Excited $0^+, T = 2$ States in $^{12}\text{Be}$, $^{12}\text{C}$, and $^{12}\text{O}$

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Abstract
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We present predictions of a relatively simple model of the low-lying $0^+$ states in $^{12}$Be, their predicted energy splitting in $^{12}$O, their cross-section ratios in $^{10}$Be$(t,p)$ and $^{14}$C$(p,t)$ and their decay widths. Comparison is made with predictions using earlier wave functions of Barker.

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A brief glance at the low-lying energy levels of $^{11}$Be [1] makes it clear that $^{12}$Be should have two low-lying $0^+$ states that are mixtures of $p$-shell [2] and $^{10}$Be $\times (sd)^2$ configurations. Barker [3] computed the expected positions of three $0^+$ states and some of their properties. He obtained an energy splitting of 2.35 MeV for the first two, with the majority of the $s^2$ component in the second state, even though he stated “these numbers should not be taken too seriously.” His lowest two $0^+$ states are not primarily admixtures of the lowest $p$-shell and lowest $(sd)^2$ $0^+$ states. They have considerably different $d^2/s^2$ ratios and opposite relative phases between $d^2$ and $s^2$ [3]. His lowest has $32\% s^2$, $29\% d^2$, whereas the second has $67\% s^2$ but only $10\% d^2$. His states resemble orthogonal mixtures of $s^2$ and $Ap^3 + Bd^2$, with his third $0^+$ being then predominantly $Bp^2 – Ad^2$.

The first excited $0^+$ state in $^{12}$Be was recently discovered at an excitation energy of 2.24 MeV [4]. (An early $0^+$ candidate [5,6] at 2.7 MeV turned out [7] to have $J^Z = 1^-$. We discuss the properties of this $0^+$ state and estimate its location in $^{12}$O. We also estimate relative cross sections for $2n$ stripping and pickup.

A $1p$ shell-model calculation [2] produces one low-lying $0^+$ state in $^{12}$Be and a $2^+$ state at 4.37 MeV above it. Coupling two $sd$-shell neutrons to a $^{10}$Be core produces two low-lying $0^+$ states and two $2^+$ states, all primarily involving only the $2s1/2$ and $1d5/2$ orbitals. With two-body matrix elements from LSF [8] and single-particle energies from $^{11}$Be (Table I), these two $(sd)^2 0^+$ states have absolute energies [relative to the physical $^{12}$Be (g.s.)] calculated to be 0.20 and 4.35 MeV (Table II). The first is predicted to be very strong in $^{10}$Be$(t,p)$, the second much less so. The corresponding second $(sd)^2 0^+$ states in $^{14,16}$C have small $(t,p)$ cross sections [9]. Experimental results from the $^{10}$Be $(t,p)$ reaction [5] confirm that $^{11}$Be(g.s.) contains significant $sd$-shell admixtures, a conclusion reached earlier in connection with $\beta$ decay [10] and $2^+ \to$ g.s. $\gamma$ decay [11].

Because the Coulomb energy shift is very sensitive to the $2s1/2$ occupancy, fitting the mass difference between $^{12}$Be and $^{12}$O ground states allows a determination of this $(2s1/2)^2$ component in the two states (assumed equal, if isospin invariance holds). Such a calculation [12] gave $53\% s^2$, with the remaining $47\%$ split among $p$-shell and $d^2$ components. Because the calculation is much less sensitive to the relative amounts of these admixtures, the Coulomb-energy calculation provided no further insight into the $d^2/p^2$ ratio. However, it is extremely likely that the physical g.s. is primarily a mixture of the $p$-shell ground state and the lower of the two $(sd)^2 0^+$ states mentioned above, which is calculated to contain $22\% d^2$, $78\% s^2$. If the physical ground state has the same $d^2/s^2$ ratio, we would then have $53\% s^2$, $15\% d^2$, and $32\%$ p shell. These components are listed in Table III, where we compare with the results of the earlier calculation by Barker. [3].

The second $(sd)^2 0^+$ state is far away, is orthogonal to the first $(sd)^2 0^+$ state, and is expected to have a very small mixing matrix element with the $p$-shell $0^+$ state. Thus, we expect the second observed $0^+$ level to be the orthogonal admixture to the ground state, viz. $32\% (sd)^2$ and the remainder p shell. In $(t,p)$, these amplitudes interfere destructively (they are constructive in the ground state). Distorted-wave cross sections [5] for $(sd)^2$ transfer are about seven times those for the p-shell amplitudes of CK for $^{10}$Be to $^{12}$Be, leading to a predicted $0^+/\text{ground state}$ ratio of 0.008. It is thus not surprising that this second $0^+$ state was not observed in $(t,p)$.

By contrast, Barker has more $s^2$ component in the excited $0^+$ state than in the ground state. Using his wave functions for the excited $0^+$ state, the predicted $0^+$/ground state $(t,p)$ cross-section ratio is 0.10, i.e., the excited state should have been strong enough to have been observed, though still considerably weaker than the ground state.

