ESSAYS ON FINANCE OF INNOVATION, FIRM DYNAMICS, AND ECONOMIC GROWTH

Sînâ T. Ateş

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Supervisor of Dissertation

Jesús Fernández-Villaverde, Professor of Economics

Graduate Group Chairperson

George J. Mailath, Professor of Economics,
Walter H. Annenberg Professor in the Social Sciences

Dissertation Committee

Ufuk Akcigit, Assistant Professor of Economics

Harold Cole, Professor of Economics

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ABSTRACT

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Sînâ T. Ateş

Jesús Fernández-Villaverde

Aggregate productivity, fundamental cause of long-run economic growth, plays a crucial role in determining economic development and living standards of nations. The main source of aggregate productivity growth is technological advances that are the outcomes of firms' and entrepreneurs' innovative activity. Complementary to the growing literature that studies how firm dynamics shape technological change, my dissertation focuses on how financial decisions of these agents affect this process. The three chapters of my dissertation provide theoretical, empirical, and quantitative investigation of the interplay between financial and innovative actions of heterogeneous firms along with its implications on aggregate productivity growth.

Chapter one studies the impact of financial system on net firm entry, an important source of aggregate productivity growth. Selective funding of most promising ideas by financial intermediaries creates a trade-off between the mass of entrant firms and their average contribution to aggregate productivity. This chapter highlights the relevance of firm heterogeneity for the relationship between finance and growth, and discusses the theoretical and empirical implications of the resulting trade-off in firm entry.

Chapter two also builds on the above mass-composition link, and uses it to study the permanent productivity losses due to sudden stops (SS). The model embeds the main mechanism into a real business cycle small open economy framework to measure the forgone productivity contribution of entrants deprived of funding. The theoretical prediction is that, during SS, smaller yet on average more productive cohorts enter the market. Chilean

plant-level data that cover the 1998 SS verify this prediction, while the calibrated model demonstrates the quantitative significance of heterogeneity and selection in measuring the long-run productivity loss.

Chapter three focuses on a specific financial intermediary that is especially relevant to innovation and growth, namely venture capital (VC) finance. It studies VC's quantitative impact on firm dynamics and economic growth using a new dynamic equilibrium model of technological change with heterogeneous firms and an explicit VC market. Distinctively, the model incorporates a unique feature of VC firms: their operational knowledge (OK) bundled with their investment. Experiments based on the estimated model highlight the quantitative relevance of OK and analyze policy implications.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

Chapter 1

Project Heterogeneity and Growth: The Impact of Financial Selection on Firm Entry

This chapter is co-authored with Felipe E. Saffie.

Abstract

In the classical literature of innovation-based endogenous growth, the main engine of productivity growth is firm entry. Nevertheless, when projects are heterogeneous, and good ideas are scarce, a mass-composition trade-off is introduced into this link: larger cohorts have a lower average contribution to aggregated productivity. Because one of the roles of the financial system is to screen the quality of projects, the ability of financial intermediaries to detect promising projects shapes the strength of this trade-off. We build a general equilibrium endogenous growth model with project heterogeneity, and financial screening to study this relationship. We use two quantitative experiments to illustrate the relevance of our analytic results. First, we show that accounting for heterogeneity and selection allows the model to conciliate two well documented and apparently contradictory effects of corporate taxation. Corporate taxation has a strong detrimental effect on firm entry while affecting the long-run growth only mildly. A second illustration studies the effects of financial development on growth. This experiment shows that size based measures of financial development (e.g. domestic credit over GDP) are not always good proxies for the ability of the financial system to select the most promising projects. Finally, we propose a novel firm level measure to assess the accuracy of financial selection across countries.

1.1 Introduction

The link between financial development and long-run economic growth is a long-lasting question in the literature. In his seminal survey, Levine (2005) summarizes the growth-enhancing functions of the financial system into five channels: *i*) ex-ante information production about investment opportunities and capital allocation, *ii*) monitoring investments and providing corporate governance, *iii*) diversification and management of risk, *iv*) pooling and channelling savings, and *v*) facilitating the exchange of goods and services. Among those, *pooling savings and providing capital to investments* has drawn most of the attention in the theoretical and empirical literature. However, in a world with vast heterogeneity among potential investments, selecting the most productive uses to allocate resources is crucial. This chapter contributes on theoretical grounds to the analysis of this selection channel and illustrate its relevance for theoretical and empirical research.

The main source of long-run growth is improvements in aggregated productivity; hence, a study of the impact of financial development on growth needs to focus on the mechanisms that link the financial system with the productivity process of the economy. Early models of innovation such as Grossman and Helpman (1991) and Aghion and Howitt (1992) provide micro-foundations of productivity growth incorporating the Schumpeterian notion of *creative destruction* into this literature. In a nutshell, entrepreneurs with a new invention (creativity) have lower production costs; and when they enter the market, they replace the former leader (destruction). Therefore, firm entry plays a central role in the determination of long-run growth.[1] But new firms need external finance in order to access the market.[2] This suggests a first link between finance and growth: more developed financial systems are able to pool more funds to finance more start-ups, allow-

[1] Bartelsman et al. (2009) use firm level data for 24 countries to study firm dynamics and the sources of productivity growth. They document that between 20% and 50% of the overall productivity growth is explained by net entry.

[2] For instance, Nofsinger and Wang (2011) document that 45% of the start ups in their 27 country panel use external funding.

ing more creative destruction and therefore more long-run growth. Nevertheless, good ideas are scarce.[3] Therefore, selecting the most promising projects is not a trivial task.

In this sense, the financial system creates value not only by pooling funds, but also by allocating resources efficiently. In fact, Fracassi et al. (2012) document a loan approval rate of only 18.2% for start-ups, using loan application data for a major venture capital in United States.[4] Moreover, the allocation of credit is not random. In fact, funded start-ups in their sample survive longer and are more profitable than rejected ones. This suggests that financial intermediation is not only about the quantity (*mass*) of the entrant cohort, but also about its quality (*composition*). Thus, a model that studies the link between the financial system and long-run economic growth needs to include both dimensions.

In order to understand how the mass and the composition of the cohorts of new firms shape long-run productivity growth, we modify the quality-ladder framework of Grossman and Helpman (1991) along two dimensions. First, we introduce *ex ante* project heterogeneity that is translated into *ex post* firm heterogeneity in the intermediate good sector. Second, we introduce a financial system, with access to a screening technology. The accuracy of the screening device represents the level of financial development of the economy. Our analytical characterization of the unique interior balanced growth path shows how creative destruction is shaped by the interaction between mass and composition of the entering cohort.

Two quantitative experiments illustrate both the strength and the relevance of the composition effect introduced in this chapter. The first experiment relates to the empirical literature on corporate taxation, firm entry, and growth. The model is able to generate mild responses in growth for a wide range of corporate taxes, and at the same time match the strong detrimental effect on entry rates. In fact, in the model, when taxes

[3]Silverberg and Verspagen (2007) document that both patent citation and returns to patenting are highly skewed toward relatively few patents.

[4]See also Benfratello et al. (2008). They use Italian firm level data to show how the development of banking affected the probability of firm innovation.

increase, a large set of projects are not enacted. Nevertheless, for financially developed economies, the marginal contribution of those forgone entrants to economic growth is negligible. However, as tax rates increase further, the contribution of the marginal entrant rises rapidly. This implies a non-linear effect of corporate taxation on economic growth.

The second quantitative illustration revisits the classical link between financial development and growth. In line with the empirical literature, the model suggests that the main source of economic growth in more financially developed economies is the efficiency in the allocation of resources rather than the mass of resources allocated. Moreover, this experiment also shows that the accuracy of the financial system is non-monotonically related to the amount of resources allocated in the economy. In particular, for countries characterized by high entry rates, an increase in the accuracy of the financial system might lead to lower domestic credit over GDP and lower entry rates. Therefore, mass related proxies are potentially misleading when trying to capture the allocative aspect of financial development.

As a first step to address the insufficiency of mass related variables to capture the selection margin of financial development, this chapter proposes a variable that could be used in the empirical literature to complement the existing proxies. Our model implies that the accuracy of the financial system is inversely related to the skewness of the ratio of value added to cost of the firms operating in the economy. The intuition behind this result is that with better selection entrants below a profitability threshold are observed infrequently. Therefore, the *bad* tail of the distribution gets thinner.

This chapter is structured as follows. Section 1.2 reviews some of the related contributions in the endogenous growth literature, then Section 1.3 presents the model and the analytical results. Section 3.4 show the two quantitative experiments that illustrate the relevance of the mechanism. Section 1.5 presents the analysis of the skewness measure, and section 1.6 concludes.

1.2 Related Literature

The role of a financial structure that evaluates investment projects has been considered in the growth literature for a long time.[5] Greenwood and Jovanovic (1990) introduced this idea into an externality driven endogenous growth model inspired by Romer (1986) to study the interdependence between financial development and economic growth. One study in that strand to which the current work particularly relates is Bose and Cothren (1996). They study how improvements in the screening technology of the financial system affect the growth rate of the economy. They develop a two type (borrowers and lenders) overlapping generation model where young borrowers seek resources to start heterogeneous projects. Financial intermediaries use screening and credit rationing to allocate the resources of the lenders. Projects differ only in their success probability (low or high), and the economy growth rate is driven by the externality generated by the average capital stock in the economy. They show that cost reducing improvements in the screening technology can decrease economic growth. Notice that heterogeneity and financial selection influence growth only through the mass of successfully enacted projects. Moreover, this class of models rely on aggregate externalities to generate an endogenous growth process, rather than providing micro-foundations for the increases in productivity.

An early innovation based endogenous growth model with heterogeneity and financial selection is proposed by King and Levine (1993a). They introduce heterogeneity to the original Aghion and Howitt (1992) model dividing the population between agents that are capable to manage an innovative project and agents that are not. The role of the financial system is to pool resources and try to identify capable individuals in order to put them in charge of projects. Hence, the better the screening device the larger the mass of firms entering the economy. Anotehr paper by Jaimovich and Rebelo (2012) builds on the non-Schumpeterian innovation tradition of Romer (1990), including heterogeneous

[5] We can trace this idea back to Bagehot (1873) and Schumpeter (1934), but a more formal exposition can be found on Boyd and Prescott (1986).

agents as in Lucas (1978) to study the non-linear relationship between taxation and long-run growth. In their model every successfully enacted project enlarges the measure of intermediate good varieties by the same amount. Nevertheless, entrepreneurs are heterogeneous in their ability to enact projects. As the ability distribution is skewed, only a few of them account for most of the generation of new varieties and, thus, output growth. Hence, as taxation discourages relatively unproductive entrepreneurs, both the mass of firms created and the growth rate of the economy decrease very mildly for a wide range of tax rates.

None of the endogenous growth models discussed above attempt to link the *ex-ante* heterogeneity with *ex-post* differences on the production side. Hence, the impact of financial selection is only driven by the mass of entrants. In particular, these models imply a monotonic relationship between firm entry and growth: the larger the mass of an entrant cohort, the higher the growth rate of the economy. In contrast, instead of using heterogeneity on the success rate, our model includes *ex ante* project heterogeneity that is also translated into *ex post* firm heterogeneity, generating a non-linear relationship between entry and growth rates.

1.3 Model

This model builds on the classical endogenous growth literature of quality-ladder models. In line with the seminal contributions of Grossman and Helpman (1991) and Aghion and Howitt (1992), a continuum of intermediate good varieties, indexed by $j \in [0,1]$, are used for final good production and the producer with the lower marginal cost monopolizes the production of its variety.[6] The engine of economic growth is the creative destruction generated by successfully enacted projects where the former leader is replaced by a newcomer with lower marginal cost. In order to disentangle the mass

[6] For a recent review of the relevance and scope of this framework see Aghion et al. (2014).

and composition effect of financial intermediation, we modify this framework to allow for project heterogeneity and financial selection. A representative financial intermediary owns a unit mass of projects, indexed by $e \in [0, 1]$, and collects deposits from the representative household to enact a portion of them. First, we introduce heterogeneity in both projects and marginal cost improvements. In particular, after enaction a successful project can generate either a drastic or an incremental innovation that leads to cost reduction in a product line. This implies that leaders have heterogeneous cost advantages over their followers. Moreover, since projects are characterized by their idiosyncratic probability of generating a drastic innovation, there is also heterogeneity before enaction. Second, we introduce financial selection by allowing the financial intermediary to access a costless yet imperfect screening device. In this section, we introduce the components of the model, define a competitive equilibrium and a balanced growth path, and derive the analytical characterization of the model.

1.3.1 The Representative Household

The representative household lends assets (a_{t+1}) to the financial intermediary at the interest rate r_{t+1} and receives the profits of the financial intermediary (π_t) as well as the revenue generated by corporate taxation (T_t), which the government levies on intermediate firms. The household supplies L units of labor inelastically, and future utility is discounted at rate β. We assume constant relative risk aversion utility to allow for a balanced growth path in equilibrium, and the inter-temporal elasticity of substitution is $\frac{1}{\gamma} \leq 1$. In particular, given the sequences of wages, interest rates, profits, lump sum transfers of tax revenue $\{w_t\,,\, r_{t+1}\,,\, \Pi_t\,,\, T_t\}_{t=0}^{\infty}$, and initial asset a_0, the representative household chooses consumption, assets $\{c_t\,,\,a_{t+1}\}_{t=0}^{\infty}$ to solve: [7]

$$\max_{\{c_t\,,a_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right\} \tag{1.1}$$

sbj. to

$$c_t + a_{t+1} \leq w_t L + a_t(1+r_t) + \Pi_t + T_t \tag{1.2}$$

$$a_{t+1} \geq 0 \tag{1.3}$$

As shown in equation (1.2), the price of consumption is set to unity since we use final good as the *numeraire*. The interior first order condition that characterizes this program is

$$\left(\frac{c_{t+1}}{c_t}\right)^{\gamma} = \beta\,(1 + r_{t+1})\,. \tag{1.4}$$

1.3.2 Final Good Sector

Using a constant returns to scale technology, the representative final good producer combines intermediate inputs to produce the final good

$$\ln Y_t = \int_0^1 \ln x_{j,t}^D dj,$$

which in turn provides resources for consumption. In particular, given input prices and wages $\{w_t\,,\,p_{j,t}\}$, the final good producer demands intermediate varieties $\left\{\left\{x_{j,t}^D\right\}_{j\in[0,1]}\right\}$ every period in order to solve

$$\max_{\left\{\left\{x_{j,t}^D\right\}_{j\in[0,1]}\right\}\geq 0} \left\{ exp\left(\int_0^1 \ln x_{j,t}^D dj\right) - \int_0^1 x_{j,t}^D p_{j,t} dj \right\}, \tag{1.5}$$

[7] Subject to the standard transversality condition.

The solution to this problem is fully characterized by the following interior set of first order conditions:

$$x_{j,t}^{D} = \frac{Y_t}{p_{j,t}}. \tag{1.6}$$

1.3.3 Intermediate Good Sector

In line with the endogenous growth literature, we assume that the amount of the intermediate good j produced, $x_{j,t}$, is linear in labor $l_{j,t}$, with constant marginal productivity $q_{j,t}$. Thus,

$$x_{j,t} = l_{j,t} q_{j,t}. \tag{1.7}$$

The efficiency of labor in the intermediate good production evolves with each technological improvement generated by successful innovation. Innovations are heterogeneous in their capacity to improve the existing technology. In particular, the evolution of technology follows

$$q_{j,t} = I_{j,t}^{d} \times q_{j,t-1}\left(1+\sigma^{d}\right) + \left(1 - I_{j,t}^{d}\right) \times q_{j,t-1}; \quad d \in \{L, H\} \tag{1.8}$$

where $I_{j,t}^{d}$ is an indicator function that equals to 1 if the product line j receives an innovation in period t of type d, and 0 otherwise implying that this period, the level of productivity is the same as in the last period. Moreover, σ^{d} is the heterogeneous step size of the innovation, with $\sigma^{H} > \sigma^{L} > 0$.[8] This implies that *high* type projects (H) improve the productivity of labor more than *low* type projects (L). Therefore, leaders are heterogeneous in their absolute distance from their closest follower.[9]

In line with the literature, we assume Bertrand monopolistic competition. This set-

[8] Incumbent heterogeneity has been introduced in step by step models even with rich incumbent dynamics, for example in Akcigit and Kerr (2010), but that literature has not studied financial selection.

[9] We allow only two types in order to summarize the composition of the product line with only one variable, the fraction of leaders with σ^{H} advantage.

up implies that the competitor with the lower marginal cost dominates the market by following a limit pricing rule, i.e. she sets her price, $p_{j,t}$ equal to the marginal cost of the closest follower. Denote the efficiency of the closest follower, by $\tilde{q}_{j,t}$, then:[10]

$$p_{j,t} = \frac{w_t}{\tilde{q}_{j,t}}. \tag{1.9}$$

In any product line j, the owner of the latest successful project of type d reaps profits $\pi^d_{j,t}$ at time t. Profits are subject to the corporate tax rate τ. A firm owner collects after-tax profits in the current period. In the next period, this firm will continue to produce if it is not replaced by a new leader. A mass M_{t+1} of projects is enacted at time $t+1$, and each of them becomes a firm with fixed probability λ. As entry is undirected, an incumbent firm will continue to produce with probability $1 - \lambda M_{t+1}$. Then, given interest rate r_{t+1}, the value $V^d_{j,t}$ of owning the product line j at time t for a type d leader is given by

$$V^d_{j,t} = (1-\tau)\pi^d_{j,t} + \frac{1-\lambda M_{t+1}}{1+r_{t+1}} V^d_{j,t+1}. \tag{1.10}$$

In this framework, incumbents are randomly replaced by more efficient entrants. This is the engine of economic growth in the model, the Schumpeterian creative destruction. Although we abstract from incumbent firms' dynamics, this channel captures an important driver of productivity growth as documented by Bartelsman et al. (2009).

1.3.4 Projects

Projects are indexed by $e \in [0,1]$. The fixed cost of enacting a project is κ units of labor. An enacted project is successful with probability λ in generating an innovation. In Aghion and Howitt (1992) potential entrants are homogeneous, and of infinite mass. One of the key novelties in this model is how heterogeneity and scarcity are introduced into this framework, and how the *ex ante* heterogeneity of projects is related to the *ex post* het-

[10] Note that, because there is no efficiency improvement by incumbents, we have $q_{j,t} = (1+\sigma^d)\tilde{q}_{j,t}$. This framework can be easily extended to allow for undirected incumbent innovations.

erogeneity of incumbents. In this economy, projects are heterogeneous in their expected cost reduction, and the ones with large expected reductions are scarce.[11] In particular, every project has an unobservable idiosyncratic probability $\theta(e) = e^{\nu}$ of generating a drastic improvement on productivity characterized by σ^H. As shown in Figure 1, the higher the index e is, the more likely it is for project e to generate a drastic (type-H) innovation, and hence, the higher the expected cost reduction. In this sense, e is more than an index, it is a ranking among projects based on their idiosyncratic $\theta(e)$, which is unobservable *ex-ante*.

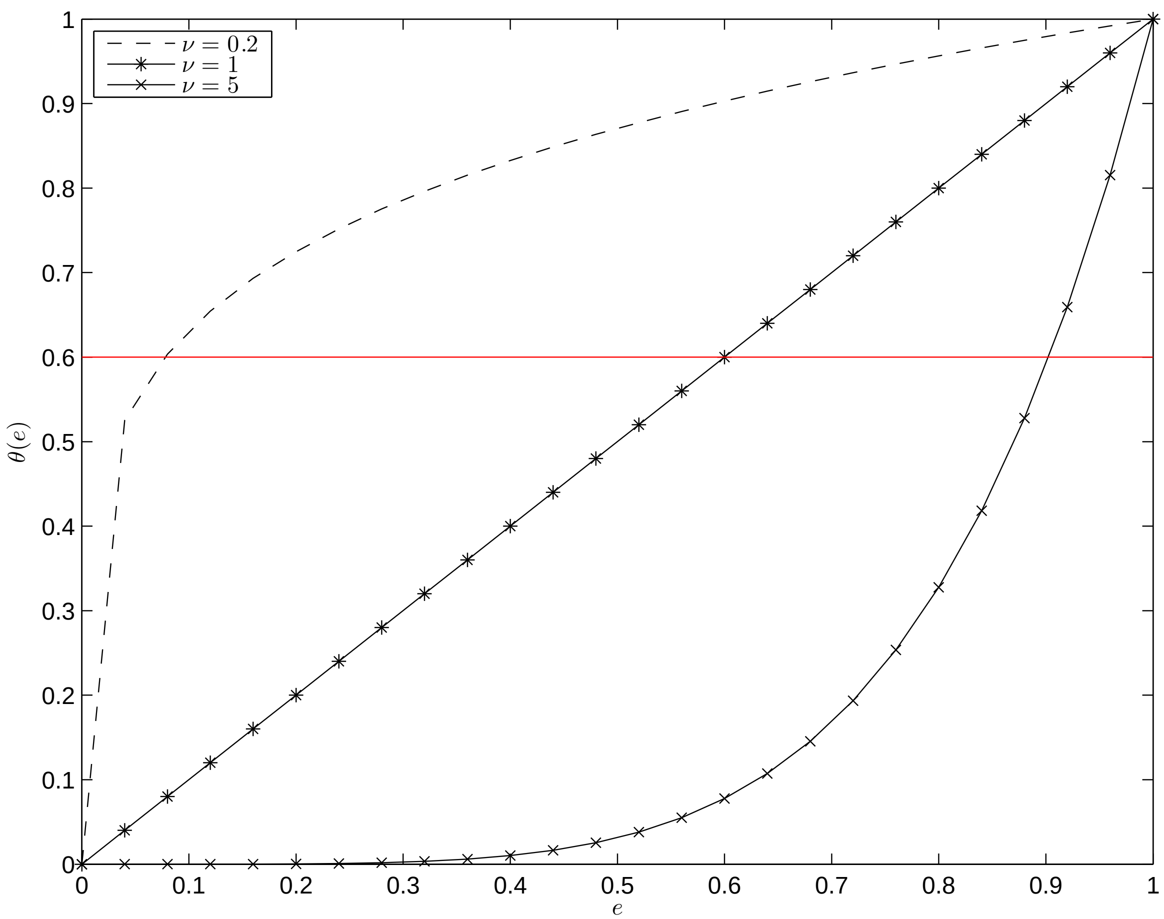


Figure 1: Project Heterogeneity

In this setting, ν governs the underlying scarcity of good projects in the economy.

[11] A similar strategy in a different framework is followed by Clementi and Palazzo (2013). They introduce *ex ante* heterogeneity linked with *ex post* firm productivity in the framework of Hopenhayn (1992) to study firm dynamics over the business cycle in a quantitative partial equilibrium model.

Figure 1 shows that for any $\bar{\theta} \in [0,1]$, the higher the value of ν the fewer projects with a probability $\theta(e) > \bar{\theta}$ of generating a type H innovation. For example, when $\bar{\theta} = 0.6$, if $\nu = 0.2$ there is a mass 0.9 of projects that deliver a drastic innovation with probability higher than 0.6, whereas when $\nu = 5$ only a mass 0.1 is above that level. Hence, ν is a measure of the shortage of projects that are likely to produce drastic innovations. Proposition 1 translates the ranking of projects into a probability distribution for θ, the proof is provided in Appendix A.1.

Proposition 1. *We can characterize the probability distribution $f(\theta)$ by*

$$f(\theta) = \frac{1}{\nu}\left(\frac{1}{\theta}\right)^{1-\frac{1}{\nu}}$$

the mean of this distribution is given by $E[\theta] = \frac{1}{\nu+1}$. Moreover, the skewness $S(\nu)$ of $f(\theta)$ is given by

$$S(\nu) = \frac{2(\nu-1)\sqrt{1+2\nu}}{1+3\nu}$$

and it is positive and increasing for $\nu \geq 1$.

We assume that good projects are scarce, this means $\nu > 1$. It translates into right-skewness of the probability distribution of drastic innovations, as suggested by the empirical research in this area. For instance, Silverberg and Verspagen (2007) use patent data to study the skewness of the patent quality distribution proxied by citations. They find that both the distribution of citations and the return to patents are highly skewed, and that the tail index is roughly constant over time.[12] The fraction of high-type improvements when enacting a set $M \in (0,1]$ of projects is given by

$$\tilde{\mu}^H = \frac{1}{M}\int_0^1 prob(e \in M) \times \theta(e)\, de$$

[12]Other firm related variables with *fat tails* are widely documented in the literature. For instance, Moskowitz and Vissing-Jorgensen (2002) find large skewness on entrepreneurial returns. Axtell (2001) shows that the size distribution of US firms closely mimics Zipf distribution, where the probability of a firm having more than n employees is inversely proportional to n. Scherer (1998) uses German patent data to show the skewness of the distribution of profits and technological innovation.

Random selection implies that for all e, $prob(e \in M) = M$. We denote by $\underline{\tilde{\mu}}^H$ the proportion of high type project on the entering cohort under random selection. Then $\underline{\tilde{\mu}}^H$ equals the unconditional probability of observing a drastic innovation:

$$\underline{\tilde{\mu}}^H = \int_0^1 e^\nu de = \int_0^1 \theta f(\theta) d\theta = \frac{1}{\nu + 1}$$

Finally, the higher ν is, the lower the proportion of high type innovations among the randomly enacted cohort. This formalizes one of the main intuitions of the model, that projects are heterogeneous and good ideas are scarce.

1.3.5 The Representative Financial Intermediary

The second key novelty of this model is the introduction of a non-trivial financial system that screens and selects the most promising projects.[13] The representative financial intermediary has access to a unit mass of projects every period. It collects deposits from households, selects in which projects to invest according to their expected value, and pays back to the household the profits generated by these projects.[14] This set up implicitly assumes that all entrants are in need of external financing as enacting any project requires investment by the intermediary.[15] Note that, if $V_{j,t}^H > V_{j,t}^L$ for any product line j, the financial intermediary strictly prefers to enact projects with higher e. In particular, if e were observable, a financial intermediary willing to finance M projects would enact only the projects with $e \in [1 - M, 1]$. However, e is unobservable. Nevertheless, the financial intermediary has access to a costless, yet imperfect, screening technology that delivers a

[13] The closest reference of a financial intermediary performing this function in an endogenous growth model is King and Levine (1993b). Nevertheless, lacking a link between *ex ante* and *ex post* heterogeneity, the focus of their model is only in the effect of the mass of entrants.

[14] Alternatively, we can assume that the representative household owns the projects but does not have access to any screening technology. Hence it sells the projects to the representative financial intermediary at the expected profits net of financing costs, and the financial intermediary earns no profits.

[15] Nofsinger and Wang (2011) use data from 27 countries, to document that 45% of start-ups use funds from financial institutions and government programs. Categories for 2003: self saving and income (39.97%), close family members (12.79%), work colleague (7.7%), employer (14.18%), banks and financial institutions (33.92%), and government programs (11.02%).

stochastic signal $\tilde{e}$ defined by:

$$\tilde{e}_t = \begin{cases} \tilde{e}_t = e_t & \text{with probability } \rho \\ \tilde{e}_t \sim U[0,1] & \text{with probability } 1-\rho \end{cases}$$

Note that $\rho \in [0,1]$ characterizes the accuracy of the screening with $\rho = 1$ implying the perfect screening case. Levine (2005) suggests that one characteristic of financial development is the *improvement in the production of ex ante information about possible investments*. In this sense, the accuracy of the financial selection technology ρ is a reflection of the financial development of an economy. There is also empirical evidence of financial selection, for instance, Gonzalez and James (2007) document that firms with previous banking relationships perform significantly better after going public than firms without such relationships.[16] Define $V_t^d = E_j\left[V_{j,t}^d\right]$ to be the expected value of successfully enacting a project with step size d. Proposition 2 shows that when the expected return of a drastic innovation is higher than the one of generating an incremental innovation, the optimal strategy is to set a cut-off for the signal. The proof is provided in Appendix A.2.

