masses. Unlike the squarks, however,
the down-Yukawa couplings of sleptons are all too small to greatly effect the RGE running of their soft masses. It follows that, at a low scale, the three families of right-handed down sleptons mass squares tend to be all positive or all negative. Splitting this degeneracy, for example, to drive only $m_{\epsilon_3}^2 < 0$, requires considerable fine-tuning. Therefore, if one wishes to consider the case where only the third family squared mass turns negative, it is necessary to alter the initial slepton mass conditions given in Section 3. This is easily accomplished as follows.

As discussed in Subsection 3.3, the boundary conditions for the RGEs of the Higgs, squarks and sleptons squared masses are greatly simplified if one chooses the initial soft masses so that both $S(0) = 0$ and $S'_0(0) = 0$, with $S$ and $S'_0$ given in (4.30) and (4.34) respectively. Hence, in this paper we always choose the initial parameters to satisfy these two conditions. However, the specific choices made in Subsection 3.3 were overly constraining, since they imposed unification of all three families of squarks and sleptons, whereas the unification of each family separately is sufficient. In particular, condition (4.32) sets

$$m_{L_i}(0)^2 = m_{e_j}(0)^2$$

for all $i,j = 1,2,3$. This leads to the difficulty discussed above. However, this constraint can easily be weakened. The simplest example is to take

$$m_{L_{1,2}}(0)^2 = m_{e_{1,2}}(0)^2, \quad m_{L_3}(0)^2 = m_{e_3}(0)^2$$

which clearly continues to solve both $S(0) = 0$ and $S'_0(0) = 0$. Expression (4.47)
then generalizes to

\[
m_{L_{1,2}}(0) = m_{e_{1,2}}(0) = c_{e_{1,2}}(0)M, \quad m_{L_3} = m_{e_3}(0) = c_{e_3}(0)M. \tag{4.109}
\]

In terms of these parameters, (4.56) becomes

\[
S'_1(0) = (1 + 2C^2 - 2A^2 - A_3^2)m_{\nu_3}(0)^2, \tag{4.110}
\]

where

\[
C = \frac{c_{\nu_{1,2}}(0)}{c_{\nu_3}(0)}, \quad A = \frac{c_{e_{1,2}}(0)}{c_{\nu_3}(0)}, \quad A_3 = \frac{c_{e_3}(0)}{c_{\nu_3}(0)}. \tag{4.111}
\]

To stay as close as possible to our previous analysis, we continue to use the values

\[
A = \sqrt{6}, \quad C = 9.12 \tag{4.112}
\]

introduced in (4.62) and (4.64) respectively. In addition, let us choose

\[
A_3 = \sqrt{3}, \tag{4.113}
\]

thus minimally changing the value of (4.58) from 5 to 5.1. It follows that equations (4.59), (4.65), and the conclusions thereof for \(U(1)_{B-L}\) breaking, do not change substantially. Similarly, equation (4.63) for \(i = 1, 2\) is minimally altered to

\[
m_{e_{1,2}}(t_{B-L})^2 = ((\sqrt{6})^2 - 5.1) m_{\nu_3}(0)^2 = 0.9 m_{\nu_3}(0)^2. \tag{4.114}
\]

However, we now find that

\[
m_{e_3}(t_{B-L})^2 = ((\sqrt{3})^2 - 5.1) m_{\nu_3}(0)^2 = -2.1 m_{\nu_3}(0)^2. \tag{4.115}
\]
That is, splitting the slepton coefficient into $A = \sqrt{6}$ and $A_3 = \sqrt{3}$ allows the mass squares of the first two families to remain positive while constraining $m_{e_3}^2 < 0$, as desired. Henceforth, (4.66) is replaced by

$$c_{e_1,2}(0) = \sqrt{6} \, c_{\nu_3}(0), \quad c_{e_3}(0) = \sqrt{3} \, c_{\nu_3}(0), \quad c_{\nu_1,2}(0) = 9.12 \, c_{\nu_3}(0).$$

(4.116)

Despite these changes in the initial conditions, $c_q(0), c_{\nu_3}(0), c_\mu(0)$ and $\tan \beta$ in (4.74) remain the four independent parameters of our analysis.

The new set of initial parameters just discussed allows for the possibility that, at the electroweak scale, all soft squared masses are positive with the exception of $m_{\nu_3}^2 < 0$ and $m_{e_3}^2 < 0$. The relevant potential for discussing the vacuum of $\nu_3$ and $e_3$ is given by

$$V(\nu_3, e_3) = m_{\nu_3}^2 |\nu_3|^2 + m_{e_3}^2 |e_3|^2 + \frac{g_{B-L}^2}{2} (|\nu_3|^2 + |e_3|^2) + \frac{g_Y^2}{2} |e_3|^4. \quad (4.117)$$

The first two terms in the potential are the soft supersymmetry breaking mass terms in (4.23), while the third and fourth terms are supersymmetric and arise from the $D_{B-L}$ and $D_Y$ in (4.21) and (4.20) respectively. Contributions to (4.117) from the relevant Yukawa couplings in (4.17) are suppressed, since $\lambda_{\nu_3}$ and $\lambda_{e_3}$ are of order $10^{-10}$ and $10^{-2}$ respectively. Hence, we ignore them.

If both $m_{\nu_3}^2 < 0, m_{e_3}^2 < 0$ at the electroweak scale, then the potential is unstable at the origin of field space and has two other local extrema at

$$\langle \nu_3 \rangle^2 = -\frac{m_{\nu_3}^2}{g_{B-L}^2}, \quad \langle e_3 \rangle = 0 \quad (4.118)$$
and

\[ \langle \nu_3 \rangle = 0, \quad \langle e_3 \rangle^2 = - \frac{m_{e_3}^2}{g_{B-L}^2 + g_Y^2} \]  

(4.119)

respectively. Using these, potential (4.117) can be rewritten as

\[
V(\nu_3, e_3) = \frac{g_{B-L}^2}{2} (|\nu_3|^2 - \langle \nu_3 \rangle^2)^2 + g_{B-L}^2 |\nu_3|^2 |e_3|^2 \\
+ \frac{g_{B-L}^2 + g_Y^2}{2} (|e_3|^2 - \langle e_3 \rangle^2)^2 .
\]

(4.120)

Let us analyze these two extrema. Both have positive masses in their radial directions. At the sneutrino vacuum (4.118), the mass squared in the \( e_3 \) direction is given by

\[
m_{e_3}^2 |\langle \nu_3 \rangle|^2 = g_{B-L}^2 |\nu_3|^2 - (g_{B-L}^2 + g_Y^2) |\langle e_3 \rangle|^2 = |m_{e_3}|^2 - |m_{e_3}|^2 ,
\]

(4.121)

whereas at the stau vacuum (4.119), the mass squared in the \( \nu_3 \) direction is

\[
m_{\nu_3}^2 |\langle e_3 \rangle|^2 = g_{B-L}^2 |\langle e_3 \rangle|^2 - g_{B-L}^2 |\nu_3|^2 = |m_{\nu_3}|^2 - |m_{\nu_3}|^2 (1 + \frac{g_Y^2}{g_{B-L}^2})^{-1} - |m_{\nu_3}|^2 .
\]

(4.122)

Note that either (4.121) or (4.122) can be positive, but not both. To be consistent with the hierarchy solution, we want (4.118) to be a stable minimum. Hence, we demand \( m_{e_3}^2 |\langle \nu_3 \rangle| > 0 \) or, equivalently, that

\[ |m_{\nu_3}|^2 > |m_{e_3}|^2 . \]

(4.123)

