Gravity Trapping on a Finite Thickness Domain Wall: An Analytic Study

Mirjam Cvetič  
University of Pennsylvania, cvetic@cvetic.hep.upenn.edu  

Marko Robnik  
University of Maribor

Follow this and additional works at: http://repository.upenn.edu/physics_papers
Part of the Physics Commons

Recommended Citation  

Suggested Citation:  

© 2008 The American Physical Society  
http://dx.doi.org/10.1103/PhysRevD.124003.

This paper is posted at ScholarlyCommons.  
http://repository.upenn.edu/physics_papers/110
For more information, please contact libraryrepository@pobox.upenn.edu.
Gravity Trapping on a Finite Thickness Domain Wall: An Analytic Study

Abstract
We construct an explicit model of the gravity trapping domain-wall potential, where for the first time we can study explicitly the graviton wave function fluctuations for any thickness of domain wall. A concrete form of the potential depends on one parameter $0 \leq x \leq \pi/2$, which effectively parameterizes the thickness of the domain wall with specific limits $x \to 0$ and $x \to \pi/2$ corresponding to the thin and the thick wall, respectively. The analysis of continuum Kaluza-Klein fluctuations yields explicit expressions for both small and large Kaluza-Klein energy. We also derive specific explicit conditions in the regime $x > 1$, for which the fluctuation modes exhibit a resonance behavior, and which could sizably affect the modifications of the four-dimensional Newton's law at distances that typically are by 4 orders of magnitude larger than those relevant for Newton's law modifications of thin walls.

Disciplines
Physical Sciences and Mathematics | Physics

Comments
Suggested Citation:

© 2008 The American Physical Society
http://dx.doi.org/10.1103/PhysRevD.124003.

This journal article is available at ScholarlyCommons: http://repository.upenn.edu/physics_papers/110
Gravity trapping on a finite thickness domain wall: An analytic study

Mirjam Cvetič1,2,* and Marko Robnik2,+1

1Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, USA
2CAMTP - Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, SI-2000 Maribor, Slovenia

(Received 14 February 2008; published 3 June 2008)

We construct an explicit model of the gravity trapping domain-wall potential, where for the first time we can study explicitly the graviton wave function fluctuations for any thickness of domain wall. A concrete form of the potential depends on one parameter \(0 \leq x \leq \frac{x_c}{2}\), which effectively parameterizes the thickness of the domain wall with specific limits \(x \rightarrow 0\) and \(x \rightarrow \frac{x_c}{2}\) corresponding to the thin and the thick wall, respectively. The analysis of continuum Kaluza-Klein fluctuations yields explicit expressions for both small and large Kaluza-Klein energy. We also derive specific explicit conditions in the regime \(x > 1\), for which the fluctuation modes exhibit a resonance behavior, and which could sizably affect the modifications of the four-dimensional Newton’s law at distances that typically are by 4 orders of magnitude larger than those relevant for Newton’s law modifications of thin walls.

DOI: 10.1103/PhysRevD.77.124003 PACS numbers: 04.60.-h, 03.65.Ge, 04.60.-m, 04.65.+e

I. INTRODUCTION

Bogomol’nyi-Prasad-Sommerfied (BPS) saturated supergravity domain walls [1], which interpolate between two BPS anti–de Sitter vacua, possess the feature that they can trap gravity [2]. The original infinitely thin, \(Z_2\) symmetric domain-wall solution [3] in five dimensions can be treated explicitly to obtain the qualitative form of corrections to the four-dimensional Newton's law [2]. Subsequent studies extended further analysis to features of the finite thickness walls (see [9,10] and references therein) as well as numerous other subsequent studies of special models, primarily employing numerical analyses of fixed thickness solutions (for recent studies see e.g., [11,12] and references therein) [13].

The purpose of this paper is to advance the analysis of gravity trapping for domain walls in an explicit analytic model, where the graviton wave function fluctuations can be studied explicitly for any thickness of domain wall. The Schrödinger potential depends on one parameter \(0 \leq x \leq \frac{x_c}{2}\), which effectively parameterizes the thickness of the domain wall. The two limits \(x \rightarrow 0\) and \(x \rightarrow \frac{x_c}{2}\) reproduce the infinitely thin and thick wall limits, respectively. This is the first analytic example where the graviton wave function can be obtained explicitly for any value of allowed \(x\), both for small and large values of Kaluza-Klein masses of fluctuations. Intriguingly, we also find resonance behavior in the regime \(x > 1\) and provide an explicit analytic study of resonances, which in turn can sizably modify the four-dimensional Newton’s law at distances that are typically 4 orders of magnitude larger than those relevant for modifications in the thin wall setup.