Proposition 2. *If $V_t^H > V_t^L$, the optimal strategy for a financial intermediary financing M_t projects at time t is to set a cut-off $\bar{e}_t = 1 - M_t$, and to enact projects only with signal $\tilde{e}_t \geq \bar{e}_t$.*

When the financial intermediary optimally uses this technology to select a mass $M_t = 1 - \bar{e}_t$ of projects, the proportion $\tilde{\mu}_t^H(\bar{e}_t)$ of high type projects in the successfully enacted λM_t mass is given by

$$\begin{aligned} \tilde{\mu}^H(\bar{e}_t) &= \frac{1}{\lambda M_t}\int_0^1 \lambda \times prob(\tilde{e}_t \geq \bar{e}_t | e_t) \times \theta\left(e_t\right) de_t \\ &= \frac{1}{1-\bar{e}_t}\left[\int_0^{\bar{e}_t}\left(1-\rho\right)\left(1-\bar{e}_t\right) e_t^{\nu} de_t + \int_{\bar{e}_t}^1 \left\{\left(1-\rho\right)\left(1-\bar{e}_t\right)+\rho\right\} e_t^{\nu} de_t\right] \\ &= \frac{1}{\nu+1}\left[1-\rho+\frac{\rho}{1-\bar{e}_t}\left(1-\bar{e}_t^{1+\nu}\right)\right]. \end{aligned} \tag{1.11}$$

Note that for any cut-off $\bar{e}$, the composition increases with the level of financial technology

[16]Keys et al. (2010) document that the lower screening intensity in the sub-prime crisis generated between 10% and 25% more defaults.

ρ and decreases with the scarcity of high type projects ν. Moreover, in terms of the resulting composition, financial selection performs at least as well as the random selection of projects. We summarize these properties in Proposition 3.[17]

Proposition 3. *The proportion of high type entrants $\tilde{\mu}^H$ exhibits the following features:*

1. *$\tilde{\mu}^H(\bar{e}_t)$ is increasing in $\bar{e}_t$. Moreover, $\tilde{\mu}^H(\bar{e}_t)$ is increasing in ρ and decreasing in ν for every $\bar{e}_t$.*

2. *$\tilde{\mu}^H(\bar{e}_t) \geq \underline{\tilde{\mu}}^H$ with $\tilde{\mu}^H(\bar{e}_t) = \underline{\tilde{\mu}}^H$ if $\rho = 0$ or $\bar{e}_t = 0$.*

3. *$\tilde{\mu}^H(\bar{e}_t) = \frac{1-\bar{e}_t^{\nu+1}}{(\nu+1)(1-\bar{e}_t)}$ if $\rho = 1$ and $\lim_{\bar{e}_t \to 1} \tilde{\mu}^H(\bar{e}_t) = \frac{1+\nu\rho}{\nu+1} \leq 1$*

In this set up, the financial intermediary collects deposits D_t from the representative household in order to enact a mass $M_t = \frac{D_t}{w_t \kappa}$ of projects every period. Proposition 3 implies that the financial intermediary will always use its screening device.[18] Then, given $\{V_t^H, V_t^L, r_t, w_t\}$ the financial intermediary chooses $\{\bar{e}_t, D_t\}$ in order to solve

$$\begin{aligned} \max_{\{D_t, \bar{e}_t\}} \quad & \left\{ \frac{\lambda D_t}{w_t \kappa} \left[\tilde{\mu}^H(\bar{e}_t) V_t^H + (1 - \tilde{\mu}^H(\bar{e}_t)) V_t^L \right] - D_t(1 + r_t) \right. \\ & \left. -\xi_1 \left(1 - \bar{e}_t - \frac{D_t}{w_t \kappa} \right) - \xi_2 \left(\frac{D_t}{w_t \kappa} - 1 \right) + \frac{\xi_3}{w_t \kappa} D_t \right\} \end{aligned} \tag{1.12}$$

where $\{\xi_1, \xi_2, \xi_3\}$ are Lagrange multipliers that control for the range of $\bar{e}$, and the equality of the households' deposits to the demand of funds by the intermediary. Note that the term that multiplies the brackets in the first line is the mass of projects that are enacted and turn out to be successful. The bracketed term is the expected return of the portfolio with composition $\tilde{\mu}^H(\bar{e})$. The intermediary needs to pay back D_t plus the interest. As the objective function is strictly concave, the first order conditions are sufficient for optimality. As Proposition 3 states, a financial intermediary with $\rho > 0$ faces a trade-off between mass and composition of the enacted pool. Now, we examine the optimal decisions of the

[17] Proof is trivial and therefore omitted.

[18] When a fixed cost is included the partial solution exhibits a kink. In general equilibrium there is a region where the equilibrium implies not screening, another region where it always implies screening, and a third region characterized by non-existence. A well behaved variable cost does not alter the results significantly.

intermediary. First order conditions regarding $\{D_t, \bar{e}_t\}$, respectively, yield

$$\begin{aligned}\frac{\lambda}{w_t\kappa}\left[\tilde{\mu}^H(\bar{e}_t)V_t^H + (1-\tilde{\mu}^H(\bar{e}_t))V_t^L\right] - (1+r_t) + \frac{\xi_1}{w_t\kappa} - \frac{\xi_2}{w_t\kappa} + \frac{\xi_3}{w_t\kappa} &= 0\\ \frac{\lambda D_t}{w_t\kappa}\left(\frac{V_t^H - V_t^L}{\nu+1}\right)\left[\frac{\rho}{1-\bar{e}_t}\left(\frac{1-\bar{e}_t^{\nu+1}}{1-\bar{e}_t} - (\nu+1)\bar{e}_t^{\nu}\right)\right] + \xi_1 &= 0.\end{aligned}$$

Note that if $\rho > 0$ then $\xi_1 < 0$ which in turn implies a positive wedge between the marginal revenue the intermediary generates and the marginal payment it needs to make to households. Therefore, the screening technology allows the intermediary to make positive profits. Furthermore, the unique interior solution ($\xi_2 = \xi_3 = 0$) is characterized by

$$\rho\bar{e}_t^{\nu} = \frac{\frac{w_t\kappa}{\lambda}(1+r_t) - V_t^L}{(V_t^H - V_t^L)} - \frac{1-\rho}{(\nu+1)} \tag{1.13}$$

The uniqueness crucially depends on ρ being larger than zero. Otherwise, there are no profits and the intermediary is indifferent when enacting any mass of projects.

This partial equilibrium result is quite intuitive. In fact, the cut-off is increasing in the enacting cost κ, the interest rate, the wages, and the scarcity of good projects ν. The cut-off is decreasing in the precision of screening technology ρ and in the value of the projects which means that, in these cases, the intermediary is willing to enact more projects.

1.3.6 Equilibrium

Having introduced the basic components of the model, we can examine its equilibrium and balanced growth path (BGP). First, we characterize the analytical relationships posed by the equilibrium conditions, then we narrow down our analysis further to state the existence and uniqueness of a BGP, and characterize it analytically.

Definition 1 (Equilibrium)**.** *A competitive equilibrium for this economy consists of quantities* $\left\{D_t, \left\{x_{t,j}^S\right\}_{j\in[0,1]}, \left\{x_{t,j}^D\right\}_{j\in[0,1]}, c_t, y_t, a_{t+1}, \left\{l_{j,t}^d\right\}_{j\in[0,1]}, \bar{e}_t\right\}_{t=0}^{\infty}$*, policy parameters*

$\{\tau, T_t\}_{t=0}^{\infty}$, values $\left\{ \left\{ V_{j,t}^{H} \right\}_{j\in[0,1]}, \left\{ V_{j,t}^{L} \right\}_{j\in[0,1]} \right\}_{t=0}^{\infty}$, prices $\left\{ w_t\,,\, r_{t+1}\,,\, \{p_{j,t}\}_{j\in[0,1]} \right\}_{t=0}^{\infty}$, financial intermediary profits $\{\Pi_t\}_{t=0}^{\infty}$, intermediate good producer's profits $\left\{ \pi_{t,j}^{d} \right\}_{j\in[0,1]\,,\,t=0}^{t=\infty}$, entrants and incumbents compositions $\{\tilde{\mu}_t\,,\, \mu_t\}_{t=0}^{\infty}$ and initial conditions $\left\{ a_0\,, \{q_{0,j}\}_{j\in[0,1]}\,, \mu_0^H \right\}$ such that:

1. *Given $\{w_t\,,\, r_{t+1},\, T_t,\, \Pi_t\}_{t=0}^{\infty}$, household chooses $\{c_t\,,\, a_{t+1}\}$ to solve* (1.1) *subject to* (1.2) *and* (1.3).
2. *Given $\{p_{j,t}\}$, final good producer chooses $\left\{ \left\{ x_{t,j}^{D} \right\}_{j\in[0,1]} \right\}$ to solve* (1.5) *every t.*
3. *Given $\{w_t\}$, and $\{q_{j,t-1}\}$ intermediate producer of good j with type d sets $p_{j,t}$ according to* (1.9)*, and earns profits $\pi_{t,j}^{d}$, for every t that she remains the leader in product line j.*
4. *Given $\left\{ V_t^H\,,\, V_t^L\,,\, r_t\,,\, w_t \right\}$, financial intermediary chooses $\{D_t,\, \bar{e}_t\}$ to solve* (1.12) *every t.*
5. *Labor, asset, final and intermediate good markets clear:*

$$\int_0^1 l_{j,t}^{d}\, dj + (1-\bar{e}_t)\kappa = L \tag{1.14}$$

$$a_t = D_t = (1-\bar{e}_t)w_t\kappa \tag{1.15}$$

$$x_{j,t}^{S} = x_{j,t}^{D} \quad \Rightarrow \quad l_{j,t}q_{j,t} = \frac{y_t}{p_{j,t}} \tag{1.16}$$

$$c_t = y_t = e^{\int_0^1 \ln x_{j,t} dj} \tag{1.17}$$

6. *$V_{j,t}^{d}$ evolves accordingly to* (1.10)*, $q_{j,t}$ evolves accordingly to* (1.8)*, and government budget is balanced every period.*
7. *The entrant's composition $\tilde{\mu}_t$ is determined by* (1.11) *and the composition of the product line μ_t evolves according to:*

$$\mu_{t+1}^{H} = \mu_t^{H} + \lambda(1-\bar{e}_{t+1})\left(\tilde{\mu}_{t+1}^{H} - \mu_t^{H}\right). \tag{1.18}$$

An important feature of this class of models is that profits, values, and labor across intermediate goods are independent of the efficiency level accumulated in product line j up to time t. As a result, the particular product line j does not matter for the determination of these values; the size of the last innovation is a sufficient statistic for them. This is summarized in Proposition 4, the derivation is in Appendix A.3.

Proposition 4. *Equilibrium:*

1. $\forall j \in [0,1]$ *and* $\forall D \in \{L, H\}$ *we have:*

$$\pi^d_{j,t} \;=\; \pi^d_t \qquad ; \qquad l^d_{j,t} = l^d_t \qquad ; \qquad V^d_{j,t} = V^d_t$$

2. *If* $\sigma^H > \sigma^L$*:*

$$\pi^H_t \;>\; \pi^L_t \qquad ; \qquad l^H_t < l^L_t \qquad ; \qquad V^H_t > V^L_t$$

Proposition 4 shows that in equilibrium we have $V^H_t > V^L_t$ and hence the financial intermediary uses a cut-off strategy when selecting projects.[19] The system of equations that characterizes the equilibrium is in Appendix A.4.

Definition 2 (BGP)**.** *Define* $Q_t = \exp\left\{\int_0^1 \ln q_{j,t}\right\} dj$ *as the average efficiency level. The economy is in a Balanced Growth Path at time T if it is in such an equilibrium that,* $\forall t > T$*, the endogenous aggregate variables* $\{C_t,\, Q_t,\, Y_t,\, a_{t+1}\}$ *grow at a constant rate, and the threshold* $\bar{e}_t$ *is constant.*

Lemma 1 states the existence and uniqueness of a BGP for this economy. The proof is provided in Appendix A.5.

Lemma 1. *Existence and Uniqueness:*

$\frac{\kappa}{L} \in [a, b]$*, where* $\{a, b\}$ *are constants that depend on the model parameters, is a sufficient condition*

[19] Note that more efficient leaders needs less labor to serve the demand of their variety. For concreteness, imagine a type H leader with a follower of productivity level $\tilde{q}$. This leader will charge the same price as a type L leader who is also followed by someone with efficiency $\tilde{q}$. This implies that both are selling the same quantity, nevertheless, the more efficient leader needs less labor to produce that quantity, and hence earns more profits.

for the existence and uniqueness of an interior BGP for this economy.

1.3.7 Mass and Composition Effect

As derived in Appendix A.5, the long-run growth of this economy is characterized by the following expression:

$$1 + g(\bar{e}) = \left[(1+\sigma^H)^{\mu^H(\bar{e})}(1+\sigma^L)^{1-\mu^H(\bar{e})}\right]^{\lambda(1-\bar{e})} \tag{1.19}$$

The economic intuition of equation (1.19) is clear: the long-run growth of this economy is the geometric mean of the efficiency improvement weighted by the composition of the entrants and scaled by the mass of entrants. The trade-off between mass and composition is manifested in this term. Lower credit standard (low $\bar{e}$) implies a larger pool of entrants that increases the exponent of this term, but also decreases the base through the indirect effect on composition $\mu(\bar{e})$. The interaction of these two margins determines the long-run growth $g(\bar{e})$. Nevertheless, $\bar{e}$ is an endogenous variable, so we should also clarify the optimization problem that determines this variable.

To understand the source of the trade-off it is useful to think about two alternative cases the intermediary could face when investing in projects: An economy with no accuracy ($\rho = 0$) where project initialization is random, and a model with no heterogeneity ($\sigma^H = \sigma^L$) where selection is useless. These two alternatives have in common that the expected step size of the marginal enacted project is constant with respect to the total enacted mass, destroying the trade-off between the enacted mass and its composition.[20] But, the full model is characterized by the decreasing expected step size of the marginal entrants with respect to the total entry, this tension introduces a trade-off between mass and composition into the model. Since this is a general equilibrium model, the economic

[20] In both cases, the financial intermediary has no profits. Nevertheless this is not the source of the composition effect, if we impose a zero expected profit condition, as long as $\rho > 0$ and $\sigma^H > \sigma^L$, all the results carry on.

impact of this trade-off should be assessed by studying the long-run comparative statics of the model. Proposition 5 shows the general equilibrium comparative statics to changes in the enacting cost κ, the patience coefficient β, and the corporate tax rate τ.[21]

Proposition 5. *General Equilibrium Comparative Statics:*

1. *An economy with higher enacting cost κ has higher lending standards, less entry but better composition. Long-run growth decreases with κ:*

$$\frac{\partial \bar{e}}{\partial \kappa} \geq 0 \quad ; \quad \frac{\partial g(\bar{e})}{\partial \kappa} \leq 0 \quad ; \quad \frac{\partial \mu^H(\bar{e})}{\partial \kappa} \geq 0$$

2. *An economy with lower patience coefficient β has higher lending standards, less entry but better composition. Long-run growth increases with β:*

$$\frac{\partial \bar{e}}{\partial \beta} \leq 0 \quad ; \quad \frac{\partial g(\bar{e})}{\partial \beta} \geq 0 \quad ; \quad \frac{\partial \mu^H(\bar{e})}{\partial \beta} \leq 0$$

3. *An economy with higher corporate tax rate τ has higher lending standards, less entry but better composition. Long-run growth increases with τ:*

$$\frac{\partial \bar{e}}{\partial \tau} \geq 0 \quad ; \quad \frac{\partial g(\bar{e})}{\partial \tau} \leq 0 \quad ; \quad \frac{\partial \mu^H(\bar{e})}{\partial \tau} \geq 0$$

Proposition 5 shows first that economies with higher enacting cost (κ) enact in equilibrium less projects and hence, exert a tighter selection. Note that those economies are characterized by a lower rate of long-run growth but a higher composition on their product line. Second, economies with a higher patience coefficient (β) save more they are able to enact more projects. Although those economies grow more in the long-run, their average composition is lower. Finally, economies with higher corporate taxes (τ) have lower entry rates and lower long-run growth, but higher composition. All these cases share an impor-

[21] We select these parameters for the intuitive relationship to the main mechanism of the model, other results are available upon request. The proof is provided in Appendix A.6.

tant result: the mass effect generated by the underlying parametric change dominates the composition effect. Nevertheless, the composition effect introduces non-linearities on the relationship between credit availability and growth. In fact, in the alternative models that lack either selection or heterogeneity any marginal amount allocated to project enaction has a constant contribution to growth. Therefore, the relationship between entry (or total credit) and growth is linear. The model presented here breaks that linearity introducing a non-trivial relationship between entry and growth shaped by the interaction between heterogeneity, scarcity, and financial selection that characterizes the economy. In fact, the strength of the selection margin that determines the magnitude of the trade-off between mass and composition rest on the accuracy of the screening technology of the financial intermediary. Before concluding this section, we study the relatively more complex effect of a better screening technology (higher ρ).

Intuitively, better selection technology can be used to avoid enacting bad projects or to aim for more high-type projects. On the one hand, we can expect economies characterized by a high entry rates to increase their lending standards (higher $\bar{e}$) in response to an increase in the accuracy of their financial system. In fact, for those economies the marginal project enacted is more likely to be of low type, so the marginal benefit of improving the overall quality of the pool by reducing its size outweighs the potential benefit of increasing its mass. On the other hand, economies that are currently enacting less projects, should be willing to relax the selection standards and aim for a larger entry, since the marginal entrant has a high probability of becoming a type H leader. Proposition 6 gives analytical support to this intuition.[22]

Proposition 6. *Financial Development:*

1. *Let $\bar{s} > \underline{s}$ be two constants that are determined by the model parameters. For any economy with an equilibrium level of selection $\bar{e} \geq \bar{s}$ a marginal increase in the accuracy of the*

[22]The proof is provided in Appendix A.6.

screening technology ρ will result in a less selective equilibrium.

$$\bar{e} \geq \bar{s} \Rightarrow \frac{\partial \bar{e}}{\partial \rho} < 0.$$

2. *For any economy with an equilibrium level of selection $\bar{e} \leq \underline{s}$ a marginal increase in the accuracy of the screening technology ρ will result in a more selective equilibrium.*

$$\bar{e} \leq \underline{s} \Rightarrow \frac{\partial \bar{e}}{\partial \rho} > 0.$$

Proposition 6 suggests that the effects of financial development are non-monotonic. In particular, the level of domestic savings shapes the marginal response of entry to changes in the accuracy of the financial system.[23] This non-monotonic relationship between domestic savings and financial development challenges the most widely used proxy for economic development in the empirical literature. In fact, most of the cross country empirical research that relates financial development and economic growth proxies the first by the credit to output ratio. If we emphasize the screening role of the financial system, this strategy is only valid for economies with low entry rates. Moreover, the ambiguous relationship between financial development and firm entry carries on to the effect in growth. For example, if an increase in ρ triggers a reduction in the entry, the final effect on growth will depend on the relative strength of the two margins: a smaller cohort *versus* a higher proportion of drastic improvements.

To sum up, this section introduced a long-run endogenous growth model that features project heterogeneity and financial selection. In this economy good ideas are scarce and the ability of the financial intermediary to select the most promising ones is limited. This induces a trade-off between mass and composition as the larger the entrant cohort is, the lower the fraction of drastic innovations in the economy. The growth rate of this economy is endogenously determined and results from the interaction between mass and

[23] Recall that equation 1.15 imply a one to one mapping between entry and savings in equilibrium.

composition effect described above.

1.4 Mass and Composition: Two Quantitative Illustrations

In this section we perform a quantitative exploration of the model to illustrate the relevance of the composition effect introduced in this chapter. After proposing a reasonable parametrization of the model, we revisit two classical development problems.

First, we study the effects of corporate taxation on firm entry and economic growth. The empirical research points to an almost insignificant negative effect on long run growth but a strong and significant negative effect on firm entry. As the trade-off between mass and composition effect implies that the marginal entrant's contribution to growth is decreasing in the size of a cohort, the model can successfully account for both facts.

Second, we study the impact of financial development in economic growth. In the baseline parametrization, financial development reduces entry but increases growth due to a better allocation of resources. In particular, more financially developed economies tight their lending standards, experiencing gains from the composition margin that outweigh the losses on the mass margin. This generates a negative relationship between the level of financial development and the size of the entrant cohort. Alternative parametrizations with lower entry rates can generate a positive relationship between mass and financial development. Interestingly, the marginal gain from reallocation is increasing in the level of financial development. Moreover, in line with the empirical literature, financial development influence growth mostly by improving the allocation of resources.

1.4.1 Parametrization of the Model

We focus the baseline parametrization in high income economies, and then in each experiment we study deviations from this set-up. We proceed this way due to the availability of empirical literature on mark-up, and manufacturing productivity for more developed economies. Table 1 shows the baseline parametrization for the quantitative experiments of this section.

κ	λ	σ^L	σ^H	β	ν	ρ	γ	τ	L
0.12	0.25	0.095	0.45	0.95	5	0.9	2	0.3	1

Table 1: Parameter Values

Given the normalization of the labor force to 1 the value of κ implies that 12% of the labor force is enough to enact all the projects in the economy. The value of λ implies that one out of every four projects are able to generate a successful innovation in some product line. When the innovation is drastic the increase in the productivity of labor is 45% while an incremental innovation just generates a 9.5% increase in productivity. Given the scarcity parameter ν, the underlying heterogeneity of the projects is such that one out of every six projects is expected to generate a drastic innovation, this implies a highly skewed distribution for the probability of generating a drastic innovation.[24] The value of ρ suggests that 90% of the projects are successfully screened by the financial intermediary. In line with the average statutory corporate tax for high income economies presented by Djankov et al. (2010), we set τ to 30%. Finally, the intertemporal elasticity of substitution is set to 0.5 and the patience coefficient β to 0.95.

Table 2 presents a summary of the long-run implications of the model under the baseline parametrization.

$\bar{e}$	μ^H	$\lambda(1-\bar{e})$	g	r	$\frac{\kappa w}{Y}$	$\kappa(1-\bar{e})$	$Av.(\sigma)$	$Sd.(\sigma)$	$Sk.(\pi)$
0.599	0.373	0.100	0.012	0.095	0.105	0.048	0.228	0.172	0.524

Table 2: Output of the Model

The resulting cut-off value implies that 40% of the projects are enacted, given the level of financial development the resulting composition on the intermediate good sector

[24]The implied skewness using Proposition 1 is 1.66, in general, any value larger than one is considered high.

is more than two times higher than the one under random selection. The entry rate of 10% is in line with the international firm level evidence for developed countries.[25] The growth rate is also consistent with the average labor productivity growth of the European Union and the United States reported by Ark et al. (2008).[26] Fracassi et al. (2012) report an average interest rate for start up loans in the United States 11.5% slightly higher than the one generated by this set of parameters.[27] According to the *Doing Business* project, the average entry cost in 2012 resulting from fees and legal procedures among the OECD countries was 4.5% of the average per capita income. Moreover, the average minimum capital requirement to start a business was 13.3% for those countries, also in 2012, so the entry cost generated by the model of 10.5% of the average income is in line with the data. Fairlie (2012) states that in 2011, according to the Kauffman index of Entrepreneurial Activity, 0.32% of adults in the United States were engaged in business creation every month. This implies that almost 4% of the adult population was engaged in entrepreneurship every year which is comparable to the 5% generated by the parametrized model. The average markup generated by the model is also consistent with the estimates of Christopoulou and Vermeulen (2012). They document an average markup of 28% for the manufacturing and construction sector in the United States between $1981-2004$ and a corresponding value of 18% for the Euro area. The standard deviation of the markup is roughly half of the one estimated by Dobbelaere and Mairesse (2005) for the French economy between $1978-2001$.[28] Finally, the resulting skewness of the profit distribution is roughly consistent with the values reported by Scherer et al. (2000).[29] The first quantitative experiment studies the effects of corporate taxation in both entry and growth rates.

[25] According to the International Finance Corporation's micro small and medium-size enterprises database the Euro area has an average entry rate of 8.9% between $2000-2007$ while United States has a 12.9% average entry rate between $2003-2005$.

[26] They report an average of 1.5% for the European Union between $1995-2005$ and 2.3% for United States over the same period.

[27] They use the complete set of start-up loan applications received by Accion Texas between $2006-2011$. This number is consistent with the 11.3% reported by Petersen and Rajan (1994) from the National Survey of Small Business Finance also in the US for the years 1988 and 1989.

[28] Their weighted markup average estimation (33%) more than doubles the one estimated for France by Christopoulou and Vermeulen (2012).

[29] Note that financial selection implies that not all the underlying skewness is passed to the composition of the intermediate producers.

1.4.2 Corporate Taxation, Firm Entry, and Growth

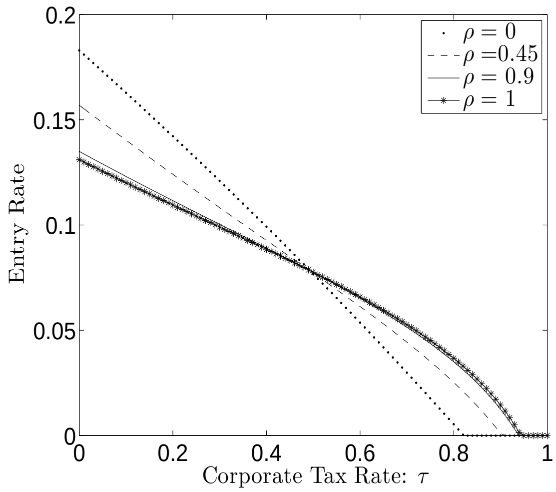
The empirical literature indicates a weak negative relationship between corporate taxes and long-run growth rates, whereas the effect on firm entry is found to be negative and sizeable. On the one hand, a cross sectional study with 85 countries performed by Djankov et al. (2010) suggests that decreasing the average tax rate from 29% to 19% would increase the average entry rate from 8% to 9.4%. Moreover, Da Rin et al. (2011a) explore a firm level panel data for 17 European countries, and find a non-linear relationship between corporate taxes and entry rates with high responses in the relevant corporate tax range. On the other hand, the empirical growth literature finds only a slightly negative effect of corporate taxation on growth. To compare the magnitude of this relationship to the former stated regularity on entry rates we can take the estimation of Gemmell et al. (2011), where a 10 percentage points corporate tax reduction could increase long-run growth by at most 0.3 percentage points.[30] In summary, the research in corporate taxation suggests a fragile negative effect on growth and an economically significant negative effect on entry.[31]

Figure 2 shows the long-run responses of entry, composition, and growth in the model to changes in corporate taxation for the baseline parametrization ($\rho = 0.9$) and three other values of ρ. Figure 2d displays the entry elasticity of growth defined as the ratio of the percentage change in growth to the percentage change in entry generated by a one percentage point increase in taxation. In particular, an elasticity smaller than one in absolute value implies that marginal increases in taxation have larger absolute marginal effects on entry than in growth. In other words, growth responds to taxation less than entry does. In line with Proposition 5, increases in marginal taxation reduce both entry

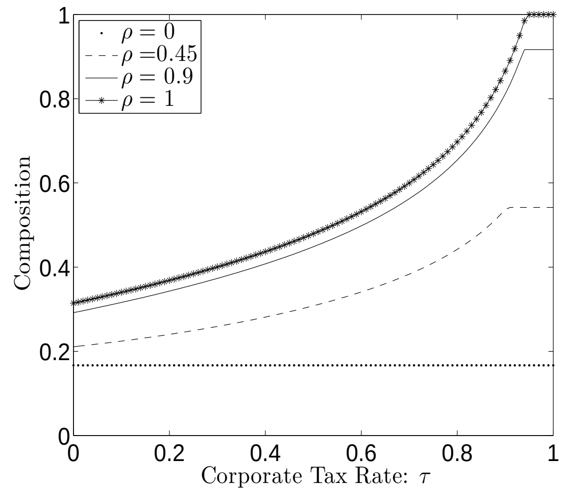
[30] Easterly and Rebelo (1993) study this relationship using a panel of 125 countries spanning over 1970 − 1988 and find that there is no robust effect of taxes on growth. Widmalm (2001), and Angelopoulos et al. (2007) establish a similar result for the OECD countries. Moreover, Levine and Renelt (1992) argue that the negative relationship documented in the literature is not robust to slight changes on the specifications of the econometric model.

