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Abstract
In this letter, we present and analyze theoretically some ideas for thin one-dimensional (1-D) cavity resonators in which a combination of a conventional dielectric material and a metamaterial possessing negative permittivity and permeability has been inserted. In this analysis, it is shown that a slab of metamaterial with negative permittivity and permeability can act as a phase compensator/conjugator and, thus, by combining such a slab with another slab made of a conventional dielectric material one can, in principle, have a 1-D cavity resonator whose dispersion relation may not depend on the *sum* of thicknesses of the interior materials filling this cavity, but instead it depends on the *ratio* of these thicknesses. In other words, one can, in principle, conceptualize a 1-D cavity resonator with the total thickness far less than the conventional λ/2. Mathematical steps and physical intuitions relevant to this problem are presented.

Keywords
Cavity resonator, metamaterials, negative index of refraction, negative permeability, negative permittivity, phase compensator, phase conjugation.

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***Abstract*—In this letter, we present and analyze theoretically some ideas for thin one-dimensional (1-D) cavity resonators in which a combination of a conventional dielectric material and a metamaterial possessing negative permittivity and permeability has been inserted. In this analysis, it is shown that a slab of metamaterial with negative permittivity and permeability can act as a phase compensator/conjugator and, thus, by combining such a slab with another slab made of a conventional dielectric material one can, in principle, have a 1-D cavity resonator whose dispersion relation may not depend on the *sum* of thicknesses of the interior materials filling this cavity, but instead it depends on the *ratio* of these thicknesses. In other words, one can, in principle, conceptualize a 1-D cavity resonator with the total thickness far less than the conventional λ/2. Mathematical steps and physical intuitions relevant to this problem are presented.**

***Index Terms*—Cavity resonator, metamaterials, negative index of refraction, negative permeability, negative permittivity, phase compensator, phase conjugation.**

I. Introduction

In the past several decades, the electromagnetic (EM) properties of complex media have been the subject of research study for many research groups [1]–[16]. Several types of EM complex media such as chiral materials, omega media, bianisotropic media, local, and nonlocal media to name a few, have been studied. Recently, the idea of composite materials in which both permittivity and permeability possess negative values at certain frequencies has gained considerable attention [17]–[21]. In 1967, Veselago theoretically investigated plane wave propagation in a material whose permittivity and permeability were assumed to be simultaneously negative [22]. His theoretical study showed that for a monochromatic uniform plane wave in such a medium, the direction of the Poynting vector is antiparallel with the direction of phase velocity, contrary to the case of plane wave propagation in conventional simple media. Recently, Smith *et al.* constructed such a composite medium for the microwave regime and demonstrated experimentally the presence of anomalous refraction in this medium [17], [18], [20], [21]. It is also worth noting that previous theoretical study of EM wave interaction with omega media reveals the possibility of having negative permittivity and permeability in omega media for a certain range of frequencies [23]. For metamaterials with negative permittivity and permeability, several names and terminologies have been suggested such as "left-handed" media [17]–[22], media with negative refractive index [17]–[22] "backward media" (BW media) [24], "double negative metamaterials" [25], [26], to name a few. The anomalous refraction at the boundary of such media and the fact that for a plane wave the direction of the Poynting vector is antiparallel with the direction of phase velocity, provide us with features that can be advantageous in design of novel devices and components. Recently, we introduced and presented in a symposium [27] one of our ideas for a compact cavity resonator. Here in this letter, we describe the details of this idea and the mathematical steps behind our analysis.

A. Metamaterials With Negative ε and μ as Phase Compensators/Conjugators

When a lossless metamaterial possesses negative real permittivity and permeability at certain frequencies, the index of refraction in such a medium attains real values. So as theoretically predicted by Veselago, the EM wave can propagate in such a medium [22]. However, for a monochromatic uniform plane wave in such a medium the phase velocity is in the opposite direction of the Poynting vector.

