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Two-Proton Decay Energy and Width of ^{19}Mg(g.s.)

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Two-proton decay energy and width of ^{19}Mg(g.s.)

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We use a weak-coupling procedure and results of a shell-model calculation to compute the two-proton separation energy of ^{19}Mg. Our result is at the upper end of the previous range, but ^{19}Mg is still bound for single proton decay to ^{18}Na. We also calculate the $2p$ decay width.

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I. INTRODUCTION

Along the proton drip line, many of those proton-rich nuclei have not had their masses determined. In almost all cases, the neutron-rich mirrors of these nuclei have well-determined masses. In the absence of experimental evidence on the proton-rich masses, a procedure that accurately calculates Coulomb energy differences is useful in estimating them. Of special interest is ^{19}Mg, which may be the best candidate among light nuclei (after ^{6}Be, of course) to be a simultaneous $2p$ emitter.

For nuclei near $A = 20$, the Coulomb energy difference for ground states (g.s.) of mirror nuclei (which have identical nuclear energies) is quite sensitive to the relative population of nucleons in the $2s1/2$ and $1d5/2$ orbitals. The larger the $2s1/2$ occupancy, the lower will the state be in the proton-rich member of the mirror pair. This effect has long been called the Thomas-Ehrman shift (TES), but it is nothing special. It comes naturally out of a single-particle type calculation in a diffuse potential well.

If core excitation can be neglected, these Coulomb energies can usually be reliably computed using shell-model wave functions and coupling s or d nucleons to appropriate core states (frequently many in number). If the nucleus consists of valence nucleons coupled to a core containing vacancies in a lower shell, the shell-model calculation is usually not so straightforward. In some of these cases, e.g., a few sd-shell nucleons coupled to one or two p-shell holes, weak-coupling procedures (suitably applied) are reasonably reliable. In certain instances, a combination of shell model plus weak coupling is appropriate. For example, such a hybrid calculation gave reasonable agreement [1] for many levels of the ^{18}O-^{18}Ne pair. In that example, the states of ^{18}O were [2] linear combinations of two-particle $(sd)^2$ configurations and a collective component that was primarily $(sd)^4(1p)^{-2}$. The assumption that the $(sd)^4$ states behaved as the low-lying positive-parity states of ^{20}Ne worked well. As the $(sd)^4$ part of the $4p$-$2h$ structure was the same for ^{18}O and ^{18}Ne, the Coulomb energy for the $4p$-$2h$ component involved only the ^{14}O-^{14}C mass difference and the weak-coupling Coulomb parameter c.

In certain other cases, weak-coupling alone (in the Banzal-French-Zamick sense [3]) might be thought to be appropriate, but further inspection reveals that not to be so. One such example is the pair ^{17}N-^{17}Ne. Both are thought to be well described as $(sd)^2(1p)^{-1}$. However, weak coupling would identify the $(sd)^2$ part with the g.s. of ^{18}O and ^{18}Ne. And those states contain significant ($\sim$10%) core excitation. To the extent that this $(sd)^4(1p)^{-2}$ component contains two holes in the $p1/2$ orbital, weakly coupling a $p1/2$ hole to that component would violate the Pauli principle. This is a clear example of a situation in which the particles in the parent state are different from those in the particle-hole state. A major manifestation of this effect is that even though ^{18}O has three low-lying 0^+ states [primarily linear combinations of $(1d\ 5/2)^2$, $(2s1/2)^2$, and $4p$-$2h$], coupling a $p1/2$ hole to them results in only two low-lying $1/2^-$ states in ^{17}N [4,5].

If we were to calculate the mass excess of ^{17}N in pure weak coupling, we would get $^{17}\mathrm{N} = {}^{18}\mathrm{O} + {}^{15}\mathrm{N} - {}^{16}\mathrm{O} + 2(a + b/4) = 7.675$ MeV, to be compared with the known mass excess [6] of 7.871 MeV for ^{17}N. (We take the value of $a + b/4$ from ^{16}N.) This 200 keV discrepancy is easily understandable from the injustice this calculation does to the Pauli principle. In reality, ^{17}N is better described as a $p1/2$ hole in the lowest $(sd)^20^+$ state, not the physical ^{18}O(g.s.). A similar calculation for ^{17}Ne misses the mass excess by a different amount. The fact that this simple (and incorrect) calculation misses ^{17}N and ^{17}Ne by different amounts means that such a calculation would also miss the ^{17}N-^{17}Ne Coulomb energy (see below).

