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Nonperturbative Yukawa Couplings from String Instantons

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Nonperturbative D-brane instantons can generate perturbatively absent though phenomenologically relevant couplings for type II orientifold compactifications with D-branes. We discuss the generation of the perturbatively vanishing $SU(5)$ GUT Yukawa coupling of type $\langle \mathbf{10}\ \mathbf{10}\ \mathbf{5}_H \rangle$. Moreover, for a simple globally consistent intersecting $D6$-brane model, we discuss the generation of mass terms for matter fields. This can serve as a mechanism for decoupling exotic matter.

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Introduction.—Nonperturbative mass generation by gauge instantons is essential to explain the pattern of meson masses in QCD. Specifically, QCD instantons generate a nonperturbative, effective fermion interaction [1] that involves N_f flavors of quark fields and breaks the perturbative $U(1)_A$ axial symmetry to a discrete Z_{2N_F} subgroup. In this way the mass of the η' meson gets generated [2,3]. In addition, gauge instantons of the weak $SU(2)$ gauge group are responsible for baryon number violating processes in the standard model of particle physics.

Following these observations, it would be very interesting if also some of the quark and lepton masses are of nonperturbative origin in theories beyond the standard model. For instance, Weinberg [4] suggested that quark masses could be due to gauge instantons of some hypercolor interactions in subquark models.

Also in string theory compactifications, quark and lepton masses, or rather the respective Yukawa couplings, can be generated by nonperturbative effects, more precisely by string world-sheet [5] or by spacetime instantons. Most importantly, string theory opens up some new perspectives in that nonperturbative effects do not only include gauge instantons of the effective gauge theory, but also entirely stringy instantons not related to effective gauge interactions. In fact, during the last year there has been some progress towards a better understanding of nonperturbative effects in $\mathcal{N} = 1$ supersymmetric four-dimensional string compactifications on Calabi-Yau orientifolds ([6–9] and further developments in [10–19]; for closely related earlier work see [20,21]). Type IIA orientifolds with intersecting $D6$-branes (see, e.g., the recent reviews [22,23] for references) receive nonperturbative corrections from Euclidean $D2$-brane instantons, short $E2$-instantons, wrapping special Lagrangian three-cycles of the internal Calabi-Yau space.

Once a conformal field theory (CFT) description of the background is available, the induced nonperturbative couplings can be studied using methods from open string theory. An analysis of the zero mode structure of such instantons shows that so-called O(1)-instantons [14–17] can generate terms in the effective four-dimensional superpotential.

Of particular interest are those induced interactions which are forbidden perturbatively due to global $U(1)$ selection rules. Under suitable circumstances, $E2$-instantons can break these global $U(1)$ symmetries to certain discrete subgroups and generate $U(1)$ violating interactions [6,8]. Important examples of these new couplings are nonperturbative Majorana mass terms for right-handed neutrinos or μ-terms in the Higgs sector for the minimal supersymmetric standard model [6,8,13,16,19].

The existence of such perturbatively forbidden type IIA couplings resolves one of the puzzles about the proposed large coupling dual description in terms of M-theory compactifications on singular G_2 manifolds [24–26]. In this picture, non-Abelian gauge symmetries are localized at an ADE-singularity over a supersymmetric three-cycle on the G_2 manifold. Clearly, the perturbative $U(1)$ gauge symmetries on the type IIA $D6$-branes become massive due to the Green-Schwarz mechanism and therefore decouple completely in the strong coupling M theory dual description, but how then should the resulting global $U(1)$ selection rules appear? The resolution to this puzzle is given by the appearance of the described $U(1)$ breaking nonperturbative terms in the type IIA picture. Therefore, each coupling present in M-theory vacua on G_2 manifolds should be realized either perturbatively or nonperturbatively in the type IIA orientifold.

