The Anaphoric Parallel Between Modality and Tense

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Comments

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The Anaphoric Parallel between Modality and Tense

MS-CIS-97-09
IRCS 97-06

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1997
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Summary
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1 Introduction
Modal statements in natural language admit quite precise construals in context. Discourse (1) illustrates this.

(1) a I want to hold a barbecue.
    b Some vegetarians may be coming.
    c What can I do for them?

A general gloss for (1c), as in (2), is unsatisfying.

(2) What actions for me to take towards vegetarians are logically possible?

Sentence (1c) means something much more specific. A better paraphrase is given in (3).

(3) Assuming vegetarians come to my barbecue, what actions will be available to me (in virtue of the properties of vegetarians and barbecues in the actual world) that will contribute towards making the event a success?

1 Thanks to Mark Steedman and Dan Hardt for extensive comments, and Filippo Beghelli, Anoop Sarkar, Beverly Spejewski, and the pragmatics and computational semantics seminars for helpful discussion. This work was supported by an NSF graduate fellowship, and an IRCS graduate fellowship, as well as NSF grant IRI95-04372, ARPA grant N6601-94-C6043, and ARO grant DAAH05-94-G0426. This paper has been submitted to Linguistics and Philosophy. May 12, 1997.
The context refines the interpretation of (1c) in at least three respects. First, context indicates that only a certain hypothetical prospect is under consideration: what to do if the vegetarians come to the barbecue. Second, context dictates that certain other general facts are to be taken as given in considering what is possible—facts about what vegetarians and barbecues are usually like. Third, context suggests that any possible action must be assessed for its relevance to my intention to have a successful barbecue.

In a seminal series of papers (notably (Kratzer, 1977), (Kratzer, 1981), and (Kratzer, 1991)), Kratzer characterizes and formalizes these contextual dependencies. Her tools are an ontology of alternative, total possible world histories, and two parameters that can be supplied to modal quantifiers over such histories. One, the MODAL BASE, describes the set of possibilities under general consideration. For (1c), the modal base picks out those possible histories where vegetarians come to the barbecue, and that constitute a world otherwise like ours. The other parameter, the ORDERING SOURCE, describes the plausibility and present relevance of the different possibilities given by the modal base. For (1c), the ordering source ranks higher those possibilities that lead naturally to a successful barbecue. Kratzer goes on to show how these parameters can be given a precise, formal role in the semantics of modals. Each alternative history is interpreted model-theoretically as a possible world, and modals are interpreted as quantifiers that range over the ordering-source-best worlds in the modal base.

Although Kratzer’s work outlines how modals can and do vary in interpretation in context, it remains largely open how language users arrive at the particular contextual interpretations they do. Answering this question is an important precondition to the construction of computational systems that participate in ordinary modal talk, and to the understanding of the play of context and coherence in discourse generally. This paper addresses one aspect of the contextual dependency of modal meaning from this perspective. This is the phenomenon of MODAL SUBORDINATION (Roberts, 1986), present in (1) but better illustrated by the famous exemplar in (4).²

(4) A wolf might walk into the house. It would eat you.

In context, the meaning of the second sentence can be paraphrased as in (5).

(5) If a wolf walked into the house, it would eat you.

The force of the assertion is restricted to a hypothetical scenario where a wolf walks in. Because of this restriction, the hypothetical wolf can serve as the referent for the pronoun it.

Here is an intuitive account of how (4) gets its meaning. The first sentence introduces consideration of a hypothetical possibility into the discourse. The next

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²(4) is based on (Roberts, 1986) (11), where it is attributed to Fred Landman. I will use Roberts’s term modal subordination to refer to the phenomenon exhibited, not its analysis; as we shall see, the examples might better be said to illustrate modal anaphora.
sentence refers to this possibility, and makes an additional claim about it. In other words, the interpretation of (4) arises from an anaphoric connection between the two modal sentences: they refer to a common semantic object, in virtue of a formal link.

The recognition that modals can refer to evoked possibilities goes back at least to (Isard, 1974). The contribution of this paper is to show how to formalize this explanation robustly and naturally, so as to describe meanings of discourses like (1) using principles familiar from other domains and a simple semantic ontology that is open to refinement. The inspiration for this account comes from treatments of tense in discourse (Partee, 1984; Hinrichs, 1986; Webber, 1988); in fact, the account presents the interpretive connections between modal clauses in discourse as exact analogs to the temporal connections between successive tensed clauses. The result is a Reichenbachian approach to modality, where modal meanings are built by imposing a limited number of relations between scenarios evoked by the point of speech, the topic of the utterance, and the description the utterance presents. To my knowledge, no previous semantic research has investigated the formal parallel between modality and tense in this way. In virtue of the success of the Reichenbachian analysis of tense, it is anticipated that the parallel approach to modal meaning will streamline the treatment of modals across theories of syntax, acquisition and computation.

Technical specialists may compare this proposal to recent work of (Kibble, 1994; Portner, 1994; Geurts, 1995; Kibble, 1995; Frank and Kamp, 1997), all of which explore an anaphoric analysis of modals, without making an explicit connection with tense. The previous proposals all take modals to refer to complex, dynamic objects—objects that must be interpreted by a semantically explicit quantifier and which set up referents by a special mechanism. In contrast, I propose to represent modal referents by abstract static objects that can be described without overt linguistic quantification and which do not set up referents in a special way. This difference in ontology is a crucial technical innovation of the present approach. The conceptual advantage of the new ontology is illustrated by its parallel account of the (a) and (b) examples of (6) and (7).

(6) a A wolf walked in. A victim was waiting. It ate him.
   b A wolf might walk in. A victim would be waiting. It would eat him.

(7) a John ate a cheesesteak. It {was, #is} very greasy.
   b John might be eating a cheesesteak. It {would be, #is} very greasy.

With the new ontology, all illustrate introduction and subsequent reference to entities in a flat model of the discourse—whether those entities are individuals, times, events or scenarios. Subsequent reference is restricted by the need to meet presuppositions. As outlined in more detail in section 5, the different types assigned to temporal and modal referents forces previous proposals to give completely separate formal
treatments to the (a) and (b) data in (6) and (7). In particular, the (b) cases require the construction of a discourse model with a special nested structure and new constraints about when access to entities in this nested structure is possible.

The empirical consequences of the new ontology are illustrated by (8):

(8) A wolf might walk in. We would be safe because John has a gun. He would use it to shoot it.

With the new ontology, this discourse describes a development that the speaker asserts is a possible continuation of real events, as a single static object. The speaker introduces entities, like John’s gun, into the DISCOURSE, not into particular scenarios. The update that adds a real gun to the discourse automatically enables subsequent discourse to refer to the gun in the wolf-scenario, because the scenario is compatible with reality. So, we straightforwardly predict the pattern of possible reference seen in (8). But in previous approaches, we can only refer to the gun in a NEW scenario to which the gun has been EXPLICITLY added. Adding the gun a new real scenario leaves the hypothetical scenario unchanged—and gunless; cf. (Roberts, 1995).

The outline of the paper is as follows. In section 2, I review the evidence for the anaphoric interpretation of tense; I show that precisely the same attributes describe the interpretation of modals. In section 3, I describe the theoretical description of tense which I take as a model for extending this parallel in the data to a parallel in formal accounts. Then in section 4, I present the new account of modals, emphasizing the high-level formal parallel between modals and tenses and the correspondence between the high-level view and its model-theoretic implementation. Section 5 observes formally how the new account remedies the awkward features of previous accounts of modal subordination. I close in section 6 with a sketch of the role the alternative proposal may play in further linguistic explanations.

2 The Parallel between Modality and Tense

We begin by a comparison between modality and tense. As with modality, tense exhibits a contextual dependence that can be described as anaphoric. But with temporal anaphora, previous research has compiled relevant linguistic judgments and traced out the connections between these judgments and theoretical constructs in great detail. This work offers a resource for organizing data and explanations for the account of modals, because modals are, in the relevant respects, JUST LIKE TENSES. In this section, I review several kinds of data characterizing temporal interpretation, and illustrate parallel examples for modals.

2.1 Similarities in connections and binding

Partee’s treatment of tense as anaphoric (Partee, 1973; Partee, 1984) depends on a number of similarities between the temporal interpretation of clauses and the interpretation of pronouns. Partee observed that tenses, like pronouns, may be used
to refer to a specific object from nonlinguistic context, as shown by the parallel between (9) and (10).

(9) She left me. (= (Partee, 1984) 1b)

(10) I didn’t turn off the stove.

Either sentence may be used felicitously out of the blue: just as she can refer to the significant woman in the speaker’s life, didn’t can describe the most recent time the speaker left home.

Like pronouns, tenses may refer to specific entities introduced earlier by definite reference. (12) resembles (11) in this respect: when John saw Mary picks out a time familiar to the addressee in the same way Sam picks out a familiar individual.

(11) Sam is married. He has three children. (= (Partee, 1984) 2a)

(12) When John saw Mary, she crossed the street. (= (Partee, 1984) 2c)

Alternatively, either tenses or pronouns may refer to entities introduced into the discourse indefinitely, as in (13) and (14).

(13) Pedro owns a donkey. He beats it. (= (Partee, 1984) 3a)

(14) Mary woke up some time during the night. She turned on the light. (= (Partee, 1984) 3b)

Moreover, the time recovered for a tense may covary as if bound by some operator. This may parallel the ordinary binding of pronouns by quantifiers, as whenever in (16) parallels every in (15).

(15) Every woman believes that she is happy. (= (Partee, 1984) 4a)

(16) Whenever Mary telephoned, Sam was asleep. (= (Partee, 1984) 5a)

Alternatively, it may parallel the discourse binding of ‘donkey anaphora’—as on a Friday in (18) parallels a donkey in (17).

(17) If Pedro owns a donkey, he beats it. (= (Partee, 1984) 6a)

(18) Whenever Mary telephoned on a Friday, Sam was asleep. (= (Partee, 1984) 6d)

For each of these examples, parallel sentences can be constructed where the interpretation of a modal auxiliary depends on a particular possible scenario that the speaker seems to refer to in using the modal. This scenario may be salient in the extralinguistic context: if a speaker utters (19) while looking at a high-end stereo in an electronics store,
(19) My neighbors would kill me.

the utterance is interpreted as asserting only that, if the speaker bought the stereo and played it a “satisfying” volume, the neighbors would be incited to violent retaliation.

Of course, explicit if-clauses may introduce some hypothetical scenario by definite reference, just as when-clauses introduce a time. Such hypotheses may be recovered for modals, within and across sentences, as shown by (20) (from the Brown Corpus).

(20) New York Central Railroad president Alfred E. Perlman said Tuesday his line would face the threat of bankruptcy if the Chesapeake & Ohio and Baltimore & Ohio Railroads merge. Perlman said bankruptcy would not be an immediate effect of the merger, but could possibly be an ultimate effect.

Modals of possibility, meanwhile, might be said to introduce a new hypothetical scenario—one where the possibility envisaged by the modal occurs—by indefinite reference. Subsequent modals may refer to such contexts, as in Roberts’s (4), or in (21) from the Brown Corpus.

(21) There may be other 1961 state committee retirements come April 18, but they will be leaving by choice of the Republican voters.

What about bound variable readings of modals? If you take possible-world semantics as given, you may regard ANY conditional as an illustration of bound-variable modality. Unless a conditional totally describes a possible history, it must be interpreted using quantification over worlds. For now, let us put aside such formal considerations, and make a simpler observation. Many if-clauses parallel ordinary when-clauses in making a claim about what might intuitively be described as a single case. (The theoretical analysis of sections 5 and 4 will substantiate this point of view formally.) For example, we might judge (22) true provided one person (perhaps the first) to prove the result is recognized:

(22) When a mathematician proved Fermat’s Last Theorem, he gained great notoriety.

Just the same, we might judge (23) true provided that we anticipate that at least the first person to prove the result will be recognized.

(23) If a mathematician proves the Riemann hypothesis, he will gain great notoriety.

Such data contrast with cases where multiple instantiations of the conditional are clearly intended—for example, in any conditional interpreted generically. In such sentences, the link between alternative instantiations of the antecedent and corresponding realizations of the consequent surely reflects a bound-variable interpretation of modality. A simple illustration is provided by (24).
If a concertgoer arrives late, he or she will not be permitted into the auditorium until intermission.