For ^{17}N and ^{17}Ne we have demonstrated [7] that coupling s^2 and d^2 nucleons to the physical g.s. of ^{15}N and ^{15}O can easily provide the known g.s. Coulomb energy, with an s^2 component of 22% (24 ± 3% in Ref. [8]), very close (but larger than) the known s^2 occupancy [2] in ^{18}O [19% of the total wave function, but 21% of the $(sd)^2$ component]. Other estimates [9] provide 40–50% s^2 in ^{17}Ne, and some [10] even suggest a preponderance of s^2 over d^2. Two different shell-model approaches [8] gave satisfactory agreement for Coulomb energies of several excited states of ^{17}Ne. These states were then used as cores to compute the g.s. mass of ^{18}Na, with reasonable success. [8] We turn now to the case of ^{19}Mg.

The nucleus ^{19}Mg is expected to be unbound with respect to $2p$ decay to ^{17}Ne, but bound for p decay to ^{18}Na. It may be the best candidate for simultaneous $2p$ decay after ^{6}Be. Grigorenko *et al.* [9] performed calculations for ^{17}Ne and ^{19}Mg. They estimate for ^{19}Mg a value $E_{2p} = 550$–850 keV

(unbound). However, they may have too much s^2 admixture in their wave function. They estimate 40–50% s^2 in ^{17}Ne, whereas we prefer (as stated above) significantly less—about 22–24%. Because of the TES, having too much s^2 will cause the proton-rich nucleus to be less unbound. In Ref. [9], the proton-rich member of a mirror pair has higher $s1/2$ occupancy than the neutron-rich member. In all our calculations, these are equal in the mirror pair. For ^{20}Mg we found [11] that the 11th and 12th proton were predominantly in the d orbital, so that the s^2 occupancy in ^{20}Mg should be approximately equal to that in ^{18}Ne. Before we attempt to calculate the ^{19}N-^{19}Mg Coulomb energy with the shell model, we estimate it using weak coupling considerations.

II. WEAK COUPLING

Usually, Coulomb energy differences of mirror nuclei will depend on the Coulomb parameter c when computed in weak-coupling formalism. However, for the heavy isotopes of N and their mirrors, there is no dependence on c. This independence of c arises because, in weak coupling, c multiplies a factor $n_{pp}n_{ph}$, where n_{pp} is the number of proton particles and n_{ph} is the number of proton holes. In $^{16+m}$N, the hole is a proton, but the $m+1$ particles are all neutrons, whereas in the mirror the hole is a neutron and the multiplicative factor is still zero. And, of course, because the mirrors are supposed to have the same nuclear energy, the mass difference is independent of the weak-coupling parameters a and b. Thus, we have the simple situation that weak-coupling evaluations of mass-excess differences between heavy N nuclei and their mirrors are independent of all three wc parameters a, b, and c—they depend only on mass excesses of ground states of neighboring nuclei, and (of course) on the assumption that weak coupling is applicable.

The nucleus ^{19}N is predominantly a $p1/2$ proton hole in ^{20}O, and ^{19}Mg a $p1/2$ neutron hole in ^{20}Mg. We have already satisfactorily computed the ^{20}O-^{20}Mg Coulomb-energy difference, using shell-model wave functions [11]. To the extent that weak coupling is a good approximation, we have

$$M(^{19}\mathrm{Mg}) - M(^{19}\mathrm{N}) = M(^{20}\mathrm{Mg}) - M(^{20}\mathrm{O}) + M(^{15}\mathrm{O}) - M(^{15}\mathrm{N}),$$

Where the M's are mass excesses. For this procedure to be valid, two conditions should be fulfilled: (1) The $(sd)^2$ structure (actually, primarily the s^2/d^2 ratio) should be the same in the four nuclei with $A = 19, 20$; (2). The g.s. of ^{20}O and ^{20}Mg should be free of $p1/2$ holes, i.e., the $p1/2$ shell should be full in these two g.s. Before dealing with ^{19}Mg, we return to the pairs ^{17}N-^{17}Ne and ^{18}N-^{18}Na. As above,

$$M(^{17}\mathrm{Ne}) - M(^{17}\mathrm{N}) = M(^{18}\mathrm{Ne}) - M(^{18}\mathrm{O}) + M(^{15}\mathrm{O}) - M(^{15}\mathrm{N}),$$

$$M(^{18}\mathrm{Na}) - M(^{18}\mathrm{N}) = M(^{19}\mathrm{Na}) - M(^{19}\mathrm{O}) + M(^{15}\mathrm{O}) - M(^{15}\mathrm{N}).$$

Results are listed in Table I.

