APEX: Autonomous Vehicle Plan Verification and Execution

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Abstract
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Keywords
autonomous vehicles, formal verification, reachability

Disciplines
Computer Engineering | Electrical and Computer Engineering | Systems and Communications | Theory and Algorithms

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Introduction

Advanced Driver Assistance Systems (ADAS) and fully autonomous vehicles (AVs) feature a complex integration between discrete and continuous controllers. The discrete controllers make decisions regarding the vehicle’s next goal (“move to left lane”), while the continuous controllers determine throttle and steering inputs which allow the vehicle to execute the decisions of the discrete controller.

Approximately 1.25 million traffic accidents occur yearly around the world; almost 90% of such accidents are attributable to driver error [1]. ADAS and AV technology has the potential to all but eliminate this burden from society; however, experts are still unsure when such technology will be ready for the consumer market. Thus, as such vehicles come to market there is a pressing need to bound and minimize the risks they might pose to other vehicles, pedestrians and infrastructure.

Legal liability is a main consideration in the design of the next generation of vehicles, and could determine the future of ADAS and AVs as a mainstream technology. The question is: who’s liable when the AV (or an ADAS feature) causes an accident or unsafe situation? Currently there is no one answer to the question: on the ADAS side, Tesla Motors has included a lane changing feature; however, it must be initiated by the human driver as a way to make the latter ultimately responsible for the outcomes.

Volvo on the other hand has recently announced it will assume full liability for its autonomous cars’ actions, and warns that “the US risks losing its leading position due to the lack of Federal guidelines for the testing and certification of autonomous vehicles” [2]. The possibility that the manufacturer might ultimately be deemed responsible for the car’s actions highlights the urgent need for a technology that can automatically and exhaustively certify the impossibility of accidents in various driving scenarios, and under well-defined conditions.

It is clear that, for now, next generation AVs will remain research projects because there is a lack of confidence that they
can be used in safety critical applications. More specifically, there are currently few tools for plan verification and execution analysis [3]. New, more practical methods, for formal verification and model based design could increase the confidence that highly autonomous systems can be put into service and can potentially reduce both development costs and time to market.

A successful tool must handle a large variety of scenarios in which the AVs will operate: for example, highway driving, entrance ramps, roundabouts, and stoplights. Furthermore, each scenario has a large number of variations: highway driving involves a varying number of cars and different starting configurations of the cars. These configurations define the traffic participants relative positions, their initial speeds, orientations, and their goals. The configuration space can be extremely large, and in fact, is often uncountably infinite, as can be seen by considering that a car’s position is a real-valued variable. In this paper, we address two questions: (1) Is it possible to investigate all configurations via symbolic abstraction? (2) If it is, how can we verify that the car’s behavior is correct in all configurations? Here, correct means that the vehicle is both safe and achieves the mobility goals of the pilot.

The most common approach today to verifying the safety and correctness of the car’s controllers is to perform a large number of simulations and tests. Every simulation varies the scenario configuration: in one simulation the cars start 0.7m apart from each other, in another they start 2m apart with equal speeds, in yet a third they start 2m apart with the lead vehicle making a left turn. However, simulation always leaves a verification gap. To illustrate this gap, we focus on a common scenario: the lane change.

**Simulating a lane change**

Consider the situations presented in Fig. 1, all of which are variations on a lane change scenario. Every car in the scenario is characterized by its state $x$ which includes at least its position, velocity, orientation and yaw rate. In general, additional variables might be used to describe the state of the car at every time instant. In this example, we use a 7-dimensional state. To simulate the scenario, we select initial values for these variables, i.e., an initial state $x(0)$ (which we called configuration). Note for this example that once an initial state is fixed, the system evolves deterministically based on the controllers of each car. The initial state can have any value in a bounded set: e.g., in Fig. 1, the initial position $(s_x, s_y)$ of the ego vehicle is in $[0, 1] \times [0, 1]$, and the velocity $v$ is in $[24, 33] m/s$. There are two distinct sources for uncertainty about the state: first, the ego vehicle will have to perform a lane change under a variety of initial states. Simulating it under only one initial state is clearly insufficient, because we expect the outcome of the scenario when the two cars start 0.5m apart to differ from its outcome when the two cars start 5m apart. The second source of uncertainty comes from errors in perception such as localization and velocity estimate. Even if we wish to start the simulation in a particular state, inaccuracies in measurements mean that the car’s state can not be exactly known. So while the control algorithms assume a given starting state, the car may actually begin from anywhere in a bounded set around that state estimate. Thus it’s important to verify that these measurement errors do not cause unsafe situations.

![Figure 1: Simulation is not sufficient to fully verify a lane change.](image)

The question then becomes: how many simulations should we perform, and which simulations should we perform? Ideally, we would simulate all configurations that produce an unsafe outcome, but this can not be guaranteed in general. Even experienced engineers might not think of corner cases, especially given the size of the vehicles configuration space. E.g., in Fig. 1, we show a lane change scenario, which has been simulated a 1000 times, including with varying numbers of vehicles. Yet, it is only the last, non-simulated, situation that reveals the collision: if the ego vehicle starts with a positive orientation and yaw rate, and attempts to change lanes while the other vehicle is slowing down, it could cause a collision. This is because the ego vehicle is unable to exactly follow the reference trajectory which the motion planner determined would be safe.