In the intended reading of (24), the scenarios evoked by the antecedent and described in the consequent vary across many different concertgoers. A more complicated illustration is given in (25):

If a submarine can not self-destruct if an enemy captures it, the enemy will learn its secrets.

Here the antecedent contains a constituent if-clause, if an enemy captures it, that varies across submarines much like the if-clause in (24) varies across concertgoers. At the same time, the modal will in the consequent describes a scenario that includes this capture of the submarine. So the interpretation of the consequent varies with the interpretation of a subconstituent of the antecedent (and not simply with the interpretation of the antecedent as a whole). Because of this, the sentence may be regarded as an exemplifying an analogue of ‘donkey’ anaphora for modals.

We conclude that, for these five phenomena that Partee considered, the interpretation of modals offers the same range of effects that characterize the interpretation of pronouns and tense. The only difference is in the type of objects involved. Where pronouns refer to individuals, tenses refer to times/events, and modals refer to hypothetical scenarios.

2.2 Parallels in presuppositions about referents

We have seen sequences such as in (26) that illustrate parallel cases where reference is possible:

a. Pedro owns a donkey. He beats it.
b. When John saw Mary, she crossed the street.
c. If the railroads merged, the line would face bankruptcy.

Equally parallel are sequences such as in (27) where reference is NOT possible.

a. #Pedro owns a donkey. She beats it.
b. #When John saw Mary, she crosses the street.
c. #If the railroads merged, the line will face bankruptcy.

In each case, the anaphoric element—the pronoun she, the present tense in crosses, the ‘vivid’ modality in will—is incompatible with its intended referent. This is because the anaphoric forms impose presuppositions that their referents must satisfy.

In the nominal domain, these presuppositions are simple properties. Pronouns must agree with their antecedents in number, person, and in some cases gender: she can refer to an entity only if the entity is a single female other than the speaker or addressee of the utterance.
In both the temporal and modal domain, these properties are better described as RELATIONS between antecedents and an entity deictically or indexically related to the point of speech. Thus the PRESENT tense may refer only to the moment of speech, while the PAST tense may refer only to times that precede the moment of speech. Since no time enjoys both characteristics, (27b) is incoherent. In a similar way, factual or REAL modals may refer only to the scenario given by the actual perspective of the speaker; real modals include ordinary present tense indicative verbs like is and the epistemic modal must. In contrast, modals I shall call VIVID may refer to possible elaborations of the speaker’s information; will and can are representative vivid modals. Counterfactuals or REMOTE modals, like would or should, may even refer to scenarios that are not regarded as real possibilities given the state of the world at the moment of speech. In example (27c), since the conditional evokes a remote scenario while will requires a vivid one, the intended reading is inconsistent.3

Presuppositions about temporal and modal referents can influence what continuations are possible in discourse in several ways. The data in (27) show that presuppositions can prevent the time or scenario described by one sentence from being recovered in another. At the same time, as shown by the parallels in (7) (repeated below), these presuppositions indirectly can prevent the INDIVIDUALS described by one sentence from being recovered in another.

(7)  
  a  John ate a cheesesteak. It {was, #is} very greasy.
  b  John might be eating a cheesesteak. It {would be, #is} very greasy.

In describing something as literally greasy at a certain time in a certain scenario, a speaker presupposes that the object has a physical existence there. Thus the time or scenario in which such a predication is made must match the time or scenario in which the object is supposed to exist. In (7a), as in (27b), we know that the verb is cannot refer to the time when John ate. Given its tense, is should refer to now. So it cannot refer to the cheesesteak, either: the cheesesteak doesn’t exist anymore, and whatever is greasy is supposed to exist now. Meanwhile, in (7b), as in (27c), we know that is cannot refer to the scenario evoked by might. As a factual verb, is must describe a scenario the speaker presupposes to be real. Again, that means it cannot refer to the cheesesteak, since the cheesesteak, unlike whatever is greasy, is not presumed to be real.

This explanation does not give the construction that introduces an entity defini-

tive control over where that referent can appear; it uses both the construction and

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3The handy term REMOTE is from (Isard, 1974). Modals are perhaps like tense in that it is difficult to formalize presuppositions of vividness or remoteness precisely and correctly as it is to get presentness or pastness exactly right for all cases. (Iatridou, peer) offers one convincing starting point for such explanations; (Frank and Kamp, 1997) considers the impact of descriptions of graded modality for definitions of modal presuppositions. Here, the informal notions will suffice, provided we observe that both tense and modals impose presuppositions that must ultimately appeal to some classification of referents by relation to a speech point.
the use of a concrete predication. This is important, and correct, for there are many properties that possible objects or future objects actually can have in the real world. For example, consider the data in (28).

(28)  
a. John will have an enormous wedding. It is his mother’s idea.  
b. John may have an enormous wedding. It is his mother’s idea.

An enormous future wedding can be John’s mother’s present idea. An enormous wedding can be her real idea even if the wedding seems quite unlikely to everyone else. Sure enough, reference in (28) is felicitous.  

2.3 Parallels in adverbials and clauses

Hinrichs (Hinrichs, 1986) observed a different range of behavior in temporal anaphora. Here too we find parallel behavior for references to hypothetical scenarios in discourse.

Hinrichs observed that tense morphemes are not the only constituents to show the sorts of contextual dependencies described by Partee. TEMPORAL ADVERBS, such as three hours later, and TEMPORAL CONJUNCTIONS, including when-clauses, are also interpreted with respect to contextually salient times. In fact, entities can be introduced by tense morphemes, temporal adverbs or temporal conjunctions; and regardless of how they are introduced, they may subsequently be referred to by constituents of any of these types.

We find the same freedom of behavior with modal adverbs such as otherwise or then and modal conjunctions such as if-clauses. We have already seen examples where modal morphemes provide antecedents for modal morphemes (e.g., (21)) and where modal conjunctions do (e.g., (23)). Examples of the remaining cases are presented below ((30), (31), and (34) are from the Brown corpus; the rest are constructed).

(29) Modal Adverb – Modal Morpheme: Otherwise, John would have walked down Walnut Street. He would have gone past the Library.

(30) Modal Morpheme – Modal Adverb: Wine stored for a long time should be on its side; otherwise, the cork dries and air enters to spoil it.

(31) Modal Conjunction – Modal Adverb: If they have trouble exporting, the international bill for their support will grow larger than it otherwise would.

(32) Modal Adverb – Modal Adverb: Otherwise, John would have gotten really upset, because then he would have eaten alone.

(33) Modal Morpheme – Modal Conjunction: A wolf might walk in. If you hid from it, it would not eat you.
(34) **Modal Conjunction – Modal Conjunction:** If you want a picture get to the corner of Adams and Clark just as fast as you can. **If you hurry** you might beat the headquarters boys.

(35) **Modal Adverb – Modal Conjunction:** Otherwise, John would have walked past Cinnabon. **If he felt hungry,** he would have bought a ’bon there.

2.4 **Parallels in modification**

Treating tense as anaphoric streamlines the interpretation of adverbials in another respect. As detailed by Hornstein (Hornstein, 1990), defining the meaning of tense in terms of Reichenbach’s three specified points enables temporal adverbials to be treated as **MODIFIERS** rather than **OPERATORS.** This more adequately describes their linguistic behavior.

Operators are functions that take sentences to sentences in an unrestricted way. In tense logic, cf. (Prior, 1967), temporal information is specified by operators like $\mathbf{P}$: $\mathbf{P}\varphi$ is true at time $t$ if and only if $\varphi$ is true at some time $t'$ before $t$. Analogously, an operator can be defined to formalize to the meaning of an adverbial phrase like *three weeks earlier*. Thus, **three weeks earlier** $\varphi$ is true at time $t$ if and only if $\varphi$ is true at some time $t'$ three weeks before $t$.

Logically, operators can be iterated. Composing definitions like those above, for example, the formula

(36) **three weeks earlier** (**four weeks earlier** $\varphi$)

is true at time $t$ just in case $\varphi$ is true at some time $t'$ seven weeks before $t$. The linguistic expressions these formulas model do not share this property, however. An English sentence like (37) is unintelligible.

(37) **#Three weeks earlier, four weeks earlier, John had left.**

Modifiers, in contrast, impose constraints on independently specified entities. As a modifier, **three weeks earlier** receives the following treatment. Given its argument time $t'$, **three weeks earlier** refers to another point $t$ (anaphorically specified) and requires that $t'$ precedes $t$ by three weeks. Naturally, alternative modifiers cannot assert contradictory properties of the same object. Similarly, for Gricean reasons, we should also disprefer cases where one modifier presupposes or entails the content of another. (In such cases, why use both?) On the other hand, we do not rule out the possibility that a speaker could construct a consistent, felicitous description using certain combinations of modifiers. In such cases, we simply predict that the modifiers in fact fuse into a single expression that imposes a compound condition on their argument. Examples like (37) and those considered by Hornstein reveal that the temporal adverbials of English share these characteristics of modifiers.
The same arguments apply for modal adverbials. Take possibly. Taking it as an operator, we say possibly p is true in a scenario w if p is true in some scenario w' possible from w. Taking it as a modifier, we say that given an argument scenario w' and reference scenario w, possibly requires that w' be possible from w—according to some particularly absolute or inclusive notion of possibility.

The operator view of possibly is very natural, but examining natural occurrences of possibly yields surprising and resounding evidence that it does modify—in patterns identical to temporal adverbs. Possibly occurs 59 times in the Brown corpus. Never does it co-occur with another modal adverbial, as in (38).

(38) #Necessarily, John will possibly come.

This is as expected if possibly constrains the interpretation of a verbal predication, for felicitous and consistent ways to explicitly describe a state of affairs in such multiple ways should be rare. In (38) for example, necessary already presupposes possibly.

Where does possibly occur? Half the time, it accompanies an adjunct or parenthetical expression, as in (39).

(39) She quickly moved into cafe society, possibly easing her conscience by talking constantly of her desire to be in show business.

Such uses are entirely compatible with a treatment of modal adverbs as modifiers. Stump (Stump, 1985) observes that the modal and temporal interpretation of adjuncts is in general varied, and suggests that pragmatics supplies a specification of time and mood for each adjunct appropriate to its context. Thus, in (39), possibly will modify the scenario recovered at easing—and, of course, influence its recovery. Such modification is just like the modification of the temporal specification inferred for an adjunct. (40) replaces possibly with later to demonstrate this.

(40) She quickly moved into cafe society, later easing her conscience by talking constantly of her desire to be in show business.

Interestingly, of the remaining cases, the large majority exhibit what Halliday calls MODAL HARMONY ((Halliday, 1970), cf. (Coates, 1983)). In these examples, possibly occurs in conjunction with a modal auxiliary that already expresses possibility.

(41) a I may possibly be a greater risk than is the normal person of my age.
    b She was eating bread and cheese just as fast as she possibly could, and washing it down with red wine.

On an operator view, these cases are particularly surprising since they seem to involve two modal operators, somehow taking scope with respect to each other, when one modal operator would express the same thing.
On a modifier view, however, the harmony between *may* or *could* and *possibly* is in fact expected; it parallels the harmony between tenses and modifiers seen with *three weeks earlier*. That is, *possibly* contributes a substantive meaning independent of *may* or *could*; the modal auxiliaries appear as a reflex of the kind of meaning *possibly* contributes.

Recall Kratzer’s observation that modal meaning depends on the context in several ways. For example, *as fast as she could* can be fleshed out in any of the ways below:

(42) a  as fast as she legally could  
    b  as fast as she physically could  
    c  as fast as she morally could  
    d  as fast as she conceivably could

We don’t see any of these modifiers as redundant, since they help specify an independent feature of the interpretation of *could*. It is natural to regard *possibly* as functioning in the same way. In (41)—and in (39) for that matter—*possibly* seems to indicate that the intended modality is some kind of absolute possibility, perhaps possibility with an empty ordering source or a modal base that takes nothing extra for granted. In other words, its use highlights that all possible alternatives are under consideration in the utterance, not just those possibilities good enough for the purpose at hand. If *possibly* offers this specific additional contribution as a modifier of modality, it is less surprising that the auxiliary is also coded for possibility. In fact, it is predicted! What arises is exactly the situation found with temporal modification in (43), where a past perfect verb indicates that the *E* point precedes the *R* point at the same time as the temporal modifier *three weeks earlier* indicates that *E* precedes its reference point by a specified amount:

(43) Three weeks earlier, John had left.

