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Abstract
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**B(E2) value and configuration mixing in 32Mg**

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I demonstrate that the $B(E2)$ value in $^{32}$Mg can be understood with a model in which both the ground and $2^+$ first-excited states are predominantly of $sd$-shell character.

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**I. INTRODUCTION**

Several groups have equated a “larger than expected” $B(E2)$ value in $^{32}$Mg with the conclusion that its ground state (gs) is dominated by an intruder configuration of two neutrons in the $fp$ shell. It is true that the $B(E2)$ value is larger in $^{32}$Mg than in $^{30}$Mg, (see Table I) but it is significantly smaller than in $^{34}$Mg—the last almost certainly consisting of the $(fp)^2$ configuration. It does appear that core excitation is larger in $^{32}$Mg (and $^{30}$Ne) than in other nearby nuclei. Except for one anomalously large value, [1] (which I ignore here) various experiments [2–5] agree on the $B(E2)$ measurements in $^{32}$Mg. The analysis of the data divides the results into two camps—depending on the magnitude of the correction for feeding from above. Table I lists the $B(E2)$’s from gs to $2^+_1$ in $^{30,32,34}$Mg. It has been recently claimed [6] that $B(E2,gs^{32}$Mg) is not dominated by the $(fp)^2$ configuration. Straightforward analysis [6] of data from the $^{30}$Mg($t,p$) reaction [7] has demonstrated that it is the excited 0 state that dominates the data from $^{34}$Mg. It has been recently claimed [6] that $B(E2,gs^{32}$Mg) is not dominated by the $(fp)^2$ configuration. Therefore, analysis [6] of data from the $^{30}$Mg($t,p$) reaction [7] has demonstrated that it is the excited 0 state that contains most of this intruder configuration.

Parameters in a shell-model calculation [8–12] can be adjusted to produce a $^{32}$Mg (gs) that is predominantly $\nu(fp)^2(sd)^2$, but that may not be necessary (or correct). It is possible that some of the shell-model calculations did not sufficiently renormalize the interaction when including different $\hbar\omega$’s. Inclusion of neutron excitations into the $fp$ shell is important for understanding the properties of neutron-rich nuclei. However, that need not imply that these excitations dominate the ground states. In some descriptions this $2\hbar\omega$ excitation arises from deformation, so that the 1/2 Nilsson orbital is significantly lowered. Other descriptions ascribe this excitation to a pairing effect in spherical nuclei. Yamagami and Van Giai [13] performed Hartree-Fock-Bogoliubov (HFB) calculations for these nuclei with modified Skyrme interactions. They state that the $B(E2)$’s and $2^+_1$ energies in $^{30,32}$Mg are well described in calculations in which both nuclei are spherical. They find an $fp$-shell occupancy of ~0.5 neutrons for $^{32}$Mg(gs). Several calculations [14,15] found $^{34}$Mg to be deformed, with $\beta$ in the range 0.3 to 0.4. $^{30}$Mg to be spherical, but $\beta$ soft; and $^{32}$Mg to be transitional with coexisting spherical and deformed shapes. With one choice of Skyrme force (SKM$^*$), they were degenerate, but with all other forces [15] the deformed state was 2 to 4 MeV above the spherical gs. Reference [14] found for $^{32}$Mg a minimum in the potential-energy surface at $\beta = 0$, and no other. A very recent paper [16] found $^{36}$Mg to be very $\beta$ soft, $^{34}$Mg to be $\gamma$ soft, and $^{32}$Mg to have two minima—at 0 and 0.33.

**II. THE MODEL**

Here, we investigate whether we can understand the $B(E2)$ value in a simple, consistent model. For $^{32}$Mg, let

\[ A = a(32^{\text{Mg}}(gs, sd\text{shell}) + b(32^{\text{Mg}}(gs, sd\text{shell}) \times \nu(fp)^2) \text{, and} \]

\[ 2^+_1 = A(2^+_1, sd\text{shell}) + B(32^{\text{Mg}}(gs, sd\text{shell}) \times \nu(fp)^2) \text{.} \]

Reference [6] found $\alpha^2 = 0.74–0.81$. Later we consider adding a third term,

\[ C(30^{\text{Mg}}(2^+_1, sd\text{shell}) \times \nu(fp)^0) \text{,} \]

to the $2^+$ state. We define $B(E2; i \rightarrow f) = M^2 / (2J_i + 1)$, so that if $J_i = 0$, $B(E2) = M^2$. Then we have for $^{32}$Mg

\[ M(E2; 32^{\text{Mg}}) = \alpha A(32^{\text{Mg}}(sd\text{shell}) + bB(32^{\text{Mg}}(fp)^2) \text{, and} \]

