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Comments
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Generative Power of CCGs with Generalized Type-Raised Categories

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Abstract

A type of ‘non-traditional constituents’ motivates an extended class of Combinatory Categorial Grammars (CCG), CCGs with Generalized Type-Raised Categories (CCG-GTRC) involving variables. Although the class of standard CCGs is proved to be equivalent to Linear Indexed Grammars and Tree Adjoining Grammars, use of variables can increase the power beyond these grammars. In order to establish a desired context with respect to computational complexity, this paper shows that there is a subclass of CCG-GTRC which is still weekly equivalent to the standard CCGs. The idea behind the proof is that the behavior of GTRCs can be simulated by appropriately setting the lexicon of a standard CCG.

1 Introduction

In Japanese, as in other SOV languages, a sequence of NPs can form a conjunct as exemplified below.

(1) John-ga Mary-o , Ken-ga Naomi-o tazuneta.
    { John-NOM May-ACC } CONJ { Ken-NOM Naomi-ACC } visited
    “John visited Mary and Ken [visited] Naomi.”

This type of ‘non-traditional constituents’ poses a problem to many grammar formalisms and parsers, including those specifically designed for Japanese, e.g., JPSG [Gunji, 1987] and JLE (based on finite-state syntax) [Kameyama, 1995]. Although these systems could be extended to cover the presented case, such extensions would not generalize to the wide range of non-traditional constituency.

Combinatory Categorial Grammar (CCG) has been proposed to account for non-traditional constituency in various areas of syntax [Ades and Steedman, 1982, Dowty, 1988, Steedman, 1985, Steedman, 1996] and also in the related areas of prosody, information structure, and quantifier scope [Steedman, 1991a, Prevost and Steedman, 1993, Prevost, 1995, Hoffman, 1995, Park, 1995, Park, 1996]. The mechanisms independently motivated to cover the wide range of non-traditional constituency can also provide an analysis for the NP-NP sequences in (1) as follows:

*I am grateful to Mark Steedman for his numerous suggestions and comments. I would also like to thank Beryl Hoffman, Anoop Sarkar, K. Vijay-Shanker, B. Srinivas, David Weir and reviewers of ACL/EACL-97 for their comments. The research was supported in part by NSF Grant Nos. IRI95-04372, STC-SBR-8920230, ARPA Grant No. N66001-94-C6043, and ARO Grant No. DAAH04-94-G0426. This technical report (IRCS-97-15) is a long version of [Komagata, 1997c] presented at ACL/EACL-97 (Student Session).
(2) John-ga [NP] Mary-o [NP] type raising
\[ S/(\lambda \cdot \text{NP}) \upharpoonright (\lambda \cdot \text{NP}) / \left( (\lambda \cdot \text{NP}) \upharpoonright \right) \]
\[ S/(\lambda \cdot \text{NP}) \upharpoonright \left( (\lambda \cdot \text{NP}) \upharpoonright \right) \]
\[ S/(\lambda \cdot \text{NP}) \upharpoonright \]

Informally, the NPs are assigned higher-order function categories associated with the basic category NP, and these functions can compose to derive another function category representing the NP-NP sequence. The two instances of such a category can then be coordinated and take the transitive verb category, \((\lambda \cdot \text{NP}) \upharpoonright \), as the argument to derive the category \( S \).

A similar type of constituents can also be formed of NPs extracted from different levels of embedding as in the following example:

(3) Japanese: Rinaryouzai-wa natoriumi-ni, \( \beta \) syadanzai-wa koukan sinkei kei-ni, kankei-no aru kouketutatu-no hito-ni kikimasu.
Gloss: \{ Diuretic-TOP sodium-DAT \} & \{ \( \beta \) blocker-TOP sympathetic nervous system-DAT \}
relevance-GEN exist hypertension-GEN person-DAT effective.
Translation: “Diuretic is effective for the person with hypertension related to sodium, and \( \beta \) blocker [is for the person with hypertension related] to sympathetic nervous system.”

The underlined part is another instance of non-traditional constituent, which includes an extraction from the relative clause. Its structure is schematically shown as follows:

(4) \([t_1 \text{ hypertension}_2 \text{-GEN person-DAT effective.}] [t_2 \ t_3 \text{ relevance-GEN exist}]\)

Due to functional composition, the category \((\lambda \cdot \text{NP}) \upharpoonright \), identical to the transitive verb category, can be derived for the above phrase with two extraction sites \( t_1 \) and \( t_3 \). As in the first example, the NP-NP sequences at the sentence-initial position receive the category \( S/(\lambda \cdot \text{NP}) \) and then the sentence category is derived after the combination with the category \((\lambda \cdot \text{NP}) \upharpoonright \).

Assuming that the competence grammar does not place a bound on the levels of embedding [Miller and Chomsky, 1963], we may have unboundedly-many extractions [Becker et al., 1991, Rambow and Joshi, 1994, Rambow, 1994]. Since no systematic constraint has been identified for the bound on the composition of such extracted constituents, we also assume that these constituents can compose without a limit, potentially resulting in an unboundedly-long NP sequence. As in the case of embedding, the degraded acceptability of long sequences can be attributed to performance issues. These assumptions calls for an infinite set of type-raised categories such as \((\lambda \cdot \text{NP}) \) associated with NP. We capture this polymorphic situation by using variables as in \( S/(\lambda \cdot \text{NP}) \upharpoonright \).

Although CCGs have also been motivated by psychological aspects, we will be concerned only with syntactic aspects in this paper.

The formal properties of the standard CCGs not involving variable (CCG-Std) are relatively well-studied. CCG-Std is proved weakly-equivalent to Head Grammars (HG), Linear Index Grammars (LIG), and Tree Adjoining Grammars (TAG), collectively called “mildly context-sensitive grammars” [Joshi et al., 1991, Vijay-Shanker and Weir, 1994]. CCG-Std is also shown to be polynomially parsable [Vijay-Shanker and Weir, 1990, Vijay-Shanker and Weir, 1991, Vijay-Shanker and Weir, 1993]. But, use of variables can destroy these properties. For example, Hoffman [1993]

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1The category \( S/(\lambda \cdot \text{NP}) \) is associated with a higher-order semantics \( \lambda P.P_a \) where \( a \) is the semantics of NP. Composition of \( \lambda P.P_a \) and \( \lambda Q.Q_b \) (left to right) is \( \lambda Q Q_b a \). In a sense, this semantics anticipates that of a two-argument predicate (simple or complex), \( Q \).

2The use of trace \( t \) is for illustration purposes only. The current approach does not assume the notion of gap or movement as the theories which employ trace.

3These assumptions need to be examined more carefully and is left for future research [cf. Joshi et al., 1994]. Since this paper assumes that the scope of coordination and adverb modification is bounded, the unbounded composition of type-raised category actually calls for a motivation involving information structure. For example, consider the following pseudo-Japanese:


The “theme” part can grow very long as can be seen. The information-structural status of the verb complex is not clear but should not affect the major division between the theme and theme across where all the categories must be combined prior to the combination of the theme and the rheme.

4It is also possible to dynamically type-raise categories as needed. We take lexical approach to avoid the procedural aspects associated with dynamic type raising. Polymorphic forms also have advantage of underspecification which avoids unnecessary proliferation of categories.
showed that a grammar involving categories of the form \((T \chi x)/(T \chi y)\) can generate a language \(a^mb^nc^nd^ne^n\) which no mildly context-sensitive grammar can generate. The use of variables in the coordination schema \(\chi+\text{conj } x \rightarrow \chi\) is also believed to generate a language \((wc)^n\) beyond LIG’s power [Weir, 1988]. At a level higher in the scale, Becker et al. [1991], Rambow and Joshi [1994], Hoffman [1995] propose formalisms which are more powerful than mildly context-sensitive grammars to account for ‘doubly’-unbounded scrambling.\(^5\) As we know that full context-sensitive capacity is too powerful to be a formal model of natural language syntax [e.g., Savitch, 1987], it is essential to identify the generative power of the formalism which interests us.

