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Necessary and sufficient conditions for localization of multiple robot platforms

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Abstract

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Comments

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NECESSARY AND SUFFICIENT CONDITIONS FOR LOCALIZATION OF MULTIPLE ROBOT PLATFORMS

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ABSTRACT

In this paper, we consider the problem of cooperatively localizing a formation of networked robots/vehicles in $\mathbb{SE}(2)$*. First, we propose necessary and sufficient conditions to establish when a team of robots with heterogeneous sensors can be localized. We then show how these conditions are analogous to well-known results in the literature on kinematics of planar mechanisms. We show how localization is equivalent to solving a system of nonlinear closure equations. Depending on what sensors are available for each robot, the multirobot formation can be modeled as a sensing graph consisting of vertices representing robots and edges corresponding to sensory information. We establish conditions that must be satisfied by this graph and show how this graph influences estimates of positions and orientations of the robots in a team through experiments and simulations.*

INTRODUCTION

In order for a team of mobile robots to navigate autonomously in some desired formations and further perform cooperative tasks, such as mapping, surveillance and target acquisition, they must be able to localize themselves in the formation as well as in a global reference frame [1, 2]. Therefore, how to estimate robots' positions and orientations (poses) in a precise and efficient way is of particular interest. Our interest in this paper is localizing a team of heterogeneous robots in the two-dimensional Special Euclidean group, $\mathbb{SE}(2)$ [3], and in localizing targets with information obtained from heterogeneous sensors. Specifically, we are interested in conditions under which all robots in the formation can be localized in the environment, and in minimizing the relative and absolute uncertainty in the estimates. Our goal in this paper is to derive necessary and sufficient conditions for localizing a formation of three or more robots in $\mathbb{SE}(2)$ from distributed measurements and quantifying the quality of the resulting estimates. The adaptation of the *sensing graph* and formation geometry to improving these estimates is discussed in [4].

Our study of team localization has benefited from extensive research on parallel mechanisms in the past decades [5–9]. In the following sections, we will show that a multi-robot formation can be modeled as a closed kinematic chain. Measurements of features in the environment constrain the robot's position and orientation in the world, in much the same way a linkage does. Similarly, measurements of one robot's position and orientation by another robot constrains estimates of the relative position and orientation and can be thought of as a linkage connecting the two robots constraining their relative configurations. Thus a multi-robot platform with distributed and often redundant measurement information can be viewed as a mechanism with closed kinematic chains [10, 11]. The task of localizing the platform in $\mathbb{SE}(2)$ based on the measurements is analogous to the forward kinematic analysis problem of parallel mechanisms [12, 13], whose goal it is to determine the platform's position and orientation relative to the base given the sensed lengths of the linkages. Whether or not a set of measurements is sufficient to *localize* each robot in the formation is analogous to asking if a parallel kinematic chain is statically stable. Thus, a system of robots that cannot be localized with a given set of measurements can be thought of as a mechanism with closed kinematic chains

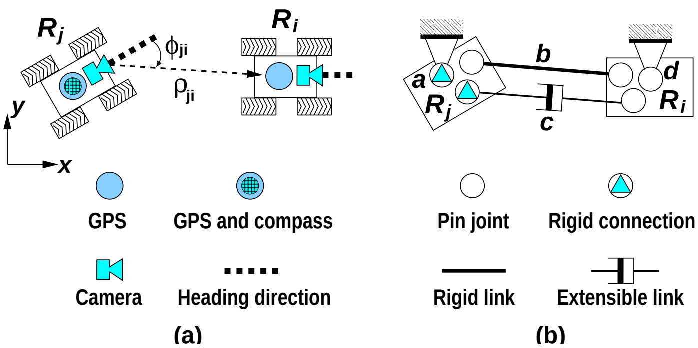

Figure 1. (a) A FORMATION OF TWO ROBOTS IN $\mathbb{SE}(2)$ WITH HETEROGENOUS SENSORS. (b) ANALOG TO A PLANAR MECHANICAL PLATFORM OF TWO RIGID BODIES CONNECTED BY RIGID AND EXTENSIBLE LINKAGES.

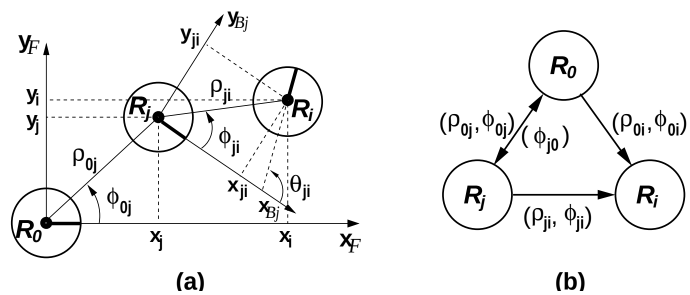

Figure 2. MODELING OF SENSORY INFORMATION: (a) A FORMATION OF TWO ROBOTS IN $\mathbb{SE}(2)$: R_j HAS ABSOLUTE POSITION AND ORIENTATION INFORMATION, AND RANGE AND BEARING INFORMATION ABOUT R_i. R_i HAS ABSOLUTE POSITION INFORMATION. A BODY REFERENCE FRAME, B_j, HAS BEEN ATTACHED TO R_j. (b) THE SENSING GRAPH ASSOCIATED WITH THE FORMATION.

that is *shaky* [14].

Also relevant to this work is the recent literature that uses graphs to model sensor networks and cooperative control schemes [15, 16]. It has been shown that results on graph rigidity theory [17–19] can be directly applied to multiple robot systems in $\mathbb{R}^2$ [20, 21]. However, relatively little attention has been paid to robot-sensor networks in $\mathbb{SE}(2)$, where orientation estimates must be derived. Such networks are particularly relevant for robots with exteroceptive sensors such as cameras.

In the next section, we will present a graphic model of multi-robot platforms, based on which we will establish the necessary and sufficient conditions for teamlocalization. We will then describe a simple least-squares estimation scheme for localization of multiple robots and illustrate these with experimental results.

MODELING

Consider a planar world, $W = \mathbb{R}^2$, occupied by a team of n robots, $R = \{R_1, R_2, \ldots R_n\}$, equipped with heterogenous sensors, such as global positioning sensors (GPS), compasses, inertial sensors (IMU) and cameras. Assume each robot can communicate with every other robot in the team. The physical configurations of the robots coupled with the characteristics of the hardware and the requirements of the sensing algorithms induce a multi-robot platform or a *formation* of n robots in $\mathbb{SE}(2)$. As an example, consider a two-robot formation in Fig.1(a). Robot R_j is equipped with GPS, a compass and a camera. The GPS and compass provide absolute position and orientation measurements. The camera provides range and bearing information about robot R_i. Similarly robot R_i has absolute position sensors via its onboard GPS.

In a static or quasi-static setting, each sensor measurement introduces one constraint on the formation geometry or the configuration of the team R. Each robot is considered as a rigid body. Each constraint can be thought of as kinematic constraints arising from a mechanical linkage. As shown in Fig.1(b), a two-robot formation in $\mathbb{SE}(2)$ can be modeled as a planar mechanical linkage. Joint a is a rigid connection of R_j with a fixed base because of the GPS and compass fix. The link b with pin joints models the ability of the camera on R_j to measure range, while the prismatic joint c incorporates the bearing information obtained by the camera. The pin joint d tells us that robot R_i can localize itself from the GPS. Note that R_i does not have orientation information but must, instead, use the kinematics of the mechanical linkage to infer this orientation.

To simplify the localization process, we assume that sensors are mounted at fixed positions on the robot platforms and all measurements made by one robot can be referred to the center of the robot. Thus the team R can be viewed as a team of point robots. Fig.2(a) gives a simplified model for the formation in Fig.1. In this simplified model, we define a global reference frame F by forming a virtual fixed robot or a fixed beacon R_0 with fixed configuration $q_0 = 0$ in the inertial frame. The configuration or the shape variables of R in F is denoted by $q = \left[q_1^T, q_2^T, \ldots, q_n^T\right]^T \in \mathbb{R}^{3n}$, where $q_i = (p_i, \theta_i)^T \in \mathbb{R}^3$ is a parameterization of $\mathbb{SE}(2)$, with $p_i = (x_i, y_i)^T$ and θ_i, the absolute position and orientation of the i^{th} robot. A body reference frame B_j at the j^{th} robot is also defined with its origin located at the center of the robot and its x-axis aligned with the direction of heading of R_j. The configuration or the *shape* of the formation, R, is described in the body-fixed frame B_j by $\tilde{q} = [(q_1^j)^T, (q_2^j)^T, \ldots, (q_n^j)^T]^T$, where $q_i^j = (p_{ji}, \theta_{ji})^T$ with $p_{ji} = (x_{ji}, y_{ji})^T$ and θ_{ji}, is the relative position and orientation of R_i about R_j, and $q_j^j = 0$.

In order to represent the sensory information, we define a directed graph called the *sensing graph*, $G = (V, E, Z, P)$, where $V = R \cup \{R_0\}$ is a finite set of vertices. The edge set $E \subset V \times V$ consists of labeled edges that represent the presence of measurements (observations) between robots. The measurement set Z

consists of three type of sensory information: (i) range between two robots, ρ_{ij}, (ii) bearing of one robot in relation to another, ϕ_{ij}, and (iii) absolute position of a robot in F, (x_j, y_j), which can be obtained by global positioning sensors, or via triangulation with fixed, known landmarks. P is a model of the uncertainties associated with the estimates in Z. Figure 2(b) shows the sensing graph of the two-robot formation given in Fig.1 and Fig.2(a).

In a sensing graph G, the j^{th} vertex has an incoming edge from the i^{th} vertex labeled by (ρ_{ij}, ϕ_{ij}) whenever robot R_i can sense robot R_j. Corresponding to types of sensory information, we use (i) a shorthand *relative range edge* (ρ_{ij}) to denote $(\rho_{ij}, null)$, (ii) a *relative bearing edge* (ϕ_{ij}) for $(null, \phi_{ij})$ and (iii) a *range-bearing edge* (ρ_{ij}, ϕ_{ij}) directed from R_i to R_j respectively. Any absolute measurements made for robot R_j can be regarded as a range-bearing edge, (ρ_{0j}, ϕ_{0j}). We will also consider errors in measurements, with P consisting of information about variances: $\sigma^2_{\rho_{ij}}$ and $\sigma^2_{\phi_{ij}}$ for range and bearing measurements, and covariance matrices for range-bearing measurements.

NECESSARY CONDITION FOR TEAM LOCALIZATION

In order to consider whether a team of robots can be localized or not, it is necessary to fuse the information available from different sensors/robots and verify if this information is adequate. For a team of n robots in $\mathbb{SE}(2)$, localization is the determination of the $3n$ coordinates that characterize the robot positions and orientations. Thus it is first necessary to see if $3n$ independent measurements are available or not.

Definition 1. *A team of n robots in* $\mathbb{SE}(2)$ *is said to be* localizable *if the* $3n$ *coordinates of the n robots can be estimated in an inertial frame.*

Remark 1. *Localizability is obviously related to observability in systems theory [22] – if a team is localizable over any time interval, the system is completely observable. However, we will use Definition 1 in an instantaneous, static setting and thus refrain from using systems theoretic notation.*

Remark 2. *We can also require the team to be localized in a relative setting [2] where it is only necessary to be able to estimate* $3n-3$ *coordinates of* $n-1$ *robots in a body reference frame.*

Every measurement specifies a constraint on the $3n$ coordinates. Specifically, for each range and bearing measurement, the constraints on the coordinates in a body reference frame B_j are given by:

$$Type\ 1: \ \rho_{ik} = \sqrt{(p_{ji}-p_{jk})^T(p_{ji}-p_{jk})} \tag{1}$$

$$Type\ 2: \ \phi_{ji} = tan^{-1}(y_{ji}/x_{ji}) \tag{2}$$

A pair of bearing measurements, ϕ_{ij} and ϕ_{ji}, involving robots R_i and R_j, results in the following Type 3 constraint:

$$Type\ 3: \ \phi_{ik} - \phi_{ki} + \pi = \theta_{ik} = \theta_{jk} - \theta_{ji} \tag{3}$$

Finally, any pair of bearing measurements, ϕ_{ij} and ϕ_{jk}, involving three robots R_i, R_j, and R_k, results in the following Type 4 constraint.

$$Type\ 4: \ \phi_{ij} - \phi_{ik} = cos^{-1}\frac{(p_{ji}-p_{jj})^T(p_{ji}-p_{jk})}{\|p_{ji}-p_{jj}\| \cdot \|p_{ji}-p_{jk}\|} \tag{4}$$

All these constraints can be written in the form:

$$L_1 \cdot z = h(\tilde{q}) \tag{5}$$

where L_1 is a linear combination of measurements, and h is a nonlinear function of the shape variables, $\tilde{q}$, in some body-fixed reference frame. It is not difficult to see that there are only four types of constraints that can be used to describe the configuration of a formation. All other equations that can be written are functionally dependent on the above constraint equations.

Categorizing the constraints into different categories enables us to establish conditions for localizability of a robot team by the following theorem.

Theorem 1. *A formation of n robots in* $\mathbb{SE}(2)$ *is localizable only if*

$$N = 3n - 2n_g - n_b - n_r \leq 0, \tag{6}$$

where n_g, n_b *and* n_r *are numbers of measurements made by inertial or global positioning sensors, bearing sensors and range sensors respectively.*

Proof. It is easy to verify that each absolute position measurement made by global positioning sensor can be directly used to estimate two state variables, and each bearing and range measurement will add at least one constraint (Type 1 – 4) on the configuration of a formation. Thus, n_g global position sensors, n_b bearing and n_r range sensors will provide at most $2n_g + n_b + n_r$ independent measurements. Since $3n$ state variables have to be estimated, $2n_g + n_b + n_r$ must be at least equal to $3n$.

Given a formation of robots in $\mathbb{SE}(2)$ with limited sensing capability, Theorem 1 provides a simple necessary condition to easily verify the localizability of a formation. Note that additional sensors such as landmark sensors, compasses and IMUs can be incorporated into this framework in a straightforward way.

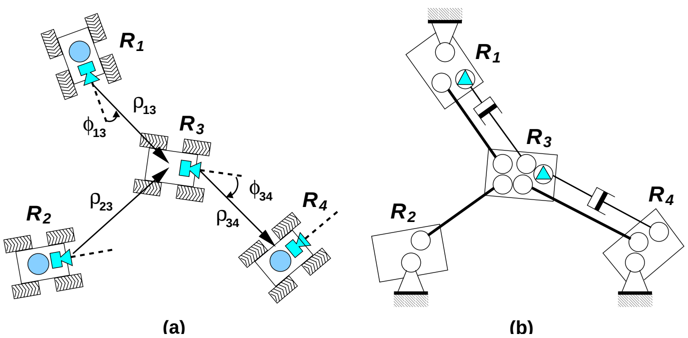

Figure 3. (a) A MULTI-ROBOT FORMATION IN $\mathbb{SE}(2)$. (b) THE ANALOGOUS LINKAGE WITH CLOSED KINEMATIC CHAINS.

This result is of course very similar to the Kutzbach-Grübler criterion that is well known in the kinematics literature [23, 24], which yields

$$N = 3\tilde{n} - 2j, \tag{7}$$

where $\tilde{n}$ is the number of virtual rigid bodies and j is the number of virtual joints.

Figure 3 shows, as an example, a formation of four robots in $\mathbb{SE}(2)$ with distributed sensor measurements and its mechanical analog. By Eqn.(6), we have $n = 4$, $n_g = 3$, $n_b = 2$, $n_r = 3$ and thus obtain $N = 1$. And by Eqn.(7), we have $\tilde{n} = 9$ and $j = 13$, which gives us the same result. Therefore this multi-robot formation is not localizable by the available sensors and is analog to a shaky mechanical framework at this point.

SUFFICIENT CONDITION FOR TEAM LOCALIZATION

Considering the formation geometry, it is obvious that not all pieces of sensory information contribute to the localization algorithm in an independent manner. A simple example is the case when two measurements, ρ_{ij} and ρ_{ji} are made by robot R_i and R_j respectively, which result in redundant constraints on the coordinates of those robots. In such a case, it is necessary to develop a test of functional independence for all constraints. In this section, accordingly, we will define a *constraint matrix* whose rank will allow us to verify if the team can be localized or not.

First, by differentiating the four constraint equations, we get expressions describing allowable small changes (equivalently velocities) of the robot coordinates.

$$\begin{bmatrix} r_{1,ik} & r_{2,ik} \end{bmatrix} \begin{bmatrix} \dot{p}_{ji} \\ \dot{p}_{jk} \end{bmatrix} = 0 \tag{8}$$

$$\begin{bmatrix} \frac{-y_{ji}}{p_{ji}^T p_{ji}} & \frac{x_{ji}}{p_{ji}^T p_{ji}} \end{bmatrix} \begin{bmatrix} \dot{p}_{ji} \end{bmatrix} = 0 \tag{9}$$

$$\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{ji} \\ \dot{\theta}_{jk} \end{bmatrix} = 0 \tag{10}$$

$$\begin{bmatrix} b_{1,jik} & b_{2,jik} & b_{3,jik} \end{bmatrix} \begin{bmatrix} \dot{p}_{ji} \\ \dot{p}_{jj} \\ \dot{p}_{jk} \end{bmatrix} = 0 \tag{11}$$

where

$$r_{1,ik} = \frac{p_{ji} - p_{jk}}{\sqrt{(p_{ji}-p_{jk})^T (p_{ji}-p_{jk})}}, \quad r_{2,ik} = \frac{p_{jk} - p_{ji}}{\sqrt{(p_{ji}-p_{jk})^T (p_{ji}-p_{jk})}}$$

$$\begin{aligned}
b_{1,jik} = {} & \frac{2[(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})]}{(p_{ji} - p_{jj})^2 (p_{ji} - p_{jk})^2} (2p_{ji} - p_{jj} - p_{jk}) \\
& - \frac{2[(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})]^2}{(p_{ji} - p_{jj})^4 (p_{ji} - p_{jk})^2} (p_{ji} - p_{jj}) \\
& - \frac{2[(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})]^2}{(p_{ji} - p_{jj})^2 (p_{ji} - p_{jk})^4} (p_{ji} - p_{jk}) \\
b_{2,jik} = {} & \frac{2[(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})]}{(p_{ji} - p_{jj})^2 (p_{ji} - p_{jk})^2} (p_{jk} - p_{ji}) \\
& + \frac{2[(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})]^2}{(p_{ji} - p_{jj})^4 (p_{ji} - p_{jk})^2} (p_{ji} - p_{jj}) \\
b_{3,jik} = {} & \frac{2[(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})]}{(p_{ji} - p_{jj})^2 (p_{ji} - p_{jk})^2} (p_{jj} - p_{ji}) \\
& + \frac{2[(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})]^2}{(p_{ji} - p_{jj})^2 (p_{ji} - p_{jk})^4} (p_{ji} - p_{jk})
\end{aligned}$$

Following this procedure for all possible constraints gives us a $m \times 3n$ constraint matrix for reference frame B_j:

$$K_f(\tilde{q})\,\dot{\tilde{q}} = 0 \tag{12}$$

where

$$K_f = \begin{bmatrix}
\cdot\cdot & r_{1,ik} & \cdot\cdot & r_{2,ik} & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
\cdot\cdot & \left(\frac{-y_{ji}}{p_{ji}^T p_{ji}} \; \frac{x_{ji}}{p_{ji}^T p_{ji}}\right) & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
\cdot\cdot & \cdot\cdot & -1 & \cdot\cdot & 1 & \cdot\cdot & \cdot\cdot \\
\cdot\cdot & b_{1,jik} & \cdot\cdot & b_{2,jik} & \cdot\cdot & b_{3,jik} & \cdot\cdot \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{bmatrix}$$

This matrix, K_f, is called the *constraint matrix* for the formation. It can be directly obtained by differentiating $h(\tilde{q})$ in Eqn.(5), since $K_f = \frac{\partial h}{\partial \tilde{q}_j}$.

The following theorem provides sufficient conditions for localizability.

Theorem 2. *Consider a formation of n robots in* $\mathbb{SE}(2)$ *with the shape* $\tilde{q} = [(q_1^j)^T, (q_2^j)^T, \ldots, (q_n^j)^T]^T$ *in* B_j*. The formation is* *localizable relative to the robot-fixed reference frame B_j if and only if*

$$rank\{K_f(\tilde{q})\} = 3n-3,\ (n \geq 2). \tag{13}$$

Proof. Any body-fixed description of a formation in $\mathbb{SE}(2)$ has a natural group symmetry. Translation of this body-fixed reference frame does not change the shape vector $\tilde{q} = [(q_1^j)^T,\ (q_2^j)^T,\ \ldots,\ (q_n^j)^T]^T \in \mathbb{R}^{3n}$. Thus there are $3n-3$ free variables that determine the shape of the formation in the frame B_j. Since the rank of K_f determines the number of independent constraints imposed by sensor measurements in the network, we must have:

$$rank\{K_f\} = 3n-3$$

in order to estimate the $3n-3$ nonzero variables in $\tilde{q}$.

Recall that the condition in Theorem 1 is only a weak necessary condition. There are other necessary conditions that must be satisfied. We are particularly interested in cases where global positioning capability is not available to most robots. For example, if n_g = 0 or 1, it follows from Theorem 2 that we need at least one range measurement leading to a Type 1 constraint and $n-1$ pairs of bearing measurements leading to Type 3 constraints. At least one Type 2 constraint must be incorporated. And finally, for localization in an inertial frame, one needs at least one global position estimate ($n_g > 0$), and at least one bearing measurement of the virtual robot (*i.e.*, a measurement ϕ_{j0}).

Special Case: Team Localization in $\mathbb{R}^2$

Without considering robots' orientations, the team localization problem is simplified to the determination of $2n$ coordinates that characterize the robots' positions in the 2D Euclidean space. It is easy to verify that the previous models and theorems can be applied to this situation by ignoring a portion of sensory information and the associated Type 3 constraints on robots' orientations. Therefore, the problem of team localization in $\mathbb{R}^2$ is treated as a special case of the localization problem in $\mathbb{SE}(2)$.

Following the proof of the previous theorems, it is straightforward to see that the necessary condition for localizing a team of n robots in $\mathbb{R}^2$ is

$$N = 2n - 2n_g - n_b - n_r \leq 0, \tag{14}$$

where n_g, n_b and n_r are numbers of measurements made by inertial or global positioning sensors, bearing sensors and range sensors respectively. And the sufficient condition for the localizability of the team in a body reference frame in $\mathbb{R}^2$ becomes

$$rank\{K_f(\tilde{q})\} = 2n-2,\ (n \geq 2), \tag{15}$$

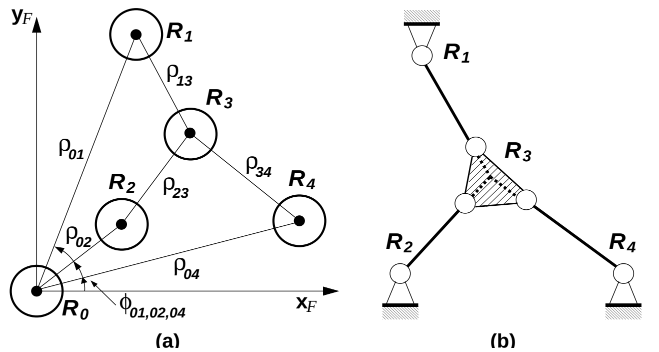

Figure 4. (a) A FORMATION OF FOUR ROBOTS IN $\mathbb{R}^2$. (b) ANALOG TO A PLANAR PLATFORM WITH PARALLEL LINKAGES.

where $\tilde{q} = [(q_1^j)^T, (q_2^j)^T, \ldots, (q_n^j)^T]^T \in \mathbb{R}^{2n}$ with $q_i^j = (x_{ji}, y_{ji})^T$ in a body reference frame B_j, and the constraint matrix K_f is derived from the constraints of Type 1, 2 and 4 only.

Figure 4(a) shows an example of the simplified model of multiple robot system in $\mathbb{R}^2$. In this particular case, we obtain $N = -1$ by checking Eqn.(14) with $(n_g = 3,\ n_b = 0,\ n_r = 3)$, or $(n_g = 0,\ n_b = 3,\ n_r = 6)$ if we model the global positioning information (x_j, y_j) by (ρ_{0j}, ϕ_{0j}). The same result can be also obtained by using Eqn.(7) to calculate the number of degrees of freedom of the analogous planar mechanical platform with parallel linkages, as shown in Fig.4(b). Note that the polygonal rigid body, which denotes R_3, has only 2 degrees of freedom since all constraints on that rigid body can be transferred to its center without being coupled with moments. Further, the localizability of the system in a robot-fixed reference frame can be verified by writing the constraint equations for ρ_{13}, ρ_{23}, ρ_{34} and checking the rank of the constraint matrix using Eqn.(15).

Observe that, for verifying the localizability of multiple robot systems in $\mathbb{R}^2$, our approach is closely related to the concepts of rigid graph [17–19] and rigid point formation [21]. It has been shown that the positions of n robots in $\mathbb{R}^2$ can be determined

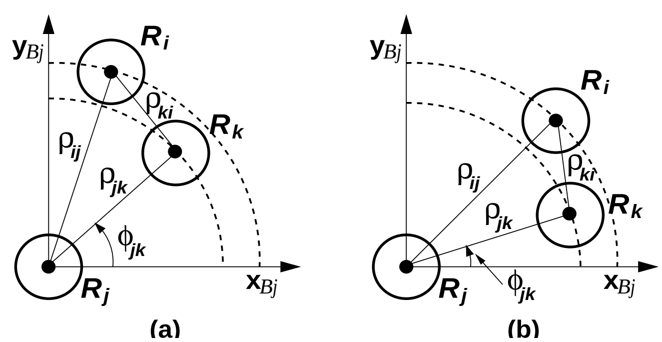

Figure 5. (a) A RIGID FORMATION FORMED BY THREE ROBOTS IN $\mathbb{R}^2$. (b) THE SAME RIGID FORMATION AFTER A ROTATION IN THE BODY REFERENCE FRAME B_j.

in a body reference frame by $2n-3$ Type 1 constraints [20]. However, the uniqueness of the solution is up to a rotation if the reference frame is formed only by a single robot. To illustrate this, we consider a team of $n=3$ robots in Fig.5. It is easy to see that the rigid formation formed by R_i, R_j and R_k with $2n-3=3$ constraints (on ρ_{ij}, ρ_{jk} and ρ_{ki} respectively) can not be uniquely localized in the reference frame B_j, unless a bearing measurement, say ϕ_{jk}, is obtained to specify a Type 2 constraint. This causes the rank condition of K_f for localizability to be higher than that of rigidity matrix for graphs or point formations.

ESTIMATION ERRORS IN TEAM LOCALIZATION

In the previous sections, we have presented a deterministic approach to verify the localizability of multi-robot platforms in $\mathbb{SE}(2)$ and $\mathbb{R}^2$. In a realistic setting, however, measurement noise always exists within the sensory information acquired, which may be caused by a dynamic environment or perturbations on sensors. Therefore, we need to find a systematic way to measure the quality of the resulting estimates from those noisy sensor data, if the system is localizable in the environment.

Assume that the measurements are independent and the measurement noise is given by a joint normal distribution. Then, for a given sensing graph, the Weighted Least Squares method can be applied to compute the best estimates of $\tilde{q}$ and the covariance matrix P for estimation errors about the nominal configuration specified by the nonlinear constraints as [4]

$$\delta\tilde{q} = (H^T R^{-1} H)^{-1} H^T R^{-1} L_1 \,\delta z \tag{16}$$
$$P = (H^T R^{-1} H)^{-1} \tag{17}$$

where H is obtained by deleting the three columns corresponding to the j^{th} robot's coordinates q_j^j in K_f, and R is the block diagonal covariance matrix of measurement noise. Since H is derived based on the measurement set Z and its associated constraints, the resulting covariance matrix P will naturally depend on the sensing graph G built by every particularly set Z considered.

The trace of the covariance matrix is a scalar utility measure that captures quality of the estimate obtained from every particular measurement set Z. This leads to a natural strategy for comparison and optimization of sensing graphs for localization. To demonstrate this, we consider a robot team ($n=3$) in $\mathbb{SE}(2)$ with its sensing graph as shown in Fig.6. The configuration of the team $R=\{R_i, R_j, R_k\}$ in B_j is $\tilde{q} = \text{column}\{q_i^j, q_j^j, q_k^j\}$ with $q_j^j = (p_{jj}, \theta_{jj})^T = 0$. The available measurements and the associated covariance matrix for their noise are

$$z = (\rho_{ij}, \rho_{ik}, \phi_{ij}, \phi_{ik}, \phi_{ji}, \phi_{jk}, \rho_{kj}, \phi_{kj})^T$$
$$R_0 = \text{diagonal}(\sigma^2_{\rho_{ij}}, \sigma^2_{\rho_{ik}}, ..., \sigma^2_{\phi_{kj}})$$

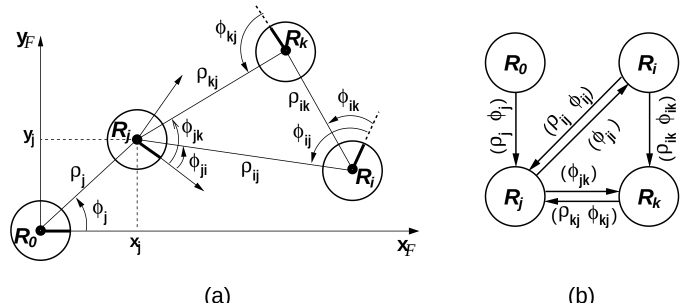

Figure 6. (a) A FORMATION OF THREE ROBOTS IN $\mathbb{SE}(2)$ WITH DISTRIBUTED SENSOR MEASUREMENTS AND A BODY REFERENCE FRAME B_j ON R_j. (b) SENSING GRAPH.

Note that we wrote the elements in Z into a vector z to align with the notation we used in [4].

Any subgraph of the sensing graph G that contains only $3n-3$ independent pieces of measurement-constraint information is said to be a minimal sensing graph. In order to keep the analysis simple, we manually choose two subsets of $3n-3$ measurements from Z and construct two minimal sensing graphs accordingly. (See Fig.7.) For graph (a), we can write the following constraint equations

$$\rho_{ij} = \sqrt{(p_{ji}-p_{jj})^T(p_{ji}-p_{jj})}, \tag{18}$$
$$\rho_{ik} = \sqrt{(p_{ji}-p_{jk})^T(p_{ji}-p_{jk})}, \tag{19}$$
$$\phi_{ij} - \phi_{ji} + \pi = \theta_{ij}, \tag{20}$$
$$\phi_{ji} = tan^{-1}(y_{ji}/x_{ji}), \tag{21}$$
$$\rho_{kj} = \sqrt{(p_{jk}-p_{jj})^T(p_{jk}-p_{jj})}, \tag{22}$$
$$\phi_{jk} - \phi_{kj} + \pi = \theta_{jk}. \tag{23}$$

By differentiating Eqn.(18) – (23) and rearranging the first order

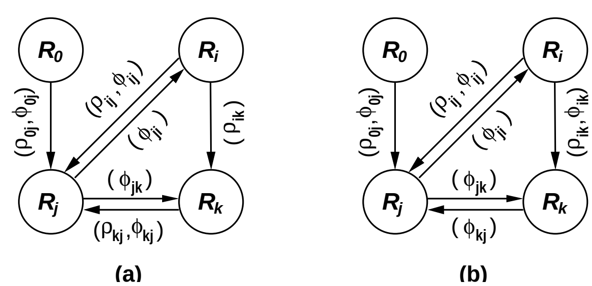

Figure 7. TWO MINIMAL SENSING GRAPHS THAT ARE SUBGRAPHS OF FIG.6(b) RESPECTIVELY.

terms, we obtain the constraint matrix for graph (a) as

$$K_{f,a} = \begin{bmatrix} r_{1,ij} & 0 & r_{2,ij} & 0 & 0 & 0 \\ r_{1,ik} & 0 & 0 & 0 & r_{2,ik} & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ \left(\frac{-y_{ji}}{p_{ji}^T p_{ji}} \;\; \frac{x_{ji}}{p_{ji}^T p_{ji}}\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{1,jk} & 0 & r_{2,jk} & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

Similarly, for Fig.7(b), we have the constraint equations as

$$\rho_{ij} = \sqrt{(p_{ji} - p_{jj})^T (p_{ji} - p_{jj})}\,, \tag{24}$$

$$\rho_{ik} = \sqrt{(p_{ji} - p_{jk})^T (p_{ji} - p_{jk})}\,, \tag{25}$$

$$\phi_{ij} - \phi_{ik} = cos^{-1} \frac{(p_{ji} - p_{jj})^T (p_{ji} - p_{jk})}{\|p_{ji} - p_{jj}\| \cdot \|p_{ji} - p_{jk}\|}\,, \tag{26}$$

$$\phi_{ij} - \phi_{ji} + \pi = \theta_{ij}\,, \tag{27}$$

$$\phi_{jk} = tan^{-1}(y_{jk}/x_{jk})\,, \tag{28}$$

$$\phi_{jk} - \phi_{kj} + \pi = \theta_{jk}\,, \tag{29}$$

which results in

$$K_{f,b} = \begin{bmatrix} r_{1,ij} & 0 & r_{2,ij} & 0 & 0 & 0 \\ r_{1,ik} & 0 & 0 & 0 & r_{2,ik} & 0 \\ b_{1,jik} & 0 & b_{2,jik} & 0 & b_{3,jik} & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{-y_{jk}}{p_{jk}^T p_{jk}} \;\; \frac{x_{jk}}{p_{jk}^T p_{jk}}\right) & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

It is not difficult to verify that $rank\{K_{f,a}\} = rank\{K_{f,b}\} = 3n-3$. Thus according Theorem 2, we can claim that this multiple robot system can be completely localized in a body reference frame with either of those two sets of distributed sensor measurements.

Further, considering the measurement noise, we can derive the H matrix from the above constraint matrices as

$$H_a = \begin{bmatrix} r_{1,ij} & 0 & 0 & 0 \\ r_{1,ik} & 0 & r_{2,ik} & 0 \\ 0 & -1 & 0 & 0 \\ \left(\frac{-y_{ji}}{p_{ji}^T p_{ji}} \;\; \frac{x_{ji}}{p_{ji}^T p_{ji}}\right) & 0 & 0 & 0 \\ 0 & 0 & r_{2,jk} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; H_b = \begin{bmatrix} r_{1,ij} & 0 & 0 & 0 \\ r_{1,ik} & 0 & r_{2,ik} & 0 \\ b_{1,jik} & 0 & b_{3,jik} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \left(\frac{-y_{jk}}{p_{jk}^T p_{jk}} \;\; \frac{x_{jk}}{p_{jk}^T p_{jk}}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Following the procedure of the nonlinear Least Squares method we presented in [4], two weighting matrices, $W_a = R_a^{-1}$ and $W_b = R_b^{-1}$ can be constructed based on the given covariance matrix R_0 for the measurement noise. Then two covariance matrices, P_a and P_b, for the estimation errors within those two sensing graphs can be computed by

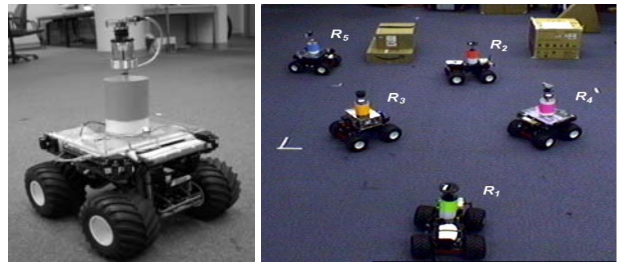

Figure 8. (a) A CAR-LIKE ROBOT PLATFORM EQUIPPED WITH OMNI-DIRECTIONAL CAMERA; (b) A FIVE-ROBOT FORMATION.

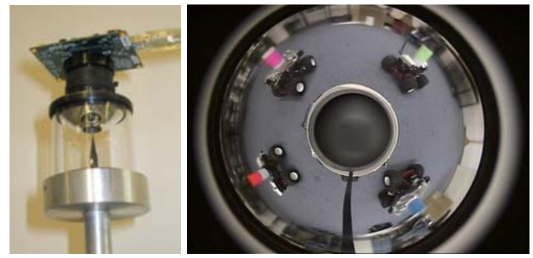

Figure 9. (a) PARABOLIC OMNI-DIRECTIONAL CAMERA; (b) A SAMPLE IMAGE FROM OMNI-DIRECTIONAL CAMERA.

$$P_a = (H_a^T R_a^{-1} H_a)^{-1}, \text{ and } P_b = (H_b^T R_b^{-1} H_b)^{-1}.$$

Eventually by comparing the traces of P_a and P_b, we can tell which sensing graph in Fig.7 is better in a sense that the estimates obtained from that sensing graph is of more accuracy.

TEAM LOCALIZATION EXPERIMENTS

In this section we show experimental results obtained from five robots with cameras and illustrate how the localization problem can be translated to solving kinematic equations for a mechanical network.

As shown in Fig.8(a), we used car-like robots equipped with omni-directional cameras (Fig.(9)(a)) as our experimental platform. In the experiments, a five-robot team maintained a static formation on the ground (see Fig.8(b)), and tried to localize each member in the formation by taking relative measurements about each other. In order to simplify visual classification and association, each robot was marked with a different color providing unique sensor identification for each robot. A calibrated overhead camera with an external computer was used to gather the ground true data for the robot locations in the environment.

With all the necessary and sufficient conditions for team localization being strictly maintained, six feasible sets of measure-

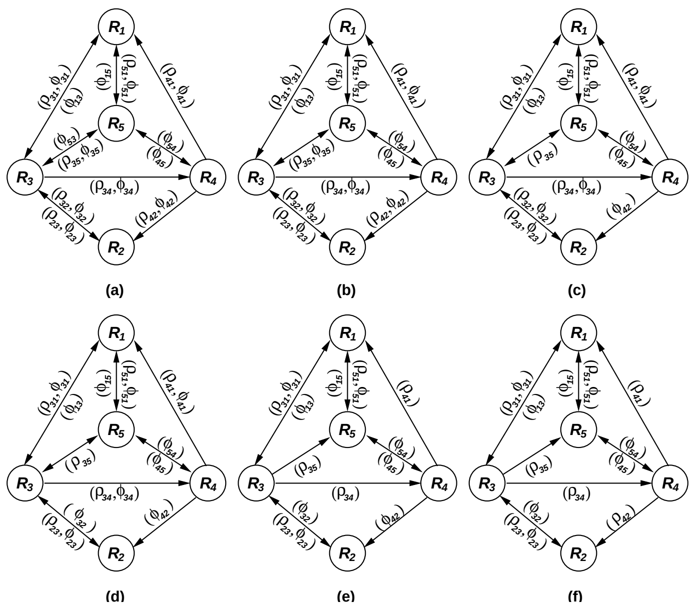

Figure 10. SENSING GRAPHS USED TO ILLUSTRATE IMPACT ON LOCALIZATION QUALITY (a) CONTAINS ALL AVAILABLE MEASUREMENTS. (b) CONTAINS ALL MEASUREMENTS EXCEPT FOR ϕ_{53}. (c), CASE b WITH ϕ_{35} AND ρ_{42} REMOVED. (d), CASE c WITH ρ_{32} REMOVED. (e) CONTAINS THE GRAPH THAT MAXIMIZES ESTIMATE QUALITY SUBJECT TO USING THE MINIMUM NUMBER OF MEASUREMENTS REQUIRED FOR LOCALIZABILITY. (f), AS IN CASE e WITH ONE BEARING MEASUREMENT ϕ_{42} SUBSTITUTED BY ρ_{42}.

ments, which are denoted by six sensing graphs in Fig.10, were used to estimate the state of five robots in the experiment respectively. Equation (17) was applied to compute the covariance matrix of estimation errors for each sensing graph considered. Table 1 shows the corresponding localization quality, quantified by the trace of the error covariance matrix.

As expected, Table 1 indicates that the localization quality was improved when more measurement information was used to construct the sensing graph. However, comparing the cases provides insight to the process of sensing graph selection. The removal of ϕ_{53} in case b does not significantly reduce localization quality. This indicates ϕ_{53} is relatively uninformative given the measurements in graph b. Considering the cost of processing additional measurements motivates selecting a measurement sub-graph when redundancy is high.

Substitution of a bearing measurement for range measurement improves the estimate quality obtained in case e over f. This outcome is expected considering the characteristics of the omni-directional camera, the other measurements used and the true shape of the robot formation. However, impact of an individual measurement cannot be determined independently of other graph assignments. Solutions to this difficult nonlinear assignment are of significant practical interest.

Table 1. ESTIMATIONS BASED ON DIFFERENT SENSING GRAPHS

	Case a	Case b	Case c	Case d	Case e	Case f
$\hat{x}_{12}(m)$	-0.3468	-0.3468	-0.4955	-0.1548	-0.1308	-0.5969
$\hat{y}_{12}(m)$	2.4448	2.4448	2.4564	2.2527	2.6649	2.2244
$\hat{\theta}_{12}(rad.)$	8.4112	8.4112	8.4137	8.4204	8.4323	8.4326
$\hat{x}_{13}(m)$	-1.2828	-1.2828	-1.3583	-1.0777	-1.5312	-1.7106
$\hat{y}_{13}(m)$	1.2907	1.2907	1.9488	1.0129	1.2013	1.5479
$\hat{\theta}_{13}(rad.)$	3.5728	3.5825	3.5680	3.5743	3.5846	3.5883
$\hat{x}_{14}(m)$	0.3020	0.3020	0.3423	0.1579	0.5942	0.6330
$\hat{y}_{14}(m)$	1.4214	1.4214	1.3642	1.6027	1.5983	1.7310
$\hat{\theta}_{14}(rad.)$	0.4097	0.4097	0.4482	0.4316	0.4467	0.4423
$\hat{x}_{15}(m)$	-2.5256	-2.5256	-2.8087	-2.8652	-2.1744	-2.6033
$\hat{y}_{15}(m)$	2.3552	2.3552	2.8717	2.8993	2.0196	2.2614
$\hat{\theta}_{15}(rad.)$	-0.3903	-0.3903	-0.3803	-0.3833	-0.3719	-0.3705
$Trace(P)$	0.0142	0.0142	0.0171	0.0228	0.1231	0.1506

CONCLUSIONS

In this paper, we presented a graphical model of multiple robot systems in $\mathbb{SE}(2)$ and $\mathbb{R}^2$, and derived the sufficient and necessary conditions for building sensing graphs and localizable formations based on distributed exteroceptive sensors. The results were proved to be compatible with the well known results for closed kinematic chains and results that are obtained from graph rigidity theory. Experimental results with multiple robots show how the topology of the network or the underlying graph affects the quality of the resulting estimates.

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REFERENCES

[1] Belta, C., and Kumar, V., 2001. "Motion generation for formation of robots: a geometric approach". In Proceedings of International Conference on Robotics and Automation, Seoul, Korea.

[2] Das, A., Spletzer, J., , Kumar, V., and Taylor, C., 2002. "Ad hoc networks for localization and control". In Proceedings

of the 41st IEEE Conference on Decision and Control, Las Vegas, NV.
[3] Murray, R., Li, Z., and Sastry, S., 1994. *A Mathematical Introduction to Robotic Manipulation*. CRC Press.
[4] Zhang, F., Grocholsky, B., and Kumar, V., 2004. "Formations for localization of robot networks". In Proceedings of International Conference on Robotics and Automation, New Orleans, Louisiana.
[5] Kumar, V., Meyer, B., and Ulrich, N., November 1989. "Mechanics and design of a planar parallel manipulator". In Proceedings of the First National Conference on Applied Mechanisms and Robotics, Cincinnati.
[6] Freeman, R., and Tesar, D., 1988. "Dynamic modeling of serial and parallel mechanisms/robotic systems: part 1 - methodology". In Trends and Developments in Mechanisms, Machines, and Robotics, ASME pub., vol. 15-3, pp. 7–18.
[7] Agrawal, S. K., and Roth, B., 1992. "Statics of in-parallel manipulators". Journal of Mechanical Design, Transactions of the ASME, **114** (4) , pp. 564–568.
[8] Pfreundschuh, G., Sugar, T. G., and Kumar, V., 1994. "Design and control of a three degree-of-freedom, in-parallel, actuated manipulator". Journal of Robotic Systems, **11** (2) , pp. 103–115.
[9] Ebert-Uphoff, I., Gosselin, C., and Laliberte, T., March 2000. "Static balancing of spatial parallel platform mechanisms - revisited". ASME Journal of Mechanical Design, **122** (1) , pp. 43–51.
[10] Kumar, V., and Waldron, K., December 1988. "Force distribution in closed kinematic chains". IEEE Transactions on Robotics and Automation, **4** (6) , pp. 657–664.
[11] Bajpai, A., and Roth, B. "Workspace and mobility of a closed loop manipulator". International Journal of Robotics Research, **5** (2), pp. 139–150.
[12] Kumar, V., September 1990. "Characterization of workspaces of parallel manipulators". In ASME Proceedings of the 21th Biennial Mechanisms Conference, pp. 321–329.
[13] Behi, F., 1988. "Kinematic analysis for a six-degree-of-freedom 3-prps parallel mechanism". IEEE Journal of Robotics and Automation, **4** (5) , pp. 561–565.
[14] Wohlhart, K., 1999. "Degrees of shakiness". *Mechanism and Machine Theory,* **34** , pp. 1103–1126.
[15] Desai, J., Kumar, V., and Ostrowski, J., 2001. "A theoretical framework for modeling and controlling formation of mobile robot". *IEEE Transactions on Robotics and Automations* .
[16] Pappas, G., Tabuada, P., and Lima, P., 2001. "Feasible formations of multi-agent systems". In Proceedings of the American Control Conference, Arlington, Virginia.
[17] Laman, G., October 1970. "On graphs and rigidity of plane skeletal structures". *Journal of Engineering Mathematics,* **4** , pp. 331–340.
[18] Roth, B., 1982. "Rigidity and flexible frameworks". *The American Mathematical Monthly,* **88** , pp. 6–21.
[19] Whiteley, W., and Tay, T., 1985. "Generating isostatic frameworks". *Structural Topology,* **11** , pp. 21–69.
[20] Olfati-Saber, R., and Murray, R., 2002. "Graph rigidity and distributed formation stabilization of multi-vehicle systems". In Proceedings of the IEEE Conference on Decision and Control.
[21] Eren, T., Belhumeur, P., Anderson, B., and Morse, A., 2002. "A framework for maintaining formations based on rigidity". In Proceedings of the IFAC Congress, pp. 2752–2757.
[22] Maybeck, P. S., 1979. *Stochastic models, estimation and control*. Academic Press.
[23] Paul, B., June 1979. *Kinematics and dynamics of planar machinery*. Prentice Hall Inc., Englewoods Cliffs, New Jersey 07632.
[24] Hunt, K., 1978. *Kinematic geometry of mechanisms*. Clarendon Press, Oxford.