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# Dispatch Sequences for Embedded Control Models

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## **Abstract**

We consider the problem of mapping a set of control components to an executable implementation. The standard approach to this problem involves mapping control blocks to periodic tasks, and then generating a schedule. This schedule is platform-dependent, and its execution requires realtime operating system support. We propose an alternative approach which involves generating a dispatch sequence of control blocks in a platform-independent manner. Our solution relies on assigning relative complexity and relative importance measures to control components, and is an adaptation of classical scheduling algorithms such as earliest-deadline-first. We show the benefits of our approach using simulation experiments on two case studies.

# Dispatch Sequences for Embedded Control Models \*

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## Abstract

*We consider the problem of mapping a set of control components to an executable implementation. The standard approach to this problem involves mapping control blocks to periodic tasks, and then generating a schedule. This schedule is platform-dependent, and its execution requires real-time operating system support. We propose an alternative approach which involves generating a dispatch sequence of control blocks in a platform-independent manner. Our solution relies on assigning relative complexity and relative importance measures to control components, and is an adaptation of classical scheduling algorithms such as earliest-deadline-first. We show the benefits of our approach using simulation experiments on two case studies.*

## 1. Introduction

Contemporary industrial control design already relies heavily on tools such as Simulink for mathematical modeling and simulation. Even though many such tools support implementation via automatic code generation from the model, many issues relevant to correctness and optimality of the implementation with respect to the timed semantics of the model are not satisfactorily addressed, and the implementation is tailored to a specific platform. Consequently, analysis results established for the model are not meaningful for the implementation and the code cannot be ported across platforms posing challenges for system integration. These challenges motivate our research.

In this paper, we focus on generating an executable implementation from a set of control blocks  $B$ . A control block computes outputs that influences other blocks or the environment being controlled. The control model has a well-defined timed semantics (either continuous or discrete) that can be used for simulation and analysis. Typically, the

implementation relies on the support offered by a real-time operating system for scheduling periodic tasks. Each control block  $B_i$  is compiled into an executable code in a host language such as C, and the control designer specifies a period  $\rho_i$  for the corresponding task. To implement the resulting periodic tasks on a specific platform, one needs to determine the worst-case-execution-time  $\tau_i$  for each block  $B_i$ , and check whether the task set is schedulable using standard scheduling algorithms such as earliest-deadline-first (EDF) or rate monotonic scheduling (c.f. [4, 17]).

While the real-time scheduling based implementation offers a separation of concerns using the abstraction of real-time tasks with periods and deadlines, it can hinder portability of control designs across platforms. As a concrete example, consider vision-based navigation of an autonomous robot trying to reach a target in a room full of obstacles. One control block computes the estimates of the obstacles while the other decides the trajectory based on the current estimates. Mapping these blocks to two tasks with specific periods introduces an abstraction that is not relevant to the high-level model or its goals. There are no *hard* real-time requirements in this application, and the performance can be measured by the time taken by the robot to reach the target. If the WCET (worst-case execution time) analysis on a particular processor reveals that the tasks are not schedulable, then in fact, the periods should be increased. If the analysis says that the tasks are schedulable, then it produces a schedule, which is a mapping from time slots to the tasks. This schedule is platform-dependent as it depends on the platform-specific WCET estimates. Moreover, executing the schedule requires real-time support from the operating system while the current trend in many application domains such as robotics is to employ commonly available computing platforms such as .NET [6]. Furthermore, since the scheduler views the tasks as periodic, it may leave the processor idle, thereby preventing improved performance.

In the proposed solution, our goal is to produce a *dispatch sequence* of blocks, rather than periodic tasks. The dispatch sequence is simply a string of control blocks, and is platform-independent. Unlike a schedule, a dispatch se-

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quence has no notion of time slots or other real-time requirements. Ideally, we would like the sequence to be such that, on any given platform, it follows the reference trajectory of the continuous time model as best as one can on that platform. This goal is hard to quantify abstractly, and even if one could find a concrete measure for specific applications (for instance, the total distance traveled in the above robot example), we are not aware of any methods to generate sequences that optimize this measure in an efficient way. In this paper, we formulate the sequence generation problem, and propose a possible solution. We associate with each control block  $B_i$  a measure  $\tau_i^r$  of *relative complexity* and a measure  $\rho_i^r$  of *relative importance*. The  $\tau_i^r$  value is supposed to capture the computation time of  $B_i$  relative to the other blocks, and the  $\rho_i^r$  value is supposed to capture in a relative manner, how updating the output of  $B_i$  impacts the environment. We use the appropriately tightly scaled versions of  $\rho^r$  values as periods and of  $\tau^r$  values as WCET estimates to generate sequences of blocks using classical real-time scheduling algorithms such as non-preemptive EDF and EDF. Since EDF is preemptive and we want to generate an executable sequence of blocks, this step requires model transformation via block-code-splitting. The output of our strategy is a platform-independent and untimed sequence of blocks: executing this sequence does not require preemption or any support from real-time scheduler, and its ability to follow the reference trajectory on a particular platform depends on the processing power of the platform.

The rest of the paper is organized as follows. Section 2 describes our model for control blocks along with a continuous time and a discrete time semantics for the same. Section 3 describes the classical real-time scheduling based approach by formalizing schedules, schedule semantics and strategies for generating schedules using period and WCET assignments. Section 4 defines the notion of a dispatch sequence, the associated semantics, and proposes strategies for generating dispatch sequences inspired by scheduling techniques, but using the notions of relative complexity and relative importance. Section 5 describes simulation experiments on two examples, one for robot navigation, and one for controlling heaters across multiple rooms, demonstrating the benefits of the proposed approach. We conclude with directions for future research in Section 6.

**Related Work.** Bridging the gap between high-level modeling or programming abstractions, and implementation platforms has been identified as a key challenge for embedded software research by many researchers (c.f. [19, 18]). Programming abstractions for embedded real-time controllers include synchronous reactive programming (languages such as ESTEREL and LUSTRE [3, 10, 9]), and the related *Fixed Logical Execution Time* (FLET) assumption used in the Giotto project [12, 13]. While these provide schedule-independent semantics, they do not ad-

dress the problem of mapping continuous time controllers to an executable implementation. Recently, the problem of generating code from timed and hybrid automata has been considered in [1, 14, 21], but in these papers the focus has been on choosing the sampling period so as to avoid errors due to switching and communication. The work on mapping Simulink blocks to Lustre focuses on signal dependencies [5]. Model-based development of embedded systems is also promoted by other projects with orthogonal concerns: Ptolemy supports integration of heterogeneous models of computation [7] and GME supports integration of multiple views of the system [16]. There is a rich literature on sampled control systems with a focus on understanding the gap between continuous and discrete controllers, determining the correct sampling period, and compensating for the computation delays in the design of control laws (c.f. [2]). In scheduling literature, while many variations of the basic periodic scheduling problem have been explored, the focus is on determining a platform-dependent mapping from time slots to tasks. The most relevant of these is control-aware scheduling [20], where periods for tasks are determined by optimizing a performance index.

## 2. Modeling Controllers

In this section, we describe the model of a real-time control system and the desired semantics for the model.