We will impose (4.123) as an additional condition for the remainder of this subsection. It then follows from (4.122) that \( m_{\nu_3}^2 |\langle e_3 \rangle| < 0 \) and, hence, the stau extremum (4.119) is a saddle point. As a consistency check, note that \( V|_{\langle \nu_3 \rangle} < V|_{\langle e_3 \rangle} \) if and
only if
\[ g_{B-L}^2 \langle \nu_3 \rangle^4 > (g_{B-L}^2 + g_Y^2) (e_3)^4 \] (4.124)
or, equivalently,
\[ |m_{\nu_3}|^2 > |m_{e_3}|^2 (1 + \frac{g_Y^2}{g_{B-L}^2})^{-1/2} . \] (4.125)
This follows immediately from constraint (4.123). Finally, note that the potential
descends monotonically along a path \( C \) from the saddle point at (4.119) to the
absolute minimum at (4.118). Solving the \( \frac{\partial V}{\partial e_3} = 0 \) equation, this curve is found to
be
\[ |e_3|_C = (\langle e_3 \rangle^2 - |\nu_3|^2 (1 + \frac{g_Y^2}{g_{B-L}^2})^{-1})^{1/2} . \] (4.126)
Note that it begins at \( \langle e_3 \rangle \) for \( \nu_3 = 0 \) and continues until it tangentially intersects
the \( e_3 = 0 \) axis at \( |\nu_{30}| = \frac{|m_{e_3}|}{|m_{\nu_3}|} \langle \nu_3 \rangle \). From here, the path continues down this axis
to the stable minimum at (4.106). We conclude that at the electroweak scale the
absolute minimum of potential (4.117) occurs at the sneutrino vacuum given in
(4.118).

Phenomenologically Allowed Regions and the Mass Spectrum:

In this subsection, we analyze our results subject to the following additional conditions.

- The third family right-handed slepton soft mass squared will be constrained
to be negative, that is, \( m_{e_3}^2 < 0 \). All other squark and slepton soft squared
masses are positive over the entire scaling range, with the exception of \( m_{\nu_3}^2 \).
• To ensure that the $B$-$L$ breaking VEV is the absolute minimum, we impose
condition (4.123),
\[
|m_{\nu_3}|^2 > |m_{e_3}|^2 ,
\] (4.127)
at the electroweak scale.

We will refer to these two conditions collectively as the $m_{e_3}^2 < 0$ mass condition.

As discussed in previous subsections, we proceed by scanning over the entire
cq(0)-cν(0) plane, at each point analyzing the associated cμ(0)-tan β plane to see
if an allowed region exists. The results are shown in Figure 4.20. As in Figures
4.11 and 4.15, the white region indicates points whose corresponding cμ(0)-tan β
plane contains no locus of electroweak symmetry breaking, whereas the yellow area
represents points whose cμ(0)-tan β plane has a region where electroweak symmetry
is broken. Finally, each point in the green area has a phenomenologically allowed
region in its corresponding cμ(0)-tan β plane satisfying the $m_{e_3}^2 < 0$ mass condition.
Since some of the initial parameters are now different to allow for a negative stau
squared mass, this green region cannot be superimposed with the blue and red
regions discussed previously. Let us analyze the properties of an arbitrary point in
the green area. For example, consider point (E) shown in Figure 4.20. This has the
values
\[
c_q(0) = 1.1 , \quad c_\nu(0) = 0.5 .
\] (4.128)
For this point, the regions of the $c_\mu(0)$-tan β plane corresponding to the different
constraints, as well as their intersection, are shown in Figure 4.21. The $m_{e_3}^2 < 0$
Figure 4.20: A plot of the $c_q(0)$-$c_{\nu_3}(0)$ plane showing physically relevant areas. The yellow and white indicate points whose corresponding $c_\mu(0)$-$\tan \beta$ plane does and does not contain a region of electroweak symmetry breaking respectively. Within the yellow area, the green shading contains all points whose $c_\mu(0)$-$\tan \beta$ plane has a non-vanishing region satisfying all experimental sparticle and Higgs bounds and for which $m_{2_\nu}^2 < 0$. (E) indicate the point analyzed in detail in the text.

mass condition is satisfied everywhere in the depicted regime.

In the yellow region both inequalities (4.75) and (4.76) are satisfied, leading to stable electroweak breaking vacua. There are two separated areas where electroweak breaking does not occur. As before, anywhere in the upper white region the first inequality (4.75) is violated, while (4.76) continues to be satisfied. This indicates stable vacua, but with vanishing Higgs VEVs. It follows that the boundary between the yellow and upper white regions corresponds to the vanishing of $M_Z^2$ in (4.72). However, as at point (B), for example, the lower right white region shown in Figure 4.21 violates both constraints (4.75) and (4.76). Hence, the origin of Higgs space is a local maximum and the potential energy is unbounded from below. There
Figure 4.21: The $c_{\mu}(0)$-tan $\beta$ plane corresponding to the point $c_{\mu}(0) = 1.1$, $c_{\nu_{3}}(0) = 0.5$. The yellow and white regions indicate where electroweak symmetry is and is not broken respectively. The individual regions satisfying the present experimental bounds for squarks and sleptons, gauginos and Higgs fields are shown in the indicated colors. The dark brown area is their mutual intersection where electroweak symmetry is broken and all experimental mass bounds are satisfied. We present our predictions for the sparticle and Higgs masses at point (T).
are no stable vacua in this regime.

The regions where the squarks/sleptons, gauginos and Higgs exceed their experimental lower bounds are depicted in the indicated colors. Any point in the intersection area, shown in dark brown, has broken electroweak symmetry and an acceptable mass spectrum. As an example, consider the point (T) indicated in this region. Our calculated values for the squark, slepton, Higgs and gaugino masses are presented in Table 4.6.

The $B$-L/Electroweak Hierarchy:

We have determined the subspace of the $c_q(0)$-$c_{\nu_3}(0)$ plane for which each point has a region in the corresponding $c_\mu(0)$-$\tan \beta$ plane satisfying 1) the $m_{\nu_3}^2 < 0$ mass condition with 2) broken electroweak symmetry and 3) phenomenologically acceptable squark, slepton, Higgs and gaugino masses. Given such a point in the $c_q(0)$-$c_{\nu_3}(0)$ plane and choosing a point in the acceptable region in the $c_\mu(0)$-$\tan \beta$ plane, one can analyze the $B$-L/electroweak hierarchy for these initial values. The analysis proceeds exactly as in previous subsections, so we simply present the results.

For point (E) in Figure 4.20, we have computed the hierarchy everywhere in the dark brown area of Figure 4.21. We find that this takes values of 7.60-7.74 along the lower boundary of the allowed region. Note that below this boundary at least one of the gaugino or Higgs masses violates their experimental bound. Hence, the lower
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Table 4.6: The spectrum at point (T) in Figure 4.21. The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for the right-handed fields. The mixing between the third family left- and right-handed scalar fields is incorporated.
values of the hierarchy are determined from the experimental data. On the other hand, as one approaches the boundary with the upper white region, the hierarchy becomes infinitely large for the reasons previously discussed. It follows that within the phenomenologically acceptable region, any value of the $B$-$L$ hierarchy in the range $7.60 \lesssim M_{A_{B-L}}/M_Z < \infty$ can be attained.