The energy shift from $^{12}$Be to $^{12}$O is also very different with the two sets of wave functions. Because Barker has more $s^2$ in the upper state, the splitting between the two $0^+$ states in $^{12}$O will be significantly smaller than in $^{12}$Be. We have computed this energy difference with our wave functions and those of Barker. Results are also listed in Table III. We see that the expected location of the second $0^+$ state in $^{12}$O is considerably different in the two models. With a splitting of 2.24 MeV in $^{12}$Be, our wave functions provide a splitting of 1.95 MeV in $^{12}$O, whereas using Barker’s wave functions yields 1.19 MeV splitting in $^{12}$O. This difference is easily understood, because it is the upper of the two that has the most $s^2$ strength in Barker’s calculation.

If we accept the result [12] that the $^{12}$Be-$^{12}$O ground-state energy difference requires about $53 \pm 3\% s^2$, with little
dependence on the $p^2/d^2$ ratio, we can look elsewhere for more information on that ratio. A recent breakup experiment [13] extracted a $d5/2$ spectroscopic factor of $0.48 \pm 0.06$ for $^{12}\text{Be}(\text{g.s.}),$ implying $0.24 \pm 0.03$ for the $d^2$ probability in that state. If this value is correct we would then have $1 - 0.53 - 0.24 = 0.23 \pm 0.05$ for the $p$-shell component—in excellent agreement with the value $0.22 \pm 0.04$ implied by the $p1/2$ spectroscopic factor of $0.44 \pm 0.08$ in Ref. [13]. This mixture of $s^2$, $d^2$, and $p$-shell components results in the same $^{12}\text{Be} - ^{12}\text{O}$ energy difference as our earlier mixture, to within $2.5 \text{keV}$. And, changing the $d^2/s^2$ ratio from $0.2/0.8$ to $0.3/0.7$ reduces the $(s^2)^2$ probability to be quite strong in $(p, t)$. Even though the majority of the CK $2^+$ strength lies considerably higher, only about 25% of the $p$-shell $2^+$ state mixed into the physical $2^+$ state could explain the observed magnitude of the $(p, t)$ cross section, without any need for any $0^+_2$ contribution. We have no reliable estimate of the amount of this mixing. The near degeneracy of the second $(sd)^2 2^+$ state and the $p$-shell one (see Table II) could complicate the mixing. It is possible that both $0^+$ and $2^+$ states are being populated. If we fit the $29.63$-$\text{MeV}$ angular distribution with a mixture of a smooth curve drawn through the $0^+$ $27.57$-$\text{MeV}$ angular distribution and the $L = 2$ curve displayed with the data in Ref. [14], a reasonable fit is obtained, with $\sigma(2^+) = 3.8 \pm 0.2 \mu\text{b/sr}$ at the average of the first two angles and $\sigma(0^+_2) = 3.6 \pm 1.0 \mu\text{b/sr}$ at $\theta \approx 30^\circ - 35^\circ$. This $2^+$ yield would correspond to about 19 \pm 9\% of the CK $2^+$ state mixed into the physical state. Some of the $\approx 200$-$\text{keV}$ width [14] could then come from overlapping levels rather than natural width. The decay branching ratios would then be difficult to untangle because both states would contribute to the decay.

In $(p, t)$ the two $0^+$ states can be populated in direct $2n$ pickup from both components of $^{14}\text{C}(\text{g.s.})$ (see Fig. 1), but both paths involve $p$-shell transfer, as we now demonstrate. For simplicity, think of $2p$ pickup to $^{12}\text{Be}$, rather than $2n$ pickup.

### Table I. Hamiltonian for low-lying $(sd)^2$ states in $^{12}\text{Be}$.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>Matrix element</th>
<th>Value (MeV)$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>$V(s^2, s^2)$</td>
<td>$-1.54$</td>
</tr>
<tr>
<td></td>
<td>$V(s^2, d^2)$</td>
<td>$-1.72$</td>
</tr>
<tr>
<td></td>
<td>$V(d^2, d^2)$</td>
<td>$-2.78$</td>
</tr>
<tr>
<td>$2^+$</td>
<td>$V(ds, ds)$</td>
<td>$-0.59$</td>
</tr>
<tr>
<td></td>
<td>$V(ds, d^2)$</td>
<td>$-0.59$</td>
</tr>
<tr>
<td></td>
<td>$V(d^2, d^2)$</td>
<td>$-1.02$</td>
</tr>
</tbody>
</table>

$^a$From LSF (Constrained II), Ref. [8].