### 2.1. Model

Let  $X$  be a finite set of *environment variables* modeling the physical world to be controlled, and  $U$  be a finite set of *control variables* to be computed by the control software. Each variable has a type, which typically is  $\mathbb{R}$ , the set of reals. A state over a set of variables  $W$  is a mapping from  $W$  to values. We use  $Q_W$  to denote the set of all states over  $W$ . A control model is given by  $M = \langle M_C, M_E \rangle$ , where  $M_C$  is the *controller model* and  $M_E$  is the *environment model*.

The controller model  $M_C$  consists of a finite set  $B$  of *control blocks*, where each control block  $B_i \in B$  has the following components:

- A set of input variables  $Y_i \subseteq (X \cup U)$ , which the block reads to do its computation.
- A set of output variables  $U_i \subseteq U$ , which the block writes after its computation.
- A relation  $f_i \subseteq Q_{Y_i} \times Q_{U_i}$ , defining the computation of the block.
- A set of initial states  $Q_i^0 \subseteq Q_{U_i}$  for the output variables of the block.

The following properties must be satisfied by  $M_C$ . Every output variable must be computed by a unique block. That

is,  $[(\forall i, j, i \neq j : U_i \cap U_j = \emptyset) \text{ and } (\cup_j U_j = U)]$ . Further, consider a directed graph  $B_G$  whose nodes are control blocks and where there is an edge from  $B_i$  to  $B_j$  if  $B_j$  reads an output variable computed by  $B_i$ . Then  $B_G$  must be acyclic.

The environment model  $M_E$  is given by

- A relation  $g_x \subseteq Q_X \times Q_U \times \mathbb{R}$  for every environment variable  $x \in X$ . This relation is used to define the rate of change of  $x$  in terms of the current state.
- A set of initial states  $Q^0 \subseteq Q_X$  for the environment variables.

We have allowed our models to be nondeterministic, but this choice is not central to this paper, and in many cases, the computation of each control block  $B_i$  is defined by a function  $f_i : Q_{Y_i} \rightarrow Q_{U_i}$ , and the rate of change of an environment variable  $x$  is given by a function  $g_x : Q_X \times Q_U \rightarrow \mathbb{R}$ .

**Robot Navigation Example.** Consider a robot  $R$  which can move on a 2-D plane (see Figure 1). Initially  $R$  is at the (fixed) starting point  $S$ . Its goal is to reach a (fixed) target point  $T$ , without colliding with any of the stationary circular obstacle-disks  $O_1, O_2$  and  $O_3$  on the plane. The robot moves in the direction  $\theta$  at a constant speed  $v_R$ . It can estimate the obstacles only approximately, and we assume that the estimate is a circle whose center coincides with the center of the obstacle  $(x_c, y_c)$  and whose radius  $r$  is always larger than the actual radius  $r_0$ . The estimation rule is given by  $r = r_0 + (\sqrt{(x_c - x)^2 + (y_c - y)^2} - r_0)^2 / 500$  where  $(x, y)$  is the current position of  $R$ . The estimate  $r$  is smaller if  $R$  is closer to the obstacle. Based on the estimated radii of the 3 obstacles from the current position, the robot computes  $\theta$  as follows : first, it checks if the direct path from the current position  $(x, y)$  to the target  $T$  faces no obstruction — if so, it proceeds in that direction. If not, it computes the slopes of the tangents from the current position to the estimated obstacle circles, and checks whether rays along the tangents face any obstruction. Then, among the rays without any obstruction, it chooses to go along that ray which makes the least angle with the direct path. Figure 1 shows a snapshot of the robot position during its motion, along with the estimated obstacle radii and the selected direction of motion.

Figure 1 also shows a block diagram of the model. The environment variables are the coordinates of the robot position,  $(x, y)$ . The initial values of  $(x, y)$  are the coordinates of  $S$ . The differential equations governing the rates of change of  $x$  and  $y$  are :  $\dot{x} = v_R \cos \theta$ ,  $\dot{y} = v_R \sin \theta$ . The control variables are  $e_0, e_1, e_2$ , and  $\theta$ , where  $e_i$  is the estimate of the radius of obstacle  $O_i$ . There are four control blocks  $B_0, \dots, B_3$ . The control block  $B_i$ , for  $0 \leq i \leq 2$ , is used to estimate radius of obstacle  $O_i$ . Its input variables are  $x$  and  $y$ , and its output variable is  $e_i$ . The control block  $B_3$  is used to calculate  $\theta$ . Its input variables are  $e_0, e_1, e_2, x,$

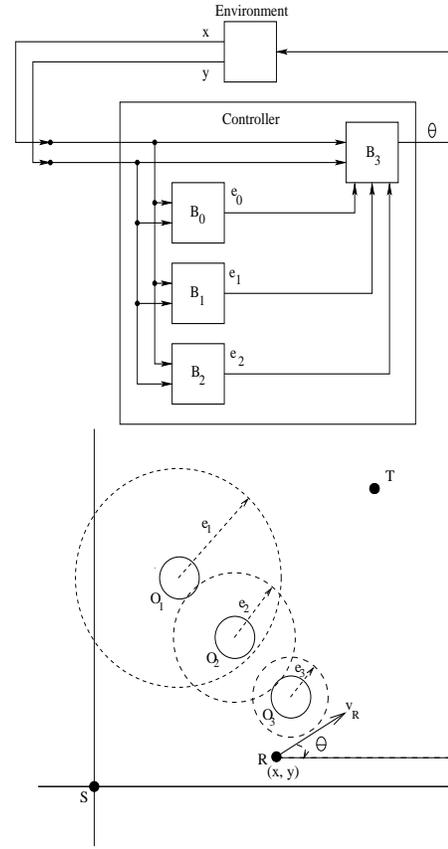


Figure 1. Robot navigation example

and  $y$ , and its output variable is  $\theta$ . The initial values of  $e_i$  are the estimates from  $S$ , and that of  $\theta$  is the angle computed using the initial values of  $e_i$ .

## 2.2. Semantics

Given a model  $M$  over variables  $X$  and  $U$ , a *trajectory* for  $M$  is a function  $\psi : \mathbb{R} \rightarrow Q_{X \cup U}$ . A *semantics* for a model  $M$ , denoted  $\llbracket M \rrbracket$ , is a set of trajectories for  $M$ . Two semantics, *continuous time* and *parameterized discrete time*, are described below.

**2.2.1. Continuous Time Semantics.** The continuous time semantics for a model  $M$ , denoted  $\llbracket M \rrbracket_C$ , evaluates all control variables at *every* point in the continuous time domain. It consists of the trajectories  $\psi$  satisfying the following constraints :  $\forall t \in \mathbb{R}, t \geq 0, \forall B_i \in B$  we have

$$\begin{aligned} \psi(0)(X) &\in Q^0 \\ (\psi(t)(X), \psi(t)(U), \dot{\psi}(t)(X)) &\in g_x \end{aligned} \quad (1)$$

$$\begin{aligned} \psi(0)(U_i) &\in Q_i^0 \\ (\psi(t)(Y_i), \psi(t)(U_i)) &\in f_i \end{aligned}$$

**2.2.2. Parameterized Discrete Time Semantics.** The parameterized discrete time semantics for a model  $M$  evaluates the *control variables* with a sampling period of  $\Delta$ , and a zero-order hold. So, all control outputs are piecewise-constant, the pieces being of length  $\Delta$ .