Another way to analyze the data is to pick a specific point in the allowed region and to compute (4.84) as a function of $c_\mu(0)$ along the fixed $\tan \beta$ line passing through it. For concreteness, choose the point (T) with $\tan \beta = 22$ for which we calculated the mass spectrum in Table 4.6. We find that the hierarchy begins at $M_{A_{B-L}}/M_Z = 7.65$ at the experimentally determined lower boundary, rises slowly to $M_{A_{B-L}}/M_Z \sim 30$ across most of the region, and then rapidly diverges to infinity as one approaches the upper boundary. Approaching both the lower and, especially, the upper boundary requires fine-tuning of $c_\mu(0)$. For “typical” values of $c_\mu(0)$, the hierarchy is naturally in the range

$$10 \lesssim \frac{M_{A_{B-L}}}{M_Z} \lesssim 30.$$  

(4.129)

4.5.4 Summary

We first note that the above classification of vacua using the sign of $m_{Q_i}^2$ and $m_{e_i}^2$ is complete. The only other squared masses are for right-handed squarks and left-handed sleptons, which enter the effective masses at the $B$-$L$ breaking VEV $\langle \nu_3 \rangle$.
as
\[
\langle m^2_{u,i} \rangle = m^2_{u,i} - \frac{1}{4} g^2_i \langle \nu_3 \rangle^2, \quad \langle m^2_{d,i} \rangle = m^2_{d,i} - \frac{1}{4} g^2_i \langle \nu_3 \rangle^2
\] (4.130)

and
\[
\langle m^2_{L,i} \rangle = m^2_{L,i} - \frac{3}{4} g^2_i \langle \nu_3 \rangle^2
\] (4.131)

respectively. Since all of these effective masses must be positive to ensure that the vacuum is a stable minimum, it follows from the minus signs in each expression that \( m^2_{u,i}, m^2_{d,i}, \) and \( m^2_{L,i} \) must all be positive. Therefore, all \( m^2 > 0, m^2_{Q,i} < 0, \) and \( m^2_{e,i} < 0 \) in subsections 4.1, 4.2 and 4.3 respectively are the only possibilities.

From the above analysis, several broad conclusions can be made. For the reasons discussed above, we limited our search to the four-dimensional space of parameters listed in (4.74). By combining the results in the \( m^2 > 0, m^2_{Q,i} < 0, \) and \( m^2_{e,i} < 0 \) regimes, we can find the generic region of this parameter space for which one obtains a phenomenologically acceptable vacuum. The full range of allowed values for the \( c_q(0) \) and \( c_{\nu_3}(0) \) parameters were presented in Figures 4.18 and 4.20. From these, we observe a maximum range of
\[
0 < c_q(0) < 1.8, \quad 0 < c_{\nu_3}(0) < 1.5.
\] (4.132)

Similarly, by examining the \( c_\mu(0)\tan \beta \) plane over the allowed values of \( c_q(0) \) and \( c_{\nu_3}(0) \), the range of phenomenologically allowed values is found to be
\[
0.8 < c_\mu(0) < 1.75, \quad 8 < \tan \beta < 33.
\] (4.133)

To obtain this result, we computed the allowed regions for numerous points in the
c_q(0)-c_{\alpha}(0) plane including, but not limited to, (A)-(E) presented in the text. Thus, even with our restrictive premises in Section 3, a phenomenologically viable B-L MSSM vacuum exhibiting an acceptable hierarchy occurs for a reasonably wide space of initial parameters.

4.6 Some $\langle \nu_3 \rangle \neq 0$ Phenomenology

The results presented in this paper allow one to compute any quantity in our B-L MSSM theory at any energy scale. In particular, we have shown that for a wide range of initial conditions there is a stable vacuum which breaks both B-L and electroweak symmetry with an acceptable sparticle and Higgs mass spectrum and B-L/electroweak hierarchy. These are important necessary conditions on the theory, but are not sufficient to guarantee that it is phenomenologically viable. In this section, we explore two more important constraints arising from lepton number and baryon number violation respectively.

4.6.1 Lepton Number Violation

The most general superpotential invariant under gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ is presented in (4.17). Assuming a flavor diagonal basis, the superpotential becomes

$$W = \mu H\bar{H} + \sum_{i=1}^{3} \left( \lambda_{u,i}Q_iHu_i + \lambda_{d,i}Q_i\bar{H}d_i + \lambda_{e,i}L_iH\nu_i + \lambda_{\nu,i}L_i\bar{H}e_i \right). \quad (4.134)$$
Recall that since $U(1)_{B-L}$ contains matter parity, the dangerous lepton and baryon number violating terms in (4.18) are forbidden. Note, however, that these results are only valid at high scales where the gauge symmetry, in particular $U(1)_{B-L}$, is exact. At low energy-momentum the gauged $B-L$ symmetry is spontaneously broken, potentially allowing these operators to “grow back”. This can be analyzed by expanding the third family right-handed sneutrino around its VEV, that is, let $\nu_3 = \langle \nu_3 \rangle + \nu_3'$. Note that

$$\mu H \bar{H} + \lambda_{\nu_3} L_3 H \nu_3 = \mu H (\bar{H} + \epsilon_3 L_3) + \ldots ,$$

where

$$\epsilon_3 = \lambda_{\nu_3} \frac{\langle \nu_3 \rangle}{\mu} .$$

This motivates performing a rotation of the down Higgs and third family lepton doublet superfields given, to leading order, by

$$\bar{H}' = \bar{H} + \epsilon_3 L_3 , \quad L'_3 = L_3 - \epsilon_3 \bar{H} .$$

Written in terms of these new superfields, and then dropping the ' for simplicity, the superpotential becomes

$$\mathcal{W} = W + \epsilon_3 \sum_{i=1}^3 \lambda_{e,i} L_3 L_i e_i + \epsilon_3 \sum_{i=1}^3 \lambda_{d,i} L_3 Q_i d_i ,$$

where $W$ is given in (4.134). As expected, the lepton number violating terms of the form

$$L_3 L_i e_i , \quad L_3 Q_i d_i$$

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have grown back. Note, however, that the baryon violating terms $u_i d_j d_k$ have *not* been regenerated by the right-handed sneutrino VEV. In this subsection, we analyze the lepton violating interactions in (4.138). The question of baryon violation will be discussed in the next subsection.

It is well-known [93, 35, 36, 53] that the lepton number violating terms in (4.138) influence the baryon asymmetry at high temperature in the early universe. The requirement that the existing baryon asymmetry is not erased before the electroweak phase transition typically implies [34] that

$$\left( \frac{\epsilon_3}{10^{-6}} \right) \left( \frac{\tan \beta}{10} \right) \lesssim 1.$$  

(4.140)

Parameter $\epsilon_3$ for a given $\tan \beta$ can be explicitly evaluated for any $B$-$L$ MSSM vacuum using (4.136). For example, consider the vacuum specified by point (P) in Figure 4.10. This has the values $\tan \beta = 18$ and $c_\mu(0) = 1.0$. RG running $c_\mu$ down to the electroweak scale, we find that $c_\mu(t_{EW}) = 0.855$ and, hence, that $\mu = 0.855 \mathcal{M}$. The VEV of $\nu_3$ can be obtained using (4.118). For the parameters of this vacuum, $\langle \nu_3 \rangle = 4.433 \mathcal{M}$. Finally, unless otherwise stated we will use the highest estimate for the third family neutrino Yukawa coupling given by $\lambda_{\nu_3} \simeq 10^{-10}$. Putting these values into (4.136) gives $\epsilon_3 \simeq 5.185 \times 10^{-10}$ and, hence,

$$\left( \frac{\epsilon_3}{10^{-6}} \right) \left( \frac{\tan \beta}{10} \right) \simeq 0.933 \times 10^{-3},$$  

(4.141)

well below the necessary bound of unity. If we sample over all five vacua (P),(Q),(R),(S),(T)
specified above, we find that

\[ 0.688 \times 10^{-3} \lesssim \left( \frac{\epsilon_3}{10^{-6}} \right) \left( \frac{\tan \beta}{10} \right) \lesssim 1.04 \times 10^{-3} , \]  

(4.142)

in each case below the bound in (4.140). Note that taking \( \lambda_{\epsilon_3} < 10^{-10} \) leads to even smaller values for \((\epsilon_3/10^{-6})(\tan \beta/10)\). We conclude that our \( B-L \) MSSM theory generically satisfies the conditions for baryon asymmetry.

