Departmental Papers (ESE)   Department of Electrical & Systems Engineering

December 2005

Evanescent Growth and Tunneling Through Stacks of Frequency-Selective Surfaces

Andrea Alù
University of Pennsylvania, andreaal@seas.upenn.edu

Nader Engheta
University of Pennsylvania, engheta@seas.upenn.edu

Follow this and additional works at: http://repository.upenn.edu/ese_papers

Evanescent Growth and Tunneling Through Stacks of Frequency-Selective Surfaces

Abstract

The presence of wave tunneling and the "growing evanescent envelope" for field distributions in suitably designed, periodically layered stacks of frequency selective surfaces (FSS) is discussed in this letter. It is known that a slab of double-negative (DNG) or left-handed metamaterial sandwiched between two vacuum half-spaces may lead to "growing" evanescent plane waves, which may be used to restore sub-wavelength information in Pendry's "perfect" lens (Phys. Rev. Lett., vol. 85, pg. 3966, 2000). In this letter, it is shown that a completely different setup allows an analogous buildup of evanescently *modulated* waves. In particular, it is shown how an interface resonance phenomenon similar to the one present at the interface between metamaterials with oppositely signed constitutive parameters may be induced by a proper choice of the periodicities of the FSS stacks and the geometrical properties of these surfaces. The analysis is performed through an equivalent transmission-line approach, and some physical insights into this phenomenon are presented. Salient features, such as the complete wave tunneling through the pair of cascaded FSS, each operating at its bandgap, are presented and discussed.

Keywords

Frequency selective surfaces, growing exponential, metamaterials, negative refraction, resonance, tunneling

Evanescent Growth and Tunneling Through Stacks of Frequency-Selective Surfaces

Andrea Alù, *Student Member, IEEE,* and Nader Engheta, *Fellow, IEEE*

Abstract—The presence of wave tunneling and the "growing evanescent envelope" for field distributions in suitably designed, periodically layered stacks of frequency selective surfaces (FSS) is discussed in this letter. It is known that a slab of double-negative (DNG) or left-handed metamaterial sandwiched between two vacuum half-spaces may lead to "growing" evanescent plane waves, which may be used to restore sub-wavelength information in Pendry's "perfect" lens (Phys. Rev. Lett., vol. 85, pg. 3966, 2000). In this letter, it is shown that a completely different setup allows an analogous buildup of evanescently *modulated* waves. In particular, it is shown how an interface resonance phenomenon similar to the one present at the interface between metamaterials with oppositely signed constitutive parameters may be induced by a proper choice of the periodicities of the FSS stacks and the geometrical properties of these surfaces. The analysis is performed through an equivalent transmission-line approach, and some physical insights into this phenomenon are presented. Salient features, such as the complete wave tunneling through the pair of cascaded FSS, each operating at its bandgap, are presented and discussed.

Index Terms—Frequency selective surfaces, growing exponential, metamaterials, negative refraction, resonance, tunneling.

I. INTRODUCTION

PENDRY in his paper on "perfect lens" made of a slab of double-negative (DNG) medium – a medium in which the real permittivity and permeability are simultaneously negative (also known as "left-handed" or "negative index" media) – theoretically showed that inside such a metamaterial slab the evanescent wave has a "growing exponential", which contributes to the higher resolution of the image [1]. This idea has generated a lot of interest in the field of metamaterials, and many groups worldwide have been recently studying this issue of reconstruction of the evanescent waves and its sub-wavelength focusing. A natural question may arise: "May the growing exponential phenomenon happen in other classes of media or in properly designed structures, perhaps arguably more easily realizable than

DNG media?" Some ideas have been already reported in the literature [2]–[4]. We have been interested to explore the answers to this question and to find other alternatives. Here, we show that a somewhat analogous phenomenon in the form of growth of evanescently modulated waves may occur inside suitably designed periodic arrangements of stacks of frequency-selective surfaces (FSS) within conventional [i.e., double-positive (DPS)] media.

Frequency selective surfaces (FSS), which are constructed as planar two-dimensional periodic arrays of metallic elements with specific geometrical shapes, or as periodic apertures in a metallic screen, have been widely studied and known for many years [5]. The transmission and reflection coefficients for these surfaces are dependent on the frequency of operation (hence their names) and may also depend on the polarization and angle of incidence. In particular, the possibility of having frequency bands at which a given FSS is completely opaque (stop-bands) and others at which the same surface allows wave transmission has attracted the attention of scientists and designers.

