Explaining a Restriction on the Scope of the Comparative Operator

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1 Introduction

Several recent discussions of comparatives (Hackl, 2000; Heim, 2001; Bhatt and Pancheva, 2004; Takahashi, 2006) note an unexplained restriction on the scope of the comparative operator which has been dubbed the “Heim-Kennedy constraint”:

(1) **Heim-Kennedy Constraint (HK):**
A quantificational DP may not intervene between a degree operator and its trace.

Suppose, following Hackl (2000) and Heim (2001), that gradable adjectives are functions from individuals to sets of degrees, and that the comparative morphemes *-er* and *less* are scope-taking elements which compare the maxima of two sets of degrees. These otherwise plausible assumptions create a puzzle: certain quantifiers do not interact with degree-denoting expressions in the expected way.

(2) Every girl is less tall than Jim is.

   a. Direct scope: every girl > less > d-tall
   \[ \forall x [\text{girl}(x) \rightarrow \max(\lambda d. \text{tall}(x)(d))] < \max(\lambda d. \text{tall}(Jim)(d)) \]
   “For every girl x, Jim’s max height is greater than x’s max height.”

   b. Scope-splitting: less > every girl > d-tall
   \[ \max(\lambda d. \forall x [\text{girl}(x) \rightarrow \text{tall}(x)(d)]) < \max(\lambda d. \text{tall}(Jim)(d)) \]
   “Jim’s max height is greater than the max degree to which every girl is tall (i.e., he is taller than the shortest girl).”

If (2) had the “scope-splitting” reading in (2b), it would be true (on this reading) if the shortest girl is less tall than Jim. However, (2) is clearly false if any girl is taller than Jim. The Heim-Kennedy constraint (1) attempts to account for this restriction (and similar facts with different quantifiers) by stipulating that the quantificational DP *every girl* may not intervene between the degree operator *less* and its trace *d-tall*. The puzzle is what syntactic or semantic principles explain the HK constraint given that structures such as (2b) are semantically unexceptionable according to standard assumptions.

My purpose here is to show that HK follows from the theory of weak islands proposed by Szabolcsi and Zwarts (1993), henceforth S&Z. “Scope-splitting” readings of comparatives with certain quantificational DPs are semantically deviant for the same reason that how many/much questions are weak island-sensitive. One of S&Z’s core claims is that amounts are weak island-sensitive because they have a more complex algebraic structure than normally assumed, and that the operations’ intersection and complement are not defined on this structure. As a result, the scope-splitting reading in (2b) is not available because computing it would require intersecting the heights of all the girls, and this operation is not available for purely semantic reasons.

2 Similarities between Weak Islands and Comparative Scope

As Hackl (2000) and Szabolcsi (2006) mention, there are considerable similarities between the limitations on degree operator scope summarized in HK and the core facts of weak islands discussed by Kroch (1989), Rizzi (1990), and Szabolcsi and Zwarts (1993). Rullmann (1995) notes the following patterns:

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*I wish to thank Chris Barker, Anna Szabolcsi, Arnim von Stechow, Emmanuel Chemla, Rick Nouwen, Roberto Zamparelli, Lucas Champollion, and audiences at the 2009 Penn Linguistics Colloquium and the ESSLLI 2009 Student Session for helpful discussion.*
(3)  
a. I wonder how tall Marcus is.  
b. I wonder how tall this player is.  
c. I wonder how tall every player is.  
d. I wonder how tall most players are.  
e. I wonder how tall many players are.  
(4)  
a. *I wonder how tall Marcus isn’t.  
b. *I wonder how tall no player is.  
c. *I wonder how tall few players are.  
d. *I wonder how tall fewer than ten players are.  
e. *I wonder how tall at most ten players are.  
(5)  
a. Marcus is taller than Lou is.  
b. Marcus is taller than this player is.  
c. Marcus is taller than every player is.  
d. Marcus is taller than most players are.  
e. Marcus is taller than many players are.  
(6)  
a. *Marcus is taller than Lou isn’t.  
b. *Marcus is taller than no player is.  
c. *Marcus is taller than few players are.  
d. *Marcus is taller than fewer than five players are.  
e. *Marcus is taller than at most five players are.  

