Peano Antennas

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***Abstract*—In this letter, we investigate the radiation characteristics of Peano antenna, i.e., a single antenna made of a thin wire, patterned after a special type of space-filling curve known as the Peano curve. We use a moment-method-based simulation code to study various properties of this antenna, namely, the radiation pattern, impedance bandwidth, current distribution, cross-polarization level, radiation efficiency, and the feed location for achieving an electrically small matched antenna. As in the case of the Hilbert antenna, the radiation patterns of a Peano antenna resemble those of a linear dipole; however, this antenna has a lower cross-polarization level than the Hilbert antenna and, for a matched Peano antenna, the resonant frequency is lower than the same order matched Hilbert antenna at the expense of a smaller input-impedance bandwidth.**

***Index Terms*—Bandwidth, Hilbert antenna, input impedance, matched antenna, miniaturized antenna, Peano curve, small antenna.**

I. Introduction

Electrically small antennas are of great interest for a variety of applications in wireless networks and personal communications systems and are being studied extensively (see, e.g., [1]–[8], [10], [11]). One approach to design an electrically small antenna is to explore the concept of space-filling curves [2]–[8]. These curves may provide resonant structures that can have very small footprints as one increases the step order in iterative filling of a two-dimensional (2-D) region. One of the most well known of these curves is the Hilbert curve, proposed by Hilbert in 1891 [9]. Several research groups in the last few years have studied the radiation properties of the antennas in the form of the Hilbert curve and have found some interesting features for such antennas (e.g., [2]–[8], [10], [11]). Like other electrically small antennas, the Hilbert antenna has a small radiation resistance and a small input resistance, in the order of a few ohms, when it is center-fed at its point of symmetry. We have previously shown, however, that a properly chosen off-center feed point can provide, regardless of how high the iteration (growth) order n, an approximate 50-ohm match at the structure's lowest resonant frequency [8].

In this letter, we consider another small wire antenna whose geometry is based on another class of space-filling curve known as the Peano curve and we numerically study its radiation characteristics. It is noteworthy that the Peano curve, proposed by Peano in 1890, was, in fact, the first set of space-filling curve [9]. One interesting feature of the Peano-curve algorithm is its relatively higher compression rate than the Hilbert-curve algorithm in filling a 2-D region, which suggests the Peano antenna may resonate at a lower fundamental resonant frequency than a comparable Hilbert antenna of the same iteration order. The question is, however, whether one can match a Peano antenna to a feed line with a given characteristic impedance without using an external matching network. In addition, it is important to study the bandwidth, current distribution, radiation pattern, radiation efficiency, and cross-polarization level of a matched Peano antenna as the iteration order of Peano algorithm increases and compare them with those of the Hilbert antenna. Some preliminary results of our study was presented in a recent symposium [10]. In addition, González-Arbesú *et al.* have recently investigated the quality factor and radiation efficieny of Peano antennas and compared them with those of Hilbert antennas and meander-line monopoles [11]. Here we present a thorough numerical study of Peano antennas and, in particular, concentrate on the feed-point effects, current distribution, pattern, cross-polarization, and other radiation characteristics of these antennas that have not been previously considered in [10], [11].

II. Peano Curve and Peano Antenna

To gain some insights about the Peano space-filling curves, in Fig. 1, we show the first four iteration-order of this algorithm. For a Peano antenna, made of a thin wire in the form of the Peano curve with side dimension L and order n, the length of each line segment d and the sum of all the line segments S are given by [9]

$$d = \frac{L}{(3^n - 1)}; \quad S = (3^{2n} - 1)d = (3^n + 1)L.$$

Because the total length of the wire is larger than that of the same order Hilbert antenna, which is $S = (2^n + 1)L$, we would expect that the Peano antenna to resonate at a lower fundamental frequency than the same order Hilbert antenna.

By applying a moment-method-based simulation code NEC-4, we analyze numerically the current distribution, bandwidth variation, radiation patterns, feed-point effects, radiation efficiency, and cross-polarization level in Peano antennas placed in free space as the iteration order increases from $n = 1$ to $n = 4$. In most parts of our analysis, unless otherwise specified, the antenna wire segments are assumed to be perfectly conducting and, thus, no ohmic loss is considered here (except in the study of radiation efficiency to be discussed at the end of this letter). For the easy comparison, the outer dimension of the antenna is taken to be 70 mm $\times$ 70 mm, similar to the Hilbert case in [8].

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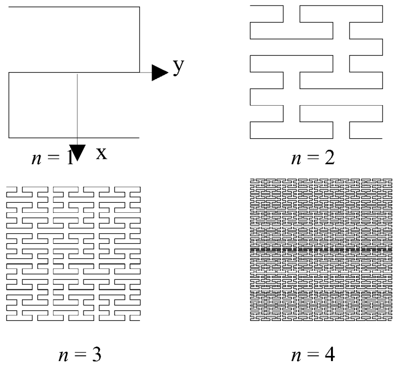

Fig. 1. First four iteration orders of the Peano curve. The plane $\phi = 0°$ is the x–z plane.

As in the case of the Hilbert antennas, when the feed point is placed at the point of symmetry of the Peano antennas, the real part of the input impedance is very low at the resonant frequencies. In our analysis, in this case the real part is only about 10 ohms for a first-order Peano antenna.

However, also as in the case of Hilbert antenna, it is possible to locate an off-center feed point that results in an approximate input resistance of between 50 and 100 ohms and negligible input reactance for all the step orders investigated here. To illustrate this feature, we have performed numerical simulations for Peano antennas of iteration orders 1–4 with a fixed outer dimension of 70 mm × 70 mm. For each iteration order of Peano antenna, a search is performed to find the feed location for which the input impedance is about 50 or 75 ohms at the first (fundamental) resonant frequency. In these simulations, the diameter of the line is chosen to be 1 mm, except for the fourth-order antenna, in which case the diameter is 0.25 mm in order to allow a nonzero spacing between the line segments. The relative location of such off-center feed points for Peano antennas are shown and mentioned in Fig. 2. (In order to see the feed location of the fourth-order antenna we had to draw the figure larger than the others, but the physical size is still similar to the others).

As in the case of the Hilbert antenna [8], here we also list the normalized total lengths of the wire (S/λ_0) for these four Peano antennas in Table I, where λ_0 denotes the first resonant wavelength (i.e., longest resonant wavelength) for each antenna. The values of (L/λ_0) are shown in Fig. 2 next to their corresponding Peano antennas. The first resonant frequency and the $\mathrm{VSWR} < 2$ bandwidth for Peano antennas of iteration orders 1–4 with fixed side dimensions 70 mm × 70 mm are listed in Table II. We see that the resonant frequency decreases drastically as the order is increasing from 1 to 4, consistent with what was obtained in [11].

We notice that for $n = 4$, the outer dimension of the antenna is only about 1/39 of its first resonant wavelength and the antenna is still matched to 50-ohm line at its fundamental resonance, while for the same order Hilbert antenna the outer dimension is about 1/16 of its first resonant wavelength [8]. In general, for a given footprint and a fixed step order n, the Peano antenna has a longer total wire length, resonates at a lower fundamental resonant frequency, thus resulting in an electrically more compact radiator than a comparable Hilbert antenna, albeit at the expense of a smaller input-impedance bandwidth.

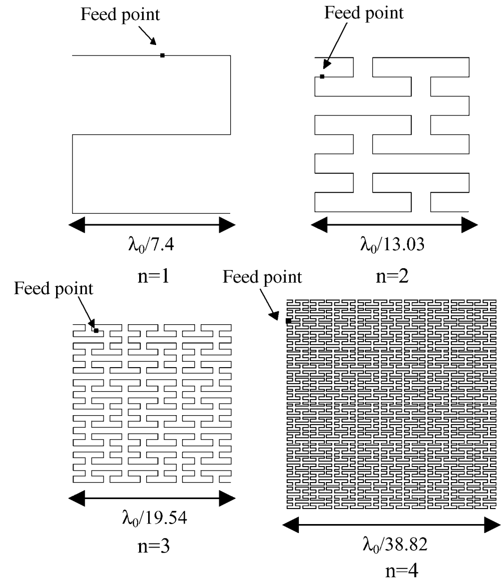

Fig. 2. The geometry and feed point of Peano antennas with approximately 50-ohm input impedance for n =1, 2, 4, and 75 ohm for n = 3 at its *fundamental* resonant frequency. The relative locations of these feed points, defined as the ratio of the distance along the wire from the feed point to the one end of antenna divided by the total length of the wire in the antenna (S), are found to be 0.143 75, 0.0578, 0.0182, and 0.009 22 for n =1 to 4, respectively. The normalized linear (side) dimension (L/λ_0) is also shown. Here λ_0 represents the first (i.e., longest) resonant wavelength for each antenna.

TABLE I
FIRST RESONANT WAVELENGTHS AND THE NORMALIZED TOTAL LENGTHS OF THE WIRE FOR PEANO ANTENNAS OF ITERATION ORDER 1–4

n	1	2	3	4
λ_0 (mm)	517.69	911.85	1367.86	2717.64
S/λ_0	0.54	0.768	1.433	2.112

TABLE II
RESULTS OF NEC SIMULATION, WHERE f_0 IS THE FIRST RESONANT FREQUENCY AND BW IS THE VSWR BANDWIDTH $(\mathrm{VSWR} < 2)$

n	1	2	3	4
f_0(MHz)	579.5	329	219.32	110.39
BW(%)	1.8 (50ohms)	0.29 (50ohms)	0.055 (75ohms)	0.0063 (50ohms)

This is expected due to the relatively higher compression rate of the Peano-curve algorithm in filling a 2-D region, as was mentioned earlier. On the other hand, for a given resonant frequency, Peano antenna has nearly the same bandwidth as that of a Hilbert antenna; this is evident by comparing, for example, the Peano antenna of order $n = 2$ in Table II, with the Hilbert antenna of order $n = 3$ in [8]. As we will discuss later, however, Peano antenna has the advantage of having a much lower cross-polarization level than the Hilbert antenna. Also, it is evident from Table II that Peano antennas of order $n = 3$ and larger will be impractical for most antenna applications due to their inherently very small bandwidth. It is noteworthy, however, that a larger bandwidth may be obtained if one uses a larger wire diameter.

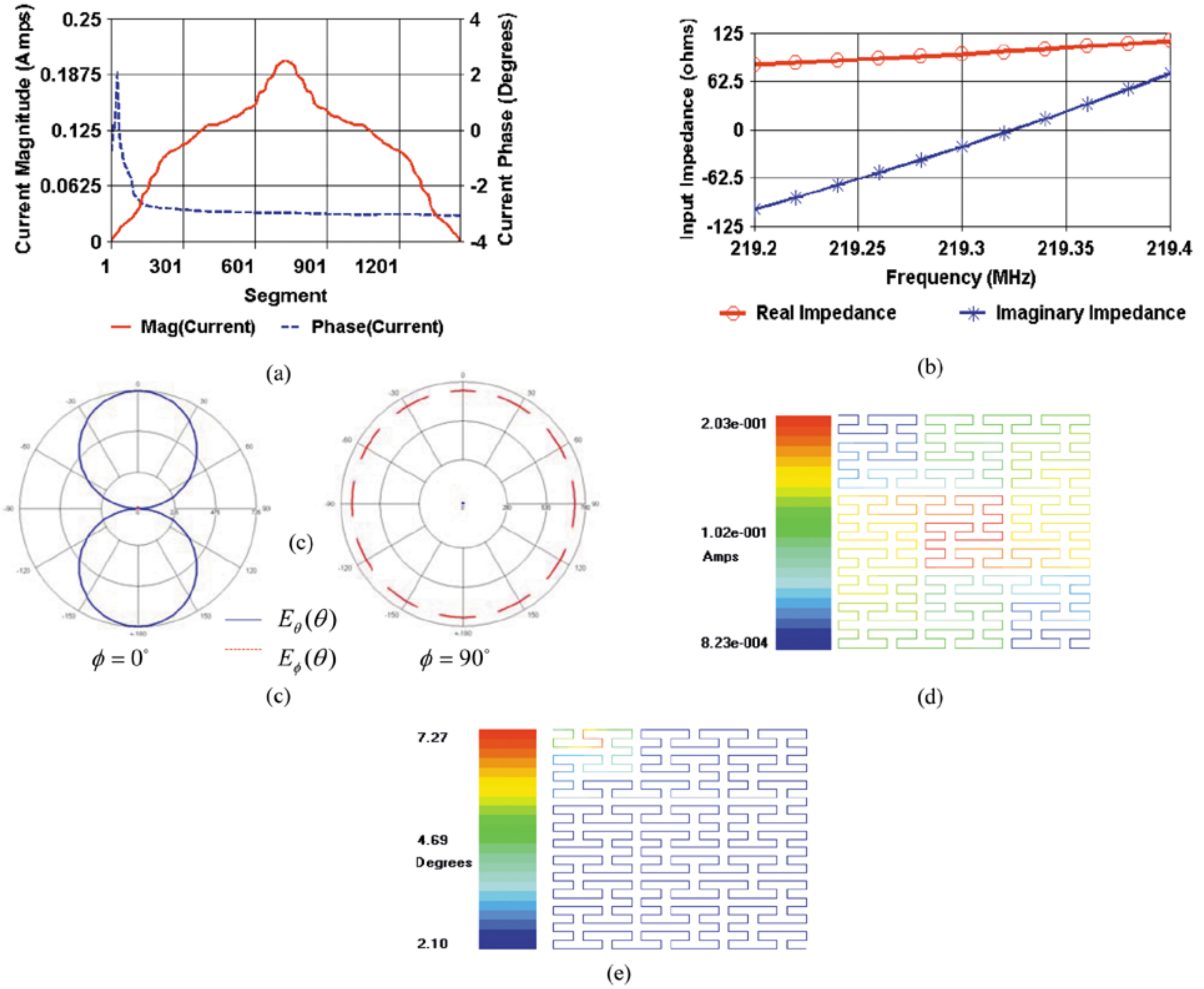

Fig. 3. Plots for the third-order off-center fed Peano antenna. (a) Magnitude and phase of current distribution along the wire segments. (b) Real and imaginary parts of the input impedance versus frequency. (c) The radiated electric field components in two orthogonal planes $\phi = 0^\circ$ and $\phi = 90^\circ$. (d) Contour plot of magnitude of current distribution. (e) Contour plot of phase of current distribution. at the lowest resonant frequency. The number of method of moments segments (cells) we used along the wire for the numerical analysis of this antenna was 1456. We note that the magnitude of the current distribution along the antenna wire is approximately centrally symmetry with respect to the antenna center, while that of the Hilbert antenna has the mirror symmetry [8].

The current distributions, radiation patterns and multiresonant features of the different iteration-order Peano antennas at their corresponding first resonances are quite similar; therefore, here we only concentrate on the characteristics of the third iteration-order Peano antenna, as shown in Fig. 3. Just like the Hilbert antenna [8], we observe the following similar features for the Peano antenna of order 3. The magnitude of the current distribution [Fig. 3(a)] is almost symmetric and the phase [Fig. 3(a)] varies slowly along the wire length. We note that the real part of input impedance is approximately 75 ohms with a $\mathrm{VSWR} = 1.25$ at the fundamental resonant frequency. The bandwidth over which $\mathrm{VSWR} < 2$, however, is only about 0.055%, which is much smaller than the same order Hilbert antenna but comparable to a Hilbert antenna of order $n = 5$ with approximately the same resonant frequency and the same electrical size of about $1/20\lambda_0 \times 1/20\lambda_0$ [8]. The radiation patterns of this antenna at its fundamental resonant frequency in the planes of $\phi = 0^\circ$ and $\phi = 90^\circ$ (Fig. 3(c)), almost resemble the patterns of a linear dipole, as expected and similar to the Hilbert antenna [8]. In Fig. 3(d) and (e), we show the color-coded contour plots of the magnitude and phase of current distribution along the wire length of this antenna. In Fig. 4 we show the multiresonant feature of the Peano antenna of order

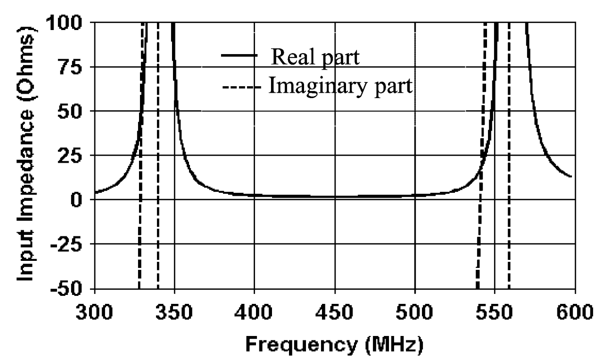

Fig. 4. Real and imaginary parts of the input impedance for the second-iteration-order Peano antenna.