Let  $t_k = k \Delta$  for  $k \in \mathbb{N}$ . Given a  $\Delta > 0$ , the discrete time semantics for  $M$ , denoted by  $\llbracket M \rrbracket_D^\Delta$ , is a set of trajectories  $\psi$  satisfying the following constraints, besides (1) :  $\forall B_i \in B$  and  $\forall k \in \mathbb{N}$ , we have

$$\begin{aligned} \psi(0)(U_i) &\in Q_i^0 \\ (\psi(t_k)(Y_i), \psi(t_k)(U_i)) &\in f_i \end{aligned}$$

$$\text{For } t_{k-1} \leq t < t_k, t \in \mathbb{R} : \psi(t)(U_i) = \psi(t_{k-1})(U_i)$$

**2.2.3. Error in Discrete Time Semantics.** We note that the continuous time semantics is the ideal semantics for any given model. The discrete time semantics introduces an error into the model because of the zero-order hold for  $\Delta$  intervals. We may want to define the error using some metric over trajectories, but it is difficult to quantify the errors abstractly. For specific applications, such as those evaluated in this paper, we can find some concrete measures to quantify the performance of a trajectory, and use them to compare any two trajectories. In our robot navigation example, total distance traveled from the start position to the target is a reasonable measure of performance.

### 3. Schedule-based Implementation

In this section, we discuss some standard implementation strategies to generate real-time tasks from a given model  $M = \langle M_C, M_E \rangle$ . We first define the notion of a *schedule* and then discuss the standard platform-dependent ways of computing schedules.

We assume henceforth that the minimum time unit of execution of a control task is 1. That is, the values of the control variables can be *updated* by any control function only in intervals of one time unit. This simplifies the notation, otherwise we would need definitions parameterized by  $\Delta$  as in case of discrete-time semantics.

#### 3.1. Schedule and Schedule Semantics

A schedule is a mapping from time slots to blocks, which indicates the block that executes in each time slot. The schedule semantics for a schedule is the set of trajectories obtained by executing the blocks according to the schedule : that is, an instantiation of a block executes only in the time slots given by the schedule; its input values are read at the beginning of the first time slot of its execution and the control outputs computed by the block are updated at the end of the last slot of its execution. This can be implemented using the time-triggered architecture [17].

Formally, a *schedule*  $sch$  for  $M$  is a function  $sch : \mathbb{N} \rightarrow B \cup B^+ \cup \perp$ , where  $B^+ = \{B_i^+ \mid B_i \in B\}$  is used to denote the completion of the current instances of the corresponding tasks, and  $\perp$  denotes *idle*. The connotation of a schedule is as follows : let *slot*  $k$  denote the time interval  $[k - 1, k]$ . Then for  $k \geq 1$ ,

$$sch(k) = \begin{cases} B_i \text{ means } B_i \text{ executes in slot } k \text{ but } f_i \text{ is not} \\ \text{yet computed.} \\ B_i^+ \text{ means } B_i \text{ executes and finishes} \\ \text{computation of } f_i \text{ in slot } k. \\ \perp \text{ means the processor is idle in slot } k. \end{cases}$$

Given a schedule  $sch$ , the semantics associated with the model  $M$ , denoted by  $\llbracket M \rrbracket_{sch}$ , is a set of trajectories obtained by executing the blocks according to  $sch$ . For example, consider the schedule  $B_0 B_1 B_1 B_0^+ B_1^+ \dots$ . Block  $B_0$  starts executing at time  $t = 0$  after reading its inputs and executes in time slot 1. It is then preempted at time  $t = 1$  when  $B_1$  starts executing. Block  $B_0$  again executes in time slot 4, and finishes its execution in that time slot. The values computed by  $B_0$  are updated at the end of slot 4. We assume that reading and updating take zero computation time. Therefore,  $\psi(t)(U_0) = q_0$  for some  $q_0 \in Q_0^0$  for  $0 \leq t < 4$ , and  $\psi(4)(U_0) = f_0(\psi(0)(Y_0))$ .

Formally,  $\llbracket M \rrbracket_{sch}$  consists of the trajectories  $\psi$  satisfying the following constraints, besides (1):  $\forall k \in \mathbb{N}, B_i \in B$ ,

$$\begin{aligned} \psi(0)(U_i) &\in Q_i^0 \\ \psi(t)(U_i) &= \psi(k-1)(U_i) \text{ for } (k-1) < t < k \\ \psi(k)(U_i) &\in f_i(\psi(l)(Y_i)) \text{ if } sch(k) = B_i^+, \text{ where } l \\ &\text{is the smallest index } l' \text{ such that} \\ &sch(l') = B_i \text{ and} \\ &\forall j : l' < j < k : sch(j) \neq B_i^+; \\ &l = k \text{ if no such index } l' \text{ exists.} \\ \psi(k)(U_i) &= \psi(k-1)(U_i) \text{ otherwise.} \end{aligned}$$

#### 3.2. Standard Ways of Computing Schedules

Given a model  $M$ , the following steps are typically followed :

1. We first generate one task  $T_i$  for each block  $B_i$  in the model. The code executed by the task will be the function  $f_i$ , and the values used as input for variables in  $Y_i$  will be the most recently computed values for those variables.
2. We then assign a period  $\rho(B_i)$ , where  $\rho : B \rightarrow \mathbb{N}$ , to each task  $T_i$ . The period  $\rho(B_i)$ , also denoted by  $\rho_i$ , is *independent of the platform* on which the tasks are going to be executed. That is, as long as the task set is schedulable, the periods remain the same. They are usually assigned by control engineers (c.f. [2]) to satisfy the performance requirements of the control model such as stability, ability to track a given trajectory, etc.. The relative deadline of  $T_i$  is equal to  $\rho_i$ .

- Then, given an execution platform  $F$ , we compute  $\tau : B \rightarrow \mathbb{N}$ , where  $\tau(B_i)$ , also denoted as  $\tau_i$ , is the Worst-Case-Execution-Time (WCET) of  $B_i$  on  $F$ . The WCETs can be estimated using well-known WCET estimation methods (c.f. [11]).
- Given  $\rho$  and  $\tau$ , we can execute the tasks using a real-time operating system (RTOS) that includes a real-time scheduler for periodic tasks.

The RTOS typically uses well-known hard real-time scheduling algorithms for executing the tasks. We use two scheduling algorithms in this paper : the earliest-deadline-first (EDF) algorithm and the non-preemptive earliest-deadline-first (NPEDF) algorithm. The EDF (c.f. [4]) algorithm is a preemptive algorithm. When a new task is released or when the current task completes execution, it schedules the task with the earliest deadline among all active tasks. The NPEDF algorithm (c.f. [15]), if the processor is idle or the currently executing task has finished execution, schedules the task with the earliest deadline among all active tasks.

For a given  $\rho$  and  $\tau$ , if the task set is schedulable by EDF, it produces a periodic schedule  $sch$ , and the semantics  $\llbracket M \rrbracket_{EDF(\rho, \tau)}$  is defined to be  $\llbracket M \rrbracket_{sch}$ . If the task set is not schedulable using EDF, then the semantics  $\llbracket M \rrbracket_{EDF(\rho, \tau)}$  is undefined. The semantics associated with the NPEDF algorithm  $\llbracket M \rrbracket_{NPEDF(\rho, \tau)}$  is defined in a similar way.