In this work, we show how by periodically stacking two sets of FSS with "dual" behaviors (e.g., one stack of FSS with holes in the metallic plate, and the other a stack of FSS with metallic patches over the surface of a very thin dielectric holder) one may achieve growing field distributions toward the interface between these two stacks, and may have complete tunneling of an electromagnetic wave through this pair, even though each stack alone would be "below cutoff" (i.e., in its stop band), similar to what we have obtained for an incident plane wave tunneling through a pair of properly chosen single-negative (SNG) bilayers [6].

II. FORMULATION OF THE PROBLEM

It is well known that an equivalent circuit model representation may be used for plane wave propagation and for its interaction with FSS [4]. In this transmission-line analogy, under a time-harmonic $e^{j\omega t}$ excitation, an FSS may be represented as a shunt load, either inductive or capacitive depending on its geometrical parameters, in a transmission line (TL) segment representing the surrounding space. A cascade of FSS with dielectric materials filling the space between them (shown in Fig. 1, top part) would therefore be modeled as a series of TL cells periodically loaded by shunt elements, as in Fig. 1, bottom part. The distance between every FSS would correspond to the distance between the loads, and the "host" TL parameters Z_c and k_{TL} depend on the angle of incidence, the frequency and the polarization of the incident plane wave. The values of the shunt admittances representing the FSS depend directly on its geometrical parameters, in addition to the frequency, polarization, and
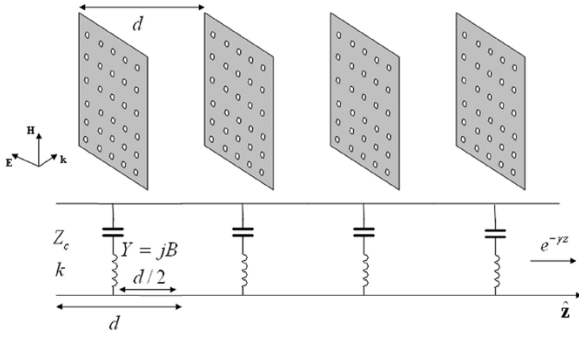
Fig. 1. (Top) Cascade of equally spaced FSS and (bottom) its circuit model representation.

angle of incidence of the incoming wave. Such parameters have been extensively studied and reported in the literature (see e.g., [5]) for several types of FSS. Obviously, in the limit of no loss, the values of the shunt admittances of FSS are assumed to be purely imaginary.

Referring to Fig. 1, therefore, the length of every single cell d equals the distance between the two neighboring FSS and every TL unit cell has been chosen to have its shunt load in the middle (at $d/2$) to provide symmetry for the ease of analysis. The shunt admittance $Y = jB$ represents the lossless FSS.

The problem of plane wave interaction with such a system, therefore, in first approximation is equivalent to finding the solution for the voltage and current propagation along such a periodically loaded TL. The solutions can be determined from the well known Bloch equation (see e.g., [7])

$$\cosh \gamma d = \cos k_{TL}d - \frac{BZ_c}{2} \sin k_{TL}d, \tag{1}$$

where γ is the Bloch wave number, which describes the $e^{-\gamma z}$-exponential dependence of the propagation/attenuation along z. A purely imaginary γ corresponds to a propagating Bloch mode, whereas a positive real γ corresponds to a decaying evanescent wave, representing the stop-band of this periodic structure. In the present work, we show how two different cascades of FSS (one with inductive FSS and the other with capacitive FSS), when properly designed, may exhibit "growing evanescently modulated" field distributions and complete tunneling of incident wave through this pair, although each cascade alone operates in its stop-band.

In our previous work [6] for a pair of only-epsilon-negative (ENG) and only-mu-negative (MNG) layers (or for a pair of DPS-DNG layers), we have derived the conditions for the transparency, resonance and complete tunneling of a wave incident on the pair, which depended on the characteristic impedances of the two metamaterials and their electrical lengths. In the case here, analogous conditions may be derived for wave transparency and interface resonance for the above-mentioned pair of cascades of dual FSS.

The Bloch impedance, which represents the ratio of voltage and current at the input section of every cell for an infinite periodically and homogeneously loaded TL, may be written as

$$Z_B = \pm jZ_c \frac{\frac{BZ_c}{2} \cos k_{TL}d + \sin k_{TL}d - \frac{BZ_c}{2}}{\sqrt{\left(\cos k_{TL}d - \frac{BZ_c}{2}\sin k_{TL}d\right)^2 - 1}}. \tag{2}$$

Not surprisingly, by comparing (1) and (2) it can be verified that a real-valued Z_B is possible only in the pass-bands, and conversely Z_B can be imaginary in the stop-bands, when no real power is transferred. The sign should be positive when we are in the pass-bands (and Z_B is real), whereas it may be positive or negative when Z_B is imaginary, depending on the capacitive or inductive behavior of the evanescent decay along the TL.