These similarities are impressive enough to suggest that a theory of the weak island facts in (4) should also account for the limitations on comparatives in (6). Rullmann suggests, in the case of how tall and taller, that the unavailability of the examples in (4) and (6) is due to semantic, rather than syntactic, facts. Specifically, both wh-questions and comparatives make use of a maximality operation, roughly as in (7):

(7)  
a. I wonder how tall Marcus is.  
   I wonder: what is the degree \( d \) such that \( d = \text{max}(\lambda d. \text{Marcus is } d\text{-tall}) \)?  
b. Marcus is taller than Lou is.  
   \((\text{id}.d = \text{max}(\lambda d. \text{Marcus is } d\text{-tall})) > (\text{id}.d = \text{max}(\lambda d. \text{Lou is } d\text{-tall}))\)

With these interpretations of comparatives and questions, we predict that the sentences in (8) should be semantically ill-formed because each contains a definite description that is undefined:

(8)  
a. *I wonder how tall Marcus isn’t.  
   I wonder: what is the degree \( d \) such that \( d = \text{max}(\lambda d. \text{Marcus is } d\text{-tall}) \)?  
b. *Marcus is taller than Lou isn’t.  
   \((\text{id}.d = \text{max}(\lambda d. \text{Marcus is } d\text{-tall})) > (\text{id}.d = \text{max}(\lambda d. \text{Lou is } d\text{-tall}))\)

If degrees of height are arranged on a scale from zero to infinity, there can be no maximal degree \( d \) such that Marcus or Lou is not \( d\text{-tall} \), and so (8a) and (8b) are undefined.

Rullmann claims that similar reasoning will explain the unacceptability of the other downward entailing expressions in (4) and (6). However, the similarities between comparatives and weak island-sensitive expressions such as how tall go deeper than Rullmann’s discussion would indicate.

S&Z point out that several of the acceptable examples in (3) do not have all the readings predicted by the logically possible orderings of every player and how tall. As it turns out, the same scopal orders are also missing in the corresponding comparatives when we substitute -er for how tall. For example,

(9)  
a. every player > how tall > d-tall  
   “For every player \( x \), I wonder: what is the max degree \( d \) s.t. \( x \) is \( d\text{-tall} \)?”  
b. how tall > every player > d-tall  
   “I wonder: what is the degree \( d \) such that \( d = \text{Max}(\lambda d. \text{every player is } d\text{-tall}) \)?”
To satisfy the speaker’s curiosity under the first reading in (9), we would have listed all the players and their heights. In contrast, an appropriate response to the second reading (9b) would be to intersect the heights of all the players and give the maximum of this set, i.e., to give the height of the shortest player. This second reading is clearly not available. Similar facts hold for the corresponding comparative:

(10) Marcus is taller than every player is.
    a. every player > -er > d-tall

    “For every player, Marcus is taller than he is.”
    b. -er > every player > d-tall

    “Marcus’ max height is greater than the max height s.t. every player is that tall, i.e., he is taller than the shortest player.”

The amount question in (9) and the amount comparative expression in (10) allow similar scopal orderings. Furthermore, Rullmann’s explanation does not exclude the unacceptable readings. Unlike comparatives with an intervening negation, there is a maximal degree d s.t. every player is d-tall on Rullmann’s assumptions, namely the height of the shortest player.

Note in addition that (10) is identical in terms of scope possibilities to our original comparative scope-splitting example in (2), although its syntax is considerably different. Like (2), (10) falls under HK, which correctly predicts the unavailability of (10b). However, HK does not address negation or wh-questions, and so leaves unexplained the systematic correspondences between comparatives and weak island-sensitive expressions. Rullmann addresses these correspondences, but cannot explain the missing readings in (9) and (10).

