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Flat 3-brane with Tension in Cascading Gravity

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Flat 3-brane with Tension in Cascading Gravity

Abstract
In the cascading gravity brane-world scenario, our 3-brane lies within a succession of lower codimension branes, each with their own induced gravity term, embedded into each other in a higher dimensional space-time. In the (6 + 1)-dimensional version of this scenario, we show that a 3-brane with tension remains flat, at least for sufficiently small tension that the weak-field approximation is valid. The bulk solution is singular nowhere and remains in the perturbative regime everywhere.

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Comments

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An old idea to address the vexing problem of the cosmological constant is to confine the visible Universe on a 3-brane in a higher-dimensional space-time: vacuum energy on the brane curves the extra dimensions, but leaves the 4d geometry flat [1]. While tantalizing, this proposal fails as soon as the extra dimensions are compactified; since 4d general relativity is recovered below the compactification scale, the theory unavoidably succumbs to Weinberg’s no-go theorem [2]. (An alternative strategy is to use compact extra dimensions to suppress radiative corrections to the cosmological constant [3].)

The situation is drastically different, and far more promising, if the extra dimensions have infinite volume [4]. In this case, gravity is approximately 4d only at short distances, thanks to an Einstein-Hilbert term on the brane, but becomes higher dimensional in the infrared. In the Dvali-Gabadadze-Porrati (DGP) scenario [5] with one extra dimension, the gravitational force law on the brane scales as the usual 1/r^2 at short distances, but the asymptotes scale as 1/r^3 at large distances. Gravity therefore behaves as a high-pass filter [6]. This weakening of gravity suggests that vacuum energy, by virtue of being the longest-wavelength source, only appears small because it is degravitated [6,7].

The degravitation phenomenon is not realized in the original DGP model because the weakening of the force law is too shallow in the infrared [7]. This motivates exploring higher-codimension branes, i.e., a higher-dimensional bulk. Realizing these higher-codimension scenarios has proven difficult. To begin with, the simplest constructions are plagued by ghost instabilities [8,9]. Second, the 4d propagator is divergent and must be regularized [10]. Furthermore, for a static bulk, the geometry for codimension N > 2 has a naked singularity at finite distance from the brane, for arbitrarily small tension [4]. (Allowing the brane to inflate gives a Hubble rate on the brane inversely proportional to the brane tension for codimension N > 2 [4].)

Recently, it was argued that these pathologies are resolved by embedding our 3-brane within a succession of higher-dimensional branes, each with their own induced gravity term, embedded into each other in a higher-dimensional space-time. In the (6 + 1)-dimensional version of this scenario, we show that a 3-brane with tension remains flat, at least for sufficiently small tension that the weak-field approximation is valid. The bulk solution is singular nowhere and remains in the perturbative regime everywhere.
Unlike the case of a pure codimension-3 DGP brane in \((6+1)\) dimensions, where the static bulk geometry has a naked singularity for arbitrarily small tension \([4]\), here the bulk metric is completely smooth. This traces back to the cascading mechanism of regulating the propagator: the presence of parent branes removes the power-law divergence in the 4\(d\) propagator.

As illustrated in Fig. 1, the 3 extra spatial dimensions are denoted by \(y, z,\) and \(w,\) with the codimension-1 brane located at \(w = 0\), the codimension-2 brane at \(z = w = 0\), and the codimension-3 brane at \(y = z = w = 0\). Indices in 7\(d\) are denoted by \(A, B, \ldots\), indices in 6\(d\) by \(a, b, \ldots\), indices in 5\(d\) by \(\alpha, \beta, \ldots\), and finally indices in 4\(d\) by \(\mu, \nu, \ldots\)

I. Scalar Green’s Functions.—In solving for the metric perturbations, it is useful to first consider the scalar Green’s functions, determined from the action

\[
S = \frac{1}{2} \int d^d x \Psi [M_5^3 \square_y + \delta(w)M_6^4 \square_z + \delta^3(y, z, w)M_3^2 \square_z + \delta^3(y, z, w)M_4^2 \square_z] \Psi,
\]

where \(M_d\) denotes the “Planck” mass in \(d\) dimensions. The model has three cross-over scales:

\[
m_5 = \frac{M_5^3}{M_4^3}, \quad m_6 = \frac{M_6^4}{M_5^4}, \quad \text{and} \quad m_\gamma = \frac{M_3^2}{M_6^2}.
\]

marking, respectively, the transition scale from 4\(d\) to 5\(d\), from 5\(d\) to 6\(d\), and finally from 6\(d\) to 7\(d\).

In the absence of the 5\(d\) and 4\(d\) kinetic terms, the propagator on the codimension-1 brane is of the DGP form \([5]\)

\[
G_6^{(0)}(z, z') = \frac{1}{M_6^3} \int \frac{d\omega}{2\pi} \frac{e^{i\omega(z-z')}}{\omega^2 + q^2 + m_\gamma \sqrt{q^2 + \omega^2}},
\]

where \(q\) is the 5\(d\) momentum, and \(\omega\) is the momentum associated with the \(z\) coordinate. The exact 6\(d\) propagator is then obtained by treating the 5\(d\) kinetic term as a perturbation localized at \(z = 0\):

\[
G_6(z, z') = \frac{G_6^{(0)}(z, z') - M_\gamma^3 G_6^{(0)}(z) q^2 G_6^{(0)}(-z')}{1 + M_\gamma^2 q^2 G_6^{(0)}(0)} + \ldots
\]

In particular the induced propagator on the codimension-2 brane is determined in terms of the integral of the higher-dimensional Green’s function:

\[
G_5^{(0)}(q^2) = G_6(0, 0) = \frac{1}{M_5^3} \frac{1}{q^2 + g(q^2)}.
\]
This equation is of the cascading form [12], as reviewed above. The asymptotically flat bulk solution is given by

$$\Phi(y, z, w) = e^{-|w|/\sqrt{\epsilon}} \Phi_0(y, z),$$

where the induced profile $\Phi_0(y, z)$ satisfies

$$\left( \Box_0 - m_7\sqrt{-\Box_0} - \frac{3}{5} \frac{\delta(z)}{m_6} \Box_5 \right) \Phi_0 = \frac{8}{5} \frac{\delta^2(y, z)}{M_5^3} \Lambda.$$  \hspace{1cm} (9)

To solve (11), we Fourier transform to momentum space and use the 6d and 5d Green’s functions given, respectively, by (3) and (5). The result is

$$\Phi_0(y, z) = \int \frac{dq_z dq_y}{(2\pi)^3} \frac{e^{i q_z y} g(q_y)}{\omega^2 + q_y^2 + m_7 \sqrt{\omega^2 + q_y^2}} \frac{1}{2} \frac{\eta y^2}{\delta^2(q_y)}.$$  \hspace{1cm} (12)

where the Fourier transform of the codimension-2 profile, $\phi(q_y) = \int dy e^{-i q_y y} \Phi_0(z = 0, y)$, satisfies

$$\left( \frac{3}{5} q_y^2 - g(q_y^2) \right) \phi(q_y) = \frac{8}{5 M_5^3} \Lambda.$$  \hspace{1cm} (13)

The solution to (13) can be expressed as the sum of a principal part $\mathcal{P}$ and two homogeneous modes:

$$\phi(q_y) = \frac{8\Lambda}{5 M_5^3} \mathcal{P} \left[ \left( \frac{1}{2} q_y^2 - g(q_y^2) \right) \right] + \sum_{\sigma=\pm} C_{\sigma} \delta(q_y - \sigma q_0),$$

where $\frac{1}{2} q_0^2 = g(q_0^2)$. Requiring the field $\Phi_0$ to be real imposes $C_+ = C_- = C$, while requiring $\Phi_0$ to fall as $y \to 0$ sets $C = 0$. Using the resulting expression for $\phi(q_y)$ into (12) and then into (10), we obtain the final expression for the scalar potential $\Phi(y, z, w)$:

$$\Phi = \int \frac{dq_z dq_y}{(2\pi)^3} \frac{e^{-|w|/\sqrt{\epsilon}} e^{i q_y z} \eta_{y z} g(q_y)}{\omega^2 + q_y^2 + m_7 \sqrt{\omega^2 + q_y^2}} \mathcal{P} \left[ \frac{1}{2} q_y^2 - g(q_y^2) \right].$$

$$\Phi = \int \frac{dq_z dq_y}{(2\pi)^3} \frac{e^{-|w|/\sqrt{\epsilon}} e^{i q_y z} \eta_{y z} g(q_y)}{\omega^2 + q_y^2 + m_7 \sqrt{\omega^2 + q_y^2}} \mathcal{P} \left[ \frac{1}{2} q_y^2 - g(q_y^2) \right].$$

$$\Phi = \int \frac{dq_z dq_y}{(2\pi)^3} \frac{e^{-|w|/\sqrt{\epsilon}} e^{i q_y z} \eta_{y z} g(q_y)}{\omega^2 + q_y^2 + m_7 \sqrt{\omega^2 + q_y^2}} \mathcal{P} \left[ \frac{1}{2} q_y^2 - g(q_y^2) \right].$$

(14)

This is our main result. Thanks to the cascading mechanism, which has regularized all potential divergences, this solution is finite everywhere. Figure 2 shows that $\Phi(y, z, w)$ is smooth everywhere and decreases with $w$.

As it stands, however, our framework has a ghost [8,9], as indicated by the poles at $q_y^2 = \pm q_0^2$. There are two ways to resolve this issue. One can introduce sufficiently large tension on both the codimension-2 and -3 branes [11]; to remove the ghost, the codimension-2 tension should be $\gtrless M_3^2 m_7^2$, whereas the corresponding bound on the codimension-3 tension is yet to be determined.

Alternatively, one can regularize codimension-2 and -3 branes and include the 6d Einstein-Hilbert term localized on these objects [9,12]. Following this route, we demonstrate in Appendix B that the poles do disappear, and that the profile for $\Phi(y, z, w)$ is qualitatively unchanged.

**III. Discussion.**—In this Letter, we have shown that a 3-brane with tension remains flat in the $(6 + 1)$-dimensional cascading gravity framework. In the weak-field approximation, we have obtained a bulk solution which is nowhere singular and remains perturbative everywhere.

These properties crucially depend on the existence of parent branes with finite Planck masses. Indeed, our solution goes outside the perturbative regime and acquires divergences in the limit $M_5, M_6 \to 0$, consistent with [4].

We are currently extending our solutions to the nonlinear regime through numerical analysis. For now, we view the present results as a tantalizing first step towards realizing the idea of Rubakov and Shaposhnikov.

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**Appendix A.**—We show that the weak-field metric can be brought to the form (10) by symmetry and gauge freedom. In the de Donder gauge, $\partial_\alpha h_B^A = \frac{1}{2} \partial_\beta h_C^C$, (7) reduces to

$$-\frac{M_5^3}{2} \Box_5 \left( h_{A B} - \frac{1}{2} \eta_{A B} h_C^C \right) = \delta(w) (T_{A B}^{(6)} - M_6^4 G_{A B}^{(6)}),$$

where the effective stress energy on the codimension-1 brane, $T_{A B}^{(6)}$, includes contributions from the 5d and 6d induced gravity terms. Since there is no stress energy along the $(a, w)$ and $(v, w)$ directions, the corresponding equations are consistently satisfied by setting $h_{a w} = 0$ and $h_{v w} = h'^e_c$, where $h'^e_c$ is the 6d trace. It follows that the induced gauge choice in 6d is given by $\partial_\alpha h_B^A = \partial_\beta h_C^C$; hence, the $(a, b)$ components of (A1) reduce to

$$-\frac{M_5^3}{2} \Box_5 (h_{a b} - \eta_{a b} h'^e_c) = \delta(w) \frac{M_6^4}{2} (\Box_5 h_{a b} - \partial_\alpha \partial_\beta h'^e_c) + \delta(w) T_{a b}^{(6)},$$

(A1)

To proceed further, it is convenient to decompose $h_{a b}$ into its trace and transverse-traceless (TT) parts:

161601-3
\[ h_{ab} = h_{ab}^{6dT} + \frac{\partial_a \partial_b}{\Box_6} h^c. \]  

(A2)