and the two terms are constructive. Now, we need to estimate $M(sd)$ and $M(fp)$. Because $M(sd)$ connects $2^+_1$ and gs in the $sd$-shell $^{32}$Mg, it must be a pure proton excitation, as the neutrons form a filled shell. If $M(fp)$ connects $\nu(fp)^0$ to $\nu(fp)^2$, it is a pure neutron excitation. Now, look at $^{30}$Mg. Is its $M(E2)$ larger or smaller than $M(sd)$? In the absence of cross-shell excitations, the proton part of $M(30^{\text{Mg}})$ should be similar to $M(sd)$ (the protons are similar in the two)., but $M(30^{\text{Mg}})$ can also contain some $sd$-shell neutron excitation. If the $2^+$ and gs in $^{30}$Mg also contain some $(fp)^2$ excitation, they will add to $M(30^{\text{Mg}})$. Because all the terms will add constructively, then we expect, quite rigorously, that $M(sd) \lesssim M(30^{\text{Mg}})$. For $^{32}$Mg we assume equality, but we return to this point later. For $^{34}$Mg, both of the complicating terms will be smaller, so we expect $M(fp) \approx M(34^{\text{Mg}})$. This might be a topic for further study.

**III. CALCULATIONS AND RESULTS**

From Table I, the $B(E2)$ in $^{30}$Mg is 295(26) $e^2$fm$^4$. The weighted average of the four large values in $^{32}$Mg is 446(31) $e^2$fm$^4$. (We have ignored the much larger value of Chiste et al. [1] and have not averaged in the two values derived with large corrections for feeding from above.) We return to this point later. The weighted average in $^{34}$Mg is 577(79) $e^2$fm$^4$. Thus, we have $M(30^{\text{Mg}}) = 17.2(5)$ and $M(34^{\text{Mg}}) = 24.0(16)$ efm$^2$. The ratio is 1.40(12). We use these temporarily as $M(sd)$ and $M(fp)$, respectively, and investigate changing them later. Thus, we have

\[ M(32^{\text{Mg}}) = \alpha A[17.2(5)] + bB[24.0(16)]. \]
The value of the ratio that arises from the combination that maximizes \( B(E2; 32\text{Mg}) \) for given \( a^2 \) and \( 2^+ \) mixing in the gs and 2, still with \( A^2 = B(E2; ^{30}\text{Mg}) \) that results from the given \( a_A \) combination. The value of the ratio that arises from the assumption of equal mixing in the gs and \( 2^+ \) states is also plotted (short dashes). The two results are not very different. This is the ratio of \( B(E2; ^{32}\text{Mg}) \) to \( B(E2; ^{30}\text{Mg}) \), where we have taken \( B(E2; ^{30}\text{Mg}) \) for the latter. Remember, Ref. [6] found \( a^2 \sim 0.75 \). Near this region, this simple model predicts a \( B(E2) \) ratio that is significantly larger than unity. In Fig. 2, we plot the predicted \( B(E2) \) (solid) and the \( \pm 1 \sigma \) limit (dashed) curves arising from uncertainties in \( M(sd) \) and \( M(fp) \). Also shown are both sets of experimental values for \( ^{32}\text{Mg} \), as solid squares and open circles, with their uncertainties. We note that if the lower experimental \( B(E2) \) value is correct, the \( B(E2) \) requires \( a^2 \geq 0.7 \). We have agreement with the larger experimental value for a wide range of \( a^2 \) up to about 0.7. The simple model agrees well. Throughout the remainder of this article, the \( 0^+ \) mixing is held constant and the \( 2^+ \) mixing is investigated. Note that \( a^2 = 0.75 \) corresponds to \( N_{fp} = 0.50 \), consistent with the calculation of Ref. [13]. Remember that the present calculation, so far, does not contain any component in the \( 2^+ \) state in which the core is excited to \( 2^+ \), i.e., the term \( C \cdot A^{30}\text{Mg}(2^+, sd \text{ shell}) \cdot v(fp)h_0^2 \). This term will serve to increase
FIG. 3. (Color online) Squares are as in Fig. 2, but are now plotted vs $C^2$, using $a^2 = 0.75$ and $b^2 = 0.25$, and with the C term (see text). The upper curve is for the $A_B$ combination that maximizes $B(E2)$ for given $C^2$, still with $a^2 = 0.75$ and $b^2 = 0.25$.

with reasonably small values of $C^2$, a decrease in $a^2$ will produce a $B(E2)$ that is too large.

Concerning our assumption that $M(sd)$ can be approximated by $M(30Mg)$, if it should turn out that $M(sd)$ is significantly smaller than $M(30Mg)$, then the calculated $B(E2)$'s presented here will be smaller. But a smaller $M(sd)$ can be compensated by a slightly larger value of $M(fp)$ or a slightly larger value of $C$. We note from Fig. 3 that, with the C term present, it should be an easy matter to fit the larger of the two $^{32}$Mg experimental values with a value for $M(sd)$ that is significantly smaller than that for $M(30Mg)$. This point is clearly made in Fig. 4, where we plot, vs $A^2$, the value of $M(sd)$ needed to produce $B(E2; ^{32}Mg) = 446(31) e^2 fm^4$, for three different values of $C^2$. As expected, with a reasonably small $C^2$, the $^{32}$Mg $B(E2)$ can be reproduced with an assumed $M(sd)$ that is quite a bit smaller than $M(30Mg)$.