[31] For concreteness, Appendix A.7 uses cross country data to illustrate that higher taxes are significantly and strongly correlated with lower entry, but the negative correlation with growth rate is extremely weak.

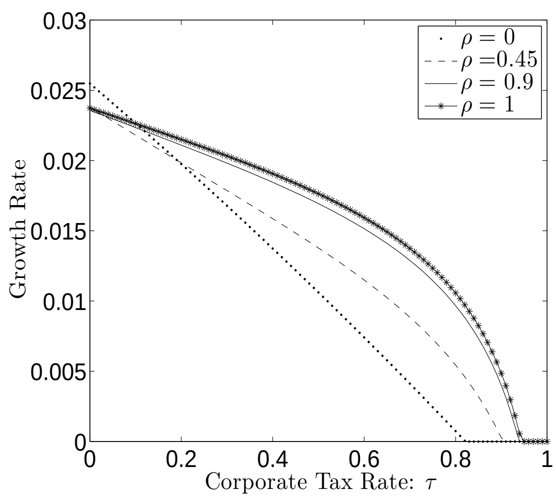
and growth, but improve the composition of the economy.[32]



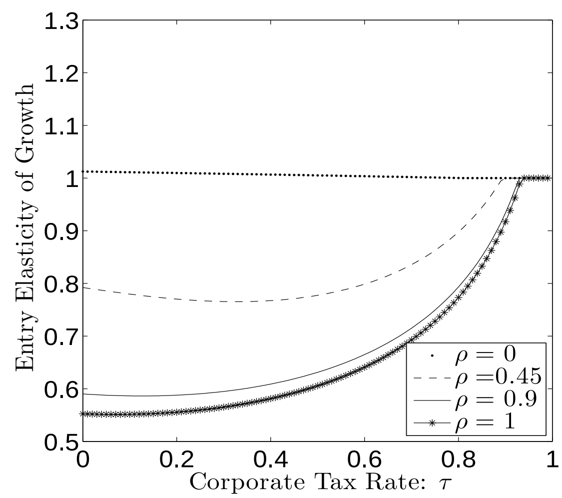
(a) Entry



(b) Composition



(c) Growth



(d) Elasticity

Figure 2: The Effect of Corporate Taxation on Growth and Entry

We first focus the analysis on the responses of the model when ρ is at its benchmark level. As Figures 2a and 2c show, the response of long-run entry and growth to changes in taxation are both highly non-linear, yet the growth rate exhibits the strongest non-linearity. Moreover, the responses of both, entry and growth are in line with the magnitudes suggested by the empirical literature discussed above. In fact, a tax cut of 10

[32]Recall that this result holds only for interior solutions. In fact, after a corner solution is met, entry and growth are both zero and do not react to extra taxation.

percentage points from the baseline parametrization of 30% increases growth from 1.98% to 2.11% while the increase in entry from 10% to 12.5% is also in line with the empirical studies. This asymmetry in the response to taxation is summarized in Figure 2d: For a wide range of tax rates, the percentage decline in the growth rate caused by a one percentage point increase in taxation is only 60% of the corresponding decline in the entry rate. The reason behind this difference is the strength of the composition effect. As seen in Figure 2b the decrease in entry induced by higher corporate taxation implies tighter lending standards and hence a higher composition. In fact, financial selection implies that the contribution of the marginal entrant to growth is decreasing in entry. Therefore, the initial reductions in entry triggered by higher corporate taxation do not impose an important cost in terms of growth to this economy. Only when the level of taxation reaches extremely high levels, the sacrificed entrants pose a sizeable challenge to the long-run growth of the economy.

In a related article, Jaimovich and Rebelo (2012) use a similar mechanism to generate extremely non-linear responses of long-run growth to taxation. Their model combines the product line expansion framework of Romer (1990) with the heterogeneous ability framework of Lucas (1978). In a nutshell, entrepreneurs are heterogeneous in their ability to create firms, and more skilled entrepreneurs have a higher rate of success when enacting a project.[33] As the distribution of ability is highly skewed, relatively few entrepreneurs explain most of the entry rate of the economy. Hence, increases in taxation discourages only marginal entrepreneurs, and both the entry and the growth rates respond mildly for a wide range of taxes. In their model there is no *ex post* heterogeneity, all the active incumbents are identical, and hence the average *per firm* contribution to growth is the same for every cohort, regardless of its size.[34] In other words, even though their model

[33]In the context of our model, the heterogeneity is not in σ but in λ. Nevertheless, as the frameworks are completely different, this comparison need to be taken cautiously. In fact, Romer (1990) engine of growth is not the Schumpeterian creative destruction of Aghion and Howitt (1992), but an expansion in the number of intermediate varieties without replacement.

[34]They focus on self selection instead of financial selection, we believe that both mechanisms are present in the data and reinforce each other.

features selection, the only engine of growth is the volume of the entrant cohort: the mass effect. The absence of a composition channel implies that their model exhibits, by construction, an elasticity equal to one for any level of taxation, so it cannot generate any asymmetry between the responses of entry and growth.[35] Hence, the composition margin is fundamental when modelling this asymmetry.

Returning to Figure 2, to illustrate the key role of financial selection in determining the strength of the composition effect, we compare the baseline parametrization with three others that only differ in the value of ρ. The dotted line represents a model with no financial selection ($\rho = 0$) where project enaction is random, and in line with Proposition 3, composition is constant, and the responses of growth and entry to taxation are linear. Moreover, as shown in Figure 2d, there is no asymmetry between the two responses. The other two parametrizations exhibit intermediate levels of financial development. Figure 2a shows that for a wide range of corporate tax rates the parametrizations with lower levels of financial development exhibit higher entry rates. Nevertheless, as seen in Figure 2c, these economies are not able to capitalize the larger entry into higher economic growth. This is a consequence of the potential strength of the composition effect, where economies with less entry can grow at a faster pace only due to a higher proportion of drastic innovation. In fact, as shown in Figure 2b, the higher the corporate tax rate, the bigger the compositional advantage of the more developed economies. Moreover, for extremely high tax rates, a more developed economy can have larger and better cohorts than a less developed one, dominating the later not only in composition but also in mass. Finally, note that more financially developed economies exhibit extremely convex responses in growth, accentuating the asymmetry between the sensitivity of growth and entry to corporate taxation. This is clear in Figure 2d, where more financially developed economies

[35] Jaimovich and Rebelo (2012) do not study the effects on entry. When interpreting their results we use the same definition as in Romer (1990) for an entrant. Nevertheless, if an entrant is defined as one entrepreneur regardless of the number of product lines that she owns, then that model also generates this asymmetry between entry and growth. In this case, the composition should refer to the average size of an entrant in terms of the number of product line per entrepreneur. Yet, still the only engine of growth is the increase in the number of product lines, and hence, a mass perspective.

have systematically lower entry elasticities to growth. Given the relevance of the financial development parameter ρ, we explore quantitatively its influence in entry and growth in the next experiment.

1.4.3 Financial Development, Resource Allocation and Output Growth

To close the quantitative section we explore the interactions between financial development, resource allocation and output growth. In particular, we emphasize the relevance of Proposition 6 when studying the empirical relationship between financial development and economic growth. To illustrate this we present our results under three parametrizations that differ only in the fixed cost of project enaction (κ). Note that in Proposition 6, κ does not enter in $\bar{s}$ or $\underline{s}$, and it affects $\bar{e}$ monotonically. Hence, different values for κ are a natural choice to illustrate the non monotonicity introduced by changes in financial development (ρ) for economies with different entry rates. In particular, economies with high κ, which are characterized by a higher $\bar{e}$ and a lower entry rate, are likely to increase entry when ρ increases, but the opposite is expected from economies with low κ.

Size Measures and Financial Development

Figure 3 shows the long-run responses of entry, growth, composition, and the ratio of entry and composition effects to changes in the accuracy of the screening technology. To calculate the last component, we start by taking the natural logarithm of equation (1.19):

$$g_{t+1} \approx \ln\left(1+g_{t+1}\right) = \lambda\left(1-\bar{e}_t\right)\left(\mu_t^H \ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right) + \ln\left(1+\sigma^L\right)\right)$$

Taking natural logarithm again we have:

$$\ln\left(g_{t+1}\right) \quad \approx \quad \ln\left(\lambda\left(1-\bar{e}_t\right)\right)+\ln\left(\mu_t^H \ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)+\ln\left(1+\sigma^L\right)\right)$$

Then, the change in this expression as a response to one percentage point shift in ρ yields approximately

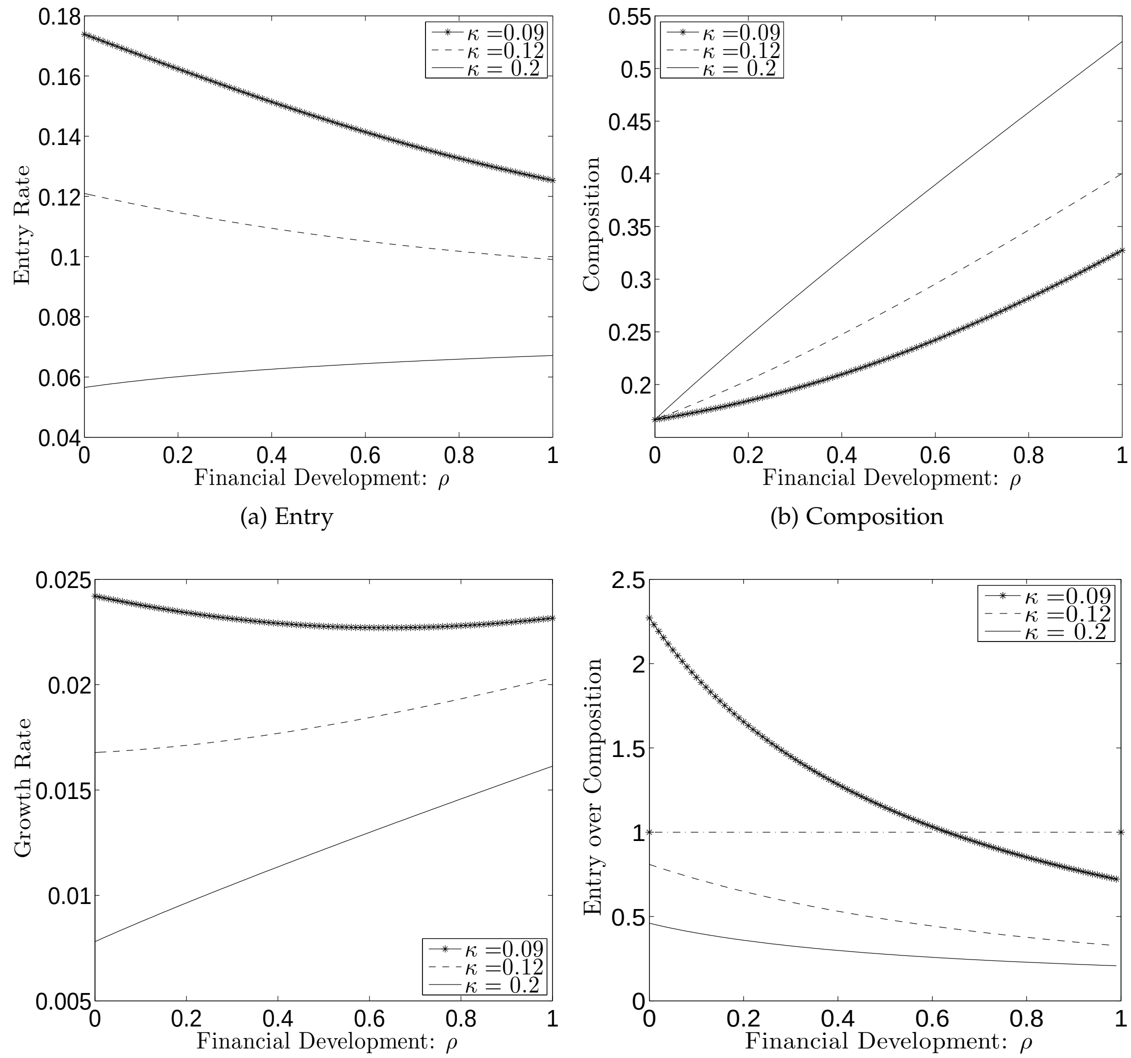
$$\Delta\%_\rho\, g \approx \Delta_\rho\%\, mass + \Delta_\rho\%\, composition$$

Now we can define the relative measure of entry and composition effects at each level of ρ:

$$\eta_\rho = \frac{|\Delta_\rho\%\, mass|}{|\Delta_\rho\%\, composition|}.$$

Note that, $\eta_\rho > 1$ implies that mass effects accounts for most of the percentage change in growth due to a marginal change in ρ, and $\eta_\rho < 1$ reflects the dominance of the composition effect.

In line with Proposition 6, Figure 3a shows that at high levels of entry (dashed and asterisk lines) entry rate decreases with financial development, while for the highest value of κ (solid line) the entry rate increases in ρ. As shown in equation 1.15, the entry rate $\lambda(1-\bar{e})$ and the level of domestic savings $(1-\bar{e})\kappa w$ are always positively related. As a result, the non-monotonicity between the entry rate and financial development implies that the relationship between domestic savings and financial development is not monotonic as well. Therefore, size based measures such as domestic credit over output do not necessary reflect the cross-country differences in the accuracy of the financial systems.



(a) Entry (b) Composition

(c) Growth (d) Relative Strength

Figure 3: The Effect of Financial Development on Growth and Entry

Resource Allocation and Financial Development

Note first that in Figure 3, under the baseline parametrization, mass and composition effects act in opposite directions: a higher level of financial development reduces the mass but improves the composition of the entrant cohort.[36] Note that as $\eta_\rho < 1$ for all the domain, the composition effect dominates the mass effect for this parametrization,

[36]Two forces explain the rise in composition: a direct one due to the increase in ρ, and an indirect one due to the reduction in entry.

and thus growth is increasing in ρ in Figure 3c. This suggests that under the baseline parametrization, the main source of growth is the reallocation of resources, and not an increase in the volume of resources allocated. In contrast, for low levels of κ (asterisk line) $\eta_\rho > 1$ in the low ρ region, the mass effect dominates, and output growth might even decrease with ρ.[37]

Figure 3d also shows that the relevance of the composition effect is always rising with ρ for every value of κ. This pattern can be explained using Figures 3a and 3b. While the proportion of high type leaders rises at increasing rates in Figure 3b, the changes in entry take place at decreasing rates for any κ. Moreover, for high levels of ρ, $\eta_\rho < 1$ at any value of κ. Hence, an increase in ρ in more financially developed economies translates into higher gains in growth mainly through the composition channel. In other words, more financially developed economies do not necessarily allocate more resources in order to grow, but they are more efficient in allocating them.

It is also interesting to note that for any given κ more financially developed economies experience larger improvements in growth due to a marginal increase in ρ than less developed economies do.[38] In fact, the effect of financial development on economic growth is highly non-linear: countries that are financially challenged benefit less from financial development than financially developed countries.

Related Empirical Work

Before reviewing some of the related empirical literature it is useful to recall the three main messages delivered by Figure 3: *i*) entry and financial development exhibit a non-monotonic relationship, *ii*) the main channel through which financial development affects

[37]Bose and Cothren (1996) find a similar result in the context of optimal contracting in an externality-driven growth model. Imposing a zero profit condition on the financial intermediary can eliminate this feature of our model.

[38]Note that at higher levels of ρ the difference in growth rates for different entry costs κ decreases. In accordance with Asturias et al. (2012), this suggests that financial development helps to overcome entry barriers, and that the importance of ρ in terms of economic growth is greater when κ is higher.

economic growth is the better allocation of resources, rather than the allocation of more resources, and *iii*) the effect of financial development on economic growth is highly non-linear, more developed countries benefit more from marginal increases in their ability to select promising projects.

On the empirical side, the seminal contribution of Rajan and Zingales (1998) examine a cross country cross industry sample and find that industries with higher financial dependency grow faster in countries with more developed financial markets. Note that, in the context of the model presented in this chapter, industries more in need of the financial system should be subject to more screening, and hence, grow more in more financially developed countries. Nevertheless, this analogy is accurate only if the empirical proxy for financial development is a good measure of the screening accuracy ρ. Rajan and Zingales (1998), as most of the literature, use a size measure in order to proxy for financial development, in particular, they use the total size of the stock market and the measure of domestic credit. But, as seen in Proposition 6, the amount of resources available in the credit market is not always positively related with the accuracy of the financial system.

Rioja and Valev (2004) explicitly mention this issue when using a 74 countries panel data to study if the effect of financial development in growth is monotonic across levels of financial development. In fact, they use three proxies for financial development, two of them centered on the size dimension (private credit and liquid liabilities) and a third measure that tries to proxy for the ability of an economy to perform a more accurate selection (the ratio of commercial bank assets over central bank assets).[39] For the two size measures they find that the effects of financial development are stronger for countries with an intermediate level of financial development than for countries with high levels. Moreover, the effect on countries with very low levels of financial development is insignif-

[39]The empirical work of King and Levine (1993b) and King and Levine (1993a) states these and other proxies for financial development. They suggest that the higher this ratio is, the stronger the screening in the economy, since commercial bank tend to exert a more thorough selection. For each of their measures they find a strong relationship between economic growth and financial development, moreover, they use case studies of financial reforms to validate them.

icant. Nevertheless, when using the third measure, they also find a significant economic effect for lower levels of financial development. All their specifications point to strong non-linearities in both the relationship between volume of credit and economic growth, and the relationship between screening intensity and economic growth. These observations are in line with the non-linearities displayed under different parametrizations in Figure (3).

In another related empirical study, Wurgler (2000) studies the efficiency of the allocation of resources for different economies. His main contribution is the development of an elasticity based index that measures the ability of an economy to increase its investment in growing industries, and decrease it in the ones that are shrinking. He finds no significant relationship between the volume of capital allocated in manufacturing and his proxy for financial development. Therefore, in line with Figure 3d, financially more developed economies grow faster mainly because of a better allocation of resources. He also finds that his measure of efficient capital allocation is strongly and positively related with the idiosyncratic (firm level) information available in the stock prices, a measure of the information available in the economy.[40]. Finally, Galindo et al. (2007) use a different approach to study the relationship between finance and the allocation of resources.[41] They use firm level panel data for 12 developing countries to build a measure of the efficiency in the allocation of resources, and then they use the chronology of financial reforms in Laeven (2003) for those countries. They find that episodes of financial liberalization are linked to better allocation of resources, but not necessarily to a larger mobilization of resources, this is again consistent with Figure 3d.

In sum, this section presented a quantitative examination of the strength and relevance of the composition effect. The first experiment showed that the composition effect

[40]The lower price *synchronicity* on the stock market, measured as in Morck et al. (2000), the higher the idiosyncratic information contained on the stock. He also finds that reallocation is more efficient when state ownership declines, and minority stockholder rights are strong

[41]They also review the cross country and firm level literature on the relationship between financial liberalization and growth. They argue that the positive effect on growth is well established, while a clear effect on the amount of resources allocated has not been found.

can overturn the mass effect and allow an economy to grow faster even when enacting less projects. We also explained how the composition effect can rationalize the empirical relationship between corporate taxation, firm entry, and economic growth. The second illustration replicated the empirically observed non-linear relationship between financial development, allocation and reallocation of resources. This experiment also exemplifies the risks associated with the use of volume based proxies for financial development.

1.5 Skewness: A Proxy for Financial Selection

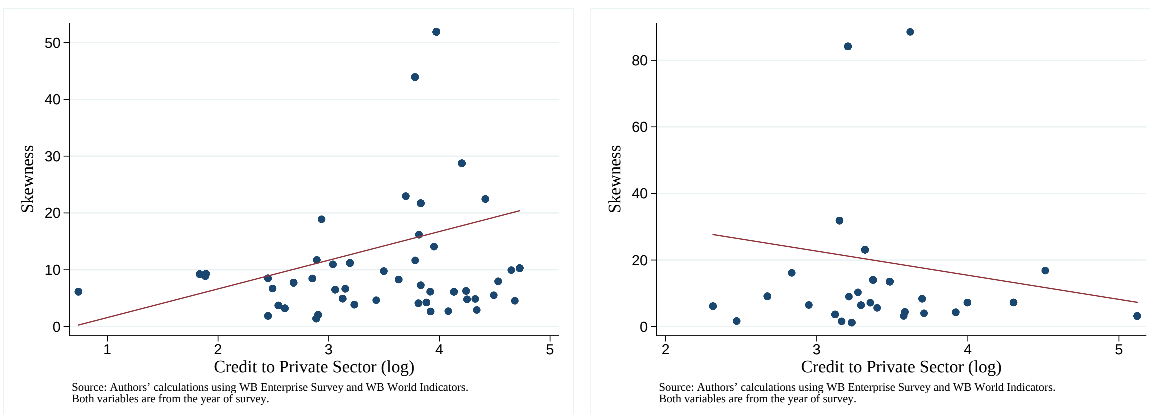
This chapter suggests that size-based measures are not appropriated proxies to assess the accuracy of the financial development in an economy when allocating resources to their best uses. In this section, we propose a micro-based measure that can potentially capture the selective function of the financial system. We also show that in line with the prediction of the model, size based measures and the proposed proxy behave non-monotonically.

The model implies that economies characterized by high accuracy (ρ) are successful at selecting projects ranked above the intended threshold ($e \geq \bar{e}$). Thus, among the enacted projects there are only few that with extremely low probability will generate a high cost reduction. This implies that the left tail of the realized ratio of value added to cost (profitability) among entrants is thin. Therefore, more financially developed economies should be characterized by low skewness in the profitability distribution of their incumbents.[42] Data to generate this variable are obtained from Private Enterprise Survey conducted by the WorldBank. This is an annual survey of about 135,000 firms in 135 countries that focuses on the financial and private sectors. Cost is defined as sales minus electricity, raw materials, and labor expenses. Value added is sales minus cost. Each observation of skewness is weighted by the firm weights specified by the survey. For the relevance of

[42] The underlying assumption is that countries of interest have similar levels of ex-ante skewness across the potential projects.

using weights, see Garcia-Santana and Ramos (2013).

According to Proposition 6 for low entry rates more financial development (lower skewness) should be associated with higher resources to firm entry, whereas the opposite should hold for countries characterized by high entry rates. We build our size based proxy to take into account the different entry costs across countries. In particular, we divide domestic credit by the average entry cost faced by firms, it reflects the total numbers of firms that could be created if all the credit is used to finance start-ups. We call this variable *Credit to the Private Sector*. Following our theoretical result we divide the sample of countries into two groups: below and above median entry, and we evaluate the relationship of the two measures.



(a) Above median entry (b) Below median entry

Figure 4: Financial Development and Private Credit

Figure 4 shows the results.[43] Our finding confirms the theoretical argument. Financial development and private credit have indeed a non-monotonic relationship, and the direction of it is governed by the level of entry rate, as suggested by our model. This again shows why a researcher should be cautious when proxying the level of financial soundness of an economy with measures that only reflect the size of funds available to firms. Such proxies capture only one side of the impact of the financial development on economic growth, yet miss another one: the selective role in allocating resources. Hence,

[43] Figure 18 in Appendix A.8 presents a version after removing the outliers. Results remain the same.

variables that capture the selective margin of the financial system should complement the empirical analysis of financial development. As a first candidate, we propose the skewness of the ratio of value added to cost observed in the economy.

1.6 Conclusion

In this chapter we introduced project heterogeneity and financial selection in an analytically tractable way to the classical endogenous growth framework of Aghion and Howitt (1992). A financial intermediary, with access to an imperfect screening device, selects *ex ante* heterogeneous projects characterized by an idiosyncratic probability of generating a drastic innovation. Following implementation of the projects, the model also delivers an *ex post* heterogeneity, where two types of incumbents have different cost advantages over their followers, and hence, earn more profits. The model has a unique interior balanced growth path shaped by the Schumpeterian creative destruction generated by new firms. The impact of creative destruction in this economy results from the interaction between the mass and the composition of the entrant cohort. The relative strength of each margin crucially depends on the underlying scarcity of drastic ideas relatively to the accuracy of the selection technology in the economy.

Two quantitative experiments illustrate the importance of including heterogeneity and financial selection into the endogenous growth framework. First, since the marginal entrant has a decreasing contribution to economic growth, changes in the entry rate are not linearly mapped into the economic growth rate of the economy. Hence, this framework can accommodate the strong negative relationship between entry rates and corporate taxation without delivering a counterfactually strong negative effect of corporate taxation on economic growth.

The second experiment addresses the widely-debated link between financial development and economic growth. Two main lessons arise from this experiment. First, size-based measures miss the selective role of financial system and therefore are not good proxies for financial development. Hence, variables that capture the selection aspect should complement empirical work that aim to assess the level of financial development. As a first step, we suggest the skewness of the ratio of value added to cost across firms. The idea is that as financial selection improves less extremely unproductive firms operate in the market, reducing the *bad* tail of the value added to cost distribution. Second, the effect of financial development in economic growth is extremely non-linear. In particular, for a country with a high degree of financial development, a marginal increase in that financial development leads to a greater increase in growth, relative to the change in firm entry.

The next chapter extends this framework to study the growth effect of a credit crunch. A stochastic version of this model is well suited for economic analysis even outside the balanced growth path. Moreover, when using firm level data from Chile and the financial crisis triggered by the Russian sovereign default of 1998 as a natural experiment to test the model, we observe a strong compositional component; in fact, cohorts born under tighter credit conditions perform significantly better than cohorts arising under laxer credit standards. We believe that this framework can be enriched and brought quantitatively to data in order to perform policy analysis. For instance, changes in corporate taxation, entry barriers or financial liberalization can be evaluated, even accounting for the economic transition between the two balanced growth paths.

Chapter 2

Fewer but Better: Sudden Stops, Firm Entry, and Financial Selection

This chapter is co-authored with Felipe E. Saffie.

Abstract

We combine the real business cycle small open economy framework with the endogenous growth literature to study the productivity cost of a sudden stop. In this economy, productivity growth is determined by successful implementation of business ideas, yet the quality of ideas is heterogeneous and good ideas are scarce. A representative financial intermediary screens and selects the most promising ideas, which gives rise to a trade-off between mass (quantity) and composition (quality) in the entrant cohort. Chilean plant-level data from the sudden stop triggered by the Russian sovereign default in 1998 confirms the main mechanism of the model, as firms born during the credit shortage are *fewer, but better*. A calibrated version of the economy shows the importance of accounting for heterogeneity and selection, as otherwise the permanent loss of output generated by the forgone entrants doubles, which increases the welfare cost by 30%.

2.1 Introduction

In August 1998, the Russian sovereign default triggered a violent sudden stop in the developing world.[1] Interest rate spreads for the seven biggest Latin American economies tripled in the weeks after this crisis, decreasing the availability of external funding by 40% between 1998 and 2002. Most of the economic analysis of these crises of interest rate spreads is centered on the short-run detrimental effects that they imposed on the real economy. Nevertheless, the empirical studies of large economic downturns by Cerra and Saxena (2008) and Reinhart and Rogoff (2014) have documented persistent output losses associated with large economic downturns, pointing to permanent losses in total factor productivity. Because firm entry is an important driver of productivity growth, and because start-ups are in need of external funding, distortions in firm entry are likely to cause part of this long-run cost. This paper develops a framework that links short-run financial crises with long-run output losses through distortions in firm entry.