Consider a slab of conventional lossless material with real permittivity $\varepsilon_1 > 0$, real permeability $\mu_1 > 0$, and the index of refraction $n_1 \equiv \sqrt{(\varepsilon_1\mu_1)/(\varepsilon_o\mu_o)}$, where ε_o and μ_o are the permittivity and permeability of the free space. Here, n_1 is taken to be a positive real quantity. The slab is infinitely extent in the x-y plane and has the thickness d_1 along the z axis. We temporarily assume that the intrinsic impedance of the dielectric material $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ is the same as that of the outside region $\eta_o = \sqrt{\mu_o/\varepsilon_o}$, i.e., $\eta_1 = \eta_o$, but its refractive index is different from that of outside, i.e., $n_1 \neq n_o$. (We will soon remove the first part of this assumption.) Let us assume that a monochromatic uniform plane wave is normally incident on this slab. The wave propagates through the slab without any reflection (because for now we are still assuming $\eta_1 = \eta_o$). As this wave traverses this slab, the phase at the end of the slab is obviously different from the phase at the beginning of the slab by the amount $n_1 k_o d_1$, where $k_o \equiv \omega\sqrt{\varepsilon_o\mu_o}$. Now, consider a slab of a lossless metamaterial with negative real permittivity and permeability, i.e., $\varepsilon_2 < 0$ and $\mu_2 < 0$ at certain frequencies. For this slab, the index of refraction is a real quantity denoted by

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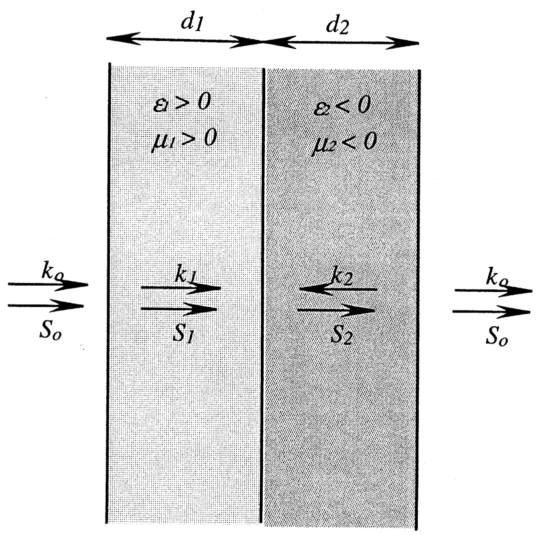

Fig. 1. A two-layer structure in which the left layer is assumed to be a conventional lossless dielectric material with $\varepsilon_1 > 0$ and $\mu_1 > 0$ and the right layer is taken to be a lossless metamaterial with negative permittivity and permeability. In the first layer, the direction of Poynting vector (S_1) is parallel with the direction of phase velocity or wave vector (k_1), whereas in the second layer, these two directions are antiparallel. With proper choice of ratio of d_1 and d_2, one can have the phase of the wave at the left (entrance) interface to be the same as the phase at the right (exit) interface, essentially with no constraint on the total thickness of the structure.

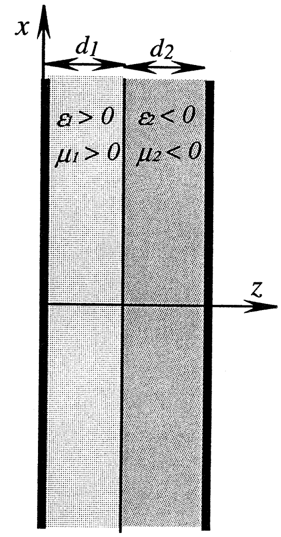

Fig. 2. An idea for a compact, subwavelength, thin cavity resonator. The two-layer structure discussed in Fig. 1 is sandwiched between the two reflectors. Our analysis shows that with the proper choice of ratio of d_1 over d_2, one can have a resonant cavity in which the ratio of d_1 and d_2 is the main constraint, *not the sum of thicknesses* $d_1 + d_2$.

