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Study of a novel range-dependent propagation effect with application to the axial injection of signals from the Kaneohe source

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A novel range-dependent propagation effect occurs when a source is placed on the seafloor in shallow water with a downward refracting sound speed profile, and sound waves propagate down a slope into deep water. Under these conditions, small grazing-angle sound waves slide along the bottom downward and outward from the source until they reach the depth of the sound channel axis in deep water, where they are detached from the sloping bottom and continue to propagate outward near the sound channel axis. This "mudslide" effect is one of a few robust and predictable acoustic propagation effects that occur in range-dependent ocean environments. As a consequence of this effect, a bottom mounted source in shallow water can inject a significant amount of acoustic energy into the axis of the deep ocean sound channel that can then propagate to very long ranges. Numerical simulations with a full-wave range-dependent acoustic model show that the Kaneohe experiment had the appropriate source, bathymetry, and sound speed profiles that allows this effect to operate efficiently. This supports the interpretation that some of the near-axial acoustic signals, received near the coast of California from the bottom mounted source located in shallow water in Kaneohe Bay, Oahu, Hawaii, were injected into the sound channel of the deep Pacific Ocean by this mechanism. Numerical simulations suggest that the mudslide effect is robust. © 2002 Acoustical Society of America. [DOI: 10.1121/1.1432983]

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I. INTRODUCTION

Sound propagation in range-dependent ocean environments has long been recognized to be a difficult modeling and prediction problem. Normal modes are in general coupled in complicated ways that defy simple physical explanation, and rays may be converted from one type to another and often back again in an exceedingly complex manner.

The recent discovery of ray chaos in underwater acoustics, and its associated finite frequency manifestations, has re-emphasized the distinction between range-independent and range-dependent propagation, since a necessary condition for chaos is range dependence of the environment. For range-independent propagation, variables separate, the ray equations are completely integrable, and in principle the solution of the acoustic wave equation can be explicitly written down at any range and then analyzed in detail. In outline, at least, the physics of range-independent propagation is fully understood, numerically computed solutions are stable and robust, and no surprising new physical effects are expected. In contrast, for range-dependent propagation there exist numerical models that march the solution in range out from the source (or from the receiver when reciprocity is invoked),

and almost every new application reveals new phenomena unique to the particular range-dependent environment where propagation is modeled.^{1,2}

Although it is unlikely that the complete set of phenomena that occur in range-dependent sound propagation will ever be classified and fully understood, it is worthwhile to look for a subset of range-dependent propagation phenomena that are robust and at least qualitatively predictable. Several examples come to mind: the cross-frontal propagation effect, the bathymetric blockage effect, and the slope enhancement effect. One purpose of this article is to add to this list a novel range-dependent propagation effect that we call the "mudslide" effect.

A second purpose of this article is to further understand the acoustic propagation effects associated with signals received at ranges of about 4000 km from the source located in Kaneohe Bay, Oahu, Hawaii. ⁴⁻⁶ A fundamental question is how can acoustic signals be efficiently injected into the axis of the deep ocean sound channel from a bottom mounted source located in shallow water? The mudslide effect described below offers one plausible answer to this question. In fact, this is what Spiesberger and Tappert⁶ showed must be happening although the specific mechanism was not identified.

The mudslide effect is explained in Sec. II in terms of the basic physics of sound propagation in range-dependent oceanic wave guides. Section III contains numerical simulations with the UMPE acoustic model of the mudslide effect in the context of the Kaneohe source. Finally, the results are discussed in Section IV.

II. MUDSLIDE EFFECT

This novel range-dependent propagation effect occurs when a source is placed on the seafloor in shallow water with a downward refracting sound speed profile, and sound waves propagate down a slope into deep water. As a rule, sound speed profiles in shallow water are downward refracting at nonpolar latitudes, so this assumption is not a major restriction. Since bottom losses would significantly attenuate the signal, the source should not be placed too far shoreward from deep water.

Under these conditions, small grazing-angle sound waves slide along the bottom downward and outward from the source until they reach the sound channel axis in deep water, where they detach from the sloping bottom and continue to propagate outward near the sound channel axis. The reason for this behavior is that the minimum sound speed in the water column in shallow water is at the bottom. Since the seafloor is a good reflector of small grazing-angle waves, and those that penetrate are refracted upward by sediment gradients of sound speed, these small angle waves are trapped in a wave guide that follows the down-sloping bathymetry toward deeper water. Waves that have steeper grazing angles are more strongly attenuated by bottom losses, and are not guided downward by this sound speed minimum.

As the nearly bottom-grazing sound waves descend the slope, the sound speed gradient g = dc/dz decreases in magnitude until it vanishes when the axis of the deep ocean sound channel is reached. At and near this depth, the sound waves no longer feel the sound speed gradient and they detach from the bottom and propagate freely near the sound channel axis without being further influenced by bottom interactions.

The term mudslide effect is used to describe this range-dependent acoustic propagation effect. The name is derived from a pictorial analogy (not physical) with undersea mudslides, or turbidity currents, whereby water containing suspended sediments flows down slopes beneath the less dense ambient seawater until a depth of neutral buoyancy is reached where the sediment-laden water moves off horizontally into the deep ocean. Turbidity currents are important in marine geology, and survey articles describing the associated hydrodynamic phenomena have recently appeared. ^{7,8} Observations of the resulting marine deposits, called turbidites, have been reported by Tucholke, ^{9,10} who observed them in the abyssal plains of the Atlantic Ocean, several thousand kilometers from their presumed place of origin.

The above physical description of the underwater acoustic mudslide effect indicates that it is robust and qualitatively predictable in real ocean environments. This effect is expected to have wide applicability in ocean acoustics. However, quantitatively accurate predictions of sound pressure

levels, depth spreads around the axis, multipath time spreads, etc., require a high-fidelity, range-dependent acoustic model and accurate environmental input data.

The above physical description can be partly substantiated by the following idealized ray theoretic analysis based on the first-order parabolic approximation and specular reflections from the sloping bottom. The usual convention calls for the depth z to increase downward, and a positive grazing angle θ to be downgoing. Let us assume that the sound speed gradient in the shallow water is constant and downward refracting, dc/dz=g=const with g<0, and that the water depth has constant slope, $z_b=z_0+sr$ with s>0. If the launch angle of a ray is $\theta_0< s$ and the ray starts at the seafloor at r=0 and $z=z_0$, then it can be shown that the maximum height of the ray above the bottom is constant and given by

$$h = \frac{c_0}{2|g|} (s - \theta_0)^2. \tag{1}$$

Physically, this result means that the rays follow the slope downward into deeper water. For example, if $g = -0.1 \text{ s}^{-1}$, s = 0.1, and a ray is launched horizontally, then h = 75 m and this ray remains within 75 m of the bottom as it proceeds outward and downward. Other realistic values of g, s, and θ_0 give comparable values of h. A similar calculation yields the travel time of an RBR (refracted-bottom-reflected)³ ray after N reflections from the bottom as

$$t = \frac{r}{c_0} - \frac{N}{3|g|} (s - \theta_0)^3. \tag{2}$$

Since $N \approx (|g|r/2c_0)/(s-\theta_0)$, it follows that the travel time to range r is approximately

$$t \approx \frac{r}{c_0} \left[1 - \frac{1}{6} (s - \theta_0)^2 \right].$$
 (3)

This shows that the multipath travel time spread of small grazing-angle rays ($|s-\theta_0| \le 1$) is quite small during the downslope portion of the mudslide effect. The mudslide effect is not qualitatively altered by the inclusion of diffraction or by invoking the parabolic approximation.

The robustness of the mudslide effect requires analysis of more general environmental conditions: variable sound speed gradients, variable bottom slopes, bottom losses, bottom roughness, and surface reflections. This is best done with a numerical acoustic model. In the next section, a real ocean acoustic problem is modeled and robustness is established. Figure 1, that is more fully discussed in the next section, gives a dramatic illustration of the mudslide effect and shows that sound waves appear to flow down a slope in a manner that resembles an undersea mudslide.

Similar plots appeared earlier in model studies of sound propagation in the Straits of Florida. 11,12 Modeled sound propagated downwards hugging the bottom slope of the Florida Terrace eastwards into deeper water. The significance for tomography, which was the purpose of those studies, was that this sound went deeper than the core of the Gulf Stream. The generality of the mudslide effect was not described by those researchers.