As discussed in [34, 95], theories with lepton number violating interactions of the form in (4.138) naturally solve many fundamental cosmological problems if the gravitino is the lightest supersymmetric partner (LSP). The lifetime of the gravitino is then found to be [34]

\[ \tau_{3/2} \simeq 10^{28} s \left( \frac{\epsilon_3}{10^{-7}} \right)^{-2} \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{m_{3/2}}{10 \text{ GeV}} \right)^{-3} . \]

(4.143)

Assuming that the lightest neutralino is the next-to-lightest superparticle (NLSP), one finds that

\[ \tau_{\text{NLSP}} \simeq 10^{-9} s \left( \frac{\epsilon_3}{10^{-7}} \right)^{-2} \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{m_{\tilde{N}_1}}{200 \text{ GeV}} \right)^{-3} . \]

(4.144)

These results are relevant to the \( B-L \) MSSM theory discussed in this paper. First, it is possible to choose parameters so that the gravitino is, indeed, the LSP. Second, as can be seen from the spectra presented in the previous section at five different points, the lightest standard model sparticle is always the neutralino \( \tilde{N}_1^0 \).

As an example, let us compute the lifetimes of the gravitino and the lightest neutralino at the point \( (P) \) in Figure 4.10. From Table 4.2, we see that \( \tilde{N}_1^0 = 100 \text{ GeV} \).
Hence, adjusting the gravitino mass to be, say, \( m_{3/2} = 80 \, \text{GeV} \), makes it the LSP while \( \tilde{N}_1^0 \) is the NLSP. Using this value for \( m_{3/2} \) and (4.141), it then follows from (4.143) that

\[
\tau_{3/2} \simeq 3.45 \times 10^{28} \, \text{s} .
\]  

(4.145)

Noting that the age of the universe is typically estimated to be 13.7 billion years, that is, \( 4.32 \times 10^{17} \) seconds, we see that the gravitino lifetime greatly exceed this. Hence, the gravitino is the primary candidate for dark matter. On the other hand, using \( \tilde{N}_1^0 = 100 \, \text{GeV} \) and (4.141), we find from (4.143) that

\[
\tau_{\text{NLSP}} \simeq 1.77 \times 10^{-6} \, \text{s} ,
\]  

(4.146)

much to short-lived to form dark matter. Let us extend these results by evaluating the LSP and NLSP lifetimes at the five points \((P),(Q),(R),(S),(T)\) specified above. Choosing \( m_{3/2} \) to be 20 \( \text{GeV} \) lighter than the corresponding \( \tilde{N}_1^0 \) mass, we find using (4.142) that

\[
1.65 \times 10^{28} \, \text{s} \lesssim \tau_{3/2} \lesssim 2.47 \times 10^{29} \, \text{s} \]  

(4.147)

and

\[
1.45 \times 10^{-6} \, \text{s} \lesssim \tau_{\text{NLSP}} \lesssim 5.52 \times 10^{-6} \, \text{s} .
\]  

(4.148)

Finally, note that choosing \( \lambda_{\nu_3} < 10^{-10} \) lowers the value of \( \epsilon_3 \) and, hence, will increase the gravitino and neutralino lifetimes. However, it is always sufficiently large so as to make the NLSP decay much more rapidly than the lifetime of the universe. For example, taking the often quoted value of \( \lambda_{\nu_3} = 10^{-12} \) changes \( \tau_{\text{NLSP}} \)
to be of order $10^{-2}s$, still very short lived. We conclude that for a gravitino LSP, our $B$-$L$ MSSM theory generically has a long-lived gravitino consistent with it being dark matter, as well as an NLSP which decays very rapidly.

4.6.2 Baryon Number Violation

Recall that since $U(1)_{B-L}$ contains matter parity, the dangerous lepton and baryon number violating interactions in (7) are disallowed in the high energy superpotential. At much lower scales, the $B$-$L$ violating VEV $\langle \nu_3 \rangle$ can potentially re-introduce these terms. As discussed above, however, this VEV induces from the dimension four superpotential only the lepton number violating interactions in (4.138). The baryon number violating $u_i d_j d_k$ terms are not regenerated. Therefore, to this order, baryon number is conserved and the proton is completely stable. However, the superpotential can contain $B$-$L$ invariant higher dimensional terms proportional to $u_i d_j d_k \nu_3$. When the sneutrino develops a non-zero VEV, this term generates an effective dimension four operator of the form

$$\frac{\langle \nu_3 \rangle}{M_c} u_i d_j d_k , \quad (4.149)$$

where $M_c$ is the compactification scale which we loosely identify with $M_u$ in (4.27). In addition, we have set any dimensionless coupling parameters to unity.

Lepton number violating terms of the form $\lambda'_{ijk} L_i Q_j d_k$ can combine with the baryon number violating interactions $\lambda''_{ijk} u_i d_j d_k$ to produce the effective operators in Figure 4.22. For light-quark external states, these operators can induce proton
Figure 4.22: Effective operators generated by the dimension 4 interactions \( \lambda'_{ijk} L_i Q_j d_k \) and \( \lambda''_{ijk} u_i d_j d_k \). When the external fields are light families, these graphs generate nucleon decay. The solid lines represent fermions while the dashed line represent scalar propagators.

decay. As discussed in [70, 16], this will be suppressed below the observed bounds if the product of the dimensionless coefficients satisfy

\[
\lambda' \lambda'' < \mathcal{O}(10^{-26})
\]  

(4.150)

for couplings leading to \( p \to \pi^+ + \nu \) and

\[
\lambda' \lambda'' < \mathcal{O}(10^{-25})
\]  

(4.151)

for couplings inducing the decay \( p \to K^+_s + \nu \). In estimating these bounds, we have taken the mass of the intermediate squarks in Figure 4.22 to be of \( \mathcal{O}(1 \text{ TeV}) \), corresponding to their derived values in Section 4.

For the vacua of the \( B-L \) MSSM theory discussed in this paper, that is, where only the third family sneutrino gets a non-vanishing VEV, the relevant lepton num-
The violating term generated in (4.138) is
\[ \epsilon_3 \sum_{i=1}^{3} \lambda_{d,i} L_3 Q_i d_i \, . \]  
(4.152)

It follows from (4.149) and (4.152) that the product of the dimensionless couplings in the B-L MSSM theory inducing the decays \( p \to \pi^+ + \nu \) and \( p \to K_s^+ + \nu \) are
\[ \lambda' \lambda'' = \epsilon_3 \lambda_{d,i} \frac{\langle \nu_3 \rangle}{M_c} \]  
(4.153)
for \( i = 1, 2 \) respectively. As an example, let us compute these products at the point (P) in Figure 4.10. As discussed above, here \( \tan \beta = 18 \), \( \mu = 0.855M \), \( \langle \nu_3 \rangle = 4.433M \) and, for the conservative value of \( \lambda_{\nu_3} \simeq 10^{-10} \), one finds \( \epsilon_3 \simeq 5.185 \times 10^{-10} \).

Using these and (4.27), we find that for the \( p \to \pi^+ + \nu \) decay
\[ \lambda' \lambda'' = \epsilon_3 \lambda_{d,1} \frac{\langle \nu_3 \rangle}{M_c} = 1.92 \times 10^{-27} < \mathcal{O}(10^{-26}) \]  
(4.154)
whereas for the \( p \to K_s^+ + \nu \) channel
\[ \lambda' \lambda'' = \epsilon_3 \lambda_{d,2} \frac{\langle \nu_3 \rangle}{M_c} = 6.89 \times 10^{-26} \lesssim \mathcal{O}(10^{-25}) \, . \]  
(4.155)
That is, the \( p \to \pi^+ + \nu \) bound is easily satisfied. However, although satisfied, our results come close to the \( p \to K_s^+ + \nu \) bound. Of course, choosing \( \lambda_{\nu_3} < 10^{-10} \) rapidly suppresses proton decay through these dimension four operators well below all experimental bounds. For example, choosing \( \lambda_{\nu_3} = 10^{-12} \) reduces each of (4.154) and (4.155) by two orders of magnitude.
Chapter 5

Appendices

Appendix A: Kaluza-Klein Modes

Here we give a very brief sketch of the ideas behind Kaluza-Klein theory. For a more in-depth discussion, please see [91] and [1]. Much of this discussion follows [1] with a few additions for clarity. We first consider a \((D + 1)\) dimensional space where one of the dimensions is periodic which can be written as \(M^D \times S^1\), where \(M^D\) is a \(D\) dimensional manifold. For a simple bosonic field to be well defined on this manifold, it must obey:

\[ \hat{\phi}(x, z) = \hat{\phi}(x, z + 2\pi R), \]

(5.1)

where we define the notation of \(\hat{\phi}\) representing a field in the \((D + 1)\) space, \(x\) is a vector in \(M^D\), likewise \(z\) is in \(S^1\), and lastly \(R\) is the radius of the compact dimension.
We can now expand this field $\dot{\phi}$ as a Fourier series in the compact dimension

$$\dot{\phi}(x, z) = \sum_{n=-\infty}^{\infty} \phi(x) e^{inz/R},$$

(5.2)

where $\phi$ represents a field in $M^D$.