$n_2 \equiv \sqrt{(\varepsilon_2\mu_2)/(\varepsilon_o\mu_o)}$. It is important to note that here we do not need to specify any sign for the operation of the square root appearing in the expression of n_2. We only need to state that n_2 is a real quantity for the lossless metamaterial with $\varepsilon_2 < 0$ and $\mu_2 < 0$ for a given frequency. (Here, n_2 can, for example, be taken to be a positive real quantity.) This slab is also infinitely extent in the x-y plane, but has a thickness of d_2 in the z direction. For now, we again assume that the intrinsic impedance of this metamaterial $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$ is also the same as that of outside region, i.e., $\eta_2 = \eta_o$. We put this slab right next to the first slab (Fig. 1). As the plane wave exits the first slab, it enters the slab of metamaterial and finally it leaves this second slab. The direction of power flow (i.e., the Poynting vector) in the first slab should be the same as that in the second one, because the power of the incident wave enters the first slab (without any reflection at the first interface), traverses the first slab, exits the second interface, enters the second slab and traverses it, and finally leaves the second slab. In the first slab, the direction of the Poynting vector is parallel with the direction of phase velocity; however, in the second slab these two vectors are antiparallel (see Fig. 1). Therefore, the wave vector $k_2(\equiv n_2k_o)$ is in the opposite direction of the wave vector $k_1(\equiv n_1k_o)$. As a result, the phase at the end of the second slab is different from the phase at the beginning of it by the amount $-n_2k_od_2$. (As was mentioned above, n_2 here is taken to be positive.) So the total phase difference between the front and back faces of this two-layer structure is $n_1k_od_1 - n_2k_od_2$. Therefore, whatever phase difference is developed by traversing the first slab, it can be decreased and even cancelled by traversing the second slab. If the ratio of d_1 and d_2 is chosen to be $d_1/d_2 = n_2/n_1$, then the total phase difference between the front and back faces of this two-layer structure becomes zero. (The total phase difference is not 2π, 4π, or 6π. But instead it is zero!) So indeed the slab of metamaterial with $\varepsilon_2 < 0$ and $\mu_2 < 0$ at given frequencies can act as the phase compensator in this structure. This also resembles the process of phase conjugation. It is important to note that such phase cancellation in this geometry does not depend on the *sum* of thicknesses $d_1 + d_2$; rather it depends on the *ratio* of d_1 and d_2. So, in principle, $d_1 + d_2$ can be any value as long as d_1/d_2 satisfies the above condition. Therefore, even though this two-layer structure is present, the wave traversing this structure would not experience the phase difference. This feature can lead to several interesting ideas in design of some devices and components.

B. Compact Subwavelength 1-D Cavity Resonators Using Metamaterials With $\varepsilon_2 < 0$ *and* $\mu_2 < 0$

What we described above can be used to conceptualize an exciting possibility of designing a compact 1-D cavity resonator. We can take the above two-layer structure and put two perfect reflectors (e.g., two perfectly conducting plates) at the two open surfaces of this structure (Fig. 2). Here, we generalize the problem by assuming that the intrinsic impedances of the first layer (conventional material) and the second layer (metamaterial with $\varepsilon_2 < 0$ and $\mu_2 < 0$ at specific frequencies) are not taken to be the same as η_o. So here, in general, we have $\eta_1 \neq \eta_2 \neq \eta_o$ and $n_1 \neq n_2 \neq n_o$. We are now interested to solve for solutions of Maxwell equations in this cavity resonator.

C. Formulation of the Problem

We use the Cartesian coordinate system (x, y, z), where the plane $z = 0$ is taken to be at the perfectly conducting plate located at the left face of the conventional material slab shown in Fig. 2. The other perfectly conducting plate is placed at $z = d_1 + d_2$, which is the right face of the metamaterial slab. Since this is assumed to be a 1-D cavity resonator, all quantities are independent of the x and y coordinates. The time dependence for the monochromatic solutions is taken to be $\exp(-i\omega t)$. Without loss of generality, we take the electric and magnetic field vectors to be oriented along the x and y direction, respectively. In the region $0 \leq z \leq d_1$, where the material is a conventional lossless material, the electric and magnetic fields can be written as

$$\begin{aligned} E_{x1} &= E_{o1} \sin(n_1 k_o z) \\ H_{y1} &= \frac{n_1 k_o}{i\omega\mu_1} E_{o1} \cos(n_1 k_o z) \end{aligned} \tag{1}$$

and in the region $d_1 \leq z \leq d_1 + d_2$, where the metamaterial with $\varepsilon_2 < 0$ and $\mu_2 < 0$ is located, the fields are written as

$$\begin{aligned} E_{x2} &= E_{o2} \sin[n_2 k_o (d_1 + d_2 - z)] \\ H_{y2} &= -\frac{n_2 k_o}{i\omega\mu_2} E_{o2} \cos[n_2 k_o (d_1 + d_2 - z)] \end{aligned} \tag{2}$$

where subscripts "1" and "2" denote the quantities in the regions "1" and "2," which are the conventional slab and the metamaterial slab, respectively. It is worth emphasizing that although in the above equations the values of indexes of refraction are taken to be positive quantities (and, hence, no ambiguities are introduced through the sign of the operation of square root for $n_2 \equiv \sqrt{(\varepsilon_2\mu_2)/(\varepsilon_o\mu_o)}$), as will be seen shortly the choice of the sign for n_1 and n_2 will be irrelevant in the final results. The choice of the solutions presented in (1) and (2) guarantees the satisfaction of the boundary conditions at the perfectly conducting plates at $z = 0$ and $z = d_1 + d_2$. To satisfy the boundary conditions at the interface between the two slabs we should have