### Table II. Calculated energies (MeV) and wave functions of three lowest $0^+$ and $2^+$ states in $^{12}\text{Be}$.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>Config.</th>
<th>Eigenvalue $E_i$</th>
<th>Wave fn.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>$(sd)^2$</td>
<td>$-3.47$</td>
<td>$d^2$</td>
</tr>
<tr>
<td>$p$ shell</td>
<td>28.69</td>
<td>1.10</td>
<td>0</td>
</tr>
<tr>
<td>$(sd)^2$</td>
<td>0.68</td>
<td>4.35</td>
<td>0.22 0.78</td>
</tr>
<tr>
<td>$2^+$</td>
<td>$(sd)^2$</td>
<td>$-0.04$</td>
<td>$ds$</td>
</tr>
<tr>
<td>$p$ shell</td>
<td>33.06</td>
<td>5.46</td>
<td>0</td>
</tr>
<tr>
<td>$(sd)^2$</td>
<td>1.75$^b$</td>
<td>5.42</td>
<td>0.12 0.88</td>
</tr>
</tbody>
</table>

$^a$Relative to physical $^{12}\text{Be}(\text{g.s.})$.

### Table III. Calculated and measured properties of first two $0^+$ states in $^{12}\text{Be}$.

<table>
<thead>
<tr>
<th>Calc.</th>
<th>Ref.</th>
<th>State</th>
<th>Wave-function intensities</th>
<th>Splitting in $^{12}\text{O}$ (MeV)</th>
<th>Cross-section ratio $^{10}\text{Be}(t, p)$</th>
<th>$^{14}\text{C}(p, t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$s^2$</td>
<td>$d^2$</td>
<td>$p$ shell</td>
<td>1.95</td>
</tr>
<tr>
<td>Present$^b$</td>
<td>g.s.</td>
<td>0.53</td>
<td>0.15</td>
<td>0.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barker$^b$</td>
<td>g.s.</td>
<td>0.17</td>
<td>0.05</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barker$^b$</td>
<td>0$^+$</td>
<td>0.325</td>
<td>0.292</td>
<td>0.384</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barker$^b$</td>
<td>0$^+$</td>
<td>0.67</td>
<td>0.10</td>
<td>0.23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$Reference [12].

$^b$Reference [3].

$^c$Our calculation with Barker’s wave function.

$^d$Reference [5].

$^e$Reference [14] and present.
to the $T = 2$ states of $^{12}$C; the nuclear structure is the same. (We assume the wave-function admixtures for the $0^+$, $T = 2$ states in $^{12}$C are the same as in $^{12}$Be.) Then, with

$$^{14}\text{C} \text{(g.s.)} = (1-\varepsilon^2)^{1/2}(^{12}\text{C}_{\text{CK}}) + \varepsilon(^{12}\text{C}_{\text{CK}}) \times (sd)^2,$$

$$^{12}\text{Be} \text{(g.s.)} = \alpha^{10}\text{Be}_{\text{CK}} \times (sd)^2 + \beta^{12}\text{Be}_{\text{CK}},$$

and

$$^{12}\text{Be}(0^{+}) = -\beta^{10}\text{Be}_{\text{CK}} \times (sd)^2 + \alpha^{12}\text{Be}_{\text{CK}},$$

the $2p$ transfer amplitudes for $^{14}$C to $^{12}$Be are

$$A(\text{g.s.)} = \beta(1-\varepsilon^2)^{1/2}A(^{14}\text{C} \rightarrow ^{12}\text{Be})_{\text{CK}}$$

$$+ \alpha\varepsilon A(^{12}\text{C} \rightarrow ^{10}\text{Be})_{\text{CK}},$$

and

$$A(0^{+}) = \alpha(1-\varepsilon^2)^{1/2}A(^{14}\text{C} \rightarrow ^{12}\text{Be})_{\text{CK}}$$

$$- \beta\varepsilon A(^{12}\text{C} \rightarrow ^{10}\text{Be})_{\text{CK}}.$$
The nature of the third $0^+$ state is also quite different in the two models. In our approximation it is predominantly the second $(sd)^2$ $0^+$ state, whereas Barker’s is nearly all $p^2$ and $d^2$. His third $0^+$ level is 8.46 MeV above the first. Ours is computed to be 4.15 MeV above the position of $(sd)^20^+_1$ before the latter mixes with the CK g.s. to form the physical ground state. With the observed separation of the two lowest $0^+$ states and our wave functions, it is a simple matter to calculate this unmixed location, which turns out to be 0.7 MeV above the physical ground state—implying our third $0^+$ level should lie near 4.8 MeV. The properties of this third $0^+$ state of $^{12}$Be in the two models are summarized in Table V.

Perhaps surprisingly, the predicted $(t,p)$ cross sections (which are quite small) are nearly identical. The biggest differences are in the excitation energy and in the expected neutron-decay properties.

In conclusion, we have presented predictions of a relatively simple model of the low-lying $0^+$ states in $^{12}$Be, their predicted energy splitting in $^{12}$O, their cross-section ratio in $^{10}$Be$(t,p)$ and $^{14}$C$(p,t)$ and their decay widths. Comparison has been made with predictions using earlier wave functions of Barker. In all cases for which the experimental quantity is known, agreement is better with our wave functions. This is especially true for the decay width of the second $0^+$ state.