Next we will look at what happens when a massless bosonic field in a manifold with a periodic dimension is expanded in such a way. We recall that such a field satisfies

$$\hat{\Box} \hat{\phi} = 0 .$$

(5.3)

We can immediately see that, once expanded, the lower dimensional field satisfies

$$\Box \phi_n - \frac{n^2}{R^2} \phi_n = 0 .$$

(5.4)

This is the equation for a bosonic field in $D$ dimensions with a mass of $|n|/R$. Thus we find an infinite “tower” of massive fields and one massless “zero” mode. In general, after integrating over the compact dimension, the $D+1$ dimensional action becomes a $D$ dimensional action composed of a massless field sector, a heavy sector, and the interaction term that connects the two

$$S^{D+1} \rightarrow S_0^D + \sum_{n=1}^{\infty} S_n^D + S_{int}^D .$$

(5.5)

At energies much below the characteristic mass $M = 1/R$, the heavy sector decouples and the interactions terms become loop corrections of order $1/M$. 181
Appendix B: The Numerical Solution for $m_H(t)^2$

In this Appendix, we present a numerical solution of equation (3.108). This is accomplished using the numerical fitting and solving packages in Wolfram’s Mathematica program [97]. The details of these packages are quite complex and we refer the reader to the documentation provided by Mathematica for further discussion.

We begin by considering equation (3.108) with the property that $m_H^2$ is positive over the entire scaling range $t_{EW} \leq t \leq 0$. With this additional condition, (3.108) is equivalent to equation (3.109). First note, using (3.24), that (3.108) contains two arbitrary parameters, $m_H(0)^2$ and $|M_3(0)|^2$. The coefficient $m_H(0)^2$ is the value of the up-Higgs soft supersymmetry breaking mass at $t = 0$ and, hence, is a natural input parameter. However, for the reasons discussed in the text, it is more transparent physically to input the value of $m_H^2$ at $t_{EW}$, given in (3.134), (3.135) and (3.136), rather than $|M_3(0)|^2$. To do this, one chooses $T$ and $\Delta^2$ subject to the constraints $T^2 \gtrsim 40$ and $0 < \Delta^2 < 1$. It then follows from (3.134) and (3.135) that we have completely specified the value of $m_H(t_{EW})^2$. For this choice of parameters, we solve the $m_H^2$ equation of motion (3.108) numerically, adjusting the value of $|M_3(0)|^2$ until the value of $m_H^2$ at $t_{EW}$ is given by $m_H(0)^2/T^2$. The plot of this value of $|M_3(0)|^2$ for a large number of choices of $T^2$ is shown in Figure 5.1. Fitting this result with a smooth curve, we find that

$$|M_3(0)|^2 = 0.0352(1 - \frac{11.5}{T^2})m_H(0)^2.$$ (5.6)
Figure 5.1: This plot shows a representative set of points of \( |M_3(0)|^2/m_H(0)^2 \) (black dots) for different values of \( T'^2 \) as well as the accuracy of the fit of equation (5.6) (blue line) to these representative points. For simplicity we show only show a few points but many more were used in the generation of this fit. The accuracy of this fit over the range of \( T'^2 \) exceeds one percent at each data point (including those not shown).

This curve is also plotted in Figure 5.1 and closely reproduces the numerical data. This justifies expression (3.138) used in the text.

Having determined relation (5.6), one can now numerically solve the \( m_H^2 \) equation (3.108) for any choice of \( m_H(0)^2 \) and \( T'^2 \). Numerical plots of \( m_H^2/m_H(0)^2 \) as a function of \( t \) are shown in Figure 5.2 for several different choices of \( T'^2 \). For each such graph, we fit the numerical results to a smooth curve. We find that for any allowed value of \( T'^2 \), the data are well represented by

\[
m_H(t)^2 = \left( 1 - \left( 1 - \frac{1}{T'^2} \right) \frac{t_{EW} - b}{t_{EW}} \right) \left( \frac{t}{t - b} \right) m_H(0)^2, \tag{5.7}
\]
Figure 5.2: These plots show representative points \( m_H(t)^2/m_H(0)^2 \) (black dots) obtained by numerical solution of (3.108) for different values of \( t \) in our scaling range as well as the fit of equations (5.7),(5.8) (blue line) to these points. We show this for three choices of \( T' \) spanning a wide range of physically interesting values.

where \( b \) is a function of \( T' \) of the form

\[
b(T') = 19.9 (1 - \frac{.186}{T' - 3.69}) . \tag{5.8}
\]

Note that at \( t = 0 \) and \( t = t_{EW} \), \( m_H^2 \) is given by \( m_H(0)^2 \) and \( m_H(0)^2/T'^2 \) respectively, as it must be. This smooth curve is plotted in Figure 5.2 for each of the choices of \( T'^2 \) and is seen to give a close fit to the numerical data. This justifies equations (3.139),(3.140) used in the text.

**Appendix C: The Relationship Between \( m_\nu(0)^2 \) and \( m_H(0)^2 \)**

As highlighted in the discussion following equations (3.155) and (3.193), squark and slepton masses evaluated around the VEVs \( \langle \nu_3 \rangle \), \( \langle H^0 \rangle \) generically depend on the two initial values, \( m_\nu(0)^2 \) and \( m_H(0)^2 \). We want to explore the region of parameter
space leading to positive squark/slepton masses over the entire scaling range, thus simplifying the discussion of symmetry breaking. This region of parameter space can be specified in terms of a simple $T^2$ dependent relationship between these two initial conditions. In this Appendix, we present a detailed derivation of this relationship.

Recall that the squark/slepton masses depend not only on the associated $m^2$ coefficients and their running, but on contributions from the $\langle \langle \nu_3 \rangle \rangle$ and $\langle \langle H^0 \rangle \rangle$ VEVs as well. These expectation values are only non-zero below certain scales, thus complicating the analysis. Furthermore, both $m_{Q_3}^2$ and $m_{u_3}^2$ contain an $m_H^2$ term and, hence, require the numerical solution described in Appendix A. As an explicit example of how the relationship between $m_{\nu}(0)^2$ and $m_H(0)^2$ effects the sign of the scalar mass terms, let us consider $\langle \langle m_{Q_3}^2 \rangle \rangle$. It follows from (3.148), (3.159) and (3.196) that

$$\langle \langle m_{Q_3}^2 \rangle \rangle \simeq \frac{1}{3} m_H^2 - \frac{2}{3\pi^2} \int_0^t g_3^2 |M_3|^2 + \frac{1}{64\pi^2} \int_0^t g_3^2 S_1' + \frac{1}{6} m_H(0)^2 + \frac{1}{4} g_4^2 \langle \langle \nu_3 \rangle \rangle^2 + \frac{1}{4} (\frac{1}{5} g_1^2 \mp g_2^2) \langle \langle H^0 \rangle \rangle^2 , \quad (5.9)$$

where $\mp$ indicates that for the up field $U_3$ and the down field $D_3$ of the doublet $Q_3$ one uses a minus sign and plus sign respectively. Let us discuss each term in this equation and its value over the entire scaling range. The first term is $m_H^2$ derived in Appendix A and given in (5.7), (5.8). It is proportional to $m_H(0)^2$ and depends explicitly on the value of $T'$. We plotted it over the scaling range for three different values of $T'$ in that Appendix. Next we have the integrals containing
Figure 5.3: Graph (a) is a plot of the term $-\frac{2}{3\pi^2} \int_0^T g_5^2 |M_3|^2$ over the entire scaling range taking $T' = 40$. Graph (b) is the term $\frac{1}{64\pi^2} \int_0^T g_4^2 S_1'$ plotted over the same range.

