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Other-Regarding Preferences and Consequentialism

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Abstract
This dissertation addresses a basic difficulty in accommodating other-regarding preferences within existing models of decision making. Decision makers with such preferences may violate the property of stochastic dominance that is shared by both expected utility and almost any model of non-expected utility. At its core, stochastic dominance requires a decision maker's behavior to conform to a basic form of consequentialism, namely, that her ranking of outcomes should be independent of the stochastic process that generates these outcomes. On the other hand, decision makers with other-regarding preferences may show a concern for procedures; that is they may care not just about what the outcomes of others are but also about how these outcomes are generated and therefore their ranking of outcomes may be intrinsically dependent on the outcome-generating process. We provide theoretical foundations for a new representation of other-regarding preferences that accommodates concerns for procedure and possible violations of stochastic dominance. Our axioms provide a sharp characterization of how a decision maker's ranking of outcomes depends on the procedure by expressing `payoffs' as a weighted average of her concerns for outcomes, and her concerns for procedure. The weight used in evaluating this weighted average, which we call the procedural weight, is uniquely determined and quantifies the relative importance of procedural concerns. In the special case in which procedural concerns are absent our baseline decision model reduces to expected utility, and our most parsimonious representation is one parameter richer than that model. We use our decision model to provide an expressive theory of voting.

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To my parents, for their courage, character, patience and love

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ABSTRACT

OTHER-REGARDING PREFERENCES AND CONSEQUENTIALISM

Abhinash Borah

Andrew Postlewaite

This dissertation addresses a basic difficulty in accommodating *other-regarding preferences* within existing models of decision making. Decision makers with such preferences may violate the property of *stochastic dominance* that is shared by both expected utility and almost any model of non-expected utility. At its core, stochastic dominance requires a decision maker's behavior to conform to a basic form of *consequentialism*, namely, that her ranking of outcomes should be independent of the stochastic process that generates these outcomes. On the other hand, decision makers with other-regarding preferences may show a concern for *procedures*; that is they may care not just about *what* the outcomes of others are but also about *how* these outcomes are generated and therefore their ranking of outcomes may be intrinsically dependent on the outcome-generating process. We provide theoretical foundations for a new representation of other-regarding preferences that accommodates concerns for procedure and possible violations of stochastic dominance. Our axioms provide a sharp characterization of how a decision maker's ranking of outcomes depends on the procedure by expressing 'payoffs' as a weighted average of her concerns for outcomes, and her concerns for procedure. The weight used in evaluating this weighted average, which we call the *procedural weight*, is uniquely determined and quantifies the relative importance of procedural concerns. In the special case in which procedural concerns are absent our baseline decision model reduces to expected utility, and our most parsimonious representation is one parameter richer than that model. We use our decision model to provide an *expressive* theory of voting.

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Introduction

> "*Who we are, our persona, is shaped by both the private and social consequences of our choices. In contrast, Decision Theory has been mainly concerned with the private side of economic choices.*"
> – Fabio Maccheroni, Massimo Marinacci and Aldo Rustichini (2008)

1.1 Motivation

The question of how an individual's behavior is influenced by those around her (the *others* in her world) has long concerned philosophers and psychologists. In recent years, this is a question that has engaged many economists as well, and the term *other-regarding preferences* has found an entry into their lexicon. A decision maker has other-regarding preferences if her choices are influenced by a concern not just about her own outcomes but others' outcomes as well. Economists have collected an impressive body of experimental evidence that suggests very strongly that such concerns matter for many decision makers. At the same time they have shown that introducing such concerns into economic models produce novel insights that are of qualitative and quantitative significance. For instance, economists have appealed to other-regarding preferences to deepen their understanding of such matters as Ricardian equivalence (Andreoni, 1989), the equity premium puzzle (Abel, 1990), the difference in redistribution policy between United States and Western Europe (Alesina and Angeletos, 2005), amongst others.

This dissertation provides new decision theoretic foundations for other-regarding preferences. In this section we clarify why this exercise is necessary. In particular, we argue that other-regarding preferences violates the property of 'consequentialism' in a form that is considered almost unchallengeable and paradigmatic in existing theories. We begin with two examples drawn from recent works in experimental economics, which illustrate the challenge in modeling other-regarding preferences.

Example 1: Probabilistic Dictator Game

Consider a decision maker who has to choose between allocating 20 euros either to herself or to some other person. Faced with this decision problem most decision makers, even altruistic ones, would perhaps prefer to keep the money. Let us assume that this is the case. [1] Now consider introducing risk in the environment. In particular, the decision maker is given the option of assigning some probability λ to the other person getting the 20 euros, while assigning the complimentary probability of $1-\lambda$ to herself getting the money. Will the decision maker choose λ equal to 0, or will she choose a positive value of λ? Recent experiments conducted by Krawczyk and Le Lec (2008) provide us with an answer to the question. They report that faced with such a choice problem, a non-trivial number of their subjects (about 30%) choose to assign some probability to the other person getting the 20 euros. On average, these subjects were willing to give a probability of about 0.09 to the other person getting the money.

[1] The argument that we make below does not depend on this assumption.

Example 2: Experimental Election

In a recent paper, Feddersen, Gailmard and Sandroni (2009) conduct an experimental election with two alternatives – call these 1 and 2. The experiment was run as follows. First, subjects were broken up into two groups, one consisting of voters and the other of non-voters. Then the voters cast their votes for 1 or 2. Finally, after all voters had cast their vote, one voter was randomly picked, whose choice became the group choice. The number of eligible voters was determined by the experimenters and was varied across different trials of the experiment. Accordingly, the probability of a voter being pivotal (the reciprocal of the number of voters) was directly controlled as a treatment variable in these experiment. As far as payoffs went, alternative 2 gave a higher monetary reward to the voters than alternative 1. On the other hand alternative 1 was better for the non-voters than alternative 2. An interesting pattern of choice that was exhibited by a non-trivial number of voters is the following. When the probability of their vote being pivotal was high, in particular when it was 1, these voters chose alternative 2. On the other hand when the pivot probability was low, they instead voted for alternative 1.

This evidence is puzzling from the perspective of models of decision making under risk (for instance, expected utility, rank dependent utility, 'betweenness' based theories, weighted utility theory, generalized expected utility, cumulative prospect theory). In Example 1 these models would dictate that the decision maker should choose λ to be equal to 0, and in Example 2 they require that a voter's choice should never switch from alternative 2 to 1 when the probability that her vote is pivotal becomes small. This disconnect arises because the decision makers in the experiments violate consequentialism in a form that these theories cannot accommodate. This violation can be explained as follows. Suppose, there are two outcomes x and y, and a decision maker reveals that she prefers the outcome x to the outcome y. Now consider two lotteries. The first one results in x with some positive probability λ and some other consequence p (which is either an outcome or a lottery) with complimentary probability $1-\lambda$, the second gives y with probability λ and p with complimentary probability $1-\lambda$. Which of the two lotteries *should* any 'rational' decision maker, who prefers the outcome x to the outcome y, choose? It stands to reason that the first lottery is the obvious choice as it gives the decision maker a better chance of getting her more preferred outcome. In fact, some may consider such a choice to be synonymous with rationality, as choosing otherwise would imply behavior inconsistent with one's goals. The above line of reasoning appears hardly controversial; however, it does appeal to a critical assumption, namely, that the decision maker's ranking over outcomes is independent of the (stochastic) process that generates these outcomes. This is the consequentialist assumption that the above mentioned theories maintain but the experimental decision makers violate. Formally, this assumption is referred to as stochastic dominance, and is defined as follows. Assume that X is a set of outcomes, $\Delta(X)$ is the set of lotteries (probability measures) on X, and $\succ$ is the decision maker's strict preference relation over lotteries in $\Delta(X)$.

Definition 1.1. *Suppose x, $y \in X$ and $p \in \Delta(X)$. Then the decision maker's preferences satisfy stochastic dominance if for any $\lambda \in (0,1]$,*

$$x \succ y \Rightarrow \lambda x + (1-\lambda)p \succ \lambda y + (1-\lambda)p,$$

where $\lambda x + (1-\lambda)p$ (resp. $\lambda y + (1-\lambda)p$) is the compound lottery that gives x (resp. y) with probability λ and p with probability $1-\lambda$.

The difference between the two lotteries $\lambda x + (1-\lambda)p$ and $\lambda y + (1-\lambda)p$ is only in the λ-probability event. Under the former the outcome is x and under the latter the outcome is y. Stochastic dominance requires that the ranking over these two lotteries must be determined solely on the basis of how the outcomes x and y are ranked. Accordingly, the choices of decision makers, who violate consequentialism in this sense, cannot be explained based on a concern for outcomes alone, *something else* other than outcomes matters. For decision makers with other-regarding preferences, it then begs the question, what is this 'something else' that matters?

The hypothesis we pursue is the following: Decision makers with other-regarding preferences may care not just about *what* others' outcomes are, but also about *how* others' outcomes are determined. This can be thought of as a concern for the process or procedure via which others' outcomes are determined. Such decision makers may value the choice of a lottery not only *instrumentally* in terms of the outcomes it may lead to, but also *non-instrumentally* as it may be associated with certain procedures for determining outcomes thats is considered desirable. For instance, an altruistic decision maker in the probabilistic dictator game may care not just about the ex-post payoffs of the other person, but also about ex-ante chance that the other person had of getting the money. Alternatively put, she may feel very differently about the ex-post allocation in which she gets the 20 euros depending on whether the other person had some chance of getting the money or none at all.

There has been a great deal of interest in the last two decades or so in incorporating other-regarding concerns like fairness, equity and envy in economic models. The discussion above contains an important insight for this research program. It demonstrates that models of other-regarding preferences that focus exclusively on outcomes may not adequately capture all the concerns of decision makers with such preferences. A leading example of such models would be 'social utility models' like those of Bolton (1991), Fehr and Schmidt (1999), Bolton and Ockenfels (2000) and Charness and Rabin (2002). In these models, a decision maker's utility is defined over her own outcomes and those of others.[2] Such models are unable to accommodate procedural concerns of decision makers. Therefore, a satisfactory analysis of other-regarding preferences requires a decision model that incorporates procedural concerns, and the manner in which concerns for outcomes and procedures interact. This dissertation introduces such a decision model.

We propose a tractable and parsimonious utility representation that clarifies the interaction between concerns for outcomes and procedures. In addition, we identify a set of axioms on behavior that are equivalent to the proposed representation. We provide these axioms with two goals in mind. First, they are meant to provide a choice theoretic foundation of our decision model. The key idea of our axioms is that once concerns for procedure are accounted for, choice behavior adheres to the spirit of the 'classical axioms' of decision theory. Therefore, they are meant to suggest the plausibility of our decision model. Second, our axioms are meant to provide the grounds on which our decision model may be falsified. Just as we used our motivating example to show a violation of stochastic dominance and argue that standard models of decision making may not be adequate in modeling other-regarding preferences, so too, choice behavior that violates our axioms would falsify our decision model. We now provide a brief sketch of our representation.

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[2]For instance, Fehr and Schmidt propose the following social utility function to evaluate the utility derived by individual 1 in a two individual world. Suppose individual 1 receives the outcome x_1 and individual 2 receives the outcome x_2 (x_1, $x_2 \in \mathbb{R}_+$). Then individual 1's utility is given by:

$$U_1(x_1, x_2) = x_1 \text{ - } \mu.\max\{x_2 - x_1,\, 0\} \text{ - } \mu'.\max\{x_1 - x_2,\, 0\},\ \mu,\ \mu' > 0.$$

The basic idea behind this functional form is to incorporate a notion of inequity aversion. The decision maker receives 'utility' from her own outcome x_1, but receives 'dis-utility' from inequities in the final allocation.

1.2 The Baseline Representation and Its Interpretation

Assume that there are n individuals, denoted $1, \ldots, n$, about whose outcomes our decision maker (DM hereafter) may care. Denote the set of DM's outcomes by the set Z, individual i's outcomes by the set Z_i, $i = 1, \ldots, n$, and let $A = \prod_{i=1}^{n} Z_i$. Let p be a simple lottery on the allocation space $Z \times A$, and let p_A denote the marginal probability measures of p over A. Under our baseline decision model, DM evaluates the lottery p by the function:

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{(1-\sigma_z)w(z,a) + \sigma_z w(z,p_A)\},$$

where $\sigma_z \in [0,1]$, for all $z \in Z$, and $p(z,a)$ denotes the probability that the lottery p assigns to the outcome (z,a).

The representation provides a tractable and parsimonious account of the interaction between concerns for outcome and concerns for procedure in determining the decision maker's choices over lotteries. Note that, under the representation, DM considers the marginal probability measure p_A over A to be the *procedure* by which others' outcomes are determined under the lottery p. Consider any outcome (z,a) in the support of p; DM's evaluation of this outcome is conditioned on the procedure p_A. The term inside the parentheses,

$$(1-\sigma_z)w(z,a) + \sigma_z w(z,p_A)$$

reflects her payoffs from the outcome (z,a) when the procedure determining others' outcomes is p_A. This payoff is a weighted average of two terms. The first term is DM's payoffs from the outcome (z,a), whereas the second term is DM's payoffs from the procedure p_A given that she receives outcome z. The same function w is used to evaluate DM's concern for outcomes as well as procedures.[3] The weight σ_z used in evaluating this weighted average is subjective; that is, we derive this weight from DM's choice behavior. We refer to σ_z as a *procedural weight*, and it quantifies the relative strength of concerns for procedure relative to concerns for outcome in determining DM's choices. Observe that the procedural weight is allowed to depend on DM's outcomes.[4] Finally, once we have evaluated these payoffs for all the outcomes (z', a') that are possible under p, we simply use an 'expected utility criterion' over these payoffs to evaluate DM's overall payoff from the lottery p.

There are three key ideas embedded in our representation. First, the representation specifies that a decision maker's evaluation of an outcome (z, a) depends on the procedure by which others' outcomes are determined. In particular, when an outcome (z, a) is realized under some lottery p, we may think of the triple (z, a, p_A) as representing the 'things DM cares about' in this situation. We call such a triple a **procedure-contingent outcome**. We use information about DM's ranking of lotteries on the allocation space $Z \times A$, which is a primitive of our model, to elicit her ranking over procedure-contingent outcomes. Second, the representation provides a simple expression for how these procedure-contingent outcomes are evaluated. It says that the concern for outcomes and procedures interact linearly. The strength of the procedural concern is captured by the parameters σ_z, which, as we mentioned above, are subjective. In particular, decision makers for whom the parameters σ_z are all 0 are expected utility maximizers. In the special case in which the σ_z's are all equal, our

[3]The domain of the function w is $Z \times \Delta_A$, where Δ_A refers to the set of simple lotteries on the set A. We abuse notation by not distinguishing between the outcome a and the degenerate lottery that gives a with probability 1.

[4]We do axiomatize a special case of our baseline representation in which the procedural weights are independent of DM's outcomes. In this case DM evaluates the lottery p by the function:

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z, a)\{(1 - \sigma)w(z, a) + \sigma w(z, p_A)\},$$

where $\sigma \in [0, 1]$.

model is parsimonious in the sense of being one parameter richer than expected utility. Third, in our representation, once DM's concerns for procedure have been accounted for by expanding the notion of outcomes to that of procedure-contingent outcomes, 'event-separability' holds over this expanded space of outcomes. This property helps to keep the representation tractable. We consider two other extensions of our baseline model. First, we allow for 'nonlinear probability weighting' in DM's evaluation of risk faced by others. Second, we present a representation in which DM's evaluation of an allocation is separable across her outcome and others' outcomes.

We conclude this discussion of our representation with a word on our axioms. Beyond the usual axioms of *weak order* and *continuity*, four other axioms characterize our baseline representation. First, we have an axiom in the spirit of the *weak axiom of revealed preference* that allows us to consistently deduce from DM's choices over lotteries her ranking of procedure-contingent outcomes. Second, we have a *monotonicity* axiom, which says that if in some event DM considers the outcome and procedure under one lottery better than that under another lottery, and 'everything else is constant,' then the first lottery is ranked higher than the second. Third, although *independence* in the usual sense 'fails' in our set-up, we are still able to recover a 'subjective version' of this axiom. Finally, we have a *dominance* axiom which requires that once DM's concerns for procedure have been accounted for, an appropriately defined property of event-wise dominance holds.

1.3 Connections to the Literature

This project relates to the decision theoretic literature on *nonseparable* models of preferences in environments of risk. In such models decision makers' preferences are *nonseparable* across mutually exclusive events in the sense that their evaluation of a prospect in a given event may be intrinsically tied to considerations relating to other events that could have occurred but did not. In the words of Machina (1989), "An agent with nonseparable preferences feels (both ex ante and ex post) that risk which is borne but not realized is gone in the sense of having been *consumed* (or "borne"), rather than gone in the sense of *irrelevant*."[5] These are decision makers who violate the *independence* axiom of expected utility theory. It is worth highlighting here that Hammond (1988) shows that the condition of independence/event-separability can be derived using consequentialist foundations. Informally speaking, a decision maker's preferences satisfies the independence condition if and only if a decision maker's choice behavior at 'any point' in a decision problem is a function only of the lotteries that are feasible at that point.[6] It is worth noting that consequentialism in the sense of stochastic dominance that was mentioned above is a strictly weaker notion than the one in Hammond.

There is a large decision theoretic literature that accommodates preferences that are nonseparable and hence violate the independence axiom. Prominent examples include rank dependent utility, *betweenness* based theories (like implicit expected utility), and generalized prospect theory.[7] The key feature of these models is that although event-separability of preferences is not required to hold on the space of all lotteries, each of these models identifies a subset of lotteries over which preferences are separable (see Chew and Epstein (1988) for an illustration of this point). Decision

[5]The emphases in the quote are as in the original.

[6]More formally, let X be a set of outcomes and $\Delta(X)$ the space of lotteries defined on X. Let Ξ be 'a rich class of decision problems.' Let $F : \Xi \rightrightarrows \Delta(X)$ denote a feasibility correspondence that specifies the feasible set of lotteries that can possibly result from the decision maker's choices in any decision problem. Let $B : \Xi \rightrightarrows \Delta(X)$ be a behavior correspondence that specifies the decision maker's choice behavior in any decision problem. The decision maker is a consequentialist if there exists a choice correspondence $C : 2^{\Delta(X)} \setminus \emptyset \rightrightarrows \Delta(X)$ such that for all $\xi \in \Xi$, $B(\xi) = C(F(\xi))$. In other words, changes in the structure of a decision problem should have no bearing on choices unless they change the feasible set. It is critical to recognize that we provide this definition for a *given* set of outcomes that we hold fixed.

[7]Refer to Starmer (2000) for a comprehensive survey of non-expected utility models.

makers whose preferences are accommodated by any of these models may then be thought to conform with consequentialism in a restricted sense. In particular, under all these models behavior retains consequentialism in the minimal sense of stochastic dominance. In contrast, the decision model we present here does not require behavior to conform to this minimal notion of consequentialism. The important point to recognize is that consequentialism (defined with respect to a *given* set of outcomes) and a concern for procedures are fundamentally conflicting notions, and therefore, consequentialism has to be given up at a very basic level to accommodate concerns for procedure. We should point out here that Karni and Safra (2002) is another paper that accommodates violations of consequentialism in the minimal sense of stochastic dominance.

Our work also relates to a literature on procedures. This line of research seeks to highlight the fact that in addition to outcomes or consequences, the way decisions are made may itself influence an individual's well being. Sen (1997) explains it thus: "Maximizing *behavior* differs from nonvolitional *maximization* because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person's preferences over *comprehensive* outcomes (including the choice process) have to be distinguished from the conditional preferences over *culmination* outcomes *given* the acts of choice."[8] In order to model such concerns for procedure, Sen suggests using "menu dependent" models of choice behavior, which allow a decision maker's preferences over outcomes to depend on the set (menu) from which the choice is made. The vast majority of work that highlights concerns for procedure have been conducted within an empirical/experimental setting. Some examples are Kahneman, Knetsch and Thaler (1986), Bies, Tripp and Neale (1993), Frey and Pommerehne (1993), Bolton, Brandts and Ockenfels (2005)

[8]The emphases in the quote are as in the original.

and Krawczyk and Le Lec (2008).

Finally, our work is part of a large literature on other-regarding preferences. A vast portion of this research has been undertaken within experimental settings, and this work has played an important role in demonstrating that other-regarding concerns matter to many decision makers. This experimental literature is too vast to adequately document here. For a recent survey of this literature, the reader may refer to Cooper and Kagel (2009). Inspired by this evidence from the 'lab,' many researchers have sought to incorporate other-regarding concerns into economic models. 'Social utility' models are a leading example of this endeavor. In these models a decision maker's utility is defined over not just her own outcomes but others outcomes as well. Different forms of other-regarding concerns like fairness, envy, altruism, etc. are incorporated into these models by writing functional forms for utility that intuitively correspond to the particular 'emotion' that is sought to be modeled. Some leading examples of such models are Bolton (1991), Fehr and Schmidt (1999), Bolton and Ockenfels (2000) and Charness and Rabin (2002). The difference between these works and ours is twofold. First, these models are outcome based and do not consider concerns for procedure. Second, these models are not based on an axiomatic treatment of choice behavior. This is an observation that holds true in general about how other-regarding preferences have been treated in the literature. Despite the interest in such preferences, there are not that many papers studying the choice theoretic foundations of such preferences. Some notable exceptions include Ok and Kockesen (2000), Gilboa and Schmeidler (2002), Karni and Safra (2002), Neilson (2006), Sandbu (2005), Maccheroni, Marinacci and Rustichini (2008) and Rohde (2009).