As discussed above, the ^{17}N-^{17}Ne case involves a small violation of the Pauli principle, so the difference is understandable. Slightly different s^2/d^2 ratios in ^{18}Ne and ^{17}Ne also contribute to the difference. For ^{18}N-^{18}Na, the parent nuclei are ^{19}O and ^{19}Na (see Fig. 1), and the first two states of those two nuclei seem to have very little $2s1/2$ occupancy. Of course, in this case, weak coupling provides an estimate of the energy of the doublets arising from coupling a $p1/2$ hole to the $5/2^+$ and $3/2^+$ states in ^{19}O and ^{19}Na. The g.s. of ^{18}Na is 1^-, and hence must involve the $3/2^+$ state of ^{19}Na, not the $5/2^+$, which is the g.s. in ^{19}O and which we took to be the g.s. in ^{19}Na. If the energy splittings in the doublet are different in ^{18}N and ^{18}Na, this will affect the weak-coupling estimate for the g.s. In any case, we expect our weak-coupling estimate for ^{19}Mg to be accurate to within 200–400 keV. We turn now to the shell-model calculations.

TABLE I. Weak-coupling estimates of mass excess (MeV) for mirrors of ^{17,18,19}N.

Nucleus	Weak coupling	Measured	Difference
^{17}Ne	16.725(15)	16.461(27) [6]	0.264(31)
^{18}Na	25.461(23)	25.06(13) [17] or 25.04(17) [18]	0.401(132)
^{19}Mg	32.389(32)	?	

III. SHELL MODEL

In Ref. [8], we used results of two different shell-model (sm) calculations to compute the energies of ^{17}Ne, given the energies in the mirror ^{17}N. Both calculations gave satisfactory agreement for the levels of ^{17}Ne that have been identified [4,12], but the two sets of results differed somewhat. We then used the sm spectroscopic factors for $^{17}\mathrm{N}^* \rightarrow {}^{18}$N(g.s.) [and hence $^{17}\mathrm{Ne}^* \rightarrow {}^{18}$Na(g.s.)], plus a potential model to compute the mass excess of ^{18}Na(g.s.). Whenever the states of ^{17}Ne were known we used the known excitation energies. Otherwise we used the calculated excitation energies (not the shell-model ones—the shell model does not give a good account of the ^{17}N excitation energies). Probably because the largest spectroscopic factors involved the low-lying states (whose excitation energies are known), the two calculations gave nearly identical values for the mass excess of ^{18}Na [8].

The situation is different for ^{19}Mg, for which we need the excitation energies in ^{18}Na, and none are accurately known. So, our first order of business is to make our best estimates of the ^{18}Na excitation energies.

A. $^{17}\mathrm{Ne}^* \rightarrow {}^{18}\mathrm{Na}^*$

Our approach is straightforward. We take as input the known excitation energies in ^{18}N [13] and shell-model S's for $^{17}\mathrm{N}^* \rightarrow {}^{18}\mathrm{N}^*$. Because we wish to be able to assess the stability of our results, we have done the calculations for the first four levels of ^{18}Na with two quite different sets of spectroscopic factors. One set is from the sm results [14] labeled sm in Ref. [8]. For the other, we have used the LSF wave functions [5] for ^{17}Ne (Table I of Ref. [8]), but weak coupling (wc) for ^{18}Na—where we have assumed that the four