In this Letter we explore two new types of phenomenologically important instanton generated couplings for type II orientifolds. In particular, we investigate the generation of the crucial $SU(5)$-like grand unified theory (GUT) model Yukawa couplings of type $\mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}_H$, which are known to be absent perturbatively [27]. In the second part of the letter we provide a globally consistent intersecting $D6$-brane model and show that an $E2$-instanton generates a mass term for certain matter fields, thus providing a new mechanism for decoupling exotic matter.

$SU(5)$ Yukawa couplings.—Grand unified $SU(5)$-like models based on intersecting $D6$-branes generically suf-

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fer from the absence of the important Yukawa coupling $\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{5}_H$ and are therefore so far not considered realistic. Such models were first generally proposed in [28] and explicitly constructed for intersecting $D6$-branes in [27,29–33].

In the sequel we abstract from concrete models and analyze the general setup of minimal intersecting $D6$-brane models realizing $SU(5)$ GUT as shown in Table I. Such a setup involves only two stacks a and b of branes giving rise to a $U(5)_a \times U(1)_b$ gauge symmetry. The $U(5)_a$ splits into $SU(5)_a \times U(1)_a$, so that there are two Abelian gauge groups $U(1)_a \times U(1)_b$. One linear combination of these is anomalous and becomes massive via the generalized Green-Schwarz mechanism. However, it survives as a global symmetry in the effective action. Matter fields transforming as $\mathbf{10}$ under $SU(5)_a$ arise at the intersections of stack a with its image a', while the matter fields transforming as $\bar{\mathbf{5}}$ as well as Higgs fields $\mathbf{5_H}$ and $\bar{\mathbf{5}}_\mathbf{H}$ are located at intersections of stack a with b and b'. For a globally consistent realization of this quiver the concrete wrapping numbers decide if the anomaly free combination $U(1)_X$ of the Abelian groups really remains massless. If not, the model is of the usual Georgi-Glashow type, while in the presence of a massless $U(1)_X$ it represents a flipped $SU(5)$ model.

From the $U(1)_{a,b}$ charges it is clear that perturbatively the two Yukawa couplings

$$\langle \mathbf{10}_{(2,0)}\bar{\mathbf{5}}_{(-1,1)}\bar{\mathbf{5}}^H_{(-1,-1)}\rangle, \qquad \langle \bar{\mathbf{5}}_{(-1,1)}\mathbf{1}_{(0,-2)}\mathbf{5}^H_{(1,1)}\rangle \tag{1}$$

are present. Focusing for concreteness on flipped $SU(5)$, these give masses to the heavy (u, c, t) quarks and the leptons. However, the Yukawa couplings for the light (d, s, b) quarks

$$\langle \mathbf{10}_{(2,0)}\mathbf{10}_{(2,0)}\mathbf{5}^H_{(1,1)}\rangle \tag{2}$$

are not invariant under the two $U(1)$s. Note that this interaction is also of key importance for the solution of the doublet-triplet splitting problem for flipped $SU(5)$. For a nonzero vacuum expectation value of the standard model singlet component in $\mathbf{10}+\overline{\mathbf{10}}$ there is no partner for the weak Higgs doublet to pair up with. For Georgi-Glashow $SU(5)$ models, the role of (u, c, t) and (d, s, b)-quarks has to be interchanged and the GUT Higgs field is usually in the adjoint representation of $SU(5)$.

TABLE I. GUT $SU(5)$ intersecting $D6$-brane model, $U(1)_X = \frac{1}{4}U(1)_a - \frac{5}{4}U(1)_b$. The multiplet $\mathbf{10}_{(2,0)}$ also contains the GUT Higgs field which should appear as a vectorlike pair.

