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Structure of $2^+$, $T = 2$ states in $A = 12$ nuclei

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Using a reasonable but simple model, properties of $2^+$ states in $^{12}$Be and $^{12}$O are calculated and compared with results of experiments.

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I. INTRODUCTION

Excitations into the $2\pi d$ shell are important at quite low excitation energies in $^{12}$Be. Several different experiments have demonstrated a large $(sd)^2$ component in the $^{12}$Be ground state (gs). In the past, we have used a simple model to describe low-lying states in nucleus $A + 2$ in terms of two neutrons in the $sd$ shell coupled to a $p$-shell core $A$. This description has been successful for $^{14,16}$C [1], $^{17}$N [2], $^{15}$C [3], $^{13}$B [4], $^{11}$Be [5], and the $0^+$ states of $^{12}$Be [6,7]. Here, we apply it to the $2^+$ states of $^{12}$Be (and, by isospin invariance, to $^{12}$C and $^{12}$O).

The model is not meant to be rigorous, but it does contain the principal elements of the nuclear structure. It uses “local” single-particle energies (spe’s) and “global” two-body residual interaction matrix elements. In the present case, we take the spe’s for $2s_1/2$ and $1d_{5/2}$ from the $1/2^+$ and $5/2^+$ states of $^{11}$Be [8]. We know those are not pure single-particle (sp) states, but this represents the simplest approach. The $(sd)^2$ two-body matrix elements (listed in Ref. [7]) are the same as we have used throughout this mass region. They first arose in a description of two-particle ($2p$) and four-particle, two-hole ($4p-2h$) states in $^{18}$O [9]. Here, for the $sd$ shell, we allow only the $1d_{5/2}$ and $2s_{1/2}$ orbitals, abbreviated $d$ and $s$, respectively. After diagonalizing the $(sd)^2$ Hamiltonians, the wave functions for the two $0^+$ and two $2^+$ states are as listed in Table I. The $(sd)^2$ states are then allowed to mix with the $p$-shell ones, for which we use the results of Cohen-Kurath [10].

In $^{10}$Be($t,p$) [11], the cross section of the first $2^+$ state is about 20 times larger than that calculated for the $p$-shell $2^+$ state, but is consistent with the state being predominantly of $(sd)^2$ character. In $^{14}$C($p,t$) [12], a peak at 2.06 MeV above the lowest $0^+ T = 2$ state appears to contain contributions from both $0^+$ and $2^+$ states. Fitting the angular distribution to the sum of $0^+$ and $2^+$ suggests [7] that the $2^+$ cross section is $19 \pm 9\%$ of that expected for the $p$-shell $2^+$ state, using amplitudes from Cohen-Kurath [10]. Even with core excitation in $^{14}$C(gs) [13], the $2n$ pickup is all from the $p$ shell [7]. So, we take as given that $2^+_1$ contains about $19 \pm 9\%$ of the $p$-shell $2^+$ state.

Our calculated energy of the lowest $2^+$ state (3.63 MeV, Table I) is significantly higher than the experimental value of 2.1 MeV. This is also true of other calculations. Blanchon et al. [14] get the first two $2^+$ states at 3.86 and 4.59 MeV. In Ref. [15], the lowest is at 3.8 MeV. The fact that the calculated energy of the $2^+_1$ state is significantly higher than the experimental energy is perhaps an indication that some collective component has not been included. The most obvious candidate is $^{10}$Be($2^+$) $\times (sd)^2_0$. Nunes et al. [16] showed that including this configuration does indeed bring the $2^+_1$ energy down. However, that configuration cannot be a major component because it has no direct one-step route in $^{10}$Be($t,p$) and (as noted above) the state is very strong there. We ignore this component for now, even though we expect it to be present at some level in all the $2^+$ states. We will return to this point later. Hamamoto and Shimoura [17] reproduce the $2^+$ energy with deformation. For $^{11}$Be, they assume the lowest $1/2^+$, $5/2^+$, and (supposed) $3/2^+$ states are members of a decoupled $1/2^+$ rotational band built on the Nilsson deformed orbital [220] $1/2^+$. These energies allow them to compute the moment-of-inertia and decoupling parameters for $^{11}$Be. They then scale the former to get a value for $^{12}$Be, leading to a $2^+$ energy of 2.09 MeV. So, fixing the $2^+$ energy is not a problem, but the fixes are outside the present scope.