We define a class of grammars involving variables called CCG-GTRC, and show that there is a subclass of CCG-GTRC which is weakly-equivalent to CCG-Std. Our intuition is that a linguistically-motivated use of variables in type raising can be well-constrained so that it does not increase the generative power. The key idea is that unbounded, but restricted permutations in CCG-GTRC can be ‘simulated’ by re-organization of categories in the lexicon. A preliminary version of the current paper appeared as [Komagata, 1997c]. The related question about parsing efficiency is addressed in [Komagata, 1997a, Komagata, 1997b].

The paper is organized as follows: Section 2 introduces Generalized Type-Raised Categories (GTRC) and defines the classes of CCG-GTRC. Section 3 proves the weak equivalence between CCG-Std and a subclass of CCG-GTRC. Section 4 includes a few simple examples.

### 2 CCGs with Generalized Type-Raised Categories

#### 2.1 Generalized Type-Raised Categories

CCG-GTRC involves the class of constant categories (Const) and the class of Generalized Type-Raised Categories (GTRC).

A constant (derivable) category \(c\) can always be represented as \(F[a_n...a_1]\) where \(F\) is an atomic target category and \(a_i\)’s with their directionality are arguments.\(^6\) We will use \(A,...,Z\) for atomic, constant categories, \(\alpha,...,\zeta\) for possibly complex, constant categories, and \(\langle \rangle\) as a meta-variable for directional slashes \(/\,\,\num{/}\). Categories are in the “result-leftmost” representation and associate left. Thus we usually write \(F[a_n...a_1]\) for \((\ldots(F[a_n])...a_1)\). We will call \(\langle a_i...a_j\rangle\) a sequence (of arguments). The length of a sequence is defined as \(\lvert a_1...a_i \rvert = i\) while the nil sequence is defined to have the length 0. Thus an atomic constant category is considered as a category with the target category with the nil sequence. We may also use the term ‘sequence’ to represent an ordered set of categories such as \(\langle c_1,...,c_2\rangle\) but these two uses can be distinguished by the context. The standard CCGs (CCG-Std) solely utilize the class of Const.

GTRC is a generalization of Lexical Type-Raised Category (LTRC) which has the form \(\langle T[a_1] \rangle \langle b_1 \rangle\) associated with a lexical category \(a[b_1...b_1]\) where \(T\) is a variable over categories with the atomic target category \(T\) [cf. Dowty, 1988, Steedman, 1996]. The target indication may be dropped when it is not crucial or all the atomic categories are allowed for the target. We assume the order-preserving form of LTRC using the following notation, \(\langle i \rangle\) and \(\langle j \rangle\), indicate that either set of slashes in the upper or the lower tier can be chosen but a mixture such as \(\langle i \rangle\) and \(\langle j \rangle\) is prohibited.\(^7\) GTRC is defined as having the form of \(\langle T[a_m...a_1] \rangle \langle b_n...b_1 \rangle\) resulting from compositions of LTRCs where \(m \geq 1, n \geq 0\), and the directional constraint is carried over from the involved LTRCs.\(^8\) When the directionality is not critical, we may simply write a GTRC as \(\langle T[a_m...a_1] \rangle \langle b_n...b_1 \rangle\). For gtrc = \(\langle T[a_m...a_1] \rangle \langle b_n...b_1 \rangle\), we define \(\lvert \text{gtrc} \rvert = n + 1\), ignoring the underspecified valency of the variable. Note that the introduction of LTRCs in the lexicon is non-recursive and thus does not suffer from the problem of the overgeneration discussed in [Carpenter, 1991].

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\(^5\) ‘Doubly’-unbounded scrambling has the following characterization: (i) there is no bound on the distance of scrambling and (ii) there is no bound on the number of unbounded dependencies in one sentence.

\(^6\) Note that the representation \(F[a_1...a_i]\) involves meta-variables for object-level constant categories but we will remain ambiguous about this object-meta distinction in this paper.

\(^7\) For a related discussion, see [Steedman, 1991b].

\(^8\) In a sense, GTRCs have two stacks. But these two sequences are not completely independent and do not behave like two stacks for a PDA, which is Turing-Machine equivalent.
These categories can be combined by combinatory rule schemata. Rules of (forward) “generalized functional composition” have the following form:

\[(6) \quad x/y \uparrow^k y|z_k|z_1 \rightarrow x|z_k|z_1\]

The integer ‘k’ in this schema is bounded by \(k_{\text{max}}\), specific to the grammar, as in CCG-Std.\(^{10}\) Rules of functional application, “\(x/y \uparrow^0 y \rightarrow x\)”, can be considered as a special case of (6) where the sequence \(z_i\)/s is nil.\(^11\) The index \(k\) may be dropped when no confusion occurs. We say “’\(x/y\)\’ derivates \(x|z_k|z_1\)” and “’\(x|z_k|z_1\) generates the string of nonterminals ’\(x/y\)\’|\(z_k\)|\(z_1\)” or the string of terminals ’\(ab\)’” where the terminals \(a\) and \(b\) are associated with \(x/y\) and \(y|z_k|z_1\), respectively. The case with backward rules involving ‘\(\uparrow\)’ is analogous.

The use of variable for polymorphic type drew attention of researchers working on Lambek calculus [Moortgat, 1988, Emms, 1993]. In particular, Emms showed decidability for an extension called Polymorphic Lambek Calculus. The use of variables in the current formulation is limited to type raising. This reflects the intuition about the choice of rules based on ‘combinators’ [Steedman, 1988]. But otherwise, we do not assume that categories are wildy polymorphic.\(^{12}\)

One way to represent this situation is to use two distinct subclasses of the type ‘category’ constructed as follows:

\[(7)\]

<table>
<thead>
<tr>
<th>Type construction</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (\text{Const} (\text{Target}, \text{Arguments}))</td>
<td>(F \setminus a_n…\setminus a_1) → (\text{Const}(F, a_n…\setminus a_1))</td>
</tr>
<tr>
<td>b. (\text{GTRC} (\text{Target}, \text{IDir}, \text{ISeq}, \text{OSeq}))</td>
<td>(T \setminus (a_n…\setminus a_1) \setminus b_n…\setminus b_1) → (\text{GTRC}(T,/,a_n…\setminus a_1,\setminus b_n…\setminus b_1))</td>
</tr>
</tbody>
</table>

Such type construction can be defined in ML style as follows:\(^{13}\)

\[(8)\]

\[
\text{datatype target } A | B | C ... (* \text{atomic categories }*)
\]

\[
\text{datatype dir} \mid \text{left} | \text{right}
\]

\[
\text{datatype complex_cat} = \text{Complex of target} * \text{arg}
\]

\[
\text{and arg} = \text{Arg of dir} * \text{complex_cat} (* \text{mutually recursive }*)
\]

\[
\text{datatype seq} = \text{Seq of arg list}
\]

\[
\text{datatype cat} = \text{Const of target} * \text{seq}
\]

\[
| \text{GTRC of target} * \text{dir} * \text{seq} * \text{seq}
\]

Then we can define the combinatory rules on instantiated categories. Theoretically, no unification of variable is required although our implementation based on the proposed formalism uses variable unification for convenience. Although dealing with more number of cases is tedious, the technique is straightforward. Although dealing with more number of cases is tedious, the technique is straightforward. This leads to a favorable result that CCG-GTRC is not only decidable but also polynomially recognizable [Komagata, 1997a, Komagata, 1997b].

Inclusion of GTRCs calls for thorough examination of each combinatory case depending on the involved category classes. All the possible combination of category classes are described below. Some cases are subdivided furthermore. Although the traditional categorial representation is used below, the complete description for the constructor format can be defined. A summary of the cases is given in Table 1.

\[(9)\]

\[
\text{Const} \uparrow \text{Const}: a/b \uparrow^k c|d_k|d_1 \rightarrow a|d_k|d_1
\]

\[(10)\]

\[
\text{GTRC} \uparrow \text{Const}
\]

\[\begin{align*}
a. & \text{Functor GTRC has an outer sequence:} \\
& T | (T | a_m…\setminus a_1) | b_n…\setminus b_1 \uparrow^k c | d_k…\setminus d_1 \rightarrow T | (T | a_m…\setminus a_1) | b_n…\setminus b_2 | d_k…\setminus d_1 \\
& \text{Example: } T \setminus (T/\text{PP}) / \text{NP} \uparrow \text{NP} \rightarrow T \setminus (T/\text{PP})
\end{align*}\]

\[b. \text{Functor GTRC has no outer sequence:} \]

\(^9\)Vijay-Shanker and Weir [1994] call the functor and input categories as primary and secondary components, respectively.