I will argue that S&Z’s account of weak islands, which is designed to handle data such as those in (4), also explains the unavailability of the ‘shortest-player’ reading of the comparative in (10), as well as the corresponding gap in our original example (2). The essential insight is that the similarities between amount comparatives and amount wh-expressions are not due to monotonicity nor to restrictions on movement of the degree operator, but to the nature of amounts: specifically, their algebraic structure.

3 Comparative Scope and the Algebraic Structure of Amounts

In Section 2 we saw that amount comparatives and amount wh-questions seem to have the same scope-taking abilities, despite their quite different syntax and overt word order. I will argue that the semantic theory of weak islands in S&Z extends to comparatives in a straightforward way that predicts that HK should hold. This theory leads to a clear notion of how amount comparatives and amount questions are “the same” in the relevant respects.

Like Rullmann (1995), S&Z argue that no syntactic generalization can account for the full range of weak islands, and propose to account for them in semantic terms. They formulate their basic claim as follows:

(11) Weak island violations come about when an extracted phrase should take scope over some intervener but is unable to.

S&Z explicate this claim in algebraic terms, arguing that weak islands can be understood if we pay attention to the operations that particular quantificational elements are associated with. For instance,

(12) Universal quantification corresponds to taking intersections (technically, meets).
    Existential quantification corresponds to taking unions (technically, joins).
    Negation corresponds to taking complements.

(12) becomes important once we assign particular algebraic structures as denotations to types of objects, since these operations are not defined for all structures. The prediction is that a sentence will be semantically unacceptable, even if it can be derived syntactically, if computing or verifying it requires performing an operation on a structure for which this operation is not defined. S&Z illustrate this claim with the verb behave, which induces weak islands:
(13)  a. How did Mario behave?
    b. *How didn’t Mario behave?
    c. How did everyone behave?
       i. For each person, tell me: how did he behave?
       ii. *What was the behavior exhibited by everyone?

*Behave* requires a complement that denotes a manner. S&Z argue that manners denote in a free join semilattice, the same structure which Landman (1991) suggests for masses.

(14) Free join semilattice

A noteworthy property of (14) is that it is closed under union, but not under complement or intersection. For instance, the union (technically, join) of \([a] \oplus [b] \oplus [c]\) is \([a \oplus b \oplus c]\), but the intersection (meet) of \([a]\) with \([b \oplus c]\) is not defined. The linguistic relevance of this observation is that it corresponds to our intuitions of appropriate answers to questions about behavior. In S&Z’s example, suppose that three people displayed the following behaviors:

(15) John behaved *kindly and stupidly.*
    Mary behaved *rudely and stupidly.*
    Jim behaved *loudly and stupidly.*

If someone were to ask: “How did everyone behave?”, interpreted with *how* taking wide scope as in (13c-ii), it would not be sufficient to answer “stupidly.” The explanation for this, according to S&Z, is that computing the answer to this question on the relevant reading would require intersecting the manners in which John, Mary and Jim behaved, but intersection is not defined on (14). This, then, is a specific example of when “an extracted phrase should take scope over some intervener but is unable to.” Similarly, (13b) is unacceptable because complement is not defined on (14). Extending this account to amounts is slightly trickier, since amounts seem to come in two forms. In the first, which S&Z label ‘counting-conscious,’ wh-expressions are able to take scope over universal quantifiers. S&Z imagine a situation in which a swimming team is allowed to take a break when everyone has swum 50 laps. In this situation it would be possible to ask:

(16) [At least] How many laps has every swimmer covered by now?

In this case it seems (on the how many-wide interpretation) that the correct answer is the number of laps covered by the slowest swimmer. Counting-conscious amount expressions, then, had better denote in a structure in which intersection is defined. The number line in (17) seems to be an appropriate choice.

(17)
Intersection and union are defined in this structure, though complement is not. This fact predicts that *how many/much* should be able to take scope over existential quantification but not negation:

(18) a. How many laps has at least one swimmer covered by now?
    [Answer: the number of laps covered by the fastest swimmer.]
    b. *How many laps hasn’t John covered by now?*

So far, then, (17) seems to be appropriate for amounts.