We call this approach *platform dependent* since the schedule depends on the WCET estimates  $\tau$ . Note that the only feature of the platform relevant in our context is its processing power, which is captured by the WCET estimates  $\tau$ .

Consider the robot navigation example again. For this model, four tasks would be generated:  $T_i$ ,  $0 \leq i \leq 2$ , for estimating the radii of the obstacles, and  $T_3$  for calculating  $\theta$  based on the estimates. An assignment of periods for the tasks, and WCET estimates on three different platforms  $F_1$ ,  $F_2$  and  $F_3$  is given in Table 1. Platform  $F_1$  is the fastest while  $F_3$  is the slowest. The tasks are schedulable by NPEDF (a schedulability test for NPEDF can be found in [15]) on  $F_1$  and  $F_2$  but not on  $F_3$ . For  $t \in [1..120]$  (120 is the LCM of the periods of the tasks), the schedule produced by NPEDF on  $F_1$  and  $F_2$  is shown in table 2. The schedule produced by EDF on  $F_2$  is also shown. The notation  $[i : t_1 - t_2]$  means that block  $B_i$  executes continuously from time slot  $t_1$  to time slot  $t_2$  but without completing its execution, and  $[i : t_1 - t_2^+]$  means that  $B_i$  executes continuously from time slot  $t_1$  to time slot  $t_2$  and completes its execution at  $t_2$ .

We first note here that the periods (and therefore deadlines) assigned to the tasks are artificial. For example, if a task set is not schedulable, the control engineer might be able to increase the periods without violating the performance requirements of the control model. Here, we can in-

| Task  | $\rho$ | $\tau$ |       |       |
|-------|--------|--------|-------|-------|
|       |        | $F_1$  | $F_2$ | $F_3$ |
| $T_0$ | 120 ms | 12 ms  | 24 ms | 28 ms |
| $T_1$ | 120 ms | 12 ms  | 24 ms | 28 ms |
| $T_2$ | 120 ms | 12 ms  | 24 ms | 28 ms |
| $T_3$ | 24 ms  | 3 ms   | 6 ms  | 7 ms  |

**Table 1. Period ( $\rho$ ) and execution times ( $\tau$ ) on different platforms for robot navigation.**

| Strategy, Platform | Schedule in [1..120]   |
|--------------------|--|
| NPEDF, $F_1$       | $[3 : 1 - 3^+]$ $[0 : 4 - 15^+]$ $[1 : 16 - 27^+]$<br>$[3 : 28 - 30^+]$ $[2 : 31 - 42^+]$ $[\perp : 43 - 48]$<br>$[3 : 49 - 51^+]$ $[\perp : 52 - 72]$ $[3 : 73 - 75^+]$<br>$[\perp : 76 - 96]$ $[3 : 97 - 99^+]$<br>$[\perp : 100 - 120]$ |
| NPEDF, $F_2$       | $[3 : 1 - 6^+]$ $[0 : 7 - 30^+]$ $[3 : 31 - 36^+]$<br>$[1 : 37 - 60^+]$ $[3 : 61 - 66^+]$ $[2 : 67 - 90^+]$<br>$[3 : 90 - 96^+]$ $[3 : 97 - 102^+]$<br>$[\perp : 103 - 120]$   |
| EDF, $F_2$         | $[3 : 1 - 6^+]$ $[0 : 7 - 24]$ $[3 : 25 - 30^+]$<br>$[0 : 31 - 36^+]$ $[1 : 37 - 48]$ $[3 : 49 - 54^+]$<br>$[1 : 55 - 66^+]$ $[2 : 67 - 72]$ $[3 : 73 - 78^+]$<br>$[2 : 79 - 96^+]$ $[3 : 97 - 102^+]$<br>$[\perp : 103 - 120]$            |

**Table 2. Schedules by NPEDF and EDF.**

crease the periods slightly to render the tasks schedulable on  $F_3$ . Further, we observe that there are a lot of idle times on  $F_1$ , whereas executing the control tasks without any idle times (that is, executing the next block in sequence immediately after a block finishes execution) can improve performance. The goal in this case is to approximate the discrete semantics  $\llbracket M \rrbracket_D^1$  (and hence the continuous semantics  $\llbracket M \rrbracket_C$ ) as best as possible given the processing constraints. Abstracting this goal to scheduling of the tasks with deadlines and periods loses too much information. The performance measure in this case is the total distance traveled, or equivalently, time to reach the target, and we would like a systematic and computationally tractable approach which will minimize this performance measure.

## 4. Dispatch Sequences

In this section, we discuss our method of implementing controllers without real-time tasks. We introduce the notion of a *dispatch sequence* (*d-sequence for short*) which is a

string of blocks indicating the order in which blocks are to be executed. Then, after defining the semantics associated with d-sequences, we describe strategies to generate them using NPEDF and EDF.

#### 4.1. Dispatch Sequence Semantics

A *d-sequence*  $\sigma \in B^*$  is a string over  $B$  which indicates the *sequence* in which the blocks should be executed repeatedly. The whole block is to be executed without preemption, and when it completes its execution, the succeeding block can start executing immediately. *Unlike a schedule, there is no notion of time in a d-sequence.* Hence, d-sequences may look like cyclic executive schedules, but are different.

Given a platform  $F$ , let  $\gamma_l, \gamma_u : B \rightarrow \mathbb{N}$  be two functions that specify lower and upper bounds respectively on the execution time of  $B_i$  on  $F$ . That is  $\tau_i$ , the execution time of  $B_i$  on  $F$ , is such that  $\gamma_l(B_i) \leq \tau_i \leq \gamma_u(B_i)$ . Note that different executions of the same block can take different amounts of time, and nothing is said about the distribution of  $\tau_i$  between the two limits.

Given a triple  $(\sigma, \gamma_l, \gamma_u)$ , the d-sequence semantics associated with a model  $M$ , denoted by  $\llbracket M \rrbracket_{(\sigma, \gamma_l, \gamma_u)}$ , is the set of trajectories obtained by executing the blocks according to  $\sigma$ , where the execution times for the blocks are chosen according to the bounds. Formally, it can be defined as follows.

Let  $|\sigma| = k$ , and let  $\sigma_i$  denote the  $i^{\text{th}}$  block in  $\sigma$  for  $i \geq 1$ . Define  $Sch(\sigma, \gamma_l, \gamma_u)$ , to be the set of all schedules  $sch: \mathbb{N} \rightarrow B \cup B^+$  such that  $\exists i_0=0 \leq i_1 \leq i_2 \leq \dots$  for which  $\forall j \geq 1$ , let  $m = j \bmod k$ , then

$$\gamma_l(\sigma_m) \leq (i_j - i_{j-1}) \leq \gamma_u(\sigma_m) \text{ and}$$

$$sch(n) = \begin{cases} \sigma_m & \text{for } (i_{j-1} + 1) \leq n < i_j \\ \sigma_m^+ & \text{for } n = i_j \end{cases}$$

then  $\llbracket M \rrbracket_{(\sigma, \gamma_l, \gamma_u)} = \bigcup_{sch \in Sch(\sigma, \gamma_l, \gamma_u)} \llbracket M \rrbracket_{sch}$ .