\(^{10}\)Weir [1988] comments that the categorial grammars defined by Friedman and Venkatesan [1986] is more powerful than CCGs due to no bound on \(k\).

\(^{11}\)Forward functional composition and application are labeled as ‘\(\uparrow^k\)’ and ‘\(\uparrow\)’, respectively, in [Steedman, 1996].

\(^{12}\)Recall that we assume conjunctives (if treated categorially) and adverbs are mapped to finite sets of categories.

\(^{13}\)Note that there is no constant constructor for atomic categories.
\[
\frac{T/(T[a_m...a_2\backslash a_1]) \triangleright^k}{c \downarrow \Gamma \quad d_k...d_1 \rightarrow c_0[d_k...d_1]}
\]
Example: \(T/(T\backslash NP\backslash NP) \triangleright S\backslash NP\backslash NP \rightarrow S\)

(11) Const\(\triangleright\)GTRC

a. \(k < \mid \text{input}\): \[a/b \triangleright^k T_1(T[c_m...c_1]|d_{n...}d_k|d_k...d_1) \rightarrow a[d_k...d_1]\]
Example: \((S/(S\backslash NP\backslash NP)) \backslash (S/(S\backslash NP\backslash NP)) \backslash (S/(S\backslash NP\backslash NP)) \rightarrow (S/(S\backslash NP\backslash NP)) \backslash (S/(S\backslash NP\backslash NP))\)

b. \(k = \mid \text{input}\) (and \(k \geq 1\)): \[a/b \triangleright^k T_1(T[c_m...c_1]|d_k...d_1) \rightarrow a[b[c_m...c_1]|d_k...d_1] \quad \text{unbounded}
Example: \(S/S \triangleright T_1/(T\backslash NP\backslash NP) \rightarrow S/(S\backslash NP\backslash NP)\)

c. \(k > \mid \text{input}\) (and \(k \geq 2\)): \[a/b \triangleright^k T_0_1(T_0_1|T_1[c_m...c_1]|d_k...d_1) \rightarrow a[T_1|(b[T_1[c_m...c_1]|d_k...d_1] \quad \text{residual} \quad \text{unbounded} \]

(12) GTRC\(\triangleright\)GTRC

a. Functor GTRC has an outer sequence \textit{and} \(\mid \text{input} \mid > k\):
\[T_1(T[a_m...a_1]|b_{n...}b_2/b_{1}) \triangleright \bigcup[U|c_{p...}c_1]|d_{n...}d_{k+1}|d_{k...}d_1\]
\[\rightarrow T_1(T[a_m...a_1]|b_{n...}b_2|d_{n...}d_1)\]

b. Functor GTRC has an outer sequence \textit{and} \(\mid \text{input} \mid = k\) (and \(k \geq 1\)):
\[T_1(T[a_m...a_1]|b_{n...}b_2/b_{1}) \triangleright \bigcup(U|c_{p...}c_1)|d_{k...}d_1\]
\[\rightarrow T_1(T[a_m...a_1]|b_{n...}b_2|d_{k...}d_1) \quad \text{unbounded}\]

c. Functor GTRC has an outer sequence \textit{and} \(\mid \text{input} \mid < k\) (and \(k \geq 2\)):
\[T_1(T[a_m...a_1]|b_{n...}b_2/b_{1}) \triangleright \bigcup[U|c_{p...}c_1]|d_{n...}d_{k+1}|d_{k...}d_1\]
\[\rightarrow T_1(T[a_m...a_1]|b_{n...}b_2|U_1|d_{n...}d_{k+1}|d_{k...}d_1) \quad \text{residual} \quad \text{unbounded}\]

d. The functor GTRC has no outer sequence \textit{and} \(\mid \text{input} \mid > k\):
   (i) \(T\) spans greater than \(U\) (\(T = U([U|c_{p...}c_1]|d_{n...}d_{k+m+1})\)):\[15\]
\[\frac{T/(T[a_m...a_2\backslash a_1]) \triangleright^k \bigcup(U|c_{p...}c_1)|d_{n...}d_{k+m+1}|d_{k...}d_1}{\frac{T}{\bigcup(U|c_{p...}c_1)|d_{n...}d_{k+m+1}|d_{k...}d_1}}\]
\[\rightarrow \bigcup(U|c_{p...}c_1)|d_{n...}d_{k+m+1}|d_{k...}d_1 \quad \text{inner seq of GTRC}\]
Example: \(T/(T\backslash NP) \triangleright U/(U\backslash PP)\;\backslash NP \rightarrow U/(U\backslash PP)\)

(ii) \(T\) spans no greater than \(U\) (\(T[a_m...a_{m-j+1}] = U\)):
\[\frac{T/(T[a_m...a_{m-j}...a_2\backslash a_1]) \triangleright^k \bigcup[a_{m-j}...a_{m-j+1}][U_0|U_j][U_0|[U_j][U_1|c_{p...}c_1]|d_{n...}d_{k+1}|d_{k...}d_1}{\frac{T}{[a_m...a_{m-j}...a_{m-j+1}][U_0|[U_j][U_1|c_{p...}c_1]|d_{n...}d_{k+1}|d_{k...}d_1}}\]
\[\rightarrow F[a_{m-j}...a_{m-j+1}][U_0|[U_j][U_1|c_{p...}c_1]|d_{n...}d_{k+1}|d_{k...}d_1] \;\text{where} \; q \geq j + p\]

\textsuperscript{14}T could also be decomposed into \(T_0_1[T_1]\) for a larger \(k\) but all of them share the same characteristics with the above scheme.

\textsuperscript{15}We only consider the most general unifier.
We define the most general form of CCG-GTRC as follows:\footnote{For the moment, we use the term ‘CCG-GTRC’ ambiguously between for the class of grammars and for a grammar instance. This situation will be rectified shortly by using different style, $\mathcal{G}$ and $G$, for a class and a member where the distinction is necessary.}

**Definition 1** A CCG-GTRC$_0$ is a five tuple $(V_N, V_T, S, f, R)$ where

- $V_N$ is a finite set of nonterminals (atomic categories)
- $V_T$ is a finite set of terminals (lexical items, written as $a, \ldots, z$)

\begin{center}
<table>
<thead>
<tr>
<th>Case</th>
<th>Functor cat</th>
<th>Input cat</th>
<th>Result cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Const</td>
<td>Const</td>
<td>Const</td>
</tr>
<tr>
<td>10a</td>
<td>GTRC</td>
<td>Const</td>
<td>GTRC</td>
</tr>
<tr>
<td>10b</td>
<td>GTRC</td>
<td>Const</td>
<td>GTRC</td>
</tr>
<tr>
<td>11a</td>
<td>Const</td>
<td>GTRC</td>
<td>const</td>
</tr>
<tr>
<td>11b</td>
<td>Const</td>
<td>GTRC</td>
<td>Const</td>
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<tr>
<td>11c</td>
<td>Const</td>
<td>GTRC</td>
<td>Const</td>
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<td>12a</td>
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<td>12b</td>
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<tr>
<td>12f</td>
<td>GTRC</td>
<td>GTRC</td>
<td>GTRC</td>
</tr>
</tbody>
</table>

Table 1: **Combinatory Cases for CCG-GTRC**

Example: $T/(T\backslash(S/\text{NP})) \mathbin{\blacktriangleleft} U\backslash(U/\text{NP}) \rightarrow S$

Example: $T/(T\backslash\text{NP}) \mathbin{\blacktriangleleft} U\backslash(U/\text{NP}) \rightarrow T/(T\backslash\text{NP}\backslash\text{NP})$

The three cases indicated by ‘*’ in Table 1 introduce categories which are neither Const nor GTRC due to the residual variables. This is an unintended, accidental use of functional composition. The closure of the system must be maintained by excluding these cases by the following condition:

$$(13) \textbf{Closure Condition:} \text{ The rule } \alpha \mathbin{\blacktriangleleft} \gamma \text{ must satisfy } |\gamma| \geq k.$$  

Note that the distinction between constant categories and GTRCs must be made. This condition is particularly important for implementation since the residual variables can behave beyond our imagination and the parser must be able to compute the length of a category distinctively for constant categories and GTRCs.

