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Constructing Radio Signal Strength Maps with Multiple Robots

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Constructing Radio Signal Strength Maps with Multiple Robots

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Abstract—Communication is essential for coordination in most cooperative control and sensing paradigms. In this paper, we investigate the construction of a map of radio signal strength that can be used to plan multirobot tasks and also serve as useful perceptual information. We show how nominal models of an urban environment, such as those obtained by aerial surveillance, can be used to generate strategies for exploration and present preliminary experimental results with our multi-robot testbed.

I. INTRODUCTION

There is a growing community of researchers in multiagent robotics and sensor networks whose goal is to develop networks of sensors and robots that can perceive their environment and respond to it, anticipating information needs of the network users, repositioning and self-organizing themselves to best acquire and deliver the information. Communication is fundamental to most multi-agent coordinated tasks, such as, cooperative manipulation [1], multi-robot motion planning [2], collaborative mapping and exploration [3], and formation control [4]. Communication links are used to control the motion of the agents and for each agent to infer its location with respect to those of its neighbors and other landmarks. On the other hand, agents may also need to control their position and orientation relative to other agents to sustain communication links. While there is significant literature on multirobot control, sensing [5], planning [2], and localization [6], most of these papers focus on control and perception and assume that robots can freely communicate with each other.

Some recent papers have considered the effects of communication constraints. Reference [7] considers distributed multirobot sensing and data collection where the individual robot's communication range is assumed to be static. Decentralized controllers for concurrently moving toward goal destinations while maintaining communication constraints are discussed in [8]. The discrete motion planning problem of moving while maintaining visibility constraints is discussed in [9].

It is difficult, in general, to predict radio connectivity a priori since it depends upon a variety of factors including transmission power, terrain characteristics, and interference from other sources [10]. This suggests if we can learn the communication characteristics of the environment online, we can generate a radio connectivity map that can be used in the planning and deployment of future tasks.

In this paper, we consider the problem of acquiring information to obtain such radio signal strength maps in an urban terrain. We formulate the problem as an exploration of an environment with known geometry, but one in which the radio transmission characteristics are unknown. We assume that overhead surveillance pictures, such as the one shown in Figure 2(a), can be used to automatically construct roadmaps for motion planning, and we formulate the radio connectivity map exploration problem as a graph exploration problem. We describe algorithms that allow small teams of robots to explore two-dimensional workspaces with obstacles to obtain a radio connectivity map. The salient feature of our work is that we reduce the exploration problem to a multirobot graph exploration problem, which we solve for teams of two and three robots.

This paper is organized as follows. In Section 2, we describe the terminology and notation used to model the problem. The methodology is described in Section 3 for the two robot and three robot problems. Section 4 and 5 summarizes the results for both the two and three robot cases and provide some discussion on the computational complexity of the proposed algorithms. Section 6 discusses some ongoing research in exploration and ideas for future work.

II. MODELING

For any given environment, denote the configuration space as \( C \) and the obstacle-free portion of \( C \) as \( C_f \), also referred as the free space. Given any two positions \( q_i, q_j \in C_f \), the radio
connectivity map is a function $\varphi : (q_i, q_j) \rightarrow \mathbb{R}$ that returns the radio signal strength between the two positions given by $q_i$ and $q_j$. To obtain a connectivity map for all pairs of positions in $C_f$ is extremely difficult, instead, we propose to construct a map for pairs of locations in the set $Q = \{q_1, \ldots, q_n\}$ such that $Q$ is a subset of $C_f$.

We assume that a convex cell decomposition can be performed on any given $C_f$ such that each location in the set $Q$ is located within a cell. Since each cell is convex, is possible to predict the signal strength between any two points given the line-of-sight property associated with points in a convex set and prior knowledge of the variation of radio signal transmission characteristics with distance. This does not necessarily mean the signal strength will be the same for other pairs of positions in those two cells. However, we can effectively use the information about signal strength between a given pair of points and the knowledge of the transmission characteristics within the cell to deploy a multirobot team that can communicate via a multi-hop network between any pair of points. Thus, we will assume the decomposition is given instead of solving the problem of determining the appropriate cell decomposition.