In our work, we assume isospin invariance, namely that the wave-function amplitudes are the same for different $T_z$ members of an isospin multiplet. The effect of the Coulomb interaction is merely to change the radial-wave function. We note, however, that Grigorenko et al. [18] found significant isospin violation, namely an $s^2$ intensity in $^{12}$O(gs) that is 1.5–2.0 times the value in $^{12}$Be(gs). Even without isospin conservation, a value of about 50% $s^2$ in $^{12}$O(gs) is necessary to explain its Coulomb energy.

We described the two lowest $0^+$ states as linear combinations of the first $(sd)^2$ state and the $p$-shell one [6,7]. If we take the first $2^+$ state to be a mixture of the lowest $(sd)^2$ $2^+$ state and the $p$-shell $2^+$ and use the $^{14}$C($p,t$) results of $19 \pm 9\%$ of the $p$-shell component in the first $2^+$, then the wave function of this state is

$$2^+_1 = 0.84\, ds + 0.32\, dd + 0.44\, p\, shell,$$

where we temporarily ignore the uncertainty in the last term. In this simple description, the second and third $2^+$ states then should be linear combinations of

$$0.41\, ds + 0.16\, dd - 0.90\, p\, shell,\text{ and }0.41\, ds - 0.93\, dd.$$  

Takashina [19] states that the lowest $0^+$ and $2^+$ states are mostly $(sd)^2$. Because the second $(sd)^2$ $2^+$ and the
TABLE I. Energies and wave-function intensities in $^{12}$Be.

<table>
<thead>
<tr>
<th>$J^+$</th>
<th>Space</th>
<th>State</th>
<th>$E_\ell$ (MeV)</th>
<th>$s^2$</th>
<th>$d^2$</th>
<th>$p$ shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>0+</td>
<td>$(sd)^2$</td>
<td>$0^+_1$</td>
<td>0.20</td>
<td>0.78</td>
<td>0.22</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$(sd)^2$</td>
<td>$0^+_2$</td>
<td>4.35</td>
<td>0.22</td>
<td>0.78</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$(sd)^2 + p$ shell</td>
<td>gs</td>
<td>0.53</td>
<td>0.15</td>
<td>0.32</td>
<td>–</td>
</tr>
<tr>
<td>2+</td>
<td>$(sd)^2$</td>
<td>$2^+_1$</td>
<td>3.63</td>
<td>0.87</td>
<td>0.13</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$(sd)^2$</td>
<td>$2^+_2$</td>
<td>5.42</td>
<td>0.13</td>
<td>0.87</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$(sd)^2 + p$ shell</td>
<td>2.11 MeV</td>
<td>0.71</td>
<td>0.10</td>
<td>0.19</td>
<td>–</td>
</tr>
</tbody>
</table>

$p$-shell $2^+$ state are close together, the mixing of the two could be considerable. However, the lowest $2^+$ state should be reasonably stable to that mixing. And, of course, the $^{10}$Be$(2^+)$ × $(sd)^2_0$ configuration provides another $2^+$ state, and this strength is probably spread among all the $2^+$ levels.

II. $^{12}$O

We now use this $2^+$ wave function to calculate the expected energy and width in $^{12}$O. Pure configuration energies are listed in Table II. With our admixture, the resulting $^{12}$O$(2^+)$ energy is 1.80 MeV. The $\pm 9\%$ uncertainty in the 19% $p$-shell intensity provides an uncertainty of $\pm 15$ keV in this energy. From other work, we have found that our Coulomb energy calculations produce energies in mirror nuclei with deviations of $<40$–70 keV from experimental values. It is well known that a state in Table II. With our admixture, the resulting $^{12}$O$(2^+)$ state are close together, the mixing of the two states are quite close together and have about the same $s$-shell intensity. We integrated over the natural width of the $^{11}$N states. The expected widths are then obtained from

TABLE II. Excitation energy (MeV) in $^{12}$O of the mirror of $^{12}$Be ($2^+$, 2.1 MeV).