We conclude this chapter by citing an important debate that was initiated by an influential paper by Harsanyi (1955). In this paper Harsanyi postulated that an individual's social or moral preference that represents her moral value judgments about allocations in society satisfies the independence axiom of expected utility theory. The question of whether this is a reasonable axiom for social or moral preferences yielded a spirited debate involving many distinguished participants, including, Strotz (1958, 1961), Diamond (1967), Keeney (1980), Broome (1982, 1984), Sen (1985) and Harsanyi (1975, 1978) himself. One of the most articulate and well-known critiques of Harsanyi's viewpoint has been presented by Machina (1989), whose counter-example has come to be known as *Machina's Mom* in the literature. The example goes as follows. A mom has an indivisible treat that she could give either to her son or her daughter. She is indifferent between the daughter getting the treat or the son getting the treat, but in a violation of the independence axiom, she strictly prefers a coin flip over each of the sure outcomes.

Chapter 2

The Basic Representations

2.1 The Framework

2.1.1 Preliminaries

We assume that our stylized society comprises of a decision maker (DM) and n other individuals, denoted $1, \ldots, n$, and associated with each individual is a well defined set of outcomes.[1] We denote the set of DM's outcomes by Z and those of individual i by Z_i, $i = 1, \ldots, n$. Further, we let $A = \prod_{i=1}^{n} A_i$ denote the set of outcome-vectors for the others (others' outcomes, for short). We will assume that each of the sets Z, Z_i, $i = 1, \ldots, n$, are connected topological sets.

We denote the set of simple probability measures (simple lotteries, or just lotteries, for short) on the sets $Z \times A$, Z and A by Δ, Δ_Z and Δ_A respectively. We will denote elements of Δ by p, q etc., those of Δ_Z by p_Z, q_Z etc., and those of Δ_A by p_A, q_A etc. We define a convex combination of lotteries in any of these sets in the standard way.[2] For any p in Δ we denote by $p_Z \in \Delta_Z$, the marginal probability measure of p

[1] To fix ideas the reader may think of these outcomes as money or some consumption good.

[2] For instance, if $p^1, \ldots, p^K \in \Delta$, and $\lambda^1, \ldots, \lambda^K$ are constants in $[0, 1]$ that sum to 1,

over Z and by $p_A \in \Delta_A$, the marginal probability measure of p over A;[3] further, we denote by $p_{A,z} \in \Delta_A$, the conditional probability measure of p over A with respect to the event that DM gets $z \in Z$.

We want to highlight two special classes of lotteries in Δ. The first consists of those $p \in \Delta$ in which DM gets some outcome $z \in Z$ for sure, i.e., $p_Z(z) = 1$ for some $z \in Z$. We will abuse notation and denote such a lottery by $p = (z,p_A)$. The second consists of those $p \in \Delta$ in which the others get some outcome-vector $a \in A$ for sure, i.e., $p_A(a) = 1$ for some $a \in A$. We will denote such a lottery (again with an abuse of notation) by $p = (p_Z,a)$. Following standard notation, we shall at times denote lotteries by explicitly listing the elements in the support along with their respective probabilities. For instance,

$$p = [(z_1, a_1), \lambda_1; \ . \ . \ . \ ; (z_K, a_K), \lambda_K]$$

shall denote a lottery in Δ that gives the outcome (z_k, a_k) with probability λ_k, $k = 1, \ . \ . \ . \ , K$. A more compact notation for this, which will be used at times, is the following:

$$p = [< (z_k, a_k), \lambda_k >_{(z_k,a_k)\in S[p]}], \text{ where } S[p] \text{ stands for the support of } p.$$

Finally, note that we will abuse notation throughout by not distinguishing between an outcome and a lottery that gives that outcome with probability 1. For instance, $(z, a) \in Z \times A$ shall stand both for an outcome as well as the lottery that gives this outcome with probability 1.

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then $\sum_{k=1}^{K} \lambda^k p^k$ denotes an element in Δ that gives the outcome $(z, a) \in Z \times A$ with probability $\sum_{k=1}^{K} \lambda^k p^k(z, a)$.

[3]When we reference lotteries like $p_Z \in \Delta_Z$ and $p_A \in \Delta_A$, it will be clear from the context whether we refer to them in the sense of marginal measures of a lottery $p \in \Delta$, or 'simply' as elements of Δ_Z and Δ_A.

2.1.2 Preference

DM's preferences are given by a weak order (a binary relation that is complete and transitive) $\succcurlyeq$ on the set Δ. The symmetric and asymmetric components of $\succcurlyeq$ are defined in the usual way and denoted by $\sim$ and $\succ$ respectively. For any $z \in Z$, we use the primitive preference relation $\succcurlyeq$ to define a weak order $\succcurlyeq_z$ on Δ_A as follows: for any p_A, $q_A \in \Delta_A$,

$$p_A \succcurlyeq_z q_A \text{ if } (z, p_A) \succcurlyeq (z, q_A).$$

The symmetric and asymmetric components of $\succcurlyeq_z$ are defined in the usual way and denoted by $\sim_z$ and $\succ_z$ respectively. We will assume that the indifference surfaces of $\succcurlyeq_z$ restricted to A are connected. That is, for any $\succcurlyeq_z$, and for any $a \in A$, $\{a' \in A : a' \sim_z a\}$ is a connected subset of A. The family of preference relations $(\succcurlyeq_z)_{z \in Z}$ is the basic building block of our decision model. The interpretation that we have in mind is that this family represents the *values* of the decision maker because any preference relation $\succcurlyeq_z$ in this family tells us about DM's preferences over the risk faced by others when she is assured some outcome $z \in Z$. We will assume that the values of DM are *contingent* on what her own circumstances are; that is, her preferences over the risk faced by others varies depending on what outcome she herself gets. Formally, we have the following condition.

Contingent Values: *If $\succcurlyeq_z$ such that $\succ_z \neq \emptyset$, then there exists $\succcurlyeq_{z'} \neq \succcurlyeq_z$, with $\succ_{z'} \neq \emptyset$, such that the following holds: for all $\widetilde{a} \in A$, there exists a, a', $a'' \in A$, with $\widetilde{a} \sim_z a$, and*

$$a' \sim_{z'} a \sim_{z'} a'', \; a' \succcurlyeq_z a \succcurlyeq_z a''.$$

Further, if a is not a maximal (resp. minimal) element of $\succcurlyeq_z$, then $a' \succ_z a$ (resp. $a \succ_z a''$).

2.2 Axioms and Representations

2.2.1 Axioms for Baseline Representation

This section presents axioms on choice behavior that are necessary and sufficient for representing $\succcurlyeq$ by a utility function $W : \Delta \to \mathbb{R}$ of the form

$$W(p) = \sum_{(z,a)\in Z\times A} p(z,a)\{(1-\sigma_z)w(z,a) + \sigma_z w(z,p_A)\},$$

where $\sigma_z \in [0,1]$, for all $z \in Z$, and $p(z,a)$ denotes the probability that p assigns to the outcome $(z,a) \in Z \times A$.

Basic Axioms

We require that DM's preferences are complete and transitive.

AXIOM: Weak Order

$\succcurlyeq$ *is complete and transitive.*

Our next axiom specifies continuity properties of DM's preferences. To state it, we first observe that the topology on the sets Z and Z_i, $i = 1, \ldots, n$, induces the product topology on the set $[Z \times A] \times [Z \times A]$.

AXIOM: Bounded Bi-Continuity

Bi-Continuity: *For any $\lambda \in [0,1]$ and $q \in \Delta$, the sets*

$$\{((z',a'),(z'',a'')) : [(z',a'),\lambda; (z'',a''),1-\lambda] \succcurlyeq q\},$$

and,

$$\{((z',a'),(z'',a'')) : q \succcurlyeq [(z',a'),\lambda; (z'',a''),1-\lambda]\}$$

are closed in $[Z \times A] \times [Z \times A]$.

Boundedness: *There exists $a^* \in A$, such that for all $p \in \Delta$ there exists $\overline{z}$, $\underline{z} \in Z$ satisfying*

$$(\overline{z}, a^*) \succcurlyeq p \succcurlyeq (\underline{z}, a^*).$$

An important implication of *bi-continuity* is that for any $z \in Z$, the preference relation $\succcurlyeq_z$ satisfies the following continuity property. For any $p_A \in \Delta_A$, the sets,

$$\{a \in A : a \succcurlyeq_z p_A\} \text{ and } \{a \in A : p_A \succcurlyeq_z a\}$$

are closed in A. This continuity property in conjunction with the condition that $\succcurlyeq_z$ satisfies stochastic dominance (which will be implied by our axioms) and the topological assumption on A implies that every $p_A \in \Delta_A$ has a certainty equivalent in A with respect to the preference relation $\succcurlyeq_z$; that is, for any $p_A \in \Delta_A$, there exists $\mathcal{CE}_z(p_A) \in A$ such that $p_A \sim_z \mathcal{CE}_z(p_A)$.

Eliciting Preferences over Procedure-Contingent Outcomes

As discussed in the introduction, decision makers that we accommodate in our model may care both about the outcomes of others as well as the procedure by which their outcomes are determined. For any lottery $p \in \Delta$, we shall consider the marginal probability measure $p_A \in \Delta_A$ over others' outcomes as the procedure determining their outcomes. The decision maker's evaluation of an ex-post outcome $(z, a) \in Z \times A$ may be contingent on the procedure. In other words, for any outcome (z, a) that is in the support of two lotteries $p, p' \in \Delta$, the ex-post evaluation of this outcome may differ depending on the respective procedures p_A and p'_A. We define the following notion to account for this difference: For any $p \in \Delta$ and (z, a) in the support of p, we will refer to the triple $(z, a, p_A) \in Z \times A \times \Delta_A$ as a **procedure-contingent outcome**. Our representation implies that DM has a ranking over procedure-contingent outcomes.

We will propose here a method of eliciting this ranking from DM's choice behavior over lotteries in Δ. To that end, we will provide a definition of what it means for DM to consider one procedure-contingent outcome to be *better* than another. The key idea behind this definition will be that in assessing any procedure-contingent outcome, say, (z, a, p_A), the decision maker shall independently evaluate the outcome that others get, a, and the procedure by which their outcomes are determined, p_A, using the preference relation $\succcurlyeq_z$. We introduce the following piece of notation that shall be useful in the exposition of the definition.

- Let (z, a), (z, a') be respectively in the support of p, $p' \in \Delta$. Then $(z, a, p_A) =^* (z, a', p'_A)$ if $a \sim_z a'$ and $p_A \sim_z p'_A$.

Consider first the following intermediary definition that defines a binary relation on Δ.

Definition 2.1. *Two lotteries,*

$$p = [(z_1, a_1), \lambda_1; \ . \ . \ . \ ; (z_K, a_K), \lambda_K] \text{ and } \tilde{p} = [(z_1, \tilde{a}_1), \lambda_1; \ . \ . \ . \ ; (z_K, \tilde{a}_K), \lambda_K]$$

in Δ *are* ***comparable at*** z_j *if for all* $k \neq j$ $(j,\, k \in \{1, \ . \ . \ . \ , K\})$*, we have* (z_k, a_k, p_A) $=^* (z_k, \tilde{a}_k, \tilde{p}_A)$.

Observe that in each of the λ_k-events, $k \neq j$, the outcome that others get under p (a_k), belongs to the same indifference class of $\succcurlyeq_{z_k}$ as the outcome that they get under $\tilde{p}$ $(\tilde{a}_k)$. Further, the procedure that determines others' outcome under p (p_A), belongs to the same indifference class of $\succcurlyeq_{z_k}$ as the procedure that determines their outcome under $\tilde{p}$ $(\tilde{p}_A)$. Accordingly, in each of these λ_k-events $(k \neq j)$, if DM independently evaluates the outcome that others get and the procedure that determines this outcome using the preference relation $\succcurlyeq_{z_k}$, then, from a preference perspective, we may conclude that DM considers the lotteries p and $\tilde{p}$ to be identical in these events; or equivalently, that DM considers the procedure contingent outcomes (z_k, a_k, p_A) and $(z_k, \tilde{a}_k, \tilde{p}_A)$ to be identical. Alternatively, the only place where DM may consider p and $\tilde{p}$ to differ is in her assessment of the procedure-contingent outcomes (z_j, a_j, p_A) and $(z_j, \tilde{a}_j, \tilde{p}_A)$. We may then draw a direct inference about how DM ranks the procedure-contingent outcomes (z_j, a_j, p_A) and $(z_j, \tilde{a}_j, \tilde{p}_A)$ from how she ranks p and $\tilde{p}$. For instance, if $p \succcurlyeq \tilde{p}$, then we may draw the inference that DM considers the procedure-contingent outcome (z_j, a_j, p_A) to be better than $(z_j, \tilde{a}_j, \tilde{p}_A)$. We may extend this method of elicitation by putting together a 'chain' of such inferences. To do so, we introduce the following notation. For any

$$p = [(z_1, a_1), \lambda_1; \ . \ . \ . \ ; (z_K, a_K), \lambda_K] \text{ and } q = [(z_1, \tilde{a}_1), \lambda_1; \ . \ . \ . \ ; (z_K, \tilde{a}_K), \lambda_K]$$

in Δ that are comparable at some z_j, we will denote the corresponding outcome-vector that others' get under p and q, when DM gets this z_j, by $a(z_j, p)$ and $a(z_j, q)$ respectively. That is $a(z_j, p)$ shall refer to the outcome-vector a_j and $a(z_j, q)$ shall refer to $\widetilde{a}_j$.

Consider lotteries p, $q \in \Delta$ that are comparable at some $z \in Z$. If DM, say, prefers p over q, then, based on the above argument, we may draw the inference that DM considers the procedure-contingent outcome $(z,a(z, p),p_A)$ to be better than the procedure-contingent outcome $(z,a(z, q),q_A)$. Similarly, if there exists p', $q' \in \Delta$ that are comparable at z and $p' \succcurlyeq q'$, then we may similarly draw the inference that DM considers $(z,a(z, p'),p'_A)$ to be better than $(z,a(z, q'),q'_A)$. If in addition, it is also the case that $(z,a(z, q),q_A) =^* (z,a(z, p'),p'_A)$, then we may connect the two inferences by appealing to the fact that from a preference perspective $(z,a(z, q),q_A)$ and $(z,a(z, p'),p'_A)$ are considered identical by DM (under the interpretation, suggested above, that she independently evaluates the outcome of others and the procedure generating this outcome using the preference relation $\succcurlyeq_z$). It then seems natural to suggest on grounds of consistency that DM considers the procedure-contingent outcome (z,$a(z,p)$,p_A) to be better than the procedure-contingent outcome (z,$a(z,q')$,q'_A). There is no reason why the 'choice chain' or 'choice sequence' has to stop at two. More generally, we have the following definition.

Definition 2.2. *Let (z,a), $(z,\tilde{a})$ be respectively in the support of p, $q \in \Delta$. The procedure-contingent outcome (z,a,p_A) is **revealed better** (resp. **revealed strictly better**) than the procedure-contingent outcome $(z,\tilde{a},q_A)$ if there exists a finite sequence $(p^k,\, q^k)_{k=1}^K \subseteq \Delta \times \Delta$ satisfying,*
(i) $p = p^1$, $q = q^K$,
(ii) p^k and q^k are comparable at $z \in Z$, for all $k = 1, \ldots, K$, and
(iii) $(z, a(z,q^k), q^k_A) =^* (z, a(z,p^{k+1}), p^{k+1}_A)$, for all $k = 1, \ldots, K-1$,
such that $p^k \succcurlyeq q^k$ for all $k = 1, \ldots, K$ (resp., $p^k \succcurlyeq q^k$ for all $k = 1, \ldots, K$, and $p^{k'} \succ q^{k'}$ for some k').
*Further, $(z,\, a,\, p_A)$ is **revealed indifferent** to $(z,\, \tilde{a},\, q_A)$ if $p^k \sim q^k$, for all k.*

The following axiom, which is in the spirit of the *weak axiom of revealed preference*, is a consistency axiom on DM's behavior. It says that if a particular sequence of choices reveals that one procedure-contingent outcome is better than another, then it should not be the case that some other sequence of choices reveals a contradictory implication.

AXIOM: Revealed Consistency

Let (z,a), $(z,\widetilde{a})$ be respectively in the support of p, $q \in \Delta$. If (z,a,p_A) is revealed better than (z,$\widetilde{a}$,q_A), then there does not exist $\widetilde{p}$, $\widetilde{q} \in \Delta$, with (z,a), $(z,\widetilde{a})$ in the support of $\widetilde{p}$, $\widetilde{q}$ respectively, and $\widetilde{p}_A = p_A$, $\widetilde{q}_A = q_A$, such that (z,$\widetilde{a}$,$\widetilde{q}_A$) is revealed strictly better than (z,a,$\widetilde{p}_A$).

Our next axiom provides a monotonicity restriction on comparable lotteries. We know that for any two such lotteries p and $\tilde{p}$ that are, say, comparable at some $z \in Z$, from a preference perspective, they differ only in the event that DM gets outcome z. Now, if in this event, the outcome that others get under p is preferred to the outcome that others get under $\tilde{p}$ according to $\succcurlyeq_z$, and furthermore, the procedure p_A is preferred to the procedure $\tilde{p}_A$ according to $\succcurlyeq_z$, then the axiom of *comparable monotonicity* requires DM to prefer p over $\tilde{p}$.

AXIOM: Comparable Monotonicity

Let p, $q \in \Delta$ be comparable at some $z \in Z$. If $a(z,p) \succcurlyeq_z a(z,q)$ and $p_A \succcurlyeq_z q_A$, then $p \succcurlyeq q$.

Subjective Mixtures and Separability

The independence axiom in the usual form is not appropriate for our setting. We introduce here a 'subjective version' of this axiom by appealing to the notion of *subjective mixtures* of lotteries. The idea is a simple one. Consider two lotteries p, $q \in \Delta$ with $p \succ q$. For any $\lambda \in [0,1]$, we want to identify an element $\lambda p \oplus (1-\lambda)q \in \Delta$ that may be considered a λ-weighted preference average of p and q. Heuristically speaking, suppose DM has a 'preference scale' in which p is assigned the value 1 and q the value 0. Then we want $\lambda p \oplus (1-\lambda)q$ to be an element on that scale that is assigned the value λ. Of course, if $\succcurlyeq$ were to satisfy the axioms of the von Neumann and Morgenstern world, then $\lambda p \oplus (1-\lambda)q$ will be any element in the indifference class of $\lambda p + (1-\lambda)q$. That is, subjective mixtures and objective mixtures coincide in that set-up. We define here a notion of subjective mixtures for three classes of lotteries.

Definition 2.3. *Given p, $q \in \Delta$ that are comparable at some $z \in Z$, we denote by*

$\lambda p \oplus (1-\lambda)q$ an element in Δ that is comparable with p (and hence with q) at z and satisfies,

$$a(z, \lambda p \oplus (1-\lambda)q) \sim_z \lambda a(z,p) + (1-\lambda)a(z,q) \text{ and } (\lambda p \oplus (1-\lambda)q)_A \sim_z \lambda p_A + (1-\lambda)q_A.$$

Further, if p, $q \in \Delta$ are such that either $p = (p_Z, a)$, $q = (q_Z, a)$ for some $a \in A$, or $p = (z, p_A)$, $q = (z, q_A)$ for some $z \in Z$, then we define,

$$\lambda p \oplus (1-\lambda)q = \lambda p + (1-\lambda)q$$

*We will say that p and $q \in \Delta$ are **mixture comparable** if $\lambda p \oplus (1-\lambda)q$ exists for all $\lambda \in [0,1]$.*

For lotteries p and q that are comparable at some $z \in Z$, the subjective mixture $\lambda p \oplus (1-\lambda)q$ is a lottery that is comparable with p and q at z, and in the event that DM gets z, the outcome that others get under this lottery is a λ-weighted preference average (according to $\succcurlyeq_z$) of the outcomes that they get under p and q, and the procedure that determines others' outcome under this lottery is a λ-weighted preference average (according to $\succcurlyeq_z$) of the respective procedures under p and q. An important point to observe is that there may be multiple lotteries in Δ that may satisfy the above conditions. If that is the case, then by *conditional monotonicity* all these lotteries must be indifferent. As stated in the definition, $\lambda p \oplus (1-\lambda)q$ denotes a *representative* of this indifference class. On the other hand, for lotteries of the type $p = (p_Z, a)$, $q = (q_Z, a)$, or, $p = (z, p_A)$, $q = (z, q_A)$, the definition of subjective mixtures coincide with that of objective mixtures. The independence axiom we propose here shall hold only for mixture comparable lotteries. Within this subclass it retains the *subjective* interpretation of the classical independence axiom of expected utility theory. In addition, it will imply that the preference relation $\succcurlyeq_z$ satisfies the classical independence axiom of expected utility theory for any $z \in Z$.