For example, consider the **round-robin (RR)** d-sequence  $\sigma^{RR} = (B_0 B_1 B_2 B_3)^*$  for the navigation example. The blocks are to be executed repeatedly in the order  $B_0 B_1 B_2 B_3$ . Table 3 gives the  $\gamma_l$  and  $\gamma_u$  values for platform  $F_2$ . This means that  $B_i$  for  $i = 0, 1, 2$  can execute for anytime between 22 ms and 24 ms, and  $B_3$  for anytime between 4 ms and 6 ms. Here, estimation takes much longer than computing the direction, and round-robin does not seem to be a desirable choice.

**Relative execution times and relative periods.** Since we do not want to commit to concrete deadlines and periods, we introduce the notion of “relative” periods and “relative” execution times. Let a controller model  $M_C$  with  $n$  blocks be given. For each block  $B_i$ , we assign a *relative execution time*  $\tau_i^r \in \mathbb{N}$  and a *relative period*  $\rho_i^r \in \mathbb{N}$  such that

| Block | $\gamma_l$ | $\gamma_u$ |
|-------|------------|------------|
| $B_0$ | 22 ms      | 24 ms      |
| $B_1$ | 22 ms      | 24 ms      |
| $B_2$ | 22 ms      | 24 ms      |
| $B_3$ | 4 ms       | 6 ms       |

**Table 3.**  $\gamma_l$  and  $\gamma_u$  for the blocks in robot navigation for platform  $F_2$ .

| Block | $\tau_i^r$ | $\rho_i^r$ |
|-------|------------|------------|
| $B_0$ | 4          | 5          |
| $B_1$ | 4          | 5          |
| $B_2$ | 4          | 5          |
| $B_3$ | 1          | 1          |

**Table 4.** Relative execution times and relative periods for blocks of robot navigation.

$\gcd(\tau_1^r, \tau_2^r, \dots, \tau_n^r) = \gcd(\rho_1^r, \rho_2^r, \dots, \rho_n^r) = 1$ . The relative execution time  $\tau_i^r$  is an estimate of the WCET of  $B_i$  on any platform, relative to the times taken by other blocks in the model. We can compute them by several approximate methods. One method is to scale the execution times of  $B_i$  on several platforms by the speeds of those platforms, and take the average of the scaled times as the estimate of  $\tau_i^r$ . The relative period  $\rho_i^r$  is an index of the importance assigned to the block, when compared to the importance of other tasks. These are to be assigned by the control engineer.

Table 4 shows a set of relative execution times and relative periods for the blocks of navigation example. Note that the WCETs of the blocks on platform  $F_2$  as given in table 3 are roughly 6 times the relative execution times as given by table 4. In general,  $\gamma_l(B_i)$  and  $\gamma_u(B_i)$  are expected to be roughly  $k$  times  $\tau_i^r$  for some scaling factor  $k$ .

The d-sequence generation problem can be stated informally as follows. Given a model  $M$  and relative measures  $\tau^r$  and  $\rho^r$ , generate a string  $\sigma$  of blocks such that, on any platform  $F$  where the lower and upper bounds  $\gamma_l$  and  $\gamma_u$  for blocks are consistent with the ratios given by  $\tau^r$ , the trajectories in  $\llbracket M \rrbracket_{(\sigma, \gamma_l, \gamma_u)}$  are as close as possible to the trajectories in  $\llbracket M \rrbracket_C$ . There does not seem to be a computationally tractable way of formulating this as a mathematical optimization problem. Hence, we settle for heuristics inspired by scheduling schemes.

## 4.2. Dispatch Sequence Generation using NPEDF

In this section, we explain our strategy to generate d-sequences using NPEDF algorithm from given model  $M_C$  and relative measures. The d-sequence, denoted by  $\sigma^{NPEDF}$ , is such that a block is always executed in its entirety.

The main steps to generate  $\sigma^{NPEDF}$  are as follows :

1. Compute the utilization  $U^r = \sum_{i=1}^n \frac{\tau_i^r}{\rho_i^r}$  of the blocks. If  $U^r > 1$ , then scale the periods  $\rho_i^r$  by the smallest integer  $p$  such that  $U^r/p \leq 1$ ; otherwise, let  $p = 1$ . Call these new periods, the scaled versions of  $\rho_i^r$ .
2. Compute  $l = \text{lcm}(p\rho_1^r, p\rho_2^r, \dots, p\rho_n^r)$ . This is the lcm of the scaled periods.
3. Run the NPEDF algorithm from time  $t = 0$  to time  $t = l$  with  $\tau_i^r$  as the execution time, and  $p\rho_i^r$  as the period of task  $B_i$  to get a schedule  $sch(NPEDF)$  of length  $l$ . Since  $U^r \leq 1$ , all the instances of the blocks released before  $t = l$  are executed before  $t = l$ .
4. In  $sch(NPEDF)$ , there may be some idle times. Collapse the schedule by disregarding the idle times to obtain a d-sequence  $\sigma'$  from  $sch(NPEDF)$ . That is, if there is any idle time between two successive blocks  $B_i$  and  $B_j$  in  $sch(NPEDF)$  then  $B_j$  follows immediately after  $B_i$  in  $\sigma'$ , and the idle time after the execution of the last block in  $sch(NPEDF)$  is discarded. The desired d-sequence  $\sigma^{NPEDF}$  is  $\sigma'$ . It is easy to see that  $\sigma^{NPEDF}$  as obtained above is indeed a string over  $B$ .

For example, consider the relative execution times and periods for the robot example given in table 4. The utilization  $U^r$  is  $\frac{17}{5}$ . We scale this by  $p = 4$ . We then obtain  $l = \text{lcm}(4, 20, 20, 20) = 20$ . We then simulate it using NPEDF algorithm from  $t = 0$  to  $t = 20$  to get the schedule  $[3 : 1 - 1^+][0 : 2 - 5^+][3 : 6 - 6^+][1 : 7 - 10^+][3 : 11 - 11^+][2 : 12 - 15^+][3 : 16 - 16^+][3 : 17 - 17^+][\perp : 18 - 20]$ . We then get  $\sigma'$  from the above schedule by removing idle times : there are three slots of idle time at the end for this schedule, and so  $\sigma^{NPEDF}$  is  $(B_3 B_0 B_3 B_1 B_3 B_2 B_3 B_3)$ .

A property of  $\sigma^{NPEDF}$  is that if the algorithm above used  $\alpha\tau_i^r$  for some  $\alpha \in \mathbb{N}$  as the execution times instead of  $\tau_i^r$ , then the d-sequence produced is the same irrespective of  $\alpha$ . In other words, if the execution times are scaled by  $\alpha$  and the periods by  $p\alpha$ , where  $p$  is as in the above algorithm, and the tasks are scheduled using NPEDF, then the schedules obtained are the same as the schedules corresponding to the d-sequence  $\sigma^{NPEDF}$ . This means that the *d-sequence generation algorithm* needs to be run only once, regardless of the platform on which the d-sequence is going to execute.

**Scaling Theorem :** Let  $M$  be a given model with a relative execution time  $\tau_i^r$  and relative period  $\rho_i^r$  for each block

$B_i$ . Let  $p \in \mathbb{N}$  be the least integer such that  $\sum_i (\frac{\tau_i^r}{p\rho_i^r}) \leq 1$ . Given any  $\alpha \in \mathbb{N}$ , let  $\forall i : (\tau_i, \rho_i) = (\alpha\tau_i^r, \alpha p\rho_i^r)$ . Let  $\tau, \rho : B \rightarrow \mathbb{N}$  such that  $\tau(B_i) = \tau_i$  and  $\rho(B_i) = \rho_i$ . Then, for any schedule  $sch$ ,  $sch \in Sch_{(\sigma^{NPEDF}, \tau, \rho)}$  iff  $sch$  is generated by NPEDF( $\tau, \rho$ ).