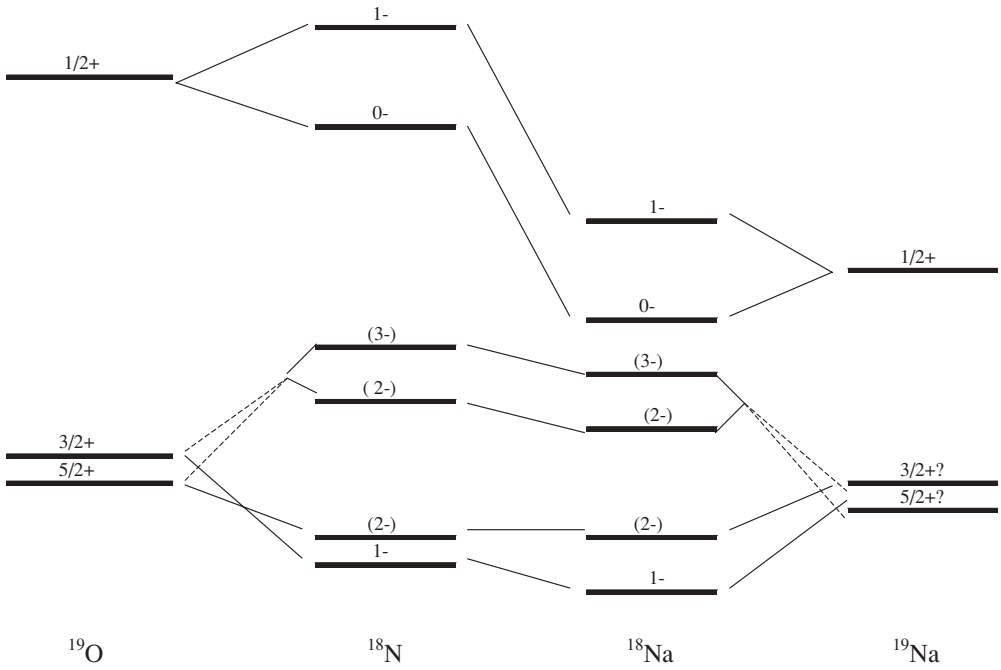

FIG. 1. Lowest six states in ^{18}N and ^{18}Na, and their parents in ^{19}O and ^{19}Na.

levels are those that arise from coupling a $p1/2$ hole to the lowest $5/2^+$ and $3/2^+$ states of ^{19}Na (Fig. 1). Furthermore, we used only the $(d5/2)^3$ component of those two states. This calculation, which we know is an extreme simplification, is labeled LSFwc in what follows. Because this extreme model contains no $2s1/2$ amplitudes from $A = 17$ to $A = 18$, we can compare with the sm results with the $\ell = 0$ S's removed.

As stated above, the sm results for excitation energies are not very good. Figure 2 displays the lowest calculated states of ^{18}N, and compares them with experiment. The ordering and spacing of the first four levels is not well reproduced. We have chosen to align the $(2J + 1)$-weighted centroid of the lowest four states. (We return to the 0^- and 1_2^- levels later.) Thus, we use experimental energies of ^{18}N [13]. The J^π is known only for the 1^- g.s., with 2^- strongly preferred for the first-excited state. The compilation lists the preferred J^π as (2^-), (3^-) in that order for the next two states. The 1^- g.s. clearly involves the $3/2^+$ state of ^{19}O, while the 2^- first-excited state probably is connected to the $5/2^+$ level. The next two states are then the other two states from these couplings. If we use the J^π's suggested in the compilation (and listed in Table II), the centroid of the $3/2^+$ doublet (1^- and 2_2^-) is at 368 keV, and the centroid of the $5/2^+$ doublet (2_1^- and 3^-) is at 484 keV, i.e., the $5/2^+$ is above the $3/2^+$—opposite to the ordering in ^{19}O. If we were to interchange the J assignments for the third and fourth state, then the $5/2^+$ centroid would be 391 keV and the $3/2^+$ becomes 467 keV—i.e., the same level ordering as in ^{19}O. However, for now we stick with the ordering in the compilation.

We performed two sets of LSFwc calculations. One, labeled yrast in Table II, involved only the lowest negative-parity state of each $J = 1/2$–$9/2$, all of whose excitation energies are known in ^{17}Ne. The other (labeled *full* in Table II) included all the LSF states, using the calculated excitation energies (Ref. [8]) in ^{17}Ne if they were not known experimentally. We emphasize that all the $17 \rightarrow 18$ spectroscopic factors in this space are for $\ell = 2$. So, the energies should represent reliable upper limits. Results are listed in columns 3 and 4 of Table II.

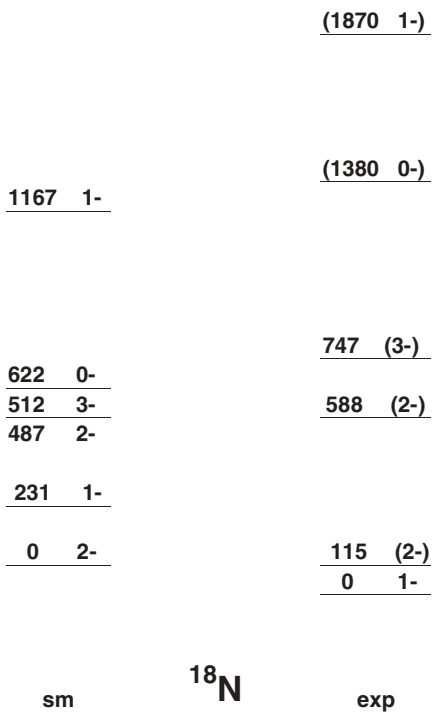

FIG. 2. Comparison of shell-model (left column) and experimental (right) low-lying levels of ^{18}N. Placement of the 0^-, 1_2^- states is discussed in the text. Shell-model spectrum has been shifted to align the $(2J + 1)$-weighted energy centroid of the lowest four levels.