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$E_\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ds$</td>
<td>1.68$^a$</td>
</tr>
<tr>
<td>$dd$</td>
<td>2.33</td>
</tr>
<tr>
<td>$p$ shell</td>
<td>1.94</td>
</tr>
<tr>
<td>Mixed$^b$</td>
<td>1.80</td>
</tr>
</tbody>
</table>

$^a(5/2^+ \times s + 1/2^+ \times d)/2.$  
$^b$Configuration in last line of Table I.

TABLE III. Widths (keV) for decay of $^{12}$O $(2^+, 1.8$ MeV).

<table>
<thead>
<tr>
<th>$^{11}$N</th>
<th>$\ell$</th>
<th>$\Gamma_{sp}$</th>
<th>$S$</th>
<th>$\Gamma_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gs</td>
<td>1/2$^+$</td>
<td>2</td>
<td>150</td>
<td>0.52</td>
</tr>
<tr>
<td>1/2$^-$</td>
<td>1</td>
<td>180</td>
<td>0.007</td>
<td>1.3</td>
</tr>
</tbody>
</table>

$\Gamma_{calc} = S\Gamma_{sp}$. They are listed in Table III. The upshot is that this $2^+$ state near 1.8 MeV should be quite narrow. Earlier, we had predicted the $^{12}$O energy of $0^+_2$ to be 1.95 MeV [7]. A recent $^{14}$O$(p,t)$ experiment [21] observed a peak at 1.8(4) MeV, with a total width of 1.6(3) MeV, where the resolution width was 1.0(5) MeV. Because the $^{12}$C$(p,t)$ reaction populated both $0^+_1$ and $2^+_1$ states, the same should be true here. By isospin invariance, the $0^+_1/2^+_1$ cross-section ratio should be roughly equal in the two reactions. Suzuki et al. [21] analyzed their peak as a single state, but we expect it contains both states. Even though narrow, the $2^+$ peak would have been about 1 MeV wide from the resolution, making it very difficult to resolve the two states.

III. $^{12}$Be

We return now to the case of $^{12}$Be. In $^{10}$Be$(t,p)$, a candidate for a second $2^+$ state was observed at an excitation energy of 4.56 MeV. Millener [20] has suggested this might instead be a $3^-$ state, or a $2^+ / 3^-$ doublet, because it is too strong to be $2^+$. Indeed, given the observed $(t,p)$ cross section for the first $2^+$ state, we find that the 4.56-MeV cross section is significantly larger than the remaining $2^+$ strength expected for the entire $d_{5/2}, s_{1/2}$, $p$-shell space. At these negative $Q$ values, $2^+$ and $3^-$ angular distributions are very similar [22], making them difficult to distinguish. However, the cross section appears to be slightly too large for a single $3^-$ state, even if this state had a pure $(1s_{1/2})(1d_{5/2})$ configuration. If it is a doublet, then the two states are quite close together and have about the same width [107(17) keV], or one of them has most of the strength. (The $3^-$ could be strong and the $2^+$ weak.) If it is all $3^-$, then the other $2^+$ state(s) are too weak to observe or are above 6 MeV. Fortune, Liu, and Alburger [11] placed an upper limit of 30 $\mu$b/sr for an unobserved narrow state below 6 MeV. However, a broad state could have had a significantly larger cross section and have been missed. One possible candidate is near 5.4 MeV, and another is on the low-energy side of the 5.70-MeV $4^+$ state. If one $2^+$ state contains the bulk of the remaining $p$-shell configuration, it should be quite strong in $^{14}$C$(p,t)$, but no candidate was observed. At this time, we are unable to say anything further about other possible $2^+$ states.