Sector	Number	$U(5)_a \times U(1)_b$ reps.	$U(1)_X$
(a', a)	$3+(1,1)$	$\mathbf{10}_{(2,0)}$	$\frac{1}{2}$
(a, b)	3	$\bar{\mathbf{5}}_{(-1,1)}$	$-\frac{3}{2}$
(b', b)	3	$\mathbf{1}_{(0,-2)}$	$\frac{5}{2}$
$(a', b$	1	$\mathbf{5}^H_{(1,1)}+\bar{\mathbf{5}}^H_{(-1,1)}$	$(-1)+(1)$

Our main result is that the coupling (2) can be generated by an $E2$-instanton of suitable zero mode structure. Concretely, the instanton has to wrap a rigid three-cycle Ξ invariant under the orientifold projection $\Omega\bar{\sigma}$ and carrying gauge group $O(1)$ [14–17]. This guarantees that the uncharged part of the instanton measure only contains the four bosonic and two fermionic modes x^μ, θ^α required for superpotential contributions. Now, from the arguments in [6,8,9] the coupling (2) requires in addition charged fermionic zero modes at intersections between Ξ and the $D6$-branes. These are responsible for an effective $U(1)$ charge of the instanton which can compensate for the excess of $U(1)$ charge of the operator (2). For intersection numbers

$$[\Xi\cap\Pi_a]^+ = [\Xi\cap\Pi_b]^+ = 0,$$
$$[\Xi\cap\Pi_a]^- = [\Xi\cap\Pi_b]^- = 1$$

we get five zero modes $\bar{\lambda}^i_{[5]}$ from the intersection of the instanton with $D6_a$ and one zero mode $\bar{\nu}_{[-1]}$ from the intersection with $D6_b$. The computation of the resulting couplings can be performed following the prescription proposed in [6] and exemplified for a concrete local model in [13]. Since the instanton lies in an $\Omega\bar{\sigma}$ invariant position, one can absorb these six matter zero modes with the three disc diagrams depicted in Fig. 1.

All charge selection rules are satisfied. Each tree-level coupling is by itself a sum over world-sheet instantons connecting the three intersection points in the disc diagrams like $10^\alpha_{[ij]}\bar{\lambda}^i\bar{\lambda}^j$, where $\alpha = 1, 2, 3$ denotes the generation index. These discs induce open string dependent terms in the instanton moduli action of type

$$\exp(-S_{\rm mod}) = \exp(C^{10}_\alpha 10^\alpha_{[ij]}\bar{\lambda}^i\bar{\lambda}^j + C^5 5_m\bar{\lambda}^m\bar{\nu}), \tag{3}$$

which are integrated over the charged fermionic measure $\int d^5\bar{\lambda}d\bar{\nu}$. Because of its Grassmannian nature, the index structure of the Yukawa coupling is

$$W_Y = Y^{\alpha\beta}_{\langle \mathbf{10}\ \mathbf{10}\ \mathbf{5}_H\rangle}\epsilon_{ijklm}\mathbf{10}^\alpha_{ij}\mathbf{10}^\beta_{kl}\mathbf{5}^H_m e^{-S_{E2}}e^{Z'}, \tag{4}$$

where the instanton action can be written as $S_{E2} = \frac{2\pi}{\alpha_{\rm GUT}}\times\frac{{\rm Vol}_{E2}}{{\rm Vol}_{D6}}$. Here we have used that the volume of the $D6_a$-brane determines the gauge coupling at the GUT scale. Note that the ratio ${\rm Vol}_{E2}/{\rm Vol}_{D6}$ depends only on the complex struc-

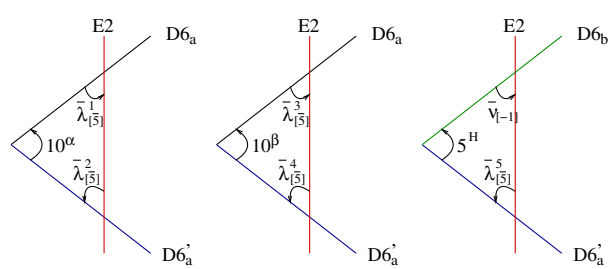

FIG. 1 (color online). Absorption of zero modes.

ture moduli, which are known to be constrained by the D-term supersymmetry conditions for the $D6$-branes. The superpotential coupling W_Y also depends on the holomorphic part of the one-loop determinant $e^{Z'}$ arising from the annulus and Möbius diagrams ending on the instanton and the $D6$-branes or O plane, respectively [6]. As shown in [10,11,18], these are related to one-loop gauge threshold corrections [34,35].

