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Abstract
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Helical Luttinger liquids and three-dimensional black holes

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We use the AdS/CFT correspondence to discuss an equivalence between a helical, strongly coupled Luttinger liquid and a fermion propagating in the background of a topologically charged black hole in three dimensions. The Fermi level is set by the topological charges, thus surmounting difficulties in low dimensions of the standard approach using Coulomb charged black holes. The construction is fully embeddable in string theory, and the microscopic Lagrangian is explicitly known. The retarded Green function at low temperature and energy arises from the geometry very near the black hole horizon, a structure that is universal for all cold, charged liquids with a dual $\mathbb{R} \times \mathrm{U}(1)^s$ invariant description in gravity. This explains a subtle relationship between Luttinger physics and the infrared behavior of higher dimensional non-Fermi liquids in the AdS/CFT correspondence.

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Recent works study the physics of strongly coupled non-Fermi liquids using the AdS/CFT correspondence [1,2]: a fermionic operator $\mathcal{O}$ interacts with a strongly coupled conformal field theory (CFT) that is represented as a gravitating anti-de Sitter (AdS) spacetime with one extra dimension. The correlation functions of a fermion moving in this spacetime are related to those of $\mathcal{O}$. A chemical potential and temperature are introduced in the gravitational picture by including a charged black hole.

A challenge is finding examples that can be embedded in string theory so that the required duality with gravity actually exists. We show how this is achieved for two-dimensional field theories with fermions described universally in the IR by effective theories of the Luttinger form. It has been suggested that the exact solvability of the Luttinger model prevents its embedding into the AdS/CFT setting. We find that because the Luttinger model is universal at low energies, with Green functions determined by conformal invariance, symmetries dictate that AdS/CFT must also give this effective low-energy description.

Specifically, we show that a Dirac fermion propagating near a three-dimensional Banados-Teitelboim-Zanelli (BTZ) black hole [3] can be dual to a helical Luttinger liquid [4], i.e. a liquid where fermions have fixed handedness. A key obstacle is introducing a chemical potential for fermions. The usual approach [1,2] of giving the AdS black hole a Coulomb charge fails, because in 2 spatial dimensions the Coulomb potential is ill defined at infinity. We propose a novel approach where the black hole has *topological* charges—holonomies for $U(1)$ vector potentials that surround the black hole. These Wilson lines control the Fermi level in the dual field theory. The bulk fermion mass controls the scaling dimension of the dual operator, which we relate to the effective couplings of the effective low-energy Luttinger liquid. Our construction is embeddable in string theory, and a Lagrangian description is available at weak coupling. The gravitational description helps in a regime where the field theory is strongly coupled, and where certain properties of the liquid are sensitive to the UV completion.

At low temperatures the three-dimensional black hole is nearly extremal and the analytic structure of the IR Green function is controlled by the near-horizon geometry, which is a two-dimensional AdS space with a constant electric field. The same geometry appears near the horizon of *any* four-dimensional and five-dimensional finite extremal black hole invariant under $\mathbb{R} \times \mathrm{U}(1)$ and $\mathbb{R} \times \mathrm{U}(1)^2$, respectively [5], and controls its IR correlation functions. In this way, the nonanalytic IR behavior of *every* non-Fermi liquid with a gravitational dual is related to the IR physics of the two-dimensional Luttinger liquid. This suggests that non-Fermi liquids in any dimension with a realization in gravity represent different UV completions of a universal IR sector. We show that the different UV completions involve geometrizing different quantities in the theory and will not be related to each other by local field redefinitions.

Consider the consistent truncation of Type IIB string theory to the three-dimensional $SU(1,1|2) \times SU(1,1|2)$ supergravity, with a metric and two $SU(2)$ Chern-Simons gauge fields. The action is $\mathcal{S} = \frac{1}{16\pi G_N}\int d^3x\sqrt{-g}(R + \frac{2}{\ell^2}) + \mathcal{S}_{\rm CS}(A_+) - \mathcal{S}_{\rm CS}(A_-)$ with $\mathcal{S}_{\rm CS} = \frac{k}{4\pi}\int \mathrm{Tr}(A\wedge dA + \frac{2}{3}A\wedge A\wedge A)$, where $k = \frac{\ell}{4G}$ is the level of the $SU(2)$ currents. The vacuum solution is AdS_3. Other solutions include the rotating BTZ black hole surrounded by Wilson lines [6]:

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2}dt^2 + \frac{r^2\ell^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2\left(d\phi - \frac{r_+ r_-}{\ell r^2}dt\right)^2;$$

$$A^3_\mp = \alpha_\mp\left(d\phi \pm \frac{dt}{\ell}\right). \tag{1}$$

The parameters $r_\pm$ are the outer and inner horizon radii. Defining the left and right temperatures $T_\pm = (r_+ \pm r_-)/2\pi\ell^2$, the mass, angular momentum, and temperature of the black hole are $M = (T_+^2 + T_-^2)\pi^2\ell^2/4G$, $J = (T_+^2 - T_-^2)\pi^2\ell^3/4G$, and $2/T = 1/T_+ + 1/T_-$. The electric term in the gauge fields $A_\pm^3$ is required because regularity in the (Euclidean) bulk imposes holomorphicity [7]. The winding of the gauge fields endows the black hole with integral topological charges $Q_\pm = k\alpha_\pm$. In the two-dimensional CFT dual to AdS_3, this black hole is described as an ensemble of microstates with left and right Virasoro levels $\frac{M\ell \pm J}{2} + \frac{k}{4}\alpha_\pm^2$, or, in the canonical ensemble, left and right temperatures $T_\pm$.

Now consider a Dirac fermion charged under the two gauge fields propagating in this background with action $\mathcal{S} = \int d^3x\sqrt{-g}(i\bar{\Psi}\Gamma^a\mathcal{D}_a\Psi - m\bar{\Psi}\Psi)$, where $\mathcal{D}$ is a gauge covariant derivative. According to the AdS/CFT dictionary, this fermion is dual to a spin-1/2 operator $\mathcal{O}_m$ of *fixed helicity* in the two-dimensional dual field theory [8]. The operator $\mathcal{O}_m$ is left handed for masses $m\ell > 0$, right handed for $m\ell < 0$. Specifically if $\gamma^{0,1}$ are the 2×2 γ matrices in two-dimensions and we take $\gamma^3 = \gamma^0\gamma^1 = \sigma^3$, then states created by the operator $\mathcal{O}_m$ are projected by $1 \pm \gamma^3$. These Weyl representations are two-dimensional analogues of fixed helicity in four-dimensions.

The BTZ black hole is just the $SL(2, \mathbb{R})$ group manifold, up to discrete identifications. This completely determines the waves propagating in the geometry. It is then a routine computation to take the ratios of outgoing and incoming waves at (conformal) infinity, with purely ingoing boundary conditions at the horizon, to obtain the retarded Green function for $\mathcal{O}_m$. For $m > 0$, taking $\psi \propto e^{-i\omega t + in\phi}\tilde{\psi}(r, \omega, n)$ and assuming noninteger $2h_\pm = m\ell + 1 \pm 1/2$, this procedure gives [9]

$$G_R(\omega, n) = -\frac{i}{2}\prod_{s=\pm}\frac{\Gamma(1-2h_s)\Gamma(h_s - i\frac{\omega_s}{4\pi T_{-s}})}{(2\pi T_{-s})^{1-2h_s}\Gamma(\tilde{h}_s - i\frac{\omega_s}{4\pi T_{-s}})} \quad (2)$$

with $\tilde{h}_\pm = 1 - h_\pm$ and $\omega_s = \omega + s(n/\ell - 2\alpha_s)$. This expression corrects a minor error in [9] where $T_\pm$ were exchanged; though in the notation used in this reference the latter corresponds to exchanging T_L and T_R. (Similar formulae with opposite conformal spin $(h_- - h_+)$ follow for $m < 0$ [9].) The Wilson lines in (1) shift the momenta $\omega \pm n/\ell$ by amounts proportional to $\alpha_\pm$, into which we have also absorbed the charges of the fermion under the two gauge fields. The temperatures $T_\pm$ are independent for left and right movers. When $2h_\pm = 1, 2, 3, \cdots$ ($|m|\ell$ 1/2 integral) the ratio of Gamma functions in (2) is multiplied by a factor involving di-Gamma functions (ψ) of the momenta:

$$\sqrt{2}[\psi(a) - \psi(n+1) + \gamma_E] + \frac{1}{\sqrt{2}}[\psi(b) + \psi(b-1)] \quad (3)$$

with $a = h_- - i\omega_-/4\pi T_+$, $b = h_+ - i\omega_+/4\pi T_-$, $n = 2h_- - 1$ and γ_E is the Euler-Mascheroni constant. (This expression is further modified for the special case $2h_- = 1$.) The singular normalization of (2) for integer $2h_\pm$ is an artifact of neglecting these di-Gamma functions. Below, for simplicity, we focus on the case of noninteger $2h_\pm$ although the integer values are in fact realized in the simplest string theoretic embeddings.

With $\alpha_\pm = 0$, a Fourier transform gives $G_R(x_+, x_-) = -i\Theta(x_+)\Theta(x_-)(\frac{\pi T_+}{\sinh \pi T_+ x_-})^{2h_+}(\frac{\pi T_-}{\sinh \pi T_- x_+})^{2h_-}$ with support in the forward light cone ($\Theta(x_+)\Theta(x_-) = \Theta(t)\Theta(t^2 - \phi^2)$) as expected. The overall numerical factor was determined such that the short distance singularity (and low temperature limit) in real space takes the canonical form so that $\langle \mathcal{O}_m(t,\phi)\mathcal{O}_m(0,0)\rangle = x_+^{-2h_-}x_-^{-2h_+}$. Thus (2) is the thermal Green function of an operator with spin $h_+ - h_- = \frac{1}{2}$ and conformal dimension $\Delta = h_+ + h_- \geq 1$. There is a tower of thermal poles at $\omega_s = -i4\pi T_{-s}(h_s + n)$ for nonnegative integer n. These poles collapse to the real line as $T_\pm \to 0$ producing nonanalytic behavior of the zero-temperature Green function $G_R(\omega, n) \propto \prod_{s=\pm}\omega_s^{2h_s - 1}$ at $\omega_s = 0$, indicating the edges of the spectral bands. At zero temperature the Fermi sea is filled up to $\omega = 0$. Thus $\omega_s = 0$ with $\omega = 0$ gives the momenta at the two edges of the Fermi surface as $n_\pm = 2\alpha_\pm\ell$.

Using the Euler reflection formula we obtain the spectral function $4A(\omega, n) = -8\,\mathrm{Im}G_R(\omega, n)$ as

$$\cosh\left[\sum_{s=\pm}\frac{\omega_s}{4T_{-s}}\right]\prod_{s=\pm}\frac{(2\pi T_{-s})^{2h_s-1}}{\Gamma(2h_s)\cos\pi h_s}\left|\Gamma\left(h_s - i\frac{\omega_s}{4\pi T_{-s}}\right)\right|^2. \quad (4)$$

This level density is plotted in Fig. 1. The sum and difference of the Wilson lines around the BTZ black hole ($\alpha_+ \pm \alpha_-$) move the spectral bands up/down and left/right in the $\omega - n$ plane. The low temperature limit $T_\pm \to 0$ of the spectral function can be extracted using $\lim_{|y|\to\infty}\frac{1}{\sqrt{2\pi}}|\Gamma(x+iy)|e^{\pi|y|/2}|y|^{(1/2)-x} = 1$. Taking $\omega_\pm/T_\mp \gg 1$ this gives

$$A(\omega, n) \approx \pi^2\cosh\left(\sum_{s=\pm}\frac{\omega_s}{4T_{-s}}\right)\prod_{s=\pm}\frac{e^{-|\omega_s|/4T_{-s}}|2\pi\omega_s|^{2h_s-1}}{\Gamma(2h_s)\cos\pi h_s}. \quad (5)$$

In the region inside the spectral bands, i.e. $\omega_+ \cdot \omega_- > 0$, the expansion of cosh gives a power law spectral density: $A(\omega, n) \propto \prod_{s=\pm}|\omega_s|^{2h_s-1}$. Similarly, outside the spectral bands ($\omega_+ \cdot \omega_- < 0$) the spectral density vanishes exponentially: $A(\omega, n) \propto \prod_{s=\pm}|\omega_s|^{2h_s-1}(\sum_s e^{-|\omega_s|/2T_{-s}})$, which rapidly declines with temperature. The structure near the edges of the spectral bands is obtained by taking $\omega_\pm \ll T_\mp$ and using that to leading order in small y $\log|\Gamma(x+iy)/\Gamma(x)|^2 = -y^2\sum_{n=0}^\infty(1/(x+n)^2) + \cdots$. The right hand side defines the Hurwitz zeta function $\zeta(2, x)$. Thus, for example, close to the spectral band boundary with

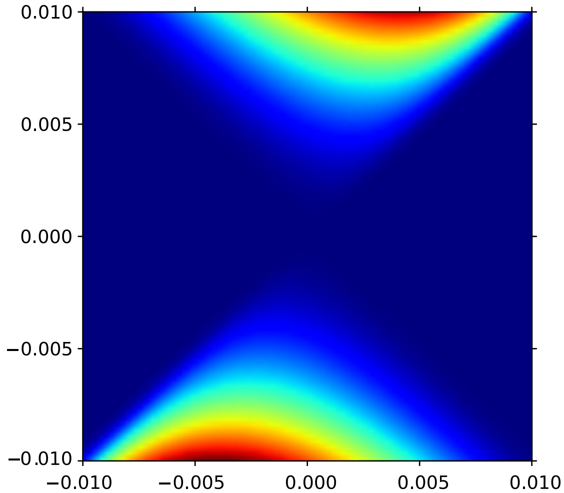

FIG. 1 (color online). Spectral density for $m = 1.2$, $T_\pm = 10^{-4}$, and $\alpha_\pm = 0$. The horizontal axis is n and the vertical ω. The spectral density vanishes rapidly outside the cones defined by $\omega \pm n = 0$. Varying $\alpha_\pm$ shifts the spectral bands in the $\omega - n$ plane.

$\omega_- = 0$, but with $\omega_+ \gg T_-$ we have $A(\omega, n) \propto |\omega_+|^{2h_+-1} \exp(\omega_-/2T_+ - (\omega_-/4\pi T_+)^2 \zeta(2, h_-))$.

The system we study can readily be embedded into full-fledged string theory, with AdS_3 appearing as a low-energy limit and AdS_2 at an even lower energy. In these detailed constructions (for a recent review, see [10]) the fermion appears with specific conformal weights. The simplest embedding is the $D1/D5$ system in Type IIB string theory, with black holes that have $AdS_3 \times S^3 \times T^4$ near-horizon geometry. In this case there are fermions in chiral primary representations with conformal weights $(h_-, h_+) = (\frac{1}{2}(\ell + 2), \frac{1}{2}(\ell + 1)) + \text{c.c.}, \ell = 0, 1, \ldots$. They have degeneracy 4. Thus the fermionic operator whose correlator we are studying is embedded in a well-defined, UV-complete field theory—it is a particular deformation of the (4, 4) supersymmetric σ model on the target space $(T^4)^k/S_k$. While the complete field theory is strongly coupled and thus not readily solvable, it could in principle be put on a lattice and studied numerically.

Another standard embedding (see the review [10]) is the chiral M5-embedding in string theory, with black holes that have $AdS_3 \times S^2 \times X$ near-horizon geometry, where X is a Calabi-Yau manifold. In this case the fermions in chiral primary representations have conformal weights of specific chirality $(h_-, h_+) = (\frac{1}{2}(\ell + 2), \frac{1}{2}(\ell + 1))$, $\ell = 0, 1, \ldots$. Their degeneracy is $2(h_{21} + 1)$, where h_{21} is a Betti number of X. It is worth noting that the simplest weights are precisely half-integer, which is the case where response functions acquire additional logarithmic behavior that is not generic. This is interesting but not mandatory since, going beyond chiral primaries, a discretuum of fermion operators with spacings of order $1/k$ can also be realized in these and more elaborate settings. Thus, fermionic operators with the properties we assume can be realized in UV-complete CFTs.