AXIOM: Comparable Independence

Let p_1, p_2, q_1, q_2 in Δ be such that p_1, p_2 are mixture comparable, as are q_1, q_2. Then, for all $\lambda \in (0, 1]$,

$$[p_1 \succ q_1,\, p_2 \sim q_2] \Rightarrow \lambda p_1 \oplus (1-\lambda)p_2 \succ \lambda q_1 \oplus (1-\lambda)q_2.$$

Our final axiom, which we call *dominance*, prescribes that once DM's concerns for procedure have been accounted for by expanding the notion of outcomes to that of procedure-contingent outcomes, 'event-separability' on this expanded notion of outcomes is normatively appealing. To present this axiom we require the following definition.

Definition 2.4. *Let $p \in \Delta$ be such that for each (z^k, a^k) in the support of p, $k = 1, \ldots, K$, the procedure-contingent outcome (z^k, a^k, p_A) is revealed indifferent to a procedure-contingent outcome $(z^k, \tilde{a}^k, \tilde{a}^k)$ for some $\tilde{a}^k \in A$. Then we shall refer to the lottery,*

$$\pi(p) = [(z^1, \tilde{a}^1), p(z^1, a^1); \ldots, (z^K, \tilde{a}^K), p(z^K, a^K)] \in \Delta$$

*as the **procedure-adjusted equivalent** of p.*

Following standard terminology, we will say that a lottery $p \in \Delta$ first order stochastically dominates a lottery $q \in \Delta$ with respect to $\succcurlyeq$ if for all $(z, a) \in Z \times A$, the probability that p assigns to outcomes that are at least as good as (z, a) (according to $\succcurlyeq$) is at least as large as the corresponding probability under q, and is strictly larger for some $(z, a) \in Z \times A$.

The axiom of *dominance* says the following.

AXIOM: Dominance

Let p, $q \in \Delta$ and $\pi(p)$, $\pi(q) \in \Delta$ be procedure-adjusted equivalents of p and q respectively. If $\pi(p)$ first order stochastically dominates $\pi(q)$ with respect to $\succcurlyeq$, then $p \succ q$.

This axiom first of all requires DM to consider a procedure-contingent outcome like $(z,\, \tilde{a},\, \tilde{a})$, in which the outcome and procedure are the same, to be equivalent to the outcome $(z,\, \tilde{a})$. Accordingly, if a procedure-contingent outcome (z, a, p_A) is revealed indifferent to a procedure-contingent outcome like $(z,\, \tilde{a},\, \tilde{a})$, then this axiom prescribes that it be evaluated the same as $(z,\, \tilde{a})$. Further, if all the procedure-contingent outcomes that are generated by a lottery can be 'equated' to outcomes via such revealed indifferences, then dominance should hold in the sense specified in the axiom.

2.2.2 Baseline Representation

We can now state our baseline representation.

Theorem 2.1. *Suppose contingent values hold. Then $\succcurlyeq$ on Δ satisfies the axioms of weak order, bounded bi-continuity, revealed consistency, comparable monotonicity, comparable independence and dominance if and only if there exists a function $w : Z \times \Delta_A \to \mathbb{R}$, satisfying*

$w(z,\lambda p_A + (1-\lambda)p'_A) = \lambda w(z, p_A) + (1-\lambda) w(z, p'_A)$, for all p_A, $q_A \in \Delta_A$, $\lambda \in [0,1]$,

and constants $\sigma_z \in [0,1]$, $z \in Z$, such that the function $W{:}\Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{(1-\sigma_z)w(z,a) + \sigma_z w(z,p_A)\}$$

represents $\succcurlyeq$.

In addition, another pair $(\tilde{w},\, (\tilde{\sigma}_z)_{z\in Z})$ represents $\succcurlyeq$ in the above sense if and only if *there exists constants* $\alpha > 0$ *and* β *such that* $\tilde{w} = \alpha w + \beta$, *and* $\tilde{\sigma}_z = \sigma_z$ *for all* $z \in Z$ *with* $\succ_z \neq \emptyset$.

For any lottery $p \in \Delta$ and (z, a) in the support of p, the representation provides a 'valuation' of the procedure-contingent outcome (z, a, p_A). This valuation is given by the expression

$$(1 - \sigma_z)w(z, a) + \sigma_z w(z, p_A)$$

It is a weighted average of DM's concern for outcomes and procedures. The subjective weight σ_z is uniquely determined in our representation as long as $\succcurlyeq_z$ is non-trivial. We call σ_z a **procedural weight**. It quantifies the strength of procedural concerns relative to concerns for outcome in determining DM's choice behavior. Under the representation, once all the procedure-contingent outcomes have been appropriately evaluated, the aggregation criterion across events is just like under expected utility. The proof of the theorem is available in the Appendix.

2.2.3 Independent Procedural Weights

In Theorem 2.1 the procedural weights are a function of the outcome that DM receives. We now provide a representation in which there is a unique procedural weight independent of DM's outcomes; that is, DM evaluates a lottery $p \in \Delta$ by the function:

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z, a)\{(1 - \sigma)w(z, a) + \sigma w(z, p_A)\}, \text{ where } \sigma \in [0, 1].$$

It should be intuitively clear that to axiomatize this case we need to impose some form of symmetry on DM's preferences. We now make precise this notion of symmetry. Note that for any $a \in A$, the indifference class of a under $\succcurlyeq_z$ (restricted to A) shall be denoted by

$$[a]_z = \{a' \in A : a' \sim_z a\}$$

Now consider the following definition.

Definition 2.5. *a, $a' \in A$ are* ***equal gains*** *w.r.t. $\succcurlyeq_z$ and $\succcurlyeq_{z'}$ if for any $\tilde{a} \in [a]_z \cap [a']_{z'}$ and $\tilde{a}' \in [a']_z \cap [a]_{z'}$*

$$[(z, \widetilde{a}), 1/2; (z', \widetilde{a}), 1/2] \sim [(z, \widetilde{a}'), 1/2; (z', \widetilde{a}'), 1/2]$$

The lotteries

$$p = [(z, a), \tfrac{1}{2}; (z', a'), \tfrac{1}{2}], \text{ and } q = [(z, a'), \tfrac{1}{2}; (z', a), \tfrac{1}{2}]$$

in Δ are ***symmetric*** *if a and a' are equal gains w.r.t. $\succcurlyeq_z$ and $\succcurlyeq_{z'}$.*

The equal gains definition gives us a condition under which the 'subjective difference' between two outcomes for the others, a, $a' \in A$, is considered the same by the decision maker under both $\succcurlyeq_z$ and $\succcurlyeq_{z'}$. To see this, consider changing the lottery from $[(z, \widetilde{a}), 1/2; (z', \widetilde{a}), 1/2]$ to $[(z, \widetilde{a}'), 1/2; (z', \widetilde{a}'), 1/2]$. Note that under both these lotteries, there is no risk facing the others, and hence, there are no procedural concerns to be considered. Suppose that $a' \succ_z a$, or equivalently $\tilde{a}' \succ_z \tilde{a}$. Then, with probability $\frac{1}{2}$ (i.e., in the event that DM gets z), the decision maker is made better off due to the change. Since the change leaves her indifferent overall, it must be that in the complementary event (i.e., when she gets z'), the change makes her worse off, and this 'negative change' must be of the same magnitude as the positive one. In other words, the improvement under $\succcurlyeq_z$ when the outcome facing the others is changed from $\tilde{a}$ to $\tilde{a}'$ (or equivalently, from a to a') is of the same magnitude as the improvement under $\succcurlyeq_{z'}$ when the outcome facing the others is changed from $\tilde{a}'$ to $\tilde{a}$ (or equivalently, from a to a'). The definition of symmetric lotteries builds on the notion of equal gains to talk about tradeoffs in lotteries in which the others face non-trivial risk (and hence outcomes and procedures need not be the same). Consider the lotteries p and q in the definition above. Note that $p_A = q_A$; that is the procedure is identical under both. Consider changing the lottery that DM faces from q to p. Suppose that $a' \succ_z a$. Then, with probability $\frac{1}{2}$ (i.e., in the event that DM gets z), the decision maker is made worse off in terms of outcomes. Further, since a and a' are equal gains, with complementary probability (i.e., in the event that DM gets z'), she is made better off (again in terms of outcomes) 'by an equal amount.' Such symmetric lotteries then allow us to elicit from behavior the strength of procedural concerns across different outcomes that DM receives.

The axiom that we need to ensure that procedural weights are independent of DM's own outcomes requires that if two allocation lotteries p, $q \in \Delta$ are symmetric, then the decision maker is indifferent between them.

AXIOM: Symmetry

If p, $q \in \Delta$ are symmetric, then $p \sim q$.

We then have the following representation result:

Theorem 2.2. *Suppose contingent values hold. Then $\succcurlyeq$ on Δ satisfies the axioms of weak order, bounded bi-continuity, revealed consistency, comparable monotonicity, comparable independence, dominance and symmetry if and only if there exists a function $w : Z \times \Delta_A \to \mathbb{R}$, satisfying*

$w(z, \lambda p_A + (1-\lambda)p'_A) = \lambda w(z, p_A) + (1-\lambda)w(z, p'_A)$, for all p_A, $q_A \in \Delta_A$, $\lambda \in [0, 1]$,

and a constant $\sigma \in [0, 1]$, such that the function $W{:}\Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z, a)\{(1-\sigma)w(z, a) + \sigma w(z, p_A)\}$$

represents $\succcurlyeq$.

In addition, another pair $(\tilde{w}, \tilde{\sigma})$ represents $\succcurlyeq$ in the above sense if and only if there exists constants $\alpha > 0$ and β such that $\tilde{w} = \alpha w + \beta$, and $\tilde{\sigma} = \sigma$ whenever there exists some $z \in Z$ with $\succ_z \neq \emptyset$.

The proof is available in the appendix.

2.3 Appendix

2.3.1 Preliminaries

A Binary Relation

We define here a binary relation. First consider the following notation. For any $p_A \in \Delta_A$ and $z \in Z$, the indifference class of p_A under $\succcurlyeq_z$ is denoted by

$$[p_A]_z = \{q_A \in \Delta_A : q_A \sim_z p_A\}$$

Further, $\Delta_A/\sim_z$ shall denote the set of all such indifference classes. We define the binary relations, $\widehat{\succcurlyeq}_z$, $\widehat{\succ}_z$, $\widehat{\sim}_z$ on $\Delta_A/\sim_z \times \Delta_A/\sim_z$ as follows:

Definition 2.6. *$([p_A]_z,\ [q_A]_z)\ \widehat{\succcurlyeq}_z$ (resp. $\widehat{\succ}_z$, resp. $\widehat{\sim}_z$) $([p'_A]_z,\ [q'_A]_z)$ if there exists procedure-contingent outcomes $(z, \tilde{a}, \tilde{p}_A)$ and $(z, \hat{a}, \hat{p}_A)$ satisfying*

$$\tilde{a} \in [p_A]_z,\ \tilde{p}_A \in [q_A]_z \text{ and } \hat{a} \in [p'_A]_z,\ \hat{p}_A \in [q'_A]_z$$

such that $(z, \tilde{a}, \tilde{p}_A)$ is revealed better than (resp. revealed strictly better than, resp. revealed indifferent to) $(z, \hat{a}, \hat{p}_A)$.

Remark 2.1. *The axiom of* revealed consistency *implies that $\widehat{\sim}_z$ and $\widehat{\succ}_z$ are respectively the symmetric and asymmetric components of $\widehat{\succcurlyeq}_z$. That is,*

$$([p_A]_z,[q_A]_z)\ \widehat{\sim}_z\ ([p'_A]_z,[q'_A]_z) \text{ iff } ([p_A]_z,[q_A]_z)\ \widehat{\succcurlyeq}_z\ ([p'_A]_z,[q'_A]_z)\ \&\ ([p'_A]_z,[q'_A]_z)\ \widehat{\succcurlyeq}_z\ ([p_A]_z,\ [q_A]_z).$$

and,

$$([p_A]_z,[q_A]_z)\widehat{\succ}_z([p'_A]_z,[q'_A]_z) \text{ iff } ([p_A]_z,[q_A]_z)\ \widehat{\succcurlyeq}_z\ ([p'_A]_z,[q'_A]_z)\ \&\ \neg\ ([p'_A]_z,[q'_A]_z)\widehat{\succcurlyeq}_z ([p_A]_z,[q_A]_z).$$

Remark 2.2. *The definition of revealed better (resp. revealed strictly better, resp. revealed indifferent) implies that if* $([p_A]_z, [q_A]_z)\ \widehat{\succcurlyeq}_z$ *(resp.* $\widehat{\succ}_z$*, resp.* $\widehat{\sim}_z$*)* $([p'_A]_z, [q'_A]_z)$ *and there exists procedure contingent outcomes* $(z, \tilde{a}, \tilde{p}_A)$ *and* $(z, \hat{a}, \hat{p}_A)$ *such that*

$$\tilde{a} \in [p_A]_z,\ \tilde{p}_A \in [q_A]_z \ \textit{and}\ \hat{a} \in [p'_A]_z,\ \hat{p}_A \in [q'_A]_z$$

then $(z, \tilde{a}, \tilde{p}_A)$ *is revealed better than (resp. revealed strictly better than, resp. revealed indifferent to)* $(z, \hat{a}, \hat{p}_A)$*.*

In the way of notation, note that we will write $[p'_A]_z\ \widehat{\succcurlyeq}_z$ (resp. $\widehat{\succ}_z$, resp. $\widehat{\sim}_z$) $[p''_A]_z$ as a shorthand for $([p'_A]_z, [p'_A]_z)\ \widehat{\succcurlyeq}_z$ (resp. $\widehat{\succ}_z$, resp. $\widehat{\sim}_z$) $([p''_A]_z, [p''_A]_z)$.

A Topological Structure on $\Delta_A/{\sim_z}$

We next endow the sets $\Delta_A/{\sim_z}$, $z \in Z$, with a topology. For any $[p'_A]_z$, $[p''_A]_z \in \Delta_A/{\sim_z}$, let,

- $][p'_A]_z, [p''_A]_z[\ = \{[p_A]_z \in \Delta_A/{\sim_z} : p'_A \succ_z p_A \succ_z p''_A\}$,
- $][p'_A]_z, \rightarrow[\ = \{[p_A]_z \in \Delta_A/{\sim_z} : p_A \succ_z p'_A\}$, and
- $]\leftarrow, [p'_A]_z[\ = \{[p_A]_z \in \Delta_A/{\sim_z} : p'_A \succ_z p_A\}$.

Since $\succcurlyeq_z$ is a preference relation, it is natural to interpret these sets as preference intervals. Let $[q^{**}_A]_z$ and $[q^*_A]_z$ denote the maximal and minimal indifference classes respectively of $\succcurlyeq_z$ in $\Delta_A/{\sim_z}$, if such elements exist. That is,

$$[q^{**}_A]_z = \{p_A \in \Delta_A : p_A \succcurlyeq_z p'_A, \text{ for all } p'_A \in \Delta_A\},$$

and

$$[q_A^*]_z = \{p_A \in \Delta_A : p_A' \succcurlyeq_z p_A, \text{ for all } p_A' \in \Delta_A\},$$

If $[q_A^{**}]_z$ and/or $[q_A^*]_z$ exist, then for any $[p_A']_z \in \Delta_A/\sim_z$ we shall write,

$$][p_A']_z, \rightarrow [\ = \ ][p_A']_z, [q_A^{**}]_z], \text{ and }] \leftarrow, [p_A']_z[\ = [[q_A^*]_z, [p_A']_z[$$

We endow the set $\Delta_A/\sim_z$ with the *order topology* of $\succcurlyeq_z$, i.e., the coarsest topology containing all sets of the form $][p_A']_z, \rightarrow [$ and $] \leftarrow, [p_A']_z[$, thus all sets of the form $][p_A']_z, [p_A'']_z[$. We endow $[\Delta_A/\sim_z]^2$ with the product topology. A set of the type $C = I \times I' \subseteq [\Delta_A/\sim_z]^2$, where I and I' are of the form $][p_A']_z, [p_A'']_z[$, or $][p_A']_z, \rightarrow [$, or $] \leftarrow, [p_A']_z[$ shall be referred to as a **cube** in $[\Delta_A/\sim_z]^2$. Our strategy in the proof of the representation results below shall be to first establish that $\widehat{\succcurlyeq}_z$ is a weak order 'locally' on such cubes, and then to extend this 'globally' by 'tying together' these cubes. Observe that if C, $C' \subseteq [\Delta_A/\sim_z]^2$ are cubes, then so is $C \cap C'$, if the intersection happens to be non-empty. Further, if we can establish that $\widehat{\succcurlyeq}_z$ is a weak order on C and C', then *revealed consistency* implies that the derived rankings must coincide on $C \cap C'$.

2.3.2 Proof of Theorem 2.1

In this subsection we prove our baseline representation result. The proof of the Theorem proceeds through several lemmas. First consider the following remark.

Remark 2.3. *Our axioms (weak order, bi-continuity and comparable independence) imply that $\succcurlyeq_z$ satisfies the three axioms of the expected utility theorem; namely*

- Weak Order*: $\succcurlyeq_z$ is complete and transitive.*
- *:* vNM Continuity*: For any p_A, p_A', $p_A'' \in \Delta_A$, if $p_A \succ_z p_A' \succ_z p_A''$, then there exists λ, $\lambda' \in (0,1)$, such that*

$$\lambda p_A + (1-\lambda)p''_A \succ_z p'_A \succ_z \lambda' p_A + (1-\lambda')p''_A$$

- vNM Independence*: For any p_A, $p'_A \in \Delta_A$, if $p_A \succ_z p'_A$, then for any $p''_A \in \Delta_A$, $\lambda \in (0,1]$*

$$\lambda p_A + (1-\lambda)p''_A \succ_z \lambda p'_A + (1-\lambda')p''_A$$

Hence $\succcurlyeq_z$ can be represented by a von Neumann-Morgenstern utility function. That is, $\succcurlyeq_z$ admits a representation $v_z : \Delta_A \to \mathbb{R}$ that satisfies: $\forall\ p_A$, $q_A \in \Delta_A$, $\lambda \in [0,1]$,

$$v_z(\lambda p_A + (1-\lambda)q_A) = \lambda v_z(p_A) + (1-\lambda)v_z(q_A)$$

In the remainder of this proof, we will therefore refer to any $\succcurlyeq_z$ as a von Neumann-Morgenstern preference (vNM preference, for short).

We next define a family of mixture set structures on $\Delta_A/{\sim_z}$ and $[\Delta_A/{\sim_z}]^2$.

Mixture Sets: Consider any $\succcurlyeq_z$. For any $[p_A]_z$, $[q_A]_z \in \Delta_A/{\sim_z}$, and $\lambda \in [0,1]$, we define a unique element $\lambda[p_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[q_A]_z \in \Delta_A/{\sim_z}$ as follows:[4]

$$\lambda[p_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[q_A]_z = [\lambda p_A + (1-\lambda)q_A]_z$$

Note that $\widehat{\oplus}_z$ is well defined since $\succcurlyeq_z$ is a vNM preference, and so if we take any $p'_A \in [p_A]_z$ and $q'_A \in [q_A]_z$, then $\lambda p'_A + (1-\lambda)q'_A \in [\lambda p_A + (1-\lambda)q_A]_z$.

Further, for any $([p_A]_z, [q_A]_z)$, $([p'_A]_z, [q'_A]_z) \in [\Delta_A/{\sim_z}]^2$, and $\lambda \in [0,1]$, we define a unique element $\lambda([p_A]_z, [q_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p'_A]_z, [q'_A]_z) \in [\Delta_A/{\sim_z}]^2$ as follows:[5]

$$\lambda([p_A]_z, [q_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p'_A]_z, [q'_A]_z) = (\lambda[p_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[p'_A]_z, \lambda[q_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[q'_A]_z)$$

[4] Formally, $\widehat{\oplus}_z : \Delta_A/{\sim_z} \times \Delta_A/{\sim_z} \times [0,1] \to \Delta_A/{\sim_z}$.

[5] Formally, $\widehat{\oplus}_z : [\Delta_A/{\sim_z}]^2 \times [\Delta_A/{\sim_z}]^2 \times [0,1] \to [\Delta_A/{\sim_z}]^2$. Observe that we are abusing notation here by using the same notation $\widehat{\oplus}_z$ to denote 'mixture operations' on the sets $\Delta_A/{\sim_z}$ and $[\Delta_A/{\sim_z}]^2$. We do so because this should not cause any confusion, and it allows us to economize on notation.