$|M_3|^2$ and $S_1'$. These can be evaluated for arbitrary $t$ using equations (3.99), (3.138) and (3.76), (3.100) respectively. Note that the first is $T'^2$ and $m_H(0)^2$ dependent, while the second is $m_\nu(0)^2$ dependent. Figure 5.3 plots their values over the scaling range, where in graph (a) we have taken $T' = 40$ for specificity. Next we have the sneutrino VEV term. It is given for any scale $t$ by minimizing the purely $\nu_3$ part of potential (3.77) using equations (3.19), (3.57) and (3.76) respectively and is plotted in Figure 5.4 over the scaling range. Note that this term depend on $m_\nu(0)^2$. Lastly, we have the Higgs VEV term. This turns out to give a very small contribution, so we omit it henceforth.

Note that each of the above terms is proportional to either $m_\nu(0)^2$ or $m_H(0)^2$. Furthermore, three of the terms are everywhere positive whereas the two remaining terms, which are proportional to $m_\nu(0)^2$, are everywhere negative. It is easy to demonstrate that for a sufficiently large ratio of $m_\nu(0)^2$ to $m_H(0)^2$, $\langle m_{Q_3}^2 \rangle$ could
be negative somewhere in the scaling range. To investigate this, we first define

\[ m_\nu(0)^2 = D^2 m_H(0)^2. \]  

(5.10)

This allows us to write each term as a function of one of these initial masses, which we choose here to be \( m_\nu(0)^2 \), and their ratio \( D^2 \). Inserting the renormalization group expression for the scaling of each term into (5.9), we find that

\[
\langle \langle m_Q^2 \rangle \rangle \approx \left\{ \frac{1}{3} \left( 1 - (1 - \frac{1}{T^2})(t_{\text{EW}} - b)(\frac{t}{t_{\text{EW}}} - b) \right) \frac{m_\nu(0)^2}{D^2} \right\} \\
- \left\{ \frac{8}{3b_3} \left( \frac{1}{(1 - g(0)^2 b_4 t) \frac{g(0)}{8\pi^2}} - 1 \right) \frac{0.0352(1 - \frac{11.5}{T^2})}{D^2} \right\} \\
+ \left\{ \frac{1}{18} \left( \frac{1}{1 - g(0)^2 b_4 t \frac{g(0)}{8\pi^2}} - 1 \right) 149 m_\nu(0)^2 \right\} + \frac{m_\nu(0)^2}{6D^2} \\
- \left\{ m_\nu(0)^2 - \frac{1}{6} \left( 1 - \frac{g(0)^2 b_4 t}{8\pi^2} - \frac{2}{\pi^2} \right) 149 m_\nu(0)^2 \right\} \\
+ \sqrt{3} \left\{ \frac{g(0)}{4 \left( 1 - g(0)^2 b_4 t \frac{g(0)}{8\pi^2} \right)^{\frac{1}{2}} \left( \frac{-2}{g(0)} \left( b_4 - \frac{g(0)}{2} \right) \right) \left( 1 - \frac{g(0)^2 b_4 t}{8\pi^2} \right)^{\frac{1}{2} \left( 1 - \frac{2}{\pi^2} \right) - 1} \right) \right\} 149 m_\nu(0)^2 \right\}.
\]  

(5.11)
Figure 5.5: In this graph, we demonstrate the dependence of the running of $\langle \langle m_{Q_3}^2 \rangle \rangle$ on $D^2$. We plot $\langle \langle m_{Q_3}^2 \rangle \rangle/m_H(0)^2$ for $D=0.75$ (green line), $D=0.85$ (blue line), and $D=0.95$ (red line). It is apparent from this plot that as $D$ increases, $\langle \langle m_{Q_3}^2 \rangle \rangle$ goes negative. For this plot, we took $T'^2 = 40$.

We have preserved the original ordering of terms for ease of reference. Recall that the coefficient $b$ is defined in (5.8). Note that we can factor out $m_\nu(0)^2$ and, hence, the properties of this equation are controlled by the value of $D^2$ and $T'^2$. Choose a fixed value for $T'^2$. If we initially assume that $D^2 \ll 1$, that is, $m_\nu(0)^2 \ll m_H(0)^2$, then $\langle \langle m_{Q_3}^2 \rangle \rangle$ is everywhere positive. However, as we increase $D^2$ the value of $\langle \langle m_{Q_3}^2 \rangle \rangle$ becomes smaller, eventually touching zero at some point $t$ in the scaling regime. For still larger $D^2$ the value of $\langle \langle m_{Q_3}^2 \rangle \rangle$ at this point, and in a range around it, is negative, see Figure 5.5. Record the value of $D^2$ at which $\langle \langle m_{Q_3}^2 \rangle \rangle$ just touches zero and repeat this for a set of choices of $T'^2$. This is plotted in Figure 5.6. We find that a smooth curve fitting the data is given by

$$D^2 = 0.864(1 - \frac{2.25}{T'^2})$$

(5.12)
Figure 5.6: This plot shows representative points of $D^2$ (black dots) for different values of $T'^2$ and the fit of equation (5.12) (blue line) to these points. The accuracy of this fit over the range of $T'^2$ exceeds one percent at each data point.

or, equivalently, using (5.10) that

$$m_\nu(0)^2 = 0.864(1 - \frac{2.25}{T'^2})m_H(0)^2.$$ \hfill (5.13)

This equation specifies the largest value of $m_\nu(0)^2$ relative to $m_H(0)^2$ for which $\langle \langle m_{Q_3}^2 \rangle \rangle$ is everywhere non-negative. An investigation of all other other squark and slepton mass squares evaluated around the B-L and Higgs VEVs shows that they are everywhere positive whenever $\langle \langle m_{Q_3}^2 \rangle \rangle$ is everywhere non-negative. That is, for $m_\nu(0)^2$ and $m_H(0)^2$ satisfying relation (5.13), all squark/slepton masses are everywhere positive, as desired. Furthermore, this is the largest value of $m_\nu(0)^2$ relative to $m_H(0)^2$ for which this is the case. This is chosen so that the B-L/EW hierarchy, which is proportional to $m_\nu(0)/m_H(0)$, is as large as possible within this context. This justifies our use of equation (3.156) in the text.
Appendix D: Comparison to the Standard Formalism and the Higgsino Mass

In this Appendix, we compare our analysis of the Higgs section with the standard MSSM results given, for example, in “A Supersymmetric Primer” by Stephen Martin [87]. Restoring the $\mu$ parameter, that is, not assuming it is necessarily sub-leading, the complete potential for the neutral Higgs fields becomes

$$V = (|\mu|^2 + m_H^2)|H^0|^2 + (|\mu|^2 + m_H^2)|\tilde{H}^0|^2 - B(H^0\tilde{H}^0 + h.c) + \frac{1}{8}(3g_1^2 + g_2^2)(|H^0|^2 - |\tilde{H}^0|^2)^2.$$  \hspace{1cm} (5.14)

Note that this potential is written in terms of the original Higgs fields, not the mass eigenstates at the origin we used in our analysis. The two constraints that must be satisfied to have a stable, non-vanishing vacuum of (5.14) are presented as equations (7.3) and (7.4) in the Supersymmetric Primer. The two minimization equations are given in (7.10) and (7.11) of that review. Note that each of these equations should be evaluated at the electroweak scale $t_{EW}$. We now show, to the order in $T^{-1}$ we are working to, that our vacuum satisfies all four of these equations.