Randomized testing, where the configurations are sampled from hypercubes of parameters, is not a scalable solution: suppose we decide to sample only 10 points in the range of every state variable. For our 7D model, and with 2 cars, this yields a total of $10^{14}$ simulations. Say we wish to simulate 10 seconds. Even if a simulation runs in real-time, this still requires $10 \times 10^{14}$ seconds = 30 million years to complete.

Thus, while simulations are a useful and intuitive method for getting a quick confidence level in the basic safety of a scenario, they are not sufficient for guaranteeing the absence of risk in a given scenario with a bounded state space.

**Contributions**

Our main contribution is a design-time approach to formally verifying the trajectory planning and trajectory tracking stacks of an ADAS/AV as they interact with potentially dynamic participants in a variety of driving scenarios. This approach is implemented in a software tool, APEX, and illustrated with examples of a lane change maneuver. The verification approach has two characteristics:

- It is formal: we are guaranteed that if APEX determines a scenario to be safe, then it is safe. No amount of simulation can find an unsafe behavior in a scenario verified as correct by APEX.
- It allows the use of an arbitrary trajectory planner, for example, it could be code or an abstraction. That is, there is
no need to model the trajectory planner, which is often very complex software. Moreover, the same trajectory planner can then be run on a real vehicle. In the case study presented in this paper, APEX uses a trajectory planner that has been tested on a real vehicle.

In APEX, the verification engineer can

- Specify the low-level dynamics of the vehicle, including the trajectory tracker. Unlike other approaches and existing tools the dynamics can be nonlinear. The default model in APEX is a 7D bicycle model.

- Provide a motion planner that takes in a starting position and end position and returns a trajectory that links the two points. The motion planner can be any piece of software: there are no restrictions on it. The default planner in APEX is a state lattice planner incorporated in ROS and tested on a real vehicle. Figure 2 shows the planner GUI available as part of Autoware [4].

- Specify a sequence of goal positions (or waypoints) that the vehicle must visit, or a behavioral planner that computes these waypoints in a reactive manner. The default behavioral planner in APEX is a simple 2-state automaton that decides whether to execute lane following or lane changing. However, we expect that designers will implement many other more complex behavioral planners.

- Specify the uncertainty sets for the ego vehicle and the other agents in the scenario.

- Specify the unsafe conditions to be avoided by the vehicle. APEX supports a rich specification language, Metric Interval Temporal Logic (MITL) for the description of unsafe behaviors [5].

APEX will then verify, in an exhaustive fashion, that the ego vehicle can complete the scenario under the specified uncertainty, or return a specific case where it fails. The engineers can then use this counter-example in order to debug the controllers, and better understand how to avoid this failure at design-time.

One real-world example of an AV software bug related to plan execution was highlighted by the first ever crash between AVs at the Urban Challenge [6]. At the time of the accident, participants noted that there are no known “formal methods that would allow definitive statements about the completeness or correctness of a vehicle interacting with a static environment, much less a dynamic one” [7]. It is beyond the scope of this paper to review the numerous developments in verification and synthesis technology; we note attempts exist to reason about the safety of autonomous vehicles in static environments via synthesis [8], but such methods cannot currently scale to realistic systems and are extremely conservative. In response the authors of [8] propose a receding horizon framework, but still rely on coarse grid-based abstractions. Others have sought to verify Adaptive Cruise Control Algorithms (ACC) which severely restrict scenarios in which the car may operate (no lane changes) [9]. Finally, some research which eschews discretization in favor of continuous linearized dynamics focuses on moving the verification task online [10].

APEX description and usage

Planning and Control for Autonomous Vehicles

In order to motivate the need for the APEX approach, we first outline the architecture of a typical ADAS/AV control system. It is not necessary that a vehicle use this particular architecture in order to be verified under APEX, but it motivates the key issues involved in obtaining a proof of safety. In the three-layer architecture paradigm [11] which we demonstrate, the planning and control of the vehicle is hierarchical in nature. Each successive layer performs a task over a shorter time horizon. Fig. 3 details this approach to AV architecture.

At the top level a mission planner is given a mobility goal. Such a goal is typically expressed as a (current location, destination) pair. Given this pair the mission planner finds an optimal (or feasible) route through the road network.

In the next layer, the behavioral planner makes local decisions about how to navigate the road network. For example, if the mission planner informs the behavioral planner that at the next intersection it will need to turn left, the behavioral planner will use a set of rules to determine that the ego vehicle must be in the left lane. It then provides a sequence of waypoints, or intermediary destinations, to the lower-level local planner.

Finally, the local planner, or trajectory planner, produces a trajectory that connects the vehicle’s current pose to the target pose at the next waypoint. Here ‘pose’ refers to the combined position, heading and velocity of the vehicle. Specifically, given
state of the AV, constraints on the configuration of the environment, and a desired behavior of the AV (like mobility goal and traffic laws), is it ever possible for the AV to violate the desired behavior? Fig. 5 summarizes a single execution of the verification engine. For each trajectory selected by the planner (highlighted in red), APEX calls dReach [13], a reachability analysis tool for nonlinear hybrid systems.

We emphasize that the verification process is offline - the vehicle does not run APEX while it is driving. At each decision point encountered by the behavioral planner there may be multiple executions of the verification engine depending on the design of the behavioral planner. Fig. 6 describes how the execution of the controller online relates to the offline verification process. In contrast to the simulation based approach outlined in the introduction, the result of the APEX approach is that we have converted a brute force search over real intervals into a finite series of tractable bounded reachability problems over a finite verification horizon.

**Tool Input**

APEX is a command line tool for verification of autonomous vehicle missions written in Python and C++. The input to the verification process is a mission definition file. The mission definition file defines the sequence of waypoints or road links which the vehicle will traverse in order to achieve a mobility goal.

The mission definition file describes the following:

- **The collection of agents in the scenario**, consisting of the ego vehicle and other cars in the scenario. The agents are described via ODEs that describe the evolution of their state with time, and their behavioral planners, which give the next waypoints for each vehicle. All agents operate in an ontology specific to the mission, in this case the world model consists of a geometric description of a road network.

- **Set of initial states** for each state variable of every vehicle.

- **The constraints that the AV should satisfy**, such as traffic laws and the unsafe conditions that ego vehicle must avoid. These are described in MITL.

- **The goal of the ego vehicle**, also expressed in MITL.

The mission definition file is part of a mission definition script. The latter manages the execution of the behavioral planner and trajectory generator. Each (state, goal) pair that is encountered on the mission generates at least one trajectory which must be verified. The mission definition script automatically updates a scenario verification instance. The scenario verification instance is a dReach (.drh) file which combines the results of the plan execution with the dynamical model of the vehicle and a low-level trajectory tracking controller. The agent definition file contains the dynamical model of the vehicle and the tracking controller is also written using the syntax of dReach, it may be manually edited in order to match mission specific vehicle models. We provide an example of the syntax of the composed scenario verification instance in Fig. 7.

Together, the constraints of the environment $\xi$ and ego vehicle goal and constraints $\phi$ constitute the specification of the mission. The mission is a success if every execution of the system (i.e., every simulation) satisfies the specification.
Tool Output

Each scenario verification instance can return either SAFE or δ-UNSAFE. SAFE means that for all possible executions of the system we can not reach an unsafe state. δ-UNSAFE means that there exists an execution of the system which comes within a δ of the unsafe region, and possibly enters it. If the system is δ-UNSAFE the tool will return a counter-example describing a tube around a concrete trajectory whose intersection with the unsafe region is not empty. Users of the APEX tool should be aware that selecting too large of a precision value (δ) may result in δ-UNSAFE results which are false positives, but any declaration of SAFE is guaranteed to be correct.

Building an Autonomous Vehicle Agent

To run APEX, we need to capture the AV dynamics, the low level tracking controller, and the planning stack which generates the trajectories for the vehicle to follow.

Modeling

The first step towards verification is a model of the AV. APEX uses the formalism of nonlinear hybrid systems to describe the AV and other vehicles. The trajectory tracking controller and AV can be described using ordinary differential equations. The discrete nature of the behavioral control layer dictates that we must capture a system with mixed continuous-discrete dynamics. We provide a list of symbols used in Table 1.

Ego Vehicle Model

APEX uses a non-linear 7 degree of freedom bicycle model [14] in order to describe the ego-vehicle. Higher order models can be supported in the future, and of course the parameters of the base model can be customized in order to match specific vehicles. See Fig. 8. The input to such a model is steering angle velocity and linear velocity, the output is vehicle state as a function of time.

The state vector describing the vehicle is described in equations (1)-(7). The variable β is the slip angle at the center of mass, ψ is the heading angle, ψ̇ is the yaw rate, v is the velocity, sx and sy are the x and y positions, and δ is the angle of the front wheel. In the formulation of [6], the inputs to the system are ax, the longitudinal acceleration, and vw the rotational speed of the steering angle.

\[ x_v = (\beta, \Psi, \dot{\psi}, v, s_x, s_y, \delta) \]  

(1)
The state equations for the system as described in [15] are:

\[
\dot{\beta} = \left( \frac{C_r l_r - C_f l_f}{m v^2} \right) \dot{\psi} + \left( \frac{C_f}{m v} \right) \delta - \left( \frac{C_f + C_r}{m v} \right) \beta \tag{2}
\]

\[
\dot{\psi} = \left( \frac{C_r l_r - C_f l_f}{l_z^2} \right) \beta - \left( \frac{C_f l_f^2 - C_r l_r^2}{l_z} \right) \left( \frac{\dot{\psi}}{v} \right) + \left( \frac{C_f l_f}{l_z} \right) \delta \tag{3}
\]

Vehicle Parameters

The parameters \(C_f, C_r\) and \(l_f, l_r\) describe respectively the cornering stiffness and distances from the center of gravity to the axles respectively; the subscripts \(f, r\) denote whether the parameter is defined for the front or rear of the vehicle. The moment of inertia, \(I_z\) and the vehicle mass, \(m\) are experimentally determined constants [16]. The kinematic bicycle model considers the two front wheels and two rear wheels of the vehicle to move in unison, with steering provided by the front wheels only. Furthermore, each abstracted wheel is located along the center of the vehicle’s body. Table 2 contains the validated vehicle parameters as given in [15]. It is possible to obtain such parameters and replace these constants in order to investigate specific vehicle characteristics.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_v)</td>
<td>-</td>
<td>Verification State Vector</td>
</tr>
<tr>
<td>(x_{sl})</td>
<td>-</td>
<td>Lattice Planning State Vector</td>
</tr>
<tr>
<td>(x_p)</td>
<td>-</td>
<td>Vehicle Pose</td>
</tr>
<tr>
<td>(x_g)</td>
<td>-</td>
<td>Goal Pose</td>
</tr>
<tr>
<td>(x_f)</td>
<td>-</td>
<td>Predicted Vehicle Pose</td>
</tr>
<tr>
<td>(p)</td>
<td>-</td>
<td>Cubic Spline Parameter Vector</td>
</tr>
<tr>
<td>(l_f)</td>
<td>s</td>
<td>Prediction Horizon</td>
</tr>
<tr>
<td>(m)</td>
<td>kg</td>
<td>Vehicle Mass</td>
</tr>
<tr>
<td>(l_r)</td>
<td>m</td>
<td>Rear Wheelbase</td>
</tr>
<tr>
<td>(l_f)</td>
<td>m</td>
<td>Front Wheelbase</td>
</tr>
<tr>
<td>(I_z)</td>
<td>kg m^2</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>(C_f)</td>
<td>N/rad</td>
<td>Front Cornering Stiffness</td>
</tr>
<tr>
<td>(C_r)</td>
<td>N/rad</td>
<td>Rear Cornering Stiffness</td>
</tr>
<tr>
<td>(\beta)</td>
<td>rad</td>
<td>Slip Angle</td>
</tr>
<tr>
<td>(\Psi)</td>
<td>rad</td>
<td>Heading Angle</td>
</tr>
<tr>
<td>(v)</td>
<td>m/s</td>
<td>Velocity</td>
</tr>
<tr>
<td>(s_x)</td>
<td>m</td>
<td>Position, x</td>
</tr>
<tr>
<td>(s_y)</td>
<td>m</td>
<td>Position, y</td>
</tr>
<tr>
<td>(\delta)</td>
<td>rad</td>
<td>Steering Angle</td>
</tr>
<tr>
<td>(e_x)</td>
<td>m</td>
<td>Tracking Error, x</td>
</tr>
<tr>
<td>(e_y)</td>
<td>m</td>
<td>Tracking Error, y</td>
</tr>
<tr>
<td>(v_w)</td>
<td>rad/s</td>
<td>Steering Angle Velocity</td>
</tr>
<tr>
<td>(a_x)</td>
<td>m/s^2</td>
<td>Longitudinal Acceleration</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>rad/m</td>
<td>Curvature</td>
</tr>
<tr>
<td>(s_f)</td>
<td>m</td>
<td>Arc Length</td>
</tr>
</tbody>
</table>

\[
\dot{\psi} = a_x \tag{4}
\]

\[
s_x' = v \cos (\beta + \psi) \tag{5}
\]

\[
s_y' = v \sin (\beta + \psi) \tag{6}
\]

\[
\delta = v_w \tag{7}
\]
**Tracking Controller**

A simple trajectory tracking controller is included with the APEX vehicle model. Trajectory tracking controllers guide a vehicle along a geometrically defined cubic spline by apply steering and longitudinal acceleration inputs. A successful path planning algorithm maintains vehicle stability and attempts to minimize the error between the desired trajectory and actual trajectory. The parameters used for verification (although it can be). The planner must run online, in real-time, therefore, lower order models are often substituted here. In this implementation we define the vehicle state equations as:

\[
\begin{align*}
\dot{x} &= v \cos(\Psi) \\
\dot{y} &= v \sin(\Psi) \\
\dot{\theta} &= \kappa \cdot v \\
\dot{\kappa} &= \frac{\dot{\Psi}}{\ell}
\end{align*}
\]

The local planner’s objective is then to find a feasible trajectory from the initial state defined by the tuple \( x_{sl} \) to a goal pose \( x_p \) defined as:

\[
x_p = (s_x, s_y, \Psi)
\]

In this formulation we limit trajectories to a specific class of parameterized curves known as cubic splines. A cubic spline is defined as a function of arc length:

\[
k(s) = k_0 + a k_1 s + b k_2 s^2 + c s^3
\]

Note that there are four free parameters \((a, b, c, s_f)\) and our goal posture has four state variables. Thus, a cubic spline is a minimal polynomial that can be assured to produce a trajectory from the current position to the goal position (if it is kinematically feasible). For any particular state, goal pair there are two steps necessary to compute the parameters. First, it is necessary to produce an initial guess. There are several approaches available such as using a neural network, lookup table, or a simple heuristic. In this case we adapt a heuristic from Nagy and Kelly [18] such that it is compatible with a stable parameter formula produced to the desired trajectory and actual trajectory. The parameters used for verification (although it can be). The planner must run online, in real-time, therefore, lower order models are often substituted here. In this implementation we define \( x_{sl} \) as:

\[
x_{sl} = (s_x, s_y, v, \Psi, \kappa)
\]

Where \( s_x \) and \( s_y \) are the x and y positions of the center of mass, \( v \) is the velocity, \( \Psi \) is the heading angle, and \( \kappa \) is the curvature. We note that the state equations involve an additional constant, \( \ell \) which is the wheelbase of the vehicle. Where the state equations are described as:

\[
\begin{align*}
\dot{s}_x &= v \cdot \sin(\Psi) \\
\dot{s}_y &= v \cdot \cos(\Psi) \\
\dot{\Psi} &= \kappa \cdot v \\
\dot{\kappa} &= \frac{\dot{\Psi}}{\ell}
\end{align*}
\]

We note that we cannot use traditional linear systems techniques or sum of squares optimizations to directly find a Lyapunov function for this system because of the obvious nonlinearity and non-polynomial form of the governing ordinary differential equations. Instead we will seek to show stability and safety properties using reachability and model checking analysis.

**Planning**

In APEX we provide a validated planning stack which can be run on a real vehicle. The planning strategy is hierarchical and includes: mission planning, behavioral planning, and local planning. In this section we will focus on the local planner because it is the layer which connects directly to the tracking controller for the vehicle. The local planner is used to generate smooth trajectories which a non-holonomic dynamically constrained vehicle is capable of following. Our planning stack utilizes the methods outlined in [17] commonly known as state-lattice planning with cubic spline trajectory generation.

Each execution of the planner requires as an input the current state of the vehicle and a goal state as defined by the behavioral planner. We note that we will call the vehicle state \( x_{sl} \) because it does not necessarily have to be the same as the model used for verification (although it can be). The planner must run online, in real-time, therefore, lower order models are often substituted here. In this implementation we define \( x_{sl} \) as:

\[
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\[
\begin{align*}
k(0) &= p_0 \\
k(s_f / 3) &= p_1 \\
k(2s_f / 3) &= p_2 \\
k(s_f) &= p_3
\end{align*}
\]
Where the parameters \((a, b, c, s_f)\) can now be expressed as:

\[
a(p) = p_0 \\
b(p) = -\frac{11p_0 - 18p_1 + 9p_2 - 2p_3}{2s_f} \\
c(p) = \frac{9 \times (2p_0 - 5p_1 + 4p_2 - p_3)}{2s_f} \\
d(p) = -\frac{9(p_0 - 3p_1 + 3p_2 - p_3)}{2s_f}
\]

(21) – (24)

Which results in the following initialization heuristic:

\[
p_0 = \kappa_0 = \kappa_i \\
p_1 = \kappa_1 = \frac{1}{49}(8b(s_f - s_i) - 26\kappa_0 - \kappa_3) \\
p_2 = \kappa_2 = \frac{1}{4}(\kappa_3 - 2\kappa_0 + 5\kappa_1) \\
p_3 = \kappa_3 = \kappa_f
\]

(25) – (28)

Finally, with an initial guess in hand, and a stable re-parameterization the local planner can solve a simple gradient descent problem to drive the vehicle to the goal posture.

Thus, we can now compute a set of parameterized trajectories which may each be evaluated to test for safety and optimality. A description of these aspects of the planner may be found in [17] and such a cost function can obviously be modified based on the goals of the design team. We note that our algorithm implementation is parallelized using OpenMP such that multiple trajectories (with goals regularly sampled around the initial goal) may be evaluated simultaneously. Furthermore, with small changes we can also support quintic splines which expand the variety of possible maneuvers and are more suitable for high speed driving. Figure 10 shows an example of a trajectory generation instance.

### Specification

Formal verification requires both a system model and a specification. This means that the project stakeholders must provide an exact definition of the desirable system properties. Furthermore, it is often the case that such properties are expressed as occurring only under certain conditions. For convenience we provide the symbols used to describe the vehicle specification in Table 4.

![Figure 10: Output of an execution (10 Hz) of the trajectory generator, a single trajectory will be chosen from this set.](image)

An example specification follows: the ego vehicle should drive in the selected lane at the speed limit unless a stop sign is encountered. We note that the traffic laws of a given region provide a partial, but informal definition of many of the high level specifications which the ego vehicle should adhere to.

### Ego Vehicle Specification

The specification for the ego vehicle has two components: safety properties and liveness properties. A specification for the ego vehicle in the case study follows:

- The ego vehicle travels at a velocity less than or equal to the speed limit
  \[\square(v_{ego} \leq v_{limit})\]

- The ego vehicle does not drive backwards
  \[\square(v_{ego} \geq 0)\]

- The ego vehicle does not collide with any of the \(n\) other objects in the environment
  \[\square\left(\sqrt{(s_{xego} - s_{xenv})^2 + (s_{yego} - s_{yenv})^2} \geq r\right)\]
  \[\forall i = 1...n\]

- If a timed lane change request is invoked, the ego vehicle completes the lane change on time.
  \[\square(LC \rightarrow (s_{yego} > w) \land (t \leq t_{max}))\]

### Environment Specification

The other vehicles operating within a scenario present both an interesting challenge and a primary motivation for formal verification. It is clear that it is impossible to know the intentions of
the agents operating such vehicles; their execution represents a significant source of non-determinism. In fact, a more complex model of such agents which includes details such as steering angle or tire friction will not enable less conservative results, for it is the control input not the plant that remains the largest unknown. Thus, we conclude that: for verifying the autonomous agent, only the perceptible behavior of other agents is important, not their internal structure.

Still it remains clear that the behavior of other agents must be part of the scenario description. As such we present a safety case which assumes that other agents will follow a certain minimal set of driving rules. For brevity we will reference the following specification as ξ in the case studies.

- Acceleration ceases when some maximum velocity is reached.
  \[ \square (v_{env} \geq v_{max} \rightarrow a = 0) \]  

- Other agents must drive in the proper direction according to their lane.
  \[ \square (v_{env} \geq 0) \]  

- The accelerations of other agents are within those rates achievable by maximum engine power
  \[ \square (a_{env} \leq a_{max}) \]  

- Other agents maintain their lanes unless explicitly specified not to.
  \[ \square (\neg LC \rightarrow (y_{min} \leq s_{y_{env}}) \land (y_{max} \geq s_{y_{env}})) \]  

- Lane changes by other agents are only permitted if the alternate lane is unoccupied or unless a degenerate scenario is being modeled.
  \[ \square (LO \rightarrow \neg LC) \]  

**APEX internals and theory**

APEX maintains an internal representation of the scenario as a hybrid system. The components of this hybrid system are:

- The behavioral planners of all vehicles involved, \( B_1, \ldots, B_m \). Fig. 11 shows the behavioral planner we used in the case study for a lane change. A behavioral planner is a finite state system. We will refer to each state of a behavioral planner as a mode.

- For every vehicle, the continuous dynamics involved in each of the modes of its behavioral planner. In general, different modes may require different dynamics: e.g. a Collision Avoidance mode which is invoked when a collision is imminent requires more stability control than a turn at a low speed. The continuous dynamics are given in terms of Ordinary Differential Equations (ODEs) \( \dot{x}_i = f_i(x_i) \), where \( x_i \in \mathbb{R}^n \) is the continuous state of the \( i^{th} \) agent.

- For each vehicle, transition conditions between the modes of the behavioral planner \( B_i \) are expressed in terms of the state vector \( x_i \). The planner transitions between two modes \( q \) and \( q' \) only if a guard condition \( G_{q,q'} \) is satisfied. In general, the guard condition for \( B_i \) is expressed as a set in the state space of all the agents, since transitions will occur based on, for example, how close two vehicles are to each other. Specifically, let \( x = (x_1, \ldots, x_n) \) combine the states \( x_i \) of the individual vehicles. So \( x \in \mathbb{R}^{n \cdot m} \). Then there’s a transition between two states \( q \) and \( q' \) of \( B_i \) only if \( x \in G_{qq'} \subset \mathbb{R}^{n \cdot m} \). For example, there’s a LF-to-LC transition only if the two cars are closer than 10m and the following car is faster than the leading car. In this case \( G_{LF,LC} = \{ x \mid ||x_1 - x_2|| \leq 10 \land v_2 > v_1 \} \).

Together, these make up a hybrid system, so-called because it combines discrete dynamics in the behavioral planner with continuous dynamics in each mode. We will refer to the \( n \) hybrid systems of the \( n \) agents in the scenario as \( H_1, \ldots, H_m \). The state of the scenario \( x \) is simply \( x = (x_1, \ldots, x_n) \).

APEX also needs to maintain a description of the scenario specification. This specification is provided by the user and can be any formula in first-order logic over the set of modes and states of all agents. See the Case Study. For example the following is a possible specification:

\[ \text{Mode}_1 = \text{LC} \rightarrow |\dot{\psi}| \leq b \]

The following sections describe how APEX verifies a property of the scenario using this internal representation.

**Execution tree and formal model**

Let \( B \) be a behavioral planner of a given vehicle. The formal model of the behavioral planner is a finite transition system \( B = (Q, q_0, \Sigma, \rightarrow) \) where \( Q \) is the finite set of modes, \( q_0 \) is the initial mode, \( \Sigma \) is a set of output labels, and \( \rightarrow \subset Q \times \Sigma \times Q \) is the labeled transition relation of the system. We write \( q \xrightarrow{\sigma} q' \) for \((q,\sigma,q')\in\rightarrow\). Fig. 11 shows the behavioral planner that is used by APEX by default for modeling a lane change controller. It can be described as \( B = ((\{LC, LF\}, LF, \mathbb{R}^n, \{(LF, LF), (LF, LC), (LC, LF)\}) \) In mode LF, the vehicle’s goal is to follow the current lane. In mode LC, the vehicle’s goal is to change lanes. In general, a mode represents a decision by the controller, a behavior that the vehicle should follow. With every transition between modes, the behavioral planner outputs a vector \( x_B \) in \( \mathbb{R}^n \): this is the destination that the vehicle must reach. The planner transitions between modes when certain guard conditions are satisfied.

The behavioral planner advances in discrete time. The discrete time advances, for example, with every update of the vehicle’s sensors. Thus \( B \) makes a decision on what to do every time its information about the environment is updated. The planner may decide to maintain the current decision, i.e., stay in the same mode, if that mode has a self-loop. Mode LF has a self-loop in Fig. 11. Let \( \Delta t > 0 \) be the update period. Since every scenario is time-limited, and every transition takes fixed non-zero time \( \Delta t \), there is a natural limit \( D \) on the number of decisions, or transitions, that can be taken in any given scenario.
In the first step of the verification process, APEX builds an execution tree: the root of the tree is the initial mode $q_0$, and every branch of the tree represents one possible sequence of decisions, i.e., one possible execution of $B$. See Fig. 13 for the execution tree of the behavioral planner of Fig. 11. Since the number of transitions is bounded by $D$ in a given scenario, this tree has a depth at most $D$.

With the execution tree built, APEX must next verify that the sequence of decisions taken by the behavioral planner can be implemented by the low-level controllers. E.g., let $(LF, LF, LC)$ be a sequence of decisions of depth 3. In every occurrence of LF, APEX must check that the vehicle can indeed follow the lane, and in every occurrence of LC, APEX must verify that the vehicle can indeed change lanes. In the next section, we define what it means to ‘follow the lane’ and ‘change lanes’ via the motion planner.

**Calling the motion planner**

After building the execution tree, APEX starts executing every branch, starting at the root, which is the initial mode $q_0$. The initial set of continuous states is $X_0$. A transition is taken if the initial set intersects its guard. Since $X_0$ may intersect more than one guard, then more than one transition are possible. APEX explores all transitions (all branches) in the execution tree. In each mode APEX enters, $B$ will output a destination $x_B$. Formally, $x_B$ is a scenario state, but in what remains, it is simpler to think of it as the position that the ego vehicle must reach.

APEX then calls the motion planner to obtain the trajectory that the vehicle will follow. Since the current state is only known as a set $X_A$, APEX sets the starting point of the trajectory to be the center $x_A$ of $X_A$. The motion planner then returns a trajectory starting at $x_A$ and ending in a neighborhood of $x_B$. The neighborhood shape and size are known to APEX and are part of the motion planner’s description. Let that neighborhood be $X_B$. Note that APEX does not place any restrictions on the motion planner’s operation and calls it as a black box. Therefore, the actual motion planner that is used on the real car can be used in the verification of the system. In this way the verification results are directly applicable to the actual deployed software.

**Verifying each trajectory**

Once a trajectory is generated connecting $x_A \in X_A$ to the neighborhood $X_B$ of $x_B$, it remains to verify that the ego vehicle will always reach $X_B$ within a specified amount of time $T$, regardless of where it starts in $X_A$. To verify that the specification is satisfied, APEX builds a reachability problem. This reachability problem is characterized by the following:

- The system: in this case, the system consists of the scenario hybrid system.
- The target set: this is the set that the system should reach. In this case the state of the ego vehicle $x_e$ should reach $X_B$, and there are no target sets for the other agents.
- The unsafe set: this is the set that the scenario hybrid system must not reach at any point in time. In this case, the ego vehicle must not get closer than $d_{min}$ to any other agent in the scenario.

We call the above a bounded reachability problem. To solve this problem, APEX passes it to dReach [12], a reachability analysis tool for nonlinear hybrid systems. dReach answers the question: is there a trajectory of the vehicle starting in $X_A$ that will violate the constraints? (e.g. will not reach the target set $X_B$ or will get too close to another vehicle). dReach returns one of two answers. If the answer dReach returns is SAFE, then this means that there exists a behavior of the ego vehicle which, when perturbed by an amount $\delta > 0$, violates the constraints. See Fig. 4. The parameter $\delta$ can be set by the user. It suffices to choose $\delta$ small enough so $\delta$-SAT means the system is not robust since a small perturbation of size $\delta$ could cause it to violate the constraints.

**Case Study**

We briefly introduce and expand on the concept of driving scenarios to help reason about inherently diverse situations and requirements which an autonomous vehicle might face.