FIG. 4. (Color online) The value of $M(sd)$ (see text) necessary to fit the experimental $B(E2) = 446(31) e^2 fm^4$ for $^{32}$Mg, plotted vs $A^2$ for three different small values of $C^2 = 0.025$ (long dashes), 0.05 (short dashes), and 0.10 (solid line). Open circles represent the experimental $M(30Mg)$.

IV. THE MIXING

In a two-state model, if the mixed states are separated by an energy $E$, and the mixing amplitude is $a$ and $b$, then the matrix element responsible for the mixing is $V = abE$. Here, I use $a^2 = 0.81$ and $b^2 = 0.19$ [6], which were obtained assuming no core excitation in $^{30}Mg$, because I intend to use the $^{30}Mg$ energies as representative of the $sd$ shell $0^+-2^+$ spacing. Then, with $E = 1.058$ MeV [7], we have $V = 0.415$ MeV. Thus, the unmixed $0^+$ states are at 0.201 ($sd$ shell) and 0.857 ($(fp)^2$) MeV. These energies are plotted in Fig. 5. For illustrative purposes we also show the $2^+$ states. In Fig. 5, we place the $sd$ shell $2^+$ state 1.48 MeV above the $sd$ shell $0^+$ state, as in $^{30}Mg$. And, we place the $(fp)^2$ 2+ state 0.654 MeV above its $0^+$ state, as in $^{34}Mg$. This almost certainly is an oversimplification, and we do not intend to claim this determines the order of the two unmixed $2^+$ states or their separation. But, it does show that the unmixed $2^+$ states are closer to one another than the unmixed $0^+$ states. Reference [2], while discussing $^{28}Ne$, mentioned that the energy shift of the $2^+$ states is larger than the $0^+$ shift, because the $2^+$ unmixed states are closer together than the unmixed $0^+$ states. The same is true here. And, of course, the mixing amplitude will be larger for $2^+$ than for $0^+$. So, it is not surprising that, in the $E2$ analysis discussed above, the $0^+$ gs was purer than the $2^+_1$ state.

The mixing preserves summed energy, so these two $2^+$ energies, combined with $E(2^+_1) = 0.886$ MeV, give the second physical $2^+$ state at 2.31 MeV. If these $2^+$ basis energies are even remotely correct, then one of the two states at 2.321 and 2.551 MeV should be $2^+$. The lower one has had several $J^+$ suggestions, the latest being $4^+$ [17]. The 2.55-MeV state

FIG. 5. Mixed and unmixed $0^+$ and $2^+$ states in $^{32}Mg$. The left-hand column depicts the physical states, the middle column depicts the $0^+$ basis states from the mixing in Ref. [6], and the right-hand column depicts one possibility for the $2^+$ basis states.
is thought [18] to be \((1^- \text{ or } 2^+)\). It is unlikely that the \(2^+\) basis energies are lower than those in Fig. 5, because there are no other known states in \(^{32}\text{Mg}\) below 2.3 MeV. If the basis \(2^+\) energies are significantly higher than those in Fig. 5, then the second \(2^+\) physical state would be above 2.6 MeV. The next known state that could be \(2^+\) is at 3.488 MeV [17].

With the \(2^+\) ordering shown in Fig. 5, the lower \(2^+\) physical state would have slightly more than 50\% \((fp)^2\). But, small changes in the energy of either can easily change the order of the \(2^+\) basis states.

V. SUMMARY

The simpler model presented here (without the \(C\) term in the \(2^+\) state) has a slight preference for the lower \(B(E2)\) value in \(^{32}\text{Mg}\), but calculated results are in rough agreement with either value. We conclude that we have no trouble understanding the \(B(E2)\) in \(^{32}\text{Mg}\) even with both \(0^+\) and \(2^+\) wave functions dominated by the \(sd\)-shell configuration. With the \(C\) term added we can reproduce the larger \(B(E2)\), even with \(M(30\text{Mg})\) considerably smaller than \(M(30\text{Mg})\). As noted in the Introduction, shell-model parameters can be adjusted to produce a \(^{32}\text{Mg}(gs)\) that is predominantly of the configuration \(\nu(fp)^2(sd)^{-2}\). Changes in these parameters should make it possible to obtain a gs that contains only about 25\% of this configuration. A very recent paper [16], using HFB, concluded that for \(^{32}\text{Mg}\), “the interpretation of “deformed ground and spherical excited 0\(^+\) states’ based on the simple inversion picture of the spherical and deformed configurations does not hold.”