One observes that the replication of the zero modes $\bar{\lambda}_i$ is entirely due to Chan-Paton indices so that each of the discs in Fig. 1 depends only on the family index and not on the pair of zero modes to which the open string operator couples. Therefore the final instanton generated Yukawa coupling factorizes into

$$Y^{\alpha\beta}_{\langle \mathbf{10}\,\mathbf{10}\,\mathbf{5}_H\rangle} = Y^\alpha Y^\beta \tag{5}$$

and the induced mass matrix for the quarks is always of unit rank. In order to exhibit nonperturbative masses for all three generations the model therefore has to possess three independent $E2$-instanton sectors.

Concerning the suppression scale of the instanton generated Yukawa coupling, for $\alpha_{\rm GUT} = 1/24$ and $\mathrm{Vol}_{E2}/\mathrm{Vol}_{D6} = (R_{E2}/R_{D6})^3$ with the moderate suppression $R_{D6} = \frac{7}{2}R_{E2}$, the main instanton suppression factor is $\exp(-S_{E2}) \simeq 3 \times 10^{-2}$. Since the $E2$-instanton lies in a $\bar{\sigma}$ invariant position it seems natural that the length scale of the internal volume is smaller than that of the $U(5)$ stack of $D6$-branes.

To summarize, we find that D-brane instantons can generate the $\langle \mathbf{10}\,\mathbf{10}\,\mathbf{5}_H\rangle$ Yukawa coupling. The described mechanism works both for Georgi-Glashow as well as flipped $SU(5)$ models. It is particularly attractive for the case of flipped $SU(5)$: Here the $E2$-instanton not only generates the desired couplings, but the complex structure dependent exponential suppression $\exp(-S_{E2})$ can explain, as a bonus, the hierarchy between the (u, c, t) quarks and the (d, s, b) quarks.

Instanton generated mass terms.—Most semirealistic string models constructed so far come with exotic vector-like states. For the phenomenological features of such models it is important to know whether these states can become massive. To date mostly perturbative mechanism have been discussed in the literature for generating such mass terms. In this section we demonstrate for a concrete globally consistent model that $E2$-instantons can also generate such mass terms. We are working with a type IIA orientifold background which serves as a simple model based on $U(4)$ gauge symmetry with a certain number of matter fields in the antisymmetric representation of $U(4)$. For a similar global model in type I theory see [12,15].

Concretely, we consider the orientifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ with Hodge numbers $(h_{11}, h_{12}) = (3, 51)$. We employ the notation of [36], to which we refer for details of the geometry and the construction of rigid cycles (see also [37]). The orbifold group is generated by θ and θ' acting as reflection in the first and last two tori, respectively.

Table II displays the wrapping numbers of the simplest globally consistent, supersymmetric model for the choice that the $O6$-plane lying parallel to the instanton is an $O6^+$-plane with the other three being $O6^-$-planes.

It involves only one stack of four bulk $D6$-branes (and its orientifold image) carrying $U(4)$ gauge group with three superfields in the adjoint representation. One can easily check that all consistency conditions are indeed satisfied and supersymmetry fixes the complex structure moduli to the sublocus $U^1 - U^2 - U^3 = U^1U^2U^3$. The model has also 32 chiral superfields in the conjugate antisymmetric representation $\bar{\mathbf{6}}$ of $U(4)$. This number can be reduced by considering tilted tori, but for simplicity we consider the untilted case in the sequel. Note that the $\mathbf{6}$ of $SU(4)$ is a real representation so that these states are chiral only with respect to the diagonal $U(1) \subset U(4)$. Since $U(1)$'s can be broken by instantons, there is a chance that mass terms are nonperturbatively generated.