The surprises, on the modifier view, are actually the cases where the modifier appears but the verb does NOT harmonize. In such cases, the modal presupposition of the verb (that the sentence describes reality) conflicts with the meaning of the modifier (that the sentence describes a possibility). Such examples are infrequent but attested:

(44) Possibly their compulsivity was not strong enough to cause them to build their own structure.

Such cases seem to resemble the narrative use of the present tense in describing past time. Hornstein’s example is (45).

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4In fact, harmonic *possibly* is almost a negative polarity item (NPI). Typical uses include *how can you possibly...?* or *you can’t possibly...*; (a) is not representative. This pattern of use corroborates the present characterization of the meaning of *possibly*. As Kadmon and Landman (Kadmon and Landman, 1993) and Eisner (Eisner, 1994) argue, NPI distribution is in part associated with the fact that NPIs refer to all conceivable exemplars, not just the expected ones.
(45) It was 1812, just before the Battle of Borodino. The anticipation of coming struggle is palpable. Napoleon has just woken. (= (Hornstein, 1990) (3) p.11)

In tense, such examples are taken to reflect flexibility in what is considered the present in interpreting a verb: the historical present is a case when the S point is taken to be something other than the actual present time. A similar flexibility in verbal modality may account for examples like (44): the scenario that grounds the modal interpretation of was is not the speaker’s current scenario, but some other possible one.

In both tense and modality, then, the evidence suggests the existence of anaphoric elements, much like pronouns. These referents figure not only in how the interpretations of verbs and adverbials depend on context, but also in how the interpretations of verbs and adverbials fit together.

3 Semantics of Tense
In light of the data outlined in section 2, I propose to aim for a formal framework for describing modal meaning that mirrors as closely as possible frameworks for describing the interpretation of tense. Such frameworks must provide formal and precise answers to three questions, namely: what are the semantic objects to which interpretations refer? how do the meanings of elements depend on those referents? and how do relations between sentences in discourse constrain what objects are associated with which elements in interpretation? Although details vary in the ontology and mechanisms involved in different accounts, views are converging on the general outline of how such an account for tense should be formulated. In this section, I briefly describe the properties of this general account, following most closely the work of Webber (Webber, 1988) and Muskens (Muskens, 1995; Muskens, 1996).

3.1 Webber’s approach to tense
For Webber, the discourse entities that figure in the interpretation of tenses are not only times but also eventualities: events, states, or processes in the world (Bach, 1986). As described by Moens and Steedman (Moens and Steedman, 1988), such eventualities can stand in a variety of relations with one another. In particular, each

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Footnote: Researchers disagree on a number of questions, including: where reference should be to times and where it should be to events; whether rules should keep only one reference point available (Partee, 1984; Hinrichs, 1986) or allow many to be accessible (Webber, 1988); whether connections need not be treated as explicitly anaphoric, but are merely derived from the need to recover coherence relations connecting the content of adjacent utterances (Lascarides, 1992; Lascarides et al., 1992) (cf. (Hobbs et al., 1988)); and whether some constraints on interpretation reflect a syntactic level for the representation of tense, not reducible to semantics or pragmatics (Hornstein, 1990). Fortunately, we can sidestep such thorny issues here: parallelism may be maintained because each choice in the theory of tense corresponds to an analogous choice in the theory of modality.
event may be associated with the preparatory process that brings it about, and with
the consequent state that the event initiates.

The interpretation of tenses in terms of these parameters follows a pattern first
suggested by Reichenbach (Reichenbach, 1947). The meaning of a tense morpheme
introduces an event \( E \) in relation to two points or intervals in time, \( S \) and \( R \). \( S \)—
abbreviating speech—indicates the time that is to be regarded as the present moment
in interpreting the tense. For main clauses, \( S \) is deictically related to the moment the
utterance or interpretation of a sentence takes place. \( R \)—abbreviating reference—
gives a time that establishes the relevance of the current statement to the matter
at hand. (Accordingly, \( R \) has been termed the TOPIC TIME by (Klein, 1992).) As
the name reference suggests, the value of \( R \) depends anaphorically on some salient
eventuality, typically one evoked in prior discourse. Given these two points, the
meaning of a tense is specified by certain temporal relations imposed between \( S, R, \)
and the time \( E \) occurs.

A good illustration of such a meaning is provided by (one sense of) the English
past perfect:

(46) John had chased the wolf.

The English past requires that \( R \) precede \( S \): the reference point is in the past. The
perfect also imposes the constraint that \( E \) precede \( R \): the eventuality described
occurs before the moment under discussion. (Note that the meaning of the perfect
in fact involves a causal relation between \( E \) and \( R \) as well as a temporal one.) Thus,
the past perfect in (46) introduces an event where John chases the wolf at some
time before the salient past moment \( R \) which the discourse has been describing (and
indicates that the relevant consequences of the chase still hold at \( R \)).

To make precise the anaphoric connection between the \( R \) point of a tense and
previous discourse, Webber introduces the following definition of anaphors.

(47) a They specify entities in an evolving model of the discourse that the
listener is constructing;

b the particular entity specified depends on another entity in that part of the
evolving “discourse model” that the listener is currently attending
to. (Webber, 1988) p.61.

The \( E \) point of tenses specify eventualities. The \( R \) point gives the time EITHER
of some eventuality directly specified by prior discourse OR of an eventuality IN-
FERRABLE using Moens and Steedman’s ontology of events from an eventuality
already specified.

The flexible dependency accounts for the variability in patterns of temporal
reference. For an example like (48), the reference time of the second tense seems
to be the event time of the first.

(48) a John played the piano.
b Mary played the kazoo. (= (Webber, 1988) (12))

The discourse suggests that the playing events were simultaneous. (For the simple past tense, the event \( E \) must take place at time \( R \).) Meanwhile, for example (49), the reference time of the second tense seems to be the duration of the consequent state of the first event.

(49) a John went into the florist shop.
    b He picked out three red roses, two white ones, and one pale pink.
    (= (Webber, 1988) (13))

Finally, examples like (50) show that the reference time may be derived from the preparatory process of the event specified in an earlier sentence.

(50) a John bought Mary some flowers.
    b He picked out three red roses, two white ones, and one pale pink.
    (= (Webber, 1988) (14))

3.2 Muskens's semantic formalism

Webber's approach can be accommodated straightforwardly into a formal system for describing semantics and anaphora. In this paper, we adopt a DYNAMIC semantics, and follow Muskens’s presentation of it as an axiomatic theory in classical type theory.

Muskens works with a many-sorted logic where special sets can be defined as primitive sorts. To represent the objects to which tenses refer, Muskens includes a type \( e \) of eventualities and a type \( \tau \) of times to complement the ordinary types \( e \) for individuals and \( t \) for truth values. (In this brief review, I suppress the limited role of possible worlds in Muskens's proposal.) Here, we accommodate the finer ontology of events needed for Webber’s account of temporal reference by dividing \( e \) into subtypes: \( \eta \) for events, \( \pi \) for processes, and \( \sigma \) for states.

To interpret the conditions tenses impose, Muskens axiomatizes familiar operations for combining times and eventualities. In particular, each eventuality \( e \) is associated with the time \( \vartheta e \) at which it occurs; and the relation \( t < t' \) holds when the interval \( t \) completely precedes the interval \( t' \). We can then define conditions \( \text{past}(s, r) \) and \( \text{nonpast}(s, r) \), \( \text{simple}(r, e) \) and \( \text{perfect}(r, e) \) that describe the Reichenbachian conditions imposed (or presupposed) between speech time \( s \), reference time \( r \), and the time \( \vartheta e \) when the eventuality takes place. We will augment this ontology with the functions on eventualities proposed by Moens and Steedman: if \( e \) is an event, \( \text{PREP}(e) \) denotes the preparatory process leading up to \( e \), and \( \text{CON}(e) \) denotes the consequent state following \( e \). (For present purposes, we follow Muskens in treating events, processes and states equally. To extend this ontology to account more generally for aspectual differences, (White, 1994) suggests using the cross-cutting ontological category of delimitedness, so that states are distinguished from
events in part because the temporal trace $\vartheta(s)$ of a state is an undelimited temporal quantity while the trace $\vartheta(e)$ of an event is a delimited one.)

Finally, to represent anaphoric connections, we give utterances semantics that describes not only their truth conditions but also their change in context: a dynamic semantics. The starting point is to introduce contextual ENVIRONMENTS as an abstract type of object representing the evolving model of discourse that the listener constructs; we abbreviate this type $s$. (Here we depart from Muskens’s overloaded use of the term state.) An environment contains entities of various types in cells, metaphorically conceived as analogous to locations in a computer memory or file system. As the discourse evolves, new elements are assigned to cells, perhaps nondeterministically, and perhaps the elements in old cells are reassigned; interpreting a constituent in terms of a cell links its value to the current state. For example, an indefinite NP will assign a cell to hold its individual referent; subsequent pronouns may then refer to it. Likewise, now, tense morphemes will assign cells to hold new eventuality referents for $e$; the $r$ point will pick out times based on the current state. Overall, the meaning of a sentence is a relation between possible input environments, where the listener is before the sentence is processed, and corresponding possible output environments, where the listener is once the sentence has been processed.

Contextual items, or DISCOURSE MARKERS, pick out the elements in cells. (This supersedes Muskens’s idiosyncratic use of the term store.) For uniformity of types, (Muskens, 1995) treats environments as primitives, and treats stores as functions on environments; a discourse marker has type $\alpha$ when the entities in the corresponding cell would have type $\alpha$. Using an axiomatic theory that makes explicit the set of discourse markers and provides names for markers, Muskens ensures that the values of markers in environments may be changed arbitrarily. Muskens then defines formulas $i[v_1, \ldots, v_k]$ that relates two environments that differ only in the values at markers $v_1$ through $v_k$.

Using this ontology, Muskens shows how the meanings of sentences can be specified using syntactic abbreviations inspired by the discourse representation structures or DRSs, of Discourse Representation Theory (Kamp, 1981; Heim, 1982; Kamp and Reyle, 1993). For present purposes, it suffices to consider DRSs built according to the following BNF syntax definition:

\[
\begin{align*}
C & ::= R\{t_1, \ldots, t_n\} \mid C, C \\
DRS & ::= [ v_1, \ldots, v_k \mid C ] \mid DRS; DRS
\end{align*}
\]

$C$ represents the type of CONDITIONS imposed in a DRS; semantically conditions are predicates of environments. In environment $i$, the atomic condition $R\{t_1, \ldots, t_n\}$ ensures that the relation $R$ holds between the values at $i$ of $t_1$ through $t_n$. The condition $C_1, C_2$ holds of exactly the environments that satisfy both $C_1$ and $C_2$.

Each DRS, meanwhile, is interpreted as a relation between environments. An atomic DRS $[ v_1, \ldots, v_k \mid C ]$ permits the output to differ from the input in having arbitrary new values of markers $v_1$ through $v_k$, so long as the output satisfies the
condition $C$. If $v_i$ is one of the new variables introduced in atomic DRS $D$, and gives a marker returning $\alpha$ values, we say $v_i : \alpha$ is in the universe of $D$, or $v_i : \alpha \in U(D)$. A complex DRS $D_1 ; D_2$ relates an input with any output that may be obtained by processing $D_1$ to obtain an intermediate environment and using that intermediate as the input to $D_2$. To indicate the link between the occurrences of $D_1$ and $D_2$ when the occurrences are constituents of an occurrence of $D_1 ; D_2$, we write $D_1 \leadsto D_2$.

These interpretations of conditions and DRSs can be made precise by a translation into the raw language of the theory of types, defined as follows. For a term $t$, the translation $t^\circ$ is a function whose value at environment $i$ is obtained from $t$ by replacing each occurrence of a free marker $v_n$ in $t$ by $v_n i$, its value at $i$. Then we have:

\begin{align*}
(R\{t_1, \ldots, t_n\})^\circ & \equiv \lambda i. R(t_1^\circ, \ldots, t_n^\circ) \\
(C_1, C_2)^\circ & \equiv \lambda i. C_1^\circ i \land C_2^\circ i \\
[v_1, \ldots, v_k \mid C] & \equiv \lambda i j. [v_1, \ldots, v_k] j \land C^\circ j \\
D_1 ; D_2 & \equiv \lambda i j k. D_1^\circ ik \land D_2^\circ kj
\end{align*}

Alternatively, we can make these interpretations precise by erasing the translation marks and then simply regarding the left hand side of each equation in (52) as an abbreviation for the right hand side. This is convenient because it allows us to freely mix DRS notation and type theoretic expressions.