**Proof :** Omitted due to space constraints.

## 4.3. Dispatch Sequence Generation using EDF

The d-sequence generation algorithm using EDF is similar to that using NPEDF, except that we use EDF to obtain  $\sigma'$ . However,  $\sigma'$  will no longer be a d-sequence over  $B$  since some blocks might be preempted. In other words, the block-code of  $B_i$  (that is, the code implementing  $f_i$ ) may need to be split. We first discuss how to handle splitting of block-code before proceeding to d-sequence generation.

**Splitting of block-code.** Given a block  $B_i$ , we assume that we can split the block-code of  $B_i$  into  $\tau_i^r$  contiguous portions such that the relative execution time of each contiguous portion is 1. We can then create  $\tau_i^r$  blocks  $B_{i1}, \dots, B_{i\tau_i^r}$  such that  $B_{il}$  executes the  $l^{th}$  contiguous portion, and  $\tau_{il}^r = 1$  for all  $l$ . The inputs of  $B_{i1}$  are the inputs of  $B_i$ , and the inputs of  $B_{il}$  for  $l > 1$  are the outputs of  $B_{i(l-1)}$ ; the outputs of  $B_i$  are the outputs of  $B_{i\tau_i^r}$ . Call  $B_{ij}$  the *split-block* of  $B_i$  and the new model  $M'$  with  $B' = \{B_{ij}\}$  as the set of blocks as the *split-model* of  $M$ . Note that  $M'$  is a semantics preserving transformation of  $M$ .

**D-sequence generation.** The main steps to generate  $\sigma^{EDF}$  are as follows :

1. Compute the utilization  $U^r = \sum_{i=1}^n \frac{\tau_i^r}{\rho_i^r}$  of the blocks. If  $U^r > 1$ , then scale the periods  $\rho_i^r$  by the smallest integer  $p$  such that  $U^r/p \leq 1$ ; otherwise, let  $p = 1$ . Call these new periods, the scaled versions of  $\rho_i^r$ .
2. Compute  $l = \text{lcm}(p\rho_1^r, p\rho_2^r, \dots, p\rho_n^r)$ . This is the lcm of the scaled periods.
3. Produce the split-model  $M'$  of  $M$ .
4. Run the EDF algorithm from time  $t = 0$  to time  $t = l$  with  $\tau_i^r$  as the execution time, and  $p\rho_i^r$  as the period of task  $B_i$  to get a schedule  $sch(EDF)$  of length  $l$ . Since  $U^r \leq 1$ , all the instances of the blocks released before  $t = l$  are executed before  $t = l$ . Now, the EDF algorithm can split the block  $B_i$  by preempting it. Thus,  $sch(EDF)$  is a mapping from  $\mathbb{N}$  to  $B \cup B^+ \cup \perp$ , and it can be viewed as a mapping from  $\mathbb{N}$  to  $B' \cup \perp$ , by replacing  $\tau_i^r$  time slots allocated to an instance of  $B_i$  by the  $\tau_i^r$  split blocks  $B_{ij}$ .
5. In  $sch(EDF)$ , there may be some idle times. Collapse the schedule by disregarding the idle times to obtain a d-sequence  $\sigma' \in (B')^*$  from  $sch(EDF)$ . That is, if there is any idle time between two successive blocks

$B_{il}$  and  $B_{jk}$  in  $sch(EDF)$  then  $B_{jk}$  follows immediately after  $B_{il}$  in  $\sigma'$ ; further, the idle time after the execution of the last block in  $sch(EDF)$  is discarded.

6. Now, note that a block  $B_i$  need not be split by the EDF algorithm into  $\tau_i^r$  split-blocks. In other words,  $B_{i(l+1)}$  may *always* follow  $B_{il}$  in  $\sigma'$ . Therefore, we can optimize splitting of  $M$  by finding maximal sequences  $B_{il}B_{i(l+1)} \dots B_{i(l+j)}$  of split-blocks of  $B_i$  which always execute contiguously in  $\sigma'$ , and combine all the blocks in a sequence into a single block. Let  $B''$  be the new set of blocks obtained after performing this optimization step, and the final schedule  $\sigma^{EDF}$  is in  $(B'')^*$ .

Note that all the steps above can be automated. As an illustration, consider again the navigation model, whose relative execution times and periods are given in Table 4. The utilization  $U^r$  is  $\frac{17}{5}$ . We scale this by  $p = 4$ . We then obtain  $l = \text{lcm}(4, 20, 20, 20) = 20$ . We then simulate it using EDF algorithm from  $t = 0$  to  $t = 20$  to get the schedule  $[3 : 1 - 1^+][0 : 2 - 4][3 : 5 - 5^+][0 : 6 - 6^+][1 : 7 - 8][3 : 9 - 9^+][1 : 10 - 11^+][2 : 12 - 12][3 : 13 - 13^+][2 : 14 - 16^+][3 : 17 - 17^+][\perp : 18 - 20]$ . We then get  $\sigma' \in B'$  from the above schedule by removing the three idle slots from the end of the schedule. We then obtain  $B''$  from  $B'$  as follows :  $B_3 \in B''$  since  $\tau_1^r = 1$ . Since  $B_0$  is split into two parts whose relative execution times are 3 and 1 respectively, the first three split-blocks of  $B_0$  can be combined into a single block  $B'_{01}$ . Similarly,  $B_{11}$  and  $B_{12}$  can be combined into a single block  $B'_{11}$ , and  $B_{13}$  and  $B_{14}$  can be combined into  $B'_{12}$ . Again, last three blocks of  $B_2$  can be combined into a single block  $B'_{22}$ . Thus,  $B'' = \{B'_{01}, B_{03}, B'_{11}, B'_{12}, B_{21}, B'_{22}, B_3\}$ . Therefore,  $\sigma'$  can be written as  $(B_3 B'_{01} B_3 B_{03} B'_{11} B_3 B'_{12} B_{21} B_3 B'_{22} B_3)$  to give the final d-sequence  $\sigma^{EDF}$ .

We note here that the *exact* splitting of block-codes to get  $B''$  is non-trivial. However, since there are no hard real-time requirements, and the purpose of the intended strategy is to improve performance, there is no need for exact splitting.

## 5. Evaluation and Experimental Results

In this section, we evaluate the performance of d-sequences generated using NPEDF and EDF, against those of round-robin d-sequences and the schedule-based platform-dependent implementation strategies. We first describe the simulator used to perform our experiments. We then examine the results in the case of two case studies : the first being the robot navigation example used in the previous sections, and the second, a house-heater system.

**Simulator.** The inputs to the simulator are the following :

- *Model*  $M = \langle M_C, M_E \rangle$ : An input file provides information about the structure of the model. It lists the environment variables, the control blocks  $B_i \in B$  in the order given by topological sort of  $B_G$  and  $U_i, Y_i, \tau_i^r, \rho_i^r, \gamma_l(B_i)$ , and  $\gamma_u(B_i)$  for every  $B_i$ . The file also indicates the function to be used. Finally, the simulator needs initial values of all the variables.
- *Simulation step*  $\delta$  : To approximate the continuous semantics, the simulator needs an integration step  $\delta$ , such that  $0 \leq \delta < 1$ .
- *Simulation time*  $N$  : It simulates from  $t = 0$  to  $t = N$ .