We further assume a connected roadmap which can be constructed from the given cell decomposition of $C_f$ and computing the set of feasible paths between neighboring cells. Figure 2(b) is an example of a cell decomposition of $C_f$ for the site shown in Figure 2(a). The undirected graph $G_1 = (V_1, E_1)$ is a representation of the roadmap where each cell is associated with a node in $V_1$ and every edge in the set $E_1$ represents a feasible path between neighboring cells. Given,

$$V_1 = \{v_1^{(1)}, \ldots, v_1^{(n_1)}\} \quad \text{and} \quad E_1 = \{e_1^{(1)}, \ldots, e_1^{(m_1)}\},$$

the total number of nodes and edges in $G_1$, are denoted as $n_1$ and $m_1$ respectively. Thus, $G_1$ is always connected and we will denote $A_1$ as the adjacency matrix for $G_1$ such that

$$A_1 = [a_{ij}] = \begin{cases} 1 & \text{if path exists between } v_i \text{ and } v_j \\ 0 & \text{otherwise} \end{cases}$$

We will call $G_1$ the roadmap graph.

Next, we define the radiomap graph, $R = (V_1, L_1)$, where $L_1$ is the set of links between nodes we would like to gather signal strength information for. The edge set $L_1$ is selected a priori based on the task objectives, the physical environment and prior knowledge of radio signal transmission characteristics and may include all possible edges in $G_1$. In other words, $R$ encodes the information that must be obtained. We will denote $A_R$ as the adjacency matrix for $R$ such that

$$A_R = [a_{R_{ij}}] = \begin{cases} 1 & \text{if signal strength between } v_i \text{ and } v_j \text{ is to be measured} \\ 0 & \text{otherwise} \end{cases}$$

The objective is to develop an optimal plan to measure the signal strength of every edge in $L_1$ given $G_1$. Thus, given the roadmap and radiomap graphs, $G_1$ and $R$, we define a third graph, which we will call the multirobot exploration graph and denote it as $G_k = (V_k, E_k)$ where $k$ denotes the number of robots. We construct the multirobot exploration graph such that obtaining an optimal plan to measure the edges in $L_1$ is equivalent to solving for the shortest path on the graph $G_k$.

We outline our methodology in the following section.

III. METHODOLOGY

Given the roadmap, $G_1 = (V_1, E_1)$, and $k$ robots we define a configuration on the graph $G_1$ as an assignment of the $k$ robots to $k$ nodes of the graph. Figure 3(b) shows some possible configurations of three robots on the roadmap graph $G_1$, shown in Figure 3(a). Here solid vertices denote the locations of the robots. Since the graph $G_1$ is connected, a path always exists for $k$ robots to move from one configuration to another. For certain configurations of $k$ robots on $G_1$, the complete graph generated by taking the locations of the robots as vertices, contains some of the edges in $L_1$. Figure 4(b) shows some three robot configurations on $G_1$ that can measure edges in $L_1$, the edge set of the radiomap graph shown in Figure 4(a). Therefore, an optimal plan to measure all edges in the set $L_1$ can be viewed as a sequence of robot configurations such that every edge in $L_1$ is measured by at least one of these configurations.

In general, given the roadmap and radiomap graphs $G_1 = (V_1, E_1)$ and $R = (V_1, L_1)$ and $k$ robots, the multirobot exploration graph, $G_k = (V_k, E_k)$, is constructed such that every node in $V_k$ denotes a $k$-robot configuration on $G_1$ that measures a subset of $L_1$. An edge, $e_k^{ij} \in E_k$, exists between any two nodes $v_k^i, v_k^j \in V_k$ if the configuration associated with $v_k^i$ is reachable from the configuration associated with $v_k^j$. Since $G_1$ is always connected, $k$ robots can always move from one configuration to another, therefore, $G_k$ is always a complete graph. To obtain an optimal plan, every edge in $E_k$ is assigned a minimum cost that represents the total number of moves required to move the robots from one configuration to another.