The final component in Muskens's architecture is a system governing the accessibility of markers in context. Environments and relations between them provide an underlying semantics to what a listener's model of discourse is, how that model evolves, and how references to this model are to be interpreted. Accessibility, in a sense, provides the corresponding proof theory. In going from a natural language discourse to its underlying semantics, each anaphoric element translates as a marker—or, given the role of inferrable referents in Webber's account, possibly as a term containing markers. We constrain this process so that only accessible terms may be used as these translations. This ensures that the anaphors will be interpreted by meanings whose context-dependency agrees with the way context has indeed been established thus far in the discourse.

Precisely, suppose we wish to use a term $t$ in an atomic DRS $D$ to represent the content of some anaphor over type $\alpha$. Then we must derive that $t$ is an accessible term of type $\alpha$ in $D$. I use a new abbreviation for this result---$t : \alpha \in D$. Such derivations combines premises of the form $v : \alpha \in U(D_1)$ (that say where markers are introduced) and premises of the form $D_1 \leadsto D_2$ (that say how values are propagated through sequences of discourse), as dictated by the form of the overall DRS in which $D$ occurs. These premises are combined by two rules of inference. The first, (53), encodes the fact that markers are accessible in the DRS where they are introduced.

\begin{align*}
& v : \alpha \in U(K) \\
\frac{v : \alpha \in U(K)}{v : \alpha @ K} & \text{axiom}
\end{align*}
The second, (54), simply ensures that whatever becomes accessible remains accessible through subsequent updates:

$$
\frac{t : \alpha @ K}{t : \alpha @ K'} \sim
$$

(54)

The accessibility of inferrable referents can be captured by the introduction of additional rules of inference. For example, (55) captures the generality that wherever an eventuality $e$ is accessible, its duration $\vartheta e$ is also accessible:

$$
\frac{x : \pi @ D}{\vartheta x : \tau @ D}
$$

(55)

Likewise, an EVENT $e$ make accessible its preparatory process $\text{PREP}(e)$ and its consequent state $\text{CON}(e)$, as captured by these rules:

$$
\frac{x : \eta @ D}{\text{PREP}(x) : \pi @ D}
\frac{x : \eta @ D}{\text{CON}(x) : \sigma @ D}
$$

(56) a $\text{PREP}$

(56) b $\text{CON}$

This approach for making inferrable objects accessible is not literally present in Muskens or other approaches to dynamic semantics and discourse representation theory; however, some authors invoke related operations. For example, in their account of plural anaphora, (Kamp and Reyle, 1993) also describe inferrable referents using rules for constructing complex terms. The only difference is that Kamp and Reyle’s rules make the terms accessible indirectly, by changing the DRS to include new markers and conditions that identify them with the complex terms, and then using the simple definition of accessibility.

### 3.3 An example

As a concrete example, consider this simple version of Webber’s (49):

(57) John entered a shop. He chose some flowers.

The first sentence is translated into the DRS in (58), ignoring the determination of its initial reference time.

(58) $\mathbb{D} = [p, e, r | \text{shop}(p), \text{enter}(e, j, p), \text{simple}(r, e), \text{past}(\text{now}, r)]$

The second sentence will be sequenced after this DRS using the $;$-operator. In this context, the second sentence may be translated using the DRS in (59)

(59) $\mathbb{D} = [f, e' | \text{flowers}(f), \text{chose}(e', j, f),$

$$\text{simple}(\vartheta(\text{CON}(e)), e'), \text{past}(\text{now}, \vartheta(\text{CON}(e)))]$$
The temporal progression is represented by using \( \vartheta(\text{CON}(e)) \) as the reference time for the second tense. The derivation in (60) witnesses its accessibility.

\[
\begin{align*}
&\text{axiom} \quad e : \eta \in U(58) \\
&\quad \Rightarrow e : \eta @ (58) \quad \Rightarrow (58) \rightsquigarrow (59) \quad \rightsquigarrow \\
&\quad \text{CON} \quad \text{CON}(e) : \sigma @ (59) \\
&\quad \text{\vartheta}(\text{CON}(e)) : \tau @ (59) \\
&\quad \Rightarrow \Rightarrow
\end{align*}
\]

A shorter derivation on the same lines justifies the use of the discourse marker \( j \) to interpret \( he \).

4 A new, parallel analysis

Our goal is a formal account where temporal and modal morphemes, like pronouns, involve reference to entities represented in a model of discourse. In the last section, we saw such an account for tenses. The relevant entities are the TIMES and EVENTUALITIES mentioned in discourse: using these eventualities and an ontological theory of the relationships between them, temporal constraints on related sentences can be captured with a simple extension of dynamic semantics and a natural formulation of tense meaning.

In this section, I establish a semantics for modals that parallels the semantics of tense. At the center is an abstract ontology of scenarios—ordinary, first-class discourse entities that give an atomic view of a partially-described possibility. The meaning of modal words is formalized in terms of primitive but parameterized operators of possibility, necessity and conditionalization that refer to scenarios; such reference figures equally in every sentence. Scenarios are governed by a straightforward regime that accumulates new values as discourse evolves.

I introduce this construction in two steps. The first step, presented in sections 4.1–4.4, is to motivate and describe a general, flexible abstraction of the dynamics of scenarios and modal meaning in discourse. The second step, presented in sections 4.5 and 4.6, shows how these abstractions can be formalized and realized concretely in a possible-worlds semantics, while preserving the simple linguistic interface that described in the first step.

4.1 Ontological background

Recall Kratzer’s approach to the meanings of modal words, as outlined in the introduction. Modals depend on two parameters, a modal base and an ordering source; modals quantify over the worlds in the modal base that are closest to the ideal given by the ordering source. While these two parameters can describe modal meanings in context, they do not make explicit provision for scenarios as discourse referents. As we see in this section, giving an independent role to scenarios requires splitting the modal base into a number of components. Far from complicating
Kratzer's approach, this move allows us to describe modal meanings with a simpler and more flexible ontology.

The modal base and ordering source provide a simple formal model for describing three kinds of variation in modal meanings across contexts. First, the ordering source describes the meaning of deontic, epistemic and circumstantial modals in a common framework. The ordering source ranks possibilities by compatibility with some ideal, like what the speaker wants, what the law provides, or just what normally happens. As a result of this flexibility, the deontic interpretation of *John may legally leave* follows from one choice of ordering source; *may* considers only the possibilities that optimize John's compliance with the law and observes that one such possibility is to leave. At the same time, the epistemic interpretation of *John may conceivably leave* follows from its choice of ordering source; *may* considers all the possibilities we can conceive of—regardless of what ideals of the real world, like compliance with the law, may be disrupted there.

Second, the modal base includes the constraints involved in reference to a salient hypothetical scenario. Thus, the modal base in Kratzer's theory is in part designed to account for the data discussed in section 2.

But the modal base plays a dual role, and thereby captures a third kind of variation in modal meaning. The modal base includes further restrictions that encode the view taken on this scenario. For example, these restrictions might identify just the physical possibilities, the practical possibilities, or the mathematical possibilities that are compatible with the scenario. Kratzer's (61) illustrates the difference.

\[(61) \quad \begin{align*}
  a & \quad \text{Hydrangeas can grow here.} \\
  b & \quad \text{There might be hydrangeas growing here. (= (Kratzer, 1991), (21))}
\end{align*}
\]

The two sentences differ in the kinds of facts from the scenario that they take into account: *can* considers what would affect the growth of hydrangeas here (taking it for granted that someone tried to grow them); *might* considers our evidence about whether in fact hydrangeas do grow here. For Kratzer, this difference in modal view is reflected in the difference between circumstantial and epistemic modal bases.

The dynamics of discourse suggest that these two functions of the modal base should be separated. For, it is the scenario, not the modal view, that is referenced anaphorically in modal subordination. Example (62) shows this:

\[(62) \quad \text{If a wolf came in, I could escape. You might be eaten, though.}
\]

The two modals refer to a common scenario, but have different modal bases. As always, the modal base for *could* combines the constraints from the scenario with constraints on what is physically possible in each world. On the other hand, *might* combines those constraints with what is EPISTEMICALLY POSSIBLE, for example in view of the speaker's available evidence.

Separating the scenario from the modal view streamlines the compositional semantics of modals as well as the dynamics of discourse. It allows us to specify the
meanings of an indicative if-clauses independently of the modality of the consequent. There are two reasons why this cannot be done given Kratzer’s assumptions about modal bases, as she illustrates with (63).

(63) a If a murder occurs, the jurors must convene. (= (Kratzer, 1991) (24))
    b [A murder occurs] ⊃ must [the jurors convene] (= (Kratzer, 1991) option 1)

First, it is wrong to translate (63a) as (63b) because we evaluate the truth of (63b) at a single, total possible world. If no murder occurs there, (63b) is automatically true. Second, the conditional and the meaning of must must interact when must contributes its modal view to the modal base. Kratzer in fact uses the murder occurs to augment the modal base that is supplied to must, in such a way that only the full modal base is represented in the semantics. Kratzer always assigns such an indirect semantics to indicative if-clauses; both meaning of the if-clause and the scenario we associate with it are not represented using explicit objects.

Suppose however that all sentences—obviously modal or not—are evaluated against scenarios. Then we can use the if clause to elaborate some initial scenario, and supply the new scenario for must. This gives a compositional semantics for sentences like (63a) with a form much like (63b). Kratzer’s objection to (63b) does not apply, because the initial scenario can be a partial object where a murder is possible but not inevitable. Indeed, we already expect a murder to be possible in the initial scenario; because of the simple present form in the antecedent and the meaning of must, (63a) represents a vivid conditional, which should presuppose the possibility of murder.

In her account of modal subordination (Roberts, 1986), Roberts follows Kratzer’s use of the modal base to encode both a reference scenario and a modal view. The indirect semantic role Kratzer assigns to scenarios forces Roberts to explain modal subordination in terms of essentially syntactic operations. Roberts offers a formal treatment in which the contents of DRSs may be copied from the matrix of a possibility operator to the implicit if-clause that provides the restrictor of a subsequent modal. These copied elements are then free to play the idiosyncratic semantic role Kratzer’s theory gives them. Although it accounts for a variety of examples of modal subordination, this analysis is ultimately unsatisfactory (as recognized by a variety of recent researchers). It explains modal subordination not as an ordinary, anaphoric process but in terms of a syntactic mechanism motivated by exceptional circumstances of language use and entirely different from mechanisms that handle all other anaphora.

Separating the scenario from the modal view, and thereby separating the semantics of conditionals and modals, also makes it possible to maintain a close parallel between indicative and counterfactual conditionals. Kratzer’s treatment of indicative conditionals as figuring into the modal base does not extend to counterfactuals. Instead, Kratzer proposes that counterfactuals represent a kind of modal
quantification. They have a UNIVERSAL modal base, which includes any conceivable possibility compatible with the content of the if-clause, no matter how remote it is from the actual world. This modal base is restricted by a REALISTIC ordering source, which counts possibilities as better when they are more like the real world.

In discourse, however, counterfactuals seem no different from indicative conditionals in operating on old scenarios to create new, slightly different ones. (Dudman emphasizes this point (Dudman, 1984; Dudman, 1988).) For example, suppose a discourse sets up a scenario by beginning with (64a):

(64)  a  A wolf may come in. It will eat you...
     b  If it enjoys you, it will eat someone else.
     c  Luckily it will dislike the experience: had it enjoyed you, it would have eaten someone else.

The discourse may then comment on this scenario by considering a richer version with additional detail, as in (64b). But it may just as well comment on the scenario by describing a related but incompatible scenario, as in (64c). Either way the assumption or scenario introduced earlier is the object of subsequent reference. As with (62), such reference is complicated on Kratzer’s account because for technical reasons the modal base is the only semantic constituent that plays a formal role in the analysis, and the modal base varies from one sentence to another. However, it seems clear that by keeping a more abstract account of how one scenario can be related to another—by elaboration or transformation—we can accommodate both (64b) and (64c) to a common paradigm.