Two aspects are key for a meaningful study of the entry margin. First, behind every firm lies an entrepreneur's idea, and ideas are not born alike. In fact, drastic innovations are a scarce resource. Second the financial system does not allocate funding randomly, and not every idea has the same chance of being granted an opportunity. Not surprisingly, when resources are scarce, banks adopt higher lending standards, and fund only the most promising projects. The main novelty of this study is the recognition that the scarcity of good ideas and the presence of financial selection induces a trade-off between the size of the entrant cohort and the average contribution of each firm within that cohort to aggregate productivity. Consistent with this intuition, we use micro-data to document that firms born during a sudden stop are *fewer, but better*. Failure to consider this trade-off would imply that discarded projects are just as productive as actual entrants, magnifying

[1] A sudden stop in capital flows is a large and abrupt decrease in capital inflows, characterized by jumps in sovereign spreads and quick reversals of current accounts deficits. See Calvo and Talvi (2005) for details of that episode.

the productivity cost of a crisis, and potentially misleading public policy. Thus, the ability of the financial system to allocate resources between heterogeneous projects needs to be taken into account when facing the main question of this paper: what is the productivity cost of the forgone entry during a sudden stop?

In order to answer this question, we generalize the real business cycle small open economy model of Neumeyer and Perri (2005) to include entry-driven endogenous growth in the tradition of Grossman and Helpman (1991) and Aghion and Howitt (1992).[2] We extend this hybrid framework in two dimensions. First, we model business plan heterogeneity and scarcity by introducing a financial intermediary with a portfolio of business plans that can generate either a drastic or a marginal productivity improvement in the production technology of an intermediate variety. Every project is characterized by its idiosyncratic probability distribution over those two improvements. Hence, projects are *ex-post* heterogeneous in terms of the productivity advantage that they enjoy after entering the industry, and they are also *ex-ante* heterogeneous with respect to their idiosyncratic probability of generating a drastic innovation. Moreover, because only a few ideas are highly likely to give birth to outstanding incumbents, promising projects are scarce. The second extension introduces financial selection. In fact, the financial intermediary cannot unveil the *ex-ante* heterogeneity of the projects in her portfolio but she can observe noisy signals of their potential. The optimal allocation of funding follows a cut-off rule based on the signal, which introduces a linkage between the size of the entrant cohort and the average efficiency gain generated by its members. The strength of this link and its implications for entry and productivity are determined by the accuracy of the screening device of the financial system. The financial intermediary borrows at the stochastic interest rate to finance start-ups. Therefore, interest rate shocks trigger entry and productivity dynamics that are absent in a traditional open economy framework. The model has a unique non-stochastic interior balanced growth path that allows for quantitative solu-

[2] This combination renders endogenous the trend distortions that Aguiar and Gopinath (2007) use to explain business cycles in small open economies.

tions of the stochastic dynamic equilibrium. In the model economy, a mass/composition trade-off (that is, a quantity/quality trade-off) arises at the cohort level: periods of high interest rates are characterized by high credit standards that give rise to smaller cohorts with higher expected average productivity.

The empirical section studies the Chilean sudden stop of 1998-2000 to validate the trade-off between mass and composition at the core of the model. We focus on Chile for three reasons: (i) it is a small open economy; (ii) plant level data for Chilean manufacturing firms is publicly available, and this data allows us to directly study entrant cohorts; and (iii) as argued by Calvo et al. (2006), the sudden stop after the Russian sovereign default is mainly exogenous to the Chilean economy. We show that firm entry in Chile from 1996 to 2007 decreased by 40% during the sudden stop, even at the three digit industry level. However, firms born in crisis are not just fewer, they are also better. In fact, the econometric analysis in Section 2.4 shows that, after controlling by individual characteristics, firms born during normal times are on average 30% less profitable during their life span than firms born during the sudden stop.

In the quantitative section of the paper, we calibrate the model to the Chilean economy between 1996 and 2007. We then use the Chilean sudden stop to assess the performance of the model, fitting the real interest rate faced by the country during this episode. This stylized model with a single shock is able to capture more than 40% of the decrease in firm entry, 20% of the conditional increase in profitability, and one-third of the observed decrease in firms' values. After validating the model, we introduce two modified economies in order to assess the role of heterogeneity and selection in shaping the effect of a sudden stop: one is a model with exogenous growth, and the other is a model with endogenous growth but no heterogeneity. We use those alternative economies to highlight the role of firm entry and financial selection when the economy is hit by a shock that increases the interest rate.

Three important features arise from the comparisons of these models. First, distortions in the entry margin trigger permanent losses in output in the models with endogenous technological change. The composition margin shapes the long-run cost of these short-run crises. In fact, the model with no heterogeneity predicts a permanent loss in output two times larger than the one predicted by the baseline model, implying a 30% larger welfare cost, in consumption equivalent terms. This is a large economic magnitude that can bias public policy during a crisis toward entry subsidies or indiscriminate government lending. Second, including endogenous technological progress amplifies the medium-run effects of a crisis. For instance, the baseline model amplifies the effects of a sudden stop in output by 30%, compared to the model with exogenous growth. Third, including heterogeneity among intermediate goods producers triggers compositional dynamics that increase the medium-run persistence of these episodes. A final experiment studies the importance of the allocative function of the financial system during these crises. More developed economies suffer more in the short run but endure much better the medium-run effects of the crisis. Moreover, they are subject to a lower permanent productivity loss. The calibrated model also suggests that the benefits of financial development are decreasing in the level of financial development.

The structure of the paper is as follows. Section 2.2 reviews the related literature. Section 2.3 introduces our model and characterizes the existence and uniqueness of an interior balanced growth path. Section 2.4 presents the analysis of the Chilean economy as a *pseudo* natural experiment for the model, exploring at the macro and micro level the consequences of the sudden stop for the Chilean economy. Section 2.5 presents the calibration of the model and the quantification of the long-run cost of a sudden stop. Finally, Section 2.6 concludes the paper and suggests avenues for future research.

2.2 Related Literature

This paper belongs to the intersection between the endogenous growth and the small open economy literature.[3] This is not the only paper introducing endogenous growth into the small open economy real business cycle framework of Mendoza (1991). For example, Queraltó (2013) studies the long-lasting productivity effects of a financial crisis; in his model an interest rate shock triggers a balance sheet channel, which harms the processes of invention and implementation. Ergo, fewer firms enter the market and fewer ideas are developed for future use. The endogenous growth model at the core of that paper is the framework that Comin and Gertler (2006) build around Romer (1990). Guerrón-Quintana and Jinnai (2014) use a similar framework to study the effect of the liquidity crash in 2008-2009 on U.S. economic growth. Gornemann (2014) combines the endogenous default model of Mendoza and Yue (2012) with the variety model of Romer (1990) to study how endogenous growth affects the decision of the sovereign to default. Because default increases the price of imported intermediate goods in his model, it decreases the expected profits of potential entrants, and, hence, depresses productivity growth.

This paper makes three contributions to the existing research. First, by introducing endogenous growth as in Aghion and Howitt (1992) instead of Romer (1990), it recognizes the dual effect of firm entry: new comers are also a destructive force that replaces incumbents. Second, it develops a tractable framework to include heterogeneity in this class of models. This dimension has proven to be key in separately analyzing, the short-run and the long-run behavior of an economy. In fact, as noted by Bilbiie et al. (2012), firm heterogeneity significantly affects the short-run fluctuations of an economy. Moreover, the quantitative literature on innovation also shows that firm heterogeneity is crucial for understanding the long-run effects of technology adoption. A salient example of the latter can be found in Akcigit and Kerr (2010). Therefore, including heterogeneity when

[3] Obstfeld (1994) studying the growth effect of international risk sharing is one of the first papers at this edge.

studying the link between short-run fluctuations and long-run productivity is a natural extension and an important element to consider. The third contribution is the use of firm level data to provide evidence of the main driving force in the model and bring discipline to the quantitative experiment. This class of models, where the main driving force is micro-funded, should be compared not only to macro aggregates, but also to firm level data. This paper is a step in that direction, but much remains to be done in linking micro data to macro models. This paper is therefore related to the empirical literature that uses firm level data to study financial crises. Two papers in that literature are particularly related to our study. Firstly, Schnabl (2012) uses the sudden stop triggered by the Russian default to document how international banks reduced lending to Peruvian banks, and how Peruvian banks diminished lending to Peruvian firms during the crisis. Secondly, Hallward-Driemeier and Rijkers (2013) evaluate the effects of recessions using firm level data from Indonesia during the Asian crisis of 1997. They do not find conclusive evidence of better reallocation among incumbents. In line with our findings, they do find an increase in the contribution of the entry margin to aggregate productivity during the crisis.

2.3 A Stochastic Open Economy Model with Entry and Selection

In this section, we introduce a tractable endogenous growth model with heterogeneity and financial selection, for a small open economy, subject to exogenous interest rate shocks. Aggregate productivity in this economy is modeled in the Grossman and Helpman (1991) and Aghion and Howitt (1992) tradition.[4] This means that we follow a Schumpeterian concept of growth, where new firms (entrants) replace established firms (incumbents). In particular, because new intermediate goods producers are more productive than incumbents, Bertrand monopolistic competition implies that the newcomer sets a

[4] A detailed review of this literature can be found in Aghion et al. (2014).

price that forces the old incumbent out of the market.[5] In order to study the role of financial selection in firm entry and productivity during a sudden stop, three main innovations are added into this traditional endogenous growth framework.

The first variation introduces *ex-ante* and *ex-post* heterogeneity in productivity improvements. A representative financial intermediary owns business plans (projects or potential firms) that can generate either a high (H) productivity improvement (step size) or a low (L) improvement in the technology for producing a particular variety of an intermediate good. Every project is characterized by its idiosyncratic probability distribution over those two outcomes. Hence, projects are *ex-post* heterogeneous in terms of the productivity advantage that they enjoy after entering the business ($\{H, L\}$), and they are also *ex-ante* heterogeneous with respect to the idiosyncratic probability of generating a drastic innovation ($P^H \in (0,1)$). This first ingredient allows us to model the underlying scarcity of the economy, where only few ideas are very likely to give birth to outstanding incumbents.

The second addition to the framework introduces an imperfect screening device to the model. The financial intermediary cannot unveil the *ex-ante* heterogeneity of its projects, but it can observe noisy signals of their potential. The optimal allocation of funding follows a cut-off rule based on the signal. This ingredient introduces a linkage between the size of the entrant cohort and the average efficiency gain generated by its members. In fact, periods of laxer credit standards (low cut-off) are characterized by a larger cohort and lower average step sizes. The strength of this link and its implications for entry and productivity are determined by the accuracy of the screening device of the financial system.

Finally, the third modification follows the framework of Neumeyer and Perri (2005) to introduce exogenous interest rate shocks into the model. This feature introduces eco-

[5] Bartelsman et al. (2009) use cross-country firm level data to quantify the importance of the entry margin for productivity growth. Its direct effect ranges from 20% to 40%.

nomic dynamics into an otherwise deterministic model. Note that, because the financial intermediary borrows at the stochastic interest rate to finance start-up businesses, interest rate shocks trigger entry and productivity dynamics that are absent in a traditional open economy framework. The next sub-section introduces the model, defines an equilibrium for this economy and proves the existence and uniqueness of an interior balanced growth path (BGP).

2.3.1 Final Good Producer

Time is discrete in this economy. We denote a history $(s_0, s_1, ..., s_t)$ by s^t, where s^t contains all the relevant past information that agents need to make decisions in period t. For instance, $Y(s^t)$ is the output at period t under history s^t, but, because capital used in production at time t is decided at $t-1$, we index it by s^{t-1}. There is a representative final good producer that combines intermediate inputs ($\{X_j(s^t)\}_{j\in[0,1]}$), indexed by $j \in [0,1]$, with capital ($K(s^{t-1})$), to produce the only final good of this economy ($Y(s^t)$). The constant return to scale production function is given by:

$$\ln Y(s^t) = \alpha \int_0^1 \ln X_j(s^t) dj + (1-\alpha) \ln K(s^{t-1}). \tag{2.1}$$

Equation (2.1) is an extension of a standard unit elastic production function, where α determines the production share of intermediate varieties. Production is subject to a working capital constraint. In particular, the final good producer needs to hold a proportion $\eta > 0$ of the intermediate goods bill before production takes place. To do so, she borrows at the interest rate at the beginning of the period and pays back just after production takes place.[6] Uribe and Yue (2006) show that this constraint can be summarized as a wedge in the cost of the input when interest rates are positive. In particular, given input

[6]This is a standard modeling assumption in the open economy literature. It is mostly used to amplify interest rate shocks using a labor channel. The main mechanism of the model does not need this feature; in fact, as Appendix B.6 shows, the long-run effect of this channel is negligible. We include it only to compare the baseline model with a standard open economy model with exogenous growth.

prices ($p_j(s^t)$), interest rate ($R(s^t)-1$), and utilization cost of capital ($r(s^t)$), the final good producer demands intermediate goods and capital in every period in order to solve:

$$\max_{\left\{X_j(s^t)\right\}_{j\in[0,1]},K(s^{t-1})} \left\{ Y(s^t) - \left(1+\underbrace{\eta(R(s^t)-1)}_{\text{Cost wedge}}\right)\int_0^1 X_j(s^t)p_j(s^t)dj - K(s^{t-1})r(s^t) \right\} \quad (2.2)$$

where the final good price is used as the *numeraire*. An interior solution to (2.2) is characterized by the following set of first order conditions:

$$X_j(s^t) = \frac{\alpha Y(s^t)}{p_j(s^t)\left(1+\eta(R(s^t)-1)\right)} \quad \forall j \quad , \quad (2.3)$$

$$K(s^{t-1}) = \frac{(1-\alpha)Y(s^t)}{r(s^t)}. \quad (2.4)$$

Both demands are unit elastic; in particular, a monopolist facing the demand in equation (2.3) would choose $p_j(s^t) \to \infty$ and hence $X_j(s^t) \to 0$. Only the existence of a potential competitor can force the intermediate producer to set a finite price.

2.3.2 Intermediate Goods Sector: *Ex-post* Heterogeneity

There is a continuum of incumbents, each producing a differentiated intermediate good indexed by j. Labor ($L_j(s^t)$) is the only input used in intermediate production and the technology has constant marginal productivity ($q_j(s^t)$). Thus, the production of variety j is given by:

$$X_j(s^t) = L_j(s^t)q_j(s^t). \quad (2.5)$$

The efficiency of labor ($q_j(s^t)$) in the production of intermediate goods evolves with each technological improvement generated by a successful entrant. Entrants are heterogeneous in their capacity to improve the existing technology. Drastic innovations (type H) improve the efficiency level by a factor of $1+\sigma^H$, while marginal innovations (type L) generate improvements with a smaller factor of $1 + \sigma^L$, where $\sigma^H > \sigma^L > 0$.[7] Innovations in this economy come exclusively from newcomers. Then, we define the indicator functions $I_j^d(s^{t-1}, s_t)$, taking the value 1 if product line j receives an entrant of type $d \in \{L, H\}$ under $s^t = (s^{t-1}, s_t)$, and 0 otherwise. We can summarize the evolution of the productivity of the most efficient firm in product line j as follows:

$$q_j(s^t) = \left[1 + I_j^H(s^{t-1}, s_t) \times \sigma^H + I_j^L(s^{t-1}, s_t) \times \sigma^L\right] \times q_j(s^{t-1}). \tag{2.6}$$

Hence, productivity in product line j remains unchanged in the next period if, and only if, no entry takes place in that product line; in that case, the last period's incumbent continues to dominate the product line.

In line with the endogenous growth literature, we assume Bertrand monopolistic competition in each product line. In order to understand how this framework allows us to abstract from the distribution of productivity along product lines, we solve the partial equilibrium problem of the intermediate good producer before continuing with the exposition of the model. This monopolistic competition set-up implies that the competitor with the lowest marginal cost dominates the market by following a limit pricing rule, i.e., she sets her price ($p_j(s^t)$) at the marginal cost of the closest follower. We denote the efficiency level of the closest follower by $\tilde{q}_j(s^t)$. Then, given wage ($W(s^t)$), the optimal price is set to:

$$p_j(s^t) = \frac{W(s^t)}{\tilde{q}_j(s^t)}. \tag{2.7}$$

Note that (2.6) implies that a leader with type d has productivity $q_j(s^t) = (1 + \sigma^d) \times \tilde{q}_j(s^t)$. Then, using the demand for varieties of the final good producer from (2.3), we derive the following expression for the profits ($\Pi_j^d(s^t)$) of the leader in product line j with

[7] We allow for only two types in order to summarize the composition of the product line with only one variable: the fraction of leaders with σ^H advantage.

productivity advantage d:

$$\Pi_j^d(s^t) \quad = \quad X_j(s^t)\left(p_j(s^t) - \frac{W(s^t)}{q_j(s^t)}\right) = \frac{\alpha\sigma^d}{(1+\sigma^d)\,(1+\eta(R(s^t)-1))}Y(s^t). \qquad (2.8)$$

Note that profits are independent of the product line, because the type of the current leader is the only relevant characteristic of product line j. Moreover, type H leaders enjoy higher profits than type L leaders in every period. Profits are subject to corporate taxation rate (τ). The value of the firm is determined by the present discounted value of its after-tax profits in the current period. Nevertheless, in the next period, the firm will continue to produce if, and only if, it is not replaced by a new leader. In fact, at time $t+1$, when a mass $M(s^t, s_{t+1}) \in (0,1]$ of projects is funded, a portion $0 < \lambda < 1$ of them will randomly enter the intermediate sector; at that time, every incumbent firm faces a time-variant survival probability of $1 - \lambda M(s^t, s_{t+1})$. Finally, using the stochastic discount factor of the representative household ($m(s^t, s_{t+1})$), the expected discounted value $V^d(s^t)$ of owning any product line j of a type d leader at time t can be defined recursively by:[8]

$$V^d(s^t) \quad = \quad (1-\tau)\Pi^d(s^t) + E\left[m(s^t, s_{t+1})\underbrace{\left(1-\lambda M(s^t, s_{t+1})\right)}_{\text{survival probability}} V^d(s^t, s_{t+1})|s^t\right] \qquad (2.9)$$

where $E\left[\bullet|s^t\right]$ denotes the conditional expectation over every possible s_{t+1} event after history s^t. Note that *ex-post* firm heterogeneity can be summarized by $d \in \{L, H\}$, since every type d leader charges the same price, hires the same number of workers, and earns the same profits. Therefore, we do not need to keep track of the distribution of labor productivity across product lines; we can instead summarize the relevant information of the intermediate sector by the fraction of leaders with step size H, namely, the time-variant fraction $\mu(s^t) \in [0,1]$.

[8] See 2.3.5 for the characterization of $m(s^t, s_{t+1})$.

2.3.3 Projects: *Ex-ante* Heterogeneity

There is a financial intermediary that owns a continuum of projects indexed by z and uniformly spread on the unit interval ($z \in [0,1]$). The fixed cost of starting (enacting) a project is κ units of labor.[9] After a successful beginning, a project materializes into a new firm generating an undirected innovation. One of the key novelties in this model is the way heterogeneity and scarcity are introduced, in particular, how the *ex-ante* heterogeneity in projects is related to the *ex-post* heterogeneity of incumbents.[10]

Projects are heterogeneous in their expected step size; every project has an unobservable idiosyncratic probability $P^H(z) = z^\nu$ ($\nu > 0$) of generating a drastic improvement in productivity characterized by step size $\sigma^H > \sigma^L$. The higher the index z, the more likely it is that project z will generate a drastic (type-H) innovation, and, hence, the higher the expected increase in productivity. In this sense, z is more than an index; it is a ranking among projects based on their idiosyncratic and unobservable $P^H(z)$. Note that ν governs the scarcity of good ideas in this economy. In fact, the implied probability distribution of P^H is given by:

$$f(P^H) = \frac{1}{\nu}\left(\frac{1}{P^H}\right)^{1-\frac{1}{\nu}}.$$

The mean of this distribution reflects the expected proportion of type H entrants when projects are enacted randomly. In fact, for any $M(s^t)$, random selection implies that, for all z, $prob(z \in M) = M$. Therefore, the fraction of high-type improvements ($\underline{\tilde{\mu}}$) when enacting a set of projects randomly is given by:

$$\underline{\tilde{\mu}} = \frac{1}{\lambda M}\int_0^1 \lambda \times prob(z \in M) \times P^H(z)\, dz = \int_0^1 P^H f(P^H) dP^H = \frac{1}{\nu + 1}$$

[9] As Klenow et al. (2013) show, cross country industry level data suggests that entry cost is mostly associated with labor. The main mechanism of the model would not change if the entry cost were instead denominated in final goods units.

[10] For the heterogeneity and scarcity of ideas, see the high skewness in firm level related variables. See, for instance, Scherer (1998) and Silverberg and Verspagen (2007).

As an example, if $\nu = 3$, then the expected proportion of type H projects in the portfolio of the financial intermediary is 25%. Therefore, if a mass of $M(s^t)$ of projects is enacted randomly, a quarter of the $\lambda M(s^t)$ entrants generate a step size σ^H. Moreover, we can characterize the skewness of $f(P^H)$ as follows:

$$S(\nu) = \frac{2(\nu-1)\sqrt{1+2\nu}}{1+3\nu}$$

Note that the skewness is fully determined by ν, and is positive and increasing for every $\nu > 1$. Intuitively, note that $\nu = 1$ implies a uniform distribution for $f(P^H)$; hence, $S(1) = 0$ because the distribution is symmetric. However, for $\nu > 1$, the skewness is strictly positive, indicating that the left tail concentrates most of the probability density. This means that only a few ideas have strong chances of generating drastic improvements in productivity. Thus, ν summarizes the underlying scarcity of *good* ideas in the economy.

2.3.4 The Representative Financial Intermediary: Selection

In this economy, projects are heterogeneous and good ideas are scarce. Therefore, as the ranking z is unobservable, project selection is not a trivial task.[11] We thus introduce a screening device in order to study the effects of financial selection.

The representative financial intermediary has access to a unit mass of projects in every period. It borrows funds and selects projects in which to invest according to the expected present value of the projects, and pays back the profits generated by its portfolio to the household every period.[12] Note that, because $V^H(s^t) > V^L(s^t)$, the financial intermediary strictly prefers to enact projects with higher z. In particular, if z were observable, a

[11] For empirical studies documenting financial selection, see, for instance Dell'Ariccia et al. (2012) and Jiménez et al. (2014). Alfaro et al. (2004) document that more developed financial system can better materialize Foreign Direct Investment into economic growth. Moreover, Holmstrom and Tirole (1997), Chan-Lau and Chen (2002), and Agénor et al. (2004), study how lending standards vary with macroeconomic conditions.

[12] Alternatively, we can assume that the representative household owns the projects but does not have access to any screening technology. Hence, in equilibrium, it sells the projects to the representative financial intermediary at the expected profits net of financing costs, and the financial intermediary earns no profits. A similar motivation is used by Jovanovic and Rousseau (2014).

financial intermediary willing to finance $M(s^t)$ projects would enact only the projects with $z \in [1 - M(s^t), 1]$. However, z is unobservable. In order to introduce selection, we define a costless, yet imperfect, screening technology that delivers the following stochastic signal $\tilde{z}$ of the underlying ranking z:

$$\tilde{z} = \begin{cases} \tilde{z} = z & \text{with probability } \rho \\ \tilde{z} \sim U[0,1] & \text{with probability } 1 - \rho. \end{cases}$$

The financial intermediary can observe the true ranking of the project with probability $\rho \in [0, 1]$; otherwise, the ranking of the signal is drawn uniformly from the unit interval. Intuitively, ρ characterizes the accuracy of the screening, with $\rho = 1$ implying the perfect screening case.[13]

Proposition 7. *The optimal strategy for a financial intermediary financing $M(s^t)$ projects at time t is to set a cut-off $\bar{z}(s^t) = 1 - M(s^t)$, and to enact projects only with signal $\tilde{z} \geq \bar{z}(s^t)$.*

Proposition 7 shows that the optimal strategy is to set a cut-off for the signal.[14] When the financial intermediary uses this technology optimally to select a mass $M(s^t) = 1 - \bar{z}(s^t)$ of projects, the proportion $\tilde{\mu}(\bar{z}(s^t))$ of high type projects in the successfully enacted $\lambda M(s^t)$ mass is given by:

$$\begin{aligned} \tilde{\mu}(\bar{z}(s^t)) &= \frac{1}{\lambda M(s^t)} \int_0^1 \lambda \times prob(\tilde{z} \geq \bar{z}(s^t)|z) \times P^H(z)\, dz \\ &= \underbrace{\frac{1}{\nu + 1}}_{\underline{\tilde{\mu}}} \times \underbrace{\left[1 - \rho + \rho \frac{1 - \left(\bar{z}(s^t)\right)^{\nu+1}}{1 - \bar{z}(s^t)}\right]}_{\geq 1}. \end{aligned} \tag{2.10}$$

Note that, for any cut-off $(\bar{z}(s^t))$, the composition of H-types $(\tilde{\mu}(\bar{z}(s^t)))$ increases with the level of accuracy (ρ) and decreases with the scarcity of high type projects (ν). Moreover, in terms of the resulting composition, financial selection performs at least as well as

[13]For instance, the battery of questions and procedures that commercial banks use to discriminate among borrowers is sometimes truly informative about the potential of the projects, but *false positives* and *false negatives* also happen.

[14]As the expected value is strictly increasing in the signal, and the enacting cost is fixed, the cut-off strategy is optimal and unique.

random selection does. Because screening is costless, the financial intermediary will always use its device to select projects. Then, the financial intermediary borrows exactly $W(s^t)M(s^t)\kappa$ in order to enact $M(s^t) = 1 - \bar{z}(s^t)$ projects every period. In particular, given $\{V^H(s^t),\, V^L(s^t),\, R(s^t),\, W(s^t)\}$, the financial intermediary chooses $\bar{z}(s^t)$ in order to solve:

$$\max_{\bar{z}(s^t)\in(0,1)} \left\{ \underbrace{\lambda(1-\bar{z}(s^t))}_{\text{Cohort's mass}} \left[\underbrace{\tilde{\mu}(\bar{z}(s^t))V^H(s^t) + (1-\tilde{\mu}(\bar{z}(s^t)))V^L(s^t)}_{\text{Cohort's expected value}} \right] - \underbrace{(1-\bar{z}(s^t))R(s^t)W(s^t)\kappa}_{\text{Total cost of enaction}} \right\}. \tag{2.11}$$

The bracketed term is the expected return of the portfolio with composition $\tilde{\mu}\left(\bar{z}(s^t)\right)$. The intermediary needs to pay back the borrowed amount plus the interest. Because the objective function is strictly concave, the first order conditions are sufficient for optimality.[15] As equation (2.10) shows, a financial intermediary with $\rho > 0$ faces a trade-off between mass and composition of the enacted pool: lower $\bar{z}_t(s^t)$ increases the mass of projects enacted, but it also decreases the average value of the entrant cohort. If an interior solution ($\bar{z}(s^t) \in (0,1)$) exists, it is unique and characterized by:

$$\bar{z}_t(s^t) = \left(\frac{\frac{W(s^t)\kappa}{\lambda}R(s^t) - \left[\frac{1}{1+\nu}V^H(s^t) + \frac{\nu}{1+\nu}V^L(s^t)\right]}{\rho(V^H(s^t) - V^L(s^t))} + \frac{1}{\nu+1} \right)^{\frac{1}{\nu}}. \tag{2.12}$$

Note that, from a partial equilibrium perspective, for $\rho > 0$ the cut-off ($\bar{z}_t(s^t)$) increases with the interest rate. Nevertheless, from a general equilibrium perspective, the interest rate also affects the intermediary's choice of cut-off through wages and values.[16] Finally, using the mass ($\lambda(1-\bar{z}(s^t))$) and composition ($\tilde{\mu}(s^t)$) of the entrant cohort, we derive the law of motion of the composition of incumbents in the intermediate sector ($\mu(s^t)$). In fact, as entry is undirected, the evolution of the composition among incumbents is given by:

$$\mu(s^t) = \mu(s^{t-1}) + \lambda\left[1-\bar{z}(s^t)\right]\left[\tilde{\mu}(\bar{z}(s^t)) - \mu(s^{t-1})\right]. \tag{2.13}$$

[15]The second derivative is given by $-\rho\nu(\nu+1)\left[V^H(s^t) - V^L(s^t)\right]\left(\bar{z}(s^t)\right)^{\nu-1} < 0$.