$$\begin{aligned} E_{x1}|_{z=d_1} &= E_{x2}|_{z=d_1} \\ H_{y1}|_{z=d_1} &= H_{y2}|_{z=d_1} \end{aligned} \tag{3}$$

which leads to

$$\begin{aligned} E_{o1} \sin(n_1 k_o d_1) - E_{o2} \sin(n_2 k_o d_2) &= 0 \\ \frac{n_1}{\mu_1} E_{o1} \cos(n_1 k_o d_1) + \frac{n_2}{\mu_2} E_{o2} \cos(n_2 k_o d_2) &= 0. \end{aligned} \tag{4}$$

In order to have a nontrivial solution, i.e., to have $E_{o1} \neq 0$ and $E_{o2} \neq 0$, the determinant in (4) must vanish. That is

$$\begin{aligned} &\frac{n_2}{\mu_2} \sin(n_1 k_o d_1) \cos(n_2 k_o d_2) + \frac{n_1}{\mu_1} \sin(n_2 k_o d_2) \cos(n_1 k_o d_1) \\ &\quad = 0 \end{aligned} \tag{5}$$

which can be simplified to

$$\frac{n_2}{\mu_2} \tan(n_1 k_o d_1) + \frac{n_1}{\mu_1} \tan(n_2 k_o d_2) = 0. \tag{6}$$

In the above dispersion relation, the quantities n_1, μ_1, n_2, μ_2, and k_o are all generally frequency dependent. It is important to note that the choice of sign for n_1 and n_2 does not affect this dispersion relation. Either choice of sign (positive or negative sign) for n_1 and n_2 will leave (6) unchanged. That is why we specifically mentioned earlier that we did not need to introduce any ambiguity regarding the choice of sign for n_2 in our analysis here. Since the first layer is assumed to be made of a lossless conventional material, its permeability μ_1 is a positive real quantity. The second layer is taken to be a lossless metamaterial with $\varepsilon_2 < 0$ and $\mu_2 < 0$. Therefore, we can write $\mu_1 = |\mu_1|$ and $\mu_2 = -|\mu_2|$. Substituting these expressions in (6), we obtain

$$-\frac{n_2}{|\mu_2|} \tan(n_1 k_o d_1) + \frac{n_1}{|\mu_1|} \tan(n_2 k_o d_2) = 0. \tag{7}$$

This implies that for a given frequency ω, if $\varepsilon_1 > 0$, $\mu_1 > 0$, $\varepsilon_2 < 0$, and $\mu_2 < 0$, a nontrivial one-dimensional (1-D) solution for this cavity is obtained when the thicknesses d_1 and d_2 satisfy the relation

$$\frac{\tan(n_1 k_o d_1)}{\tan(n_2 k_o d_2)} = \frac{n_1 |\mu_2|}{n_2 |\mu_1|}. \tag{8}$$

This relation does not show any constraint on the sum of thicknesses of d_1 and d_2. It rather deals with the ratio of tangent of these thicknesses (with multiplicative constants). So, in principle, d_1 and d_2 can conceptually be as thin or as thick as otherwise needed as long as the above ratio is satisfied. If we assume that ω, d_1 and d_2 are chosen such that the small-argument approximation can be used for the tangent function, the above relation can be simplified as

$$\frac{d_1}{d_2} \cong \frac{|\mu_2|}{|\mu_1|}. \tag{9}$$

This relation shows even more clearly how d_1 and d_2 should be related in order to have a nontrivial 1-D solution with frequency ω for this cavity. So conceptually, what is constrained here is d_1/d_2, not $d_1 + d_2$. Therefore, in principle, one can have a thin subwavelength cavity resonator for a given frequency, if at this frequency the second layer acts a metamaterial with negative permittivity and permeability and the ratio d_1/d_2 satisfies the above condition. For example, for frequency of 2 GHz, if a metamaterial with negative permittivity of $-0.5\varepsilon_o$ and negative permeability of $-0.5\mu_o$ can be constructed as the second slab and if the conventional material slab is assumed to be air with ε_o and μ_o, then $n_1 = 1$ and $n_2 = 0.5$ and, thus, the required ratio of d_1 over d_2 should be $d_1/d_2 \simeq 0.5$. If, in principle, this metamaterial slab can be made thin for this frequency, e.g., $d_2 = \lambda_o/10$, where λ_o is the free-space wavelength of operation, then the air slab should be made with thickness $d_1 = \lambda_o/20$. Thus, the total thickness of such a thin cavity would be $d_1 + d_2 = 3\lambda_o/20$, which for this example of 2-GHz frequency of operation would be 2.25 cm! This is, of course, thinner than the conventional air cavity size of $\lambda_o/2$, which would be 7.5 cm for 2 GHz.