TABLE II. Predictions for energies (keV) of lowest four levels of ^{18}Na in various spaces.

J^π	$E_x(^{18}\mathrm{N})$	$E_p(^{18}\mathrm{Na})$			
		LSFwc[a]		Shell model[d]	
		(yrast)[b]	(full)[c]	(no $\ell = 0$)	full
1^-	0	1538	1547	1447	1382
(2^-)	115	1623	1668	1618	1521
(2^-)	588	2086	2097	1985	1919
(3^-)	747	2206	2243	2210	2133

[a]Used LSF for ^{17}Ne, wc for ^{18}Na.
[b]Used only lowest negative-parity state of each J in ^{17}Ne.
[c]Used full LSF space.
[d]Reference [14].

Most of the action seems to be contained in the lowest states, from the fact that *yrast* and *full* results are similar.

The full sm results are listed in the last column of Table II. As expected, these energies are significantly lower than those from LSFwc—largely because of the $\ell = 0$ components, which have smaller Coulomb energies. To investigate whether all the difference is due to these $\ell = 0$ S's, we repeated the sm calculations with these $\ell = 0$ S's removed, giving the results in the next-to-last column of Table II. The sm results with $\ell = 0$ removed are nearly identical to those from LSFwc(yrast) for 2_1^- and 3^-, which will turn out to be most important for ^{19}Mg.

Two other states are important for computing the ^{19}Mg mass. These are the 0^- and 1_2^- states (Fig. 1) whose dominant structure in ^{18}N is that of a $p1/2$ hole coupled to the $1/2^+$ state at 1.47 MeV in ^{19}O. This $1/2^+$ state is primarily $(d_0^2)s$. However, these two states are not known in ^{18}N and hence present a problem. In the sm calculation for ^{18}N, they are so low in excitation that the 0^- would become the first- or second-excited state of ^{18}Na—because of the large TES. This situation appears unlikely because of the location [15] of the $1/2^+$ level at 0.746(2) MeV in ^{19}Na. Hence, we performed two sets of calculations for this 0^-, 1_2^- pair. In one we used the sm energies [14] in ^{18}N. In the other we placed the 0^-, 1_2^- centroid at 1.38 MeV above the 1^-, 2_2^- centroid (1.38 is 1.47 MeV minus the $3/2^+$ energy in ^{19}O). We kept the 0^-, 1_2^- splitting at the value of 490 keV from the sm calculation. Results are listed in Table III. When we computed the ^{19}Mg mass, we compared results with the two sets of energies.

TABLE III. Energies (keV) for the 0^-, 1_2^- pair of states.

J^π	source	$E_x(^{18}\mathrm{N})$	$E_p(^{18}\mathrm{Na})$	$E_x(^{18}\mathrm{Na})$
0^-	sm	662	1574	192
	wc	1380	2102	720
1^-	sm	1167	2081	699
	wc	1870	2638	1256

B. ^{18}Na* → ^{19}Mg(g.s.)

The shell-model structures of these nuclei are relatively simple. They all have small $1d3/2$ occupancies, which we include in the numerical calculations, but which we temporarily ignore for the sake of a simple argument. Within the space of $1d5/2$, $2s1/2$ orbitals (abbreviated d, s for now), the g.s. of ^{20}O has only two terms: d^4 (which is the same as d^{-2}) and d^2s^2, in which each pair is coupled to 0^+. There is no d^3s term because d^3 cannot couple to $1/2^+$. The s^2/d^2 occupancy ratio of ^{20}O is about 0.15. So, if we write g.s. $= Ad^4 + Bd^2s^2$, we have $A^2 \sim 0.74$, $B^2 \sim 0.26$.