At the same time, separating counterfactual from modal meaning clears up a troubling gap from Kratzer’s theory. For Kratzer, (65) raises a formal quandary.

(65)  If Reagan had died in the assassination attempt, Hinckley (still) should not have been executed, because of his insanity.

Intuitively, there is no problem here; the conditional evokes the scenario most like the real world but where Reagan dies, and the modal describes what happens in the most ideal possible continuations of that scenario. But Kratzer’s approach requires (65) to reconcile seemingly incompatible uses of the ordering source. In parallel with (63a), this sentence should exhibit a single modal operator should with its usual ordering source and a modal base modified by the hypothesis that Reagan dies. But in parallel with (64c), we should use an ordering source that compares possible worlds to reality. How and why must we reconcile these considerations to use a single, right ordering source? It is unclear.

These arguments show that to capture the linguistic data, we should give judgments the same types whether made factually or conditionally, and we should give indicative and counterfactual conditionals a parallel semantics in terms of reference to old scenarios and introduction of new ones. We need an ontology of the following sort. Scenarios must be partial objects; all sentences, including realistic ones,
must be descriptions of an incomplete possibility. (Abstract models of this general kind are now familiar from situation semantics (Barwise and Perry, 1983) and data semantics (Veltman, 1981; Landman, 1986).) Scenarios as partial objects must remain subject to the richly parameterized relationships of possibility and necessity that Kratzer has developed. Moreover, they must be related by developments of elaboration and transformation, in which if-clauses refer to one scenario and modify it so as to explicitly introduce a new scenario into the discourse model. (Such relationships extend those so far developed in situation semantics or data semantics.) In the next sections, I articulate abstractly how to flesh out such an ontology for scenarios, and how to use such a theory to account for the parallel between modality and tense observed in section 2.

4.2 An abstract view

The abstract view begins with a new primitive type \( \omega \) of hypothetical scenarios, and new discourse markers of type \( \omega \). I notate these markers by variables \( \omega_i \). These are the entities modals evoke, refer to, and describe.

As a formal counterpart to the evocation of new scenarios, we have a new DRS that introduces scenario markers. Given a DRS \( D, D' = \text{if}(\omega, \omega', D, \ast) \) is also a DRS. \( D' \) updates the environment so that \( \omega' \) is a new scenario modifying or extending \( \omega \) by the update \( D \) (respecting some contextual parameterization \( \ast \)). The resulting environment reflects not only the introduction of \( \omega' \), but also the update \( D \) performed in obtaining it. Because of this update, we can invoke a formal link \( D \sim D_1 \) for every link \( D' \sim D_1 \) from the \text{if} statement to subsequent discourse. Moreover, we can represent the semantic relationship between the two scenarios \( \omega \) and \( \omega' \) described by the \text{if} statement by a formal link of its own: \( \omega \sim \omega' \). This is the proof-theoretic counterpart to the semantic constraint that \( \omega' \) represents a development of \( \omega \).

Scenarios, like times, are referenced each time a statement is checked for truth. Accordingly, the first argument of each atomic condition is a scenario marker: \( R\{\omega, t_1, \ldots, t_n\} \). In environment \( i \), this condition is true exactly when \( \omega \) provides enough information about the values of \( t_1 \) through \( t_n \) in environment \( i \) to decide that they satisfy the relation \( R \) there.

A scenario marker is also invoked each time an entity is introduced into the environment; the entity is presumed to exist at that scenario, and others accessible from it, but not necessarily elsewhere. Accordingly, each DRS is now represented \( [\omega : v_1, \ldots, v_k | C] \); the markers \( v_1, \ldots, v_k \) are introduced at \( \omega \). If \( v \) has type \( \alpha \), we indicate its declaration in a DRS by the formula \( v : \alpha \in U(\omega; D) \). (For if-DRSs, we record the introduction of \( \omega' \) by \( \omega' : \omega \in U(\omega'; D) \).)

These declarations contribute to a formal account of existence presuppositions that extends the ordinary inference rules for accessibility.\(^6\) The new formal account

---

\(^6\)As observed in section 2.2, a similar relativization is needed in principle for times as well. We will suppress this here. In the temporal case, it is more difficult to give this relativization an
of accessibility determines whether we can represent an anaphor over type $\alpha$ in a predication that imposes an existence presupposition at scenario $\omega$ as part of an atomic DRS $D$ by a term $t$. These conclusions are abbreviated $t : \alpha @ \omega; D$, and are derived according to a system of inference that invokes premises about the formal structure of introduction of referents in the overall DRS containing $D$: $v : \alpha \in U(\omega_1; D_1), D_1 \vDash D_2$ and $\omega_1 \vDash \omega_2$. The three basic rules are an axiom of accessibility:

$$
\frac{v : \alpha \in U(\omega; K)}{v : \alpha @ \omega; K} \text{ axiom}
$$

(66)
a rule propagating accessibility through UPDATES:

$$
\frac{t : \alpha @ \omega; K \quad K \vDash K'}{t : \alpha @ \omega; K'} \vDash U
$$

(67)
and a rule propagating accessibility through CONDITIONALIZATION:

$$
\frac{t : \alpha @ \omega; K \quad \omega \vDash \omega'}{t : \alpha @ \omega'; K} \vDash C
$$

(68)

The rules given in (55) and (56) can be adapted simply by relativizing them to a scenario marker as well as a DRS.

4.3 Modal meaning and the meaning of modals

Modal meaning is formalized in terms of reference to three scenarios: $\omega_s$, the present information of the speaker; $\omega_r$, a reference scenario; and a new scenario $\omega_e$ where the content of the sentence holds. Each modal asserts the relationship between $\omega_r$ and $\omega_e$ using new MODAL CONDITIONS of possibility and necessity: $\text{poss}(\omega_r, \omega_e, \ast)$ and $\text{necc}(\omega_r, \omega_e, \ast)$. These characterize environments $i$ where $\omega_r$ gives enough information about $\omega_e$ to conclude that $\omega_e$ is a possibility (or necessity, respectively) according to the standards and ideals represented by the contextual parameter $\ast$. These conditions are used to describe the relation between the scenario referred to by a modal verb like must, should and may and the scenario described by the content of its clause.

For example, consider may:

$$
(69) \quad \text{John may leave.}
$$

Under this account, the interpretation of (69) proceeds by the following steps. Some reference scenario $\omega_r$ is obtained from the context, and the scenario $\omega_e$ just like $\omega_r$, but where John leaves is constructed and introduced into the discourse:

---

interesting formal role. On the one hand, an entity existing at a given time is typically presumed to exist arbitrarily before and after; on the other, lexical semantics and pragmatics commonly overrides that presumption.
The force of *may* is to assert that \( \omega_e \) is possible from \( \omega_r \) according to parameter *:

\[
(70)\quad \text{if}(\omega_r, \omega_e, \text{John leave}, \ast)
\]

\( (69) \) may be taken as a statement of what is permitted for John. In that case, * indicates that scenario \( \omega_e \) is possible only if it respects both the facts that hold in \( \omega_r \) and the obligations that are imposed on John there. Alternatively, \( (69) \) may be taken as a description of our ignorance of John's intentions. In that case, the possibility assessed by * indicates only that the scenario \( \omega_e \) is simply compatible with the facts in \( \omega_r \).

For a simple sentence like *John will leave*, the corresponding two-step translation is correct:

\[
(72)\quad \text{if}(\omega_r, \omega_e, \text{John leave}, \ast); [ | \text{necc}(\omega_r, \omega_e, \ast) ]
\]

But it is overkill. The necessity condition forces \( \omega_r \) and \( \omega_e \) to be identical, under the relevant contextual parameter *., just as simple tenses identify reference and event times. We therefore adopt translations where *will*, *would* and *is* predicate the content of the clause directly of their reference scenario.

Modal words in English are carriers both of modal meaning and of tense. Following Hornstein (Hornstein, 1990), bare modals like *will* and *may*—and also *would* and *might*—bear the same simple nonpast tense of English "present-tense" forms. Including this feature allows us to flesh out in the translation of \( (69) \) completely:

\[
(75)\quad \text{if}(\omega_r, \omega_e, \left[\omega_e : e \mid \text{leave}\{\omega_e, j, e\}, \text{simple}\{\omega_e, r, e\}, \text{nonpast}\{\omega_e, s, r\}\right], \ast); [ | \text{vivid}(\omega_e, \omega_r, \ast), \text{poss}(\omega_r, \omega_e, \ast) ]
\]
This translation reflects that the interpretation of (69) is not complete without identifying a topic time \( t \) appropriately related to event and speech times. Note that in keeping with the overall ontology we have now relativized the tense meaning to the scenario \( \omega_e \); should we choose to attribute such temporal properties to events necessarily, we could eliminate this relativization.

Again following Hornstein, complex modals like will have, may have, would have and might have differ from corresponding simple forms by bearing simple past tense instead of simple present tense. So, for example, John may have left translates as in (76).

\[
\begin{align*}
(76) \quad & \text{if}(\omega_r, \omega_e, [\omega_e : e | \text{leave}(\omega_e, t, e), \\
& \quad \text{simple}(\omega_e, r, e), \text{past}(\omega_e, s, r) ], *,) ; \\
& \quad [ \text{vivid}(\omega_s, \omega_r, *), \text{poss}(\omega_r, \omega_e, *) ]
\end{align*}
\]

As this discussion indicates, the meanings of modal words is a complex affair that ties together a number of components in addition to modal meaning proper. This discussion should be regarded as a suggestive illustration of how these different features might be reconciled in this framework. Our central concern here remains the dynamics of anaphora. As shown in the next subsection, what is important is to break down the meanings of modals into a construction (if) that introduces scenarios and a construction (like necc or poss) that refers to them.

4.4 Abstract examples

The behavior of this analysis can now be illustrated on some of the data from section 2. Example (4) is a good starting point:

(4) A wolf might walk into the house. It would eat you.

Suppose the initial reference scenario for might is \( \omega \), the initial reference time, \( r \); let \( \omega_{\text{now}} \) name the speech scenario and \( \text{now} \) the moment of speech. Then the first sentence corresponds to the DRS in (77); the superscripts are an expository device to label the occurrences of sub DRSs.

\[
\begin{align*}
(77) \quad & \text{if}(\omega, \omega', [\omega' : u, e | \text{wolf}(\omega', u), \text{walk-in}(\omega', u, e), \\
& \quad \text{simple}(\omega', r, e), \text{nonpast}(\omega', \text{now}, r) ], *,) ; \\
& \quad [ \text{remote}(\omega_{\text{now}}, \omega_r, *), \text{poss}(\omega_r, \omega_e, *) ]^3
\end{align*}
\]

This formula introduces a new scenario \( \omega' \) from \( \omega_r \), where a wolf walks in, according to some contextual considerations; it observes that this new scenario is a contextual possibility. The poss and remote conditions represent the modal contribution of might; the simple and nonpast conditions represent its temporal contribution.

In sequence with this is the DRS in (78) representing the content of the second sentence:

\[
\begin{align*}
(78) \quad & [\omega' : e' | \text{eat}(\omega', u, you, e'), \\
& \quad \text{simple}(\omega', \theta(\text{CON}(e)), e'), \text{nonpast}(\omega', \text{now}, \theta(\text{CON}(e))) ]^4
\end{align*}
\]
This translation reflects the fact that \( \omega' \) is taken as the reference scenario for the modal \textit{would}, and incorporates our convention that \textit{would} predicates the content of its clause directly of its reference scenario. Here \textit{would} supplies its reference point \( \omega' \) to \textit{eat}, so that the eating takes place in the hypothetical scenario.

The use of the term \( u \) corresponding to the pronoun \textit{it} is justified by the links among the sequenced DRSs using the derivation in (79):

\[
\frac{u : e \in U(\omega'; 1)}{u : e @ \omega' ; 1 \quad \text{axiom}} \quad \frac{1 \sim 3}{\sim U} \quad \frac{u : e @ \omega' ; 3}{3 \sim 4 \quad \sim U}
\]

The use of the term \( \vartheta(\text{CON}(e)) \) corresponding to the reference point of the tense of \textit{would} is justified by combining an argument parallel to (79) with an inference about term-construction parallel to that ending (60).