The paper is organized in the following way. In Sec. II, we present the concrete form of gravity potential in the background of the BPS domain with a finite thickness, interpolating (in \(Z_2\) invariant way) between two anti–de Sitter vacua with negative cosmological constant \(\Lambda\). We discuss in detail matching conditions on the metric and the explicit form of the potential that depends only on one free parameter \(0 \leq x \leq \frac{x_c}{2}\), effectively parameterizing the thickness of the wall. In Sec. III, we study graviton wave function fluctuations and, in particular, focus on the explicit form of the probability density at the center of the wall. In Sec. III A, we obtain analytic expressions for the probability density at the center of the wall both for small and large values of Kaluza-Klein masses. In Sec. III B, we analyze quantitatively the conditions under which the wave function exhibits resonances. In Sec. IV, we study the implications for the modification of Newton’s law, and, in particular, obtain the analytic form of Newton’s law modifications both in thin \((x < 1)\) and thick \((x > 1)\) regimes. Conclusions and proposals for further studies are presented in Sec. V.

II. FINITE THICKNESS DOMAIN-WALL MODEL

For the sake of simplicity, the model that we shall discuss is chosen to arise from a \(Z_2\) symmetric finite thickness domain wall that interpolates between two BPS vacua with negative cosmological constant \(\Lambda\) in five dimensions [16]. Our motivation is to present an explicit, integrable model of the Schrödinger potential for graviton wave function modes, which would be explicitly parameterized by the thickness of the wall. Such a model would in turn allow for an explicit analysis of graviton fluctuation modes and the subsequent analysis of a modification of the four-dimensional Newton’s law, thus providing a unifying, analytic approach, which addresses these effects as a function of the wall thickness.

*cvetic@cvetic.hep.upenn.edu
*robnik@uni-mb.si
We choose the following form of the potential:

\[ V(z) = -V_0; \quad |z| \leq \frac{d}{2}, \]
\[ V(z) = \frac{15}{4(z + \beta)^2}; \quad |z| \geq \frac{d}{2}. \]  

(1)

This potential enters the Schrödinger equation for the graviton wave function (for the derivation of the wave equation and other details see, e.g., [10]):

\[ -\frac{d^2 \psi_m(z)}{dz^2} + V(z) \psi_m(z) = m^2 \psi_m(z), \]  

(2)

where the graviton wave function \( h_{\mu \nu} = \eta_{\mu \nu} \psi(z) \exp(ik_{\mu}x^\mu) \) and \( k_{\mu}k^\mu = m^2 \) parameterizes the Kaluza-Klein energy associated with the four-dimensional momentum \( k_{\mu} \).

The potential \( V(z) \) is related to the conformal factor \( A(z) \) of the domain-wall metric

\[ ds^2 = \exp[-A(z)] \left[-dt^2 + dz^2 + \sum_{i=1}^{3} dx_i^2 \right] \]  

(3)

through the following relation (see, e.g., [10]):

\[ V(z) = \frac{9}{16} \left( \frac{dA(z)}{dz} \right)^2 - \frac{3}{4} \frac{d^2 A(z)}{dz^2}. \]  

(4)

Along with the boundary conditions \( A(0) = 0, A(0)^\prime = 0 \) and \( A(z) \rightarrow 2 \log(k|z|) \) as \( |z| \rightarrow \infty \) the relation (4) determines for the potential (1) the following form of the metric:

\[ A(z) = -\frac{4}{3} \log(\cos(\sqrt{V_0}|z|)); \quad |z| \leq \frac{d}{2}, \]
\[ A(z) = \log[k^2(|z| + \beta)^2]; \quad |z| \geq \frac{d}{2}. \]  

(5)

where \( k = \sqrt{\frac{2}{3}} \) (see, e.g., [14]) and the five-dimensional Planck constant \( M_{\text{Planck}} \) was set to 1.