[16]Random selection ($\rho = 0$) boils down to a zero profit condition with constant composition $\underline{\tilde{\mu}}$, with the intermediary either at a corner, or indifferent between any cut-off.

Note that, given last period's composition, and the value of this period's cut-off, we can pin down this period's composition.

2.3.5 The Representative Household

There is a representative consumer in this economy, and it is modeled following the open economy literature that builds on Mendoza (1991). In particular, as in Neumeyer and Perri (2005) and Uribe and Yue (2006), we include both capital adjustment costs and a bond holding cost. Capital adjustment costs are very popular in the business cycle literature, and they become particularly important in an open economy set-up with an exogenous interest rate. Without them, moderate fluctuations in the interest rate can generate implausible variations in investment. Bond holding costs are even more important in this literature because a fundamental indeterminacy arises between consumption and bond holdings.[17] Schmitt-Grohé and Uribe (2003) discuss several alternatives to solve this issue, and show that every method delivers the same quantitative results. From an economic perspective, bond holding costs can be thought to capture legal and bureaucratic issues related to levels of debt that differ from their usual long-run level.[18] In particular, the household chooses state-contingent sequences of consumption $C(s^t)$, labor $L(s^t)$, bond holding $B(s^t)$, and investment $I(s^t)$, given sequences of interest rate $R(s^t)$, wages $W(s^t)$, capital rental rates $r(s^t)$, and initial bond and capital positions, in order to solve:

$$\max_{\{B(s^t),C(s^t),L(s^t),I(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E\left[u(C(s^t),L(s^t))|s_0\right] \tag{2.14}$$

[17] In a nutshell, because the interest rate is completely inelastic with respect to the demand for bonds, consumption shows excessive smoothing and its level cannot be pinned down independently of the amount of bond holdings. This becomes critical in a dynamic setting as the Lagrange multiplier associated with the bond holding decision exhibits a unit root. Then, in the absence of bond holding costs, when a shock hits the economy, the level of debt never returns to its stationary value.

[18] Strong precautionary motives can alter this conclusion. Nevertheless, in this set-up, precautionary savings are very limited because there is no borrowing constraint.

subject to:

$$C(s^t) \leq W(s^t)L(s^t) + r(s^t)K(s^{t-1}) + B(s^{t-1})R(s^{t-1}) + T(s^t) + \Pi(s^t) - I(s^t) - B(s^t) - \Psi(\bullet) \tag{2.15}$$

$$I(s^t) = K(s^t) - (1-\delta)K(s^{t-1}) + \Phi(\bullet). \tag{2.16}$$

where $E\left[\bullet|s_0\right]$ is the expectation over history s^t, conditional on the information at $t = 0$; $0 < \beta < 1$ is the constant discount factor; investment is subject to convex adjustment costs $\Phi(\bullet)$; and bond holdings are subject to the convex cost function $\Psi(\bullet)$. The household also receives the profits of the financial intermediary $\Pi(s^t)$, as well as the revenue generated by corporate taxation $T(s^t)$, which the government levies on intermediate firms. As shown in the sequences of budget constraints defined by equation (2.15), the price of consumption is set to unity because we use the final good as the *numeraire*. The program also requires the transversality conditions on capital and bond holdings.

Following Neumeyer and Perri (2005), we modify Greenwood et al. (1988) preferences (GHH) to allow for a balanced growth path equilibrium. However, in our set-up, because aggregate labor productivity $(A(s^t))$ grows at an endogenous rate, the scaling is time-variant.[19] We also take from them the functional forms for Ψ and Φ:

$$\begin{aligned} u(C(s^t), L(s^t)) &= \frac{1}{1-\gamma}\left(C(s^t) - \Theta_l A(s^t)\left(L(s^t)\right)^{\chi}\right)^{1-\gamma} && (2.17)\\ \Psi(B(s^t), Y(s^t)) &= \frac{\psi}{2}Y(s^t)\left(\frac{B(s^t)}{Y(s^t)} - \bar{b}\right)^2 && (2.18)\\ \Phi(K(s^{t-1}), K(s^t)) &= \frac{\phi}{2}K(s^{t-1})\left[\frac{K(s^t)}{K(s^{t-1})} - (1+g_{bgp})\right]^2. && (2.19) \end{aligned}$$

where $\Theta_l > 0$ is the labor weight, $\chi > 1$ determines the Frisch elasticity of labor $\left(\frac{1}{\chi-1}\right)$,

[19] The usual economic intuition used to justify the scaling of labor dis-utility by labor productivity is that the opportunity cost of labor consists mostly of home production. Therefore, if non-market labor productivity grows at the same rate as market labor productivity, the dis-utility of labor must be scaled by it. Benhabib et al. (1991) study how home production shapes participation in the formal labor market, and how that intuition can be modeled by the preferences used in this model.

γ is the utility curvature, and $\phi > 0$ and $\psi > 0$ determine the convex cost functions. Note that, because $\bar{b}$ is the long-run household debt-output ratio and g_{bgp} the long-run growth of the economy, the household pays neither adjustment nor bond holding costs along the balanced growth path. In order to characterize the interior first order conditions of this problem, we define the stochastic discount factor of the household ($m(s^t, s_{t+1})$) as:

$$m(s^t, s_{t+1}) = \beta \frac{\frac{\partial u(C(s^{t+1}), L(s^{t+1}))}{\partial C(s^{t+1})}}{\frac{\partial u(C(s^t), L(s^t))}{\partial C(s^t)}}$$

where

$$\frac{\partial u(C(s^t), L(s^t))}{\partial C(s^t)} = \left(C(s^t) - \Theta_l A(s^t) \left(L(s^t)\right)^{\chi}\right)^{-\gamma}.$$

Then, the interior first order conditions can be stated as:

$$B(s^t): \quad 1 + \psi\left(\frac{B(s^t)}{Y(s^t)} - \bar{b}\right) = E\left[m(s^t, s_{t+1})|s^t\right] R(s^t) \tag{2.20}$$

$$K(s^t): \quad E\left[m(s^t, s_{t+1}) \frac{r(s^t, s_{t+1}) + (1-\delta) - \frac{\phi}{2}\left(\left[1 + g_{bgp}\right]^2 - \left[\frac{K(s^t, s_{t+1})}{K(s^t)}\right]^2\right)}{1 + \phi\left[\frac{K(s^t)}{K(s^{t-1})} - \left(1 + g_{bgp}\right)\right]} |s^t\right] = 1 \tag{2.21}$$

$$L(s^t): \quad W(s^t) = \Theta_l A(s^t) \chi \left(L(s^t)\right)^{\chi - 1} \quad \Rightarrow \quad L(s^t) = \left(\frac{W(s^t)}{\Theta_l A(s^t) \chi}\right)^{\frac{1}{\chi - 1}}. \tag{2.22}$$

Note that, as equation (2.22) shows, if wage and aggregate productivity grow at the same rate in the long-run, then labor supply is constant. Therefore, these preferences can support a balanced growth path. Moreover, labor supply is independent of household consumption due to the lack of an income effect on the labor decision; therefore, the efficiency adjusted wage $\left(\frac{W(s^t)}{A(s^t)}\right)$ is always positively correlated with the labor supply.

2.3.6 Interest Rate Process and Open Economy Aggregates

In this small open economy, the interest rate is completely exogenous, and we use the following AR(1) process to model it:[20]

$$\ln\left(\frac{R(s^t)}{\bar{R}}\right) = \rho_r \ln\left(\frac{R(s^{t-1})}{\bar{R}}\right) + \sigma_r \epsilon_t \quad \text{where} \quad \epsilon_t \overset{iid}{\sim} N(0,1), \tag{2.23}$$

where $\bar{R}$ is the long-run interest rate in the economy. We can easily define net exports as the difference between production and all its uses (i.e., consumption, investment, and the bond holding cost):[21]

$$NX(s^t) \quad = \quad Y(s^t) - C(s^t) - I(s^t) - \Psi(B(s^t), Y(s^t)). \tag{2.24}$$

We can also define the foreign debt of the country as the sum of the debt of the household, the debt that the final good producer incurs in holding working capital, and the debt that the financial intermediary holds in order to enact projects in every period.[22]

$$D(s^t) \quad = \quad B(s^{t-1}) \underbrace{-\eta \frac{\alpha Y(s^t)}{1+\eta(R(s^t)-1)} - (1-\bar{z}(s^t))\kappa W(s^t)}_{\text{Working Capital and Project Enaction}}. \tag{2.25}$$

2.3.7 Total Factor Productivity and Growth

In the remainder of this section, we derive the expression for the total factor productivity (TFP) in this economy; we then define an equilibrium for the stationary version of

[20]Neumeyer and Perri (2005) use two uncorrelated autoregressive processes: one for the spread and one for the international interest rate. Uribe and Yue (2006) use a VAR to estimate the determinants of the domestic interest rate, and then feed it into their model. Neither procedure alters the qualitative behavior of the model.

[21]Intermediate goods can be thought of as specialized labor, and hence as non-tradable goods.

[22]A country as an aggregate can only borrow at the domestic interest rate, because the rest of the world is not subject to the same spread. Thus, an important point that must hold along the equilibrium path is that the country should always be a net borrower ($D(s^t) < 0$), so that private savings should not be enough to fund the domestic sector.

the economy; and finally we state the existence and uniqueness of an interior balanced growth path for the model.

We can re-write the production function from equation (2.1) using equation (2.5), recognizing that intermediate labor depends only on the step size of the incumbent.

$$Y(s^t) = \underbrace{\left(A(s^t)\right)^{\alpha}}_{\text{TFP}} \left[\left(L^H(s^t)\right)^{\mu(s^t)} \left(L^L(s^t)\right)^{1-\mu(s^t)} \right]^{\alpha} \left(K(s^{t-1})\right)^{1-\alpha} \tag{2.26}$$

where $A(s^t)$ is defined as:

$$\ln(A(s^t)) \equiv \int_0^1 \ln q_j(s^t) dj.$$

The TFP in this economy is endogenous and we can characterize it using the evolution of firm level labor productivity in equation (2.6), together with the entry rate of the economy. In particular, the following expression for TFP growth explicitly accounts for both mass and composition of the entrant cohort:

$$\begin{aligned} \ln\left(\frac{A(s^t)}{A(s^{t-1})}\right) &= \int_0^1 \ln\left(\frac{\left(1 + I_j^H(s^{t-1}, s_t)\sigma^H + I_j^L(s^{t-1}, s_t)\sigma^L\right) q_j(s^{t-1})}{q_j(s^{t-1})}\right) dj \\ &= \int_0^1 \ln\left(1 + I_j^H(s^{t-1}, s_t)\sigma^H + I_j^L(s^{t-1}, s_t)\sigma^L\right) dj \\ &= \lambda(1 - \bar{z}(s^t)) \left[\tilde{\mu}(s^t) \ln\left(1 + \sigma^H\right) + (1 - \tilde{\mu}(s^t)) \ln\left(1 + \sigma^L\right)\right]. \end{aligned} \tag{2.27}$$

We get the following intuitive expression that characterizes TFP growth:

$$1 + a(s^{t-1}, s_t) = \frac{A(s^{t-1}, s_t)}{A(s^{t-1})} = \left[\left(1 + \sigma^H\right)^{\tilde{\mu}(s^t)} \left(1 + \sigma^L\right)^{1-\tilde{\mu}(s^t)}\right]^{\lambda(1-\bar{z}(s^t))}.$$

Note that TFP growth boils down to a scaled geometric weighted average of the step sizes, where the weights are given by the fraction of each type in the entrant cohort (composition) and the scale is given by the size of the cohort (mass). This highlights once again the interplay between mass and composition effects in the determination of the productivity growth of this economy.

2.3.8 Stationary System and Definitions

In order to render the model stationary, we adopt the following convention: any lower case variable represents the TFP scaled version of its upper case counterpart; for instance, the stationary transformation of output is given by $y(s^t) = \frac{Y(s^t)}{A(s^t)}$. This transformation is performed for consumption, bond holdings, capital, wages, intermediate goods production, investment, and output.[23] With this transformation, we define a stationary competitive equilibrium for this economy:

Definition 3. *A competitive equilibrium for this small open economy, given an initial efficiency level $q_j(0)$ for every product line, an initial fraction of type H incumbents, and initial levels of bond holding and capital for the household is given by:*

1. *Household optimally chooses $\{c(s^t)\,, b(s^t)\,, k(s^t)\,, L(s^t)\}$ given prices to solve* (2.14) *subject to* (2.15) *and* (2.16).
2. *Final good producer optimally chooses $\left\{\{x_j(s^t)\}_{j\in[0,1]}\,, k(s^{t-1})\right\}$ given prices to solve* (2.2).
3. *Intermediate good producers optimally choose $\{p_j(s^t), L_j(s^t)\}_{j\in[0,1]}$ given wages and their type following the pricing rule in* (2.7).
4. *Financial intermediary optimally chooses $\{\bar{z}(s^t)\}$ given values and prices in order to maximize* (2.11).
5. *Government budget is balanced in every period.*
6. *Capital markets clear in every history, and intermediate good markets clear in every history and product line.*

[23] Appendix B.1 derives the normalized system that characterizes the model, and provides a proof for the Lemma 5.

7. *Labor, asset, and final good markets clear in every history:*

$$L(s^t) = \tilde{\mu}(s^t)L^H(s^t) + (1-\tilde{\mu}(s^t))L^L(s^t) + \kappa(1-\bar{z}(s^t)) \tag{2.28}$$

$$d(s^t) = \frac{b(s^{t-1})}{1+a(s^{t-1},s_t)} - \eta\frac{\alpha y(s^t)}{1+\eta(R(s^t)-1)} - (1-\bar{z}(s^t))\kappa w(s^t) \tag{2.29}$$

$$nx(s^t) = y(s^t) - c(s^t) - i(s^t) - \psi(b(s^t)-\bar{b})^2 \tag{2.30}$$

8. $\{v_j(s^t), q_j(s^t)\}_{j\in[0,1]}$ *and* $\mu(s^t)$ *evolve according to* (2.6), (2.9), *and* (2.13).

9. *Transversality and non-negativity conditions are met.*

We can also define a balanced growth path (BGP) for this economy as follows:

Definition 4. *A BGP is a non-stochastic ($\sigma_r = 0$) equilibrium where $\{\bar{z}(s^t)\}$ is constant, and consumption, bond holdings, capital, wages, intermediate goods production, investment, net exports, and output grow at a constant rate.*

Appendix B.1 derives the BGP for this economy and shows that, as the long-run growth is determined by the growth rate of productivity, every normalized endogenous variable is constant. Moreover, that section also proves the following theorem:

Theorem 5. *There is a well-defined parameter space where this economy has a unique interior BGP ($\bar{z} \in (0,1)$).*

Theorem 5 is fundamental for the quantitative analysis in Section 2.5. In fact, it allows us to use a perturbation method to solve the stochastic system that characterizes this economy, centered on its unique BGP. Before exploring the quantitative implications of the model, Section 2.4 uses plant level data from the Chilean sudden stop of 1998 to provide empirical evidence of the mass-composition trade-off at the heart of the model.

2.4 The Chilean Case: Fewer, but Better

This section explores Chilean microeconomic data to assess empirically the main mechanism of the model, i.e., the existence of a mass-composition trade-off on the entry margin. We focus the analysis on Chile for three reasons. First, it is a small open economy with detailed macroeconomic data. Second, the violent sudden stop triggered by the Russian default provides the perfect *natural experiment* to test our mechanism. Third, we have access to detailed plant level panel data that can be used to directly study firm entry. We introduce first the firm level data set, and then we show that firms born in crisis are not just *fewer*, they are also *better*.

2.4.1 The Sudden Stop

In August 1998, the Russian government declared a moratorium on its debt obligations to foreign creditors. This default triggered a sudden and radical increase in the interest rates faced by emerging markets.[24] Latin America was not an exception. Calvo and Talvi (2005) present a detailed analysis of the impact of the Russian default on the seven biggest economies of the region. One of the most successful economies of Latin America, Chile, also suffered the consequences of the Russian default.[25] The real interest rate peaked in 1998:III, increasing by 5 percentage points in a quarter. The interest rate spread, as reported by Calvo and Talvi (2005), increased from 120 basis points before the crisis to 390 basis points in October 1998, triggering a 47% decrease in cumulative external financial flows between 1998 and 2002. The macroeconomic consequences of a sudden stop in emerging markets have been widely studied, but the effects of the firm entry dynamics triggered by these episodes have not. From a Schumpeterian point of view, those changes in entry are harmful even in the long-run, when the well-studied

[24] For a detailed time-line of the Russian default, see Chiodo and Owyang (2002).

[25] See Appendix B.2 for a general picture of the Chilean Economy and the macroeconomic effects of the Sudden Stop.

short-run effects are no more. In this section, while presenting empirical support for the composition effect, we aim to contribute to the empirical research on the microeconomic consequences of a sudden stop.

2.4.2 Mass and Composition during a Sudden Stop

There was no change in the domestic fundamentals of Chile that could have caused or predicted an increase in the interest rate as sudden and substantial as the one observed in the data. In fact, the average annualized real GDP growth of Chile between 1990:IV and 1997:IV was 8.6%, its fiscal policy was steady and sober, and the monetary policy of its autonomous Central Bank was not expansionary. Moreover, as argued by Calvo et al. (2006), the generalized and synchronized nature of the increase in spreads charged in emerging markets also points to an exogenous and common origin for this episode. Thus, taking the Russian crisis as an exogenous shock, unrelated to Chilean fundamentals, and completely unforeseen by firms and authorities, we perform a *pseudo* natural experiment in order to test the main intuition of the model: cohorts born during the sudden stop window should be smaller but more profitable.

Chile's National Institute of Statistics (INE) performs a manufacturing census (ENIA) every year, collecting plant level data from every unit with more than ten employees.[26] The survey contains yearly plant information on sales, costs, value added, number of workers, energy consumption, and other variables. For the empirical analysis in this section, we use the information in the surveys between 1995 and 2007 to build a panel.[27] We take the first appearance in the data as the entry year and the last appearance as the

[26] In 1996, 95% of firms in the survey were single plants.

[27] We restrict attention to this period because the questionnaire and the identification number of each firm are practically invariant.

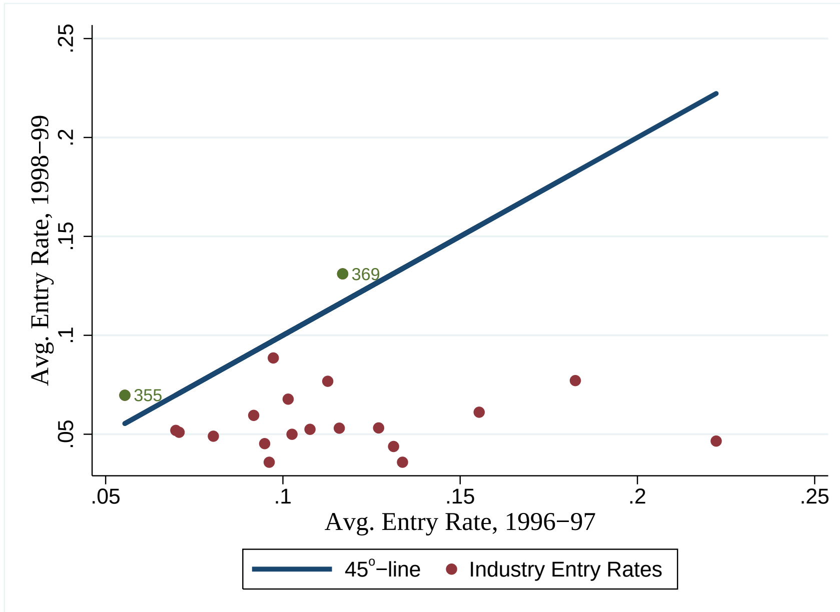


Figure 5: Mass (quantity)

exit date.[28] The sample contains practically 4000 plants and 18,000 observations.[29]

We first calculate entry rates at year t at the industry level for each cohort, dividing the number of new plants in year t by the average of the total plants in years t and $t-1$. Table 18 in Appendix (B.3) presents two-year average entry rates for every industry in the sample. Figure 5 plots two-year average entry rates by industry for the two years preceding the crisis and the first two years of the sudden stop. Every industry below the 45^o line decreased its two-year average entry rate during the crisis.

For all industries but two (355 (rubber based products) and 369 (other non-metallic products)), the average entry rate in $1998-99$ is lower than in $1996-97$. Moreover, Table 18 shows that, for practically every industry, entry rates remain low until $2002-03$. Entry dropped dramatically at the industry level during the Chilean sudden stop. In fact, the

[28]Note that a small firm might appear in the panel after passing the threshold of ten employees, and it should not be counted as an entry. To minimize this issue, we focus on plants with more than eleven workers. The results are also robust to a threshold of fifteen workers. Because of lack of entry in some industries, we restrict our attention to 20 of the 29 industries. For example, the tobacco industry is characterized by only $1-2$ plants, and we observe a positive entry in only two years.

[29]Appendix (B.3) shows the details of the data construction, and a summary of the variables used in the analysis grouped by clusters of cohorts (born before, during, and after the crisis).

average percentage change in the entry rate is -40% between $1996 - 97$ and $1998 - 99$.

Although it is clear that *fewer* firms are born during the crisis, we still have to analyze whether they are *better*. In this sense, we want to show that firms born during the sudden stop are intrinsically more profitable. To capture the profitability of each plant every year, we build the following measure:

$$P_{i,t} = \frac{Revenue_{i,t} - Cost_{i,t}}{Revenue_{i,t}}.$$

Define a firm that is one standard deviation above the mean profitability of its industry, at its first year of life (second observation), as a *superstar* entrant. The two moments are calculated using every plant operating in a given year.[30] We estimate the probability of being a superstar firm using the following logit specification:

$$\begin{aligned} Pr(\text{Superstar} = 1 | \text{age} = 1) &= \frac{e^{x_i'\beta}}{1 + e^{x_i'\beta}} \quad \text{and} \\ x_i'\beta &= \alpha + \alpha_j + \alpha_r + \beta \ln(L_{i,0}) + \gamma_{\text{cohort}} + u_{i,t}, \end{aligned} \tag{2.31}$$

where α_j is an industry control, α_r is a geographical control, and $L_{i,0}$ uses workers at entry to control for size. The cohort coefficient indicates whether a firm was born during the sudden stop window or another cohort specific characteristic. Table 3 presents the results for five alternative regressions.

The first regression compares cohorts born during the crisis (1998-2000) against every other cohort. Firms born during the crisis are statistically more likely to become superstars in their industries. In fact, evaluating the regression at the mean for the most populated region (central) and two-digit industry code (31), we find that the probability of being a superstar is 21% for firms born during the episode, while the probability for a firm born outside this window is 13.4%. The second specification shows that allowing cohorts born before and after the episode to differ does not change the results. In line

[30] In particular, we do not drop the firms born before 1995 from the sample to calculate these moments.

with the *fewer but better* hypothesis, the third specification shows that larger cohorts at the industry level are associated with lower probability of being a superstar. The fourth and fifth specifications show that the results do not change when the probability of being a superstar is evaluated at the year of entry or two years after entering.[31]

	(1)	(2)	(3)	(4)	(5)
	Superstar at age 1			Superstar at age 0	Superstar at age 2
Crisis Dummy	0.540***			0.295***	0.312**
	(0.110)			(0.0970)	(0.135)
In Crisis		0.697***			
		(0.134)			
After Crisis		0.240*			
		(0.126)			
$\text{entry}_{j,0}$			-1.575**		
			(0.803)		
$\ln(L_{i,0})$	0.222***	0.216***	0.209***	0.146***	0.153**
	(0.0527)	(0.0526)	(0.0521)	(0.0436)	(0.0605)
Ind. Control (α_j)	Yes	Yes	Yes	Yes	Yes
region Control (α_r)	Yes	Yes	Yes	Yes	Yes
Constant (α)	Yes	Yes	Yes	Yes	Yes
Observations	3197	3197	3197	4220	2618

Standard errors in parentheses, bootstrapped (250)
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Probability of a superstar firm

Although this exercise is suggestive, the prediction of the model is stronger. In fact, the model predicts that firms born during crises are on average more profitable during their entire life, even after controlling by *after entry* decisions. In this context, we must explore both the continuous nature of the profitability variable and the panel dimension of the data. In general, we would like to estimate the following equation:

$$P_{i,t} = \alpha + \beta_1 X^1_{i,t} + \beta_2 X^2_{i,t} + \gamma_1 Z^1_i + \gamma_2 Z^2_i + \mu_i + u_{i,t} \tag{2.32}$$

[31]If a cohort dummy is introduced year by year, beside the three crisis years, only firms born in 2006 have a significant coefficient (but of lower magnitude than the crisis years). Controlling by initial capital instead of initial workers also does not change the results. Results are available upon request.

where $X^1_{i,t}$ represents exogenous time-varying variables (e.g., vacancy index of the economy), $X^2_{i,t}$ refers to endogenous time-variant variables (e.g., number of workers), Z^1_i correspond to exogenous time-invariant variables (e.g., region of the country), and Z^2_i are endogenous time-invariant variables (e.g., workers in the entry year). Note that variables with a superscript 2 are endogenous in the sense that they are likely to be correlated with the unobserved fixed effect μ_i. The main challenge of this panel estimation is that the variable of interest, *being born in crisis*, is not only time-invariant, but also endogenous. On the one hand, coefficients on time-invariant variables can be consistently and efficiently estimated by random effects regression, but the estimation is not consistent when the variable is also endogenous. On the other hand, fixed effects panel regression can consistently estimate every coefficient associated with the time-variant variables, but it cannot identify the coefficients of the time-invariant variables. In this situation, the Hausman and Taylor (1981) procedure delivers consistent and efficient estimators for every coefficient in equation (2.32).[32]

Table 4 presents the results for six different specifications. In the first four regressions, the dependent variable is $P_{i,t}$. The only difference in the first three specifications is the coefficient of interest. In the first regression, we use a single dummy to determine whether the cohorts born in $1998-2000$ perform better than every other cohort. In the second regression, we use two dummies in order to allow a differential effect for cohorts *pre* and *post* crisis. The third specification studies the effect of the three-digit industry entry rate at the moment of entry. This means that it is a continuous variable common to every firm in the same industry born in the same year and is also time-invariant. Note that all the coefficients of interest are associated with time-invariant endogenous variables, because better firms (with a higher observable fixed effect μ_i) are expected to enter in years of

[32]See Appendix B.3.5 for a succinct explanation. Intuitively, we can think that this procedure aims to remove the endogenous component from the original regression in order to meet the main assumption of random effects. More details on this method can be found in Wooldridge (2010), Chapter 11. STATA software has built-in routines for both procedures; see Schaffer and Stillman (2011). After every estimation, we perform the Sargan-Hansen test to assess the validity of the instrumental variables procedure at the core of Hausman and Taylor (1981). The null hypothesis is that the instruments are valid, so the higher the p-value, the better.

crisis. In the case of the third specification, when *fewer* firms enter, we expect them to be *better*.

The fourth regression changes the initial size measurement from workers to capital; the aim of this is to verify that the results are not driven by the initial collateral held by the firm.[33] The last two regressions focus on alternative measures of firm quality. In particular, the fifth regression uses output per worker as a measure of labor productivity and the sixth specification studies the propensity to accumulate physical capital.

We use as time-variant exogenous variables ($X^1_{i,t}$) four macroeconomic aggregates: an index of manufacturing production, the unemployment rate, an index of wholesale producer prices, and an index of the cost of labor.[34] The coefficients associated with these variables are stable across the profitability regressions. The signs of the significant coefficients suggest that profitability is higher when production is high, labor costs are low, and inflation in producer prices is also low. Note that the fifth specification suggests that labor productivity increases in bad times. In particular, a high unemployment rate is associated with higher labor productivity.[35] There are four endogenous time-variant variables ($X^2_{i,t}$),: electricity consumption, number of workers, capital stock, and the age of the plant. The signs of the significant coefficients suggest that older firms and firms that are increasing their capital stock are more profitable. Also note that the fifth and sixth specifications point to a strong complementary relationship between labor and capital. We use five geographic regions and two-digit industry controls as time-invariant exogenous variables (Z^1_i). Besides the coefficients of interest, we include the initial size of the plant, specified as the initial number of workers or the initial capital holdings. In order to control for competition at the moment of entry, we also include the Herfindahl-Hirschman concentration index of the industry at the particular region in the year of entry among the time-invariant endogenous variables (Z^2_i). In line with the firm dynamic's literature,

[33]In fact, it is plausible that firms with high collateral are more likely to enter during the crisis.

