Jacob’s Game

I. Preface

Although religion and science often contradict one another, knowledge of two different types of truth can strengthen both science/mathematics and religion. I believe that using game theory, we can understand choices that religious figures made, and in doing so, gain important insights into the stories told about them.

II. The Story

In the Bible, Jacob flees from his parents’ home only to arrive at the scene of shepherds struggling to remove a stone from a well near the city of Charan. Charan, coincidentally, is where Laban, (Jacob’s mother’s brother) lived with his two daughters, Leah and Rachel:¹

Jacob went to the land of the easterners. He looked, and behold – a well in the field... Jacob said to the shepherds, “Do you know Laban the [grand]son of Nahor?” And they said “We know.” Then he said, “Is he doing well?” And they answered, “He is well, and see – his daughter Rachel is coming with the flock.”

...While he was still speaking with them, Rachel had arrived with her father’s flock, for she was a shepherdess. And it was, when Jacob saw Rachel, daughter of Laban his mother’s brother, and the flock of Laban his mother’s brother, Jacob came forward and rolled the stone off the mouth of the well and watered the sheep of Laban his mother’s brother. Then Jacob kissed Rachel and he raised his voice and wept. Jacob told Rachel that he was her father’s brother, and that he was Rebecca’s son, and she ran and told her father.

And it was, when Laban heard the news of Jacob his sister’s son, he ran toward him, embraced him, kissed him, and took him to his house... and he stayed with him a month’s time.

Then Laban said to Jacob, “Just because you are my relative, should you serve me for nothing? Tell me: What are your wages?”

Laban had two daughters. The older one Leah and the younger one Rachel. Leah’s eyes were tender, while Rachel was beautiful of form and beautiful of appearance. Jacob lover Rachel, so he said, “I will work for you for seven years, for Rachel your younger daughter.” Laban said, “It is better that I give her to you than I give her to another man; remain with me.”

So Jacob worked for seven years for Rachel and they seemed to him a few days because of his love for her.

Jacob said to Laban, “Deliver my wife for my term is fulfilled, and I will come to her.” So Laban gathered all the people of the place and made a feast. And it was in the evening, that he took Leah his daughter and brought her to him; and he came to her...

And it was, in the morning, that behold it was Leah! So he said to Laban, “What is this you have done to me? Was it not for Rachel that I worked for you? Why have you deceived me?” Laban said, “Such is not done in our place, to give the younger before the elder. Complete the week of this one and we will give you the other one too, for the work which you will perform for me yet another seven years.”

So Jacob did so and he completed the week for her; and he gave him Rachel his daughter to him as a wife... He came also to Rachel and loved Rachel even more than Leah, and he worked for him yet another seven years.

The Talmud in Tractate Megilah 13b discusses Jacob’s “mistake” in telling Rachel that he is her father’s brother, instead of saying that he is the son of her father’s sister, as was truly the case.²

To resolve this inconsistency the Talmud relates the following story:

Jacob, having fallen in love with Rachel, asked her to marry him. She responded that she would,
but warned Jacob that her father was devious man and that he would most certainly prevent the two from marrying. To this Jacob replied, “I am your father’s brother in trickery.” Meaning Jacob could easily outwit Laban. Rachel, surprised that such a good man could be as deceitful as her father, remained skeptical because her older sister Leah remained unmarried and therefore was a more suitable match for Jacob. In order to prevent this, Jacob decided that at the wedding, the veiled bride would have to show Jacob the signs that he and Rachel had agreed upon before he would take her as his wife. In this way Jacob would be sure of the identity of his bride.

Knowing that Laban would send Leah to the altar in her place, Rachel became embarrassed with the idea of her sister’s public rejection. Thus, she decided to show Leah the secret signals. Jacob, thinking he was marrying Rachel, was surprised the next morning when it turned out the bride he had married was in fact Leah.

III. Objective

This story raises a number of questions: Firstly, why was Jacob surprised? After spending seven years in Laban’s house, he should have recognized Rachel’s kind attitude and expected her to give the signs to Leah. Perhaps Jacob should have even preferred if Rachel gave her sister the signs, as it indicates Rachel's kindness, surely a positive feature to add to Rachel’s desirability as a wife. Under these assumptions, it might even be reasonable to consider that Jacob should have chosen the sister who did not show him the signs, for if Rachel were kind, and therefore worth more to Jacob, she would have been the sister who did not show signs, and if Rachel were not kind in this regard, she would be less valuable to Jacob.