The slope enhancement effect¹³ has recently been thoroughly explored in a master article by Dosso and

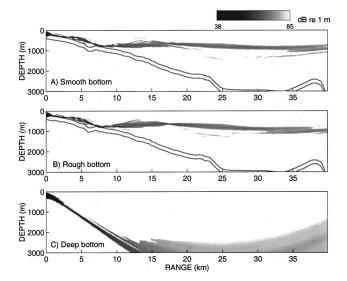


FIG. 1. The mudslide effect. (A) Downward propagating sound inclined near 5 degrees in about a one-degree beam width is modeled from the depth of the Kaneohe source into the ocean along smooth water/sediment and sediment/basement interfaces. Only the top 6 dB of sound pressure levels are shown at each range step to emphasize the location of the loudest sound. The top and bottom lines show the water/sediment and sediment/basement interfaces. (B) Same as (A) except the standard deviation of the water/sediment and sediment/basement interfaces are 2 and 8 m, respectively. (C) Same as (A) except the bottom is moved to 5000 m. No energy is trapped near 800 m depth without the mudslide effect arising from the sound speed gradients near the water/sediment interface.

Chapman, 14 which includes experimental data and a fullwave PE modeling effort that yields excellent agreement with the data. The slope enhancement effect differs from the mudslide effect in two important ways. First, in the slope enhancement effect the source is near the sea surface and is horizontally located where the depth of the minimum of sound speed intersects the bathymetric slope. In the mudslide effect, the source is mounted on the bottom and is in shallower water. Second, in the slope enhancement effect the downward refracting sound speed profile in shallow water plays no significant role, whereas it is essential to the mudslide effect. Thus, although both of these effects inject significant amounts of energy from a source into the axis of the deep water sound channel, the mechanisms of injection are distinctly different with the mudslide effect allowing the source to be far removed from the point where the axis of the sound channel intersects the slope.

III. KANEOHE SOURCE

The University of Miami Parabolic Equation (UMPE) acoustic model¹⁵ is used to perform numerical simulations of propagation from the Kaneohe source.⁴ This full-wave UMPE model uses the efficient Split-Step Fourier (SSF) algorithm¹⁶ to march the acoustic field outward from the source, and the efficient Fourier synthesis technique to obtain pulse response functions in the time domain.¹⁷ The propagator used in this study is the recently developed wide-angle c_0 -insensitive parabolic approximation¹⁸ that is fully second order accurate.¹⁹ Convergence of the UMPE model has been

tested for the cases presented below by refining the meshes in depth and range. The range mesh is 25.0 m and the depth mesh is 7.8 m.

The Kaneohe source, 4-6 that was active in the years 1983–1989, was located at 21.512 35°N latitude and 202.228 48°E longitude. Acoustic signals from this source, received at megameter ranges, were used to investigate possible global warming. Since the source was bottom-mounted in shallow water at about 180 m depth and had only about 180 dB source level, it was not initially obvious that signals could be transmitted to megameter ranges with enough power to be detectable. However, this proved to be possible. 4.5

The center frequency of the omnidirectional source was 133 Hz, and the bandwidth was 16 Hz. The geodesic on the ellipsoidal Earth from the source to the northern California receiver, whose location was indicated in Ref. 6, has a bearing at the source of 48.826° with respect to true North. Along this geodesic track, high resolution bathymetry near Hawaii was extracted from a detailed nautical chart compiled with Sea Beam²⁰ data and was then slightly smoothed. This deterministic bathymetry is shown in the upper two panels of Fig. 1. The same bathymetry was used in Ref. 6. Sound speed profiles along the same geodesic were obtained from the Levitus²¹ springtime data base using Del Grosso's sound speed formula as described in Ref. 6. The sound channel axis is at a depth of about 800 m (Fig. 3) within 100 km of the source.

In the numerical Fourier synthesis from frequency space to time, a Hann window (raised cosine) having bandwidth of 33 Hz is used in frequency space to generate the source function. ¹⁸ This gives an effective bandwidth of 16 Hz, corresponding to a transmitted pulse length of about 60 ms. ^{5,6}

The techniques for modeling the geoacoustic bottom interactions are described in Ref. 22. The thickness of the sediment layer out to 100 km from the Kaneohe source is assumed to have the constant value of 200 m. The ratio of the compressional wave speed at the top of the sediment layer to the sound speed (range-dependent) at the bottom of the water column is 1.02, and the sound speed gradient within the penetrable sediment layer is $g = 1.0 \text{ s}^{-1}$. The density ratio of the sediment to water is 1.7, and the attenuation is 0.02 dB/ km Hz. The semi-infinite basement layer is modeled as a fluid with enhanced attenuation to account for shear wave conversion losses. The constant properties of the basement are modeled as in Ref. 23: sound speed ratio is 2.0; density ratio is 2.5; and the attenuation is 0.5 dB/km Hz. Bottom and basement roughness is modeled as described in Ref. 22. The horizontal correlation length of the power-law spectrum is 2000 m (unless noted otherwise), the rms displacement of the water-sediment interface is 2.0 m, and the rms displacement of the sediment-basement interface is 8.0 m. Modeling is also done with the roughness turned off, and is labeled "smooth bottom" in this case.

Three traditional PE field plots^{1,2} displaying the CW transmission loss (in units of dB re 1 m) at center frequency 133 Hz out to the range of 40 km are shown in Fig. 1. In order to clearly show the mudslide effect from the Kaneohe source, the omnidirectionality of that source is replaced with

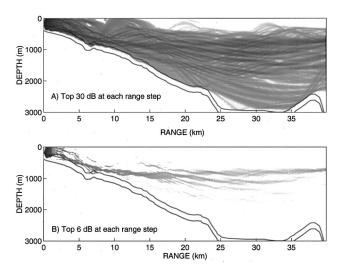


FIG. 2. Same as Fig. 1(A) except for an omnidirectional source. Panel (A) shows the top 30 dB of transmission loss at each range step. Panel (B) shows the top 6 dB of transmission loss at each range step.

an acoustic beam inclined downwards at 5 degrees in a 1 degree beam width. It is more difficult to see the mudslide effect if the acoustic model has an omnidirectional source (Fig. 2), because sound emitted from the real Kaneohe source also reflects from the surface and bottom slope of Oahu before becoming trapped in the acoustic wave guide in the deep ocean. Referring then to panel (A) in Fig. 1, some of the acoustic field propagates out from the Kaneohe source along a smooth bottom. Panel (B) shows the same process with a rough bottom. Both field plots dramatically illustrate the mudslide effect. Sound "slides" down the slope to the range of about 7 km and then propagates in deep water near the depth at which the sound speed is a minimum, i.e., 800 m (Fig. 3). Although the influence of bottom and sub-bottom roughness changes the details of the acoustic field, the mudslide effect is seen to be robust. There is no acoustic energy trapped near the depth of minimum sound speed when the source is placed at the same depth in deep water [Fig. 1(C)]. Panel (B) is computed for a horizontal roughness length of 2000 m. Very similar results are obtained with a horizontal

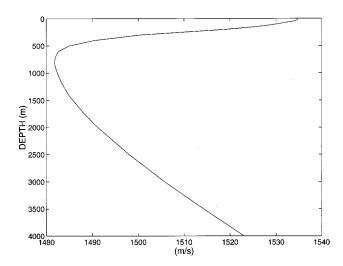


FIG. 3. The average speed of sound versus depth near the Kaneohe source from Spring (Ref. 21). The depth at which the speed is minimum is 800 m.

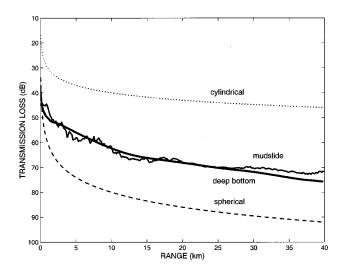


FIG. 4. The minimum transmission loss at each range step for panels (A) and (C) of Fig. 1. For reference, spherical and cylindrical losses are indicated. These losses are defined to be $10 \log_{10}(R^2)$ and $10 \log_{10}(R)$, respectively, where R is the distance from the source in meters. Note that the transmission loss from the mudslide effect is within a few decibels of that from sound which propagates into deep water.

roughness length of 200 m, so the mudslide effect is robust for these shorter undulations of the bottom too (not shown).

The amount of energy transmitted by the mudslide effect to the depth of minimum sound speed is within a few decibels of that obtained by placing the same source in deep water where the sound does not interact with a bathymetric slope (Fig. 4). These results are obtained from identical sources that beam energy downwards at 5 degrees in a 1 degree beam width as shown in panels (A) and (C) of Fig. 1. Thus the mudslide effect efficiently injects a large amount of acoustic power into near-axial waves that can then propagate to very long ranges, as observed by Spiesberger *et al.*^{5,6} Once the sound waves are injected into the sound channel, they hardly interact further with the surface or bottom; the only significant loss mechanism thereafter is cylindrical spreading and a small amount of loss due to seawater absorption

As opposed to the single frequency runs discussed above, a narrow band of frequencies over 16 Hz are modeled with the same full wave model to mimic the frequencies emitted from the Kaneohe source. It is possible to then examine the temporal aspects of modeled signals from this source using an inverse Fourier transform.

The time interval between the last and first arriving paths at distances of about 40 and 105 km is small (Fig. 5). At a distance of 40 km, the energy arrives within about 0.06 s. This is the same as the resolution of the transmitted pulse, i.e., 1/(16 Hz) which is 0.06 s. Thus, despite the complicated bottom interactions that occur during the mudslide process, the multipath time spread is actually quite small, just as predicted by the simplified analysis leading to Eq. (3). Some of the energy near 800 m depth comes from the mudslide effect and it arrives before the energy at other depths. This arrival order is opposite to that found for propagation in deep water in the northeast Pacific Ocean where the energy near the depth of minimum sound speed arrives last. The mudslide effect appears to contribute to the small time spread of paths

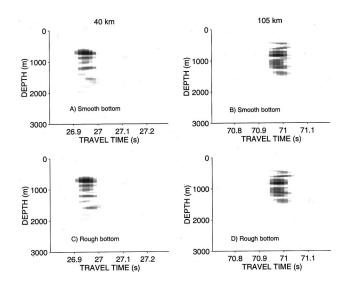


FIG. 5. Simulated time fronts from the Kaneohe source at distances of 40 and 105 km using smooth and rough water/sediment and sediment/basement interfaces. Note that the travel times at about 800 m depth correspond to the energy from the mudslide effect, and it arrives before the steeper inclined energy. Without a mudslide effect, the steeper energy arrives first in the northeast Pacific Ocean. The top 16 dB of energy is shown at each range with loudest as black. For the "rough" cases, the rms values of the water/sediment and sediment/basement interfaces are 2 and 8 m, respectively. The correlation distance is 200 m for these roughness values. The rms values of the roughness are zero for the "smooth" cases. Results are computed for an acoustic center frequency of 133 Hz and a bandwidth of 16 Hz.

at the receiver at 3709 km distance.⁶ As the time fronts propagate out to the receiver at 3709 km, the energy near 800 m near the source is eventually overtaken by the energy from rays at steeper angles. At the receiver, the more steeply inclined energy (rays) arrives first.⁶ Bottom and sub-bottom roughness has little influence at 40 and 105 km, and thus the mudslide effect is again seen to be robust (Fig. 5).

IV. DISCUSSION

The mudslide effect occurs naturally in the ocean in the vicinity of continental slopes, and is conjectured to be of general significance in underwater acoustics. It is one of a few robust and predictable acoustic propagation effects that occur in range-dependent ocean environments.

Numerical simulations of the mudslide effect have been performed using the University of Miami PE (UMPE) model. Using the real ocean environment near Kaneohe Bay, Oahu, Hawaii, it has been shown that the mudslide effect facilitates the injection of acoustic signals from a bottom-mounted source in shallow water into the deep water sound channel. These signals can then propagate to very long ranges with relatively small losses. Although the version of UMPE used in this work is two dimensional with no azimuthal coupling, it is believed that fully three-dimensional broadband model predictions would not differ significantly from those presented here.

There are different kinds of environmental situations that are not modeled here. For example, sometimes the water's sound speed at the bottom is greater than the sound speed at the top of the sediment layer. Then there is usually a minimum of sound speed near the top of the sediment layer, and the mudslide effect should occur for the same

reasons it occurs in the cases modeled in this article. Because the attenuation coefficient for shear waves is so much greater than that for compressional waves near 133 Hz,²⁴ it appears that compressional waves, modeled here, should be the dominant process by which sound propagates in this experiment. This may not be so at much lower frequencies.²⁵

The transition from shallow-water propagation to deepwater propagation is known to be a difficult modeling problem, as is the reciprocal problem (deep to shallow). It is hoped that the numerical modeling results presented here will contribute to an improved understanding of this important problem.

There appears to be two mechanisms by which the Kaneohe source injected energy near the depth of minimum speed in the Pacific Ocean. One is the mudslide effect. The other is due to rays bouncing from the surface and steep slope of Oahu. 6,23 Some of this reflected energy leaves the Oahu slope at a flat angle at the depth of minimum speed. It is not known what the relative contributions of these two effects may be, but this study indicates that the mudslide effect is significant in that experiment.

One conclusion of this study is that it is not necessary to place a bottom-mounted source at a distance from the shore that is near the point where the axis of the sound channel intersects the slope in order to couple the signals efficiently into the sound channel. Utilization of the mudslide effect accomplishes the same goal, and is more economical with sources cabled to shore because those cables are shorter. This observation, based on numerical simulations and experimental data from the Kaneohe source, has application to longrange ocean acoustic tomography, underwater noise, and possibly the so-called "T-phase" associated with sounds detected from underwater seismic events. Direct experimental verification of the mudslide effect is needed.

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