[34]Because this method relies on $X^1_{i,t}$ to build instruments, and because they are all aggregated variables, we cannot include year dummies, which are perfectly correlated with our instruments.

[35]This seems to point more to a cleansing rather than a sullying effect of recessions.

	(1)	(2)	(3)	(4)	(5)	(6)
	$P_{i,t}$	$P_{i,t}$	$P_{i,t}$	$P_{i,t}$	$\log \frac{Y_{i,t}}{L_{i,t}}$	$\frac{K_{i,t}-K_{i,t-1}}{K_{i,t}}$
Crisis dummy	0.0877**			0.0814***	0.325**	0.0527**
	(0.0423)			(0.0313)	(0.136)	(0.0233)
In Crisis		0.0861**				
		(0.0397)				
After Crisis		0.00952				
		(0.0241)				
avg. Entry$_{j,t_0}$			-0.682**			
			(0.337)			
log Manu. Prod.$_t$	0.125***	0.121***	0.120***	0.127***	-0.332***	0.134***
	(0.0387)	(0.0387)	(0.0382)	(0.0437)	(0.113)	(0.0471)
Unemp. Rate$_t$	0.211	0.198	0.210	0.229	1.427***	0.0277
	(0.147)	(0.149)	(0.148)	(0.150)	(0.428)	(0.220)
log PPI/WPI$_t$	-0.871**	-0.844**	-0.873**	-0.923**	4.11***	0.886
	(0.0408)	(0.409)	(0.410)	(0.453)	(1.51)	(0.584)
log Labor Cost$_t$	-0.335**	-0.360**	-0.388**	-0.288	-0.379	-0.266
	(0.159)	(0.166)	(0.161)	(0.182)	(0.528)	(0.231)
log Elec. Con.$_{i,t}$	-0.0804	-0.0786	-0.0732	-0.0891	5.90***	-0.690**
	(0.256)	(0.257)	(0.257)	(0.273)	(0.865)	(0.275)
log labor$_{i,t}$	-0.113	-0.114	-0.113	-0.118		0.261***
	(0.0734)	(0.0734)	(0.0735)	(0.0719)		(0.0688)
log Capital$_{i,t}$	0.669**	0.663**	0.661**	0.690**	3.40***	
	(0.269)	(0.269)	(0.269)	(0.283)	(0.814)	
log age$_{i,t}$	0.103*	0.118*	0.146***	0.0799	-0.139	-0.507***
	(0.0550)	(0.0668)	(0.0542)	(0.0501)	(0.170)	(0.0431)
HHI$_{j,t_0,r}$	0.451***	0.428***	0.326***	0.220**	0.678	0.0432
	(0.137)	(0.148)	(0.101)	(0.0937)	(0.524)	(0.0713)
log labor$_{i,t_0}$	0.453***	0.430***	0.351***			
	(0.0971)	(0.111)	(0.0692)			
log Capital$_{i,t_0}$				0.119***	0.656***	0.0554***
				(0.0238)	(0.119)	(0.0177)
Ind. Control	Yes	Yes	Yes	Yes	Yes	Yes
Region Control	Yes	Yes	Yes	Yes	Yes	Yes
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Relative effect at means[a]	−31.2%	−31.3%	–	−28.4%	−32.5%	−29.2%
Sargan-Hansen (p)	0.4545	0.2333	0.1230	0.0476	0.0395	0.7702
Observations	16834	16834	16834	16371	15583	16388

Standard errors in parentheses (bootstrapped (250), clustered by firm)
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
[a] No crisis prediction at means divided by crisis (or pre-crisis) prediction at means minus 1, evaluated at the most populated region (central) and industry (31). For regression five, it is the difference between the predictions.)

Table 4: Hausman and Taylor

larger entrants are more profitable, more productive, and accumulate capital faster than smaller entrants. Finally, firms that enter into more concentrated industries are more profitable than firms facing more competition.

Back to our main question: are those *fewer* firms born in crisis *better*? The first specification shows that firms born in crisis are significantly more profitable than firms born in normal times. In fact, after controlling for initial size, macroeconomic conditions, and post-entry decisions, firms born during the sudden stop have, on average, a profitability index 8.8 percentage points higher. This coefficient is robust to allowing post-crisis cohorts to differ from before-crisis cohorts (specification 2) and is also robust when we control for initial capital (specification 4). Table 4 also shows the relative effect evaluated at the means, that is, the predicted profitability of a firm born during normal times divided by the predicted profitability of a firm born during the crisis, minus one. The baseline regressions suggest that, if we focus on a fictitious firm, setting every observable at its mean and changing only the period of entry, we find that being born in normal times implies 31% lower profitability.[36] The third specification is more general in the sense that it aims to directly unveil a mass-composition trade-off at the entry level. Although the Sargan-Hansen test is barely 5%, the coefficient suggests that firms born in smaller cohorts have a permanently positive effect in their profitability measure. In particular, every extra percentage point in entry decreases the profitability of the firm by 0.68%. Note that specifications five and six show that this result is robust to other performance measures, as firms born during the crisis are permanently more productive and more prone to capital accumulation.

One caveat related to *post-entry* selection can be added to the preceding results. If firms born during crisis were more likely to die early, then those cohorts would seem more profitable after that initial selection. Appendix B.4 estimates a proportional hazard model in order to evaluate this concern. The main empirical question in the Appendix is

[36]The raw data in Appendix B.3 shows that firms born before the crisis have an average lifetime profitability of 23%, while firms born during the crisis have an average lifetime profitability of 24%.

whether firms born during the crisis window are more likely to exit. The answer is not only negative, but, if anything, firms born during crisis have lower hazard rates in each of their first six years of life. A second concern with the analysis might be due to the nature of selection. In fact, one might think that those cohorts are better just because of self-selection: when the interest rate is high, only good firms apply for credit. Although it is likely that some self-selection arises during these episodes, the hypothesis of complete self-selection is at odds with the real world. In fact, this argument implies that every firm that applies for credit is granted a loan. This is clearly not true in the data.[37]

Summarizing, the Chilean sudden stop had strong macroeconomic consequences. At the firm level, the effect is relatively more complex. Cohorts born during the crisis, and in its aftermath, are 40% smaller; nevertheless, firms born in normal times are at least 30% less profitable after controlling for observables. Hence, taking the average quality of the entrant cohort as a reference to evaluate the forgone entry is extremely misleading, as the unborn firms are substantially *worse* than the observed ones. As these unborn firms are often the excuse for policy interventions, such as indiscriminate government credit, it is crucial to correctly assess the economic cost of that forgone entry. For this reason, we proceed to calibrate our model and quantify the long-run cost imposed by a sudden stop.

2.5 Quantitative Exploration: The Role of Financial Selection

This section presents a quantitative exploration of the model. First, we calibrate the baseline model using Chilean data. To assess the performance of the calibrated model, we feed it with a smoothed series of the quarterly interest rate observed in the data. Although the model is stylized, with its single shock it is able to approximate the non-targeted regularities of Section 2.4. The model can account for roughly 40% of the decrease in

[37]For instance, according to Eurostat firm level data for 20 countries (showing access to finance for small and medium-sized enterprises in the European Union), 28% of firms applied for loans in 2007 (before the 2009 crisis), with a success rate of 84%. In 2010, although more firms were applying for loans (31%), the success rate decreased to only 65%.

entry and more than 17% of the increase in profitability during the Chilean sudden stop. Then, in order to assess the role of heterogeneity and selection in shaping the effect of a sudden stop, we introduce two modified economies: a model with exogenous growth and a model with endogenous growth but no heterogeneity. We use those alternative economies to highlight the role of firm entry and financial selection when the economy is hit by an interest rate shock. For instance, including heterogeneity and selection amplifies the medium-run effects of a sudden stop in output when compared to the exogenous growth model. Moreover, the shock generates a long-run permanent loss in output due to the distortion in the entry market. The composition effect plays a considerable role in shaping the long-run cost of the crisis. In fact, the model with no heterogeneity doubles the estimation of the permanent loss. Thus, even at the macro level, the existence of selection is fundamental when assessing the cost of the forgone entrants.

2.5.1 Calibration to the Chilean Economy

Externally Calibrated Parameters

The twenty parameters of the model are calibrated to Chilean data on a quarterly basis. A first group of six parameters is externally calibrated according to the real business cycle small open economy literature. In particular, the capital share $(1-\alpha)$, the inter-temporal elasticity of substitution $(1/\gamma)$, and the Frisch elasticity of labor supply $(1/(1-\chi))$ are set in accordance with Mendoza (1991). The working capital requirement (η) is set to the value used by Neumeyer and Perri (2005), which implies that the final good producer needs to keep as working capital 100% of the cost of intermediate goods. The parameter governing the debt adjustment cost (ψ) is set to a small number that guarantees stationarity in every experiment.[38] A second group of eight parameters is calibrated directly to Chilean data. The depreciation rate of capital (δ) is set at 8% annually, consis-

[38]The debt adjustment cost is in the same order of magnitude as the one used by Uribe and Yue (2006).

tent with the study by Bergoeing et al. (2002) of the Chilean economy. The corporate tax rate (τ) is set to 17%, in line with Chilean legislation of that time; the long-run interest rate ($\bar{R}$), the persistence of the interest rate process (ρ_r) and the dispersion of the shocks (σ_r) are estimated using the quarterly real Chilean interest rate on loans performed by

Parameter	Symbol	Value	Source
Capital share	$1-\alpha$	0.32	Mendoza (1991)
Elasticity of Substitution ($1/\gamma$)	γ	2	Mendoza (1991)
Frisch Elasticity ($1/(1-\chi)$)	χ	1.455	Mendoza (1991)
Working Capital	η	1	Neumeyer and Perri (2005)
Debt adjustment cost	ψ	0.0001	Low
Depreciation rate	δ	1.94%	Bergoeing et al. (2002)
Corporate tax rate	τ	0.17	Data
Long-run interest rate	$\bar{R}$	1.015	Chilean Central Bank Data
Persistence of interest rate	ρ_r	0.836	Chilean Central Bank Data
Dispersion of interest rate shock	σ_r	0.33%	Chilean Central Bank Data
Long-run debt to GDP ratio	$\bar{b}$	$4*(-0.44)$	Chilean Central Bank Data
Low profitability ($\sigma^L/(1+\sigma^L)$)	σ^L	14.5%	ENIA
High profitability ($\sigma^H/(1+\sigma^H)$)	σ^H	55.5%	ENIA

Table 5: Externally Calibrated Parameters

commercial banks between 1996:I and 2008:IV. Because no debt holding costs are incurred along the balanced growth path, we set $\bar{b}$ to the quarterly debt-to-GDP ratio of Chile. The step sizes (σ^L, σ^H) are calibrated to match the 25% and 75% percentiles of the pre-crisis profitability distribution in Table 15. The calibrated step sizes point to a significant heterogeneity in mark-ups in the Chilean economy; specifically, drastic ideas are three times more productive than incremental ones. These values are in line with the empirical studies of Navarro and Soto (2006) for the Chilean manufacturing sector. Table 5 presents the values for every externally calibrated parameter.

Internally Calibrated Parameters

The remaining seven parameters are internally calibrated to match salient features of the Chilean economy. For a given long-run growth rate (a), there is only one value of the patience parameter (β) consistent with no payment of debt holding cost along the balanced growth path.[39] Five of the remaining six parameters $(\lambda, \kappa, \Theta_l, \rho, \nu)$ are set to match the five moments summarized in Table 6.[40] Although every moment is related to the whole set of parameters, we can point to some strong relationships between targets and parameters. The success probability (λ) is highly related to the long-run entry rate of start-ups in the model; we set that target to the average entry of the pre-crisis years in our sample, 2.71% per quarter. The average cost of starting a firm as a proportion of the gross national income is obtained from the *Doing Business Indicators* from the *World Bank* database, which pins down the cost of enacting a project (κ). In the model, we set this target to 12.1% of 2004, the earliest year available. The dis-utility of labor (θ_l) is set to match a long-run labor supply of 33%. The novel parameters ρ and ν are more challenging to calibrate. The accuracy of the financial system (ρ) governs the proportion of firms in the entrant cohort that are below the threshold set by the financial intermediary. In the data, a proportion of the entrant cohort dies during their first year; we use that percentage as a proxy for the firms that were able to enter, although their true type was below the threshold. We set the former target to 15%, the average of that proportion in the pre-crisis portion of the data. Moreover, equation (2.27) relates the two step sizes of the model and the scarcity of drastic ideas (ν) to the long-run growth rate of the economy. We

[39] See equation (B.23) in Appendix B.1.

[40] We minimize the sum of the percentage absolute distance between model and data. The calibration is robust to different starting points.

follow Neumeyer and Perri (2005) and target a yearly long-run growth rate of real GDP of 2.5%. Finally, the parameter governing the capital adjustment cost (ϕ) is set to match the standard deviation of the real investment growth rate in Chile of 3.7%.[41] The model is able to match the targets successfully. Table 6 presents the performance of the model regarding the five targets and Table 7 presents the calibration for the seven internally calibrated parameters in the model.

Target	Model	Data	Expression
Entry	2.71%	2.71%	$\lambda\left(1-\bar{z}\right)$
Entry Cost	12.1%	12.1%	$\kappa(w/y)$
Working time	33.0%	33.0%	L
Fast exit	15.0%	15.0%	$\left(1-\rho\right)\bar{z}$
Growth	0.62%	0.62%	$a=\left(\left(1+\sigma^{H}\right)^{\mu^{H}}\left(1+\sigma^{L}\right)^{1-\mu^{H}}\right)^{\lambda(1-\bar{z})}-1$

Table 6: Targets: Model and Data

Parameter	Symbol	Value	Main identification
Patience parameter	β	0.9975	$\beta=\left(1+a\right)^{\gamma}/\bar{R}$
Success probability	λ	5.36%	Entry
Enaction cost	κ	6.65%	Entry cost
Labor dis-utility level	Θ_l	1.73	Working time
Screening accuracy	ρ	69.7%	Fast exit
Scarcity	ν	4.51	Growth
Capital adjustment cost	ϕ	20	Investment volatility

Table 7: Internally Calibrated Parameters

[41]Because the balanced growth path is independent of ϕ, we calibrate it separately and match the desired volatility perfectly.

The scarcity of good ideas implies that, under random selection, two out of eleven ideas (18.2%) generate a high step size. Nevertheless, given that the screening accuracy is 70%, the financial intermediary sets its credit standards to $\bar{z} = 49.4\%$, securing an *ex-post* fraction of high types equal to $\mu^H = 30\%$. This points to a non-trivial amount of selection by financial intermediaries. The fixed cost of enacting a project is 6.65% units of labor; this implies that 10.2% of the total working hours are used in project implementation. This is in line with the data from the first entrepreneurship survey for Chile (*Encuesta de Microemprendimiento*), where 13% of the Chilean workforce declared themselves to be entrepreneurs in 2011. In terms of macro moments, the model generates an investment-output ratio of 23.8% and an annualized capital-output ratio of 2.34, slightly above the 23% and 2.0 averages during the period $1996 - 2007$, as shown in Table 13 in the Appendix B.2. Therefore, the calibration is consistent with the macroeconomic aggregates of the Chilean economy. Before exploring the role of firm heterogeneity and financial selection during sudden stops, we evaluate the quantitative performance of the calibrated model.

2.5.2 Validation of the Model: The Chilean Sudden Stop

In this section, we test the firm-based implications of the model when compared to the empirical regularities documented in Section 2.4. In particular, we feed the model with a smoothed version of the quarterly real interest rate, and we plot several non-targeted time series between 1998:I and 2002:I. We transform the interest rate of Figure 19 using the following function:

$$R_t = 0.5R_{t-1} + 0.5R_{t-2}$$

We chose to use this smoothed series for two reasons. First, decisions such as investment or labor are not taken every quarter; thus, instead of introducing a *time to adjust friction*, as in Uribe and Yue (2006), or a delay in the pass-through between the external shocks and the domestic variables, as in Neumeyer and Perri (2005), we decided to smooth the effect of the financial crisis. The second reason has to do with the empirical analysis in Section 2.4. Because our firm level data is annual, at every observation we have entrants that were subject to the interest rate of different quarters; hence, a two quarter backward-looking moving average seems to be a parsimonious alternative. We set the state variables of the model at their balanced growth path level in 1998:I.[42]

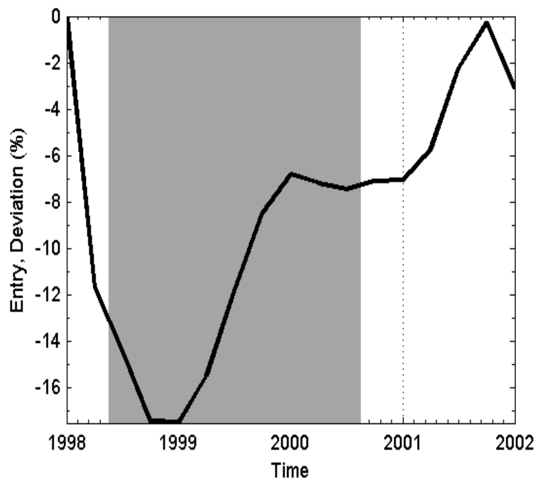
The model presented in this paper, unlike traditional small open economy real business cycle models, has strong predictions for the mass of new firms, the average profitability of each cohort, and the average value of incumbents.[43] We use those novel features to validate the calibration of the economy. Therefore, we use the empirical results of Section 2.4 to assess the ability of the model to capture the mass-composition trade-off at the heart of this paper. Figure 6 shows the implied path for quarterly entry and the average profitability of the entrant cohort, expressed as percentage deviations from their balanced growth path level. The third graph in Figure 6 shows the logarithm of the Chilean stock index (IPSA) in real terms.[44] Note that the long-run calibration previously introduced did not target any of the information in these time series.

As Figure 6a shows, the change in the interest rate can account for roughly a 17.5% decrease in the entry rate. This is more than 40% of the average decrease observed at the industry level on Table 18. Figure 6b shows the increase in the average profitability of the entrant cohort. Cohorts born in the worst part of the crisis are 6.5% more profitable than an average cohort (1.3 percentage points difference). Therefore, the model is in line with the raw data in Tables 16 and 17, and it explains 20% of the conditional increase in profitability documented in Table 4. Moreover, during the crisis, the fraction of high types in the entrant cohort increases by 18%. This is in line with the logit regressions in

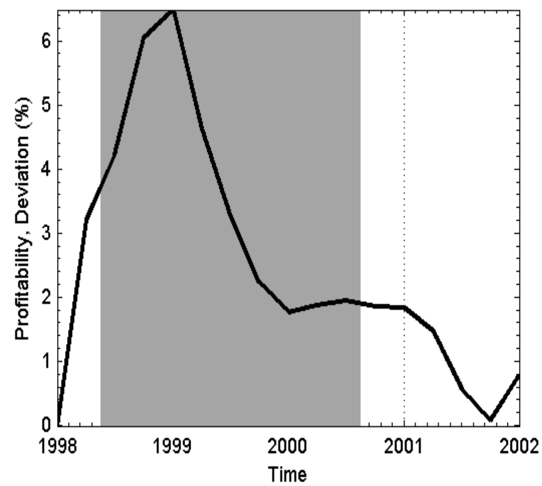
[42]The model is solved by second order perturbations using Dynare. We choose this solution method for two reasons. First, as discussed in Aruoba et al. (2006), higher order perturbation methods are appropriate for smooth systems with strong non-linearity and large shocks. Second, it allows for a meaningful welfare analysis.

[43]Appendix B.5 show the results for hours, consumption, trade balance divided by GDP, and investment.

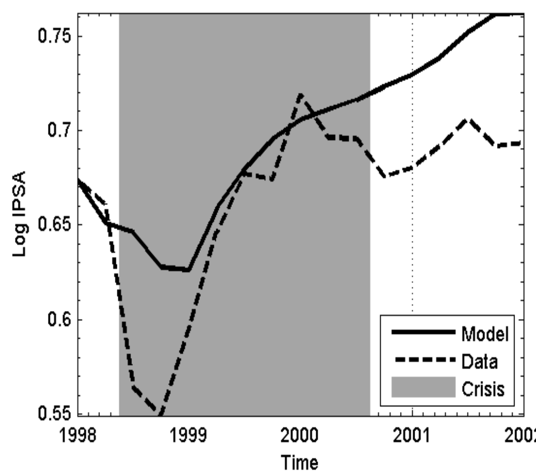
[44]The corresponding model-generated series for the IPSA is the average value of an intermediate good producer. We set the economy to its BGP on 1998:I and normalize the initial level.



(a) Entry (mass)



(b) Average cohort profitability (composition)



(c) Stock Index

Figure 6: Model Performance

Table 3, where the probability of a superstar arising during crises increases by roughly 50%. Finally, with only an interest rate shock, the model can mimic the bust and recovery of the Chilean stock market with reasonable success. In fact, this calibration suggest that the interest rate shock explains at least 1/3 of the decrease in firm values. Note that this framework is simple enough to introduce more shocks and frictions in order to close the gap between model and data in these and other aggregated variables. As the aim of this paper is to study the effects of an interest rate shock in the medium and long-run, we prefer the parsimony of a model with only one shock and a limited number of frictions. Having provided evidence to support the quantitative behavior of the model, we finally focus our attention on quantifying the role of financial selection in shaping the long-run cost of a sudden stop.

2.5.3 Long-Run Loss, Amplification, and Persistence

In this section, we study the long-run cost imposed by a sudden stop on an economy where heterogeneous entrants subject to financial selection contribute to the process of productivity accumulation. To highlight the relevance of endogenous growth and financial selection when analyzing these episodes, we introduce two modified versions of the baseline model. The first version (*Exo*) is a model with exogenous growth, and no financial intermediation. In particular, *Exo* has no entry on its intermediate product line; it experiences a constant growth rate equal to the balanced growth path of the *Baseline*. We can think of *Exo* as Neumeyer and Perri (2005) with intermediate goods, and a constant mixture of H and L incumbents equal to the BGP level of the *Baseline*. The second version (*NoHet*) is a model with no heterogeneity and with endogenous growth. In this model, there is neither *ex-ante* nor *ex-post* heterogeneity, because every entrant has the same step size. This unique step size is set so that *NoHet* exhibits the same long-run growth as the baseline. Finally, *Exo* and *NoHet* share every common parameter with the *Baseline* model. Before conducting a quantitative exploration of the models, note that growing variables, such as output or investment, are normalized by A_t in all the models. Denoting log-deviations of a variable H from its last period value by a hat ($\hat{H}_t = \ln(H_t/H_{t-1})$), let's focus on output to highlight the source of the long-run cost:

$$y_t = \frac{Y_t}{A_t} \Rightarrow \hat{Y}_t \approx \hat{y}_t + \hat{A}_t \tag{2.33}$$

Along the non-shocked path, because y_t is constant, we get $\hat{Y}_t = \hat{A}_t \approx a_{ss}$. Hence, for scaled variables, we define the distance at time t between the non-shocked economy and the one subject to the shock as $\tilde{x}_t^Y$:

$$\tilde{x}_t^Y \approx \sum_{i=1}^{i=t} \left\{ \hat{y}_i + \hat{A}_i \right\} - t * a_{ss} \tag{2.34}$$

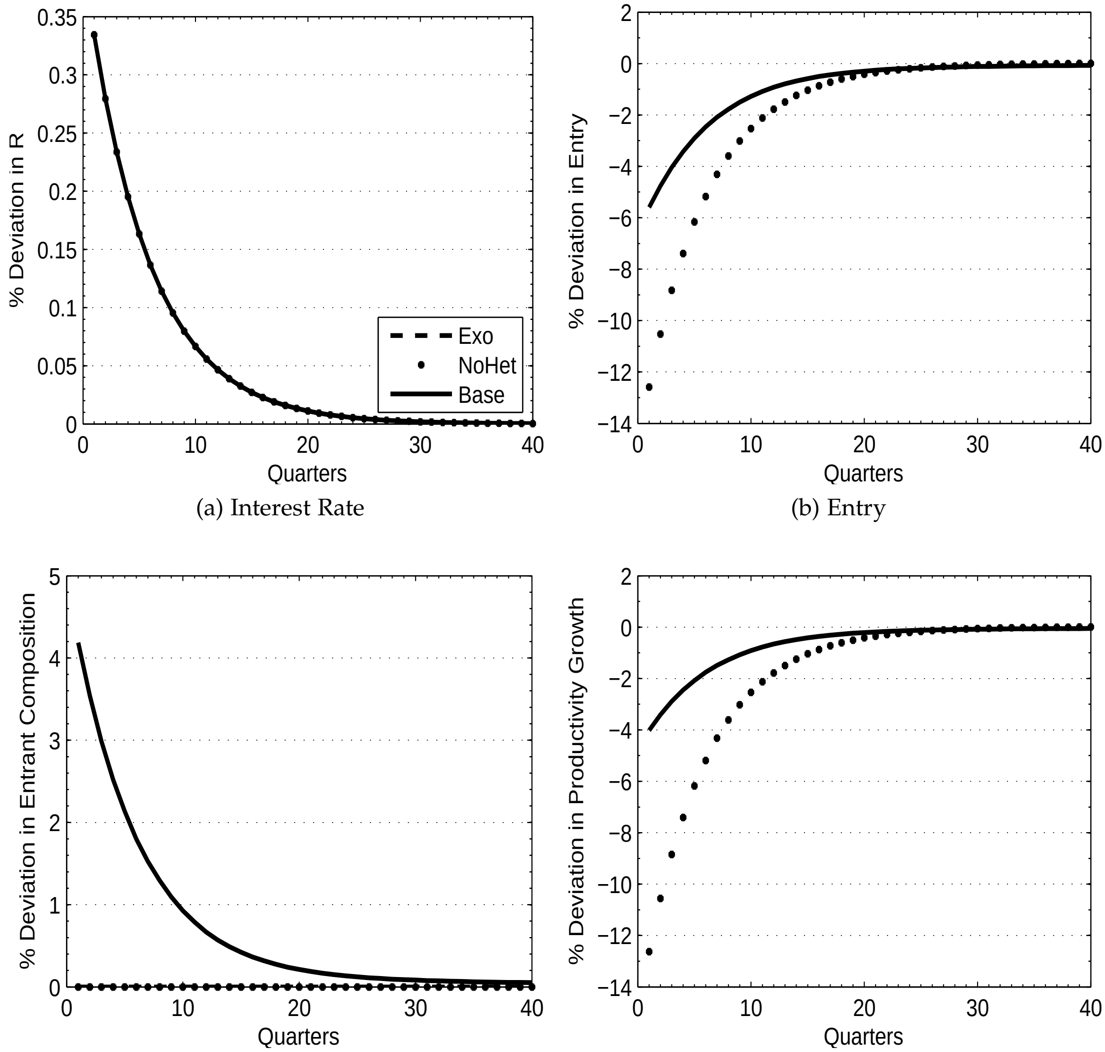
The main difference between *Exo* and the other two models is that, because growth is exogenous, $\hat{A}_t \approx a_{ss}$, and then $\tilde{x}_t^Y = \sum_{i=1}^{i=t} \hat{y}_i$. Because y_t is stationary, this term converges to zero when time goes to infinity. This illustrates why there is no long-run cost of a sudden stop for a model with exogenous growth. But a model with endogenous growth has a long-run cost (*LRC*), in any normalized variable, approximately equal to:

$$LRC \approx \lim_{t \to \infty} \left\{ t * a_{ss} - \sum_{i=1}^{i=t} \left\{ \hat{A}_i \right\} \right\} < \infty \tag{2.35}$$

Note that, as $\hat{A}_t$ converges to a_{ss}, this long-run cost is finite. Moreover, as is clear from equation (2.34), this long-run cost arises only for variables that exhibit long-run growth. Therefore, the analysis of a sudden stop in a model with endogenous growth needs to consider the long-run impact that comes through this TFP-driven loss. Moreover, because *NoHet* and *Baseline* have the same long run growth, the path of $\hat{A}_i$ fully determines their relative long-run cost. Having defined the long run-cost of a shock, we turn to the quantitative response of the models to a one standard deviation innovation in the interest rate R (33 basis points). Figures 7, 9, 10, and 11 show the responses of the models to this shock. The units on the y-axis are the percentage deviation from a counter-factual non-shocked path.

Figure 7a displays a one-time 33 basis point increase in the interest rate, for the three economies.[45] Figure 7b shows the response of firm entry in the two models that feature this margin. The decrease in entry is more than two times larger than for *NoHet*

[45] The deviations are calculated by averaging 600 simulations with a horizon of 1200 periods. For each simulation we draw a series of interest rate shocks, drop the first 100 periods to build the stochastic state of the economy at the moment of the shock. The average across simulations of the difference between the path with a one time 33 basis point shock and the original path is used to generate the impulse response functions.



(a) Interest Rate

(b) Entry

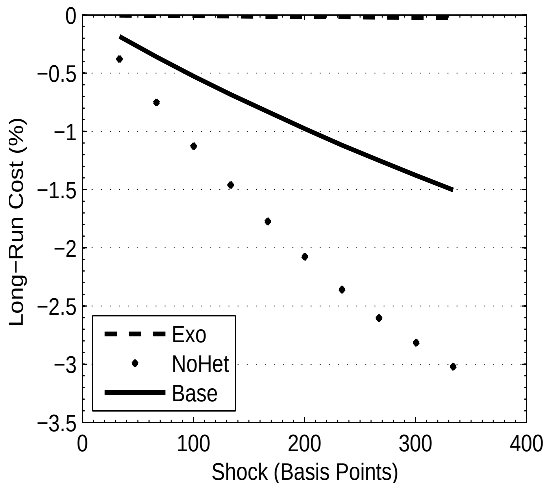
(c) Composition

(d) TFP Growth

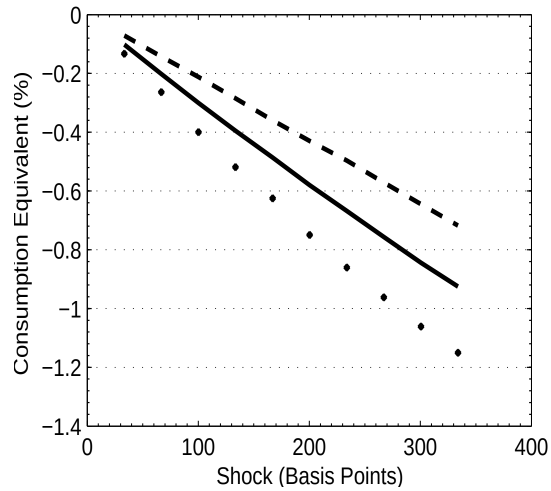
Figure 7: 33 basis point Increase in Interest Rate.

when compared to the baseline model. The main reason behind this difference lies in the compositional dynamics displayed in Figure 7c. In fact, the proportion of high type entrants in each cohort in *NoHet* is constant, while the baseline economy is able to adjust this margin. In particular, the entrant cohort contains an extra 4% of high type leaders at the time of impact. The last panel of Figure 7 shows the change in productivity growth generated by the disruption in the entry margin. Note that *NoHet* exhibits a large decrease in productivity growth at the time of impact of 15%, while the decrease in the baseline model is less than 4%. The reason behind this difference is seen in Figures 7b and 7c; in fact, when the shock hits the economy, the only margin of adjustment for *NoHet* is the mass of the entrant cohort. Because there is only one step size, the contribution of the forgone entrants to growth is the same as the contribution of the actual entrants. But the compositional dynamics of the baseline model imply that, on average, the contribution to productivity of the forgone entrants is lower than the contribution of the selected projects; hence, the productivity cost is smaller. Figure 7d can also be used to illustrate the long-run effect of a sudden stop in the three models; we can calculate *LRC* for each model using Equation 2.35. In this particular case, the long-run cost is 0.38% for *NoHet* and 0.18% for the baseline. Therefore, the model with no heterogeneity generates a two times larger long-run cost. We can also illustrate this result by calculating the consumption equivalent welfare cost of the interest rate shock. Figure 8a shows the long-run cost of each model for different shock sizes measured as the permanent distance between the shocked path and the BGP, in percentage terms. Figure 8b shows the welfare costs for different shock sizes, the maximal fraction of BGP consumption that the household would sacrifice to avoid the shock. Note that the long-run cost is a measure that abstracts from the short-run impact of the shock and the preferences of the representative household, while the welfare cost measure includes the effects of the shock at every horizon and uses the inter-temporal preferences of the representative household to quantify the loss.

As expected, in Figure 8a we see that *Exo* is not subject to a long-run cost. Therefore, in Figure 8b, the distance between the consumption equivalent welfare cost of *Exo* and the models with endogenous growth approximates the welfare cost of the long-run loss. When using the baseline model as a benchmark, the long-run cost contributes to 30% of the welfare cost; if *NoHet* is used, the long-run cost explains 45% of the welfare cost. Regardless of the size of the shock, *NoHet* doubles the estimated long-run cost of a sudden stop, increasing the consumption equivalent welfare cost by 30% when compared to the baseline model. For instance, a 330 basis point increase in the interest rate implies, under *NoHet*, a long-run cost of 3% and a welfare cost of 1.15% of consumption, considerably



(a) Long-run Cost



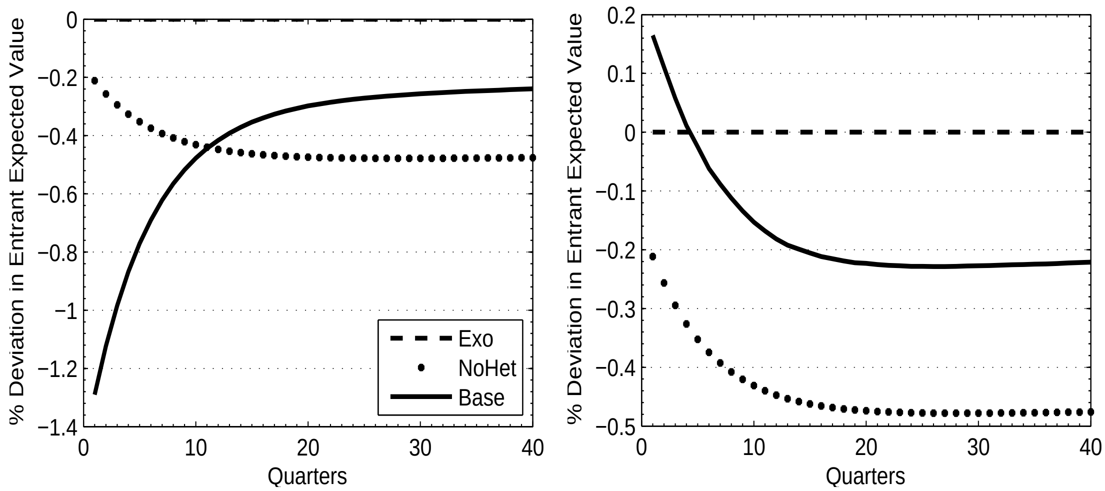
(b) Consumption Equivalent Welfare Cost

Figure 8: The Impact of Selection.

higher than the 1.5% long-run cost and the 0.93% consumption equivalent welfare cost suggested by the baseline model. Therefore, modeling heterogeneity and selection is particularly relevant when studying the absolute effect of large shocks.

In order to understand the role of selection for the financial intermediary's decision problem, Figure 9a displays the deviations of the average expected revenue per entrant under random selection: $\underline{\tilde{\mu}} V^H(s^t) + (1 - \underline{\tilde{\mu}}) V^L(s^t)$. Note that, for the baseline model, the average value of a randomly enacted project drops five times more on impact than for *NoHet*. In this sense, the *pure* decrease in values for the baseline model is more violent than for the model without selection. An important part of this difference comes from the sharp drop in entry exhibited by *NoHet*. In fact, lower entry implies higher survival probabilities, and hence more valuable product lines. Going back to Figure 9a, the higher return of a randomly enacted project in *NoHet* in comparison to the baseline implies that, if the financial intermediary in the baseline model had no access to selection, she would enact even fewer projects.[46]

[46]Note that the analysis from the cost side does not reverse this partial equilibrium intuition; as seen in Figure 11d, the marginal cost of enacting a project decreases more for *NoHet*.



(a) Expected Entrant Value (Random Selection)

(b) Expected Entrant Value

Figure 9: The Impact of Selection.

Figure 9b shows how this relationship is reversed when we take into account the change in the composition of the entrant cohort in the baseline model. In fact, financial selection allows the financial intermediary to increase the average value of each member of the entrant cohort and counteract the decrease in the value of product lines. The difference in the average value of an entrant displayed in the second panel of Figure 9 illustrates why the financial intermediary decreases project enactment in *NoHet* by a factor more than two times larger than the decrease implied by the baseline model.

Having characterized the source and magnitude of the long-run cost, we focus our attention on the response of output. Figure 10a shows the response of output for the three models. Following equation (2.26), we can distinguish the following three components of output:

$$Y(s^t) = \underbrace{\left(A(s^t)\right)^{\alpha}}_{\text{A Component}} \underbrace{\left[\left(L^H(s^t)\right)^{\mu(s^t)} \left(L^L(s^t)\right)^{1-\mu(s^t)}\right]^{\alpha}}_{\text{L Component}} \underbrace{\left(K(s^{t-1})\right)^{1-\alpha}}_{\text{K Component}} \tag{2.36}$$

Figures 10b, 10c, and 10d display the evolution of those three components for each model.

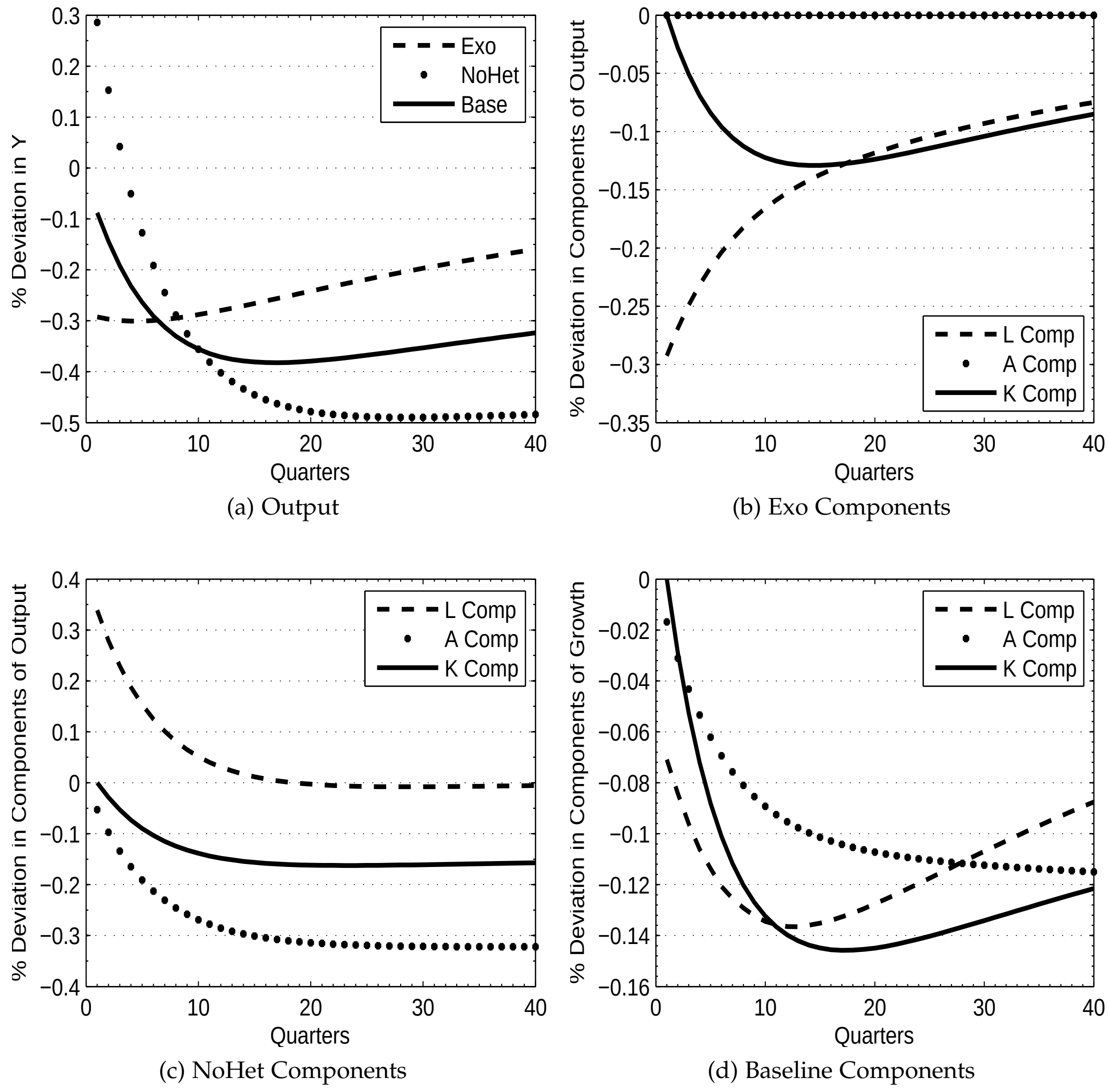


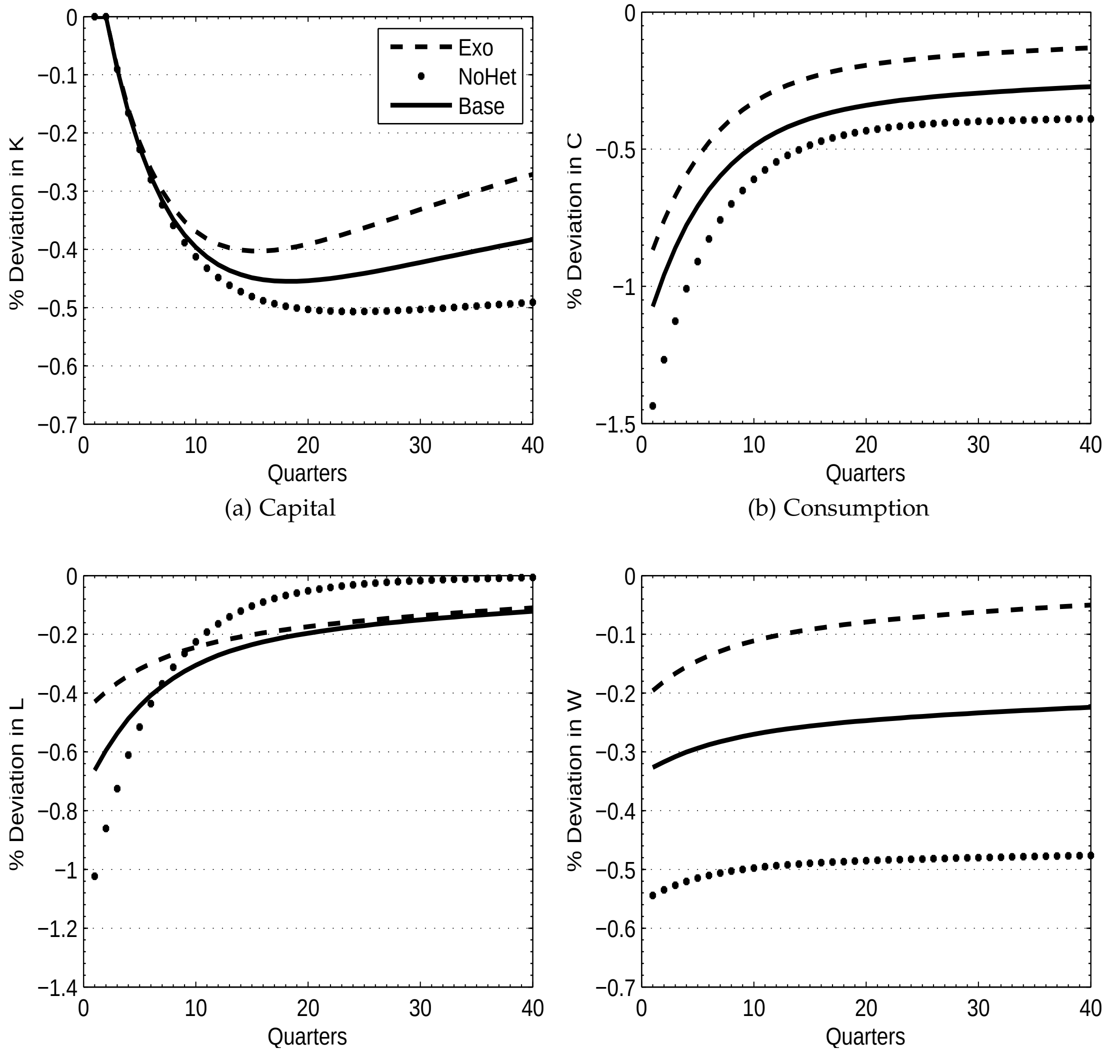
Figure 10: The Sources of Output.

The most striking fact in Figure 10a is the positive contemporaneous response of output under *NoHet*. This counter-factual response is explained by the relative changes of labor supply with respect to entry-driven labor demand. In fact, the radical decrease in entry in *NoHet* releases much more labor than the quantity that is absorbed by the contraction in labor supply. As a result, the use of labor in the intermediate good production rises in the short run, generating an increase in production. Note that the L component in Figure 10c increases by more than 0.3%, reversing the decrease in the other two components in the short run. This means that, from the point of view of the intermediate good producers, the drop in their costs due to the decrease in wages is more powerful than the drop in their benefit due to the decrease in the demand for intermediate goods triggered by the working capital constraint. This short-run mitigating effect of labor is considerably weaker in the baseline model because the reduction in the labor demand for project enaction is much lower. The effect is in line with economic intuition, as the interest rate shock mostly affects the entry margin and current output is produced by incumbents. In this sense, the reallocation of labor from entry to intermediate production implies a lower short-run effect, but more severe medium and long-run effects due to the permanent productivity loss. In fact, because labor is a stationary variable, it returns in the long-run to its balanced growth path level. On the contrary, the K and A components feature a long-run loss that drives the shocked path of output to be permanently lower in the long run. In the medium-run horizon, the baseline model exhibits an amplitude that is roughly 30% larger than that of *Exo*. Moreover, medium-run persistence is also higher, as can be seen from the delay in the lower point of the path in Figure 10a.

Comparing the sources of output in both models, we can identify the drivers of both effects. First, the gradual decrease of the A component, absent in *Exo*, has a first order effect on the depth of the crisis. The amplification is also driven by the extra drop in the K component; this is, in turn, due to the decrease in the return of capital triggered by the reduction in aggregate productivity.[47] Second, the persistence is mostly due to the hump shape of the L component. This shape is driven by the compositional dynamics in the intermediate product line. In fact, the slow convergence of $\mu(s^t)$ delays the return of the L component to its long-run value. The lack of a strong recovery in capital due to the permanent TFP loss also contributes to the medium-run persistence of the crisis. In contrast, investment and labor recover quickly in *Exo* to catch up with the TFP level in the economy; therefore, *Exo* features a full neoclassical recovery, not only in labor, but also

[47]Note that *NoHet* exhibits a lower decrease in the K component, although the loss in TFP is higher. The reason lies once again in the rise of the L component in *NoHet*, as the complementarity between inputs increases the marginal productivity of capital.

in physical capital. To complete the macroeconomic picture triggered by this episode, Figure 11 presents the deviations of capital, consumption, total hours, and wages for the three models.



(a) Capital

(b) Consumption

(c) Total Hours

(d) Wages

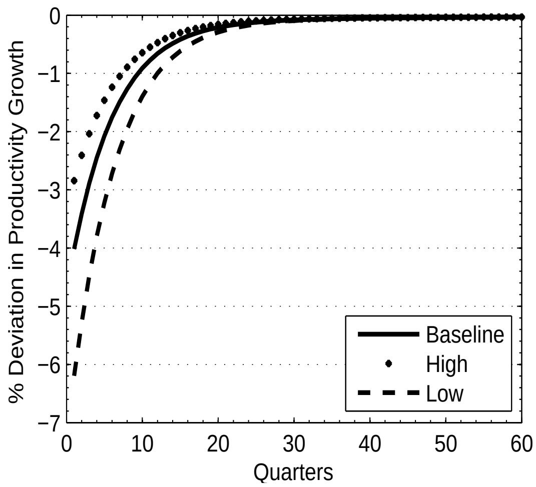
Figure 11: Macroeconomic Aggregates.

Figure 11c shows the response of total hours in the three models. On impact, the model with exogenous growth has the lower decrease in labor (-0.4%), while labor decreases 50% more in the baseline model, and the model with no heterogeneity shows an even larger decrease (-1%). Two reasons lay behind this amplification. First, the decrease in labor productivity in the models with endogenous growth amplifies the decrease in efficiency-adjusted wages and so, given GHH preferences, unambiguously reduces the labor supply of the household. Second, without entry, interest rates mostly affect labor by the working capital channel; in the presence of entry, the financial intermediary acts as a second channel that links interest rates to the labor market and the real economy. Summarizing, models with endogenous growth exhibit a long-run cost of a sudden stop in every growing variable, but failing to account for heterogeneity and selection doubles the estimation of this cost. The baseline model also generates persistence and amplification of interest rate shocks, while the model with no heterogeneity can deliver counter-factual predictions due to the violent behavior of entry.

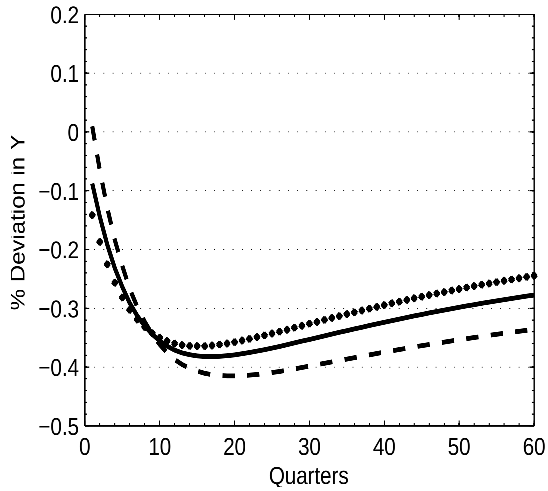
Finally, it is interesting to explore how different accuracy levels of the financial system shape the long-run cost of a crisis and its medium-run characteristics. Figure 12 compares the deviation of productivity and output of the baseline calibration with two alternative calibrations: one where the fraction of entrants with types below the threshold is 5% along the balanced growth path (instead of the original 15% target that resulted in the calibration at $\rho = 0.71$), and one where that fraction is 25%. This corresponds to a high accuracy calibration with $\rho = 0.91$ and a low accuracy calibration with $\rho = 0.49$.[48]

In line with economic intuition, Figure 12a shows that the long-run cost of a sudden stop decreases with the accuracy of the financial system. In fact, the better the selection, the stronger the composition effect, and, hence, the less detrimental the decrease in entry. Moreover, Figure 12b shows that economies with better selection technology suffer more in the short run, but endure the crisis much better in the medium run. this is because output not only drops less, but also recovers faster and is subject to a lower long-run loss. This result is in line with Cerra and Saxena (2008)'s findings, where the level of development shapes the magnitude and persistence of extreme crises. Moreover, note that

[48]Because the long-run growth rate changes with both calibrations, we also modify β to guarantee that no bond holding costs are incurred in the long run. In particular, in the original calibration $\beta = 0.9975$, for the high accuracy β increases to 0.9977, and in the low accuracy it decreases to 0.9973.



(a) TFP Growth



(b) Output

Figure 12: Financial Development.

more financially developed economies exhibit less volatile trends in response to interest rate shocks. This sheds some light on the drivers of the relative importance of trend shock documented across developed and developing countries by Aguiar and Gopinath (2007). As a final remark, the long-run cost decreases from 0.18% in the baseline to 0.15% in the high accuracy case, but it increases to 0.25% in the low accuracy case. This suggests that the benefits of policy interventions aiming to foster financial development are subject to decreasing returns.

2.6 Concluding Remarks

In this paper, we revisited the effects of a sudden stop, introducing a new element into consideration: the effect of the crisis on firm entry. With that aim, we presented an open economy endogenous growth model subject to interest rate shocks. The engine of growth in this economy is the creative destruction induced by new entrants. But, as potential entrants are heterogeneous, and promising entrants are scarce, financial selection introduces a trade-off between the mass (quantity) and the composition (quality) of the entrants. In particular, an interest rate shock increases credit standards, giving rise to a smaller cohort with higher productivity during the crisis. We use the Chilean sudden stop to test the main intuition of the model. In sum, although *fewer* firms are born during the crisis, they are *better*.

The model is able to convey some interesting insights about the role of firm entry during a financial crisis. For instance, in the quantitative section, we explore the long-run cost of a sudden stop driven by the endogenous changes in TFP growth that the crisis triggers. An increase in the interest rate has a permanent effect on output, investment and consumption. Not accounting for heterogeneity and selection doubles the estimation of this long-run cost, which has non-trivial welfare consequences, as the consumption equivalent welfare cost increases by 30%. As governments often use forgone entry as an excuse for policy, a correct assessment of that cost is critical. This model provides a tractable framework where those policies can be analyzed.

A second interesting point from the quantitative analysis is the role of the financial system in an interest rate crisis. In fact, more developed financial systems are able to take better advantage of the trade-off between mass and composition, reducing both the medium-run and long-run impact of a financial crisis, but suffering a larger contemporaneous output decrease. In this sense, financial reforms that increase the ability of the financial system to better allocate resources, such as the reforms empirically studied by Jayaratne and Strahan (1996) and Galindo et al. (2007), are potentially desirable, not only from a balanced growth path perspective, but also as a buffer against large crises. This paper provides a framework where the long-run macroeconomic consequences of banking competition can be quantified.

The scope of this model is far beyond sudden stop episodes, or the particular Chilean experience. In fact, the mass-composition trade-off at the core of this study can be triggered by any economic shock that disrupts the entry margin. The long-run economic cost of those fluctuations depends on the ability of the financial system to efficiently allocate scarce resources to the most promising projects. Note that interest rate shocks are particularly suited for this task, as they decrease the benefit of entry and increase the cost faced by entrants. However, traditional stationary adverse TFP shocks reduce both the cost and benefits of entry, having a minor impact on the TFP accumulation process. In this sense, not every stationary shock produces a sizable long-run TFP cost. Future research is needed to continue closing the gap between the quantitative firm dynamics-innovation literature and the stochastic open economy models. This is not only relevant for developing countries, where the distinction between short-run fluctuations and medium to long-run trends seems rather arbitrary, but also for developed economies. Indeed, the Great Recession challenged traditional macroeconomic models by exhibiting persistent effects in aggregate productivity, diminishing potential output even at long horizons.

Chapter 3

Beyond Cash: Venture Capital, Firm Dynamics, and Economic Growth

Abstract

This paper presents a new dynamic general equilibrium model of innovation with heterogeneous firms that incorporates an explicit venture capital (VC) market. The data show that VC financing accounts for a disproportionate share of sales and employment in the US compared with its limited share of total investment. VC firms invest heavily in young and innovative firms, bringing *operational knowledge*, together with financing, to their portfolio companies. The goal of this paper is twofold. First, I measure the particular channels through which VC firms influence their undertakings, using a structural model. Second, I explore the implications of VC investments for aggregate productivity and innovation policy. To address these goals, I combine and structurally estimate an endogenous technical change model with a VC setting that includes (i) the new feature of expertise, and (ii) the endogenous matching market where firms and VCs meet. In this model, firms improve the quality of their innovative product through risky R&D. VC expertise raises the efficiency of product development, and firms obtain VC financing at the cost of selling an endogenously determined share of the company. The entry cost that VC companies face also introduces a selection margin: VCs invest in firms that present a high potential for growth. The estimated model captures certain features of the VC matches and innovation observed in the US data. Counterfactual experiments imply that operational knowledge accounts for about 1/3 of VCs' impact on aggregate growth. Policy experiments suggest that changes affecting the VC market could result in a 7 basis point gain in the long-run growth rate of the economy.