TABLE III
LEVEL OF CROSS POLARIZATION (DECIBELS)

n		1	2	3	4
Cross -Pol level (dB)	Phi=0	-11.8	-60.3	-57.2	-60.1
	Phi=90	-13.7	-60.3	-57.2	-60.1

TABLE IV
RADIATION EFFICIENCY (%) OF PEANO AND HILBERT ANTENNAS. THE WIRE IS ASSUMED TO BE MADE OF COPPER

n	1	2	3	4	5
Hilbert Antenna	98.94	93.83	75.18	41.06	17.2
Peano Antenna	97.06	72.25	17.5	-	-

2, again analogous to that of the Hilbert antenna. As can be seen, a second resonance occurs at about 540 MHz. It is also clearly seen that the antenna is matched to an approximately 50-ohm line at the fundamental resonant frequency of 329 MHz.

One disadvantage of many electrically small resonant antennas, such as Hilbert antennas, is their relatively large cross-pol levels (CPL). Unlike the Hilbert antenna, the Peano antenna, however, has a very low CPL, as shown in Table III for various iteration orders. For example a Peano antenna of order 2 has cross-pol levels of less than -60 dB in both planes of polarization, whereas a Hilbert antenna of order 3, which has almost the same electrical footprint and the same resonant frequency, has a CPL of -9.5 dB in the $\phi = 0^\circ$ plane [8]. This difference in the CPL between the Hilbert and the Peano antennas is mainly due to the fact that the Hilbert geometry has the mirror symmetry (with respect to the y–z plane in [8, Fig. 1]) while the Peano curve has the central symmetry with respect to its center (i.e., the coordinate origin in Fig. 1 here). This will lead to the analogous symmetries for the distributions of the currents magnitudes in these two antennas, resulting in lower CPL in the Peano antennas. In addition, for $n > 1$ the Peano antenna has nearly identical CPL values in both planes, which can be attributed to its centrally symmetric current distribution, mentioned above and shown in Fig. 3. For the Hilbert antenna the level of cross-polarization present in $\phi = 90^\circ$ plane is much smaller than that in the $\phi = 0^\circ$ plane because of the mirror symmetry with respect to the $\phi = 90^\circ$ plane.

Finally, we have also analyzed the radiation efficiency of the above off-center fed Peano antennas and compared it with that of the Hilbert antenna. For this analysis, we assume the wire is made of copper, instead of perfectly electric conductor (PEC). Table IV presents these quantities for the first few iteration orders in both antennas. We note that the efficiency in both cases decreases when we increase the iteration order, as expected and consistent with the results in [11]. Furthermore, the antennas of nearly comparable electrical footprint (e.g., Peano of $n = 2$ and Hilbert of $n = 3$, both being about $\lambda/13$, as well as Peano of $n = 3$ and Hilbert of $n = 5$, both being about $\lambda/20$) have approximately the same radiation efficiencies. It is important to note that when the copper wire is used in our analysis, the resonant frequencies are slightly shifted, but still a proper feed point can be found to keep the antenna matched to either 50 or 75 ohm lines. A more detailed investigation of the radiation efficiencies of the Hilbert and Peano antenna are reported in [11].

III. CONCLUSION

In this letter, the feed-point location, current distributions, impedance bandwidth, far-field patterns, radiation efficiency, and cross-polarization characteristics of Peano antennas have been numerically studied and compared with those of the Hilbert antennas. It was shown that a Peano antenna can be matched to a 50- or 75-ohm line for various iteration orders of Peano algorithm. We demonstrated, using our analysis, that for a given footprint and a fixed-iteration order, the Peano antenna resonates at a lower fundamental resonant frequency resulting in an electrically more compact radiator than a comparable Hilbert antenna, but at the expense of a smaller input-impedance bandwidth. On the other hand, for a given electrical footprint, a Peano antenna of a lower iteration order resonates at an approximately the same fundamental frequency and has a comparable bandwidth as of a higher order Hilbert antenna. The Peano antenna, however, has the advantage of having a much smaller cross-polarization level than the Hilbert antenna. The radiation efficiency in both antennas decreases as the iteration order increases.

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