The use of the conditionalization rule in a derivation of accessibility combines flexibly with uses of the update rule such as seen in derivation (79). This property underlies the abstract analysis of examples like (8), repeated below:

(8) A wolf might walk in. We would be safe because John has a gun. He would use it to shoot it.

(80) gives a formalization of the example ignoring temporality and causality:

\[
\text{if}(\omega, \omega', [ \omega' : u, e | \text{wolf}(\omega', u), \text{walk-in}(\omega', u, e), * ]^2 ; [ | \text{remote}(\omega_{\text{now}}, \omega), \text{poss}(\omega, \omega', *) ]^3 ; [ | \text{safe}(\omega', \text{we}) ]^4 ; [ \omega : g | \text{gun}(\omega, g), \text{have}(\omega, j, g) ]^5 ; [ \omega' : e' | \text{shoot}(\omega', j, u, g, e') ]^6
\]

As advertised, the analysis describes only two scenarios: reality, represented by \( \omega \); and if a wolf walks in, represented by \( \omega' \). We have \( \omega \rightarrow \omega' \). The derivation in (81) shows how to exploit this fact to obtain the accessibility of the gun \( g \) in DRS 6 for scenario \( \omega' \):

\[
\frac{g : e \in U(\omega; 5)}{g : e @ \omega; 5 \quad \text{axiom}} \quad \frac{5 \sim 6}{\sim U} \quad \frac{g : e @ \omega; 6}{\omega \rightarrow \omega' \quad \sim U}
\]

The fact that \( \omega' \) elaborates on \( \omega \) is used in this derivation in considering DRS 6. This fact is a permanent semantic property of the static objects to which \( \omega \) and \( \omega' \) refer throughout the discourse. Thus we are entitled to apply the fact long after the introduction of \( \omega' \).

Key premises are not always available, of course. When premises are unavailable, the analysis predicts impossibility of reference. An example is (82)—on the natural reading where the speaker warns of a general danger and not of a specific wolf pacing outside.
(82) #A wolf might walk in. It is hungry.

On this reading, the first sentence continues to correspond to the DRS in (77). Given the factual presupposition of is, the second can only correspond to the following DRS:

\[
(83) [\omega_s : e' | \text{hungry}\{\omega, u, e'\}, \text{simple}\{\omega, s, e'\}, \text{nonpast}\{\omega, s, s\}]^7
\]

The use of \(u\) for it here requires the derivation of \(u : e @ \omega : 7\) because being hungry in scenario \(\omega\) presupposes existing in scenario \(\omega\). No such derivation exists, because we have no premise \(\omega' \sim \omega\).

Suppose there was no existence presupposition (as perhaps in (28b)). Then we would be free to choose any \(\omega\) that allowed the referent \(u : e @ \omega : 7\) to be derived. So we could use \(\omega = \omega'\): the hypothetical wolf would remain accessible.

4.5 A realization using possible worlds and orderings

The abstract ontology of section 4.2 offers a simple and natural account of modal subordination and its parallel with temporal interpretation. The high level description is intended to capture the discourse-semantic properties of modals while remaining compatible with a variety of more detailed proposals for conditional and dynamic logics. That it meets this intent is the argument of this section: I give a concrete interpretation of the new proposal using sets of possible worlds to represent scenarios concretely.

In the concrete realization, we adopt a new type \(w\) for the type of possible worlds; so that the type \(\omega\) is now regarded as an abbreviation of \(wt\), the type of sets of possible worlds. The structure of worlds parallels that given in (Muskens, 1995). In particular, all possible worlds share a common domain, but an predicate \(u\ in\ w\) is true of precisely those \(u\) that actually exist at a world \(w\).

We also adopt a new type, \(s^\omega\), that abbreviates the type \(s \times w\); each element of this type is a pair \((i, w)\) where \(i\) is an environment and \(w\) is a possible world. The type \(s^\omega\) provides a type of concrete environments in an adjoint step to providing a concrete type for \(a\); the type \(s^\alpha\) likewise provides a new type for discourse markers, which in the realization are always concrete.

These steps suffice to interpret terms and atomic conditions concretely. For a term \(t\), the new translation \(t^\omega\) is a function whose value at a concrete environment \((i, w)\) is obtained from \(t\) by replacing each occurrence of a free discourse marker \(v_n\) by \(v_n(i, w)\). This is as in section 3.2, except that the environments and discourse markers are concrete. For atomic conditions, the translation is as follows:

\[
R\{\omega, t_1, \ldots, t_n\}^\omega \equiv \lambda i. \forall w'(\omega(i, w)w' \supset Rw'f(i, w') \ldots f_n(i, w'))
\]

This translation realizes the barrier of abstraction that allows structured possibilities to be treated as atomic and allows concrete environments to be represented by ordinary ones. In (84), we quantify over all the concrete possibilities (possible
worlds) picked out by the abstract possibility, and at each construct a concrete environment and test that the relation holds among the values of the specified terms there.

Changes respecting this barrier of abstraction must be imposed on transition formulas between environments. In particular, we appeal to a scenario marker in constructing new environments, as anticipated by the addition of a marker to each atomic DRS. For a scenario marker \( \omega' \), the formula \( i[\omega : \omega']j \) ensures that \( i \) and \( j \) agree on all discourse markers but \( \omega' \). Further, at any world \( w \) among the concrete possibilities picked out by \( \omega, \omega'(j, w) \) may pick out any set of worlds; at other worlds \( \omega'(j, w) \) must be empty. The restriction ensures that \( \omega' \) is a possibility relative to \( \omega \). In terms of Muskens's family of predicates \( \text{mk}_\alpha \) true of the meaningful discourse markers of type \( \alpha \), this definition of \( i[\omega : \omega']j \) corresponds to the conjunction of the two conditions in (85).

\[
\text{(85)} \quad \begin{align*}
&\text{a} \quad \text{For all storage types } \text{mk}_\alpha, \forall v(\text{mk}_\alpha(v) \land v \neq \omega' \supset \forall w(v(i, w) = v(j, w))) \\
&\text{b} \quad \forall w_1(\neg \exists w_0(\omega(i, w_0)w_1) \supset \neg \exists w_2(\omega'(j, w_1)w_2))
\end{align*}
\]

For other markers \( \delta \), \( i[\omega : \delta]j \) imposes the condition that the value of \( \delta \) must exist in each world in \( \omega \); this gives it the formalization in (86).

\[
\text{(86)} \quad \begin{align*}
&\text{a} \quad (\text{Same as (85a).}) \\
&\text{b} \quad \forall w_0(\omega(j, w)w' \supset \delta(j, w') \text{ in } w').
\end{align*}
\]

As these definitions of assignment show, a revised axiom is required to ensure that there are enough abstract environments to record the changes to concrete environments that we wish to perform. For each storage type \( \alpha \), 'having enough states' means being able to update an abstract environment so that any variable \( v \) takes on any combination \( f \) of values across possible worlds.

\[
\text{(87)} \quad \forall ivf(\text{mk}_\alpha(v) \supset \exists j.i[v]j \land \forall w(v(j, w) = f w))
\]

(Here \( i[v]j \) does appeal to the original notion of environments that differ only in the value of \( v \).) The other axioms of Muskens's logic of change need no alteration, however.

The final component of the concrete proposal is semantics for the modal connectives \( \text{if}(\omega, \omega', D, \ast) \), \( \text{poss}(\omega, \omega', \ast) \) and \( \text{necc}(\omega, \omega', \ast) \). We follow the ideas of Lewis on conditionals (Lewis, 1973) and Kratzer on parameterized modals (Kratzer, 1991) as closely as possible in this new framework.

Lewis explains conditionals using a ternary relation \( \text{closer} \) on worlds; \( \text{closer}(w, w', w'') \) holds when a world \( w' \) is as similar or more similar to some reference world \( w \) than \( w'' \) is. Given some fixed starting point \( w \), the relation given by \( \lambda w' \lambda w''. \text{closer}(w, w', w'') \) is then an ordering on possible worlds that can act as a Kratzer-style ordering source in modal quantification. That is, to determine the truth of \( \text{if } p \text{ then } q \) at a world \( w \), find the set of closest worlds to \( w \) that satisfy \( p \), and
make sure that \( q \) is true in all. This set of closest worlds is the scenario evoked by the conditional.

We must adapt this definition to describe an update \( \text{if}(\omega_1, \omega_2, D, \text{closer}) \). First, we observe that, given the earlier concrete definitions of assignment and updates, the relation

\[
(88) \quad \lambda i \lambda j. \exists k(i[\omega_1 : \omega_2]k \land Dk_j)
\]

updates the context so that in each world \( w \) associated with \( \omega_1, \omega_2(j, w) \) is any set of worlds for which a \( D \)-update has succeeded. Of course, there is no requirement that \( \omega_2 \) contain all and only the closest worlds with this property. We can impose this requirement, however, by comparing \( \omega_2(j, w) \) against all other sets \( \omega_2(h, w) \) obtained similarly. If no world in any such \( \omega_2(h, w) \) is strictly closer to \( w \) than any world in \( \omega_2(j, w) \), \( \omega_2(j, w) \) must contain only closest worlds. And if every world in all \( \omega_2(h, w) \) is at least as far as some world in \( \omega_2(j, w) \), then \( \omega_2(j, w) \) must contain all closest worlds. So we can realize Lewis's condition by the following definition for \( \text{if}(\omega_1, \omega_2, D, \text{closer}) \):

\[
(89) \quad \lambda i \lambda j. (\exists k(i[\omega_1 : \omega_2]k \land Dk_j) \land \forall h(\exists k(i[\omega_1 : \omega_2]k \land Dkh) \supset \forall w w' (\omega_1(i, w')w \supset \forall w_h w_j (\omega_2(h, w)w_h \land \omega_2(j, w)w_j \land \text{closer}(w, w_h, w_j) \supset \text{closer}(w, w_j, w_h)))) \supset \forall w_h (\omega_2(j, w)w_h \supset \exists w_j (\omega_2(j, w)w_j \land \text{closer}(w, w_j, w_h))))
\]

As promised, this meaning for \( \text{if} \) includes an update of the overall context by its argument \( D \). This justifies the formal connection between \( D \) (and the referents it introduces) and subsequent discourse. On the other hand, this meaning for \( \text{if} \) does not necessarily ensure that objects known to exist throughout worlds identified by \( \omega_1 \) exist throughout the worlds identified by \( \omega_2 \). We still must justify the formal relationship between \( \omega_1 \) and \( \omega_2 \). In some cases, we do so by supposing that the ranking \( \text{closer} \) has this relationship to \( \omega_1 \): Any two worlds in \( \omega_1 \) are closer to each other than either is to any world not in \( \omega_1 \). Under these circumstances, every possible world picked out by \( \omega_2 \) in \( j \) will already be picked out by \( \omega_1 \) in \( j \). So everything that exists at the \( \omega_1 \) worlds will exist at the \( \omega_2 \) worlds. This condition can be used sensibly with indicative conditionals and modals of possibility and necessity. All evoke new scenarios by adding, rather than by altering, if possible; this is all the condition formalizes. This justifies the postulate for formal accessibility that \( \omega_1 \leadsto \omega_2 \) for these modals. In introducing counterfactual scenarios, however, we must be content to regard the postulate \( \omega_1 \leadsto \omega_2 \) merely as a default with exceptions both in the model theory and perhaps, on occasion, in pragmatics.

The conditions for \( \text{necc}(\omega_1, \omega_2, (M, \leq)) \) and \( \text{poss}(\omega_1, \omega_2, (M, \leq)) \) are more straightforward. The contextual parameters of these conditions are given in two parts: an additional modal base \( M \) indicating a perspective to take on \( \omega_1 \) and \( \omega_2 \);
and a ternary ordering source \( \leq \), where \( w' \leq_w w'' \) indicates that \( w' \) is as close or closer than \( w'' \) to some ideal for \( w \). These two parameters can give an account of these terms that follows Kratzer, with two changes. The possibility \( \omega_1 \) supplies part of the modal base; and the conditions describe a possibility \( \omega_2 \) rather than a proposition \( p \). In particular, necessity represents the following constraint:

\[
\lambda i. \forall w' (\omega_1 (i, w')) w \wedge M w \wedge \\
\forall w'' (\omega_1 (i, w') w'' \wedge M w'' \wedge w'' \leq w' w \supset w \leq w'' w') \supset \omega_2 (i, w') w
\]

For an environment \( i \) to satisfy (90), at any relevant world \( w' \), the \( \leq_w \)-closest worlds satisfying both \( \omega_1 (i, w') \) and \( M \) must also be \( \omega_2 (i, w') \) worlds. Possibility is dual to necessity (at each world):

\[
\lambda i. \forall w' \exists w (\omega_1 (i, w') w \wedge M w \wedge \\
\forall w'' (\omega_1 (i, w') w'' \wedge M w'' \wedge w'' \leq w' w \supset w \leq w'' w') \wedge \omega_2 (i, w') w)
\]

It requires of each \( w' \) that some \( \leq_w \)-closest world satisfying both \( \omega_1 (i, w') \) and \( M \) also satisfies \( \omega_2 (i, w') \).