Continuity of the metric and its derivative at the junction \( |z| = \frac{d}{2} \) imposes the following two conditions among parameters \( V_0, d \) and \( \beta \):

\[ \left( \frac{d}{2} + \beta \right) = \cos \left( \frac{\sqrt{V_0}d}{2} \right)^{-2/3}, \]
\[ \left( \frac{d}{2} + \beta \right)^{-1} = \frac{2}{3} \sqrt{V_0} \tan \left( \frac{\sqrt{V_0}d}{2} \right). \]  

(6)

These two conditions can be viewed as fixing \( \frac{d}{2} + \beta \) and \( V_0 \) in terms of one free parameter

\[ x = \frac{\sqrt{V_0}d}{2}. \]  

(7)

The parameter \( x \) has a range \([0, \frac{\pi}{2}]\). The infinitely thin wall corresponds to the following parameter limit:

\[ x \rightarrow 0; \quad d \rightarrow 0, \quad V_0 \rightarrow \infty, \quad V_0d = \frac{3}{\beta} = 3k, \]  

(8)

while the infinitely thick wall corresponds to

\[ x \rightarrow \frac{\pi}{2}; \quad d \rightarrow \infty, \quad V_0 \rightarrow 0, \quad \sqrt{V_0}\beta \rightarrow -x, \quad (\sqrt{V_0}\beta + x)^5 \left( \frac{k}{\sqrt{V_0}} \right)^3 = \frac{9}{4}. \]  

(9)

### III. Wave Function

The Schrödinger equation solution (2) has eigenvalues \( m^2 \geq 0 \), with \( m = 0 \) corresponding to the graviton bound state (see e.g., [10])

\[ \psi_0(z) = N_0 e^{-(3/4)A(z)}, \]  

(10)

where the normalization constant \( N_0 \) is fixed by the following relationship:

\[ \frac{k}{N_0^2} = \frac{2}{3} \sin(x) \cos(x)^{-5/3} (x + \sin(x) \cos(x)) + \cos(x)^{4/3}. \]  

(11)

In the thin wall limit \( (x \rightarrow 0) \) and thick wall limit \( (x \rightarrow \frac{\pi}{2}) \) the normalization coefficient \( N_0 \) takes the following respective forms:

\[ \frac{k}{N_0^2} \rightarrow 1, \quad \text{as} \quad x \rightarrow 0; \]
\[ \frac{k}{N_0^2} \rightarrow \frac{\pi}{3 \cos(x)^{5/3}}, \quad \text{as} \quad x \rightarrow \frac{\pi}{2}. \]  

(12)

The solution \( \psi_m(z) \) of (2) in the continuum \( m^2 > 0 \) has the following form:

\[ \psi_m(z) = A_m \cos(K|z|); \quad |z| \leq \frac{d}{2}, \]
\[ \psi_m(z) = N_m \sqrt{|z| + \beta [a_m Y_2(m|z| + \beta) \]
\[ + b_m J_2(m|z| + \beta)]}; \quad |z| \geq \frac{d}{2}. \]  

(13)

Here, \( K \equiv \sqrt{m^2 + V_0} \) and the coefficients \( a_m \) and \( b_m \) satisfy \( a_m^2 + b_m^2 = 1 \). The normalization constant \( N_m \) is determined by employing a regulator \( |z_c| \rightarrow \infty \) along with the asymptotic form of \( \psi_m(z) \) as \( |z| \rightarrow \infty \):

\[ \psi_m(z) = N_m \left[ a_m \sin \left( m(|z| + \beta) - \frac{5\pi}{4} \right) \right. \]
\[ \left. + b_m \cos \left( m(|z| + \beta) - \frac{5\pi}{4} \right) \right]. \]  

(14)

In the limit \( |z_c| \rightarrow \infty \), the asymptotic wave function (14) ensures dominant contribution to the wave function proba-
bility, and thus \( N_m = \sqrt{\frac{m}{2\zeta_c}} \), which in the continuum limit becomes \( N_m = \sqrt{\frac{m}{2}} \), i.e. \( \zeta_c \) is replaced by \( \pi \).