### 2.2 CCG-GTRC

We define the most general form of CCG-GTRC$_0$ as follows:\footnote{For the moment, we use the term ‘CCG-GTRC’ ambiguously between for the class of grammars and for a grammar instance. This situation will be rectified shortly by using different style, $\mathcal{G}$ and $G$, for a class and a member where the distinction is necessary.}

**Definition 1** A CCG-GTRC$_0$ is a five tuple $(V_N, V_T, S, f, R)$ where

- $V_N$ is a finite set of nonterminals (atomic categories)
- $V_T$ is a finite set of terminals (lexical items, written as $a, \ldots, z$)
• $S$ is a distinguished member of $V_N$

• $T$ is a countable set of variables\textsuperscript{17}

• $f$ is a function that maps elements of $V_T$ to finite subsets of “Const ∪ LTRC”\textsuperscript{18}

• $R$ is a finite set of rule instances of Generalized Functional Composition observing Closure Condition (i.e., those summarized in Table 1 except for those with ‘*’).\textsuperscript{19}

CCG-GTRC\textsubscript{0} differs from CCG-Std in some crucial respects.

(14) a. The set of arguments is not bounded. Not only the inner sequence of GTRC is unbounded, but also an argument of a constant category can be unboundedly long.

b. Combinatory rules cannot be specified in a ‘finite’ manner as described in [Vijay-Shanker and Weir, 1994].\textsuperscript{20}

The reason is that both functor and input categories can be unboundedly long unlike CCG-Std.

From both complexity and parsing points of view, this situation seems to require more ‘power’ to deal with. The conjecture is that this grammar is not equivalent to CCG-Std nor polynomially parsable. What I will do in the following is to find a subclass of CCG-GTRC\textsubscript{0} which still satisfies the original motivation and can be proved weakly-equivalent to CCG-Std. We discuss the following three problems in turn: (i) the bound of the arguments of constant categories, (ii) mixed directionality in GTRC inner sequence, and (iii) the behavior of GTRC outer sequences.

First, we want to apply the same techniques of CCG-Std to the “Const\textsuperscript{\textcircled{1}}Const” case. For this purpose, the set of arguments must be bounded [Vijay-Shanker and Weir, 1994, Vijay-Shanker and Weir, 1990]. Thus we place a bound on the length of an argument.\textsuperscript{21}

(15) Bounded Argument Condition: Every argument except for the inner sequence of GTRC must be bounded by the grammar.\textsuperscript{22}

Then the rules indicated as ‘unbounded argument’ in Table 1 must be restricted to those satisfying the condition while the inner sequence of GTRCs can grow without limit. We now have the following property:

(16) The set of arguments of a constant category and the set of arguments of the inner and outer sequences of a GTRC are all finite. We denote the set of all these arguments as $\text{Args}$.

Another option to the Bounded Argument Condition is to place a bound on the length of GTRC inner sequence. But then we need to re-evaluate our assumption about the unbounded NP sequence and the system degenerates to CCG-Std since every instance of GTRC can be represented as a constant.

The new subclass of CCG-GTRC\textsubscript{0} is defined as follows:

Definition 2 CCG-GTRC\textsubscript{bound-arg} is a subclass of CCG-GTRC where the Bounded Argument Condition is observed.\textsuperscript{23}

The second problem is with the mixed directionality of the GTRC. For example, consider a GTRC $T / (T/a_m.../a_2\backslash a_1)$ derived from “$T / (T \setminus a_1) \leftrightarrow (T \setminus T / a_2) \leftrightarrow ... \leftrightarrow (T \setminus T / a_m)$ “. This may proceed with the following derivation:

“$T / (T / a_m.../a_2\backslash a_1) \rightarrow c / a_m.../a_2\backslash a_1 | d_k...| d_1$”. Although the input category, $c / a_m.../a_2\backslash a_1 | d_k...| d_1$, seeks the arguments $a_2,...,a_m$ to its right, the arguments are actually found on the left of the category. In addition, although the GTRC

\textsuperscript{17}Each instance of GTRC must be assigned a new variable when the GTRC is instantiated at a particular string position in order to avoid unintended variable binding.

\textsuperscript{18}Our definition does not include the empty string in the domain of $f$ as in [Vijay-Shanker and Weir, 1993] but unlike [Vijay-Shanker and Weir, 1994].

\textsuperscript{19}Due to the introduction of GTRC, the rule instances may involve variables even at the first argument of the functor category and at the input category.

\textsuperscript{20}This ‘finiteness’ corresponds to the instantiation of the input categories. The functor category of a combinatory rule still needs a meta-variable since categories can grow without limit.

\textsuperscript{21}Alternatively, we can limit the instances of argument to those which can occur at a ‘bounded’ argument position.

\textsuperscript{22}Formally, for the rule “$x \triangleright y$” where (i) $x$ is not a GTRC with $|x| = 1$ and (ii) $|y| = k, y$ must be instantiated. This condition is weaker than the one stated in [Komagata, 1997c] in that it does not completely exclude the rule application potentially involving unbounded argument. This is preferable because if we prohibit, say, the rule 118 altogether, the following adverbial modification is impossible: $S \rightarrow \cdot | T / (NP,NP) \rightarrow S / (S,NP,NP)$.

\textsuperscript{23}The polynomial recognition algorithm works for this general class [Komagata, 1997a, Komagata, 1997b].
The inner sequence of a GTRC must have the uniform directionality as in:
\[ T / (T / a_m) \] 24

This condition is closely related to the linguistic aspect of long-distance ‘movement’ across the functor. Our motivation does not depend on these phenomena. For example, the gapping conjuncts of two underlined NPs in English, “John helped Mary, Bill, Rose,” might involve \( S / (S \backslash NP / NP) \) from \( S / (S \backslash NP) \backslash (S \backslash NP) / (NP) \)” [Steedman, 1990].

Definition 3 CCG-GTRC\(_{uni}\) is a subclass of CCG-GTRC\(_{bound\_arg}\) where the Unidirectional GTRC Condition is observed.

The third problem is related to ‘quasi-island’ condition exemplified as follows:

(a) CCG-GTRC: \( S / A \uparrow (T / A) / B \downarrow B \to S \)

(b) CCG-GTRC: \( B \downarrow S / A \uparrow T / (T / A) \backslash B \to * \)

(c) CCG-Std: \( S / A \uparrow (A / B) \downarrow B \to S \)

(d) CCG-Std: \( B \uparrow (S / A) / A / B \to S \)

With respect to the interaction with input categories of constant class, GTRCs behave like an island. But we do not have a general way in CCG-Std proper to exactly capture the effect. Our next step is to exclude outer sequence from the GTRCs altogether.

Definition 4 CCG-GTRC\(_{no\_outer}\) is a subclass of CCG-GTRC\(_{uni}\) where no GTRC has outer sequence.

This limits the instances of GTRCs to a finite set since the inner argument of a GTRC is ‘frozen’. It can only act as its own. Although the expressiveness is greatly limited, it can still represent the example we started with in addition to the coverage of CCG-Std.

These conditions may appear unnatural. But note that they are applied when the grammar is constructed and do not change the way the grammar is used to recognize a string in CCG-GTRC. Thus they are legitimate way to ‘define’ subclasses of grammar. In the rest of this paper, we focus on CCG-GTRC\(_{no\_outer}\) and prove its weak equivalence to CCG-Std. The only relevant cases are now (9), (10b), (11a,b), and (12dii,e) where no outer sequence of GTRC is present.