Many authors have assumed that the amounts that are compared in comparative constructions always denote in (17). Indeed, the problem we began this essay with—why can’t *Every girl is less tall than John* mean “The shortest girl is shorter than John”?—was motivated by the assumption that it should be possible to intersect sets of degrees. The fact that intersection is defined on (17), and yet universal intervention is not available in (2), has led authors to various levels of stipulation (HK, in Heim, 2001) or abandoning degrees in the analysis of comparatives (Schwarzchild and Wilkinson, 2002).

I would like to suggest an alternative: heights and similar amounts do not denote in (17), but in a poorer structure for which intersection is not defined, as S&Z claim for island-sensitive amount *wh* -expressions. As S&Z note, such a structure is motivated already by the existence of non-counting-conscious amount *wh*-expressions which are sensitive to a wider variety of interveners than *how many* was in the examples in (16) and (18). This is clear for heights, for example:

(19) How tall is every student in your class?
    a. For every student in your class, how tall is he/she?
    b. *What is the maximum height shared by all of your students, i.e., how tall is the shortest student?*

The unacceptability of (19b) is surprising given that the degree expression was able to take wide scope in the overtly similar (16). S&Z account for this difference by arguing that, unless counting is involved, amount expressions denote in a join semilattice:

(20) Join semilattice

\[
\begin{array}{c}
[a + b + c + d] (= 4) \\
[a + b + c] (= 3) [d] \\
[a + b] (= 2) [c] \\
[a] (= 1) [b]
\end{array}
\]

(20) should be seen as a structure collecting arbitrary unit-sized bits of stuff, and abstracting away from their real-world identity, like adding cups of milk to a recipe (S&Z:247–8). An important formal property of (20) is that “if p is a proper part of q, there is some part of q (the witness) that does not overlap with p” (S&Z:247). As a result, intersection is not defined unless the objects intersected are identical. S&Z claim that this fact is sufficient to explain the unavailability of (19b), since the heights of the various students, being elements of (20), cannot be intersected.

This explanation for (19b) relies on a quite general proposal about the structure of amounts. As a result, it predicts that amount-denoting expressions should show similar behavior wherever they appear in natural language, and not only in *wh*-expressions. The similarities between amount-denoting *wh*-expressions and comparatives, then, are explained in a most straightforward way: certain operations are not defined on amount-denoting expressions because of the algebraic structure of their denotations, regardless of the other details of the expressions they are embedded in. So, returning to (2b) (repeated in (21)),

\[\text{Note that this example from S&Z constitutes a counter-example to HK stated as an LF-constraint as in (1): the degree operator *how many* intervenes between the quantificational DP *every swimmer* and its trace. This constitutes further evidence for the algebraic approach advocated here, since the details of the expressions' denotations are relevant to the acceptable scopal relations, and not merely their structural configuration. It is unclear why this example is not as readily available with comparatives, however.}\]
(21) Every girl is less tall than Jim is.
Scope-splitting: less > every girl > d-tall
\[ \text{max}(\lambda d. \text{tall}(\text{Jim})(d)) > \text{max}(\lambda d. \forall x [\text{girl}(x) \rightarrow \text{tall}(x)(d)]) \]
“The max degree to which Jim is tall is greater than the max degree to which every girl is tall.”

This interpretation is not available because the term \( \text{max}(\lambda d. \forall x [\text{girl}(x) \rightarrow \text{tall}(x)(d)]) \) is undefined: on S&Z’s theory, there can be no such degree. I conclude that the puzzle described by the Heim-Kennedy constraint was not a problem about the scope of a particular type of operator, but was generated by incorrect assumptions about the nature of amounts. Amounts are not simply points on a scale, but rather elements of (20). This proposal is independently motivated in S&Z, and it explains the restrictions captured in HK as well as other similarities between comparatives and weak islands.