The case of ^{19}O in the d, s space has been treated fully by Lawson [16]. The $3/2^+$ state is primarily d^3, with no pair having $J = 0$, but only 2 and 4. The only other component is $d_2^2 s$. The $5/2^+$ state can have three components: d^3, $d_2^2 s$, and ds^2. The $1/2^+$ state has only one component, viz. $d_0^2 s$. Clearly, then, the $3/2^+$ state has no single-nucleon connection to ^{20}O(g.s.). The $1/2^+$ state is connected to the small d^2s^2 component. The $5/2^+$ state connects to both through the d^3 term (to d^4) and through the ds^2 term (to d^2s^2) (both with $\ell = 2$).

Now, we couple a $p1/2$ proton hole to ^{20}O to make ^{19}N and to ^{19}O to make ^{18}N. The $3/2^+$ state of ^{19}O produces 1^- and 2^- states that have no single-nucleon connection to the $1/2^-$ ^{19}N(g.s.). The $1/2^+$ state of ^{19}O leads to 0^- and 1_2^- states that will have (small) S's to ^{19}N(g.s.). The majority S is to the 2^- and 3^- states coming from the $5/2^+$ state of ^{19}O.

All these remarks for ^{19}N and ^{18}N apply to ^{19}Mg and ^{18}Na if isospin is conserved. The sm S's (Table IV) for $^{19}\mathrm{N} \rightarrow {}^{18}\mathrm{N}^*$ support the simple picture above. The total $S(\ell = 0)$ is 0.49, of which 0.46 is to the lowest 0^- and second 1^- state. The total $S(\ell = 2)$ is 3.46, of which 3.17 is to the lowest four states. The total S to 1^-, 2_2^- is only 2.4% of the total to all four states. For the 2_1^-, 3^- pair, the ratio of S's is 1.38, compared to the wc expectation of $7/5 = 1.4$. For the 0^-, 1_2^- pair the ratio is 3.08, compared to the wc value of 3.0.

One consequence of all this is that using only the lowest six states in ^{18}Na to compute the g.s. mass of ^{19}Mg should be sufficient. If the energies of these six states were known, we would be confident in our calculation of the ^{19}Mg mass. Thus, estimating the uncertainty in our final number primarily involves estimating the uncertainty in the ^{18}Na

TABLE IV. Spectroscopic factors for $^{19}\mathrm{N} \rightarrow {}^{18}\mathrm{N}^*$ [14].

Core State	S	
	$\ell = 0$	$\ell = 2$
1_1^-	0.0048	0.029
2_1^-	–	1.298
2_2^-	–	0.050
3^-	–	1.794
0^-	0.113	–
1_2^-	0.348	0.003
Sum	0.466	3.17
Excluded	0.026	0.29

TABLE V. Resulting $2p$ energies (keV) for ^{19}Mg(g.s.).

Core[a]	model	energy
4 states	LSFwc	1065
	sm	973
6 states	LSFwc	970
	sm	871

[a]See text.

energies. Our calculations of those energies are presented in an earlier section. Reference [17] suggests the first-excited state (presumably 2_1^-) of ^{18}Na is at 240(50) keV, and another state (presumably 2_2^-) is at 0.60(7) MeV. Thus, three of the first four states may have been identified. As we see below, inclusion of the next two states is important, but their exact location is not.

With the calculated energies of ^{18}Na levels and spectroscopic factors connecting those states to ^{19}Mg(g.s.), we computed the mass of the latter. As before, we used the ^{18}N energies and constrained the potential to reproduce the ^{19}N(g.s.) mass, then used the computed ^{18}Na energies, with the same potential, plus a Coulomb term, to compute the ^{19}Mg(g.s.) mass. Results are listed in Table V. These results are reasonably robust. Interchanging the 2_2^- and 3^- states in ^{18}N raises E_{2p} by 3 keV. Using sm energies for 0^- and 1_2^- (rather than wc ones) raises it by 8 keV. As stated earlier, we consider the LSFwc result a reliable upper limit for E_{2p}. In attempting to estimate the uncertainty in our calculations, we note that the same type of calculation [8] gave values of 25.132–25.152 MeV for the mass excess of ^{18}Na, for which the central value experimentally is 25.04 [18] or 25.06 [17] MeV, with uncertainties of 0.17 and 0.13 MeV, respectively. We did not state a model uncertainty there, but it was probably about 80 keV. In the present case, we have far fewer core states to consider, but must use the calculated Coulomb energies for them in ^{18}Na, because the experimental energies are not known. We think 70 keV is a reasonable value for the model uncertainty in our present calculation of E_{2p}, using shell-model wave functions.