For the configuration given by the nodes $\{2, 3, 4\}$ as shown in Figure 3(b), the cost to move to the configuration given by nodes $\{1, 2, 3\}$ is 2. The optimal plan would then be a sequence of configurations, such that moving through all configurations in the sequence results in covering all edges in $L_1$ while minimizing the number of total moves. In other words, finding an optimal plan is equivalent to solving for a minimum cost path on $G_k$ that covers all the edges of $L_1$.
We outline methods to construct $G_k$, for the two robot and three robot cases and solve for the respective optimal plans in the following sections.

A. Two Robot Problem

Given the roadmap and radiomap graphs $G_1 = (V_1, E_1)$ and $R = (V_1, L_1)$ and two robots, the maximum number of links that can be measured for any configuration is one. For the two robot case, the radio exploration graph $G_2 = (V_2, E_2)$ can be constructed such that each node in $G_2$ corresponds to one edge in the set $L_1$. For example, given the roadmap and radiomap graphs shown in Figure 5. Figure 6(a) shows the mapping of every edge in $L_1$ to a node in $G_2$. By computing the cost to move between every pair of nodes in $G_2$, we obtain the weight of every edge in $E_2$ as shown in 6(b). The minimum cost to move from the configuration $\{2, 6\}$ to $\{1, 5\}$, denoted by nodes $4'$ and $1'$ respectively in Figure 6(b), is equal to 2.

Thus, the edge $e_{\{4',1'\}}$ has a weight of 2. From this example, the optimal plan for a start configuration given by node $1'$ is the path $\{1',4',2',3'\}$ with a total cost of 6 moves. For the two robot case, an optimal plan requires the traversal of every node on $G_2$ at most once. This is equivalent to solving a traveling salesman problem on the graph $G_2$.

Algorithm 1 describes the method used to obtain the optimal plan for the 2-robot case. To determine the weight of every edge in $E_2$, we compute the shortest path between every pair of nodes in $G_1$. The adjacency and cost matrices for $G_2$ are obtained by considering the set of allowable moves given by $G_1$ and the set of edges given by $R$. Once we have the adjacency and cost matrices for $G_2$, the optimal plan is obtained by solving an open path traveling salesman problem on $G_2$. Although the Traveling Salesman Problem is known to be NP-hard, there are known approximation algorithms that solves for the minimum cost path in polynomial time [11]. For small graphs the problem can be solved using branch and bound techniques [12].

B. Three Robot Problem

Given the roadmap and radiomap graphs, $G_1$ and $R$, the set of nodes in $V_3$ is obtained by considering all 3-robot configurations on the graph $G_1$ that contain at least one edge in $L_1$. For the roadmap and radiomap graphs given in Figure 5, Figure 7(a) shows some configurations that contain some edges in $L_1$. The configuration given by nodes $\{1,5,6\}$ would correspond to node $1'$ on $G_3$. Figure 7(b) is a subgraph of $G_3$ with the nodes associated with the configurations shown in Figure 7(a) as its vertices. The algorithm to obtain the vertex set $V_3$ is outlined in Algorithm 2.

Similar to the two robot case, shortest path computation between every node in $G_1$ is required to determine the weight of every edge in $E_3$. The algorithm used compute the cost and adjacency matrices for $G_3$ is outlined in Algorithm 3. Unlike the two robot case, every edge in the set $L_1$ may potentially be associated with more than one node in $V_3$. Thus, the optimal plan for the three robot case would result in a path that contains a subset of the nodes in $V_3$. For this example, an optimal plan starting at the configuration given by node $1'$ is the path $\{1',2',4\}$ with a total cost of 4. Note the path does not contain node $3'$. Given a starting node on $G_3$, a greedy algorithm is
Algorithm 1 Computation of the optimal plan for 2-robots

Construction of the vertex set \( V_2 \)
Given \( G_1, A_1 \) and \( R, A_R \)
\( V_2 = 0 \)
for each node \( v_1^1, \ldots, v_1^{n_1} \) do
for each node \( v_1^1, \ldots, v_1^{n_1} \) do
if \( A_R(i, j) = 1 \) then
\( V_2 = V_2 \cup v_2^i \), where \( v_2^i \) denotes the vertex associated with \( v_1^i \) and \( v_1^j \)
end if
end for
end for

Computing the cost, \( C_2 \), and adjacency, \( A_2 \), matrices for \( G_2 \)
for each node \( (v_2^1, \ldots, v_2^{n_2}) \) do
for each node \( (v_2^1, \ldots, v_2^{n_2}) \) do
if \( v_2^i \neq v_2^j \) then
  determine number of moves required to move from \( v_2^i \) to \( v_2^j \) using \( A_1 \)
  \( A_2(i, j) = 1 \)
  \( C_2(i, j) = \) number of moves
end if
end for
end for

Compute minimum cost open path on \( G_2 \) such that each node in \( V_2 \) is traversed only once

Algorithm 2 Construction of the vertex set of \( G_3 = (V_3, E_3) \)

Given \( G_1, A_1 \) and \( R, A_R \)
\( V_3 = 0 \)
for each node \( (v_1^1, \ldots, v_1^{n_1}) \) do
for each node \( (v_1^1, \ldots, v_1^{n_1}) \) do
  for each node \( (v_1^1, \ldots, v_1^{n_1}) \) do
    if \( v_1^i \neq v_1^j \neq v_1^k \) then
      if \( (i, j, k) \) or \( (j, k) \) are in \( L_1 \) then
        \( V_3 = V_3 \cup v_3^i \), where \( v_3^i \) denotes the vertex associated with \( v_1^i \), \( v_1^j \), and \( v_1^k \)
      end if
    end if
  end for
end for

Algorithm 3 Computation of the adjacency and cost matrices, \( A_3 \) and \( C_3 \), for \( G_3 = (V_3, E_3) \)

Initialize \( A_3 \), \( C_3 \)
for each node \( (v_3^1, \ldots, v_3^{n_3}) \) do
  for each node \( (v_3^1, \ldots, v_3^{n_3}) \) do
    if \( v_3^i \neq v_3^j \) then
      Calculate minimum number of moves from \( v_3^i \) to \( v_3^j \)
      \( A_3(i, j) = 1 \)
      \( C_3(i, j) = \) minimum number of moves
    end if
  end for
end for

MOOT site can be found in [13] and [14]. We assume a cell decomposition of the free space as shown in Figure 2(b). The roadmap and radiomap graphs are shown in Figure 8. Using the procedure outlined in the previous sections, we construct the graphs \( G_2 \) and \( G_3 \) and solve for their optimal plans. To improve on the computation time of our algorithm we only considered edges in \( E_2 \) with weights less than or equal to two moves and edges in \( E_3 \) with weights less than or equal to six moves.

A. Two Robot Problem

Using the methodology outlined in the previous section and restricting the edge set of \( E_2 \) to edges with cost no more than two moves, we compute a total of 23 nodes and 75 edges for the multirobot exploration graph \( G_2 \). The minimum cost open path starting with one robot at node 5 and one at node 6 as shown in Figure 8(a) requires a total of 28 moves to cover every last edge shown in Figure 8(b). Figure 10 shows the step by step execution of the plan.