The matching of the wave function and its first derivative at \( |z| = \frac{\pi}{4} \) determines the coefficients \( a_m, b_m(a_m + b_m = 1) \) and \( A_m \). In particular, the key expression in determining the deviations of Newton’s law is the probability density of the wave function at the center of the wall

\[
|\psi_m(0)|^2 = A_m^2 = \frac{1}{\pi^2} \frac{1}{(J_2^2 - J_2^2)^2 + (Y_2^2 - Y_2^2)^2}.
\]

(15)

where

\[
\tilde{C} = \cos(\tilde{y}_0), \quad \tilde{S} = -K \sin(\tilde{y}_0),
\]

and

\[
\tilde{J}_2 = \sqrt{\frac{y_0}{2}} J_2(y_0), \quad \tilde{J}_2' = \frac{d\tilde{J}_2}{dy_0},
\]

\[
\tilde{Y}_2 = \sqrt{\frac{y_0}{2}} Y_2(y_0), \quad \tilde{Y}_2' = \frac{d\tilde{Y}_2}{dy_0},
\]

(16)

where \( J_2 \) and \( Y_2 \) are Bessel functions of order 2. The arguments \( y_0 \) and \( \tilde{y}_0 \) are defined as

\[
y_0 = \frac{M}{\cos(x)^{2/3}}, \quad \tilde{y}_0 = x\sqrt{1 + \frac{4M^2\sin(x)^2}{9\cos(x)^{10/3}}}
\]

(18)

and coefficients \( M \) and \( K \) are defined as

\[
M = m, \quad K = \sqrt{1 + \frac{9\cos(x)^{10/3}}{4M^2\sin(x)^2}}
\]

(19)

Note that the expression for \( A^2_m \) (15) is a function of \( x \) and \( M \), only. This allows us to fully explore the analytic behavior of the probability density \( A^2_m \), which we shall do in the following subsections. In particular, in the thin wall limit \( (x \to 0) A^2_m \) takes the form

\[
A_m^2 = \frac{2}{\pi^2 M[J_2^2(M) + Y_2^2(M)]}
\]

(20)

For the sake of completeness, we also present the expression for the ratio

\[
a_m = \frac{\tilde{J}_2 \tilde{S} - \tilde{J}_2^2 \tilde{C}}{\tilde{Y}_2 \tilde{S} - \tilde{Y}_2^2 \tilde{C}},
\]

(21)

and \( b_m = 1/\sqrt{1 + (a_m/b_m)^2} \).

Note that for the thick wall \( x > 1 \), the expression for the probability density of the wave function varies significantly over the range of the wall. In this case, the probability density, averaged over the domain of the wall thickness, may be more appropriate:

\[
|\psi_m|^2 = A^2_m F_m, \quad F_m = \frac{1}{2} \left( 1 + \frac{\sin(2\tilde{y}_0)}{2\tilde{y}_0} \right)
\]

(22)

Note that \( F_m \) is a mild function of \( x \), and while \( F_m \to 1 \) as \( x \to 0 \), for \( x \to \frac{\pi}{2} \), it approaches the asymptotic value \( \frac{1}{2} \).

### A. Small and large Kaluza-Klein energy expansion

The expression (15) can be readily expanded in different regimes of values of \( x \) and \( M \) parameters.

The expression (15) has a universal behavior

\[
A^2_m \to \frac{1}{\pi^2}, \quad \text{as } M \to \infty.
\]

(23)

On the other hand, one can also explicitly expand (15) in the limit of small \( M \). We have done so up to the fourth order in \( M \) expansion

\[
A^2_m = M\alpha_0(1 + M^2\alpha_1) + \mathcal{O}(M^5), \quad \text{for } M \ll 1,
\]

(24)

where coefficients \( \alpha_0 \) and \( \alpha_1 \) are the following explicit functions of \( x \):

\[
\alpha_0 = \frac{9\cos(x)^{10/3}}{2(\cos(x)^3 + 2x \sin(x) + 2\cos(x)^2)\sin(x)}, \quad \alpha_1 = \frac{p_1}{p_2},
\]

(25)

where

\[
p_1 = \cos(x) \left( 8 - 34 \cos(x)^2 \right)
\]

\[
- \cos(x)^4 \left( 1 + 54 \log \left( \frac{2\cos(x)^2/3}{\exp(\gamma)} \right) \right)
\]

\[
+ 2x \sin(x)(4 - 9 \cos(x)^2 - 4 \cos(x)^4),
\]