3 Equivalence of CCG-GTRC and CCG-Std

This section presents the proof of the equivalence of CCG-Std and CCG-GTRC\(_{no\_outer}\) (CCG-GTRC hereafter). Let \( G_{std} \) and \( G_{gtrc} \) be the classes of CCG-Std and CCG-GTRC, respectively. A grammar is represented by \( G_{index} \) where the subscript is optionally used to distinguish grammars. The proposition to prove is the following:

Formally, for “GTRC\(_1\) /\(\uparrow\)GTRC\(_2\)” where \( |GTRC_1| = 1, |GTRC_2| = k \), the directionality of the inner sequences must be identical. This condition was called “Order-Preserving Condition” in [Komagata, 1997c]. I also stated that it is derivable from the principles in [Steedman, 1991b] but it is not correct.

It is not clear to me why is \( GTRC_1 \) /\(\uparrow\)GTRC\(_2\) allowed in English in this case.

A weaker restriction such as the following may seem possible: the directionality of the outer sequence must equal those of the inner sequence. But we do not have a general method of achieving this effect either.

The case (12dii) may result in decomposition of the inner argument in a restricted way. This will be treated as Bounded GTRC in a later section.
Proposition 1 \( G_{gtrc} \) is weakly equivalent to \( G_{std} \).

Since any \( G_{gtrc} \in G_{std} \) is also \( G_{gtrc} \) by definition, we only need to show that for each \( G_{gtrc} \in G_{gtrc} \), there is a \( G_{std} \in G_{std} \) such that \( G_{gtrc} \) and \( G_{std} \) generate the same language, i.e., \( L(G_{gtrc}) = L(G_{std}) \). The proof is by the following lemma with the start category set to \( S \).

Lemma 1 (Main Lemma) For any \( G_{gtrc} \in G_{gtrc} \), there is a \( G_{std-sim} \in G_{std} \) such that a terminal string \( w \) is generated by a constant category \( c \) in \( G_{gtrc} \) iff \( w \) is generated by \( c \) in \( G_{std-sim} \) corresponding to \( c \) in \( G_{gtrc} \).

We will construct \( G_{std-sim} \) from \( G_{gtrc} \) so that \( G_{std-sim} \) simulates \( G_{gtrc} \).\(^{28}\) The process starts by translating \( G_{gtrc} \) to the base CCG-Std, \( G_{std-base} \) as follows:

\( G_{gtrc} \) \( \rightarrow \) \( G_{std-base} \)

(19) a. Copy all the constant categories in \( G_{gtrc} \) to \( G_{std-base} \) assigned to the same terminal symbol.

b. For each LTRC \( T \langle T \rangle a \) in \( G_{gtrc} \), add an atomic category \( \langle a \rangle \) to the lexicon of \( G_{std-base} \) assigned to the same terminal symbol.\(^{29}\) Note that the use of atomic category is to avoid decomposition of the inner argument. This is possible since the inner arguments of GTRC never unifies with a target category and are never decomposed in the current formulation.\(^{30}\)

Then \( G_{std-base} \) is extended to \( G_{std-sim} \) to simulate \( G_{gtrc} \). This situation is shown schematically as follows:

\( G_{gtrc} \) \( \rightarrow \) \( G_{std-base} \) \( \downarrow \) \( G_{std-sim} \)

Since CCG-GTRC extends the way CCGs captures phenomena including unbounded, but restricted ‘permutation’, it is crucial to identify the properties of GTRCs and provide appropriate methods for simulation. Once we have the right simulation, the equivalence can be shown by the set inclusion for both directions by invoking the simulation as needed. Two simulation techniques, ‘wrapping’ and ‘bounded GTRC’, and the proof of both directions will be described in the following subsections.

3.1 Wrapping

CCG-GTRC allows permutation as observed in the following example:

\( a \)
\( \begin{array}{c}
T/(T\backslash A) \\
T/(T\backslash B) \\
S\backslash A\backslash B \\
S\backslash A \\
S \\
S
\end{array} \\
\begin{array}{c}
T/(T\backslash B) \\
T/(T\backslash A) \\
S\backslash A\backslash B \\
S\backslash B \\
S
\end{array} \\
\begin{array}{c}
(\text{permutation})
\end{array}
\)

First, we attempt to simulate such a permutation by wrapping the arguments of a lexical category [cf. Bach, 1979, Dowty, 1979]. For example, \( \backslash A \) in \( S\backslash A\backslash B \) can wrap across \( \backslash B \) with the result of \( S\backslash B \). We use ‘\( \backslash \)’ to represent the wrapped argument as an atomic category which will unify with the GTRC-translated category also represented in the same way. This corresponds to the permutation of (21b) as follows:

\(^{28}\)The word ‘simulation’ is also used to describe operations involved in the process.
\(^{29}\)The directionality of LTRC can be captured by features such as ‘\( \leftarrow \) left’ or ‘\( \rightarrow \) right’. We will ignore this aspect for simplicity.
\(^{30}\)This depends on the ‘no-outer sequence’ condition.
The above-mentioned technique of wrapping arguments only applies to local permutation within a lexical category. But CCG-GTRC allows permutations across lexical categories as seen below.

\[
\begin{align*}
(T \backslash B) & \rightarrow T / (T \backslash A) \rightarrow S \backslash A \backslash B \rightarrow S \backslash B \\
\end{align*}
\]

\[
\begin{align*}
S & \rightarrow \langle A \rangle \rightarrow S \backslash B \backslash \langle A \rangle \rightarrow S \backslash B \\
\end{align*}
\]

Since we assume that GTRCs can compose without limit, there is no bound on the composition of the input to GTRCs.

\[
\begin{align*}
T / (T \backslash A) \rightarrow T \backslash B \rightarrow S \backslash A \backslash T \rightarrow S \backslash A \backslash B \rightarrow S \backslash B \\
\end{align*}
\]

Then, we want to obtain a wrapped category like \( S \backslash A_1 \langle A \rangle \ldots \langle A_2 \rangle \). This situation can be captured by using the technique of argument passing as follows:

\[
\begin{align*}
S \backslash B \backslash \langle A \rangle \rightarrow T \backslash B \rightarrow S \backslash B \backslash \langle A \rangle \rightarrow T \backslash B \\
\end{align*}
\]

Note: Since subscripts are frequently used for indexing the categories in this paper, the features are placed as superscript.

\[
\begin{align*}
S \backslash A_1 \langle A \rangle \ldots \langle A_2 \rangle & \rightarrow \\
& \left( T \backslash A_1 \rightarrow \left( T \backslash A_2 \rightarrow \left( \ldots \rightarrow \left( T \backslash A_1 \langle A \rangle \ldots \langle A_2 \rangle \rightarrow \left( S \backslash A_1 \langle A \rangle \ldots \langle A_2 \rangle \right) \right) \right) \right) \right)
\end{align*}
\]

The arguments which are crossed by wrapping are placed as a feature on the target category and on the first argument. They are then passed on to the category corresponding to a deeper position of the composed category. As in the case of \( \langle \rangle \), we consider the category with passed arguments as an atomic category. This also applies for the case where the canceled category is complex such as: \( (S/A)^{\backslash C} \).\end{enumerate}

This simulation depends on the fact that the list of passed arguments is bounded. First, observe (a) below. A particular argument can be crossed by any number of arguments by wrapping, which is the source of unbounded permutation. On the other hand, an argument can cross only finite number of arguments by wrapping as seen in (b).

This latter case is bounded by \( k_{max} \) of functional composition.

\[
\begin{align*}
B & \rightarrow T / (T \backslash A_n) \rightarrow (T \backslash A_1) \rightarrow S \backslash A_n \ldots \backslash A_1 \rightarrow B \\
B & \rightarrow \langle A_n \rangle \rightarrow \langle A_1 \rangle \rightarrow S \rightarrow B \langle A_n \rangle \ldots \langle A_1 \rangle \\
\end{align*}
\]

\[
\begin{align*}
T / (T \backslash A) & \rightarrow S \backslash A \rightarrow B_1 \ldots B_1 \rightarrow \langle A \rangle \\
\end{align*}
\]

Recall that the set of arguments \( \text{Args} \) is bounded. Thus at any juncture of rule application, there are only finitely many possibility of argument passing. We add all these cases to the lexicon.

\[
31 \text{Argument passing is conceptually similar to the techniques found in grammar formalism and logic including: SLASH feature of GPSG/HPSG [Gazdar et al., 1985, Pollard and Sag, 1994] and assume/discharge of natural deduction [Hepple, 1990]. But it is finite and limited in its power.}
\]
To describe wrapping concisely, we introduce the following notation: Depending on how we divide a category into the ‘function’ and the ‘arguments’, a category \( c = F[a_m...a_1] \) can be viewed with different valencies, i.e., \( c = f_i^{a_m...a_1} \). Let us refer to \( f_i \) as the functional forms of \( c \). The functional forms with every valency can then be represented as follows: \( c = F[a_m...a_1] = f_i a_i \) where \( 0 \leq i \leq m \) and \( a_i = [a_i,...,a_1] \).