Earlier, we estimated the amount of $s^2$ in $^{12}$Be and $(^{12}$O$)$ ground states by computing the $^{12}$Be, $^{12}$O mass difference, which is quite sensitive to this component. Our result was 53% for the $s^2$ intensity [6]. With a reasonable, but simple, shell-model calculation, we suggested an $s^2/d^2$ ratio of 0.78/0.22, and hence 68% $(sd)^2$, 32% $p$ shell for $^{12}$Be(gs). Navin et al. [23], in a subsequent experiment, coincidentally suggested the identical configuration admixture—68% $(sd)^2$, 32% $p$ shell. If $^{11}$Be(gs) were pure $2s_{1/2}$, the spectroscopic factor for $^{12}$Be(gs) would be just twice this $s^2$ intensity, and for $2^+$, $S$ would be equal to the $ds$ intensity. However, $^{11}$Be(gs) is only about 74%
TABLE IV. Spectroscopic factors in $^{11}\text{Be}(d,p)$ for lowest three states.

<table>
<thead>
<tr>
<th>State</th>
<th>$S_{gs}$ (Ref. [24])</th>
<th>Calculated (present)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Simple</td>
</tr>
<tr>
<td>$g_s$</td>
<td>0.28$^{+0.03}_{-0.07}$</td>
<td>1.06</td>
</tr>
<tr>
<td>$0^+_1$</td>
<td>0.73$^{+0.07}_{-0.09}$</td>
<td>0.50</td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>0.10$^{+0.05}_{-0.07}$</td>
<td>0.70</td>
</tr>
</tbody>
</table>

$^{10}\text{Be} \times 2S_{1/2}$. So, the $S$'s above need to be reduced by this factor. These numbers are listed in the Simple and Reduced columns in Table IV.

A very recent experiment [24] investigated the $^{11}\text{Be}(d,p)$ reaction in inverse kinematics, at a center-of-mass bombardment energy of 8.5 MeV. They measured $S$ for the lowest three states of $^{12}\text{Be}$. Because the $0^+_2/2^+_2$ states were not resolved, they used $\chi^2$-squared minimization to fit the doublet angular distribution to a sum of $\ell = 0$ and 2 distorted-wave curves. Their spectroscopic factors are also listed in Table IV. We note that the experimental $S$'s for the gs and $2^+$ are significantly smaller than the calculated ones, while $S(0^+_2)$ is larger than calculated. All reasonable shell-model calculations predict $S(2^+_1)$ to be $\sim 0.5$, in rough agreement with our value of 0.52. Various theoretical values in Ref. [24] are 0.41, 0.50, and 0.55. It is extremely difficult to envision a scenario in which this spectroscopic factor could be as small as 0.10 (1.0σ upper limit 0.19), found in Ref. [24]. Part of the problem could be an incorrect separation of the $0^+_2/2^+_2$ components of the unresolved doublet. However, the authors state that at the $2\sigma$ level, all the doublet strength could be $2^+$, and they arrive at $S = 0.25$—still a very small value. If isospin is not conserved and $^{12}\text{Be}(gs)$ has a smaller $s^2$ occupancy than $^{12}\text{O}(gs)$, the gs spectroscopic factor would be smaller than the calculated value in Table IV. However, the dominance of $(sd)^2$ over $p$-shell components is established from the $^{10}\text{Be}(t,p)$ reaction (and confirmed by other work). So, we would not expect a great reduction from the values in Table IV.

IV. SUMMARY

For the first $2^+$ state at 2.1 MeV in $^{12}\text{Be}$, the large cross section observed in the $^{10}\text{Be}(t,p)$ reaction is totally incompatible with the small spectroscopic factor claimed for it in the $^{11}\text{Be}(d,p)$ reaction. As both the gs and $2^+_1$ spectroscopic factors in Ref. [24] are smaller than expected in most models, it is conceivable that something is wrong with the absolute cross-section scale in Ref. [24]. We encourage another look at this reaction, difficult though it may be.

The supposed $2^+$ state at 4.56 MeV has too much strength in $(t,p)$ for another $2^+$ state. It is more likely to be $3^-$. In $^{14}\text{C}(p,t)$, the data are consistent with the first $2^+ T = 2$ state having about 20% of the strength expected for the pure $p$-shell $2^+$. There is no evidence in that reaction for another $2^+$ state with most of the remaining $p$-shell strength.

In $^{12}\text{O}$, the first $2^+$ state is expected near 1.8 MeV and should be narrow (width $\sim 80$ keV). The second $0^+$ state should be near 1.95 MeV, with a width of about 800 keV. A better $^{14}\text{O}(p,t)$ experiment might be able to separate the two.