That is,

$$\lambda([p_A]_z,\,[q_A]_z)\;\widehat{\oplus}_z\;(1-\lambda)([p'_A]_z,\,[q'_A]_z) = ([\lambda p_A + (1-\lambda)p'_A]_z,\,[\lambda q_A + (1-\lambda)q'_A]_z)$$

Any subset of $[\Delta_A/\sim_z]^2$ that is itself a mixture set shall be referred to as a mixture subset of $[\Delta_A/\sim_z]^2$. Note that because $\succcurlyeq_z$ satisfies the vN-M independence condition, any cube $C \subseteq [\Delta_A/\sim_z]^2$ is a mixture subset of $[\Delta_A/\sim_z]^2$. In addition note the following result about mixture subsets of $[\Delta_A/\sim_z]^2$. (The proof is standard, and hence omitted).

Lemma 2.1. *Every mixture subset of $[\Delta_A/\sim_z]^2$, in particular $[\Delta_A/\sim_z]^2$ itself, is connected.*

We shall now collect some useful notation to aid the exposition of the subsequent results. We shall denote the restriction of $\widehat{\succcurlyeq}_z$ to any set $\widehat{\Omega}$ in $[\Delta_A/\sim_z]^2$ by $(\widehat{\succcurlyeq}_z)_{\widehat{\Omega}}$. Further, let

$$int(\Delta_A/\sim_z) = \{[p_A]_z \in \Delta_A/\sim_z : [p_A]_z \neq [q_A^{**}]_z,\,[q_A^*]_z\}$$

$$D^* = \{([q_A]_z,\,[q_A]_z) \in [\Delta_A/\sim_z]^2 : [q_A]_z \in \Delta_A/\sim_z\}$$

$$D = \{([q_A]_z,\,[q_A]_z) \in [\Delta_A/\sim_z]^2 : [q_A]_z \in int(\Delta_A/\sim_z)\}$$

$$\Omega = \Delta_A/\sim_z \times int(\Delta_A/\sim_z), \text{ and } \Omega^* = \Omega \cup D^*.$$

Note that if $\widehat{\succcurlyeq}_z$ does not have any extremal elements then, $\Delta_A/\sim_z = int(\Delta_A/\sim_z)$ and $D^* = D$. In that case $D^* \subseteq int(\Delta_A/\sim_z) \times int(\Delta_A/\sim_z) = \Omega$ and so $\Omega^* = \Omega$.

Lemma 2.2. *Let $\succ_z \neq \emptyset$. For any $([p_A]_z,\,[q_A]_z) \in \Omega$ there exists a cube C containing $([p_A]_z,\,[q_A]_z)$ such that $\widehat{\succcurlyeq}_z$ restricted to C (denoted $(\widehat{\succcurlyeq}_z)_C$), satisfies the following.*

1. *Weak Order: $\widehat{\succcurlyeq}_z$ is complete and transitive on C.*

2. *vNM Continuity: Let* $([p_A]_z,\, [q_A]_z)$, $([p'_A]_z,\, [q'_A]_z)$, $([p''_A]_z,\, [q''_A]_z) \in C$ *be such that* $([p_A]_z,\, [q_A]_z)\ \widehat{\succ}_z\ ([p'_A]_z,\, [q'_A]_z)\ \widehat{\succ}_z\ ([p''_A]_z,\, [q''_A]_z)$*. Then there exists* $\lambda,\, \lambda' \in (0,1)$ *such that*

   $\lambda([p_A]_z,[q_A]_z)\ \widehat{\oplus}_z\ (1-\lambda)([p''_A]_z,[q''_A]_z)\ \widehat{\succ}_z\ ([p'_A]_z,[q'_A]_z)\ \widehat{\succ}_z\ \lambda'([p_A]_z,[q_A]_z)\ \widehat{\oplus}_z\ (1-\lambda')([p''_A]_z,[q''_A]_z)$.

3. *vNM Independence: Let* $([p_A]_z,\, [q_A]_z)$, $([p'_A]_z,\, [q'_A]_z) \in C$ *be such that* $([p_A]_z,\, [q_A]_z)\ \widehat{\succ}_z\ ([p'_A]_z,\, [q'_A]_z)$*. Then for any* $([p''_A]_z,\, [q''_A]_z) \in C$*,* $\lambda \in (0,1]$*,*

   $$\lambda([p_A]_z,[q_A]_z)\ \widehat{\oplus}_z\ (1-\lambda)([p''_A]_z,[q''_A]_z)\ \widehat{\succ}_z\ \lambda([p'_A]_z,[q'_A]_z)\ \widehat{\oplus}_z\ (1-\lambda)([p''_A]_z,[q''_A]_z).$$

4. *Monotonicity: for any* $([p'_A]_z,\, [q'_A]_z)$, $([p''_A]_z,\, [q''_A]_z) \in C$*,*

   $$[p'_A]_z\ \widehat{\succsim}_z\ [p''_A]_z \text{ and } [q'_A]_z\ \widehat{\succsim}_z\ [q''_A]_z \Rightarrow ([p'_A]_z,\, [q'_A]_z)\ \widehat{\succsim}_z\ ([p''_A]_z,\, [q''_A]_z).$$

5. *Non Degeneracy:* $\widehat{\succ}_z \neq \emptyset$*.*

Proof. We first consider the case of $([p_A]_z,\, [q_A]_z) \in \Omega$ for which $[p_A]_z \neq [q^{**}_A]_z$ or $[q^{*}_A]_z$.

- $(\widehat{\succsim}_z)_C$ is complete and transitive, for an appropriately defined cube C.

Pick any $([p_A]_z,\, [q_A]_z) \in \Omega$. There may be two possibilities. First, $p_A \nsim_z q_A$, and second $p_A \sim_z q_A$. For the first case assume without loss of generality that $p_A \succ_z q_A$. We can then find $a,\, a' \in A$ such that $a \sim_z p_A \succ_z q_A \succ_z a'$. The fact that we may find a as specified follows from the fact that any lottery in Δ_A has a certainty equivalent with respect to $\succsim_z$. On the other hand a' exists as specified because $([p_A]_z,\, [q_A]_z) \in \Omega$ and so $q_A \notin [q^*_A]_z$. Further, since $\succsim_z$ is a vNM preference, it follows that there exists $\lambda^* \in (0,1)$ such that,

$$[a,\, \lambda^*;\, a',\, 1-\lambda^*] \sim_z q_A$$

Now consider the case where, $p_A \sim_z q_A$. In this case pick a, $a' \in [q_A]_z$ (It is possible that $a = a'$). Then for any $\lambda \in [0,1]$, since $\succcurlyeq_z$ is a vNM preference, we have that

$$[a, \lambda; a', 1-\lambda] \sim_z q_A$$

In this case take any $\lambda^* \in (0,1)$. In either case therefore we can find a, $a' \in A$, and some $\lambda^* \in (0,1)$ such that the above preference indifference condition holds. Henceforth, without loss of generality, we shall consider $q_A = [a, \lambda^*;\, a', 1-\lambda^*]$.

We know by the assumption of *contingent values* that there exists $\succcurlyeq_{z'} \neq \succcurlyeq_z$, with $\succ_{z'} \neq \emptyset$, such that for an appropriate choice of a, a', there exists $\overline{a}$, $\underline{a}$ and $\overline{a}'$, $\underline{a}'$ that satisfy,

$$\overline{a} \sim_{z'} a \sim_{z'} \underline{a} \text{ and } \overline{a} \succ_z a \succ_z \underline{a},$$

$$\overline{a}' \sim_{z'} a' \sim_{z'} \underline{a}' \text{ and } \overline{a}' \succ_z a' \succ_z \underline{a}'$$

In particular, *bi-continuity* allows us to choose $\overline{a}$, $\underline{a}$ and $\overline{a}'$, $\underline{a}'$ in such a way that:

$$\overline{q}_A \equiv [\underline{a}, \lambda^*;\, \overline{a}', 1-\lambda^*] \succ_z q_A \succ_z [\overline{a}, \lambda^*;\, \underline{a}', 1-\lambda^*] \equiv \underline{q}_A.$$

We can now define the cube $C \subseteq \Omega$ that the statement of the lemma requires us to do. Define,

$$C =][\underline{a}]_z, [\overline{a}]_z[\times][\underline{q}_A]_z, [\overline{q}_A]_z[$$

Further, let,

$$I_a = \{\widehat{a} \in [a]_{z'} : \overline{a} \succcurlyeq_z \widehat{a} \succcurlyeq_z \underline{a}\}, \;\&\; I_{a'} = \{\widehat{a}' \in [a']_{z'} : \overline{a}' \succcurlyeq_z \widehat{a}' \succcurlyeq_z \underline{a}'\}.$$

Define a subset M of Δ as follows:

$$M = \{[(z, \widehat{a}), \lambda^*; (z', \widehat{a}'), 1-\lambda^*] \in \Delta : \widehat{a} \in I_a,\, \widehat{a}' \in I_{a'}\}.$$

Consider any $p' = [(z, \widehat{a}), \lambda^*; (z', \widehat{a}'), 1 - \lambda^*] \in M$. Since, $\widehat{a} \in I_a \subseteq [a]_{z'}$, $\widehat{a}' \in I_{a'} \subseteq [a']_{z'}$ and $\succcurlyeq_{z'}$ is a vNM preference, it follows that

$$p'_A = [\widehat{a}, \lambda^*; \widehat{a}', 1 - \lambda^*] \sim_{z'} [a, \lambda^*; a', 1 - \lambda^*] = q_A.$$

Therefore, for any p', $p'' \in M$,

$$[a(z', p')]_{z'} = [a(z', p'')]_{z'} = [a']_{z'} \text{ and } [p'_A]_{z'} = [p''_A]_{z'} = [q_A]_{z'}.$$

That is, any p', $p'' \in M$ are comparable at z, and accordingly if $p' \succ p''$, then the procedure-contingent outcome $(z, a(z, p'), p'_A)$ is *revealed strictly better* than the procedure-contingent outcome $(z, a(z, p''), p''_A)$, and if $p' \sim p''$, then $(z, a(z, p'), p'_A)$ is *revealed indifferent* to $(z, a(z, p''), p''_A)$. Hence,

$$p' \succ p'' \Rightarrow ([a(z, p')]_z, [p'_A]_z) \mathrel{\widehat{\succ}_z} ([a(z, p'')]_z, [p''_A]_z)$$

$$p' \sim p'' \Rightarrow ([a(z, p')]_z, [p'_A]_z) \mathrel{\widehat{\sim}_z} ([a(z, p'')]_z, [p''_A]_z)$$

Consider any $([\widehat{p}_A]_z, [\widehat{q}_A]_z) \in C$. Since, $\overline{a} \succ_z \widehat{p}_A \succ_z \underline{a}$, it follows that there exists $\widehat{a} \in I_a$ such that $\widehat{a} \sim_z \widehat{p}_A$.[6] Further, since $\succcurlyeq_z$ is a vNM preference, it follows that

$$[\widehat{a}, \lambda^*; \overline{a}', 1 - \lambda^*] \succ_z \overline{q}_A \succ_z \widehat{q}_A \succ_z \underline{q}_A \succ_z [\widehat{a}, \lambda^*; \underline{a}', 1 - \lambda^*].$$

It follows from *bi-continuity* that there exists $\widehat{a}' \in I_{a'}$ such that

$$\widehat{q}_A \sim_z [\widehat{a}, \lambda^*; \widehat{a}', 1 - \lambda^*].$$

That is for any $([\widehat{p}_A]_z, [\widehat{q}_A]_z) \in C$, there exists

$$p' = [(z, \widehat{a}), \lambda^*; (z', \widehat{a}'), 1 - \lambda^*] \in M$$

such that $\widehat{p}_A \sim_z a(z, p') = \widehat{a}$ and $\widehat{q}_A \sim_z p'_A = [\widehat{a}, \lambda^*; \widehat{a}', 1 - \lambda^*]$. Accordingly, $\widehat{\succcurlyeq}_z$ is a weak order on C.

[6] This follows since $[a]_{z'}$ is a connected subset of A. Note that

$$W_1 = \{\widetilde{a} \in [a]_{z'} : \widetilde{a} \succcurlyeq_z \widehat{p}_A\}, \,\&\, W_2 = \{\widetilde{a} \in [a]_{z'} : \widehat{p}_A \succcurlyeq_z \widetilde{a}\}$$

form a separation of $[a]_{z'}$, and hence their intersection must be nonempty.

- $(\widehat{\succcurlyeq}_z)_C$ satisfies vNM Continuity.

First we establish that for any $\widetilde{p}$, $\widehat{p} \in M$, any $\lambda \in [0,1]$, $\lambda\widetilde{p} \oplus (1-\lambda)\widehat{p} \in M$. Let

$$\widetilde{p} = [(z,\tilde{a}),\, \lambda^*;\, (z',\tilde{a}'),\, 1-\lambda^*] \;\&\; \widehat{p} = [(z,\hat{a}),\, \lambda^*;\, (z',\hat{a}'),\, 1-\lambda^*]$$

Further let $a_\lambda \in I_a$ be such that,

$$a_\lambda \sim_z [\tilde{a}, \lambda; \hat{a}, 1-\lambda].$$

Let,

$$\widetilde{q}_A = \lambda[\tilde{a},\, \lambda^*;\, \tilde{a}',\, 1-\lambda^*] + (1-\lambda)[\hat{a},\, \lambda^*;\, \hat{a}',\, 1-\lambda^*]$$

Since $\succcurlyeq_z$ is a vNM preference, it follows that

$$[a_\lambda,\, \lambda^*;\, \overline{a}',\, 1-\lambda^*] \succcurlyeq_z \overline{q}_A \succcurlyeq_z \widetilde{q}_A \succcurlyeq_z \underline{q}_A \succcurlyeq_z [a_\lambda,\, \lambda^*;\, \underline{a}',\, 1-\lambda^*],$$

with strict preference holding at least somewhere. *Bi-ontinuity* in conjunction with the fact the $[a']_{z'}$ is a connected subset of A implies that there exists, $a'_\lambda \in I_{a'}$, such that

$$[a_\lambda,\, \lambda^*;\, a'_\lambda,\, 1-\lambda^*] \sim_z \widetilde{q}_A.$$

Hence,

$$[(z,a_\lambda),\, \lambda^*;\, (z',a'_\lambda),\, 1-\lambda^*] = \lambda\widetilde{p} \oplus (1-\lambda)\widehat{p}.$$

We now establish that $(\widehat{\succcurlyeq}_z)_C$ satisfies the vN-M Continuity axiom. Note that this is equivalent to proving the following: For any p, p', $p'' \in M$ such that $p \succ p' \succ p''$, there exists λ, $\lambda' \in (0,1)$, such that:

$$\lambda p \oplus (1-\lambda)p'' \succ p' \succ \lambda' p \oplus (1-\lambda')p''$$

Suppose otherwise – say that $p' \succcurlyeq \lambda p \oplus (1-\lambda)p''$ for all $\lambda \in (0,1)$. We proved above that for all $\lambda \in [0,1]$ there exists $a_\lambda \in I_a$, $a'_\lambda \in I_{a'}$ such that,

$$[(z,\, a_\lambda),\, \lambda^*;\, (z',\, a'_\lambda);\, 1-\lambda^*] = \lambda p \oplus (1-\lambda)p''.$$

Denote,

$$p = [(z, \tilde{a}),\, \lambda^*;\, (z', \tilde{a}'),\, 1-\lambda^*]$$

We may then construct a sequence $(a_{\lambda_k},\, a'_{\lambda_k})_{k\in\mathbb{Z}_+} \subseteq I_a \times I_{a'}$ converging to $(\tilde{a},\, \tilde{a'}) \in I_a \times I_{a'}$, such that for all $k \in \mathbb{Z}_+$,

$$p' \succcurlyeq \lambda_k p \oplus (1-\lambda_k)p'' = [(z,\, a_{\lambda_k}),\, \lambda^*;\, (z',\, a'_{\lambda_k});\, 1-\lambda^*]$$

Let

$$\Xi = \{(a_{\lambda_k},\, a'_{\lambda_k}) \in I_a \times I_{a'} : p' \succcurlyeq [(z,\, a_{\lambda_k}),\, \lambda^*;\, (z',\, a'_{\lambda_k});\, 1-\lambda^*]\}$$

By *Bi-continuity* the set Ξ is closed in $I_a \times I_{a'}$. It then follows that $(\tilde{a},\, \tilde{a'}) \in \Xi$, that is $p' \succcurlyeq p = [(z, \tilde{a}),\, \lambda^*;\, (z', \tilde{a}'),\, 1-\lambda^*]$, which is absurd.

- $(\widehat{\succcurlyeq}_z)_C$ satisfies vNM Independence.

Now, let $([p^1_A]_z,\, [q^1_A]_z)$, $([p^2_A]_z,\, [q^2_A]_z)$, $([p^3_A]_z,\, [q^3_A]_z) \in C$ be such that $([p^1_A]_z,\, [q^1_A]_z) \mathrel{\widehat{\succ}_z} ([p^2_A]_z,\, [q^2_A]_z)$. We need to establish that for any $\lambda \in (0,1]$,

$$\lambda([p^1_A]_z, [q^1_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p^3_A]_z, [q^3_A]_z) \mathrel{\widehat{\succ}_z} \lambda([p^2_A]_z, [q^2_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p^3_A]_z, [q^3_A]_z).$$

We know from the analysis above that there exists $p,\, p',\, p'' \in M$ such that $([a(z,p)]_z, [p_A]_z) = ([p^1_A]_z,\, [q^1_A]_z)$, $([a(z,p')]_z,\, [p'_A]_z) = ([p^2_A]_z,\, [q^2_A]_z)$, $([a(z,p'')]_z\, , [p''_A]_z) = ([p^3_A]_z, [q^3_A]_z)$, and $p \succ p'$. By *comparable independence*, it follows that for any $\lambda \in (0,1]$

$$\lambda p \oplus (1-\lambda)p'' \succ \lambda p' \oplus (1-\lambda)p''$$

Accordingly, it follows that

$$\begin{aligned}([a(z, \lambda p \oplus (1-\lambda)p'')]_z, [(\lambda p \oplus (1-\lambda)p'')_A]_z) &\mathrel{\widehat{\succ}_z} \\ ([a(z, \lambda p' \oplus (1-\lambda)p'')]_z, [(\lambda p' \oplus (1-\lambda)p'')_A]_z).\end{aligned}$$

Or,

$$([\lambda a(z,p) + (1-\lambda)a(z,p'')]_z, [\lambda p_A + (1-\lambda)p''_A]_z) \mathrel{\widehat{\succ}_z} ([\lambda a(z,p') + (1-\lambda)a(z,p'')]_z,\ [\lambda p'_A + (1-\lambda)p''_A]_z).$$

That is,

$$(\lambda[a(z,p)]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[a(z,p'')]_z,\ \lambda[p_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[p''_A]_z) \mathrel{\widehat{\succ}_z} (\lambda[a(z,p')]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[a(z,p'')]_z,\ \lambda[p'_A]_z \mathbin{\widehat{\oplus}_z} [p''_A]_z).$$

Or,

$$(\lambda[p^1_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[p^3_A]_z,\ \lambda[q^1_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[q^3_A]_z) \mathrel{\widehat{\succ}_z} (\lambda[p^2_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[p^3_A]_z, \lambda[q^2_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[q^3_A]_z).$$

Or,

$$\lambda([p^1_A]_z, [q^1_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p^3_A]_z, [q^3_A]_z) \mathrel{\widehat{\succ}_z} \lambda([p^2_A]_z, [q^2_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p^3_A]_z, [q^3_A]_z).$$

Hence, $(\widehat{\succsim}_z)_C$ satisfies the vN-M Independence axiom.

- $(\widehat{\succsim}_z)_C$ satisfies Monotonicity.

This follows immediately from *conditional monotonicity*

- $(\widehat{\succsim}_z)_C$ is Non Degenerate.

This follows immediately from the assumption made in the lemma that $\succ_z \neq \emptyset$.

The proof for the case when $[p_A]_z$ is equal to either $[q^{**}_A]_z$, $[q^*_A]_z$ is exactly along similar lines. When $[p_A]_z = [q^{**}_A]_z$, take $\overline{a} = a$ in the above proof, and define the cube C as follows:

$$C = \,][\underline{a}]_z,\ [a]_z] \times\,][\underline{q}_A]_z,\ [\overline{q}_A]_z[\ .$$

The rest of the details are exactly identical. Similarly, when $[p_A]_z = [q^*_A]_z$, take $\underline{a} = a$ in the above proof, and define

$$C = [[a]_z,\, [\overline{a}]_z[\; \times\;][\underline{q}_A]_z,\, [\overline{q}_A]_z[\ .$$

$\square$

Lemma 2.3. $(\widehat{\succsim}_z)_{\Omega^*}$ *is a weak order. Further, there exists*

(i) a function $v_z : \Delta_A \to \mathbb{R}$ *that represents* $\succsim_z$ *and satisfies: for all* $\lambda \in [0,1]$, p_A, q_A $\in \Delta_A$,

$$v_z(\lambda p_A + (1-\lambda)q_A) = \lambda v_z(p_A) + (1-\lambda)v_z(q_A),\ \textit{and}$$

(ii) a constant $\sigma_z \in [0,\, 1]$,

such that the function $V_z : \Omega^* \to \mathbb{R}$ *given by*

$$V_z([p_A]_z,\, [q_A]_z) = (1-\sigma_z)v_z(p_A) + \sigma_z v_z(q_A)$$

represents $(\widehat{\succsim}_z)_{\Omega^*}$. *Further, another pair* $(\widetilde{v}_z,\, \widetilde{\sigma}_z)$ *represents* $(\widehat{\succsim}_z)_{\Omega^*}$ *in the above sense iff* $\widetilde{v}_z$ *is a positive affine transformation of* v_z *and* $\widetilde{\sigma}_z = \sigma_z$, *for all* $z \in Z$ *such that* $\succ_z \neq \emptyset$.