Let us consider now the case of two different periodically loaded TL segments, with Bloch impedances Z_{B1} and Z_{B2} and electrical Bloch lengths $g_1 = N_1\gamma_1 d_1$ and $g_2 = N_2\gamma_2 d_2$, where N_1 and N_2 are the numbers of unit cells in the first and second cascades, stacked one after the other as in Fig. 2(a). We are not limiting ourselves to be only in the stop-bands or only in the pass-bands, i.e., the Bloch parameters may be in general complex. The input impedance at the connection between the two segments looking to the right may be found by the conventional TL formula

$$Z_{in}(z = N_1 d_1) = Z_{B2} \frac{Z_c + Z_{B2}\tanh g_2}{Z_{B2} + Z_c\tanh g_2}. \tag{3}$$

The input impedance at the input of the whole system becomes therefore

$$Z_{in}(z = 0) = Z_{B1} \frac{Z_{in}(z = N_1 d_1) + Z_{B1}\tanh g_1}{Z_{B1} + Z_{in}(z = N_1 d_1)\tanh g_1}. \tag{4}$$

Requiring that the system is totally transparent to the incident wave implies that $Z_{in}(z = 0) = Z_c$. By inspection, the necessary and sufficient conditions to this equation are given by the two alternative solutions

$$\begin{cases} Z_{B1} = -Z_{B2} \\ g_1 = g_2 \end{cases} \quad \begin{cases} Z_{B1} = Z_{B2} \\ g_1 = -g_2 \end{cases} \tag{5}$$

which are formally equivalent to those derived in [6] for metamaterial bilayers, even though here they relate to the Bloch wave propagation rather than to the plane wave. For passive loads, the condition on the right clearly corresponds to Bloch segments in their pass-bands (with imaginary γ and positive real Z_B), whereas the condition on the left requires the two segments to be both in the stop-band (with positive γ and conjugate imaginary Z_B).