Jacob’s decision can be modeled as a Bayesian game if we take into consideration the two factors on which Jacob’s choices were based:

1. Jacob’s payoffs from Rachel and Leah
   - Marrying Leah = a
   - Marrying a kind Rachel = b
   - Marrying a selfish Rachel = c

2. Jacob’s beliefs about the probability that Rachel was kind enough to give the signs to Leah:
   - Probability that Rachel would give Leah the signs = p
   - Probability that Rachel would not give Leah the signs = 1-p

We already know the outcome – that Jacob took the sister who gave him the signs, so by testing the following model with different possibilities for Jacob’s payoffs [a,b,c] we can determine the range of probabilities [p] that Jacob assigned to the chance that Rachel would give Leah the signs.
IV. The Model

We also know that Jacob loved Rachel more than he loved Leah. Thus both b and c must be greater than a. Jacob also received strong warning to marry well. While this does not necessarily mean that Jacob would favor a kind wife over an unkind wife, the combination of warnings from his parents and common sense would mean that a kind Rachel is at least weakly preferred to an unkind Rachel. Thus \( b > c \).

Thus, Jacob’s expected payoffs are:
- Expected payoff from marrying the girl who gives the signs = \( a \cdot p + c \cdot (1-p) \)
- Expected payoff from marrying the girl who doesn’t give the signs = \( b \cdot p + a \cdot (1-p) \)

We know that Jacob married the girl who gave the signs, which means that:
\[ a \cdot p + c \cdot (1-p) > b \cdot p + a \cdot (1-p) \]
which leads to
\[ 1 > (c-a)/(b-a) > p/(1-p) \text{ or } 1/2 > p. \]

We can also analyze these numbers by focusing on the importance of the difference in Jacob’s payoffs between marrying Leah, marrying a kind Rachel, and marrying a selfish Rachel. In other words, how much Jacob loved Rachel more than Leah (y) and how much he valued Rachel’s kindliness (z). In this case, Jacob’s payoffs are as follows:
- Marrying Leah = a
- Marrying a selfish Rachel = \( c = a + y \)
• Marrying a kind Rachel = b = a + y + z
where a, y, and z are each greater than zero, for
the reasons stated above. In fact note that for
Jacob to choose as he did, the higher z is, the
lower p must be. In other words, the more Jacob
values kindness, the lower the probability he must
assign to Rachel’s making an act of sacrifice by
giving the signs. This implies that in order for
Jacob to choose the sister who gave him the signs
(as he did,) he must have either:
  • Valued Rachel equally whether or not she
was kind to her sister (b=c)
  • Assigned a small probability to Rachel’s
giving the signs to Leah (if b>c), as discussed above.

The first of these possibilities, that b=c and
therefore p=1/2, should not be discounted. The
Biblical text discusses Jacob’s affection towards
Rachel a number of times. First, when Jacob is at
the well, he kisses Rachel after seeing her for the
first time. The second mention of Jacob’s love of
Rachel immediately follows a description of the
two sisters – mentioning that Leah’s eyes were
weak and Rachel was very pretty. The text also
reveals that his love for her made the seven years
Jacob worked for Rachel seem like a mere few
days. All of these descriptions indicate a physical
attraction unrelated to Rachel’s kindness. This
suggest that perhaps Jacob took the sister who gave
him the signs because, Rachel’s kindness was
irrelevant, so he went for the sister who, at first
glance, seems to be Rachel.

A more rational approach to Jacob’s character
follows the second possibility: Jacob did not expect
Rachel to give the signs to Leah, in fact, as
discussed earlier, the higher z is, the lower p must
be. But why would Jacob have low expectations
for Rachel’s character? In the seven years he spent
in her father’s house, did Jacob not recognize
Rachel’s kindness earlier? It must be that regardless
of Jacob’s opinion of Rachel’s character traits, the
probability that someone would give up love to
save her sister from embarrassment was so unusual
that Jacob did not assign it a strong probability.
According to this understanding, Rachel’s action
was uniquely kind, and thus this explanation helps
clarify the following story, as told by Rabbi
Solomon Yitchaki:

The Patriarchs and the Matriarchs went to appease
God, concerning the sin of Manasseh, who placed an
image in the Temple, but He was not appeased. Rachel
entered and stated before Him, “O Lord of the
Universe, whose mercy is greater, Your mercy or the
mercy of a flesh and blood person? You must admit
that Your mercy is greater. Now did I not bring my
rival into my house? For all the work that Jacob
worked for my father, he worked only for me. When
I came to enter the nuptial canopy, they brought my
sister, and it was not enough that I kept my silence,
but I gave her my password. You, too, if Your children
have brought Your rival into Your house, keep Your
silence for them.” He said to her, “You have defended
them well. There is reward for your deed and for your
righteousness, that you gave over your password to
your sister – there will be an end to the exile and your
sons (the nation of Israel) will return to their land.

Jacob did not expect Rachel to give the signs to
her sister; it was an unprecedented and
unanticipated activity, but specifically because of Rachel’s unusual kindness, God promised to reward her and her descendents with mercy.

Through a game theoretical analysis of this story, we can deduce why Jacob acted the way he did. Furthermore, and perhaps more importantly, the analysis also elucidates the reasoning behind God’s promise and why Rachel was so deserving of reward. In this way mathematics and science are not necessarily antithetical, but can work together to reveal new meaning behind the stories of the Bible.

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2 Genesis 29:12
3 For clarification, I have provided a condensed genealogy of the characters involved: