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Comment on "Green's function theory for infinite and semi-infinite particle chains"

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In this Comment, we argue that the criticism of our previous paper, which was recently articulated by Hadad and Steinberg, is unwarranted.

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In a recently published paper,[1] Hadad and Steinberg (HS) have presented a number of analytical results for the electromagnetic Green's function in infinite and semi-infinite periodic chains of small polarizable particles. We find these results to be interesting and useful but also feel that this paper contains an unjustified criticism of our previous paper on the subject.[2,3] Thus, on p. 2, HS wrote

> "Furthermore, we show that the 'extraordinary' wave ('light-line' mode) *does not exist* in longitudinal polarization; the corresponding poles exist only in transverse polarization (consistent with Ref. 4). The continuous spectrum wave, due to a nearby branch point, is responsible for an excitation in the longitudinal polarization, which was misinterpreted as an 'extraordinary' wave in Refs. 2 and 3."

Here, the reference numbers correspond to the list at the end of this Comment. Similar criticism can be found on p. 8 of Ref. 1. In what follows, we argue that this criticism is unwarranted.

First, the above criticism is aimed at the use of terminology, not at physical results. Thus, HS do not dispute the validity of data shown in Figs. 2 and 3(a) of Ref. 2 in which we illustrate what we have termed the extraordinary surface plasmon polariton (SPP) for the longitudinal polarization. Rather, HS oppose our use of the word extraordinary for the longitudinal polarization. However, this criticism is misplaced because the terminology in question was, in fact, introduced by us in Ref. 2. This terminology simply did not exist at the time when our paper was written. Incidentally, the paper by Alu and Engheta,[4] which discusses a similar physical phenomenon (albeit with a focus on dispersion relations rather than on Green's functions), was submitted for publication a few weeks later than Ref. 2. Therefore, we could not possibly misinterpret or mislabel the longitudinally polarized SPP because the very phenomenon was discovered by us and no other relevant term or interpretation existed at that time.

From a physical point of view, we also see no reason for the criticism. Our classification of SPPs (that is, ordinary vs extraordinary) is based on different rates of spatial decay of the corresponding excitation and on the fact that the ordinary SPPs are well explained by the electromagnetic interaction in the near zone, whereas, the extraordinary SPPs require accounting for interaction in the radiation zone.

HS insist that the classification of SPPs must be based only on the poles or branch cuts of the inverse Z transform and that any other classification amounts to a misinterpretation. But we see no reason to adhere to this point of view. There may be several classification schemes, all equally valid. Our classification is based on the physical behavior of an observable and is, therefore, practically relevant. HS's classification is based on the analytical properties of the Z transform and might be mathematically convenient. Why the latter should take precedence over the former, is not clear.

We now discuss the HS results from a more technical point of view. The mathematical formalism of HS is obtained from the equation published by us[2] by a simple change of variables. Thus, we have given the expression,

$$G_n = \int_{-\pi/h}^{\pi/h} \frac{\exp(iqhn)}{1/\alpha - S(q)} \frac{h\,dq}{2\pi}. \tag{1}$$

Here, G_n is the Green's function in an infinite chain, n labels the particles, h is the chain period, and S is the dipole sum,[2,5,6] otherwise, we use HS's notations. Introducing a new integration variable $Z = \exp(iqh)$, we can transform this equation to the form

$$G_n = \frac{1}{2\pi i} \oint_{|Z|=1} \frac{Z^{n-1} dZ}{1/\alpha - S[q(Z)]}. \tag{2}$$

The mathematical advances reported by HS are based on analysis of this formula. HS take advantage of the fact that the integral (2) is taken over a closed contour, whereas, Eq. (1) is taken over a finite segment of the real axis. It may seem that, if the zeros of the denominator in Eq. (2) are known, the integral can be computed analytically without further approximations, whereas, to evaluate Eq. (1), we have resorted to the quasiparticle pole approximation and numerical simulations.

Unfortunately, Eq. (2) is not as simple as it appears. While the poles of the integrand can, indeed, be identified, albeit only numerically, the function $S[q(Z)]$ has a branch cut in the complex Z plane, which also contributes to G_n. The branch-cut contribution, denoted by HS as $G_n^{(b)}$, is additive [see Eq. (16) of Ref. 1] and cannot be disentangled from the contributions of the poles, denoted by HS as $G_n^{(p_m)}$. Moreover, $G_n^{(b)}$ cannot be computed analytically except in the asymptotic regime $|n| \to \infty$ when $G_n^{(b)} \sim O(1/n)$ [Eq. (21) of Ref. 1].

HS claim that the branch-cut contribution $G_n^{(b)}$ is a novel mode, which is characterized by a continuous spectrum and which has not been reported before. In fact, all modes reported by us previously[2] are characterized by a continuous spectrum when evaluated without approximation, e.g., numerically. The continuous spectrum of wave numbers is an inherent property of a perfectly periodic infinite system. The modes in such systems are delocalized, and the spectrum is necessarily continuous. A discrete spectrum can result from the effect of localization, e.g., due to disorder (considered by us in Ref. 2), but HS restrict consideration to perfectly periodic infinite or semi-infinite chains.[1]

It is true that, in the quasiparticle pole approximation, we make an assumption that only one wave number is dominating for a certain range of propagation distances, and this assumption is, in some cases, well justified. In quantum mechanics, there is a similar concept of quasidiscrete or quasistationary energy levels[7] where the imaginary part of an energy level corresponds to its "width." A similar situation exists in our case: The quasiparticle pole approximation results in complex quasidiscrete values of q, whose imaginary parts can be used to estimate the rate of spatial decay of the ordinary SPP. For simplicity, we have referred to this dominating wave number as the wave number of the mode, e.g., on p. 4. However, we have explained that the quasiparticle pole approximation is inapplicable for the extraordinary SPPs, and the same point was made by us earlier.[6] In any case, we did not really claim that only a single wave number contributes to any given mode either in orthogonal or in longitudinal polarization. For example, on p. 3 of Ref. 2, we have written

> "It follows from formula (9) that the wave numbers q of SP excitations that can propagate effectively in an infinite periodic chain are such that $1/\alpha \approx S(k,q)$."

Note the words "wave numbers" and the approximate equality sign. Note that the abbreviation SP in the above quote can be used interchangeably with the abbreviation SPP used in this Comment and elsewhere.

Regarding the differences between the extraordinary SPPs with orthogonal and longitudinal polarizations, which have very different amplitudes (as noted by us on p. 6 of Ref. 2), we have never claimed that the appearance of the extraordinary SPP in longitudinal polarization is caused by the same reason as in the case of orthogonal polarization, namely, by the denominator in Eq. (1) turning to zero at the resonant wave number q_2, which in this polarization, does not exist. This is clearly stated on p. 4 of Ref. 2. To quote directly (emphasis added),

> "In the case of oscillations polarized along the chain, the resonance condition can be satisfied only at $q = q_1 \approx 0.45\pi/h$. This is the wave number of an *ordinary quasistatic* SP which depends on k only weakly, as long as $kh \lesssim 0.2\pi$. However, the extraordinary (nonquasistatic) SP can be excited even for longitudinal oscillations. Mathematically, this can be explained by observing that $\partial\,\mathrm{Re}\,S(k,q)/\partial q$ diverges at $q = k$ while $\partial\,\mathrm{Im}\,S(k,q)/\partial q$ is discontinuous at $q = k$ and performing integration (9) by parts."

Thus, we have provided a mathematical explanation for the existence of extraordinary SPPs in longitudinal polarization, which does not require the denominator in Eq. (1) turning to zero. Of course, the existence of the branch cut in HS formalism is mathematically related to the divergence of $\partial S/\partial q$. Therefore, this mode can be explained either by the branch-cut contribution to integral (2) in HS formalism or by the divergence of $\partial S/\partial q$, and we see no over-riding physical significance in preferring one explanation over the other.

It should be mentioned that Ref. 1 contains one criticism of our paper, which we find valid. This concerns comparison of the Green's functions in infinite chains and in semi-infinite chains excited at the terminal dipole. Namely, HS wrote on p. 7 of Ref. 1:

> "In the domain near the source ... G for infinite and semi-infinite chains differs only by a constant multiplication factor ...However, the two solutions deviate for larger distances. Hence, the statement made in Ref. 2 that the Green's function of infinite and semi-infinite chains differs only by a multiplication factor is somewhat incomplete; it is correct only within the limited region near the source."

Indeed, we have neglected to mention that the proportionality coefficients are somewhat different for the ordinary and extraordinary SPPs. This was not so much a mistake but an omission. A relevant example is given in Fig. 1 where we plot the amplitude of the dipole moment as a function of distance from the point of excitation. In one case, the source is located at the end, and in the other case, the source is located in the center of a long but finite chain containing $N = 2001$ dipoles. Orthogonal polarization and the same parameters as in Fig. 2 of Ref. 2 have been used. Note that, in Fig. 1, we plot the dipole moment amplitudes directly and do not use normalization to the amplitude of the illuminated dipole as was done previously by us[2] and also by HS, particularly, in Fig. 8 of Ref. 1. Such normalization tends to magnify the difference between the two curves and makes it appear more significant than it is in reality.

It can be seen that the differences are truly insignificant as was correctly mentioned by us earlier.[2] The proportionality between the two curves holds but with different coefficients for the ordinary (fast decaying) and the extraordinary (slow decaying) SPPs. In the first case, the coefficient is ≈1.7 (for the ratio of amplitudes), and in the second case, it is ≈0.92. Thus, HS's discussion of this topic is also somewhat incomplete: The

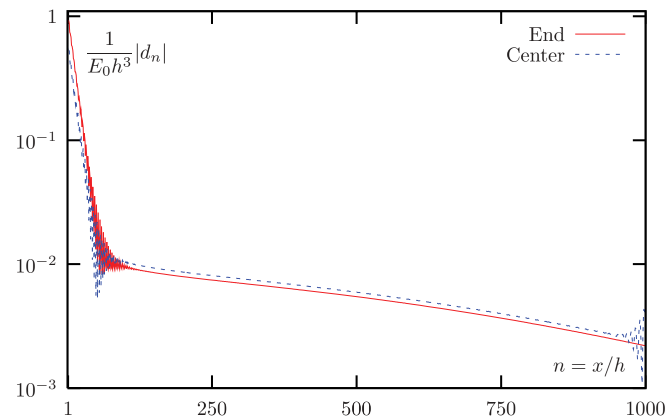

FIG. 1. (Color online) Amplitudes of dipole moments as functions of distance from the point of excitation, measured in units of the chain period h. The excitation point is located either at the end or at the center of a long chain as labeled. E_0 is the amplitude of the illuminating electric field.

proportionality holds not "only within the limited region near the source," but also in the range where one of the SPPs (either ordinary or extraordinary) is dominating. The proportionality does break in the crossover region.

To summarize, it was always understood that all modes discussed by us consist of a spectrum of wave numbers, all differently decaying. Our focus, however, was not on these mathematical intricacies but on the physical behavior. Consequently, HS's criticism that we have misidentified or missed some important excitation modes is not justified. Finally, we note that Ref. 3, which was also critically mentioned by HS, does not simulate or discuss the extraordinary SPPs at all, apart from a brief remark that these excitations have been previously observed in numerical simulations. Therefore, Ref. 3 should have not been mentioned by HS in this particular context.

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