When the left condition is applied, if one FSS cascade in its stop-band has an inductive behavior (i.e., $Z_{B1} = j|Z_{B1}|$) the other should have a capacitive behavior ($Z_{B2} = -j|Z_{B2}|$). An "interface resonance" in this case may occur in the Bloch wave domain, analogous to what we have shown for the ENG-MNG bilayers [5] in the plane wave domain, leading to the condition for having "growing exponential" modulated waves. Proper selection of electrical lengths for the two cascades satisfying the condition $N_1\gamma_1 d_1 = N_2\gamma_2 d_2$, moreover, leads to a complete tunneling of the evanescent Bloch waves through the overall system. In other words, if each of the two FSS cascades alone is illuminated by a plane wave, most of the incident energy is reflected back because it is designed to be at its band gap. However, by pairing the two FSS cascades that satisfy the left condition given in (5), an incident wave may tunnel through completely, resulting in no reflection from this pair. We also observe a "growing evanescent envelope" for the field distribution inside this pair. It is interesting to note that: 1) the condition does not
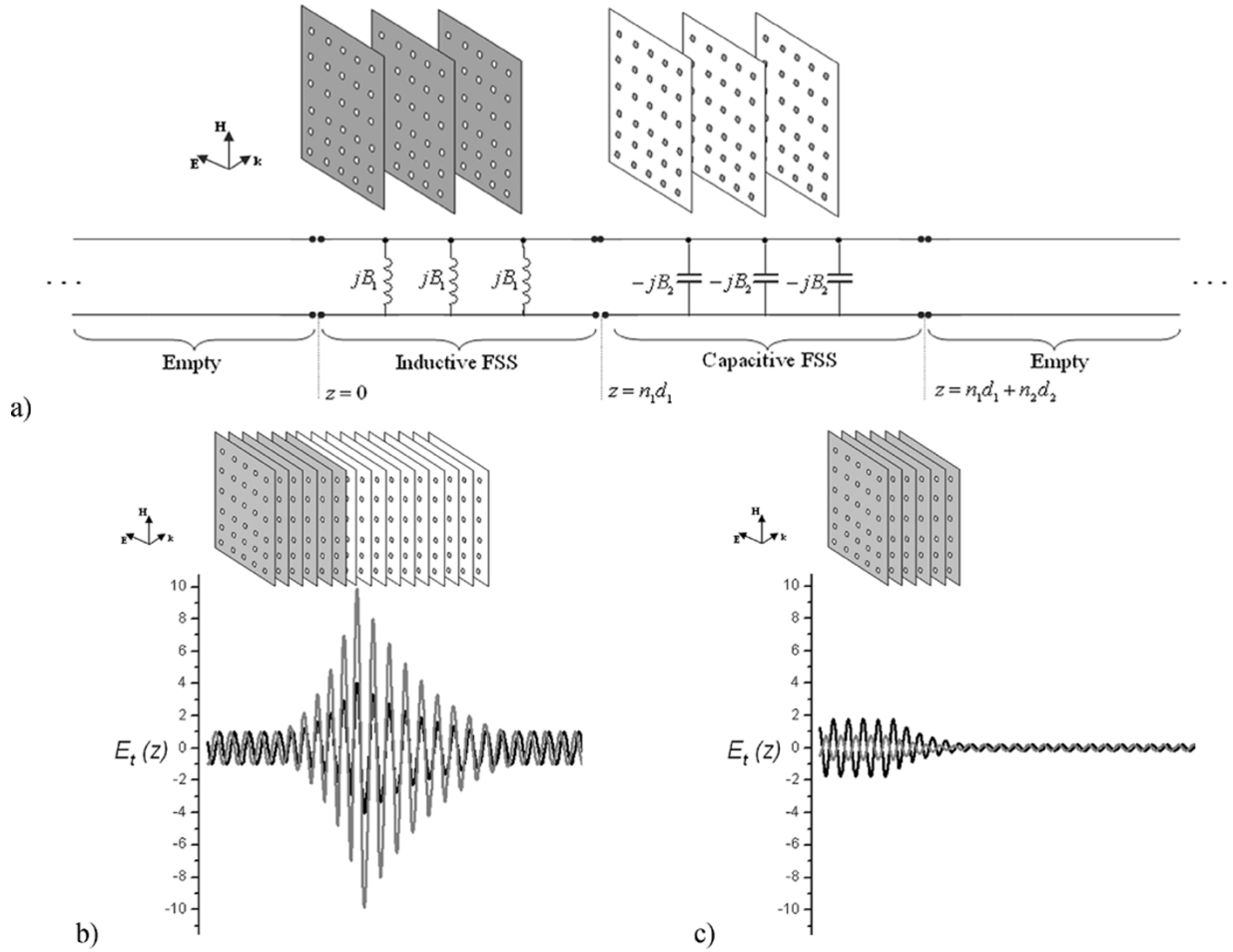
Fig. 2. (a) Stacked FSS cascades with "dual" behavior and their circuit model, e.g., one stack of FSS with holes in the metallic plate, and the other a stack of FSS with metallic patches over the surface of a very thin dielectric holder. (b) Voltage, i.e., transverse electric field (real and imaginary part), distribution for two stacks satisfying the first of (5), $k_1 d_1 = 7\pi/4$, $k_2 d_2 = 0.49 + 2\pi$, $N_1 = 6$, $N_2 = 10$, $B_1 Z_c = 0.99$, $B_2 Z_c = -0.587$. (c) The same, but the second cascade has been removed, i.e., $B_1 Z_c = 0$.

depend on the outer medium, i.e., the system is transparent independently on what surrounds it as long as the outside media on both sides are the same, and 2) there is potentially no limitation on the total length of the whole system, but the condition is on the ratio between the two lengths. Of course, when realistic losses are considered a higher tunneling is expected for shorter segments, due to absorbing phenomena and to the fact that the condition on the characteristic impedances can no longer be satisfied exactly, however, the phenomenon still remains conceptually unchanged. Both these peculiar characteristics had been shown in [6] for the ENG-MNG bilayer.

Fig. 2 presents the voltage distribution along the pair of FSS cascades shown in Fig. 2(a). The TL representation of the system has also been depicted. Fig. 2(b) clearly shows that the voltage at the entrance face is completely recovered at the exit face, thus it has "tunneled through", and at every cell of the first cascade the Bloch mode experiences an exponentially "growing" envelope for the field distribution. In Fig. 2(c), the second cascade is effectively removed and only the first cascade is present. As can be seen, the voltage distribution in the remaining cascade exhibits a decaying envelope, and the reflected wave is non zero. A similar effect may be observed if we remove the first cascade and keep the second one only. By

comparing Fig. 2(b) and (c), we see the effect of pairing the two FSS cascades that results in the growing evanescent envelope at the interface.