In the field theories we discuss, the fermion of interest interacts strongly with all the other excitations in the theory. The collective effects of these interactions endow the fermion with an anomalous dimension. The virtue of the AdS/CFT correspondence is that the strong interactions are conveniently resummed in this setting in terms of free propagation in a curved extra dimension. Consider carrying out this resummation directly in the field theory at finite temperature by integrating out all the other fields. This will yield a complicated Lagrangian for our fermion, with many higher order terms. However, upon running this Lagrangian down to the IR, the physics will be dominated by the marginal operators allowed at the interacting IR fixed point. These operators realize an effective Luttinger theory whose parameters we wish to relate to the dual AdS theory.

For spin-1/2 operators in two-dimensions, these have been exhaustively studied (see the review [11]). The only permitted marginal operators are those that preserve helicity and the discrete symmetries. To write a local interaction for a Weyl fermion we must introduce some other field. The simplest possibility is to assume time-reversal (TR) invariance, with a kinetic term $H_0 = -i \int dx(\psi^\dagger \partial_x \psi - \bar{\psi}^\dagger \partial_x \bar{\psi})$, and a four fermion dispersive interaction coupling the two directions of motion

$$H_{\text{int}} = g_2 \int dx \psi^\dagger \psi \bar{\psi}^\dagger \bar{\psi}, \tag{6}$$

with spin label omitted since it is fixed by the $1 \pm \gamma^3$ projection. This is the *helical Luttinger liquid*. In this realization (for which the fields exist in the TR-invariant D1/D5 theory), the fermions of primary interest (ψ) scatter off "secondary" fermions moving in the opposite direction ($\bar{\psi}$) realized in the bulk as a Dirac fermion with negative mass and opposite conformal spin (so the system is TR invariant). In other realizations (including the $M5$ embedding), the primary fermion must interact with more general antiholomorphic currents.

The Luttinger liquid permits an exact solution by bosonization. The free fermion ($g_2 = 0$) is represented as a scalar on a circle with radius R_{free}, and then interactions are taken into account by changing the radius to $R^4 = \frac{1+g_2/2\pi}{1-g_2/2\pi} R^4_{\text{free}}$. Interactions modify the conformal weights $(0, \frac{1}{2})$ of the free fermion to $(h_-, h_- + \frac{1}{2})$, where

$$h_- = \frac{1}{8}\left[\frac{R^2}{R^2_{\text{free}}} + \frac{R^2_{\text{free}}}{R^2} - 2\right] \simeq \frac{g_2^2}{32\pi^2}. \tag{7}$$

The latter expression is for small coupling, but the full formula is exact. Comparing with the AdS formula

$$h_- = \frac{|m|\ell}{2} + \frac{1}{4} \geq \frac{1}{4}, \tag{8}$$

we get a relation between the mass of the fermion in the dual three-dimensional gravity theory and the coupling constant of the Luttinger liquid. Note that the free theory ($R = R_{\mathrm{free}}$ or $g_2 = 0$) is never realized, since $|m| \geq 0$.

The nonanalytic structure in the low temperature Green function is due to IR physics. Low temperature can be attained by taking one or both of $T_\pm \to 0$ (recall $2/T = 1/T_+ + 1/T_-$). A limit where only one of these temperatures goes to zero leaves the field theory in a state with finite chiral momentum and corresponds in AdS_3 to an extremal, rotating BTZ black hole. The AdS/CFT correspondence reorganizes energy scales in the field theory geometrically so that IR physics in the field theory is associated to dynamics near the black hole horizon. Thus, we can extract the IR structure by examining the *near-horizon limit* of the geometry and wave equations.

The extremal ($T_- = 0$) black hole metric is $ds^2 = \ell^2 d\eta^2 + \ell^2 e^{2\eta} dw^+ dw^- + r_+^2 (dw^+)^2$, where $w^\pm = \phi \pm t/\ell$ and $r^2 = r_+^2 + \ell^2 e^{2\eta}$. The near-horizon geometry can be isolated via a scaling limit $w^- \to w^-/\lambda$ and $e^{2\eta} \to \lambda e^{2\eta}$ as $\lambda \to 0$. The form of the metric remains invariant in this limit, but w^- effectively decompactifies, giving the "self-dual orbifold" of AdS_3 [12]. We must preserve the topological charges $Q_\pm$ associated to our Wilson lines, and that is achieved by also taking $\alpha_+ \to \lambda \alpha_+$. The Dirac equation is invariant in form under this scaling limit, and so the $T_- \to 0$ Green function is