Proof. First consider those $z \in Z$ for which $\succ_z \neq \emptyset$. From Lemma 2.2 it follows that for any $([q_A]_z,\, [q_A]_z) \in D$, there exists a cube containing $([q_A]_z,\, [q_A]_z)$, which we can take to be

$$C_{[q_A]_z} =][\underline{q}_A]_z,\, [\overline{q}_A]_z[\; \times\;][\underline{q}_A]_z,\, [\overline{q}_A]_z[\; \subseteq [\Delta_A/{\sim_z}]^2$$

such that $\widehat{\succsim}_z$ restricted to $C_{[q_A]_z}$ satisfies the five axioms of the Anscombe Aumann Theorem (for finite states) – *weak order*, *vN-M continuity*, *vN-M independence*, *monotonicity* and *non-degeneracy*. It follows that there exists a function $v^{q_A}_z :][\underline{q}_A]_z,\, [\overline{q}_A]_z[\to \mathbb{R}$ that is unique up to positive affine transformation, and a constant $\sigma^{q_A}_z \in [0,\, 1]$ that is unique, such that the function $V^{q_A}_z : C_{[q_A]_z} \to \mathbb{R}$ defined by,

$$V_z^{q_A}([p'_A]_z,\, [q'_A]_z) = (1 - \sigma_z^{q_A})v_z^{q_A}(p'_A) + \sigma_z^{q_A}v_z^{q_A}(q'_A)$$

represents $(\widehat{\succsim}_z)_{C_{[q_A]_z}}$. Thats is for all $([p'_A]_z,\, [q'_A]_z),\, ([p''_A]_z,\, [q''_A]_z) \in C_{[q_A]_z}$,

$$([p'_A]_z,\, [q'_A]_z)\ \widehat{\succsim}_z\ ([p''_A]_z,\, [q''_A]_z) \text{ if and only if } V_z^{q_A}([p'_A]_z,\, [q'_A]_z) \geq V_z^{q_A}([p''_A]_z,\, [q''_A]_z)$$

Further note that the function v_z satisfies: for all $\lambda \in [0,1]$, $[p_A]_z, [p'_A]_z \in][\underline{q_A}]_z, [\overline{q_A}]_z[$,

$$v_z(\lambda[p_A]_z\ \widehat{\oplus}_z\ (1-\lambda)[p'_A]_z) = \lambda v_z([p_A]_z) + (1-\lambda)v_z([p'_A]_z).$$

In addition, for any $([p'_A]_z,\, [q'_A]_z) \in C_{[q_A]_z}$, there exists $[\widehat{q}_A]_z \in][\underline{q}_A]_z, [\overline{q}_A]_z[$ such that $([p'_A]_z,\, [q'_A]_z)\ \widehat{\sim}_z\ ([\widehat{q}_A]_z,\, [\widehat{q}_A]_z)$.

Note that $\widehat{\succsim}_z$ restricted to D^* is complete. This follows since, any two degenerate lotteries like $[(z,a),1]$ and $[(z,a'),1]$ are comparable at z, and accordingly

$$([a]_z,\, [a]_z)\ \widehat{\succ}_z\ ([a']_z,\, [a']_z) \text{ if } (z,a) \succ_z (z,a'),$$

or,

$$([a]_z,\, [a]_z)\ \widehat{\sim}_z\ ([a']_z,\, [a']_z) \text{ if } (z,a) \sim_z (z,a').\ ^7$$

Now define $O = (\cup_{[q_A]_z \in D}\ C_{[q_A]_z}) \cup D^*$. We will next show that $\widehat{\succsim}_z$ restricted to O is a weak order. Pick any $([p'_A]_z,\, [q'_A]_z) \in C_{[q_A]_z}$, $([p''_A]_z,\, [q''_A]_z) \in C_{[p_A]_z}$. We know that there exists $[\widehat{q}_A]_z, [\widehat{p}_A]_z \in \Delta_A/\sim_z$ such that

$$([p'_A]_z,\, [q'_A]_z)\ \widehat{\sim}_z\ ([\widehat{q}_A]_z,\, [\widehat{q}_A]_z) \text{ and } ([p''_A]_z,\, [q''_A]_z)\ \widehat{\sim}_z\ ([\widehat{p}_A]_z,\, [\widehat{p}_A]_z).$$

Accordingly, it follows that

$$([p'_A]_z,\, [q'_A]_z)\ \widehat{\succ}_z\ ([p''_A]_z,\, [q''_A]_z) \text{ if } ([\widehat{q}_A]_z,\, [\widehat{q}_A]_z)\ \widehat{\succ}_z\ ([\widehat{p}_A]_z,\, [\widehat{p}_A]_z),$$

or,

$$([p'_A]_z,\, [q'_A]_z)\ \widehat{\sim}_z\ ([p''_A]_z,\, [q''_A]_z) \text{ if } ([\widehat{q}_A]_z,\, [\widehat{q}_A]_z)\ \widehat{\sim}_z\ ([\widehat{p}_A]_z,\, [\widehat{p}_A]_z).$$

[7] Note that $\{[a]_z \in \Delta_A/\sim_z\, :\, a \in A\} = \Delta_A/\sim_z$.

Hence, $(\widehat{\succsim}_z)_O$ is a weak order.

Now consider any two cubes $C_{[q_A]_z}$ and $C_{[p_A]_z}$ that intersect. Pick $([q'_A]_z,\ [q'_A]_z)$, $([q''_A]_z,\ [q''_A]_z) \in C_{[q_A]_z} \cap C_{[p_A]_z}$, $[q'_A]_z \neq [q''_A]_z$, and recalibrate the function $v_z^{p_A}$ by setting

$$v_z^{p_A}([q'_A]_z) = v_z^{q_A}([q'_A]_z) \text{ and } v_z^{p_A}([q''_A]_z) = v_z^{q_A}([q''_A]_z)$$

Note that by the uniqueness result of the Anscombe Aumann Theorem, the pair $(v_z^{p_A},\ \sigma_z^{p_A})$ continues to represent $(\widehat{\succsim}_z)_{C_{[p_A]}}$. Further, $v_z^{p_A} = v_z^{q_A}$ on $][\underline{p}_A]_z,\ [\overline{p}_A]_z[\ \cap\ ][\underline{q}_A]_z,\ [\overline{q}_A]_z[$. Hence it follows that $\sigma_z^{p_A} = \sigma_z^{q_A}$. Next consider $[q_A]_z$, $[p_A]_z$ such that cubes $C_{[q_A]_z}$ and $C_{[p_A]_z}$ do not intersect. Since the set D is connected, $([q_A]_z,\ [q_A]_z)$ and $([p_A]_z,\ [p_A]_z)$ can be *linked* by finitely many cubes; that is there are finitely many cubes $C_{[p_A^1]_z}, \ . \ . \ . \ , C_{[p_A^m]_z}$, such that $C_{[p_A^1]_z} = C_{[q_A]_z}$, $C_{[p_A^m]_z} = C_{[p_A]_z}$, and each subsequent pairs of $C_{[p_A^j]_z}$'s intersect. Further, we can take $C_{[p_A^j]_z} \cap C_{[p_A^{j-k}]_z} = \emptyset$ for every $k \geq 2$. We can then repeat the above re-calibration exercise over pairs of intersecting cubes in the link. This exercise allows us to define a function v_z on $int(\Delta_A/\sim_z)$, as well as establish $\sigma_z^{q_A} = \sigma_z^{p_A} = \sigma_z$, for all $q_A \neq p_A$, $[q_A]_z$, $[p_A]_z \in int(\Delta_A/\sim_z)$. Finally, for $[\overline{p}_A]_z = [q_A^{**}]_z$, or $[q_A^*]_z$ define

$$v_z([\overline{p}_A]_z) = \lim_{\lambda \to 1} v_z(\lambda[\overline{p}_A]_z \widehat{\oplus}_z (1-\lambda)[p_A]_z),$$

where $[p_A]_z$ is any element of $int(\Delta_A/\sim_z)$.

We next establish the following claim: for any $([p_A]_z,\ [q_A]_z) \in \Omega^*$ there exists $([p'_A]_z,\ [p'_A]_z)$ in D^* such that $([p_A]_z,\ [q_A]_z) \mathrel{\widehat{\sim}}_z ([p'_A]_z,\ [p'_A]_z)$. To that end, define the function $V_z : \Omega^* \to \mathbb{R}$ by

$$V_z([p_A]_z,\ [q_A]_z) = (1-\sigma_z)v_z([p_A]_z) + \sigma_z v_z([q_A]_z)$$

where v_z and σ_z are as defined above. For any $[\widehat{q}_A]_z \in int(\Delta_A/\sim_z)$, let

$$J_{\widehat{q}_A} = \{([p_A]_z, [q_A]_z) \in \Omega : V_z([p_A]_z, [q_A]_z) = V_z([\widehat{q}_A]_z, [\widehat{q}_A]_z)\}$$

We claim that for all $([p_A]_z, [q_A]_z), ([p'_A]_z, [q'_A]_z) \in J_{\widehat{q}_A}$, $([p_A]_z, [q_A]_z) \widehat{\sim}_z ([p'_A]_z, [q'_A]_z)$. To see this note that, Lemma 2.2 guarantees that for any $([p_A]_z, [q_A]_z) \in J_{\widehat{q}_A}$, there exists a cube C containing $([p_A]_z, [q_A]_z)$ such that $(\widehat{\succcurlyeq}_z)_C$ satisfies the three vN-M axioms of Weak Order, vNM Continuity and Independence on the mixture set $(C, \widehat{\oplus}_z)$. Accordingly $(\widehat{\succcurlyeq}_z)_C$ can be represented by a von Neumann-Morgenstern utility function. Consider two such cubes C_1 and C_2 that intersect. Because of the axiom of *revealed consistency*, it follows that for any $([p_A]_z, [q_A]_z), ([p'_A]_z, [q'_A]_z) \in C_1 \cap C_2$,

$$([p_A]_z, [q_A]_z)\ (\widehat{\succcurlyeq}_z)_{C_1}\ ([p'_A]_z, [q'_A]_z) \text{ iff } ([p_A]_z, [q_A]_z)\ (\widehat{\succcurlyeq}_z)_{C_2}\ ([p'_A]_z, [q'_A]_z).$$

Further note that if V_{C_1} and V_{C_2} are two vN-M utility functions that represent $(\widehat{\succcurlyeq}_z)_{C_1}$ and $(\widehat{\succcurlyeq}_z)_{C_2}$ respectively, these functions can be re-calibrated (in a manner similar to that used in Step 2) and set equal on $C_1 \cap C_2$.

Now, consider the cube $C_{[\widehat{q}_A]_z}$ around $([\widehat{q}_A]_z, [\widehat{q}_A]_z)$. We have already established above that $(\widehat{\succcurlyeq}_z)_{C_{[\widehat{q}_A]_z}}$ is represented by the function V_z. Further, $J_{\widehat{q}_A}$ is connected. Accordingly, $([\widehat{q}_A]_z, [\widehat{q}_A]_z)$ can be *linked* to any $([p_A]_z, [q_A]_z) \in J_{\widehat{q}_A}$ using a finite number of cubes. On each pair of intersecting cubes $\widehat{\succcurlyeq}_z$ must coincide as suggested in the last paragraph. Furthermore the vN-M representations of $\widehat{\succcurlyeq}_z$ on these cubes can be re-calibrated and brought in line with V_z. Hence, we may conclude that for all $([p_A]_z, [q_A]_z), ([p'_A]_z, [q'_A]_z) \in J_{\widehat{q}_A}$, $([p_A]_z, [q_A]_z) \widehat{\sim}_z ([p'_A]_z, [q'_A]_z)$.

Note that if $\sigma_z \neq 1$, or if $[q_A^{**}]_z$ and $[q_A^{*}]_z$ do not exist, then we are done establishing our claim. However, if $\sigma_z = 1$, and either $[q_A^{**}]_z$ or $[q_A^{*}]_z$ exists then members of the set

$$B = \{([p_A]_z,\, [q_A]_z) \in \Omega : [p_A]_z = [q_A^{**}]_z \text{ or } [q_A^*]_z\}$$

are not indifferent to any element of D. In this case it is straightforward to verify that for any $([q_A^{**}]_z,\, [q_A]_z) \in B$, $([q_A^{**}]_z,\, [q_A]_z) \,\widehat{\sim}_z\, ([q_A^{**}]_z,\, [q_A^{**}]_z)$. Similarly, for any $([q_A^{*}]_z,\, [q_A]_z) \in B$, $([q_A^{*}]_z,\, [q_A]_z) \,\widehat{\sim}_z\, ([q_A^{*}]_z,\, [q_A^{*}]_z)$.

Now consider any $([p_A]_z,\, [q_A]_z)$, $([p'_A]_z,\, [q'_A]_z) \in \Omega^*$. From the argument just made, we know that there exists $([\widehat{q}_A]_z,\, [\widehat{q}_A]_z)$, $([\widetilde{q}_A]_z,\, [\widetilde{q}_A]_z) \in D^*$, such that $([p_A]_z,\, [q_A]_z) \,\widehat{\sim}_z\, ([\widehat{q}_A]_z,\, [\widehat{q}_A]_z)$ and $([p'_A]_z,\, [q'_A]_z) \,\widehat{\sim}_z\, ([\widetilde{q}_A]_z,\, [\widetilde{q}_A]_z)$. Hence,

$$([p_A]_z,\, [q_A]_z) \,\widehat{\succsim}_z\, ([p'_A]_z,\, [q'_A]_z) \text{ iff } ([\widehat{q}_A]_z,\, [\widehat{q}_A]_z) \,\widehat{\succsim}_z\, ([\widetilde{q}_A]_z,\, [\widetilde{q}_A]_z).$$

Clearly it also follows that,

$$([p_A]_z,\, [q_A]_z) \,\widehat{\succsim}_z\, ([p'_A]_z,\, [q'_A]_z) \Leftrightarrow V_z(([p_A]_z, [q_A]_z)) \geq V_z(([p'_A]_z, [q'_A]_z)).$$

Note that we may (with an abuse of notation) define the function v_z on Δ_A by simply giving all elements of an equivalence class, say $[p_A]_z$, the value $v_z([p_A]_z)$. It then follows that for all $\lambda \in [0, 1]$, p_A, $q_A \in \Delta_A$,

$$v_z(\lambda p_A + (1 - \lambda) q_A) = \lambda v_z(p_A) + (1 - \lambda) v_z(q_A).$$

The uniqueness statement is simply a re-statement of the essential uniqueness result in the first half of the proof. This then completes the proof for those $z \in Z$ for which $\succ_z \neq \emptyset$.

The proof for those $z \in Z$ for which $\succ_z = \emptyset$ is trivial. Note that for this case $[\Delta_A/\sim_z \times \Delta_A/\sim_z]$ is a singleton. We can take v_z to be any constant function, and σ_z to be any number in [0, 1]. □

Remark 2.4. *Note that the function $v_z : \Delta_A \to \mathbb{R}$ in a von-Neumann Morgenstern utility representation of the preference relation $\succsim_z$. We know that if there is some other function $\hat{v}_z$ that also happens to be a vN-M utility representation of $\succsim_z$, then v_z*

and $\hat{v}_z$ must be positive affine transformations of one another. This is a fact that we shall draw on below.

We next establish that any procedure-contingent outcome (z, a, p_A) is revealed indifferent to a procedure contingent outcome $(z, \widetilde{a}, \widetilde{a})$ in which the outcome and procedure are the same. The result follows immediately when we combine the conclusion of the last lemma with the following one.

Lemma 2.4. *Suppose $\succcurlyeq_z$ satisfies stochastic dominance and vN-M continuity. If $\succ_z$ $\neq \emptyset$, then there does not exist $p \in \Delta$ such that (a) $p_A \succ_z p_{A,z}$ and p_A is the maximal element of $\succcurlyeq_z$, or (b) $p_{A,z} \succ_z p_A$ and p_A is the minimal element of $\succcurlyeq_z$.*

Proof. Consider any $q_A \in \Delta_A$. Note the following mutually exclusive possibilities:
[A] $a \sim_z a'$ for all a, a' in the support of q_A: In this case it must be that $q_A \sim_z a$ for any a in the support of q_A. To see this assume otherwise – say $q_A \succ_z a$. Given that $\succ_z \neq \emptyset$, it follows that there exists a'' satisfying $a'' \succ_z a$ or $a \succ_z a''$. Assume it is the former.[8] Note that a'' first order stochastically dominates q_A; hence we have: $a'' \succ q_A \succ_z a$. But then by vN-M Continuity, there exists $\lambda \in (0, 1)$, such that $q_A \succ_z \lambda a''$ + $(1 - \lambda)a'$. But the lottery $\lambda a''$ + $(1 - \lambda)a'$ first order stochastically dominates q_A!
[B] $a \succ_z a'$ for some a, a' in the support of q_A: In this case there exists $\overline{a}$, $\underline{a}$ in the support of q_A such that $\overline{a} \succ_z q_A \succ_z \underline{a}$. This claim is again easily established by appealing to the fact that $\succcurlyeq_z$ satisfies stochastic dominance; so we omit the details here.
Now we proceed to prove the Lemma. Suppose there exists $p \in \Delta$ such that $p_A \succ_z$ $p_{A,z}$ and p_A is the maximal element of $\succcurlyeq_z$. Then we know from above that there exists a in the support of $p_{A,z}$ such that $p_{A,z} \succcurlyeq_z a$. Hence, $p_A \succ_z a$. Clearly, a is in the support of p_A. This implies (following Case B above) that there exists a' in

[8] The latter case is dealt analogously

the support of p_A such that $a' \succ_z p_A$. But this contradicts that p_A is the maximal element of $\succcurlyeq_z$. The case of $p_{A,z} \succ_z p_A$ and p_A is the minimal element of $\succcurlyeq_z$ not being possible can be handled analogously. $\square$

The last Lemma together with Lemma 2.3 allow us to conclude:

Lemma 2.5. *For any $p \in \Delta$, and (z, a) in the support of p, the procedure contingent outcome (z,a,p_A) is revealed indifferent to a procedure contingent outcome $(z, \widetilde{a}, \widetilde{a}) \in \Delta$ that is unique in the following sense: if $(z, \widehat{a}, \widehat{a})$ is another procedure contingent outcome that is revealed indifferent to (z,a,p_A), then $\widetilde{a} \sim_z \widehat{a}$. Further, there exists a function $v_z : \Delta_A \to \mathbb{R}$, and a constant $\sigma_z \in [0, 1]$ such that*

$$v_z(\widetilde{a}) = (1 - \sigma_z)v_z(a) + \sigma_z v_z(p_A)$$

The function v_z is unique up to positive affine transformation, and the constant σ_z is unique for all z such that $\succ_z \neq \emptyset$.

Proof. Lemma 2.3 and Lemma 2.4 allows us to conclude that there exists $[q_A]_z \in \Delta_A/{\sim_z}$ such that

$$([a]_z, [p_A]_z) \mathrel{\widehat{\approx}_z} ([q_A]_z, [q_A]_z).$$

Further, we know that there exists $\widetilde{a} \in A$ such that $\widetilde{a} \sim_z q_A$. Thus,

$$([a]_z, [p_A]_z) \mathrel{\widehat{\approx}_z} ([\widetilde{a}]_z, [\widetilde{a}]_z).$$

and it follows from Remark 2.2 that the procedure-contingent outcome (z,a,p_A) is revealed indifferent to the procedure contingent outcome $(z, \widetilde{a}, \widetilde{a})$. $\square$

The following corollary then follows.

Corollary 2.1. *Every lottery $p \in \Delta$ has a procedure-adjusted equivalent $\pi(p) \in \Delta$.*

We now state three intermediary results that leads us towards our representation. First, by *boundedness* it follows that there exists $a^* \in A$, such that for all $p \in \Delta$ there exists $\overline{z}$, $\underline{z} \in Z$ satisfying $(\overline{z}, a^*) \succsim p \succsim (\underline{z}, a^*)$. This fact allows us to prove the following lemma.