**An unsafe lane change scenario**

The following example describes a lane change scenario in the context of a mission and mobility goals. In this description we imply a valid local planning solution, and seek to verify that all possible individual trajectories which are selected in the execution of the plan are safe. First, in Scenario 1 we will demonstrate a dangerous condition that could have been missed under testing or simulation. Next, in Scenario 2 we will show how a refinement in the requirements on the perception system or a refinement in the behavioral controller can lead to a provably safe maneuver. Finally, in Scenario 3 we demonstrate how a change in manufacturer specification can be accurately assessed for safety. To perform verification, we employ dReach version 3.15.10.02 on a Mac OSX laptop with Intel(R) Core i7(2) 2.60GHz CPU and 16 GB memory, and the results are provided in Table 5.

**Scenario 1 (A simple lane change and goal)** As shown in Fig. 12, the ego vehicle is driving in the right lane of a uni-directional two lane road network. Another car is driving in front of the ego vehicle at a lower speed. We include

![Figure 11: An automaton describing a simplistic behavior planner for lane changes](image-url)
the extreme case where the environmental vehicle stops. We highlight that when there is significant uncertainty regarding the ego vehicle's orientation and that it may deviate (initially) from the reference trajectory (dashed line) while the tracking controller recovers. We note that the specification of the environment and the ego-vehicle in this scenario are defined as $\xi$ and $\phi$ respectively.

**Behavioral controller**

We associate a behavioral controller $B_1$ with Scenario 1. Figure 11 details the controller, where LC means “Lane Change” and LF means “Lane Follow”. Table 5 records the parameters. It is a simple finite state deterministic automaton. We note, that this particular behavior controller is almost surely too simplistic to cover all of the scenarios faced by an actual vehicle, nevertheless it illustrates how we may formally represent a set of rules which instantiate certain behavior classes on an autonomous vehicle. Similar examples have been published by Darpa Urban Challenge participants []. Both controllers generated via reinforcement learning and reactive behavior controllers created via synthesis may be represented as deterministic finite automatons. As our current goal is to demonstrate that verification is possible, rather than the richness of the scenarios that the behavioral controller can handle, we find this controller suitable.

Given any deterministic finite automaton it is possible to express as a computational logic tree. Such a tree is rooted in a single state, is infinite in size, and represents a branching notion of time; that is each state (moment in time) may split into multiple possible future worlds. As we will explain in the following sections, such a representation is at the heart of the APEX approach and verification occurs over a bounded search depth on such a computation tree.

We present the initialization of the scenario and the results of the verification. Table 5 contains the initialization of each parameter.

**Verification and Result**

Finally, for the lane change case, we define an additional constraint set $R_{unsafe}$ as well as a goal set representing the maximum allowable deviation from the goal state. $R_{unsafe}$ expresses that the system fails if it still hasn’t changed lanes within 2 sec or it collides with the car ahead of it.

$$((s_y < w) \land (t > 2)) \lor ((s_y < w) \land (s_x - \epsilon > s_{x_n}))) \quad (38)$$

Then, using APEX we attempt to show that there is no execution of the system which can enter $R_{unsafe}$. However, because the system is incorrectly designed dReach returns $\delta$-UNSAFE.

**A safe lane change scenario**

Using the information and counterexample from the previous scenario it is easy to see that the behavior controller must be corrected in order to guarantee safety of the lane change scenario.

**Scenario 2 (A more conservative behavioral controller)**

We begin with Scenario 1. In order to ensure the forward safety of the vehicle we propose a small modification to the behavioral controller of the vehicle, and furthermore require that the ego vehicle’s localization system return estimates with less uncertainty. Namely, we first increase the size of variable buffer, so that the ego vehicle is forced to initiate a lane change maneuver earlier. Secondly, we decrease the size of the initial sets. Speed $v$ now starts anywhere in $[10.9, 11]$ and $s_y$ starts in $[0.0, 0.05]$.

With these changes, dReal returns SAFE, meaning that no trajectory of the system violates the constraints.

**A supplier issues a specification change**

Given that a safe controller has been found a supplier wishes to know if they may reduce the accuracy of several key sensors associated with localization of the ego vehicle. Such a specification change is known to add significant uncertainty to the estimate of the ego vehicle’s heading angle during the planning phase.

**Scenario 3 (Large perception errors)** We begin with Scenario 2. In order to reflect the change in supplier specification
we update the localization system return estimates to reflect greater uncertainty. Namely, we increase the size of the initial set for ego vehicle heading such that $\Psi$ starts in $[0.0,0.2]$.

The result of this modification is again $\delta$-UNSAFE, because the ego vehicle clips the rear bumper of the environmental vehicle while executing the lane change maneuver. Again the engineer in charge of the project may use the new information to refine the controller design or reject the suppliers specification change. In this way formal verification efforts can be a useful tool in determining the requirements which sensors and perception systems must meet given a particular control algorithm.

**Conclusion**

APEX is a tool for formally verifying the trajectory planning and tracking stacks of ADAS/AV cars. It can perform formal verification on realistic autonomous vehicle planning stacks. In this paper we demonstrate a case study which formally verifies a lane change maneuver. Future work will incorporate more complex behavioral controllers for other scenarios, including synthesized planners, and will add a GUI to the tool as well as a means of visualizing complex counterexamples.

**REFERENCES**