Grigorenko *et al.*, obtained limits on E_{2p} of 860 and 580 keV, with s^2 fractions of 5.5%(6.2%) and 50.8%(61%), respectively, in ^{19}N(^{19}Mg). They stated that the "dependence of Coulomb energy on s^2/d^2 ratio is practically linear." [We assume they meant the s^2 *fraction*, not the ratio (which becomes infinite as the s^2 fraction approaches unity).] In our calculations, the energy with no s^2 component was about 100 keV higher than the energy with the s^2 value from the sm results.

Values of E_{2p} and E_p are the natural output of our model calculations. Converting to mass excess requires knowledge of the mass excess(es) of the relevant core(s). If we use the latest tabulated value [6] for ^{17}Ne of 16461(27) keV, our ^{19}Mg mass excess is 31910(75) keV. (See Table VI.) The wc value (Table I) was 32389 keV. The newest mass tabulation [6] lists 33040(250) for ^{19}Mg from an extrapolation. As noted above, our value is at the upper end of the range allowed by [9].

TABLE VI. Results of present calculations for ^{19}Mg.

Quantity	Value
E_{2p}	871(70) keV (unbound)
E_p	−511(70) keV (bound)
Mass excess	31910(75) keV[a]
$\Gamma_{\rm sim}$ [b](941 keV)	11 meV
(871 keV)	3.3 meV
(801 keV)	0.84 meV

[a]Using ^{17}Ne mass excess [6] of 16461(27) keV.
[b]Using a two-nucleon spectroscopic factor of $S = 0.5$.

IV. WIDTH FOR *2p* DECAY

We have calculated the width as a function of energy for decay by emission of a mass-two, charge-two cluster. We used a Woods-Saxon potential with $r_0 = 1.3$ fm, $a = 0.60$ fm [$R = r_0(17)^{1/3}$], plus the Coulomb potential of a uniform sphere. The two-proton cluster spectroscopic factor S from ^{19}Mg(g.s.) to ^{17}Ne(g.s.) depends on the s^2/d^2 ratio, as exhibited in Fig. 3. With the small values of this ratio that we prefer, S is in the vicinity of 0.5, which we use here. As the final widths will simply scale with the value of S, our results can be converted to any other value of S.

The next step in the procedure is to convolute the calculated widths with the profile function of the energy of relative motion of the two protons. For this profile, we use the zero-degree curve from [19], which is presented in graphical form for 0–3 MeV. We have appended an exponential tail in order to obtain an overall normalization. The equation for width is then

$$\Gamma(E_{2p}) = \frac{S\int \Gamma(E_{2p} - E_{\rm int})\,{\rm Pr}(E_{\rm int})\,dE_{\rm int}}{\int {\rm Pr}(E_{\rm int})\,dE_{\rm int}},$$

where the integral in the denominator extends from 0 to infinity. In the numerator, the integrand decreases extremely rapidly with a decrease in the argument of Γ, so that only small values (∼0–300 keV) of the argument of the profile function are

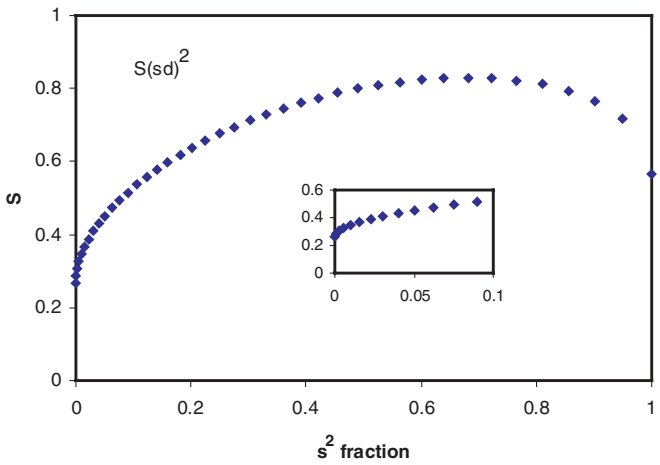

FIG. 3. (Color online) The $(sd)^2 L = 0$ two-nucleon cluster spectroscopic factor (with $d3/2$ removed) is plotted vs. the s^2 fraction. The inset shows an expansion of the left-hand corner of the graph.

relevant. Because the slope in $Pr(E)$ is large for small values of E, the uncertainty in the convoluted width could be larger here than in other cases for which the integrand peaks near the maximum of $Pr(E)$ (near 500–600 keV). We return to this point below.