Lemma 2.6. *For any $p \in \Delta$, there exists $z^*(p) \in Z$ such that $p \sim (z^*(p), a^*)$.*

Proof. Pick any $p \in \Delta$. As stated above, there exists $a^* \in A$, such that for all $p \in \Delta$ there exists $\overline{z}$, $\underline{z} \in Z$ satisfying $(\overline{z}, a^*) \succsim p \succsim (\underline{z}, a^*)$. If either of those preferences is an indifference, then we are done. So assume that $(\overline{z}, a^*) \succ p \succ (\underline{z}, a^*)$. Then, since $\succsim_{a^*}$ satisfies vNM continuity, it follows that there exists λ', $\lambda'' \in (0,\, 1)$, $\lambda' > \lambda''$, such that

$$\lambda'(\overline{z}, a^*) + (1 - \lambda')(\underline{z}, a^*) \succ p \succ \lambda''(\overline{z}, a^*) + (1 - \lambda'')(\underline{z}, a^*).$$

Let,

$$\Lambda = \{\lambda \in [0,\, 1] : p \succ \lambda(\overline{z}, a^*) + (1 - \lambda)(\underline{z}, a^*)\}$$

and let $\lambda^* = sup\ \Lambda$. We claim that $p \sim \lambda^*(\overline{z}, a^*) + (1 - \lambda^*)(\underline{z}, a^*)$. To see this, suppose otherwise.

First, suppose that $p \succ \lambda^*(\overline{z}, a^*) + (1 - \lambda^*)(\underline{z}, a^*)$. This implies that $\lambda^* \in \Lambda$. Note that,

$$\lambda'(\overline{z}, a^*) + (1 - \lambda')(\underline{z}, a^*) \succ p \succ \lambda^*(\overline{z}, a^*) + (1 - \lambda^*)(\underline{z}, a^*)$$

This implies that there exists $\lambda \in (0,\, 1)$, such that letting $\widetilde{\lambda} = \lambda\lambda' + (1 - \lambda)\lambda^*$, we have by vNM continuity of $\succsim_{a^*}$ that

$$p \succ \widetilde{\lambda}(\overline{z}, a^*) + (1 - \widetilde{\lambda})(\underline{z}, a^*).$$

But note that $\lambda' > \lambda^*$, and hence $\widetilde{\lambda} > \lambda^*$. But at the same time $\widetilde{\lambda} \in \Lambda$, which contradicts the fact that $\lambda^* = \sup \Lambda$

Next, suppose that $\lambda^*(\overline{z}, a^*) + (1 - \lambda^*)(\underline{z}, a^*) \succ p$. Then by vNM continuity of $\succsim_{a^*}$, there exists $\lambda \in (0, 1)$ such that letting $\widehat{\lambda} = \lambda\lambda^* + (1 - \lambda)\lambda''$,

$$\widehat{\lambda}(\overline{z}, a^*) + (1 - \widehat{\lambda})(\underline{z}, a^*) \succ p$$

It follows that $\widehat{\lambda}$ is an upper bound of Λ. But at the same time, since $\lambda^* > \lambda''$, $\lambda^* > \widehat{\lambda}$, which contradicts the fact that $\lambda^* = \sup \Lambda$. Finally note that there exists $z^* \in Z$ such that $(z^*, a^*) \sim (\lambda^*\overline{z} + (1 - \lambda^*)\underline{z}, a^*) = \lambda^*(\overline{z}, a^*) + (1 - \lambda^*)(\underline{z}, a^*)$, which allows us to establish the claim of the lemma. $\square$

Let,

$$p = [(z^1, a^1), p(z^1, a^1); \; . \; . \; . \; ,(z^K, a^K), p(z^K, a^K)] \in \Delta,$$

and let,

$$\pi(p) = [(z^1, \tilde{a}^1), p(z^1, a^1); \; . \; . \; . \; ,(z^K, \tilde{a}^K), p(z^K, a^K)] \in \Delta$$

be its procedure-adjusted equivalent. We know from Lemma 2.6 that for each $(z^k, \tilde{a}^k)$, there exists $(z^*(z^k, \tilde{a}^k), a^*) \in Z \times A$ such that $(z^k, \tilde{a}^k) \sim (z^*(z^k, \tilde{a}^k), a^*)$. The axiom of *dominance* then allows us to prove the following lemma.

Lemma 2.7. *Let,*

$$p = [(z^1, a^1), p(z^1, a^1); \; . \; . \; . \; ,(z^K, a^K), p(z^K, a^K)] \in \Delta,$$

and let,

$$\pi(p) = [(z^1, \tilde{a}^1), p(z^1, a^1); \; . \; . \; . \; ,(z^K, \tilde{a}^K), p(z^K, a^K)] \in \Delta$$

be its procedure-adjusted equivalent. Then,

$$p \sim [(z^*(z^1, \tilde{a}^1), a^*), p(z^1, a^1), \; . \; . \; . \; , \; (z^*(z^K, \tilde{a}^K), a^*), p(z^K, a^K)].$$

Proof. Observe that if the support of p is singleton, that is, p is a degenerate lottery, then the conclusion follows immediately from Lemma 2.6. So assume otherwise. Further, denote

$$q^* \equiv [(z^*(z^1, \tilde{a}^1), a^*), p(z^1, a^1), \ . \ . \ . \ , (z^*(z^K, \tilde{a}^K), a^*), p(z^K, a^K)],$$

and suppose towards a contradiction that $p \nsim q^*$ – say $p \succ q^*$ (The case of $q^* \succ p$ is treated analogously). Suppose first that there exists (z, a^*), (z', a^*) in the support of q^*, denoted $S[q^*]$, such that $(z, a^*) \succ (z', a^*)$. Let $(z'', a^*) \in S[q^*]$ be such that $(z'', a^*) \succcurlyeq (z, a^*)$ for all $(z, a^*) \in S[q^*]$. *Dominance* then implies that $(z'', a^*) \succ p$, and so $(z'', a^*) \succ p \succ q^*$. By vNM continuity of $\succcurlyeq_{a^*}$ it follows that there exists $\lambda \in (0,\ 1)$ such that $p \succ \lambda(z'', a^*) + (1 - \lambda)q^*$. By *dominance* though, $\lambda(z'', a^*) + (1 - \lambda)q^* \succ p$, which is absurd.

Next consider the case where $(z, a^*) \sim (z', a^*)$ for all (z, a^*), $(z', a^*) \in S[q^*]$. We know (from continuity) that for any $p \in \Delta$ there exists $(z'', a^*) \in Z \times A$ such that $(z'', a^*) \succcurlyeq p \succ q^*$. It follows that there exists $\lambda \in (0, 1)$ such that $p \succ \lambda(z'', a^*) + (1 - \lambda)q^*$. But by *dominance* $\lambda(z'', a^*) + (1 - \lambda)q^* \succ p$, which is absurd. □

Finally, the following Lemma is an immediate consequence of *comparable independence*. The proof is straightforward and we omit the details.

Lemma 2.8. *Let* $(z,\ p_A)$, $(z,\ q_A) \in \Delta$. *Then, for any* $\lambda \in [0, 1]$,

$$\lambda(z, p_A) + (1 - \lambda)(z, q_A) \sim \lambda(z^*(z, p_A), a^*) + (1 - \lambda)(z^*(z, q_A), a^*),$$

where $(z^*(z, p_A), a^*)$, $(z^*(z, q_A), a^*) \in Z \times A$ *are such that*

$$(z^*(z, p_A), a^*) \sim (z, p_A) \text{ and } (z^*(z, q_A), a^*) \sim (z, q_A).$$

Now define,

$$\Delta_{a^*} = \{(p_Z, a^*) \in \Delta : p_Z \in \Delta_Z\}.$$

and let $\succcurlyeq^*$ be the restriction of $\succcurlyeq$ to Δ_{a^*}. Note that our axioms imply that $\succcurlyeq^*$ satisfies the axioms of the expected utility theorem, and so it follows that there exists an expected utility functional $W : \Delta_{a^*} \to \mathbb{R}$ that represents $\succcurlyeq^*$. Further, following Lemma 2.6 we may 'extend' the function to the whole of Δ. Define $W : \Delta \to \mathbb{R}$ as

$$W(p) = W(z^*(p), a^*)$$

where $(z^*(p), a^*) \in Z \times A$ is such that $p \sim (z^*(p), a^*)$. Clearly, the function W represents $\succcurlyeq$.

Let,

$$\pi(p) = [(z^1, \tilde{a}^1), p(z^1, a^1); \ . \ . \ . \ ,(z^K, \tilde{a}^K), p(z^K, a^K)] \in \Delta$$

denote the procedure-adjusted equivalent of

$$p = [(z^1, a^1), p(z^1, a^1); \ . \ . \ . \ ,(z^K, a^K), p(z^K, a^K)] \in \Delta.$$

Applying Lemma 2.7 then gives us that

$$W(p) = \sum_{k=1}^{K} p(z^k, a^k) W(z^*(z^k, \tilde{a}^k), a^*) = \sum_{k=1}^{K} p(z^k, a^k) W(z^k, \tilde{a}^k)$$

Next, define the function $w : Z \times \Delta_A \to \mathbb{R}$ as follows: for any $p = (z, p_A) \in \Delta$, let

$$w(z, p_A) = W(p)$$

For any $z \in Z$, it follows from Lemma 2.8 that the function $w(z, .) : \Delta_A \to \mathbb{R}$ is a von Neumann-Morgenstern representation of $\succcurlyeq_z$. Accordingly, $w(z, .)$ is a positive affine transformation of the function $v_z : \Delta_A \to \mathbb{R}$ that we derived in Lemma 2.3. Hence it follows that

$$W(p) = \sum_{(z,a)} p(z, a)\{(1 - \sigma_z) w(z, a) + \sigma_z w(z, p_A)\}$$

represents $\succsim$. This completes the proof of sufficiency of the axioms. Necessity of the axioms as well as the proof of the uniqueness statement is straightforward and we do not provide the details here.

2.3.3 Proof of Theorem 2.2

Begin with a pair $(w, (\sigma_z)_{z\in Z})$ that represents $\succsim$ in the sense of Theorem 2.1. Consider any z, $z' \in Z$ with $\succ_z, \succ_{z'} \neq \emptyset$. There are two cases to consider. First suppose that there exists $\widehat{a} \in A$ such that

$$\{a \in A : a \sim_z \widehat{a}\} \neq \{a \in A : a \sim_{z'} \widehat{a}\}$$

In this case, there exists $a \in [\widehat{a}]_{z'}$, $a' \in [\widehat{a}]_z$ satisfying $a \succ_z \widehat{a}$ and $a' \succ_{z'} \widehat{a}$. In order to establish that $\sigma_z = \sigma_{z'}$, all we need to do is find p, $q \in \Delta$ such that p and q are symmetric with respect to z, z', for once we do that, the result follows immediately from the axiom of *symmetry*. We now proceed to establish that there exists such p and q.

We will now show that there exists $\widetilde{a} \in A$ such that the pair $\widehat{a}$, $\widetilde{a}$ is equal gains with respect to $\succsim_z$ and $\succsim_{z'}$. Consider p', $q' \in \Delta$, where

$$p' = ([z, \tfrac{1}{2}; z', \tfrac{1}{2}], a) \text{ and } q' = ([z, \tfrac{1}{2}; z', \tfrac{1}{2}], a').$$

In case $p' \sim q'$, pick any $\widetilde{a} \in [a]_z \cap [a']_{z'}$. Then, the pair $\widehat{a}$, $\widetilde{a}$ is equal gains with respect to $\succsim_z$ and $\succsim_{z'}$. On the other hand suppose $p' \nsim q'$, and without loss of generality, suppose $p' \succ q'$. Then we have that $p' \succ q' \succ ([z, \tfrac{1}{2}; z', \tfrac{1}{2}], \widehat{a}) \equiv p''$, where the final strict preference follows from *comparable monotonicity*. It then follows from *bi-continuity* that there exists some $\lambda \in (0, 1)$, such that

$$\lambda p' \oplus (1 \text{ - } \lambda)p'' = ([z, \tfrac{1}{2}; z', \tfrac{1}{2}], \mathcal{CE}_z([a, \lambda; \widehat{a}, (1-\lambda)]) \sim q'.$$

Let $a'' = \mathcal{CE}_z([a, \lambda; \widehat{a}, (1-\lambda)])$. Now pick any $\widetilde{a} \in [a'']_z \cap [a']_{z'}$. It follows that $\widehat{a}$, $\widetilde{a}$ is equal gains with respect to $\succcurlyeq_z$ and $\succcurlyeq_{z'}$. Now define p, q as follows:

$$p = [(z, \widehat{a}), \tfrac{1}{2};\, (z', \widetilde{a}), \tfrac{1}{2}] \text{ and } q = [(z, \widetilde{a}), \tfrac{1}{2};\, (z', \widehat{a}), \tfrac{1}{2}].$$

Clearly, p and q are symmetric with respect to z and z'.

On the other hand if

$$\{a \in A : a \sim_z \widehat{a}\} = \{a \in A : a \sim_{z'} \widehat{a}\}$$

for all $\widehat{a} \in A$, then by the *contingent values* assumption, there exists $\succcurlyeq_{z''}$, with $\succ_{z''}$ $\neq \emptyset$ for which there exists $\widehat{a} \in A$ such that

$$\{a \in A : a \sim_z \widehat{a}\} \neq \{a \in A : a \sim_{z''} \widehat{a}\}$$

and,

$$\{a \in A : a \sim_{z'} \widehat{a}\} \neq \{a \in A : a \sim_{z''} \widehat{a}\}.$$

Based on the argument in the last paragraph, we can then conclude that $\sigma_z = \sigma_{z''}$, and $\sigma_{z'} = \sigma_{z''}$, and hence $\sigma_z = \sigma_{z'}$.

Chapter 3

Extending the Basic Representations

3.1 Non-linear Other-regarding Preferences

So far in the analysis we have assumed that the preference relations $\succcurlyeq_z$, $z \in Z$, satisfies the independence condition of expected utility theory. This condition implies that the decision maker's preferences over the risk faced by others, when she is guaranteed some outcome z, is linear in probabilities. Here we want to allow for the possibility that these preferences may be non-linear in probabilities. Our motivation behind doing this exercise comes from existing models of decision making under risk (for example, rank dependent utility and generalized prospect theory) that emphasize the distinction that decision makers may make between raw probabilities and subjective decision weights. For instance, in this literature it has been highlighted that a decision maker may overweight small chances of receiving a 'good outcome,' and underweight large chances. Such subjective probability weighting has been used to explain phenomena such as the Allais paradox. We think that subjective weighting of probabilities may play a role when a decision maker with other-regarding preferences evaluates the risk faced by others. For instance, it may well be the case that an altruistic decision maker may overweight a small chance that someone she cares about has of getting a good outcome.

In the representation that we provide here, DM evaluates a lottery $p \in \Delta$ by the function:

$$W(p) = \sum_{z \in Z} p_Z(z)\{(1 - \sigma_z)w(z, p_{A,z}) + \sigma_z w(z, p_A)\}$$

where $\sigma_z \in [0, 1]$, $z \in Z$, is a measure of DM's concern for procedures.

We now make precise the exact structure that we will impose on $\succcurlyeq_z$. For that, we will adapt to our environment of risk a definition that Ghirardato and Marinacci (2001) have provided in the context of uncertainty.

Definition 3.1. *A preference relation $\succcurlyeq_z$, with $\succ_z \neq \emptyset$, is biseparable if it satisfies stochastic dominance,*[1] *and admits a representation $V_z : \Delta_A \to \mathbb{R}$ (that is unique up to positive affine transformation), for which there exists a strictly increasing bijection φ_z: $[0, 1] \to [0, 1]$ that satisfies $\varphi_z(0) = 0$, $\varphi_z(1) = 1$, such that, if we let $v_z(a) = V_z(a)$ for all $a \in A$, then for all a', $a'' \in A$, $a' \succcurlyeq_z a''$, and all $\lambda \in [0, 1]$,*

$$V_z([a', \lambda; a'', 1 - \lambda]) = \varphi_z(\lambda)v_z(a') + (1 - \varphi_z(\lambda))v_z(a'').$$

As the name suggests, biseparable preferences introduce event-separability in a very limited sense; viz. lotteries that put positive probability on only two outcomes (for the others) are evaluated by the decision maker in the spirit of generalized expected utility. Other than that the only restriction on $\succcurlyeq_z$ is that it respects stochastic

[1] We say that a lottery $p_A \in \Delta_A$ first order stochastically dominates a lottery $q_A \in \Delta_A$ with respect to $\succcurlyeq_z$, if for all $a \in A$, the probability that p_A assigns to outcomes that are at least as good as a (according to $\succcurlyeq_z$) is at least as large as the corresponding probability under q_A, and is strictly larger for some $a \in A$. $\succcurlyeq_z$ satisfies stochastic dominance if whenever $p_A \in \Delta_A$ first order stochastically dominates $q_A \in \Delta_A$, we have $p_A \succ_z q_A$.

dominance. The function φ_z is referred to as a probability weighting function. The probability weighting function has the interpretation that it transforms 'raw' or objective probabilities into decision weights that capture the attitude that DM has toward the chance or risk faced by others.

AXIOM: Biseparability

$\succcurlyeq_z$ is biseparable for all $z \in Z$ with $\succ_z \neq \emptyset$.

In the subsequent analysis, we shall not distinguish between the functions V_z and v_z, and use the latter to denote both. Further, we will call v_z a biseparable representation of the biseparable preference $\succcurlyeq_z$. Some prominent examples of biseparable preferences include expected utility, rank dependent utility, and Gul's 'disappointment averse' preferences.

Ghirardato et al. (2003) have shown that the structure of biseparable preferences can be used to define 'subjective mixtures' or 'preference averages.' That is, for any two outcomes a, $a' \in A$, we can identify an outcome $\overline{a} \in A$ that can be considered as the 'mid-point' on DM's 'preference scale' between a and a'. Given that the biseparable representation v_z of $\succcurlyeq_z$ is unique up to positive affine transformation, such an $\overline{a}$ is characterized by the equation,

$$v_z(\overline{a}) = \frac{1}{2}v_z(a) + \frac{1}{2}v_z(a').$$

Ghirardato et al. (2003) have shown, in the context of uncertainty, that the notion of a preference average can be equivalently defined from behavioral primitives. We now provide a similar definition in our setting of risk. In the way of notation, note that, for any $p_A \in \Delta_A$, $\mathcal{CE}_z(p_A) \in A$ shall denote the certainty equivalent of p_A with respect to the preference relation $\succcurlyeq_z$; that is, $\mathcal{CE}_z(p_A) \sim_z p_A$.

Definition 3.2. *For any $a,\ a' \in A$, if $a \succcurlyeq_z a'$, we say that $\overline{a} \in A$ is a preference average of a and a' with respect to $\succcurlyeq_z$, denoted $\frac{1}{2}a \oplus_z \frac{1}{2}a'$, if for all $\lambda \in [0,1]$,*

$$[a,\ \lambda;\ a',\ 1-\lambda] \sim_z [\mathcal{CE}_z([a,\lambda;\overline{a},1-\lambda]),\ \lambda;\ \mathcal{CE}_z([\overline{a},\lambda;a',1-\lambda]),\ 1-\lambda].$$

If $a' \succcurlyeq_z a$, $\overline{a}$ is said to be a preference average of a and a' if it is a preference average of a' and a.

For a discussion of why the criterion above constitutes a meaningful definition of preference average, the reader is encouraged to refer to the lucid commentary in Ghirardato et al. (2003). A couple of comments are in order. The first is that the preference average of any $a,\ a' \in A$ need not be unique; there may be multiple elements in A that are a preference average of such a and a'. All such preference averages form an indifference class of $\succcurlyeq_z$ (see Lemma 3.1 below), and by $\frac{1}{2}a \oplus_z \frac{1}{2}a'$ we shall denote a *representative* of this indifference class. Second, since $\succcurlyeq_z$ satisfies stochastic dominance, if $\overline{a}$ is a preference average of a and a', then $a \succcurlyeq_z \overline{a} \succcurlyeq_z a'$, and this holds with strict preference if $a \succ_z a'$. The next Lemma ties down the behavioral and utility approaches of defining preference averages by showing that for biseparable preferences they coincide. (The proof of the Lemma mimics the proof of Proposition 1 in Ghirardato et al. (2003), and the details are omitted.)

Lemma 3.1. *Let v_z be a biseparable representation of (the biseparable preference) $\succcurlyeq_z$. For any $a,\ a' \in A$, $\overline{a} \in A$ is a preference average of a and a' with respect to $\succcurlyeq_z$ if and only if*

$$v_z(\overline{a}) = \frac{1}{2}v_z(a) + \frac{1}{2}v_z(a').$$

Further, preference averages of a and a' exist for any $a,\ a'$ in A, and they form an indifference class. That is, if $\overline{a}$ and $\widehat{a}$ are both preference averages of a and a', then $\overline{a} \sim_z \widehat{a}$.

Note that we can now easily define iterated averages like $\frac{1}{2}a \oplus_z [\frac{1}{2}a \oplus_z \frac{1}{2}a']$, which is equivalent to a $\frac{3}{4} : \frac{1}{4}$ mixture of a and a', and denoted $\frac{3}{4}a \oplus_z \frac{1}{4}a'$. More generally, continuity makes it possible to identify from behavior a $\lambda : 1-\lambda$ mixture of a and a' for any $\lambda \in [0,1]$. In addition, we can extend the notion of subjective mixtures to the whole of Δ_A. For any p_A, $p'_A \in \Delta_A$, if $p_A \succcurlyeq_z p'_A$, we say that $\overline{p}_A$ is a $\lambda : 1-\lambda$ mixture of p_A and p'_A with respect to $\succcurlyeq_z$, denoted $\lambda p_A \oplus_z (1-\lambda)p'_A$, if

$$\mathcal{CE}_z(\overline{p}_A) = \lambda \mathcal{CE}_z(p_A) \oplus_z (1-\lambda)\mathcal{CE}_z(p'_A).$$

We now define an appropriate notion of subjective mixtures and mixture comparability for the current setting.

Definition 3.3. *Given p, $q \in \Delta$ that are comparable at some $z \in Z$, we denote by $\lambda p \oplus (1-\lambda)q$ an element in Δ that is comparable with p (and hence with q) at z and satisfies,*

$$a(z, \lambda p \oplus (1-\lambda)q) \sim_z \lambda a(z,p) \oplus_z (1-\lambda)a(z,q) \text{ and } (\lambda p \oplus (1-\lambda)q)_A \sim_z \lambda p_A \oplus_z (1-\lambda)q_A.$$

Further, if p, $q \in \Delta$ are such that $p = (p_Z, a)$, $q = (q_Z, a)$, for some $a \in A$, then we define

$$\lambda p \oplus (1-\lambda)q = \lambda p + (1-\lambda)q$$

Finally, if p, $q \in \Delta$ are such that $p = (z, p_A)$, $q = (z, q_A)$, for some $z \in Z$, then we define

$$\lambda p \oplus (1-\lambda)q = (z,\, \lambda p_A \oplus_z (1-\lambda)q_A)$$

We will say that p and $q \in \Delta$ are ***mixture comparable*** *if $\lambda p \oplus (1-\lambda)q$ exists for all $\lambda \in [0,1]$.*

Comparable independence now applies with respect to lotteries that are mixture comparable as per the above definition.

AXIOM: Comparable Independence*

Let p_1, p_2, q_1, q_2 in Δ be such that p_1, p_2 are mixture comparable, as are q_1, q_2. Then, for all $\lambda \in (0,1]$,

$$[p_1 \succ q_1,\, p_2 \sim q_2] \Rightarrow \lambda p_1 \oplus (1-\lambda)p_2 \succ \lambda q_1 \oplus (1-\lambda)q_2.$$

In addition, the notion of a procedure-adjusted equivalent also needs to be appropriately defined. To do that we introduce the following piece of notation. For any p in Δ we will refer to the set

$$\{(z,\, p_{A,z},\, p_A) : z \text{ is in the support of } p_Z\}$$

as the collection of **risk profiles** under p, and for any particular z in the support of p_Z, we will refer to $(z,\, p_{A,z},\, p_A)$ as the **risk profile at** z under p.

Definition 3.4. *Let $p \in \Delta$ be such that for each risk profile $(z, p_{A,z}, p_A)$, z in the support of p_Z, there exists a procedure-contingent outcome $(z, \tilde{a}, \tilde{p}_A)$, with $p_{A,z} \sim_z \tilde{a}$ and $p_A \sim_z \tilde{p}_A$, that is revealed indifferent to a procedure-contingent outcome $(z, \hat{a}_z, \hat{a}_z)$, for some $\hat{a}_z \in A$. Then we call the lottery,*

$$\pi(p) = [< (z, \hat{a}_z), p_Z(z) >_{z \in S[p_Z]}] \in \Delta, \textit{ where } S[p_Z] \textit{ denotes the support of } p_Z,$$

*the **procedure-adjusted equivalent** of p.*

The dominance axiom can then be stated in terms of the above definition of procedure-adjusted equivalents.

AXIOM: Dominance*

Let p, $q \in \Delta$ and $\pi(p)$, $\pi(q) \in \Delta$ be procedure-adjusted equivalents of p and q respectively. If $\pi(p)$ first order stochastically dominates $\pi(q)$ with respect to $\succcurlyeq$, then $p \succ q$.

Theorem 3.1. *Suppose contingent values hold. Then $\succcurlyeq$ on Δ satisfies the axioms of weak order, bounded bi-continuity, revealed consistency, comparable monotonicity, biseparability, comparable independence* and dominance* if and only if there exists*

- *a function $w : Z \times \Delta_A \to \mathbb{R}$, such that $w(z, .) : \Delta_A \to R$ is a biseparable representation of $\succcurlyeq_z$, and*
- *constants $\sigma_z \in [0, 1]$, $z \in Z$,*

such that the function $W : \Delta \to \mathbb{R}$, given by

$$W(p) = \textstyle\sum_{z \in Z} p_Z(z)\{(1 - \sigma_z)w(z, p_{A,z}) + \sigma_z w(z, p_A)\}$$

represents $\succcurlyeq$.

In addition, another pair $(\tilde{w}\ , (\tilde{\sigma}_z)_{z \in Z})$ represents $\succcurlyeq$ in the above sense if and only if there exists constants $\alpha >$ 0 and β such that $\tilde{w} = \alpha w + \beta$, and $\tilde{\sigma}_z = \sigma_z$ for all $z \in Z$ with $\succ_z \neq \emptyset$.

Once again, if we impose the axiom of symmetry, then we can establish that $\sigma_z = \sigma$ for all $z \in Z$ and that this σ is unique as long as there exists $\succcurlyeq_z$ with $\succ_z \neq \emptyset$. The proof of the theorem is available in the Appendix.

3.2 Tastes and Values

In the representations presented so far, the utility that DM receives from a pair like $(z, a) \in Z \times A$ is non-separable across what she receives and what the others receive. We introduce here a representation in which there is separability across these terms. Our primary motivation for doing this exercise is to make our representation more amenable to applications. Under the representation that we will present here, DM evaluates a lottery $p \in \Delta$ by the function:

$$W(p) = \sum_{(z,a)\in Z\times A} p(z,a)[u(z) + (1-\sigma_z)v_z(a) + \sigma_z v_z(p_A)],$$

where $\sigma_z \in [0,1]$. We think of the function u, which provides DM's subjective ranking of the outcomes that she may get (independent of any consideration of what the others get) as reflecting DM's *tastes*. On the other hand, the family of functions v_z reflects DM's *values* or *morals*.[2]

In order to achieve the desired separability, we will have to expand the domain of preferences. We assume that DM's preferences are given by a preference relation (weak order) $\succcurlyeq$ on the set,

$$\Delta^+ = \Delta \cup \Delta_Z.$$

We will provide here an axiomatization that implies that $\succcurlyeq_z$ is a von Neumann-Morgenstern preference. An alternative axiomatization where $\succcurlyeq_z$ is biseparable can be provided along similar lines.

AXIOM: Weak Order**

$\succcurlyeq$ *on* Δ^+ *is complete and transitive.*

The *boundedness* condition needs to be appropriately modified to account for the expanded domain.

AXIOM: Bounded Bi-Continuity**

Bi-continuity: *For any* $\lambda \in [0,1]$ *and* $q \in \Delta$, *the sets*

$$\{((z',a'),(z'',a'')) : [(z',a'),\lambda;\, (z'',a''),1-\lambda] \succcurlyeq q\},$$

and,

[2]The distinction between tastes and values is motivated by the following quote from Kenneth Arrow in 'Social Choice and Individual Values': "In general, there will then be a difference between the ordering of social states according to the direct consumption of the individual and the ordering when the individual adds his general standards of equity. We may refer to the former ordering as reflecting the tastes of the individual and the latter as reflecting his values. The distinction between the two is by no means clear cut...no sharp line can be drawn between tastes and values."

$$\{((z', a'), (z'', a'')) : q \succcurlyeq [(z', a'), \lambda; (z'', a''), 1-\lambda]\}$$

are closed in $[Z \times A] \times [Z \times A]$.

Boundedness: *There exists* $a^* \in A$*, such that for all* $l \in \Delta^+$ *there exists* $\overline{z}$, $\underline{z} \in Z$ *satisfying* $(\overline{z}, a^*) \succcurlyeq l \succcurlyeq (\underline{z}, a^*)$.

We now extend the notion of a procedure-adjusted equivalent for the current set-up.

Definition 3.5. *Let* $p \in \Delta$ *be such that for each* (z^k, a^k) *in the support of* p, $k = 1, \ldots, K$, *the procedure-contingent outcome* (z^k, a^k, p_A) *is revealed indifferent to a procedure-contingent outcome* $(z^k, \tilde{a}^k, \tilde{a}^k)$ *for some* $\tilde{a}^k \in A$. *Then we call the lottery,*

$$\pi(p) = [(z^1, \tilde{a}^1), p(z^1, a^1); \ldots, (z^K, \tilde{a}^K), p(z^K, a^K)] \in \Delta$$

the procedure-adjusted equivalent of p.
Further, for $p_Z \in \Delta_Z$*, the procedure-adjusted equivalent* $\pi(p_Z)$ *is given by* p_Z *itself.*

The notion of a procedure-adjusted equivalent remains the same (as in the baseline model) for lotteries in Δ. On the other hand, for lotteries in Δ_Z, since they specify only outcomes for DM, the question of procedure does not arise. Accordingly, for such a lottery p_Z, it is its own procedure-adjusted equivalent. The axiom of dominance says the following.

AXIOM: Dominance**

Let l, $l' \in \Delta^+$ *and* $\pi(l)$, $\pi(l') \in \Delta^+$ *be procedure-adjusted equivalents of* l *and* l' *respectively. If* $\pi(l)$ *first order stochastically dominates* $\pi(l')$ *with respect to* $\succcurlyeq$, *then* $l \succ l'$.

We can now state our representation result, which separates tastes from values.

Theorem 3.2. *Suppose contingent values hold. Then* $\succcurlyeq$ *on* Δ^+ *satisfies the axioms of weak order**, bounded bi-continuity**, revealed consistency, comparable monotonicity, comparable independence and dominance** if and only if there exists a function* $u : Z \to \mathbb{R}$*, functions* $v_z : \Delta_A \to \mathbb{R}$*,* $z \in Z$*, that satisfy*

$$v_z(\lambda p_A + (1-\lambda)p'_A) = \lambda v_z(p_A) + (1-\lambda)v_z(p'_A), \text{ for all } \lambda \in [0,1],$$

and constants $\sigma_z \in [0,\, 1]$*,* $z \in Z$*, such that the function* $W : \Delta^+ \to \mathbb{R}$ *defined by*

- $W(p) = \sum_{(z,a)\in Z\times A} p(z,a)[u(z) + (1-\sigma_z)v_z(a) + \sigma_z v_z(p_A)]$, $\forall\, p \in \Delta$, *and*

- $W(p_Z) = \sum_{z\in Z} p_Z(z)u(z)$, $\forall\, p_Z \in \Delta_Z$.

represents $\succcurlyeq$.

In addition, any triple $(\tilde{u},\, (\tilde{v}_z)_{z\in Z},\, (\tilde{\sigma}_z)_{z\in Z})$ *represents* $\succcurlyeq$ *in the above sense if and only if there exists constants* $\alpha > 0$ *and* β *such that* $\tilde{u} = \alpha u + \beta$*,* $\tilde{v}_z = \alpha v_z$*, for all* $z \in Z$*, and* $\tilde{\sigma}_z = \sigma_z$ *for all* $z \in Z$ *with* $\succ_z \neq \emptyset$*.*

Once again if we impose the axiom of symmetry, then we can establish that $\sigma_z = \sigma$ for all $z \in Z$, as long as there exists $\succcurlyeq_z$ with $\succ_z \neq \emptyset$. Further, instead of assuming that $\succcurlyeq_z$ are vNM preferences, we could have assumed that they are biseparable. In this case, we have a representation under which the utility of a lottery $p \in \Delta$ is given by

$$W(p) = \sum_{z\in Z} p_Z(z)[u(z) + (1-\sigma_z)v_z(p_{A,z}) + \sigma_z v_z(p_A)]$$

where $\sigma_z \in [0,1]$, and v_z is a biseparable representation of $\succcurlyeq_z$.

3.3 Appendix

3.3.1 Proof of Theorem 3.1

The proof of Theorem 3.1 proceeds along similar lines as the proof of Theorem 2.1. We do not repeat the arguments here. The parts that need elaboration are briefly discussed below.

We need to define the appropriate family of mixture set structures on $\Delta_A/\sim_z$ and $[\Delta_A/\sim_z]^2$. We define these here. Consider any $\succcurlyeq_z$. For any $[p_A]_z$, $[q_A]_z \in \Delta_A/\sim_z$, and $\lambda \in [0,1]$, define

$$\lambda[p_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[q_A]_z = [\lambda p_A \oplus_z (1-\lambda)q_A]_z$$

Similarly, for any $([p_A]_z, [q_A]_z), ([p'_A]_z, [q'_A]_z) \in [\Delta_A/\sim_z]^2$, and $\lambda \in [0,1]$, define

$$\lambda([p_A]_z, [q_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p'_A]_z, [q'_A]_z) = (\lambda[p_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[p'_A]_z, \lambda[q_A]_z \mathbin{\widehat{\oplus}_z} (1-\lambda)[q'_A]_z)$$

That is,

$$\lambda([p_A]_z, [q_A]_z) \mathbin{\widehat{\oplus}_z} (1-\lambda)([p'_A]_z, [q'_A]_z) = ([\lambda p_A \oplus_z (1-\lambda)p'_A]_z, [\lambda q_A \oplus_z (1-\lambda)q'_A]_z)$$

With the mixture set structure defined, we can prove a corresponding version of Lemma 2.2 stated with respect to this mixture set structure. Observe that a vNM preference satisfies stochastic dominance and is biseparable. We essentially used these two properties in the proof of Lemma 2.2. In particular, whenever we have a biseparable preference relation $\succcurlyeq_{z'}$, if $\widehat{a}, \widetilde{a}, \widehat{a}', \widetilde{a}' \in A$ are such that $\widehat{a} \sim_{z'} \widetilde{a}$ and $\widehat{a}' \sim_{z'} \widetilde{a}'$, then it follows that for any $\lambda \in [0,1]$,

$$[\widehat{a}, \lambda, \widehat{a}', 1-\lambda] \sim_{z'} [\widetilde{a}, \lambda, \widetilde{a}', 1-\lambda]$$

It is this property that we used in Lemma 2.2 to establish that $\widehat{\succcurlyeq}_z$ is 'locally' a weak order on the appropriately defined cubes. Once we establish the appropriate properties (namely the five Anscombe-Aumann axioms) of $\widehat{\succcurlyeq}_z$ locally, we can establish the corresponding version of Lemma 2.3.

Lemma 3.2. $(\widehat{\succcurlyeq}_z)_{\Omega^*}$ *is a weak order. Further, there exists*

(i) a function $v_z : \Delta_A \to \mathbb{R}$ that represents $\succcurlyeq_z$ and satisfies: for all $\lambda \in [0,1]$, p_A, q_A $\in \Delta_A$,

$$v_z(\lambda p_A \oplus_z (1-\lambda)q_A) = \lambda v_z(p_A) + (1-\lambda)v_z(q_A), \text{ and}$$

(ii) a constant $\sigma_z \in [0,\, 1]$,

such that the function $V_z : \Omega^* \to \mathbb{R}$ given by

$$V_z([p_A]_z,\, [q_A]_z) = (1-\sigma_z)v_z(p_A) + \sigma_z v_z(q_A)$$

represents $(\widehat{\succcurlyeq}_z)_{\Omega^*}$. Further, another pair $(\widetilde{v}_z, \widetilde{\sigma}_z)$ represents $(\widehat{\succcurlyeq}_z)_{\Omega^*}$ in the above sense iff $\widetilde{v}_z$ is a positive affine transformation of v_z and $\widetilde{\sigma}_z = \sigma_z$, for all $z \in Z$ such that $\succ_z \neq \emptyset$.

Lemma 3.2 helps to establish that every $p \in \Delta$ has a procedure-adjusted equivalent. Further, dominance implies the following result.

Lemma 3.3. *Let $p \in \Delta$, and let,*

$$\pi(p) = [(z^1, \hat{a}^1), p_Z(z^1),\ .\ .\ .\ ,\ (z^K, \hat{a}^K), p_Z(z^K)] \in \Delta$$

*be the **procedure-adjusted equivalent** of p. Then*

$$p \sim [(z^*(z^1, \hat{a}^1), a^*), p_Z(z^1),\ .\ .\ .\ ,\ (z^*(z^K, \hat{a}^K), a^*), p_Z(z^K)],$$

where $(z^*(z^k, \hat{a}^k), a^*) \in Z \times A$ are such that $(z^*(z^k, \hat{a}^k), a^*) \sim (z^k, \hat{a}^k)$, for $k = 1,\ .\ .\ .\ , K$.

Finally, Lemma 2.8 will be replaced by the following:

Lemma 3.4. *Let* $(z,\, p_A)$, $(z,\, q_A) \in \Delta$. *Then, for any* $\lambda \in [0, 1]$,

$$\lambda(z, p_A) \oplus (1 - \lambda)(z, q_A) \sim \lambda(z^*(z, p_A), a^*) + (1 - \lambda)(z^*(z, q_A), a^*),$$

where $(z^*(z, p_A), a^*)$, $(z^*(z, q_A), a^*) \in Z \times A$ *are such that*

$$(z^*(z, p_A), a^*) \sim (z, p_A) \text{ and } (z^*(z, q_A), a^*) \sim (z, q_A).$$

The rest of the proof follows along the lines of the proof of Theorem 2.1.

3.3.2 Proof of Theorem 3.2

Once again the details of the proof are similar to that of the proof of Theorem 2.1, and we do not repeat the arguments here. The parts that require elaboration are discussed below.

Observe that replicating the steps of the proof of Theorem 2.1, gives us a pair (w, $(\sigma_z)_{z\in Z}$) that represents $\succcurlyeq$ restricted to Δ. Next note that, the *boundedness* implies that for any $p_Z \in \Delta_Z$, there exists $z^*(p_Z) \in Z$ such that $p_Z \sim (z^*(p_Z), a^*)$. Then we can define the extension of the function W to Δ_Z by defining $W(p_Z) = W(z^*(p_Z), a^*)$. Further define the function $u : Z \to \mathbb{R}$ as $u(z) = W(z)$, where $W(z)$ denotes the value of the function W for the degenerate lottery in Δ_Z that gives z with probability 1. Dominance then implies that

$$W(p_Z) = \textstyle\sum_{z\in Z} p_Z(z)u(z),\ \forall\ p_Z \in \Delta_Z$$

Finally, define $v_z : \Delta_A \to \mathbb{R}$ to be $v_z(q_A) = W(z, q_A) - u(z)$. It then follows that

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z, a)[u(z) + (1 - \sigma_z)v_z(a) + \sigma_z v_z(p_A)],\ \forall\ p \in \Delta.$$

Chapter 4

Modeling Non-consequentialist Voting Behavior

In this chapter, we use our decision model to provide some foundations for non-consequentialist voting behavior. It has long been recognized that an individual's decision of whether or not to vote, and what alternative to vote for, if she indeed does vote, may be influenced by considerations other than how her choice impacts electoral outcomes. She may consider such choices to be intrinsically valuable. This has motivated expressive theories of voting which seek to incorporate a consumption or intrinsic value of voting. In much of the literature, this consumption value of voting is taken as an exogenous feature of the model and therefore independent of what outcomes are. What is the source of this consumption value of voting and is it reasonable to take this value to be independent of outcomes? For instance, consider a rigged election. Would voters, who otherwise derive a consumption value of voting, still derive these payoffs in such an election? It may well be that they do not. Therefore, it is a useful exercise to "endogenize" the consumption value of voting. The decision model that we have developed in the earlier chapters allow us to do this.

Our motivation behind providing this expressive theory of voting is two-fold. The first goal is choice theoretic. When viewed from the perspective of a consequentialist paradigm, voting behavior of non-consequentialist voters may appear to be paradoxical and indeed against self-interest. The voting model we introduce here allows us to rationalize voting behavior which when viewed from a consequentialist view point may appear to be against one's self-interest. Second, we address the question of what impact the presence of such non-consequentialist voters may have on electoral outcomes. To help us address these issues, we begin by considering some experimental evidence from a recent paper by Feddersen, Gailmard and Sandroni (2009).

The basic hypothesis that their work proposes is that large elections may exhibit a moral bias, namely, alternatives understood by voters to be morally superior are more likely to win in large elections than in small ones. To make this point, they conduct an experimental election with two alternatives – call these the *moral option* and the *selfish option*. The basic details of their experiment are as follows. First, subjects were divided into two groups, one consisting of voters and the other of non-voters. Then the voters cast their votes. Finally, after all voters had cast their vote, one voter was randomly picked, and the choice she reported became the outcome of the election. Observe that under this particular method of determining the outcome of the election, the probability that any given voter's vote is pivotal, i.e., that her vote determines the outcome of the election, is given by the reciprocal of the number of voters. The experimenters varied the number of eligible voters across different trials of the experiment and, by so doing, the probability of a voter being pivotal was directly controlled as a treatment variable in the experiment. As far as payoffs went, the selfish option gave a higher monetary reward to the voters than the moral option. On the other hand, the moral option was better for the non-voters than the selfish option. An interesting pattern of choice exhibited by a non-trivial number of voters is the following. When the probability of their vote being pivotal was high, in particular when it was 1 (i.e., they were dictatorial), these voters chose the selfish option. On the other hand, when the pivot probability was low, their vote switched to the moral option. Overall, the data from across different trials of the experiment showed a strong (statistically significant) positive relationship between the probability of the moral option being the electoral outcome and the size of the electorate.

One may make the case that the voters mentioned above voted against their self-interest based on the following kind of argument. When these voters were dictatorial, they chose the selfish option. This choice *reveals* that they *prefer* the selfish option to the moral one. At the same time, in elections where the probability that their vote is pivotal was low, they ended up voting for the moral option which, if one were to go by their revealed preference inferred from the first choice, is their less preferred alternative. It is important to recognize though that such an argument is based on consequentialism. That is, it assumes that voters have a ranking over the electoral outcomes independent of the process by which these outcomes are generated. Such a consequentialist argument is not appropriate for voters whose behavior is influenced by non-consequentialist considerations. For instance, such voters may derive 'utility' from the very *act* of voting for a particular choice owing to motivations like a sense of civic obligation or a desire to act 'morally' by making certain choices. Such individuals may be said to derive an *expressive* value from voting. We will now use our decision model to sketch out such an expressive theory of voting. We will show how concerns for procedure can rationalize voting behavior which, when viewed from a consequentialist standpoint, appear to be against one's self-interest. Further, we will show that the presence of such non-consequentialist voters is qualitatively significant in terms of electoral outcomes.

4.1 A Simple Model

We consider an election with two alternatives, 1 and 2, in a society consisting of n voters. We think of each of the alternatives as determining an outcome for each of the individuals in society. Accordingly, the alternatives can be thought of as determining the allocation for this society. We treat n as a parameter of the model. In this simple model, we assume that there are no costs to voting. This will ensure that everyone votes in the election. Further, the result of the election will be determined by the following mechanism, which mimics the one used by Feddersen et al. First, all voters cast their votes. After all voters have reported their choice, one voter is drawn at random, and the choice she reported determines the outcome of the election.

We make the assumption that all voters are identical in terms of their preferences. This simplifies the analysis, since it allows us to conduct it in the context of a 'representative voter.' Let us now describe what the problem looks like when viewed from the perspective of one such representative voter (RV). As mentioned above, she can vote for either alternative 1 or alternative 2. If alternative 1 is the group choice, the resulting allocation is $(z^1, a^1) \in Z \times A$,[1] where z^1 refers to the outcome for RV, and a^1 refers to the vector of outcomes for everyone else. Similarly, if alternative 2 is the group choice, the resulting allocation is $(z^2, a^2) \in Z \times A$, where again z^2 refers to the outcome for RV, and a^2 the outcomes for everyone else.

Note that under the electoral mechanism, the probability that RV is pivotal is given by $\alpha = 1/n$. Further, let β denote the probability that alternative 1 is the outcome of the election when RV is not pivotal.[2] Then the probability distribution

[1] We continue using the notation that the set Z denotes the outcomes of the decision maker (who in this case is the representative voter under consideration), A_i, $i \neq RV$, denotes the set of outcomes of individual i, and $A = \prod_{i \neq RV} A_i$.

[2] Of course, β is an 'endogenous object'

over final allocations generated by RV choosing alternative 1 is given by:

$$p^1 = [(z^1, a^1),\ \alpha + (1-\alpha)\beta;\ (z^2, a^2),\ 1-\alpha-(1-\alpha)\beta],$$

and that by choosing alternative 2 is given by:

$$p^2 = [(z^1, a^1),\ (1-\alpha)\beta;\ (z^2, a^2),\ 1 - (1-\alpha)\beta]$$

Note that if RV's preferences satisfy stochastic dominance, then her vote choice is independent of pivot probabilities or, equivalently, of the number of voters. To understand this claim, suppose, she prefers the allocation (z^2, a^2) to (z^1, a^1), that is, she would choose alternative 2 if the choice *were* completely left to her. Now consider any situation in which she is pivotal with probability $\alpha = 1/n$. In this case, taking the other voters' choices as given (that is, taking β as given), her vote for alternatives 1 and 2 generates respectively the lotteries p^1 and p^2 over final allocations (listed above). Since she prefers the allocation (z^2, a^2) to (z^1, a^1), stochastic dominance requires that she must prefer the lottery p^2 to the lottery p^1, and hence must vote for alternative 2 irrespective of what α and β are. Accordingly, assuming that RV has a strict preference for one of the alternatives (in the above sense) we have:

Proposition 4.1. *If voters' preferences satisfy stochastic dominance, then there exists a unique Nash equilibrium (in dominant strategies) that is independent of n in which either everyone votes for alternative 1 or everyone votes for alternative 2.*

We now contrast this result with one that is implied by our decision model in which decision makers may have procedural concerns. In this analysis, we will use the 'tastes and values' model of Chapter 3 with the family of v_z functions taking a biseparable form. That is, any lottery p is evaluated by the functional:

$$W(p) = \textstyle\sum_z p_Z(z)[u(z) + (1-\sigma)v_z(p_{A,z}) + \sigma v_z(p_A)]$$

where $\sigma \in [0,1]$, and v_z is a biseparable representation of $\succcurlyeq_z$. We will assume that RV considers alternative 1 to be better on grounds of her values or morals. That is,

$$v_H = v_z(a^1) > v_z(a^2) = v_L, \text{ for } z = z^1,\, z^2.$$

Further, we will assume that the preference relations $\succcurlyeq_z$, $z = z^1$, z^2, are cardinally equivalent. This means that there exists a probability weighting function, that is, a strictly increasing bijection $\varphi : [0,1] \to [0,1]$ that satisfies $\varphi(0) = 0$, $\varphi(1) = 1$, such that a lottery of the type $[a^1,\, \lambda;\, a^2,\, 1-\lambda]$ is evaluated as,

$$v_z([a^1,\, \lambda;\, a^2,\, 1-\lambda]) = \varphi(\lambda)v_z(a^1) + (1-\varphi(\lambda))v_z(a^2), \text{ for } z = z^1,\, z^2.$$

As discussed in Chapter 3 the probability weighting function has the interpretation that it transforms objective probabilities into decision weights. These decision weights capture the attitude that DM has toward the chance or risk faced by others. We will further assume that the procedural weight σ is equal to $\frac{1}{2}$.

We define,

$$\nu = \tfrac{u_H - u_L}{v_H - v_L}$$

and assume that:

- $[V1]$ $\nu > 1$.
- $[V2]$ There exists $\underline{\lambda}$, $\overline{\lambda} \in (0,1)$, such that for all $\widetilde{\lambda} \in (0,\underline{\lambda}) \cup (\overline{\lambda}, 1)$, φ is differentiable, and $\varphi'(\widetilde{\lambda}) > 2\nu - 1$. Further, φ is concave on the interval $[0,\underline{\lambda})$.

[V1] can be rewritten as

$$u_H + v_L > u_L + v_H$$

The left-hand side gives RV's payoffs under our representation from the allocation (z^2, a^2), whereas the right-hand side gives her payoffs from the allocation (z^1, a^1).

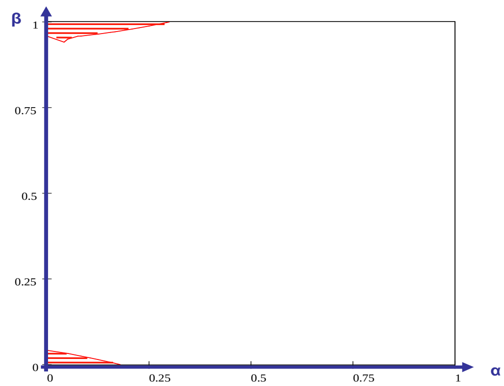
This condition therefore states that RV prefers the allocation (z^2, a^2) to the allocation (z^1, a^1), when these allocations are considered by themselves (that is, each is viewed as realizing with probability 1). Accordingly, if RV were a dictator who could decide the election outcome on her own, she would choose alternative 2.

Assumptions [V1] and [V2] together imply that for all $\widetilde{\lambda} \in (0,\underline{\lambda}) \cup (\overline{\lambda}, 1)$, $\varphi'(\widetilde{\lambda}) > 1$. It follows that there exists a neighborhood of 0 in which $\varphi(\widetilde{\lambda}) > \widetilde{\lambda}$, and there exists a neighborhood of 1 in which $\varphi(\widetilde{\lambda}) < \widetilde{\lambda}$. in other words, the representative voter tends to overweight small probabilities and underweight large probabilities of her morally preferred outcome for others, a^1, being realized. This phenomenon of over-weighting small probabilities, and under-weighting large ones, which is referred to as regressive probability weighting, has been extensively documented in the literature on decision making under risk, starting with the important contribution of Kahneman and Tversky (1979).

Proposition 4.2. *Under assumptions [V1] and [V2], there exists positive integers $\underline{n}$ and $\overline{n}$, $\underline{n} < \overline{n}$, such that for all $n \leq \underline{n}$, everyone voting for alternative 2 is the unique symmetric Nash equilibrium (in pure strategies), and for all $n \geq \overline{n}$, everyone voting for alternative 2 is the unique symmetric Nash equilibrium (in pure strategies).*

The proof is available in the Appendix. Here, we briefly go over the reasoning that drives the result. Consider Figure 4.1, which has been constructed by taking particular values of u_H, u_L, v_H, v_L and functional form for the probability weighting function that are consistent with assumptions [V1] and [V2]. The figure shows the payoff difference for our representative voter from voting for alternatives 1 and 2 as a function of α, the pivot probability, and β, the probability that alternative 1 will be chosen when RV is not pivotal. The shaded area represents those values of α and β for which the payoff of voting for alternative 1 exceeds that of voting for alternative

Figure 4.1: Payoff difference between voting for alternatives 1 and 2.



2. The incentives that RV has for voting for alternative 2 for high values of α is quite apparent given that she prefers alternative 2 to alternative 1. The interesting feature of our model is that for low values of α, and for suitable values of β, her vote choice shifts from alternative 2 to 1. In particular, there are two regions in the α-β box of the figure in which the payoff of voting for alternative 1 exceeds that of voting for alternative 2. This vote switch is brought about by the role that procedures play in her evaluation of prospects.

Consider first the lower south-west region where both α and β are small. In this scenario, RV knows that alternative 2 is the likely electoral outcome. Further, this is true irrespective of which way she votes, since the probability α that her vote is pivotal is small. Thus, her vote is relatively insignificant in terms of determining actual *outcomes*. But given that she cares about procedures, her vote holds a significance beyond its ability to influence the outcome of the election. Observe that since alternative 1 is her morally preferred outcome, she can be made better off in the event that alternative 2 is the electoral outcome if alternative 1 had a higher ex-ante chance of being realized. So by voting for alternative 1 she can increase this ex-ante chance and receive higher payoffs with respect to her procedural concerns. What makes this increase in 'procedural payoffs' significant (relative to the increase in 'outcome payoffs' if she votes for alternative 2) is the fact that she over-weights small chances of her morally preferred outcome for others, a^1, being realized. So to sum up, voting for alternative 1 is almost identical to voting for alternative 2 via her concerns for outcomes. On the other hand, voting for alternative 1 is comparatively much better than voting for alternative 2 via her concerns for procedure. Accordingly, under this scenario, she votes for alternative 1.

Now consider the north-west corner of the α-β box. In this scenario alternative 1 is the likely electoral outcome, and given that α is small, this is true irrespective of which way RV votes. Therefore, voting for alternative 1 is almost identical, once again, to voting for alternative 2 in terms of outcomes. On the other hand, voting for alternative 1 is relatively better than voting for alternative 2 via her concerns for procedure. To see this, note that if she were to vote for alternative 2, it would reduce the ex-ante chance of alternative 1 being realized by α. Given that the chance of alternative 1 being realized is close to 1, the regressive nature of probability weighting close to 1, namely, that probabilities are under-weighted, makes this reduction in ex-ante chance unattractive for her. Accordingly, under this scenario, she votes for alternative 1

Given the structure of payoff differences, it should now be obvious why our result follows. In particular, note that when everyone else is voting for alternative 1 ($\beta = 1$), for small pivot probabilities, RV's best response is to vote for alternative 1.

4.2 A Model with Costly Voting and Private Information

In this section we will introduce two changes to the simple model of the last section. First, we will consider the case in which the outcome of the election is determined by plurality rule. That is the outcome of the election is determined by which ever alternative receives the greater number of votes. In case of a tie, we will assume that alternative 2 is the electoral outcome.

Second, we consider a situation in which individuals have a cost of voting. Given positive costs of voting, it may now be that it is in an individual's best interest to abstain from voting. In other words, apart from deciding about which alternative to vote for, an individual has to decide whether she wants to vote or abstain. We will assume that the cost of voting is separable from her evaluation of a lottery. In particular, if p is a lottery over allocations that is engendered (ex-ante) by the profile of individual choices, her payoffs is given by

$$\widetilde{W}(p,c) = W(p) - c, \text{ if she votes}$$

and,

$$\widetilde{W}(p,c) = W(p), \text{ if she abstains.}$$

In other words, in terms of our representation, while considering allocations, the decision maker only considers her own costs and ignores the cost incurred by others. Further, she evaluates her "outcomes" and costs separably.

We will assume that an individual's cost of voting is private information. In the language of Bayesian games, an individual's cost is then her type. We will assume that the costs of voting for each individual is (independently) distributed according to the distribution function F, which has support $[0, \overline{c}] \subseteq \mathbb{R}$, where $\overline{c} > 0$. We will assume that the function F is common knowledge.

Formally, a strategy is a mapping:

$$s : [0, \overline{c}] : \rightarrow \{\text{Abstain, Vote for 1, Vote for 2}\}$$

We will restrict attention here to symmetric Bayesian equilibrium. That is Bayesian equilibrium in which all individuals of the same cost type take the same decision. All individuals choosing voting strategy s is a Bayesian Equilibrium if and only if for all cost types $c \in [0, \overline{c}]$, the decision $s(c)$ is a best response given that all other individuals play s. Further, we will call any such symmetric Bayesian equilibrium a *Values equilibrium* if all cost types that vote, vote for alternative 1.

Proposition 4.3. *There exists $\overline{n}$, such that for all $n > \overline{n}$, there exists a unique Values equilibrium.*

4.3 Appendix

4.3.1 Proof of Proposition 4.2

Recall that the representative voter is pivotal with probability $\alpha = \frac{1}{n}$, and β denotes the probability that alternative 1 is the outcome when she is not pivotal. Then the probability distributions over final allocations generated by the representative voter choosing 1 and 2 are respectively,

$$p^1 = [(z^1, a^1),\, \alpha + (1-\alpha)\beta;\, (z^2, a^2),\, 1 - \alpha - (1-\alpha)\beta],$$

$$p^2 = [(z^1, a^1),\, (1-\alpha)\beta;\, (z^2, a^2),\, 1 - (1-\alpha)\beta]$$

Under out representation these two lotteries are evaluated as:

$$U(p^1) = u_H + \tfrac{v_L}{2} - (\alpha + (1-\alpha)\beta)[u_H - u_L - \tfrac{1}{2}(v_H - v_L)] + \tfrac{1}{2}[\varphi(\alpha + (1-\alpha)\beta)v_H$$
$$+ (1 - \varphi(\alpha + (1-\alpha)\beta))v_L]$$

and,

$$U(p^2) = u_H + \frac{v_L}{2} - (1-\alpha)\beta[u_H - u_L - \frac{1}{2}(v_H - v_L)] + \frac{1}{2}[\varphi((1-\alpha)\beta)v_H + (1-\varphi((1-\alpha)\beta))v_L]$$

Subtracting the two gives,

$$U(p^2) - U(p^1) = \alpha[u_H - u_L - \frac{1}{2}(v_H - v_L)] - \frac{1}{2}(v_H - v_L)[\varphi(\alpha + (1-\alpha)\beta) - \varphi((1-\alpha)\beta)]$$

Accordingly,

$$U(p^2) - U(p^1) \geq 0 \Leftrightarrow g(\alpha) = \alpha(2\nu - 1) - (\varphi(\alpha + (1-\alpha)\beta) - \varphi((1-\alpha)\beta)) \geq 0$$

Now suppose everyone other than RV votes for alternative 1; i.e., $\gamma = 1$. Then,

$$g(\alpha) = \alpha(2\nu - 1) - (1 - \varphi(1 - \alpha))$$

and, for $\alpha \in (0, 1 - \overline{\lambda})$,

$$g'(\alpha) = 2\nu - 1 - \varphi'(1 - \alpha)$$

Let $\lambda' = \min\{1 - \overline{\lambda}, \underline{\lambda}\}$. Then for all $\alpha \in (0, \lambda')$, $g'(\alpha) < 0$. Further, $g(0) = 0$. Hence, $g(\alpha) < 0$ for all $\alpha \in (0, \lambda')$. Let $\overline{n}$ be any integer greater than $\frac{1}{\lambda'}$. Then,for all $n > \overline{n}$, everyone voting for alternative 1 is a Nash equilibrium.

Now consider the case when everyone other than RV votes for alternative 2. That is $\beta = 0$. Then,

$$g(\alpha) = \alpha(2\nu - 1) - \varphi(\alpha) = \alpha[2\nu - 1 - \frac{\varphi(\alpha)}{\alpha}]$$

Note that, for $\alpha < \lambda'$, $\varphi'(\alpha) > 2\nu - 1$, and since φ is concave over this range, $\frac{\varphi(\alpha)}{\alpha} > \varphi'(\alpha)$. Accordingly, for $\alpha < \lambda'$, $g(\alpha) < 0$, and everyone voting for alternative 2 can not be a Nash equilibrium. Hence, for all $n \geq \overline{n}$, everyone voting for alternative 1 is the unique symmetric Nash equilibrium (in pure strategies).

Further, note that when $\beta = 0$, $g(1) = 2\nu > 0$. By continuity of g, there exists an interval $(\lambda_1, 1]$, such that for all $\alpha \in (\lambda_1, 1]$, $g(\alpha) > 0$, and accordingly everyone voting for alternative 2 is a Nash equilibrium.

Finally, note that when $\beta = 1$, $g(1) = 2\nu - 2 > 0$. Once again by the continuity of g, there exists an interval $(\lambda_2, 1]$, such that for all $\alpha \in (\lambda_2, 1]$, $g(\alpha) > 0$, and accordingly everyone voting for alternative 1 is not a Nash equilibrium. Let, $\lambda'' = \max\{\lambda_1, \lambda_2\}$, and $\underline{n}$ be any integer less than $\frac{1}{n''}$. It follows that for all $n \leq \underline{n}$, everyone voting for alternative 2 is the unique symmetric Nash equilibrium (in pure strategies).

4.3.2 Proof of Proposition 4.3

We will show below that the Values equilibrium strategies will be of a cutoff type, namely, all individuals who have cost below some level c will vote for the alternative 1, while those above will abstain. Let us consider what any individual's decision problem looks like given that all the other individuals are following such a cutoff strategy, with cutoff cost $c \in (0, \overline{c}]$.

Let, p^1, p^A denote the probability distribution over final allocations if the decision maker votes for alternative 1 and abstains respectively. In particular,

$$p^1 = [(z^1, a^1),\, 1],$$

$$p^A = [(z^1, a^1),\, 1 - \alpha;\, (z^2, a^2),\, 1 - \alpha],$$

where α denotes the probability that all the other $n-1$ individuals have a cost above c; that is

$$\alpha(c) = (1 - F(c))^{n-1}.$$

Note that, for any $c \in (0, \overline{c})$, α converges monotonically to 0. So by Dini's theorem it follows that the sequence of functions $(\alpha(.))_{k\in\mathbb{N}_+}$ converges uniformly to the constant function 0.

Now let,

$$g(\alpha) = W(p^1) - W(p^A)$$

Elementary calculations imply that

$$g(\alpha) = \frac{v_H - v_L}{2}\{1 - \varphi(1-\alpha) - \alpha(2\nu - 1)\}.$$

It further follows that

$$g'(\alpha) = \frac{v_H - v_L}{2}\{\varphi'(1-\alpha) - (2\nu - 1)\}.$$

Under our assumptions for α sufficiently small, we have that $g'(\alpha) > 0$. Given that g is continuous and $g(0) = 0$, it follows that for α sufficiently small, $g(\alpha) > 0$. Given the uniform convergence of the sequence of functions $(\alpha(.))_{k\in\mathbb{N}_+}$, it follows that for n large, $W(p^1) > W(p^A)$.

A similar set of calculations can be used to establish that for n sufficiently large, the payoffs of voting for alternative 1 exceeds that of alternative 2. We then have that there exists some n^* such that for all $n > n^*$,

1. Payoffs of voting for alternative 1 exceeds that of voting for alternative 2, and

2. $W(p^1) > W(p^A)$.

Accordingly, the individual votes for alternative 1 if and only if

$$W(p^1) - W(p^A) \geq c',$$

where c' denotes the individual's cost of voting. Define the function $G : (0, \bar{c}]$ by

$$G(c) = g(\alpha(c)).$$

Then it follows that the cutoff strategy c is a Values equilibrium if

$$G(c) = c.$$

Note that $G(c)$ is strictly decreasing. This along with the fact that $G(\bar{c}) = 0$, allows us to conclude that for all $n > n^*$, there exists a unique Values equilibrium.

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