The simulator simulates  $M$  as per the continuous time and parameterized discrete time semantics, as per the platform-dependent NPEDF and EDF strategies as described in section 3, and by using the d-sequences generated by the round-robin, NPEDF, and EDF platform-independent strategies as described in section 4. Each of these cases is briefly discussed below :

*Continuous-time semantics* (`cont`) : The simulation is carried out in steps of  $\delta$ . At the end of each  $\delta$ -interval, all the environment variables are evaluated in parallel, and then the control outputs serially as per the topological sort of the blocks. The Euler method of integration is used for updating the environment variables in steps of an integration step  $\delta$ . In all the other cases below, the environment variables are evaluated in the same way.

*Discrete-time semantics* (`disc`) : This is same as the `cont` case except that the control variables are updated only in intervals of the parameter  $\Delta$ . The simulator uses  $\Delta = 1$ .

*Round-robin d-sequence* (`rr`) : The blocks are executed as per the d-sequence  $\sigma^{RR}$ . The order of blocks in  $\sigma^{RR}$  is given by the topological sort of  $B_G$ . The execution time of  $B_i, \tau_i$ , is chosen uniformly at random between  $\gamma_l(B_i)$  and  $\gamma_u(B_i)$  for each execution of  $B_i$ . The block  $B_i$  samples the values of  $Y_i$  when it starts execution, and the variables in  $U_i$  are updated at the end of the execution.

*NPEDF d-sequence* (`npedf_pi`) : The execution of this is the same as that of `rr` except that the d-sequence used is  $\sigma^{NPEDF}$ .

*EDF d-sequence* (`edf_pi`) : This case is interesting because of the need to simulate splitting of block-codes. Let the split-blocks produced for  $B_i$  be  $B_{ij}$  (after optimization). Let the relative execution time of  $B_{ij}$  be  $\tau_{ij}^r$ . Assuming that the relative execution time  $\tau_i^r$  corresponds to an actual execution time of  $\gamma_u(B_i)$ , the execution time of  $B_{ij}$  is  $(\tau_{ij}^r \times \gamma_u(B_i)) / \tau_i^r$ . Now, for any particular execution of  $B_i$ , the execution time  $\tau_i$  may be less than  $\gamma_u(B_i)$ . In such a case, we execute the blocks in the order  $B_{i1}, B_{i2}, \dots$  until an execution time of  $\tau_i$  is consumed, and the remaining blocks are not executed.

*NPEDF schedule* (`npedf_pd`) and *EDF schedule* (`edf_pd`) : These are simulated using the NPEDF and EDF periodic algorithms. A min-priority queue is used to extract the block with the earliest deadline.

The outputs of the simulator are the following :

- For each variable  $v$ , the value of  $v$  after each  $\delta$ -interval from time  $t = 0$  to time  $t = N$ .
- The value of a measure  $opt$  for each strategy. This measure is used for assessing the performance of the strategies. It is application specific, and is calculated a function of control outputs during the course of simulation.

## 5.1. Robot Navigation Example

The performance measure  $opt$  in this case is the total distance  $D$  traveled by the robot from the source  $S$  to the target  $T$ . The simulation parameters are  $N = 500$ ,  $\Delta = 0.1$ ,  $S = (0, 0)$ ,  $T = (200, 200)$ , `ROBOT_SPEED` = 2.0, `MINRAD` = 10.0,  $O_0 = (90, 110)$ ,  $O_1 = (260, 50)$ , and  $O_2 = (50, 260)$ . The relative execution times and relative periods are those in table 4. The actual periods used are those in table 1.

**Experimental results.** The simulation results for three sets of simulations using the above parameters for different  $\gamma_l$  and  $\gamma_u$  are shown in table 5 for all 7 strategies. The notation used is  $\gamma(B_i) = [\gamma_l(B_i), \gamma_u(B_i)]$ . The execution times for I and II are taken from table 1 and in both cases,  $\gamma_l(B_i) = \gamma_u(B_i)$ . The  $\tau_i^r$ 's are scaled by 3 for I, and by 6 for II. For III, the  $\tau_i^r$ 's are scaled *roughly* by 6 so that  $\gamma_l(B_i) < \gamma_u(B_i)$ .

The lower the value of  $D$ , the better the strategy is. It can be seen, as expected, that `cont` and `disc` always perform much better than the other strategies. In all cases (except that of `edf_pi` of I), round-robin performs worse than the platform-independent strategies. The platform-independent strategies introduce computation of  $\theta$  in between the estimation of obstacle radii, and this helps the robot to take advantage of recently computed obstacle radii to change its course. This also demonstrates that taking into account the relative periods of the tasks can improve control performance. Next, observe that in I, the platform-independent NPEDF strategy performs better than the platform-dependent NPEDF strategy because the latter has a lot of idle times, while the former has none. In III, the platform-independent strategies perform better than the platform-dependent ones. This is because while the platform-independent strategies schedule the next block immediately after the current block finishes execution, platform-dependent strategies have to wait for the task to be released at the beginning of its period. Thus, the former can take advantage

of tasks finishing earlier than their worst possible execution times.

## 5.2. Heater Example

Ivancic and Fehnker [8] discuss benchmarks for verification of hybrid systems, and this example is adapted from one of their benchmarks.

**Description of the example.** The benchmark deals with a set of rooms in a house being heated by a limited number of heaters and sharing the heaters so as to maintain some minimum temperature in all the rooms. The number of heaters is strictly less than the number of rooms. The temperature  $x_i$  of a room  $R_i$  depends linearly on the temperatures of the adjacent rooms, on the outside environment temperature  $u$ , and on  $h_i$ , which is 1 if a heater is present in the room and switched on, and 0 otherwise. The equation governing the rate of change of  $x_i$  is

$$\dot{x}_i = c_i h_i + b_i (u - x_i) + \sum_{i \neq j} a_{i,j} (x_j - x_i)$$

where  $a_{i,j}, b_i, c_i$  are constants. Each room  $R_i$  has two thresholds  $on_i$  and  $off_i$  such that the heater, if present in the room, is switched on if  $x_i$  is below  $on_i$  and switched off if  $x_i$  exceeds  $off_i$ . Each room may have at most one heater. If  $R_i$  does not have one, a heater can be fetched from an adjacent room  $R_j$  provided  $R_j$  has a heater,  $x_i$  is below a certain threshold  $get_i$  and  $x_j - x_i \geq diff_i$ . If there are more than one such rooms  $R_j$ ,  $R_i$  can choose non-deterministically to fetch from any of those rooms.

Our example has 3 rooms,  $R_0, R_1$  and  $R_2$ , where  $R_1$  is adjacent to  $R_0$  and  $R_2$ , and  $R_0$  and  $R_2$  are not adjacent. There are two heaters, initially switched on and in  $R_0$  and  $R_1$ . The outside temperature is constant at  $u = 4$ , and  $x_i = 20$  initially for all  $i$ . The thresholds are the same for all the rooms and are  $off = 21$ ,  $on = 20$ ,  $get = 18$ , and  $diff = 1$ .