At this point there are several important gaps in the account. The first is that S&Z do not work out their account compositionally, and this needs to be done. The second problem is that it remains unexplained why certain modals and intensional verbs are able to intervene between a degree operator and its trace both in amount comparatives and amount questions, as discussed at length in Heim (2001). (22) and (23) illustrate:

(22) (This draft is 10 pages.) The paper is required to be exactly 5 pages longer than that. (Heim, 2001:224)
   a. required > exactly 5 pages -er > that-long
      \[ \forall w \in \text{Acc} : \text{max} (\lambda d : \text{long}_w(p,d)) = 15 \text{pp} \]
      “In every world, the paper is exactly 15 pages long.”
   b. exactly 5 pages -er > required > that-long
      \[ \text{max} (\lambda d [\forall w \in \text{Acc} : \text{long}_w(p,d)]) = 15 \text{pp} \]
      “The max common length of the paper in all accessible worlds, i.e. its length in the world in which it is shortest, is 15 pages”

(23) How long is the paper required to be?
   a. required > how long > that-long
      “What is the length s.t. in every world, the paper is exactly that long?”
   b. how long > required > that-long
      “What is the max common length of the paper in all accessible worlds, i.e. its length in the world in which it is shortest?”

These data support the present theory in that comparatives and weak island-sensitive \textit{wh}-expressions pattern similarly in yet another way. However, on the assumption that \textit{require} involves universal quantification over accessible worlds, the (b) readings of these examples are problematic. S&Z suggest that these operators are acceptable interveners because they do not involve algebraic operations. This is perhaps too drastic a step, but detailed investigation is needed to account for the complicated and subtle data involved, including the fact that some modals and intensional verbs involving universal quantification (must, require) can intervene while others (should, be supposed to) cannot (cf. Heim, 2001).

4 Notes on Density and Informativity

In this section I discuss very briefly the relationship between the present analysis and an influential proposal by Fox and Hackl (2006). I show that the algebraic account is not in direct competition with Fox and Hackl’s theory, but that there are some complications in integrating the two approaches.

Fox and Hackl argue that amount-denoting expressions always denote on a dense scale, effectively (17) with the added stipulation that, for any two degrees, there is always a degree that falls between them. The most interesting data from the current perspective are in (24)–(26):

(24) a. How fast did you drive?
b. *How fast did you not drive?
(25)  a. How fast are we not allowed to drive?
    b. *How fast are we allowed not to drive?

(26)  a. How fast are we required not to drive?
    b. *How fast are we not required to drive?

The contrast in (24) follows straightforwardly from S&Z’s theory, but (25) and (26) do not: on S&Z’s assumptions, there is no maximal degree \( d \) such that you are not allowed to drive \( d \)-fast, and yet (25a) is fully acceptable. In addition, (25a) and (26a) do not ask for maxima but for minima (the least degree which is unacceptably fast, i.e., the speed limit). Fox and Hackl show that the minimality readings of (25a) and (26a), and the ungrammaticality of (25b) and (26b), follow if we assume (following Dayal, 1996 and Beck and Rullmann, 1999) that \( \text{wh} \)-questions do not ask for a maximal answer but for a maximally informative answer, defined as follows:

(27) The maximally informative answer to a question is the true answer which entails all other true answers to the question.

Fox and Hackl show that, on this definition, upward monotonic degree questions ask for a maximum, since if John’s maximum height is 6 feet, this entails that he is 5 feet tall, and so on for all other true answers. However, downward entailing degree questions ask for a minimum, since if we are not allowed to drive 70 mph, we are not allowed to drive 71 mph, etc. \(^2\)

This is not as deep a problem for the present theory as it may appear. S&Z assume that \( \text{wh} \)-questions look for a maximal answer, but it is unproblematic simply to modify their theory so that \( \text{wh} \)-questions look for a maximally informative answer. Likewise, we can just as easily stipulate that a join semilattice (20) is dense as we can stipulate that a number line (17) is dense. This maneuver would replicate Fox and Hackl’s result about minima in downward entailing contexts. It is possible simply to combine S&Z’s theory with Fox and Hackl’s. In fact, this is probably independently necessary for Fox and Hackl, since their assumption that amounts always denote in (17) fails to predict the core data of the present paper: the fact that How tall is every girl? and Every girl is less tall than John lack a ‘shortest-girl’ reading.