For now, take \text{real} to represent the same condition as \text{necc} and \text{vivid} to represent the same condition as \text{poss}. For \text{remote}, we can either take the \text{remote} condition to represent the negation of the \text{vivid} condition; or, invoking Gricean principles in accounting for the distribution of remote forms, take it to represent the always true relation.

### 4.6 A Concrete Example

Let us return to the abstract analysis of (4)

(4) A wolf might walk into the house. It would eat you.

which we left with the abstract analysis below:

\[
\text{if}(\omega, \omega', [\omega': u, e | \text{wolf}(\omega', u), \text{walk-in}(\omega', u, e), \\
\text{simple}(\omega', r, e), \text{nonpast}(\omega', \text{now}, r) ]^1, \ast)^2;
[ | \text{remote}(\omega_{\text{now}}, \omega), \text{poss}(\omega, \omega', \ast) ]^3;
[ \omega' : e' | \text{eat}(\omega', u, \text{you}, e')], \\
\text{simple}(\omega', \partial(\text{CON}(e)), e') \text{ nonpast}(\omega', s, \partial(\text{CON}(e))) ]^4
\]

Given the concrete proposal in section 4.5, we can regard (92) as an abbreviation for an expression describing changing states and sets of possible worlds. Each part of this concrete expression plays a role in setting up and propagating referents that agrees with an intuitive description of reference to scenarios and with the intended interpretation of the abstractions of section 4.2.

Thus, according to the definition in (89), updating by DRS 2 sets up a state \( j \) in which referents \( \omega', u \) and \( e \) have two key properties. First, at each possible world \( w \) in \( \omega', u(i, w) \) picks out a wolf that exists in \( w \) and that enters in a future event
picked out by $e(i, w)$. Second, all the best worlds of $\omega$ where a wolf walks in are represented in $\omega'$. So in $j$, $\omega'$ represents the scenario just like $\omega$ but where a wolf walks in; and further $u$ and $e$ are discourse referents that pick out the hypothetical wolf and its hypothetical walk.

Meanwhile, DRS 3 incorporates a Kratzer-style requirement of contextually-relativized possibility between $\omega$ and $\omega'$. Suppose, plausibly, that $\omega$ represents a scenario that refers to the speaker’s present information. To realize such a scenario as a concrete object, we treat the speaker’s current information as a set of worlds $W$ that are indistinguishable in terms of what the speaker knows there; at the current state $j$, in each world $w$ (with $Ww$), $\omega(j, w)$ is just $W$. Since the epistemic modal base is already reflected in the definition of our hypothesis, we can assume that no constraints are imposed by the modal base $M$ local to DRS 3. If we also suppose that the speaker is taking no ideal for granted, we can give DRS 3 an ordering source that treats all worlds as incomparable. Under all these assumptions, the interpretation of DRS 3 according to (91) just asserts that there is a world in $W$—a world compatible with what we know in $\omega$—where a wolf comes in, as described by $\omega'$. This is as desired. After updating by DRS 3, we remain in state $j$.

Finally, DRS 4. We return to the worlds in $\omega'$, the scenario like $\omega$ but where a wolf comes in. At each of these worlds $w$, the translation introduces a new event represented by $e'(j, w)$, and ensures that this is an consequent event to the entrance $e(j, w)$, in which $u(j, w)$ eats you. According to the update that constructed $j$, $u(j, w)$ is the wolf; and $e(j, w)$ is the event of its walking in. Thus, we have: if a wolf walks in, the wolf then eats you.

5 A critique of current approaches

In section 4, we took pains to hide the model-theoretic structure of scenarios from the dynamics of discourse. The reason is that it is difficult to describe such quantification over possible worlds as modals may do, using the overt quantification of dynamic semantics. Overt dynamic quantification introduces and characterizes temporary environments that differ in the values of relevant discourse markers. Previous accounts (Kibble, 1994; Portner, 1994; Geurts, 1995; Kibble, 1995; Frank and Kamp, 1997) adopt dynamic quantification in accounts of modal subordination, to the detriment of simplicity and coverage.

At the heart of the problem is that the formal treatment of pluralities and quantification is vastly different from the treatment of ordinary entities in dynamic semantics. Thus, treating scenarios in terms of explicit sets throws off the parallel with tense from the start. The problem is revealed by considering a simple, counterfactual sentence, such as (93).

(93) If a wolf came in, it would eat you.

Assuming a possible-worlds representation, many possible worlds are under consideration in (93): all the possible worlds which have a wolf that comes in and which
are "closest" to some initial possibilities. These worlds differ, among other factors, in the identity of the wolf that comes in. Given this ontology and a usual dynamic semantics, (93) must be translated by a quantificational expression. Roughly, this expression will construct a set of TEMPORTARY environments that list the current world and a wolf that comes in there; and it will make sure that in each, the listed wolf eats you. The temporary environments will play no role outside this expression, and hence the possible worlds, and the wolves, will be added only temporarily. Modal anaphora will be impossible.

Let me be more precise, by first introducing a presentation of modal meaning that is formulated to work with dynamic objects, environments and updates, in addition to worlds and propositions. (Similar presentations are fleshed out with varying details in (Kibble, 1994; Portner, 1994; Geurts, 1995; Kibble, 1995; Frank and Kamp, 1997).)

To characterize generic if- and when-clauses, dynamic semantics uses an operator ⇒ relating two updates. p ⇒ q is a new update; intuitively, this update makes sure that all current p-extensions have q-extensions. That is, p ⇒ q outputs its input i, when for any j that you can get from i by updating on p, there is an environment k you can get to from j by updating on q. Formally, this is represented as the following translation or abbreviation:

\[(p \Rightarrow q)^o \equiv \lambda i. \lambda i'. i = i' \land \forall j(p^o ij \supset \exists k.q^o jk).\]

As a test, any environment that p ⇒ q outputs is identical to its input; for some inputs, however, there is no output. Since the environment does not change, no new referents are introduced into a model of discourse using this operation. While the discourse model may evolve as i is transformed into j and then into k, these models are discarded after the test is complete.

For other conditionals, we must lift this propositional connective to a quantificational connective incorporating reference to possible worlds. Following (Chierchia, 1992), these too involve tests over updates. They take the form \(Q(P, Q)\) where both P and Q are DYNAMIC properties, functions that take an entity to return an update. Using Q* to represent the static quantifier corresponding to Q, the interpretation of Q as a dynamic quantifier is given in (95).

\[(95) \quad Q(P, Q) \equiv \lambda i. \lambda i'. i = i' \land Q^*(\lambda x. \exists j.Pxij, \lambda x. \exists jk.Pxij \land Qxjk)\]

According to this definition, the restrictor of Q* is the set of entities for which a P-update is possible, and the nuclear scope is the set of entities for which a P-update can be followed by a Q-update. Thus, for a sentence like (96a), where the entities are ordinary individuals, the definition gives an interpretation that can be paraphrased as (96b).

\[(96) \quad a \quad \text{Every man who owns a donkey beats it.} \]
\[(96b) \quad \text{Every man who owns a donkey beats some donkey he owns.}\]
For counterfactuals, where the entities are possible worlds, $Q^*$ says the consequent is true in all closest worlds where the antecedent is true; we can apply this quantifier to Chierchia’s presentation in analyzing (93). The resulting expression relates $i$ and $i$ at $w$ by this condition:

\[(97) \text{ In all closest worlds to } w \text{ where } i \text{ can be updated to find a wolf that comes in, } i \text{ can be updated to find a wolf that comes in and eats you.}\]

This condition does indeed have the characteristic described informally earlier: no overall update occurs.

Something has to be added to the dynamic representation of the meaning of (93). We must store and retrieve entities from the environment. The irony is that accomplishing such storage, previous proposals actually increase the divergence between temporal anaphora and modal anaphora. Because of the complex type of the quantifier, there is no simple entity to be recovered. Unlike a time, when a modal referent is recovered, it must be used both to find a set of worlds and to supply needed referents. The association between scenarios and reintroduction of referents leads to empirical difficulties and theoretical asymmetries.

In the literature, examples like (7b), repeated below, are typically taken to show that the prediction of impossibility of modal reference is generally right.

\[(7) \begin{align} a & \text{ John ate a cheesesteak. It \{was, \#is\} very greasy.} \\ b & \text{ John might be eating a cheesesteak. It \{would be, \#is\} very greasy.} \end{align}\]

The claim is that modal subordination represents a special exception and that it is therefore natural to treat scenarios as complex, dynamic objects that supply referents. The parallel between modality and tense suggests the opposite. Nobody would argue that the impossibility of reference in (7a) was best explained by assuming that times had a quantificational structure that blocked off further reference, or that times had a dynamic structure that supplied referents by a separate mechanism in extraordinary circumstances. The simple and uniform analysis of (7) is that atomic entities are incrementally added to a single evolving model of the discourse and that modal presuppositions about referents, like temporal ones, modulate possibilities for further anaphora. Importantly, the uniform analysis is crucially needed to account for data like (28), repeated below.

\[(28) \begin{align} a & \text{ John will have an enormous wedding. It is his mother’s idea.} \\ b & \text{ John may have an enormous wedding. It is his mother’s idea.} \end{align}\]

These sentences emphasize that possibilities for anaphora are determined by lexical semantics and presuppositions rather than by a syntactic, formal structure that judiciously reintroduces referents.

The different explanations of temporal and modal anaphora in (7) is not the only divergence. As observed by a number of researchers, including (Portner, 1994; Kibble, 1995) and indirectly (Spejewsky, 1994), the bundling of referents and
updates requires new characterizations of continuity and coherence in discourse. To
see this, it will help to introduce Geurts’s \(+\)-notation for describing updates (Geurts,
1995) and thereby describe the particular entities different theories propose to store
in the discourse model. We introduce variables \(p, q, \text{etc.}\) to represent sets of (world,
environment) pairs. If \(D\) is a DRS (that is implicitly or explicitly parameterized by
a world variable), the condition

\[
q = p + D
\]

is intended to indicate that \(q\) is interpreted by the set of (world, environment) pairs
obtained by updating the (world,environment) pairs in \(p\) by the dynamic meaning
of \(D\). Variables such as \(p\) and \(q\) can provide the restrictors and scopes for modal
quantifiers:

\[
3j.(w, i) \in p \land Dwij \text{ if and only if } \exists j. (w, j) \in p + D.
\]

With this ontology and notation, the anaphoric interpretation of \textit{might} \(D\); \textit{would} \(D'\) is formalized as in (99):

\[
q = p + D; \text{poss}(p, q); r = q + D'; \text{necc}(r, q)
\]

These ordinary, dynamic variables are used for modal anaphora in (Kibble, 1994;
Geurts, 1995; Frank and Kamp, 1997).

Meanwhile, (Portner, 1994; Kibble, 1995) use variables that essentially refer to
the updates themselves, rather than a particular result of the update. This introduces
conditions of the form \(M = \lambda p.p + D\). Particular results of the update are obtained as
part of processing modal conditions, by applying update variables to a representation
of the current state, which we can notate by \(\dagger\). The use of updating dynamic variables
leads to the following kind of formalization for \textit{might} \(D\); \textit{would} \(D'\):

\[
M_1 = \lambda p.p + D; \text{poss}(\dagger, M_1(\dagger)); M_2 = \lambda q.q + D'; \text{necc}(M_1(\dagger), M_2(M_1(\dagger)))
\]

Now, consider the parallel data in (6), repeated below.