(26)

\[
p_2 = 18 \cos(x)^{10/3}(\cos(x)^3 + 2x \sin(x) + 2\cos(x)).
\]

(27)

The thin wall limit \( (x \to 0) \) produces the following values of \( \alpha_0 \) and \( \alpha_1 \):

\[
\alpha_0 = \frac{1}{2}, \quad \alpha_1 = -\frac{1}{2} + \log \left( \frac{M\exp(\gamma)}{2} \right), \quad \text{as } x \to 0.
\]

(28)

Note that (27) is a valid expansion for \( x < \frac{\pi}{2} \); however, in the thick wall limit \( (x \to \frac{\pi}{2}) \), \( A_m \) has further suppressions for small values of \( M \):

\[
\alpha_0 \to \frac{9}{2\pi} \left( \frac{\pi}{2} - x \right)^{10/3},
\]

\[
\alpha_1 \to \frac{2}{9} \left( \frac{\pi}{2} - x \right)^{-10/3}, \quad \text{as } x \to \frac{\pi}{2}.
\]

(29)

and thus \( A_m^2 \to \frac{1}{\pi^2} M^3 \).

In Figs. 1 and 2, \( A^2_m \) is plotted for a small and an intermediate value of \( x \), respectively.

For the sake of completeness, we also give the expansion

\[
\frac{a_m}{b_m} = M^2 \beta_0(1 + M^2 \beta_1) + \mathcal{O}(M^6), \quad \text{for } M \ll 1,
\]

(30)

where
\[ \beta_0 = -\frac{3}{4}, \quad \beta_1 = -\frac{3}{4} + \frac{1}{2} \log \left( \frac{M \exp(\gamma)}{2} \right), \quad \text{as } x \to 0; \]
\[ \beta_0 \to \frac{3}{4} \left( \frac{\pi}{2} - x \right)^{5/3}, \quad \beta_1 \to -\frac{5\pi}{72} \left( \frac{\pi}{2} - x \right)^{-13/3}, \quad \text{as } x \to \frac{\pi}{2}. \quad (34) \]

B. Resonances

Another interesting behavior of \( A_m \) takes place in a particular window for the combination where parameter \( x > 1 \) and for small values of \( M \):

\[ y_0 = \frac{M}{\cos(x)^{2/3}} \ll 1, \quad \tilde{y}_0 = x \sqrt{1 + \frac{4M^2 \sin(x)^2}{9 \cos(x)^{10/3}}} \ll 1, \quad (35) \]

or

\[ \frac{M}{\cos(x)^{2/3}} \ll 1, \quad \frac{M \tan(x)}{\cos(x)^{2/3}} \gg 1. \quad (36) \]

The probability density (15) exhibits a resonance behavior [17]. The resonance then appears when \( \tilde{Y}_2 \tilde{S} - \tilde{Y}_2 \tilde{C} \sim 0 \), which for the regime (36) takes the form

\[ \tilde{s} \tilde{C} \sim -\frac{3}{2\tilde{y}_0}. \quad (37) \]

For the above expression, we have employed the small argument \( y_0 \ll 1 \) expansion of \( \tilde{Y}_2 = -\frac{1}{\pi y_0} + O(y_0^{1/2}) \) and \( \tilde{Y}_2 = \frac{3\sqrt{2}}{\pi y_0} + O(y_0^{-1/2}) \). Equation (37) is satisfied approximately when \( \tilde{y}_0 \sim \frac{1}{2}(2n + 1) \) \((n = 1, 2, \cdots)\) or

\[ M_n \sim 3\cos(x)^{5/3}\sqrt{n(n + 1)}, \quad n = 1, 2, \cdots. \quad (38) \]

Note that the above relation for the position of resonances becomes less valid as \( n \) increases.

When condition (37) is satisfied, the probability density (15) is highly peaked at

\[ A_m^2 \sim \frac{18}{\pi^2 \tilde{y}_0^5} = \frac{18 \cos(x)^{10/3}}{\pi^2 M_n^2} \gg 1. \quad (39) \]

To obtain the above expression, we set in (15) \( \tilde{Y}_2 \tilde{S} - \tilde{Y}_2 \tilde{C} \sim 0 \) and employed the small \( y_0 \) argument expansion of \( \tilde{J}_2 = \frac{(\sqrt{2}\pi y_0)}{16} + O(y_0^{1/2}) \) and \( \tilde{J}_2 = \frac{5\sqrt{2}\pi y_0}{32} + O(y_0^{1/2}) \), which yields [along with (37)] \( \tilde{J}_2 \tilde{S} - \tilde{J}_2 \tilde{C} \sim -\frac{\gamma}{\sin(2x)} \). Note the sharp falloff of the density amplitude with the increase of the discretely valued \( M_n \) (38).