For $E_{2p} = 871$ keV, and $S = 0.50$, we obtain $\Gamma(871\text{ keV}) = 3.3$ meV. Adding 70 keV, we find $\Gamma(941\text{ keV}) = 11$ meV, while subtracting 70 keV results in $\Gamma(801\text{ keV}) = 0.84$ meV. Thus, the relatively small uncertainty in E_{2p} causes a very large uncertainty in the calculated width (not surprisingly, of course). This uncertainty certainly swamps any uncertainty arising from the profile function and/or the convolution procedure, preventing any need to estimate that uncertainty. Without the convolution, our calculated width is 1.5 eV for zero energy of relative motion of the $2p$.

Our calculated width for the central value of E_{2p} corresponds to a mean life of 0.20 ps. Reference [9] stated their lifetimes were 0.5–60 ps for their energy range of 860–580 keV, corresponding to widths of 1.3×10^{-3} eV and 1.1×10^{-5} eV. Their earlier, slightly different, calculation [20] gave somewhat smaller widths for slightly smaller energies: 3.7×10^{-4} eV at 838 keV and 8.7×10^{-7} eV at 548 keV. The nonobservation of ^{19}Mg in a fragmentation experiment put an upper limit on the half-life of 22 ns. [21] Our widths are compared with earlier ones [9,20] in Fig. 4.

Even though the g.s. of ^{19}Mg is bound with respect to ^{18}Na(g.s.) $+p$, there is a possible sequential mode through the extreme low-energy tails of ^{18}Na $=^{17}$Ne $+p$ resonances that correspond to excited states of ^{18}Na, but with widths such that they extend well below ^{18}Na(g.s.).

The only likely possibilities are the 0^- and 1^- s-wave resonances, whose energies are not known in ^{18}Na. The energies of the mirror states in ^{18}N are also not known. These two states were discussed earlier.

For present purposes, we consider the 0^- member, which will lie lower than the 1^-. The product of spectroscopic factors for sequential decay through the 1^- state will be larger than for the 0^-, but this fact is largely offset by the higher 1^- energy. For the test calculations, we put the 0^- state at $E_x = 720$ keV in ^{18}Na. As might be expected, it is the details of the ^{17}Ne $+p$ profile function at very low proton energies that are relevant. Using a wide variety of profile functions and doing the convolutions provides sequential decay widths of 1.5 μeV–1.2 meV—quite a wide range. Decays through the 1^- could be of similar magnitude. These are to be compared to our simultaneous $2p$ width of 3.3 meV at our central energy.

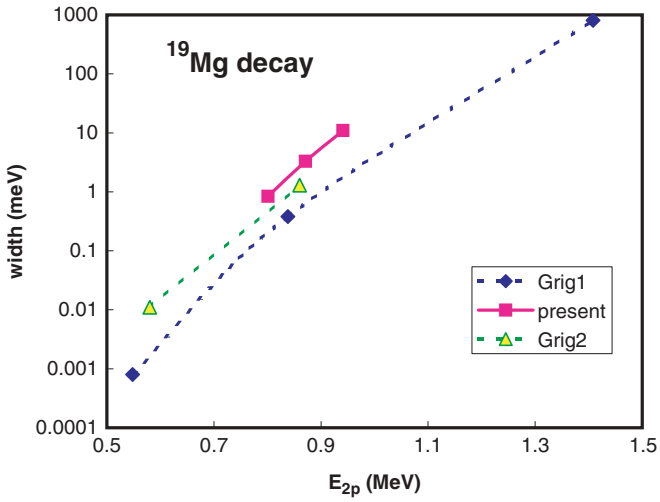

FIG. 4. (Color online) Lines serve only to connect the points. Calculated widths for simultaneous $2p$ decay of ^{19}Mg vs. the available energy: squares (present), triangles (Ref. [9]), and diamonds (Ref. [20]).

We conclude that sequential decays through tails of higher-lying resonances will not dominate, but could participate sufficiently that they should be included.

V. CONCLUSIONS

In conclusion, our use of shell-model wave functions for ^{18}Na and ^{19}Mg(g.s.) in a Coulomb-energy calculation for ^{19}Mg(g.s.) provides a two-proton energy of $E_{2p} = 871(70)$ keV (unbound). At our central energy, the width computed for simultaneous $2p$ decay is 3.3 meV, corresponding to a mean life of 0.20 ps. Our results for the energy are at the upper end of the range calculated previously by others [9]. Our possible lifetimes cover a range of only a factor of 13, narrowing the previous range of a factor of 120 [9]. Sequential decay might also be present at some level, but should not dominate.

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