We notice from this example that it is not necessary to design the two stacks of FSSs to be the "perfect conjugate" of one another. The necessary and sufficient conditions for inducing this resonant tunneling phenomenon derived here do not in fact require that each single FSS of the first stack enters into resonance with each single FSS of the second stack, separately designed to have an opposite shunt impedance. That would have been a more trivial case of tunneling. Here instead the two stacks are composed with a different number of FSSs and with unrelated geometrical properties, but still the two evanescent Bloch waves supported in the two periodic layers can be designed to satisfy condition (5) by properly adjusting the periods in the two stacks. This is similar to the distinction given in [6] between the tunneling through a generic ENG-MNG bilayer and the special case of a "conjugate-matched" bilayer, composed of two slabs with same thickness and oppositely signed constitutive parameters.

Another interesting feature of this setup, as seen in Fig. 2, is represented by the fact that the electromagnetic waves indeed propagate in the free space regions between the FSSs and they are just *"modulated"* by the Bloch wave resonance of the

system. This implies that the radiation experiences a growing evanescent modulation, even though in each free-space region the waves by themselves are not evanescent. In other words, the "growth" of electromagnetic waves in a passive system, which has been shown to be possible for evanescent waves in negative-parameter slabs [1], [6], has been obtained here for *propagating* waves in a properly designed system supporting evanescent Bloch modes without utilizing any negative-parameter media.

The second condition in (5) corresponds to the propagating Bloch modes (i.e., when the FSS are both operating at their pass-band) with opposite behaviors; one having a forward propagation, while the other having a backward propagation. Also in this case, somehow analogous to the pair of DPS-DNG slabs [8]–[10], a total transparency may be achieved with a proper design of the line parameters.

The phenomenon underlying this anomalous behavior, as also in the case of Pendry's lens, relies on the "interface resonance" present at the junction of the two FSS cascades. We should remember, in fact, that this analysis is valid in the steady-state regime, when the multiple reflections at the resonant interface have been already built up. A detailed explanation of the circuit equivalence of such growing evanescent fields for Pendry's lens, has been given by us in [11].

III. Summary and Conclusion

Here we have shown how even a simple collection of FSS with conjugate (dual) behavior, which is indeed easily constructible, may support the "growing" exponential modes that have attracted so much attention in recent years. In particular, it has been shown how a resonance phenomenon similar to the one present at the interface between ENG and MNG or DPS and DNG media may be induced by a proper choice of the spacing and the geometrical properties of the frequency selective surfaces. Interesting features, such as the presence of "growing" exponential envelopes and complete tunneling through the pair of stacked FSS, while each alone operating at its band gap, are presented. These results are considered a first step toward sub-wavelength imaging systems employing FSS structures.

References

[1] J. B. Pendry, "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, vol. 85, no. 18, pp. 3966–3969, 2000.

[2] C. Luo, S. G. Johnson, J. D. Joannopoulos, and J. B. Pendry, "Subwavelength imaging in photonic crystals," *Phys. Rev. B*, vol. 68, Paper No. 045 115, Jul. 2003.

[3] S. Maslovski and S. Tretyakov, "Phase conjugation and perfect lensing," *J. Appl. Phys.*, vol. 94, no. 7, pp. 4241–4243, Oct. 2003.

[4] S. Maslovski, S. Tretyakov, and P. Alitalo, "Near-field enhancement and imaging in double planar polariton-resonant structures," *J. Appl. Phys.*, vol. 96, no. 3, pp. 1293–1300, Aug. 2004.

[5] B. A. Munk, *Frequency Selective Surfaces: Theory and Design*. New York: Wiley, 2000.

[6] A. Alù and N. Engheta, "Pairing an epsilon-negative slab with a mu-negative slab: Anomalous tunneling and transparency," *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, pp. 2558–2571, Oct. 2003.

[7] R. E. Collin, *Field Theory of Guided Waves*. Piscataway, NJ: IEEE Press, 1991.

[8] A. Alù and N. Engheta, "Guided modes in a waveguide filled with a pair of single-negative (SNG), double-negative (DNG), and/or double-positive (DPS) layers," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 1, pp. 199–210, Jan. 2004.

[9] ——, "Polarizabilities and effective parameters for collections of spherical nano-particles formed by pairs of concentric double-negative (DNG), single-negative (SNG) and/or double-positive (DPS) metamaterial layers," *J. Appl. Phys.*, vol. 97, Paper No. 094 310, May 2005.

[10] ——, "Achieving transparency with plasmonic and metamaterial coatings," *Phys. Rev. E*, vol. 72, Paper No. 016 623, Jul. 2005.

[11] ——, "Physical Insight into the 'Growing' Evanescent Fields of Double-Negative Metamaterial Lenses using their Circuit Equivalence," IEEE Trans. Antennas Propag., to be published.