One can also estimate the approximate width \( \Delta M_n \) of the resonances by determining the change in parameter \( M \) (by \( \frac{1}{2} \Delta M_n \)) when \( \tilde{Y}_2 \tilde{S} - \tilde{Y}_2 \tilde{C} \sim \tilde{J}_2 \tilde{S} - \tilde{J}_2 \tilde{C} \sim -\frac{\gamma}{3\sin(2x)} \), using lo-
IV. MODIFICATION OF NEWTON’S LAW

The analytic form of the probability density in turn allows us to study explicitly the deviation of Newton’s law in the presence of continuum states. The (bound state) graviton wave function (10) determines the four-dimensional Newton’s law, while continuum Kaluza-Klein fluctuation modes (13) are responsible for corrections to Newton’s law. The four-dimensional gravitational potential between two pointlike particles with $M_1$ and $M_2$, a distance $r$ apart, can be written in the form (see, [18])

$$V = G_N \frac{M_1 M_2}{r(1 + \delta)}, \quad (42)$$

where $G_N = M_{\text{Planck}}^3 / N_0^2$ and $\delta$, the correction to Newton’s law is of the form

$$\delta = \frac{1}{N_0^2} \int_0^\infty e^{-mr} |\psi_m|^2 dm. \quad (43)$$

Note that since $N_0^2/k$ [Eq. (11)] is only a function of $x$, and $|\psi_m|^2$ [Eq. (22)] is only a function of $x$ and $M = m/k$, the correction to Newton’s law can be rewritten as an integral over $M$,

$$\delta = \frac{k}{N_0^2} \int_0^\infty e^{-(Mr)} |\psi_m|^2 dM, \quad (44)$$

and is thus only a function of $kr$ and $x$.

Note again that the averaged wave function density $|\psi_m|^2$ [Eq. (22)] is of form $A_m^2 F_m$, where $F_m$ is a mild function of $x$ and $M$, with $F_m \rightarrow 1$ and $F_m \rightarrow 1/2$ as $x \rightarrow 0$ and $x \rightarrow \frac{\pi}{2}$, respectively. In the following, we shall therefore not address detailed wave function effects away from the center of the wall, but shall focus on the modification of Newton’s law due to the wave function probability density near the center of the wall. Namely, our focus shall be on the $A_m^2$ factor and its effect on the structure of $\delta$. However, further detailed studies of the nontrivial wave function profile would be of interest.

The amplitude $A_m^2$ [Eq. (15)] has a small $M$ expansion of the form (24) and (27) and asymptotic to $\pi^{-1}$ as $M \rightarrow \infty$ [Eq. (23)]. For $x \ll 1$, it is a reasonable approximation to split the integral (44) into two intervals $M = [0, M_C]$ and $M = [M_C, \infty]$, where for the form of $A_m^2$, the small $M$ and large $M$ expansion is used, respectively. A reasonable choice $M_C \sim (\pi \alpha_0)^{-1}$ produces an approximate result, which is in good agreement with the numerical one

$$\delta = \frac{k}{N_0^2} \left\{ \frac{\alpha_0}{(kr)^2} \left[ 1 - e^{-(Mckr + 1)} \right] + \frac{\alpha_0 \alpha_1}{(kr)^3} \left[ 6 - e^{-(Mckr)} + 3(M_c kr)^2 + 6Mc kr + 6 \right] + \frac{1}{\pi kr} e^{-(Mc kr)} \right\}. \quad (45)$$

Again, note that for $r \gg (kM_c)^{-1}$, the leading correction is of the form
\[ \delta = \frac{k}{N_0^2} \left( \frac{1}{(kr)^2} + \frac{6\alpha_1}{(kr)^2} \right) \]  \tag{46}