$$G_R = C \omega_+^{2h_+ - 1} \left| \Gamma\left(h_- - \frac{i\omega_-}{4\pi T_+}\right) \right|^2 \sin\pi\left(h_- + i\frac{\omega_-}{4\pi T_+}\right), \tag{9}$$

where C is a temperature dependent normalization constant. In order to match this IR Green function with the UV theory, we take λ to be finite and small (rather than strictly zero). Then the IR ω_+ in (9) is related to the UV lightcone momentum as $\lambda\omega_+ = \omega_+^{\mathrm{UV}}$, reflecting the redshift between the near-horizon and asymptotic part of the black hole geometry. The dependence of the Green function on the chemical potential T_+ constitutes nontrivial dynamical information characterizing the system that the primary fermions interact with.

It is instructive to compare our results in $D = 1 + 1$ to the related study of cold, non-Fermi liquids in $D = 2 + 1$ [2]. The latter involve *charged* black holes in AdS_4 vs our *rotating* black holes in AdS_3. In both studies, the IR dynamics (a near-horizon limit in AdS space) are matched with the UV dynamics (the asymptotic geometry) to construct retarded Green functions, and the crucial part of the near-horizon geometry is AdS_2 with an electric field. The final result (in Appendix D of [2]) for the nonanalyticity that leads to non-Fermi liquid behavior is

$$G_R^{\mathrm{nFL}}(\omega) = C' \omega^{2\nu} |\Gamma(\nu - iqe_2)|^2 \sin\pi(\nu + iqe_2), \tag{10}$$

where C' is a normalization constant, q is the fermion charge, and e_2 is the electric field (parametrizing the chemical potential). This precisely matches the form of (9), with the identifications $T_+ = (4\pi e_2)^{-1}$, $\omega_+ = \omega$ and $\omega_- = q$, and $h_- = \nu \equiv \sqrt{m_2^2 R_2^2 - q^2 e_2^2}$. Recalling that $h_- = |m|\ell + \frac{1}{2}$, we relate the AdS_2 mass of the non-Fermi liquid to the mass of our three-dimensional fermion.

This precise agreement between low temperature correlation functions confirms that the AdS_2 near-horizon geometry is responsible in both cases for the IR behavior. The parameters of the different UV completions are then related by comparing physical quantities in the low-energy effective theory. Despite this simple picture, the IR sector of the 2 + 1 dimensional non-Fermi liquid in [2] cannot be simply mapped into a Luttinger liquid. This is due to a subtle sensitivity to the UV theory. The AdS_4 black brane in [2] is charged under an auxiliary electric field, while the electric field in our AdS_2 is geometrized as an extra dimension which is the direction of the momenta of the Luttinger liquid. Thus, Luttinger modes of different momenta appear in AdS_2 as a tower of particles with integrally spaced charges and masses, while the momentum modes of the 2 + 1 dimensional field theory in [2] appear in AdS_2 as a tower of particles with different masses but fixed charge. These differences imply different spectra for $\omega_- \equiv q$ in the two cases. This feature complicates the relationship between the two-dimensional physics of the Luttinger liquid and of four-dimensional non-Fermi liquids with gravity duals.

In summary, the IR structure of correlation functions in the holographic approach to cold Fermi liquids always derives from the omnipresent near-horizon AdS_2 geometry. The full black hole geometry is analogous to the UV completion of an IR field theory [2,13]. The BTZ black holes presented here are the most transparent UV completion. While convenient, our three-dimensional completion may not capture all interesting phenomena. For example, a superconducting instability can usually be implemented in AdS in terms of a charged boson with a mass that is stable in the UV but tachyonic in the AdS_2 near-horizon geometry [14]. An interesting feature of our AdS_3 completion is that here the stability bound is exactly the same in the UV and the IR, since changes in AdS radius are precisely compensated by a change in the Breitenlohner-Freedman bound. Thus condensation by this mechanism appears impossible. It would be interesting to understand in more detail what features of the UV completion drive specific low-energy phenomena in the holographic description of condensed matter systems.

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