We begin by examining the constraint (7.3) in [87]. This equation, which ensures that the potential is bounded from below, and is given by

$$2B < 2|\mu|^2 + m_H^2 + m_{\tilde{H}}^2.$$  \hspace{1cm} (5.15)
Using (3.136), this can be rewritten in the form
\[
\frac{1}{T} = |\mu|^2 + \frac{m_H^2}{m_H^2 - m_{H^*}^2} + \frac{1}{2}.
\] (5.16)

It then follows from (3.133),(3.134),(3.135) and (3.136) that this inequality is satisfied by our vacuum over the entire range $T \sim 6.32$ and $0 < \Delta^2 < 1$ for any value of the parameter $\mu$. Next, consider the constraint equation (7.4) in [87] given by
\[
B^2 < (|\mu|^2 + m_H^2)(|\mu|^2 + m_{H^*}^2).
\] (5.17)

This ensures that the origin will not be a stable minimum of the potential. Using (3.133),(3.134),(3.135) and (3.136) this becomes
\[
1 > (1 + \frac{|\mu|^2}{m_H^2})(1 + \frac{|\mu|^2}{m_{H^*}^2(0)}) \frac{1 - \Delta^2}{(1 - \frac{1-\Delta^2}{T^2})^2}.
\] (5.18)

For the time being, take $\mu$ to be sub-dominant to the Higgs mass parameters and ignore it, as we did in the text. Later in this Appendix we will keep it in our analysis and observe the consequences. We are left with a simple inequality
\[
(1 - \frac{1-\Delta^2}{T^2})^2 > 1 - \Delta^2.
\] (5.19)

We want to examine if this holds over the entire range $0 < \Delta^2 < 1$ and $T \sim 6.32$ of our vacuum solution. Note that (5.19) is trivially satisfied as $\Delta^2 \to 1$. However, in the $\Delta^2 \to 0$ limit this inequality clearly does not hold. To find what bound is set by the above equation, solve (5.19) for $\Delta^2$ keeping the relevant root. We find that the inequality will be satisfied for
\[
\Delta^2 > 0 + \frac{2}{T^2} + \mathcal{O}\left(\frac{1}{T^4}\right)
\] (5.20)
for any value of $T$. To the order we are working in this paper, 0 is in fact the lower bound and inequality (5.19) is satisfied over the whole range of $\Delta^2$. However, to next order in $T^{-1}$, a constraint appears. This is due to the fact that we only took the first order approximation in equation (3.131). Had we kept higher orders, then relation (3.137) would be more complicated and, hence, the lower bound on $\Delta^2$ different. It is straightforward to verify that, to higher order in $T^{-1}$, the lower bound would be identical to (5.20). We conclude that, to the order we are working, constraint (7.4) in [87] is satisfied over the entire range $T \sim 6.32$ and $0 < \Delta^2 < 1$ of our vacuum.

We now show that identities (7.10) and (7.11) in [87], which are derived from the minimization conditions of potential (5.14), are also satisfied by our vacuum. First consider (7.10) given by

$$\sin 2\beta = \frac{2B}{m_H^2 + m_H^2 + 2|\mu|^2}.$$  \hspace{1cm} (5.21)

This equation can be rewritten as

$$\tan \beta = \frac{2}{C(1 - (1 - \frac{1}{C})^{1/2})},$$  \hspace{1cm} (5.22)

where

$$C^{-1} = \left(\frac{B}{m_H^2 + m_H^2 + 2|\mu|^2}\right).$$  \hspace{1cm} (5.23)

Expressing $C$ in terms of $T$ using (3.136), we find that (5.22) becomes

$$\tan \beta = T(1 + \frac{2}{T^2} + O(\frac{1}{T^4})).$$  \hspace{1cm} (5.24)
Hence, to leading order, $T = \tan \beta$. This is identical to expression (3.183) in our vacuum solution. Note that this is true for any value of the parameter $\mu$. It also follows from (5.24) that to higher order in $T^{-1}$, the relationship of $T$ to $\tan \beta$ becomes non-linear. Finally, consider the minimization equation (7.11) in [87]. For our purposes, this is most conveniently re-expressed for large $\tan \beta$ in (7.12) as

$$m_Z^2 = -2(m_H^2 + |\mu|^2) + \frac{2}{\tan^2 \beta}(m_H^2 - m_H^2) + O\left(\frac{1}{\tan^4 \beta}\right).$$  (5.25)

It is clear from (3.133),(3.134),(3.180),(3.183) that as long as $\mu$ is chosen to be sub-dominant to $m_H$, our vacuum explicitly solves (5.25). We conclude that, to the order we are working, minimization equations (7.10) and (7.11) in [87] are satisfied over the entire range $T \gtrsim 6.32$ and $0 < \Delta^2 < 1$ of our vacuum.

An assumption in this paper was that $\mu$, while non-vanishing, was sub-leading to the Higgs mass parameters at $t_{EW}$ and, hence, ignorable in the vacuum solution to leading order. Can one quantify how much this assumption restricts the size of $\mu$ and, hence, physical quantities such as the Higgsino mass? To do this, note from this assumption and (3.133),(3.134) and (3.135) that

$$|\mu|^2 \ll m_H^2 \simeq \frac{1 - \Delta^2}{T^2} m_H(0)^2 \ll m_H^2 \simeq m_H(0)$$  (5.26)

in our vacuum. Therefore, a reasonable parameterization of $\mu$ which is sub-leading to the Higgs masses is

$$\frac{|\mu|^2}{m_H^2} = \frac{\alpha(1 - \Delta^2)}{T^2},$$  (5.27)
where $\alpha$ is a constant of $\mathcal{O}(1)$. It then follows from (3.134) that

$$\frac{|\mu|^2}{m_H^2(0)} = \frac{\alpha(1 - \Delta^2)^2}{T^4}.$$  \hfill (5.28)

Putting this into inequality (5.18), one finds the condition

$$\Delta^2 > \frac{2 + \alpha}{T^2}$$  \hfill (5.29)

to the first non-vanishing order in $T^{-1}$. To see how this limits the size of $\mu$, calculate the ratio of $\mu$ to $M_Z$ using (3.180) and (5.27). We find

$$\frac{|\mu|}{M_Z} = \sqrt{\frac{\alpha}{2}} \frac{(1 - \Delta^2) 1}{\Delta} \frac{1}{T}.$$  \hfill (5.30)

Applying constraint (5.29), it follows that

$$\frac{|\mu|}{M_Z} < \sqrt{\frac{\alpha}{2(\alpha + 2)}} \left(1 - \frac{2 + \alpha}{T^2}\right)$$  \hfill (5.31)

which, for the range $T \gtrsim 6.32$ and taking, for example, $\alpha = 1$, gives

$$|\mu| < \frac{1}{\sqrt{6}} M_Z = 37.5 GeV.$$  \hfill (5.32)

We conclude that, even subject to our assumption that it be sub-leading to the Higgs mass parameters at $t_{EW}$, the $\mu$ parameter and, hence, such quantities as the Higgsino mass, can be relatively large, with an upper bound approaching 40% of the $Z$-mass.

**Appendix E: Mass Diagonalization**

Obtaining the mass eigenstates of the matter fields contained in the theory considered in this paper is non-trivial. This is due to the mixing induced by terms
contained in the soft SUSY breaking terms given in (4.22), (4.23), (4.24) and (4.25).

We note that the theory presented in this paper is similar to the MSSM and thus portions of this presentation can be found in discussions of the MSSM spectrum; see for example [87]. We perform the present analysis to illustrate the differences and similarities of the MSSM case to our own. It follows

We begin by discussing the mass eigenstates of the neutralinos. Their mass matrix will be of the form

\[
M_\tilde{N} = \begin{pmatrix}
M_1 & 0 & \frac{g_Y \langle H \rangle}{\sqrt{2}} & \frac{g_Y \langle H \rangle}{\sqrt{2}} \\
0 & M_2 & \frac{g_2 \langle R \rangle}{\sqrt{2}} & \frac{g_2 \langle H \rangle}{\sqrt{2}} \\
\frac{g_Y \langle H \rangle}{\sqrt{2}} & \frac{g_2 \langle R \rangle}{\sqrt{2}} & 0 & -\mu \\
\frac{g_Y \langle H \rangle}{\sqrt{2}} & \frac{g_2 \langle H \rangle}{\sqrt{2}} & -\mu & 0
\end{pmatrix}.
\]  
(5.33)

In the above matrix, the \( \mu \) terms are from the Higgsino portion of the \( \mu \) term. The \( M_i \) entries are from the associated soft breaking terms and the off diagonal terms are derived from the gauge coupling portion of the SUSY invariant Lagrangian. Next, we diagonalize this matrix and label the mass eigenstates as \( M_\tilde{N}_i \) for \( N = 1, \ldots, 4 \) with masses \( m_\tilde{N}_i \). There is a fifth neutral gaugino, that is the super-partner of the \( A_{B-L} \) boson. However, this field does not mix with the other neutralinos. This can be seen when we observe the Higgs gauge part of the SUSY Lagrangian written in terms of superfields

\[
H^\dagger \text{Exp}\left(-i\frac{g_Y}{2}A_Y - ig_2\tau_i B^i\right)H + \bar{H}^\dagger \text{Exp}(i\frac{g_Y}{2}A_Y - ig_2\tau_i B^i)\bar{H}.
\]  
(5.34)

From these terms, we see that the \( B - L \) gauge superfield does not couple with
the Higgs superfields and thus, does not mix with the Higgsinos, nor with other gauginos. We will call this fifth neutralino, \( \tilde{A}_{B-L} \). We have found no bound on the mass of \( \tilde{A}_{B-L} \) in the literature, so we will impose the most stringent lower bound for a neutralino listed above.

Next, we consider the mass matrix for the Charginos. This mass matrix is exactly that of the MSSM, see for example [87] equation 7.41, so we will not discuss it here.

Next, we consider the squark and slepton mixings. Recall that in this discussion, we have assumed that the squark and slepton mass matrices are diagonal (4.19), thus the only mixing possible is between each field’s left and right handed parity states. This mixing is produced by the scalar triplet coupling and couplings in the scalar potential generated by the elimination of the F terms. The mass contributions from the scalar triplet couplings are clearly produced when the Higgs scalar obtains a VEV. To see how the F terms contribute, we note that the F terms for the “up” type fields \( \phi_u \) come from the equation

\[
|\frac{\partial}{\partial H} W|^2 = |\mu \bar{H} + \lambda \phi^* u,L \phi_{u,R}|^2
\]  

(5.35)

and likewise for the “down” type fields. Once again we have assumed that the Yukawa couplings are diagonal. Since the Yukawa couplings for the third family are considerably greater than the others, it is reasonable to drop all but the third family contributions to mixing. We also note that previously, it was assumed that the A couplings were proportional to the corresponding Yukawa couplings. Thus
we drop all mixing save for those generated by third family squarks and sleptons. In the basis of \((\phi_L, \phi_R)\), we get the mixing matrices \(m_t^2\), \(m_b^2\), and \(m_{\tau}^2\), where

\[
\begin{align*}
    m_t^2 &= \begin{pmatrix} m_{Q_3}^2 + m_t^2 & \langle H \rangle A_t^* - \mu \lambda_t \langle \bar{H} \rangle \\ \langle H \rangle A_t - \mu^* \lambda_t \langle \bar{H} \rangle & m_{u_3}^2 + m_t^2 \end{pmatrix} \\
    m_b^2 &= \begin{pmatrix} m_Q^2 & \langle \bar{H} \rangle A_b^* - \mu \lambda_b \langle H \rangle \\ \langle \bar{H} \rangle A_b - \mu^* \lambda_b \langle H \rangle & m_{d_3}^2 \end{pmatrix} \\
    m_{\tau}^2 &= \begin{pmatrix} m_L^2 & \langle \bar{H} \rangle A_{\tau}^* - \mu \lambda_{\tau} \langle H \rangle \\ \langle \bar{H} \rangle A_{\tau} - \mu^* \lambda_{\tau} \langle H \rangle & m_{\tilde{e}_3}^2 \end{pmatrix}.
\end{align*}
\]

These can be diagonalized by the unitary matrix

\[
\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s_{\tilde{t}} \\ s_{\tilde{t}} & c_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}.
\]

To obtain this matrix, one must diagonalize the matrix \(m_t^2 \dagger m_t^2\) and then take the square root of the eigenvalues. We note that if any of these are degenerate, one must use a process known as the Takachi diagonalization process [37].

**Higgs Mass**

Next we consider the mass of the Higgs boson. As is well known, in the MSSM, the tree level mass of the lightest Higgs is bounded above by the mass of the Z boson [87]. The existence of a Higgs boson with a mass below the Z mass in the MSSM model has been ruled out by experimentation, thus we need to explore the one-loop
corrections to the Higgs mass. It turns out the corrections are quite sizable and allow for the MSSM to still be a viable, albeit a fairly restricted, theory of nature.

We note that there exists an extensive discussion in the literature on the various corrections to the MSSM Higgs mass. Leading corrections to this mass have been explored up to the three-loop level [66] where they report an error of less than 500 MeV. For the purposes of this paper, we explore the lightest Higgs mass to the first few leading terms in the one-loop correction, as quoted by [40]. Thus, for this paper, we use the following for our approximation of the lightest Higgs mass.

\[
m^2_h = \frac{1}{2} (m^2_A + M^2_Z + \omega_t) - \sqrt{\frac{(m^2_A + M^2_Z)^2 + \omega_t^2}{4} - m^2_A M^2_Z \cos^2 2\beta + \frac{\omega_t \cos 2\beta}{2} (m^2_A - M^2_Z)}
\]

(5.40)

where

\[
\omega_t = \frac{3}{4\pi^2} \sin^2 \beta \lambda^2 m_t \left\{ \ln \frac{m_t^2 m_{t_2}^2}{m_t^4} + c_t^2 s_t^2 \ln \frac{m_{t_2}^2 - m_{t_1}^2}{m_{t_2}^2} \ln \frac{m_{t_2}^2}{m_{t_1}^2} \right\} + c_t^4 s_t^4 \frac{(m_{l_2}^2 - m_{l_1}^2)^2}{m_t^4} (1 - \frac{m_{l_2}^2 + m_{l_1}^2}{m_{l_2}^2 - m_{l_1}^2} \ln \frac{m_{l_2}^2}{m_{l_1}^2})
\]

(5.41)

For reference, we give the tree level masses for the other Higgs bosons. They are

\[
m^2_A = 2|m|^2 + m^2_H + m^2_{H^\pm}
\]

(5.42)

\[
m^2_H = \frac{1}{2} + \sqrt{(m_A^2 + m_Z^2)^2 + 4m_Z^2 m_A^2 \sin^2(2\beta)}
\]

(5.43)

\[
m^2_{H^\pm} = m_A^2 + m_W^2.
\]

(5.44)
Note that we follow the standard notation of referring to the Higgs mass eigenstates by the labels $h(H)$ for the lightest (heaviest) neutral Higgs, $A$ for the CP odd neutral Higgs, and $H^\pm$ for the respective charged Higgs.
Bibliography


T.P. Cheng, Marc Sher, Phys.Rev.D35 11 (1987);

Marc Sher and Yao Yuan, Phys.Rev. D44 5 (1991);


[99] F. Zwirner, review presented at the NO-VE International Workshop on Neutrino Oscillations, Venice, Italy, April 17, 2008.