The intriguing possibility takes place for \( x > 1 \) where the resonance behavior takes place. Note that in this case, the resonances are highly peaked, and the integral over \( M \) can be approximated by a sum of expressions \( kN_0^{-2} A_m^2 \Delta M_n x^{-M_n kr} \) over \( n \). Employing the analytic expressions from Sec. III B, we obtain the following correction to Newton’s law:

\[ \delta \sim \frac{2}{\pi} \sum_n \sqrt{1 + \frac{1}{4(n+1)}} e^{-M_n kr}, \]  \tag{47}

where \( M_n \) are locations of resonances (38), and the approximately constant prefactor is obtained by using the limit \( x \to \frac{\pi}{2} \) in (11): \( kN_0^{-2} \to \frac{\pi}{2} \cos(x)^{5/3} \) and (41):

\[ (A_m^2 \Delta M_n) \sim \frac{\alpha}{\pi} \cos(x)^{5/3} \sqrt{1 + \frac{1}{4(n+1)}}. \]  Note that the prefactor is approximately constant and of order 1.

\( M_n \)'s (38) are typically in the range of \( 10^{-2} \), and thus, for a choice of small \( k \) and range of distance \( r \sim (M_n k)^{-1} \), the leading resonances can potentially contribute a sizable effect. In particular, when choosing \( k \sim 10^6 \) GeV, at distances \( r \sim 10^{-4} \) GeV\(^{-1} \) the thick wall limit with resonances \( M_n \sim 10^{-2} \) would produce an order 1 modification of Newton’s law, while for the walls with \( x \approx 1 \), the corrections (45) and (46) would be negligible, i.e. \( O(10^{-4}) \).

V. CONCLUSIONS

We have presented the first explicit model of a finite thickness domain wall, interpolating between \( Z_2 \) symmetric five-dimensional anti–de Sitter vacua, where the graviton wave function fluctuations can be studied explicitly for any thickness of the wall, parameterized by \( x = \{0, \frac{\pi}{2}\} \). This allows us to explicitly determine the probability density of Kaluza-Klein fluctuations both for the small and large values of Kaluza-Klein energy. Notably, for \( x > 1 \) resonance behavior emerges, which is most pronounced in the range \( x = \{1.35, 1.55\} \) and can be analyzed explicitly. For a specific range of Kaluza-Klein momenta (and anti–de Sitter cosmological constant \( \Lambda \)) this resonance behavior can significantly modify four-dimensional Newton’s law by an effect of order \( 10^4 \) larger than that of thin walls.

While the concrete model is based on a specific, analytically solvable Schrödinger potential for the graviton fluctuation modes, we have not addressed the origin of the supergravity Lagrangian that would result in a BPS domain-wall solution whose metric leads to the proposed Schrödinger potential. Certainly, this is an important outstanding issue.

The analytic solvability of the model for graviton fluctuations lends itself to the further study of fermionic and spin-one fluctuation modes. In addition, while we chose to study fluctuation modes for codimension one (domain wall) configuration in five dimensions, extending the study to other dimensions is readily available and is relegated to future work.

ACKNOWLEDGMENTS

We thank Valery Romanovski for useful discussions. This research was supported by a Senior Scientist Research Grant (M.C.) of the Slovenian Research Agency (M.R.) and by the Ministry of Higher Education, Science and Technology of Slovenia (M.R.), and in part by Fay R. and Eugene L. Langberg endowed Chair (M.C.) and Department of Energy Grant No. EY-76-02-3071 (M.C.). M.C. would like to thank The Center for Applied Mathematics and Theoretical Physics for hospitality.

[3] For a detailed discussion of supergravity domain walls in four-dimensions, including the thin wall limit, see [4,5]. Related thin wall analysis in five- and D-dimensions was provided in dimensions [6–8], respectively.
[13] The study of possible string theory origin, within the gauged supergravity context, of such walls was initiated in [14,15].
[16] Studies of codimension one objects in other dimensions with an analogous structure of the potential would also be interesting, but are relegated to further studies.
[17] The appearance of resonances is analogous to that of a
simpler setup with a square-well potential, where however both the depth and the thickness are free parameters. In our case, it is remarkable that even though we have only one free parameter $x$ that specifies the potential, the resonances can nevertheless appear. [18] A. Brandhuber and K. Sfetsos, J. High Energy Phys. 10 (1999) 013.