(6) a A wolf walked in. A victim was waiting. It ate him.

b A wolf might walk in. A victim would be waiting. It would eat him.

We intuitively regard (6a) this way. The first sentence describes an event; the result
state of that event serves as the reference time for both subsequent utterances. The
formalization of temporal reference presented in section 3 matches this intuitive
view precisely; each statement in turn contributes to the evolution of a single dis-
course model, and the referents introduced at each stage are accessible subsequently,
including the event referents. These convergent views are naturally exploited in
accounts like centering that describe the coherence of sentences in discourse in
terms of continuity of reference (Grosz et al., 1995; Kameyama et al., 1993).

We have seen that there is no reason to regard (6b) any differently. In the model
presented in section 4, the first sentence describes a new scenario; that scenario
serves as the referent for \textit{would} in both subsequent utterances. However, previous
formal accounts are inconsistent with this view. There, the referent set up by the
first sentence defines a set of possible worlds and an update that finds a wolf in each. The third sentence may describe the same set of possible worlds, but it must invoke a different update—one that finds a wolf AND A VICTIM in each. In symbols, taking $W$, $V$, and $E$ to represent the propositional content of the successive sentences, the ordinary dynamic theory must represent (6b) as (101a) rather than (101b):

\[(101) \begin{align*}
  a & \quad q = p + W; r = q + V; s = r + E \\
  b & \quad q = p + W; r = q + V; s = q + E
\end{align*} \]

The updating dynamic theory requires a similar chaining of applications. So the third sentence must get its referent from the second sentence, which must update the accessible updates, as it were. Examples like (6b) suggest that modal higher-order referents must be governed by different, and less intuitive, pragmatic regimes from ordinary temporal referents, as well as different, less intuitive, semantic ones.

Again the divergence results in an empirical difficulty for previous approaches. Consider (8):

\[(8) \quad \text{A wolf might walk in. We would be safe because John has a gun. He would use it to shoot it.} \]

For the proposal of section 4, this example presents no trouble. The scenario introduced in the first sentence is again described in the second and third. The second sentence also introduces a real object into the discourse. Since that object exists throughout the larger scenario of reality, it exists and is available for reference in any component scenario that includes a possible way reality might develop. Only the discourse model specifies entities for reference, but presupposed semantic relationships among scenarios can determine the scenarios where entities are presupposed to exist.

Meanwhile, (8) remains mysterious when scenarios are realized as complex objects that supply referents. On the ordinary dynamic view, we are actually stuck. Schematizing the clauses by $W, S, G, U$, we must represent the first three clauses as in (102):

\[(102) \quad q = p + W; r = q + S; s = p + G \]

We must take $s$ to extend $p$ because of its factual mood. If we took $s$ to extend $r$, then we would know only that John would have a gun if a wolf came in. But John does in fact have a gun. Meanwhile, by representing the discourse this way, we have ruled out the possibility of referring to the gun and the wolf at the same time. $U$ can refer to the wolf only if it extends $q$ or $r$; it can refer to the gun only if it extends $s$.

For the updating dynamic approaches there is a possible escape, but it is very unnatural. We record the updates represented by the first three clauses thus:

\[(103) \quad M_1 = \lambda p.p + W; M_2 = \lambda p.p + S; M_3 = \lambda p.p + G \]
Now, the content of the third sentence must be that $M_3(t)$ is necessary given $t$—that John has a gun given current information. However, because the discourse has stored updates, these updates could potentially be reapplied to different arguments in different orders in interpreting $U$. For example, in the state represented by $M_3(M_2(M_1(t)))$, there is a wolf and a gun and a victim. This application, however, is not faithful to the history of the discourse because $M_3$ is applied to the scenario where a wolf comes in, not to reality. Similarly, wolf, gun and victim are available in the state $M_2(M_1(M_3(t)))$. Here each update applies to the same set of possible worlds as it did in the original discourse. This strategy is unfaithful to the history of the discourse because the updates must be processed in a new order. No previous approach has worked out in a satisfying way how such novel sequences of updates should be defined or constrained.

The quantificational analysis of sentences like (93) is not a theoretical necessity. It follows only from the decision to “expose” the logical ontology of possible worlds as the linguistic ontology too. Contrast this with (104):

\[(104) \text{ When a wolf came in, it ate you.}\]

Just because the event and time of the wolf’s entrance can be represented by an atomic formal object, and quantification thereby avoided, the formal theory assigns (104) a meaning in which the relevant entities are permanently added into an evolving environment. Temporal anaphora follows straightforwardly. Taking the same step with scenarios will give the same straightforward account of modal anaphora: by abstracting the partiality of scenarios throughout the semantic analysis, I avoid the need for explicit linguistic quantification over compatible totalities. There is then no need to handle (93) by a different formal technique from (104), nor (6b) differently from (6a).

6 Issues and Prospects

In this paper, I have argued that tense and modality are parallel in referring anaphorically to primitive semantic objects and in describing them in terms of their relation to one another and to a point of speech; I have presented a formal approach that realizes the parallel intuitively but precisely. This referential account of modality promises other exciting applications in describing the interface between syntax, semantics and pragmatics. These applications can be described particularly simply using the abstract characterization of modality developed here.

For example, Portner (Portner, 1994) accounts for SUBJECTIVE SENTENCES in discourse in terms of reference to scenarios.\(^7\) Subjective sentences “present the
thoughts, perceptions, and inner states of characters in the story” ((Wiebe, 1994) p.233). A simple constructed example appears in (105).

(105) Sally imagined a frightening scene. A lion was chasing after her.

(= (Portner, 1994) (2))

In (105), the second sentence reports what occurred in the scene Sally imagined, not something that actually happened to Sally. In other words, the sentence takes Sally’s PSYCHOLOGICAL POINT OF VIEW. Naturally occurring examples are (not surprisingly) more complicated. The appropriate psychological predicate typically must be inferred from the content of the clause and the circumstances of its subject—not directly recovered the way imagination is in (105). In principle, inferring these states and reasoning about the role of the reports in narrative requires a theory of folk psychology, describing how characters form appraisals of their environment and intentions for the future and how those appraisals and intentions in turn inform characters’ emotions and actions. (Promising outlines can be found in Bratman (Bratman, 1987) for intention and Ortony et al. (Ortony et al., 1988) for appraisals and emotion.) Two representative examples discussed by Wiebe (Wiebe, 1994) illustrate some of this complexity. In (106a), the final sentence reports Zoe’s negative appraisal of Joe: this judgment is effectively the content of the fury described earlier.

(106) a “What are you doing in here?” Suddenly she [Zoe] was furious with him [Joe].
   “Spying, of course.”
   “Well of all the dumb things! I thought you ran away.” Joe Bunch was awful. [Oneal, War Work, p. 130] (=Wiebe, 1994) (4)

b He [Jeff] could see her walking the other way. If he wanted to avoid notice, he would have to act with the same deliberate manner as all the robots around him. He lengthened his stride and gave chase without otherwise altering his body language. [Wu, Cyborg, p. 71] (=Wiebe, 1994) (7)

In (106b), the second sentence describes the course of an episode of practical reasoning for Jeff: he adopts the intention to move in a particular way to satisfy his goal of escaping detection. The third sentence describes, objectively, how he realizes this intention.

Wiebe goes on to describe a variety of features that can indicate when a sentence in narrative is subjective; she provides an algorithm for interpreting the point of view of narrative sentences that integrates these features in context. In (Portner, 1994), however, Portner invokes the anaphoricity of modality to address a more basic

parameters, but it is not clear to me at present how to make such an attempt compatible with the ideas about mood outlined above.” (p. 21)
question: how and why indicative sentences permit subjective interpretations at all. In a nutshell, he proposes that all constructions with propositional complements introduce particular discourse markers of the same type as those discussed earlier for modals. The indicative mood of subjective sentences refers to these discourse referents. Since sentences make a claim about the hypothesis they refer to, this reference accounts for the subjective force of subjective sentences.

Meanwhile, Beghelli (Beghelli, 1996) explores the idea that the modal scope of noun phrases is interpreted referentially. This gives a novel characterization of the well-known ambiguity between the *de re* and *de dicto* interpretations of embedded noun phrases. The ambiguity is that the content of the embedded noun phrase can describe real properties of a real entity, or subjective properties (perhaps of entities that exist only subjectively). For example, in (107), Pat’s belief is interpreted either that *x* called (where *x* is known otherwise to be a salesman) or just that some indefinite salesman-calling took place.

(107) Pat thinks that a salesman called.

Quantificational accounts of modality must account for the difference in terms of movement to a different scope, but the kind of movement needed is *ad hoc* and unbounded. In contrast, a referential analysis allows ordinary anaphoric mechanisms to determine the scenario in which a *salesman* introduces a referent and ensures that it refers to a salesman. The advantages of the referential account are not limited to its standard machinery. Presuppositions about modal referents imposed in the description of an entity can contribute to disambiguating its modal scope, along the lines observed in section 2.2, allowing the theory to capture additional constraints. In addition, since noun phrases also receive a referential temporal interpretation (Enç, 1986), the theory offers another parallel between modality and time.

Finally, Stone and Hardt (Stone and Hardt, 1997) extend the parallel between tense and modality to a simple theory of attentional state. This theory explains data as in (108) (examples (4) from that paper), where tenses and modals are implicated in sloppy anaphora:

(108) a [VP [Tense]] You Past [ think I [ Past] was crazy]. You probably still Pres do VPE.

b [Modal [NP]] John would use slides if [ [ he] had to give the presentation]. Bill would just use the chalkboard.

Anaphora is sloppy when the entity evoked by an anaphor differs from the referent of its antecedent because an anaphor contained in the antecedent is interpreted differently. For example, in (108a), the anaphor is the VPE; its antecedent is the main VP of the previous sentence. The sloppy reading of the elided VP allows the reference point of the embedded verb *was crazy* to be reinterpreted as referring to the moment of speech. Dually, in (108b), the anaphor is the modal verb; on the
sloppy reading, the pronoun he in the description of its scenario referent refers first to John, then to Bill.

In the theory of (Hardt, 1996), sloppy anaphora is a consequence of two assumptions. First, an environment specifies a distinguished CENTER of each type that to which speaker and hearer’s attention is most directed. Second, all anaphoric items can access so-called DYNAMIC discourse referents whose values in an environment are functionally related to these centers. When the value of the center changes, values of dynamic discourse referents change in a parallel way. For (108b), the modal would can access a dynamic discourse referent whose value is a function of the NP center and changes as the center changes from John to Bill. Thus, by treating sloppy identity as a general feature of the resolution of anaphoric devices, including tense and modals, (Stone and Hardt, 1997) give a simple, unified treatment of the sloppy identity patterns observed with tense and modals.

The present paper introduced, in section 4.2, a simple, abstract semantics that can serve as a foundation for these applications. Markers for scenarios are primitive. They are introduced directly into the evolving discourse model by the update \( \text{if}(\omega, \omega', D, *) \), which introduces \( \omega' \) as what would happen in \( \omega \) if \( D \). They are evoked like other entities; they are described by a handful of linguistically-motivated, atomic conditions: \( \text{poss}(\omega, \omega', *) \) and \( \text{necc}(\omega, \omega', *) \); \( \text{real}(\omega, \omega', *) \); \( \text{vivid}(\omega, \omega', *) \) and \( \text{remote}(\omega, \omega', *) \). This abstract view crystallizes intuitions and formalism in a natural form that makes the account accessible beyond specialists in dynamic modal semantics and logic, for use and criticism on linguistic grounds. It is hoped that such research may address the open problems in this account, in particular the formulation of a compositional semantics for referential modals with theoretical interest and justification.

The abstract view is justified by a concrete formalization in section 4.5. This formalism remains open for revision by specialists in linguistic semantics and commonsense reasoning. It can be anticipated that devising an abstract, neutral representation of modality for linguistic research, as has been proposed here, will facilitate such revision—for it is doubtless inappropriate. The uniform schemes for modal meaning used in 4.5 do not always match contemporary research—exemplified by (Morgenstern and Stein, 1988; Ortiz, 1996; Gelfond and Lifschitz, 1993; Sandewall, 1994) and even Kratzer’s own (Kratzer, 1989)—which instead emphasizes the subtlety of rankings among possible worlds and the inherently different roles different kinds of knowledge play in constructing and evaluating possible alternatives.

References