The environment variables are  $x_i$  and the differential equations governing behavior of  $x_i$  are given by

$$\begin{aligned} \dot{x}_1 &= -0.9x_1 + 0.5x_2 + 0.4u + 6h_1 \\ \dot{x}_2 &= 0.5x_1 - 1.3x_2 + 0.5x_3 + 0.3u + 7h_2 \\ \dot{x}_3 &= 0.5x_2 - 0.9x_3 + 0.4u + 8h_3 \end{aligned}$$

The controller has two blocks,  $B_0$  for shifting heaters from one room to another if necessary and  $B_1$  for switching on or switching off all the heaters. There are six boolean control variables :  $hp_0, hp_1$  and  $hp_2$  indicating the presence of heaters in the rooms, and  $hs_0, hs_1$  and  $hs_2$  which are 1 iff there is a heater in the room and is switched on. The block diagram of the model is shown in Figure 2.

**Experimental results.** We measured the *minimum temperature*  $\eta$  reached in any of the rooms during the simulation, and the *total duration*  $\xi$  for which the the tempera-

|          | I                                  | II                                 | III                                |
|----------|------------------------------------|------------------------------------|------------------------------------|
|          | $\gamma(B_{\{0,1,2\}}) = [12, 12]$ | $\gamma(B_{\{0,1,2\}}) = [24, 24]$ | $\gamma(B_{\{0,1,2\}}) = [21, 24]$ |
|          | $\gamma(B_3) = [3, 3]$             | $\gamma(B_3) = [6, 6]$             | $\gamma(B_3) = [4, 6]$             |
|          | $D$                                | $D$                                | $D$                                |
| cont     | 346.52                             | 346.52                             | 346.52                             |
| disc     | 348.42                             | 348.42                             | 348.42                             |
| rr       | 599.08                             | 967.34                             | 914.78                             |
| npedf_pi | 499.10                             | 428.88                             | 419.46                             |
| npedf_pd | 575.44                             | 428.90                             | 518.54                             |
| edf_pi   | 605.80                             | 518.72                             | 512.06                             |
| edf_pd   | 579.38                             | 649.50                             | 560.86                             |

Table 5. Simulation results for robot example.

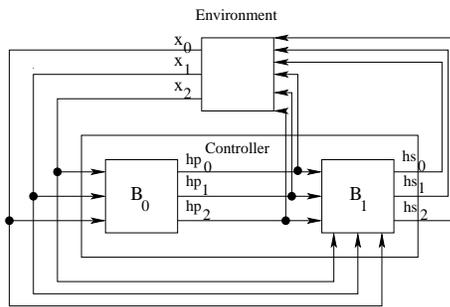


Figure 2. Block diagram of heater model.

ture in one of the rooms was below a certain threshold temperature  $x_{min}$ . The simulation parameters are  $N = 100$ ,  $\Delta = 0.01$ ,  $x_{min} = 13$ ,  $(\tau_0^r, \tau_1^r) = (4, 1)$ ,  $(\rho_0^r, \rho_1^r) = (4, 1)$ , and  $(\rho_0, \rho_1) = (24, 6)$ . The value of  $x_{min}$  was chosen to be 13, slightly below the min temperature attained by `disc`. The relative period of  $B_0$  is much higher than that of  $B_1$  because we expect update of heater state in a room to be more important than shifting of heaters. However, the actual results vary a lot depending on the choice of these simulation parameters.

The simulation results for four sets of simulations using the above parameters for different  $\gamma_l$  and  $\gamma_u$  are shown in table 6 for all 7 strategies. The  $\tau_i^r$ 's are scaled by 1 in I, and by 2 in II and for both I and II,  $\gamma_l(B_i) = \gamma_u(B_i)$ . In III,  $\tau_i^r$ 's are scaled *roughly* by 2 so that  $\gamma_l(B_i) < \gamma_u(B_i)$ . In IV, the values are chosen such that the task set was not schedulable using the platform-dependent NPEDF strategy.

Now, the higher the  $\eta$  value, and the lower the  $\xi$  value, the better the strategy is. It can be seen that the performance of `rr` is worse than that of `npedf_pi` and `edf_pi` as in the navigation example. Next, observe that if  $\gamma_l(B_i) < \gamma_u(B_i)$ , we expect the platform-dependent strategy to perform worse than that of independent strategies because the former always assumes that  $B_i$  takes  $\gamma_u(B_i)$  time. This can

be seen from the results in III. Next, in IV, while the tasks are not schedulable using NPEDF-dependent strategy, the platform-independent NPEDF strategy performs quite well, that is, much better than round-robin.

## 6. Discussion and Conclusions

We have proposed an approach to generate a dispatch sequence, instead of a schedule based on real-time tasks with deadlines and periods, from a set of interacting control blocks. This proposal is relevant when there are no hard real-time deadlines, or when the implementation platform does not offer support for real-time tasks. The generation strategy itself uses relative measures inspired by scheduling literature, and our preliminary simulation experiments suggest that it outperforms naive methods such as round-robin in optimizing application-level performance metrics.

There are many directions for future work. Extensive experimental validation and fine tuning of the proposed approach will be necessary. In particular, we are integrating the d-sequence generation strategy in the system ROCI developed for robotics applications [6]. In our examples, the d-sequence is supposed to imitate the timed model as best as one can, and there are no hard real-time requirements. However, a more general framework would integrate d-sequence generation with application-level real-time constraints. The current generation strategy does not take into account the interdependence among control blocks due to their inputs and outputs. Also, we have assumed that there is a single processor dedicated to the controller, and this can be relaxed. Finally, it is worth exploring if control design and d-sequence generation can be integrated so that some optimality guarantees of performance of the generated d-sequence can be obtained.

|          | I  |       | II   |       | III  |       | IV   |       |
|----------|--|-------|--|-------|--|-------|--|-------|
|          | $\gamma(B_0) = [4, 4]$<br>$\gamma(B_1) = [1, 1]$ |       | $\gamma(B_0) = [8, 8]$<br>$\gamma(B_1) = [2, 2]$ |       | $\gamma(B_0) = [6, 8]$<br>$\gamma(B_1) = [1, 2]$ |       | $\gamma(B_0) = [9, 9]$<br>$\gamma(B_1) = [3, 3]$ |       |
|          | $\eta$   | $\xi$ | $\eta$   | $\xi$ | $\eta$   | $\xi$ | $\eta$   | $\xi$ |
| cont     | 15.74  | 0     | 15.74  | 0     | 15.74  | 0     | 15.74  | 0     |
| disc     | 13.31  | 0     | 13.31  | 0     | 13.31  | 0     | 13.31  | 0     |
| rr       | 7.79   | 61.01 | 6.88   | 67.78 | 6.91   | 58.71 | 6.58   | 76.24 |
| npedf_pi | 11.20  | 51.59 | 9.64   | 48.7  | 10.88  | 41.72 | 7.14   | 59.44 |
| npedf_pd | 9.90   | 50.01 | 9.64   | 50.67 | 9.69   | 49.08 | -  | -     |
| edf_pi   | 10.99  | 48.31 | 9.61   | 40.71 | 10.43  | 43.94 | 8.66   | 44.04 |
| edf_pd   | 9.90   | 50.01 | 7.35   | 59.99 | 8.68   | 57.31 | 8.66   | 52.36 |

**Table 6. Simulation results for heater example.**

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