The process of wrapping is now presented as follows:

(27) Wrapping: Consider functional forms of a lexical category \( c = F[a_m...a_1] = f_i a_i \) where \( a_i = [a_i,...,a_2] \) and \( \text{’}[\text{’} \rangle \) indicates the optionality. In case \( a_1 \) is not nil, consider all the possible sequence of arguments \( \llbracket [d_k] \rrbracket d_1 \) (as passed arguments) where \( \llbracket [d_k] \rrbracket \leq \kappa_{\text{max}} \). For a concatenation of \( \kappa_{\text{max}} \) (including \( \llbracket [d_k] \rrbracket = 0 \)), apply all the possible wrapping. Note the use of \( \langle a_i \rangle \) to represent the wrapped argument \( a_i \). Optionally, designate the last \( j \leq \kappa_{\text{max}} \) arguments as \( \zeta \), and place them as the feature on \( a_i \). The process can be abbreviated as follows: \( f_i a_i \Rightarrow f_i^{[\zeta]} a_i \) where \( a_i \) is obtained by wrapping as described above and the both categories are assigned to the same terminal. Categories with passed arguments are considered as atomic categories.

Categories including a wrapped argument and/or a passed argument, do not interact with constant category until these features are canceled.\(^{32}\) For example, the following unintended cases all fail.

(28) a. \( D \downarrow S(B) \downarrow (D) \rightarrow * \)
   b. \( C(/(S/B/A)) \downarrow S(C \langle A \rangle) \rightarrow * \)
   c. \( S(B) \downarrow S(B) \downarrow S(B) \rightarrow * \)

The use of \( \langle \rangle \) avoids overgeneration of the following kind as well:

(29) a. \( S(C) \langle A \rangle \downarrow \langle A/B\rangle \uparrow B \rightarrow S/C \) (potential overgeneration)
   b. \( S(C) \langle A \rangle \downarrow \langle A/B\rangle \uparrow B \rightarrow * \) (implemented)

### 3.2 Bounded GTRC

When GTRCs appear as input category, their instances are bounded (16). Thus we can replace the variables with constants. For example, suppose that coordination is lexical, defined for each instance of conjunct category, and the set of conjuncts is bounded. Coordination of non-traditional constituents might need the conjunctive category like \( S(S\langle NP \rangle N) \rightarrow (S/S\langle NP \rangle N) \). Then we can derive \( S/S\langle NP \rangle N \) as “\( S/S\langle NP \rangle \uparrow N ) \). Both of the instances must be added to the lexicon since \( G_{\text{std-sim}} \) has no other way to represent this non-traditional constituency. Since we are motivated to deal with unboundedly-long inner sequence of GTRC, we cannot apply this technique to (12e). Wrapping has been introduced for this purpose. The procedure of adding GTRC instances is described as follows:

(30) Bounded GTRC:

11a): Suppose that the whole GTRC \( \tau \langle \langle T \rangle a_m...a_1 \rangle \) unifies with some argument of a category, i.e., a member of the set of arguments Arg in \( G_{\text{str}} \). The GTRC must be derived uniquely from a sequence of LTRCs, \( T_m \langle T_m \langle a_m \rangle \rangle, ..., T_1 \langle T_1 \langle a_1 \rangle \rangle, ..., T_m \langle T_m \langle a_m \rangle \rangle \), depending on the directionality (cf. Lemma 3).\(^{33}\) Add the ground instances of the LTRCs to the lexicon of \( G_{\text{std-sim}} \).

11b): Since we have set a bound on the instances on \( b \langle c_m...c_1 \rangle \), add the LTRCs which derives \( b \langle c_m...c_1 \rangle \).

12dii): The only possibility is the following: “\( \tau / (T \downarrow a) \rightarrow UNION (U \llbracket U \llbracket c_m...c_1 \rrbracket \rrbracket \rightarrow F[a_m...a_{m-\rho}] d_k...d_1 \)”. The function category must be an LTRC and the instances of \( a \) is bounded. We add those instances in the lexicon.

\(^{32}\) Although an additional feature was introduced to control potential overgeneration in [Komagata, 1997c], the current formulation with \( (\angle) \) avoids the feature and simplifies the analysis.

\(^{33}\) This can be proved by induction on the length of the inner sequence.
3.3 Proof: \( L(G_{gtrc}) \subseteq L(G_{std-sim}) \)

Now the simulation is established for the given CCG-GTRC. The proof of the direction from \( G_{gtrc} \) to \( G_{std-sim} \) is by induction on the height \( h \) of a derivation in \( G_{gtrc} \). The primary recursion (for both directions) deals only with constant categories (of CCG-GTRC) since we are concerned with derivations of a constant category, \( S \) in particular. The current direction also involves GTRCs as the source derivation and these are handled by Lemma 3 and wrapping handled by Lemma 4 introduced below. The latter lemma sets a mutually-recursive situation with this direction of the main lemma (Lemma 2).

Lemma 2 The direction \( L(G_{gtrc}) \subseteq L(G_{std-sim}) \) of the Main Lemma.

Base case \((h = 0)\): \( c \) is a lexical category. Then \( c \) is also in \( G_{std-sim} \) assigned to the same terminal symbol.

Induction hypothesis (IH2): The lemma holds for all \( h' \leq h - 1 \).

Induction step \((h \geq 1)\): We only consider the following relevant cases where the result category is Const.

(31)  
\( a. \) (Const\textarrow{\textrightarrow} Const, 9) The same derivation is available in \( G_{std-sim} \). For the left and right categories, which are constant categories of smaller height, apply the induction hypothesis (IH2). The pair of strings obtained by the application of the induction hypothesis in the same order can be concatenated to provide the desired string in \( G_{std-sim} \).

\( b. \) (GTRC\textarrow{\textrightarrow} Const, 10b)
\[
T/(T \backslash a_m \ldots \backslash a_1) \textarrow{\textrightarrow} c_i \, |d_k| \, |d_1| \rightarrow c_0 |d_k| \, |d_1|
\]
This case requires the simulation. Note that \( c \) is unbounded. Lemma 4 provides us the wrapped form \( c_0 |d_k| \, |d_1| \backslash a_m \ldots \backslash a_1 \) from \( c_0 |a_m \ldots |a_1| |d_k| |d_1| \). Lemma 3 shows that there is a sequence of the categories with the corresponding string which can combine with \( c_0 |d_k| \, |d_1| \backslash a_m \ldots \backslash a_1 \) in the same order with the same string. Thus, after applying each category of the sequence to the wrapped category, we have the desired result \( c_0 |d_k| \, |d_1| \) with the same string.

\( c. \) (Const\textarrow{\textrightarrow} GTRC, 11a)  
\[
a/b \textarrow{\textrightarrow} T|T|c_m \ldots |c_1| \rightarrow a
\]
Since the GTRC is bounded, we have the corresponding category in \( G_{std-sim} \) by (30). The rest is similar to the previous case.

\( d. \) (Const\textarrow{\textrightarrow} GTRC, 11b)  
\[
a/b \textarrow{\textrightarrow} T|T|c_m \ldots |c_1| \rightarrow a|b|c_m \ldots |c_1|
\]
Since the GTRC is bounded by the stipulated Bounded Argument Condition, we have the corresponding category in \( G_{std-sim} \) by (30). The rest is similar to (a).

\( e. \) (GTRC\textarrow{\textrightarrow} GTRC, 12dii)  
\[
T/(T \backslash a_i \ldots \backslash a_1) \textarrow{\textrightarrow} U|(U|c_p \ldots |c_1|) \rightarrow a_0
\]
Since \( a \) is bounded, the process is similar to the previous case.

Lemma 3 If the derivation of \( T/(T \backslash a_m \ldots \backslash a_1) \) from the string \( \sigma \) can combine with \( x \backslash a_m \ldots \backslash a_1 \) in \( G_{gtrc} \), \( \langle a \rangle, \ldots , \langle a_1 \rangle \) which is associated with the same terminal string can combines with \( x \backslash y_m \ldots \backslash y_1 \) for some \( x \) in \( G_{std-sim} \) where \( y_1 \) may be \( a \) or \( \langle a \rangle \).

Proof: By induction on the height \( h \) of derivation.\(^{34}\)

Base case \((h = 0)\): The category must be an LTRC, \( T/(T \backslash a) \). Thus there is \( \langle a \rangle \) and \( a \) assigned to the same terminal in \( G_{std-sim} \) by the simulation. Then either \( \langle a \rangle \) or \( a \) can combine with \( x \backslash a \) or \( x \backslash \langle a \rangle \) as desired.

Induction hypothesis: The lemma holds for \( h' \leq h - 1 \).

Induction step \((h \geq 1)\): The GTRC \( T/(T \backslash a_m \ldots \backslash a_1) \) must be derived as “\( T/(T \backslash a_m \ldots \backslash a_{i+1}) \textarrow{\textrightarrow} U|(U|a_i \ldots |a_1|) \rightarrow T/(T \backslash a_m \ldots \backslash a_i) \)” for some \( i \) (12e). Apply the induction hypothesis to the input category. Then we have a sequence of \( \langle a_i \rangle , \ldots , \langle a_1 \rangle \) which generates the same string as \( U|(U|a_i \ldots |a_1|) \). Since each of \( \langle a_i \rangle \) has the corresponding \( a_i \), the

\(^{34}\) Induction on the length of the inner sequence also works for this case.
sequence can apply to $x' \backslash y_1 \ldots \backslash y_1$ in series to derive some $x' = x \backslash y_m \ldots \backslash y_{i+1}$ in $G_{std-sim}$. Next, apply the induction hypothesis to the functor category and $x \backslash y_m \ldots \backslash y_{i+1}$ to obtain $x$ in $G_{std-sim}$ from the same string as desired.

Lemma 4 Consider a category $c[a_m \ldots a_1 | d_k \ldots d_1]e$ derivable in $G_{gtrc}$ where $k \leq k_{\text{max}}$ and $m \geq 0$. If this category combines with a GTRC $T/(T \backslash a_m \ldots \backslash a_1)$ to reduce to $T/(T \backslash a_m \ldots \backslash a_1) \triangleright c[a_m \ldots a_1 | d_k \ldots d_1] \rightarrow c[d_k \ldots d_1] e$, then there is a category $c[d_k \ldots d_1 | y_m \ldots y_1]$ in $G_{std-sim}$ where $y_i$ is either $a_i$ or $\langle a_i \rangle$ which generates the same terminal string as $c[a_m \ldots a_1 | d_k \ldots d_1] e$ in $G_{gtrc}$.

The proof is by the following lemma which is a more general version.

Lemma 5 Consider a category $x := c[a_m \ldots a_1 | d_k \ldots d_1] e$ in $G_{gtrc}$ where $k \geq 1$, $k \leq k_{\text{max}}$, $m \geq 0$, $\langle e \rangle$ may be nil. $c$ and $e$ may be associated with passed arguments as a feature. If $T/(T \backslash a_m \ldots \backslash a_1) \triangleright c[a_m \ldots a_1 | d_k \ldots d_1] e \rightarrow c[d_k \ldots d_1] e$, there is a category $y := c^{(O)} | b_j | b_1 \langle a_m \ldots \langle a_1 \rangle \rangle^{(2)}$ in $G_{std-sim}$ where $j \leq k_{\text{max}}$, $m \geq 0$, $0$ and $\|$ are sequences of arguments shorter than $k_{\text{max}}$ (possibly nil) such that $y$ derives the same terminal string as $x$.

Proof: By induction on the height $h$ of derivation.

Base case ($h = 0$): $c[a_m \ldots a_1 | d_k \ldots d_1] e$ is a lexical category. By (27), there is $c[d_k \ldots d_1 | \langle a_m \ldots \langle a_1 \rangle \rangle] e$ in $G_{std-sim}$ which is associated with the same terminal.

Induction hypothesis: The lemma holds for $h \leq h - 1$.

Induction step ($h \geq 1$): Consider the following cases:

(32) \(a\) Reduction: $c[a_m \ldots a_1 | d_k \ldots d_1] e \leftarrow c[a_m \ldots a_1 | d_k \ldots d_1] f \triangleright f[d_p \ldots d_1] e$

By induction hypothesis, $c[a_m \ldots a_1 | d_k \ldots d_1] f$ has the corresponding

$c[d_k \ldots d_1] \langle a_m \ldots \langle a_1 \rangle \rangle / f[d_p \ldots d_1] e$ which generates the same string, and $f[d_p \ldots d_1] e$ has the corresponding

$f[d_p \ldots d_1] e$ which generates the same string. This case may involve a GTRC as the input category ($p = 0$). But such a case is limited to a bounded form. We can thus consider the bounded instances as if they are constants.

(b) Reduction: $c[a_m \ldots a_1 | d_k \ldots d_1] e \leftarrow c[a_m \ldots a_1] f \triangleright f[a_i \ldots a_1 | d_k \ldots d_1] e$

By induction hypothesis, $c[a_m \ldots a_1] f$ has $c[d_k \ldots d_1] \langle a_m \ldots \langle a_1 \rangle \rangle / f[d_p \ldots d_1] e$, and $f[a_i \ldots a_1] | d_k \ldots d_1] e$ has $f[d_p \ldots d_1] \langle a_i \ldots a_1 \rangle e$.

c. Reduction: $c[a_m \ldots a_1 | d_k \ldots d_1] e \leftarrow c \triangleright f[a_m \ldots a_1 | d_k \ldots d_1] e$

By induction hypothesis, $f[a_m \ldots a_1] | d_k \ldots d_1] e$ has $f[d_k \ldots d_1] \langle a_m \ldots \langle a_1 \rangle \rangle e$. By the induction hypothesis of the main lemma (IH2) there is a constant category which generates the same string as $c/f$.

3.4 Proof: $L(G_{gtrc}) \supseteq L(G_{std-sim})$

We will use the following classification for the categories in $G_{std-sim}$.

(33) \(a\) Const\(_2\): Categories translated from Const of $G_{gtrc}$. Exclusive of the following.

\(b\) GTRC: Categories translated from GTRC of $G_{gtrc}$. Represented as $\langle x \rangle$.

\(c\) Wrap: Categories obtained by Wrapping. They may include wrapped argument represented as $\langle x \rangle$ and/or passed argument $\{p\}$.

\(d\) BGTRC: Categories obtained by Bounded GTRC.

Note that ‘Const\(_2\)’ in this classification stands in relation to ‘Const’ in $G_{gtrc}$ and that all the categories in $G_{std-sim}$ are constant. We will drop the subscript on Const\(_2\) where no confusion arises.

The proof is by induction on the height $h$ of a derivation in $G_{std-sim}$. The primary recursion is on Const and we introduce Lemma 7 to have a mutually-recursive situation on wrapped categories.
Lemma 6  The direction $L(G_{gtrc}) \supseteq L(G_{std-sim})$ of the Main Lemma.

Base case ($h = 0$): By the definition of Const$_2$ above, there is a corresponding constant lexical category with the same terminal string in $G_{gtrc}$.

Induction hypothesis (IH6): The lemma holds for $h' \leq h - 1$.

Induction step ($h \geq 1$): We consider the following cases which result in Const.