Following [13], the background of this model exhibits one class of 8×8 rigid $O(1)$ instantons, whose bulk part is also shown in Table II. The multiplicites are due to the two possible positions of the instanton in each torus and the different twisted charges, respectively. The intersection number of these $E2$-instantons with the bulk $D6$ branes is exactly one. Taking into account the Chan-Paton label of the gauge group $U(4)$, there are four fermionic zero modes localized at the intersection of $E2$ and the matter branes. As shown in Fig. 2, these four fermionic zero modes λ can be saturated via two disc diagrams thereby generating mass terms for the matter fields in the $\bar{\mathbf{6}}$ representation.

We denote the 32 matter superfields as $\Phi_I^A = \phi_I^A + \theta\psi_I^A$, where the lower index $I = 1, \ldots, 8$ refers to the various intersections on T^6 and $A = 1, \ldots, 4$ counts the different orbifold images.

As in the previous section, we can compute the disc diagrams in Fig. 2. Taking also the Grassmannian nature of the fermionic zero modes into account, the overall structure of the generated mass terms is

$$\mathcal{L}_{\rm mass} = C' M_s e^{-S_{E2}} \epsilon_{ijkl} M^{I,J}_{A,B} (\psi_I^A)_{ij} (\psi_J^B)_{kl} \tag{6}$$

with the instanton action $S_{E2} = \frac{2\pi}{\alpha_{SU(4)}} \frac{V_{E2}}{V_{D6}}$. Moreover, C' includes all angle dependent constants due to the CFT computation as well as due to integration over all bosonic and fermionic zero modes [13,18]. Since the four instanton zero modes arise from the Chan-Paton factors of $U(4)$, the mass matrix due to each individual instanton factorizes

TABLE II. Wrapping numbers of $U(4)$ global model.

Stack	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$	I_{E2x}
$U(4)$	8	$(1, -1) \times (1, 1) \times (1, 1)$	1
E_2	1	$(1, 0) \times (0, 1) \times (0, -1)$	

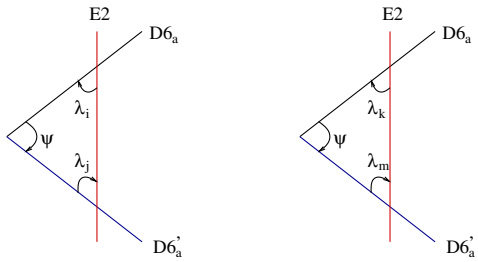

FIG. 2 (color online). Absorption of the zero modes.

into

$$M_{A,B}^{I,J} = h_A^I h_B^I, \tag{7}$$

where these factors are essentially the disc amplitudes in Fig. 2 containing a sum over world-sheet instantons. Each of the 8 choices for the position of the instanton results in a different matrix of this type, which in turn is unaffected by the twisted charges. The final mass matrix upon summing up all contributions is then in general of rank 8. This exemplifies that string instantons can also generate mass terms for exotic matter fields.

We have shown that the phenomenology of intersecting D-brane models in type II orientifolds requires the inclusion of string instanton effects. These do generate important couplings that are often absent perturbatively due to the strong $U(1)$ selection rules present for D-brane models. This becomes even more striking when the instanton generated couplings are known to have certain hierarchies with respect to perturbative couplings. Of particular interest is the appearance of a (flipped) $SU(5)$ Yukawa coupling. It would be very important to find globally consistent semirealistic string vacua exhibiting this effect. A straightforward implementation into the models of [27,29–33] is hampered by the necessity of absorbing dangerous instanton zero modes due to nonrigidity of the cycles in the used backgrounds. We hope to return to this question in the future.

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