The electric and magnetic field expressions for the nontrivial solutions in this 1-D cavity are given as

$$\begin{aligned} E_{x1} &= E_o \sin(n_2 k_o d_2) \sin(n_1 k_o z) \\ H_{y1} &= \frac{n_1 k_o}{i\omega\mu_1} E_o \sin(n_2 k_o d_2) \cos(n_1 k_o z) \\ E_{x2} &= E_o \sin(n_1 k_o d_1) \sin[n_2 k_o (d_1 + d_2 - z)] \\ H_{y2} &= -\frac{n_2 k_o}{i\omega\mu_2} E_o \sin(n_1 k_o d_1) \cos[n_2 k_o (d_1 + d_2 - z)] \end{aligned} \tag{10}$$

where $\mu_1 = |\mu_1|$ and $\mu_2 = -|\mu_2|$.

It is worth noting that if both layers 1 and 2 had been made of two conventional lossless dielectric materials, the form of the dispersion relation in (6) would have remained unchanged.

However, both quantities μ_1 and μ_2 would have been positive real and, thus, if $\tan(n_1 k_o d_1)$ had been positive, the other term $\tan(n_2 k_o d_2)$ must have been negative in order to fulfill the dispersion relation in (6). This implies that for such a case, if $d_1 < \pi/2n_1 k_o$ for $\tan(n_1 k_o d_1)$ to be positive, then d_2 must have been greater than $\pi/2n_2 k_o$ in order to have $\tan(n_2 k_o d_2)$ negative, which puts some limits on the sum of thicknesses d_1 and d_2. In the case under study in this letter, however, since $\mu_1 > 0$ and $\mu_2 < 0$, the two functions $\tan(n_1 k_o d_1)$ and $\tan(n_2 k_o d_2)$ can be both positive (or both negative) and, as a result, there is no constraint on the sum of d_1 and d_2.

Finally, one notes that the electric field expressions may possess discontinuous first derivatives (with respect to z) at the boundary between the two layers, i.e.,

$$\left.\frac{\partial E_{x1}}{\partial z}\right|_{z=d_1} \neq \left.\frac{\partial E_{x2}}{\partial z}\right|_{z=d_1}.$$

This should not cause any concern since according to the Maxwell equations, the tangential component of the magnetic field in this problem, which is expressed as $H_y = \frac{1}{i\omega\mu}\frac{\partial E_x}{\partial z}$ are continuous at such a boundary. This implies that $\frac{1}{i\omega\mu_1}\frac{\partial E_{x1}}{\partial z} = \frac{1}{i\omega\mu_2}\frac{\partial E_{x2}}{\partial z}$. Since in our problem here, $\mu_1 > 0$ and $\mu_2 < 0$, the first z-derivatives of electric field $\partial E_{x1}/\partial z$ and $\partial E_{x2}/\partial z$ have opposite signs at the interface between the two layers.

II. Summary

We have introduced ideas for 1-D cavity resonators utilizing the concept of lossless metamaterials in which both permittivity and permeability possess negative real values at given frequencies. We have shown that a slab of metamaterial having negative permittivity and permeability can function as a phase compensator/conjugator. Our analysis has also shown that when the cavity is filled with two layers of materials; the first layer assumed to be a lossless conventional material and the second layer is taken to be the metamaterial with negative permittivity and permittivity, the nontrivial 1-D solutions for such a cavity, in principle, depend on the ratio of thicknesses of the two layers, not the sum of thicknesses. In other words, the cavity can conceptually be thin and can still be resonant, as long as the ratio of thicknesses is satisfied in the special dispersion relation. This can, in principle, provide possibility for having subwavelength thin compact cavity resonators. Such subwavelength cavity resonators can lead to very interesting designs for various compact subwavelength devices and components.

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