The only real barrier to a simple marriage of these theories is the fact, already noted in the previous section, that S&Z do not have an explanation for the occasional acceptability of universal modal interveners with non-counting amount questions. This is actually not a problem with respect to (25)–(26): the questions ask for numbers of miles per hour, suggesting that (17) is appropriate, and the maximally informative answer is the same in all worlds. However, corresponding questions with non-counting amount questions are a mixed bag: S&Z can account for (28), but fails to predict (29) for the same reasons that it failed with (26)–(26).

\(^2\) A problem which Fox and Hackl do not note is that (i) below should be unacceptable, since their account of modal intervention assumes that a 70 mph speed limit denotes a closed interval from 70 to infinity.

(i) How fast are we allowed to drive? 70 mph.

(ii) How fast are we not allowed to drive? 70 mph.

But (i) is at least as good as (ii), and probably even more natural. This poses a problem for Fox and Hackl’s explanation of modal intervention: on their account, if (ii) is acceptable then (i) should be ruled out, and vice versa.

\(^3\) A further problem is that the crucial data in (23)–(24) can be replicated in the domain of manners, where an account in terms of density would be implausible.

(i) How are we required not to behave?

(ii) *How are we not required to behave?

(iii) How are we not allowed to behave?

(iv) *How are we allowed not to behave?

Fox and Hackl’s account of (25)–(26) cannot be extended to these data. Abrusán (2007) also notes these data, though not the connection with Fox and Hackl’s key argument; she gives a competing account which generates these data, but does not extend straightforwardly to the Heim-Kennedy phenomena.
A RESTRICTION ON THE SCOPE OF THE COMPARATIVE OPERATOR

(28)  a. How much pain are you not able to endure?
     b. *How much pain are you able not to endure?
(29)  a. How much pain are you required not to inflict?
     b. *How much pain are you not required to inflict?

I conclude that neither theory is complete: Fox and Hackl’s theory needs to be supplemented by an account of the difference between counting and non-counting amount questions along the lines of S&Z; but S&Z lack an account of universal modal intervention. Nevertheless, the two theories are broadly compatible. Note that this is not an endorsement of Fox and Hackl’s central thesis; it merely shows that if their theory is correct, this fact does not constitute a reason to abandon the current approach to the HK phenomena.

Finally, note that the maximal informativity hypothesis in (27), whatever its merit in \(wh\)-questions and other environments discussed by Fox and Hackl, is not appropriate for comparatives: here it appears that we need simple maximality.

(30)  a. How fast are you not allowed to drive?
     b. *You’re driving faster than you’re not allowed to.

A simple extension of the maximal informativity hypothesis to comparatives would predict that (30b) should mean “You are exceeding the speed limit.” In contrast, the maximality-based account predicts that (30b) is unacceptable, since there is no maximal speed which is not allowed. This appears to be the correct prediction.

5 Conclusion

To sum up, the traditional approach on which amounts are arranged on a scale of degrees fails to explain why the constraint HK in (1) should hold. However, S&Z’s semantic account of weak islands predicts the existence of this constraint and the numerous similarities between amount comparatives and amount-denoting \(wh\)-expressions. To be sure, important puzzles remain. Nevertheless, the algebraic approach to comparative scope offers a promising explanation for a range of phenomena that have not been previously treated in a unified fashion.

Furthermore, if S&Z’s theory turns out to be wrong, all is not lost. The most important lesson of the present paper, I believe, is not that S&Z’s specific theory of weak islands is correct—as we have seen, there are certainly empirical and technical challenges—but rather that weak island phenomena are not specific to \(wh\)-questions. In fact, we should probably think of the phenomena summarized by the Heim-Kennedy constraint as comparative weak islands. However the theory of weak islands progresses, evidence from comparatives will need to play a crucial role in its development.

References


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4 Thanks to an ESSLLI '09 reviewer for bringing the contrast in (30) to my attention.

5 An additional important question which I have not discussed for space reasons is the relationship between the algebraic theory and interval-based theories such as Abrusán (2007) and Schwarzchild and Wilkinson (2002).


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