From (A1), the 6dT components satisfy

\[ -\frac{M_7^2}{2} \left( \Box_e + \frac{\delta(w)}{m_7} \Box_6 \right) h_{ab}^{6dT} = \delta(w) \left( T_{ab}^{(6)} - \frac{1}{5} \eta_{ab} T^{(6)} \right) \]

\[ + \frac{1}{5} \partial_a \partial_b \partial_6 T^{(6)} . \]  

(A3)

The symmetries of the problem allow a simple expression for the 5d components of the 6dT part:

\[ h_{a\beta}^{6dT} = -\frac{1}{4} \Phi \eta_{a\beta} - \left( \Box_5 - \frac{5}{4} \right) \frac{\partial_a \partial_5}{\Box_6} \Phi. \]  

(A4)

This follows from setting \( h_{a\beta}^{6dT} = 0 \), which is consistent with the equations of motion for the case of interest. Substituting into (A3), and using \( T_{a\beta}^{(5)} = -\delta^\mu_a \delta^\nu_\beta \Lambda \eta_{\mu\nu} \delta(y) \), the resulting equation of motion for \( \Phi \) agrees with (9).

We can now be explicit about the form of the various metric components. Combining (A2) and (A4), we get

\[ h_{a\beta} = -\frac{1}{4} \Phi \eta_{a\beta} - \left( \Box_5 - \frac{5}{4} \right) \frac{\partial_a \partial_5}{\Box_6} \Phi. \]  

(A5)

And since everything is independent of \( x^a \), we get \( h_{a\mu} = 0 \) and \( h_{\mu\nu} = -\frac{1}{4} \Phi \eta_{\mu\nu} \). Similarly, from (A2) we obtain

\[ h_{yz} = \frac{\partial_y \partial_z}{\Box_6} (h^c - \Phi); \quad h_{zz} = \frac{\partial_z^2}{\Box_6} (h^c - \Phi) + \Phi; \]

\[ h_{yy} = \frac{\partial_y^2}{\Box_6} (h^c - \Phi) + \Phi. \]  

(A6)

This is equivalent to (8) after a small diffeomorphism.

Appendix B.—One way to cure the ghost of higher-codimension DGP models \[8,9\] is to consider a higher-dimensional Einstein-Hilbert model localized on the regularized brane \[9,12\]. Following this prescription, we will consider this prescription, we will.

103, 161601 (2009)  

PHYSICAL REVIEW LETTERS  

week ending 16 OCTOBER 2009

with \( T_{aa}^{(6)} = 0 \), \( T_{zz}^{(6)} = M_3^2 \delta(z) R_5/2 \), and

\[ T_{ab}^{(6)} = -M_3^2 \delta(z)[C_{ab}^{(5)} + \frac{1}{2} \Box_5 \eta_{ab} \delta(y) - \delta_a \delta_b \eta_{zz}] \]

\[ - \delta(z) \delta(y) \Lambda \eta_{\mu\nu} \delta^\mu_\alpha \delta^\nu_\beta \]  

(B2)

Using this in the 6d part of the Einstein equations, we get

\[ h_{zz} = -\psi, \quad \Box_5 \delta_{yy} = -4 \Box_5 \psi + \delta_y^2 \delta_\alpha \delta^\alpha_\alpha, \quad h_{\mu\nu} = 0, \]

\[ \Box_5 h_{\mu\nu} = \Box_5 \delta_\psi \eta_{\mu\nu} + \delta_\mu \delta_\nu \delta_\alpha \delta^\alpha_\alpha, \]  

with

\[ \left[ \Box_5 + \frac{\delta(w)}{m_7} \Box_6 + \frac{\delta^2(w, z)}{m_7 m_5} \Box_5 \right] \psi = \frac{2}{5} \frac{\delta^3(w, z, y)}{M_7^5} \Lambda. \]  

(B3)

We notice that the kinetic term for \( \psi \) is now everywhere positive, signaling that the ghost has been cured. Equation (B3) is similar to (9) for \( \Phi \), except for a redefinition of \( m_6 \) and \( m_7 \). The profile for \( \psi(y, z, w) = -\frac{2\Lambda}{5M_7^4} \) satisfies the weak-field approximation, in a ghost-free setup.