B. Three Robot Problem

For the three robot problem, we compute a total of 139 nodes and 6045 edges for the multirobot exploration graph \( G_3 \) by considering edges with cost no more than six moves. The minimum cost path starting with robots at nodes 6, 7 and 9 as shown in Figure 8(a) traverses a total of 13 nodes in \( G_3 \).
Fig. 8. (a) Roadmap graph for the site shown in Figure 2(a). (b) Roadmap graph for the site shown in Figure 2(a).

Fig. 9. Radio frequency map obtained by manually placing robots at each location associated with each node in \( G_2 \). Radio signal strength is normalized to a scale of 0 - 65 dB.

with a minimum cost of 31 moves. Figure 11 shows the step by step execution of the optimal plan.

Figure 9 shows a radio connectivity map for the MOUT site where the radio signal strength between any two locations are denoted by the different edges.

V. DISCUSSION

Without considering the cost of computing a solution for the traveling salesman problem, the adjacency and cost matrices for \( G_2 \) given \( G_1 \) and \( R \) can be obtained in \( O(n_2^3) \), where \( n_2 \) denotes the number of nodes in \( G_2 \). This is due to the need to compute shortest paths for all pairs of nodes in \( G_2 \). However, depending on the topology of \( G_1 \) and \( R \), we could decrease the computation time by considering edges with weights no more than \( x \) number of moves. Similarly, for the three robot case, without considering the computation of the shortest path on \( G_2 \), the proposed methodology requires a run time of \( O(n_2^3) \) where \( n_2 \) is the total number of nodes in \( G_2 \). It is worth noting that depending on the topology of \( R \), it is possible to further reduce both the number of nodes and the number of edges in \( G_3 \) by enforcing stricter selection criterion when generating the vertex set outlined in Algorithm 2 and considering edges weighing no more than \( y \) number of moves in Algorithm 3.

For example, if we only consider the set of nodes in \( G_1 \) such that every edge in the complete graph induced by the 3 robots is contained in \( L_1 \), then the number of nodes for \( G_3 \) can be reduced to a total of 15.

The difficulty in obtaining an optimal plan under the proposed methodology is the need to compute a minimum cost path on \( G_k \) such that every node on the path leads to measurement of every edge in \( L_1 \). Such minimum cost path computations are known to be extremely inefficient since the complexity is exponential in the number of nodes. For small graphs, the problem is solvable using branch and bound techniques. In general, the computational cost for finding a path on any \( G_k \) can be expensive and thus heuristic approaches need to be pursued.

VI. FUTURE WORK

In this work, we have addressed the case where the locations whose connectivity we wish to explore are given a priori. We hope to be able to address the problem of automatically selecting locations to be explored either by using overhead images which provide partial maps, or in the context of an online exploration process. Here we envision that we may want to consider the problem of selecting promising sites for communication relays. If we were able to identify and explore these locations efficiently we may choose to forgo the more laborious task of discovering the complete radio map of the site in favor of finding a set of locations that form an effective communication "skeleton" which allows us to span the site with communication links.

Similarly we can imagine focusing our exploration strategies to discover communication pathways that support the transmission of information from a particular area of interest back to the base station. This might be appropriate in situations where the users are interested in monitoring a particular area of the site.

Furthermore, it is often the case that the exploration of the radio map of the scene is being carried out concurrently with other activities such as environmental monitoring or situational awareness. Thus, another area which we plan to address is pursuing the radio mapping with other objectives and which must be effectively balanced against the other mission goals.

The ability to measure the strength of radio links between members of our mobile robot teams opens up many avenues for future work. We can imagine using the measurements gleaned from the robots to construct models for the transmission characteristics of the site. Since the rate of signal strength falloff with distance depends upon the composition of the materials in the environment and the geometry of the scene, it may be difficult to predict this relationship accurately before exploration. However, once the robots start their exploration we may be able to model this relationship effectively from measurements. These models could then be used to predict radio connectivity between locations that have not been visited.

Additional details and figures are available at http://www.seas.upenn.edu/~mya/publications/scra04-tech.pdf.

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