(34)  a. Const$\rightarrow$Const: Apply the induction hypothesis (IH6) to the functor and input categories. Then the same strings can be generated from the corresponding categories in $G_{gtrc}$. Since we can apply the same rule in $G_{gtrc}$, we generates the same string from the same category.

b. Const$\rightarrow$BGTRC, BGTRC$\rightarrow$Const, and BGTRC$\rightarrow$BGTRC: By the simulation, any bounded instance of GTRC in $G_{std-sim}$ has the corresponding GTRC in $G_{gtrc}$. Apply IH6 to the Const. Then this case has the corresponding derivation. Note that there is no formal distinction between BGTRC and Const. Thus there may be ambiguous case where a single derivation may need be considered for both cases, where only one of them may apply.

c. Const$\rightarrow$Wrap, Wrap$\rightarrow$Const, Wrap$\rightarrow$BGTRC, BGTRC$\rightarrow$Wrap: These cases do not apply. Regardless of the position of the indication of wrapping (either $\langle x \rangle$ or passed argument), either they fail to unify with the other category or would remain in the result category.

d. GTRC$\rightarrow$<any class>, BGTRC$\rightarrow$GTRC, Const$\rightarrow$GTRC: Not applicable. GTRC-translated category $\langle x \rangle$ can only combine with the identical argument of a wrapped category.

e. Wrap$\rightarrow$Wrap: The only applicable case is the following: “$a/b^{[p]} \rightarrow b^{[p]}c \rightarrow d^n$” (other instances of wrapping are not applicable for the same reason as (3)). Apply Lemma 7 to both categories.

f. Wrap$\rightarrow$GTRC: The rule application takes the form: “$a/\langle b \rangle \rightarrow \langle b \rangle \rightarrow a'$”. By the simulation, the same string can be generated by the corresponding categories in $G_{gtrc}$.

For the case where the result category is Wrap, consider the following lemma.

Lemma 7  For a wrapped category $c$ in $G_{std-sim}$, there is a constant category $c'$ in $G_{gtrc}$ which generates the same terminal string.

Proof: By induction on the height $h$ of derivation.

Base case ($h = 0$): $c$ is a lexical category in $G_{std-sim}$. There must be a category $c'$ in $G_{gtrc}$ by wrapping (27).

Induction hypothesis: The lemma holds for $h' \leq h - 1$.

Induction step ($h \geq 1$):

(35)  a. Wrap$\rightarrow$Wrap: The derivation takes the form: “$f^{(\circ)}A/b^{[p]} \rightarrow b^{[p]}c[d] \rightarrow f^{(\circ)}A/c[d]$”. Either $\circ$ or $\Pi$ is non-nil. Apply the induction hypothesis to both categories. We have the corresponding $f\circ A'/b$ and $b^{[p]}c'[d]$ where $A'$ and $C'$ are the result of removing $P$ and $\Pi$ from $A$ and $C$, respectively. They can derive: “$f\circ A'/b \rightarrow b^{[p]}c'[d] \rightarrow f\circ A'/c'[d] = f\circ A/C'[d]$”.

b. Const$\rightarrow$Wrap, Wrap$\rightarrow$Const: Apply IH6 to Const and the induction hypothesis to Wrap. The rest is similar to the above.

c. No other case can result in Wrap.

4  Example of Simulation

Example 1  English heavy NP-shift
“John gave the book to Mary.”

“[John gave to Mary] the book which ....”

\[ f_{gtre} = \{ (j\text{ohn}.NP), (j\text{ohn},T \langle (T \backslash NP) \rangle ), (\text{the book}.NP), (\text{the book},T \langle (T \backslash NP) \rangle ), (\text{to mary},PP), (\text{to mary},T \langle (T \backslash PP) \rangle ), (\text{gave},S\langle NP \rangle PP\langle NP \rangle ... \} \]

\[ f_{std}^{base} = \{ \text{replace the GTRCs with} (\text{john},\langle NP \rangle ), (\text{the book},\langle NP \rangle ), (\text{to mary},\langle NP \rangle ), \} \]

\[ f_{std}^{sim} = \{ \text{add the following to the above} \}
\{ (\text{gave},S\langle NP \rangle PP\langle NP \rangle ), (\text{gave},S\langle NP \rangle \langle NP \rangle / PP ... \} \]

\[ \text{John gave to Mary} \quad \text{the book which} ... \]
\[ \frac{S/\langle NP \rangle \langle NP \rangle / PP \langle NP \rangle}{S/\langle NP \rangle} \]

\[ \text{Example 2} \quad \text{Japanese long-distance extraction} \]

“Mary-nom John-nom Mary-acc helped-comp thought.”

“Mary-acc [Mary-nom John-nom helped-comp thought].”

\[ f_{gtre} = \{ (\text{john-nom}.NP_{nom}), (\text{mary-nom}.NP_{nom}), (\text{john-acc}.NP_{acc}), (\text{mary-acc}.NP_{acc}), (\text{john-nom},T / (T \backslash NP_{nom})), (\text{mary-nom},T / (T \backslash NP_{nom})), (\text{john-acc},T / (T \backslash NP_{acc})), (\text{mary-acc},T / (T \backslash NP_{acc})), (\text{helped},S\langle NP_{nom} \rangle NP_{acc}), (\text{helped},S\langle NP_{nom} \rangle \langle NP_{acc} \rangle ), (\text{comp},S\langle NP_{nom} \rangle S',) ... \}
\]

\[ f_{std}^{base} = \{ \text{add the following to the above} \}
\{ (\text{john-acc},\langle NP_{nom} \rangle ), (\text{mary-nom},\langle NP_{nom} \rangle ), (\text{mary-acc},\langle NP_{nom} \rangle ), \}
\{ \text{the rest is the same} \}
\]

\[ f_{std}^{sim} = \{ \}
\{ (\text{thought},S\langle NP_{nom} \rangle NP_{acc}), (\text{thought},S\langle NP_{nom} \rangle \langle NP_{acc} \rangle ) ... \}
\]

\[ \text{Mary-acc} \quad \text{Mary-nom} \quad \text{John-nom} \quad \text{helped} \quad \text{thought} \quad \text{-comp} \]
\[ \frac{S\langle NP_{acc} \rangle \langle NP_{nom} \rangle / \langle NP_{acc} \rangle}{S\langle NP_{nom} \rangle \langle NP_{acc} \rangle \langle NP_{nom} \rangle \langle NP_{acc} \rangle} \]

\[ \frac{S\langle NP_{acc} \rangle \langle NP_{nom} \rangle / \langle NP_{nom} \rangle}{S\langle NP_{nom} \rangle \langle NP_{acc} \rangle \langle NP_{nom} \rangle} \]

\[ \text{5 Conclusion} \]

Motivated by unbounded composition of type-raised categories, we have introduced CCGs with Generalized Type-Raised Categories involving variables as an extension to the standard CCGs. Through the investigation of CCG-GTRC in detail, a subclass of CCG-GTRC is shown to be equivalent to CCG-Std. This is done by way of simulating unbounded, but restricted ‘permutations’ of CCG-GTRC by lexical wrapping and argument-passing across categories. This contrasts with the formalisms involving ‘doubly’-unbounded scrambling, which are strictly more powerful than CCG-Std. Thus CCG-GTRC can be used in place of CCG-Std to account for non-traditional constituents including the ones shown in the introduction without proliferation of type-raised categories with the same computational properties.

The most restrictive condition for the choice of the studied subclass seems to be ‘no outer sequence’. This is also associated with the limitation that the instances of GTRCs are finite. Naturally, we want T / (T \backslash NP) / NP for English prepositions and T / (T \backslash NP) \ NP for Japanese particles and to derive categories freely. Inclusion of outer
sequence seems to increase the power since that class cannot be simulated by CCG-Std due to the fact that CCG-Std cannot simulate certain `island'–like behavior of GTRCs. But in practice, CCG-Std calls for additional mechanism such as conditions on rule application for various linguistic reasons. These conditions cannot be in general expressed in CCG-Std proper either. We are thus at the borderline of mildly context sensitivity. To find out where exactly we are is another question we want to ask.

The subclass with outer sequence but limited to unidirectional GTRCs is used as the base for an implementation of a parser which is capable of analyzing non-traditional constituents in a practical way.

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