"... We were cash positive. We didn't have a year where we lost money... We eventually ... sold 5% of the company for a million dollars ... just to get a venture capital company to join our board and give us some adult advice... That money sat in the bank."

Bill Gates, ex-chairman and co-founder of Microsoft

3.1 Introduction

Investments by venture capital (VC) companies have a disproportionate impact on the US economy. In 2010, the revenues of firms that had ever received VC support accounted for 21% of GDP, and their employment share was 11% of total private sector employment, although VC investments to their portfolio companies amount to less than 0.2% of GDP.[1] VC financing is of particular relevance for firm creation and innovation because VC firms strive to find young and innovative firms that lack market experience. VC firms are unique in that they do not only provide financing: They also actively engage in management by bringing their *operational knowledge* to bear in their investments.[2] Despite this distinctive structure, the contribution of operational knowledge to firm productivity and its implications for aggregate economic growth lack a thorough investigation. This paper presents a rigorous quantitative framework to explore the distinct mechanisms by which VC firms influence innovative firms and, through them, aggregate productivity.

Investing in a young company that needs to develop an innovative business idea entails considerable uncertainty and is subject to pervasive moral hazard problems (Gom-

[1]National Venture Capital Association (NVCA, 2013). Well-known examples include Amazon, Google, Starbucks, and FedEx.

[2]By operational knowledge, I refer to a general body of expertises concerning organizational structure, marketing, product development, and other business domains. This role of venture capital financing reflects the idea "that the typical founder is an incomplete businessman, with gaps in experience in matters such as financial management and marketing. An active board of directors, staffed by representatives of the [venture capital] investors, is expected to help fill these gaps" (Bartlett, 1995). For evidence on different methods, see Gorman and Sahlman (1989), Sahlman (1990), Gompers (1995), and Hellmann and Puri (2000), among others. Da Rin et al. (2011b) provide an extensive survey.

pers, 1995). In such environments, Casamatta (2003) shows that the *optimal contract* specifies a "dual role" for the VC firm. The optimal contract bundles financing and advice so that a VC firm's financial stake in the company motivates it to provide valuable advice. By contrast, consulting firms are not preferred by young and innovative firms because they do not acquire stakes in the latter.[3] As a consequence of this lack of "skin in the game," entrepreneurs have to pay a very high price in order to obtain valuable advice from the consultant. Therefore, young and innovative companies prefer VC advising to consulting advice.

Taking the structure of the optimal contract as given, I develop a structural model containing an explicit VC market. This model serves two main purposes: Firstly, I use the model to measure the importance of the VCs' *operational expertise* to firm growth. To identify this channel, it is fundamental to separate it from the provision of financing and the overall selection of "portfolio" firms by VCs. Establishing a unified structure that accounts explicitly for different aspects of the VC market, the structural model is an effective tool to accomplish this task. Quantifying the operational knowledge channel is useful for evaluating the advantages commonly attributed to VC finance in fostering firm productivity and growth. To the extent that VC companies add valuable knowledge to their undertakings, they become a more efficient option for financing innovation than more traditional financing sources such as bank loans. Secondly, the model provides a suitable ground to shed light on various policy discussions such as the relationship between an active public equity market and VC financing.

To address these issues, I propose a new dynamic general equilibrium model of innovation with heterogeneous firms, in the tradition of Romer (1990), Grossman and Helpman (1991), and Acemoglu et al. (2013), among others.[4] In this model, entrepreneurs/private firms produce differentiated goods of heterogeneous quality which they can improve

[3] The result assumes that the entrepreneurial effort is cheaper and is key for the success of the project.

[4] The model shares features such as product variety expansion with Romer (1990), quality latter structure with Grossman and Helpman (1991), and innovation by incumbent firms with Acemoglu et al. (2013), whose details are explained below.

through risky research and development. The efficiency of this development process can only be increased with the help of a VC and is otherwise fixed. I introduce to this model a detailed venture capital market through (i) the feature of VC expertise and (ii) the endogenous matching market for firms and VCs. Every private firm that is not in a relationship with a VC can search for VCs and meet them in the matching market. VCs improve the efficiency of product development through their operational knowledge. They also provide financing and relax the cost of inputs into the production of goods. This financial support to a priori unconstrained firms reflects the dual structure of the optimal contract for VC investment. The heterogeneity in the quality level of private firms determines the magnitude of the improvement that VC firms can potentially create. VC firms are subject to entry costs, which induce them to select firms for investment that present more room for growth. Thus, in addition to financing and operational knowledge channels, the VC setting also accounts for the effects of selection by VC firms. Because the preferred option for VC firms to exit their portfolio companies is to sell them via initial public offerings, the model also includes a public equity market. To complete the general equilibrium framework, the rest of the structure builds on the shoulders of endogenous technical change models in which entrepreneurs own intermediate product lines. Entrepreneurs enter the market with a new product line while intermediate good producers who cannot develop the quality of their good sufficiently exit the economy. Together with entry and exit margins, the innovations generated by these intermediate good firms determine the endogenous rate of growth of the aggregate economy.

The main identification problem in this model is to distinguish the financing channel from operational knowledge in their relative influence on firms receiving VC investment. The assumption that disentangles these two channels is that the former mainly affects the level of the profits while the latter changes its growth rate. The financial help of VC decreases the cost of inputs in the intermediate good production. Therefore, the entrepreneur earns higher profits for a given quality level of the intermediate good that the firm produces. Operational knowledge, on the other hand, directly affects the efficiency of the entrepreneur's effort in generating innovations that increase the product quality.[5] To discipline the size of the financial impact in the model, I target the ratio of VC investment to GDP. Determining the size of VCs' contribution to efficiency in the data is a more delicate task. A well-known concern is selection: VCs might be "cherry-picking" already good firms instead of improving them in some other way. I address this issue by applying the method of indirect inference in my estimation. To do so, I utilize the findings of Puri and Zarutskie (2012). In their empirical study, Puri and Zarutskie (2012) provide statistics on growth rates of VC-backed and non-VC-backed private firms, controlling for selection on observable characteristics. In a nutshell, Puri and Zarutskie (2012) create samples of private firms with and without VC support that are matched on some measurable features. Following similar steps, I create the analogues of such samples from the stationary firm distribution of my model. Finally, I use the model-generated samples to match the regression statistics on VCs' effect on firm growth provided in the same paper.

I estimate this model by the method of moments, using US data on the venture capital market, public equity issuances, and research and development expenditures. The model does a successful job in matching moments that pertain to venture capital and innovation aspects of the model, such as the duration of VC matches, firm age at the issuance of initial public equity, and aggregate share of R&D. Moreover, the model-generated regression results accurately predict the coefficient estimate found in Puri and Zarutskie (2012). Successfully hitting this target via indirect inference is crucial because it determines the scope of the influence of the operational knowledge channel. Before using the estimated model for counterfactual analyses, I compare its auxiliary predictions to data moments in order to obtain out-of-sample validation. This comparison reveals that the model is

[5]Gonzalez-Uribe (2014) is a recent empirical work that points towards the efficiency enhancing role of VC. By using the introduction of the Prudent Investor Rule (PIR) across states as a source of exogenous variation in VC financing, she first documents a 50% rise in the annual citations of patents of a firm after it obtains VC financing. More interestingly, she shows that the probability of receiving a citation from a company in the portfolio of the same VC firm increases twice as much as from a company outside the VC's portfolio. This result indicates that VCs facilitate the diffusion of knowledge among their portfolio companies. Similarly, Lindsey (2008) argues that by mitigating informational and contractual problems, VC firms increase the probability of strategic alliances among their portfolio companies. The empirical estimates imply a 70% increase in the probability of R&D alliances, a significant constituent of strategic cooperations.

very precise in capturing the high IPO frequency among VC-backed firms and the share of IPOs issued by VC-backed companies, both of which are definitive characteristics of the VC market in the US.

The first set of counterfacual experiments determines the relevance of VCs' operational knowledge to firm and aggregate growth. I create hypothetical economies in which I strengthen particular channels of VC finance in each experiment. Comparing the responses of the aggregate growth rate to these changes demonstrates that the knowledge channel accounts for 1/3 of VCs' impact. Hence, the conclusion is that VC support matters significantly beyond financing. Then I consider a 15% increase in the fixed cost of IPO to capture the average level of underwriting spreads in the US before their secular decline after the 1980s.[6] As a result, fewer private firms issue IPOs, and the equilibrium probability of a match with a VC firm decreases. Thus, the increase in the fixed IPO cost results in a smaller share of VC-backed private firms in the economy. This leads to a 1.5 basis point loss in the long-run growth rate.

As an example for the policy implications of the model, I consider a recent regulation that the European Union introduced in 2013 regarding European VC firms. In order to decrease the fundraising costs of VC firms, this policy aims to harmonize the legislative environment these firms face when investing across the borders of European countries. I map this change into the model as lower entry costs for VC firms through lump-sum subsidies. I find that this policy can increase the long-run growth rate by 7 basis points at a cost of subsidies that corresponds to approximately 8% of the VC investment in the model. This increase in the growth rate hinges on the reallocation of private firms towards the VC market. Moreover, a rise in the median duration of the VC-firm relationship amplifies the effect of the operational knowledge. These results highlight the significance of the general equilibrium effects for the policy evaluation.

[6] An underwriting spread, also known as gross spread, measures the fees paid to the underwriter of the issue in compensation for expenses such as legal expenses, management fees, etc. as a fraction of the total proceeds raised. This spread is a direct cost associated with the issue, which I model as a fixed cost.

Related Literature. This paper draws on several strands of the literature. First, by embedding the VC market into the endogenous technical change environment, it contributes to the literature that concentrates on innovation and firm dynamics (Klette and Kortum, 2004; Akcigit, 2010; Akcigit and Kerr, 2010; Lentz and Mortensen, 2008; Acemoglu et al., 2013).[7] Lentz and Mortensen (2008) and Acemoglu et al. (2013) are recent examples that particularly focus on allocation of resources across firms with heterogeneous capacities to innovate. This paper contributes to the analysis by introducing a link between this heterogeneity and the financing decisions of innovative firms. In that regard, this paper also relates to work on finance of innovation (Aghion and Tirole, 1994; Aghion et al., 2004; Kortum and Lerner, 2000; Lerner et al., 2011; Brown et al., 2009; Amore et al., 2013).[8] As an example in the setting of endogenous growth, Itenberg (2014) explores the effect of developments in the US public equity market on R&D decisions of small firms. By contrast, the focus of this paper is the venture capital market.

This study extends the venture capital literature by analyzing VC financing in a dynamic quantitative framework. The theoretical work in this area uncovers the conditions in a static setting that leaves room for the use of venture capital in the existence of alternative financing or in advising agents such as banks and consultants (Amit et al., 1998; Casamatta, 2003; Ueda, 2004; de Bettignies and Brander, 2007).[9] While my work acknowledges these theoretical foundations, and borrows the features of the optimal VC contract from this literature, it focuses on quantitative analysis of the VC market. In particular, my model improves the understanding of VC financing in a dynamic general equilibrium setting that enables the measurement of the distinct channels through which VC firms affect firm dynamics. A realistic structure for the VC market in a model of endogenous

[7]For a detailed discussion of innovation and firm dynamics in the context of Schumpeterian growth theory, see Aghion et al. (2014).

[8]Hall and Lerner (2009) is a seminal survey on this topic.

[9]There are a few recent papers that include venture capital in a dynamic setting. Silveira and Wright (2013) model VC firms in a search and matching environment and analyze its theoretical predictions for the life cycle of VC firms. Pinheiro (2012) examines the theoretical underpinnings of the optimal duration in matches between VC firms and their undertakings. Opp (2014) analyzes the cycles in the venture capital market. None of these studies consider the effects on firm-level innovation and economic growth.

firm dynamics also allows the analysis of venture capital from the perspective of innovation policy. Due to these characteristics, this work also contributes to answering empirical questions in VC literature.[10] In particular, Kortum and Lerner (2000) show the significant effect of VC finance on firm-level innovation in terms of both patent counts and citations, whereas papers such as Hellmann and Puri (2000, 2002) examine the effectiveness of particular management practices applied by VC firms using hand-collected data. My paper advances these exercises to quantify VCs' impact by using a new structural model as a measurement tool that takes into account important margins such as selection and reallocation. Furthermore, the setting should also be helpful in shedding light on various policy debates, such as the relationship between public equity and VC markets (Black and Gilson, 1998; Bottazzi and Rin, 2002).

Finally, a related literature focuses on the role of the financial system in evaluating and selecting investment projects. For instance, Jayaratne and Strahan (1996) show empirically that interstate branch reform in the US banking system has led to a tighter selection in lending through increased competition. This in turn has resulted in higher lending quality and growth rates in liberalized states. In the context of VC, Casamatta and Haritchabalet (2007) provide evidence on how VC firms use syndication practices to obtain a second opinion when deciding on early rounds of investment. By estimating a theoretical framework using Bayesian methods, Sørensen (2007) finds important effects of assortative matching in the VC market. In relation to this literature, my paper formalizes the idea that VC firms search for high growth potential by embedding an endogenous search and matching market that accounts for selection. This aspect is integral to identifying the effect of "value-adding" practices of VC firms in the model. Furthermore, building on the endogenous growth framework, this paper relates to the literature that analyzes the effects of selection on economic growth (King and Levine, 1993a; Jaimovich and Rebelo, 2012).[11] A recent paper in this strand, Ateş and Saffie (2014), argue that financial selection

[10] For a recent survey, see Da Rin et al. (2011b).

[11] Levine (2005) provides an excellent survey on this topic, and on the relationship between finance and growth, in general.

introduces a trade-off between the mass and the quality in firm entry, and analyzes its impact on aggregate productivity growth in the context of sudden stops.[12] In comparison to this literature, my work focuses on a specific financial intermediary that is especially relevant to innovation and growth, namely venture capital finance.

The next section introduces the main ingredients of my model. Section 3.3 explains the data used in the estimation, and discusses identification. Section 3.4 presents counterfactual and policy experiments, and Section 3.5 concludes.

3.2 Model

In this section, I present the components of the model economy. Entrepreneurs produce differentiated intermediate goods and sell them to a representative final good producer that combines these intermediate goods into a final output. The entrepreneurs enter the business with an innovative product, the quality of which they can improve over time by investing in risky research and development (R&D) activities. The efficiency of the R&D process is a firm characteristic that is fixed unless the firm uses the additional business expertise of a VC. VC support also entails a reduction in the cost of intermediate good production. An intermediate good producer can search for and match with a VC firm in the endogenous search and matching market. The contribution of the VC to firm growth stems from increased R&D efficiency that makes product developments likelier. However, it comes at the cost of firm dilution, and carries an exogenous risk of running the project idle. Any private firm can issue public equity to expand the size of its enterprise, but there is an associated fixed cost. By improving the efficiency of product development, VC firms help their undertakings raise adequate resources faster to afford the IPO cost.

[12]In a related paper, Ateş and Saffie (2013) provide a theoretical characterization of the nexus between financial selection and the long-run productivity growth by analyzing the mass-quality trade-off in a closed economy setting.

3.2.1 Preferences

Consider the following closed economy in discrete time. The representative household maximizes the expected discounted sum of the period utility from consumption with the following preferences

$$U_t = \sum_{\tau=t}^{\infty} \beta^{\tau} \frac{C_{\tau}^{1-\varepsilon} - 1}{1-\varepsilon}$$

where C_t denotes consumption at time t. β is the discount factor and ε is the coefficient of relative risk aversion. The household consumes a final good, and supplies labor inelastically to the final good producer, which I normalize to 1 without loss of generality. Households own all the firms in the economy, and their budget constraint is

$$C_t \leq \int_{j \in J} \Pi_{jt} dj + w_t \tag{3.1}$$

where Π_{jt} is the flow profit of the intermediate firm j in the interval J of actively operating firms, and w_t is the wage level at time t.

3.2.2 Final Good Production

The final good, which is used for consumption, R&D, and intermediate good production, is produced in a perfectly competitive market. The production technology combines labor and differentiated intermediate varieties in the following structure:

$$Y_t = \frac{1}{1-\alpha} L_t^{\alpha} \int_{j \in J} q_{jt}^{\alpha} k_{jt}^{1-\alpha} dj. \tag{3.2}$$

Here, L_t denotes the labor input, k_{jt} refers to intermediate good $j \in J$ at time t, and q_{jt} is the associated quality of product j. $(1-\alpha)$ stands for the physical factor share. Y_t is the numeraire good in the economy.

The representative final good producer chooses a bundle of intermediate goods and labor in order to maximize its profits. Taking the price of the intermediate product, p_{jt}, as given, the problem of the final good producer reads as:

$$\Pi_{Y,t} = \max_{L_t, \{k_{jt}\}_{j \in J}} \left\{ \frac{1}{1-\alpha} L^{\alpha} \int_{j \in J} q_{jt}^{\alpha} k_{jt}^{1-\alpha} dj - \int_{j \in J} p_{jt} k_{jt} dj - w_t L_t \right\}. \tag{3.3}$$

The solution of this maximization problem yields in equilibrium the following inverse demand for intermediate good j:

$$p_{jt} = q_{jt}^{\alpha} \left(k_{jt}^{d} \right)^{-\alpha} \tag{3.4}$$

where k_{jt}^{d} is the optimal amount of good j demanded by the final good producer.

3.2.3 Intermediate Good Firms

Intermediate firms are distributed across product lines whose measure, J_t, is determined endogenously. There are three types of intermediate good firms: private firms that are not matched with a VC, private firms that are matched with a VC, and public firms. Each firm is characterized mainly by two state variables which are the product quality and the R&D efficiency. An entrepreneur that has an innovative project enters a product line as a private firm without a VC. First I introduce decisions that are common to all intermediate good firms, and then I go into the specific choices of different types of firms.

Production

Each intermediate good is a monopolist in producing its differentiated good k_{jt}. To maximize the operating profits, the monopolist solves the following problem

$$\Pi_{jt}\left(q_{jt}\right) = \max_{k_{jt}} \left\{ p_{jt}k_{jt} - C_k\left(k_{jt}\right)\right\} \quad \textit{subject to}$$
$$p_{jt} = q_{jt}^{\alpha}\left(k_{jt}^{d}\right)^{-\alpha}$$

where $C_k\left(k_{jt}\right)$ denotes the cost of inputs to produce k_{jt} amount of intermediate good in terms of the final good and has the following form:

$$C_k\left(k_{jt}\right) = \eta_j k_{jt}.$$

In this specification, $\eta_j \in \{\eta^H, \eta\}$ denotes the marginal cost of production with $\eta^H > \eta$. For any firm that does not have VC support, this parameter has the higher value. Therefore, this structure captures the financial contribution of VC firms to their undertakings. An interpretation for this structure is that it reflects cash-in-hand constraints in a reduced form way. In line with reality, VC relaxes this financial constraint with its monetary commitment.

In equilibrium, the optimal level of intermediate good production becomes

$$k_{jt} = \alpha\left[\frac{1-\alpha}{\eta_j}\right]^{\frac{1-\alpha}{\alpha}} q_{jt}. \tag{3.5}$$

With a constant mark-up over price, this optimal quantity generates profits that are linear in product quality q_{jt}. Thus, $\Pi_{jt} = \pi_j q_{jt}$ where

$$\pi_j = \begin{cases} \pi^L & \text{if } C_k\left(k_{jt}\right) = \eta^H k_{jt} \\ \pi^H & \text{if } C_k\left(k_{jt}\right) = \eta^L k_{jt} \end{cases}$$

is a constant depending on the marginal cost η_j. Hence, the operational profits are higher if the marginal cost of intermediate good production is lower due to VC support.

Research and Development

Each firm invests in R&D to improve the quality of its product and hence to increase its operating profits. Let i_{jt} and $Z\left(i_{jt}\right)$ denote the (process) innovation rate and the R&D effort required to generate this rate, respectively. The R&D cost function in terms of the final good has the following form:

$$Z\left(i_{jt}\right) = \frac{h\left(i_{jt}\right)}{\theta_j} q_{jt}$$

where $h\left(\cdot\right)$ is a convex, strictly increasing function. In this specification, θ_j denotes the efficiency in developing the product quality.

In order to analyze the effect of VC firms' operational knowledge, the parameter θ can take three different values:

$$\theta_j = \begin{cases} \theta^L & \text{for private firms without VC} \\ \theta^H & \text{for private firms with VC} \\ \theta_j^{pb} \in \left\{\theta^L, \theta^M\right\} & \text{for public firms} \end{cases}$$

In this economy, the private firms conduct R&D with low efficiency, θ^L, unless they receive help from a VC firm. Once matched, a VC firm raises the efficiency level of its portfolio company to θ^H thanks to its expertise. In turn, a higher efficiency in generating innovations increases the expected growth rate of the private firm. In addition to this direct effect, I allow for the possibility that VC firms also cause a permanent effect. Product development efficiency after IPO, θ_j^{pb}, depends on whether the firm used VC finance or not. The underlying motivation is that, although the entrepreneur separates from the VC, she retains some of the operational skills brought to the firm by the venture capitalist. Therefore, for a firm that becomes public under the supervision of a VC, I assume that $\theta_j^{pb} = \theta^M$. Although determined by the data through estimation, it is expected that $\theta^H > \theta^M > \theta^L$, as validated in Section 3.3.4. The ordering $\theta^H > \theta^M$ reflects the loss of VC supervision, whereas $\theta^M > \theta^L$ reflects the VC's permanent impact on the firm's operational knowledge stock. The product development efficiency remains constant unless the firm changes its type due to a financial decision.

A successful process innovation improves the product quality of the firm by an amount that is taken to be proportional to the average quality of the firm, $\bar{q}_t$:

$$q_{jt+1} = q_{jt} + \lambda \bar{q}_t.$$

where $\bar{q}_t \equiv \int_{j \in J} q_{jt} dj$. If the R&D is unsuccessful, q_{jt} remains the same. These additive increments in product quality introduce decreasing returns to innovation, and imply smaller incentives to innovate for larger firms. Limiting the growth potential of larger firms, this structure enables the model to generate a stationary size distribution in equilibrium.

Free Exit

Every intermediate firm has an outside option $\chi_t^o = \chi^o \bar{q}_t$ which is proportional to the average quality $\bar{q}_t$. If the value of the firm goes below this level, the firm exits the economy. Notice that the option value grows at the rate of the aggregate economy. Therefore, if a firm fails to innovate for a long period of time, it will necessarily exit the market. Therefore, in addition to the profit enhancing motive, there is another incentive to motivate, namely to survive in the business.

Free Entry

The economy has a unit measure of potential entrants. These outside firms need to generate an innovation to enter the market. An entering firm observes the initial quality of its product upon successful innovation. This initial quality is drawn from the stationary distribution of the previous period, but from a range that is small enough such that the entrant does not go public immediately.[13] An entrant opens a new intermediate product line and starts with the low level of product development efficiency θ^L.[14]

The cost of generating a product innovation for entry is quadratic in probability of innovation, x_t:

$$C_e(x) = \chi^e \bar{q}_t f(J_{t-1}) x_t^2$$

where χ_e denotes the scale of the cost function. There are two important features in this cost structure. First, it is proportional to aggregate productivity level $\bar{q}_t$. Since the expected value of an innovation also shares this proportionality the optimal innovation rate becomes independent of $\bar{q}_t$. Second, the cost depends on the previous measure of the intermediate firms through a convex and increasing function.[15] This structure relates the measure of firms to the size of entry, and enables the economy to reach a stable size.

Timing of Events

The timing of events is summarized in Figure 13. The period starts with the entrants' decision to pay the entry fee and draw their productivity. Then, the private intermediate

[13] Given the median age of US firms at the IPO stage, this assumption is a plausible one. Moreover, the average size of entry firms in the US are drastically smaller than the average size of incumbent firms (Scarpetta et al., 2004; Bartelsman et al., 2009).

[14] The fact that entrants open new product lines introduces a source for growth due to expanding product markets, à la Romer (1990). However, as explained below, the measure of intermediate product lines, J, remains constant in a balanced growth path equilibrium.

[15] In the estimation, $f(\cdot)$ is assumed to have a quadratic form. This type of relationship has the interpretation that the resources to innovate are scarce, and the costlier it is for entrants to use these resources, the larger the share of incumbent firms becomes.

good producers make their financial decisions, which are searching for VC and going public. Then, the final good producer and intermediate good producers decide on production. Intermediate good producers also determine their innovation intensities. Lastly, the R&D outcomes are realized, and intermediate producers make their exit decisions.

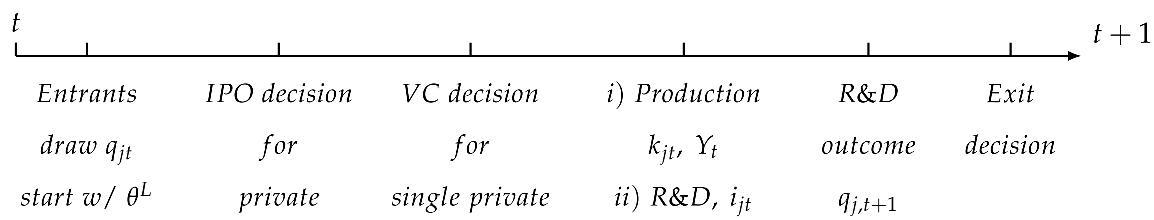


Figure 13: Flow of Events

Next, I explain the different types of intermediate firms and their specific financial choices. In particular, a public firm will consider only the decisions introduced above. Every private firm considers going public at the onset of a period, in addition to the aforementioned common decisions. Lastly, a private firm that is not matched with a VC can search for a VC if it has already chosen to remain private.

Firm Types and Financial Decisions

Public Firm. A private firm can choose to go public by issuing an initial public offering (IPO) and raise public funds to expand its operations.[16] I assume that a firm cannot look for a VC and cannot raise any public funds once it is public. Therefore, the only decisions that a public firm needs to consider are production, R&D, and exit decisions. Specifically, let V_t^{pb} denote the value of a public company. Then the problem

[16] This type of modelling IPO is in line with the investment financing explanation of equity finance. Using extensive data on initial and seasoned equity offerings across 38 countries during the period 1990-2003, Kim and Weisbach (2008) show that firms subsequently use proceeds from selling equity for R&D and CAPEX investments. When I explain the decision to do an IPO below, it will be clear that going public also provides an exit channel for the VC.

of the public firm becomes

$$V_t^{pb}\left(q_{jt},\theta_j^{pb}\right)=\max_{i_{jt}}\left\{\begin{array}{c}\pi^L q_{jt}-h\left(i_{jt}\right)/\theta_j^{pb}\cdot q_{jt}+\frac{1}{1+r_{t+1}}\times\\ \left[i_{jt}W_t^{pb}\left(q_{jt}+\lambda\bar{q}_t,\theta_j^{pb}\right)+\left(1-i_{jt}\right)W_t^{pb}\left(q_{jt},\theta_j^{pb}\right)\right]\end{array}\right\} \tag{3.6}$$

where r_{t+1} denotes the interest rate. The continuation value is defined as

$$W_t^{pb}\left(q_{jt+1},\theta_j\right)=\max\left\{\chi^o\bar{q}_t,\mathbb{E}V_{t+1}^{pb}\left(q_{jt+1},\theta_j^{pb}\right)\right\}. \tag{3.7}$$

Every period, the public firm collects flow profits and decides on the optimal size of process innovation.[17] In case of successful R&D, it increases its product quality with which it starts the next period, unless it chooses to use the outside option and exits. If R&D efforts do not result in an incremental innovation, the product quality remains the same. In this regard, product development enables the firm to decrease the likelihood of exiting the market, besides increasing the profits. Note that the product development efficiency remains constant in any case.

VC decision. A private firm without VC backing can search for a VC in every time period. To understand the benefit of becoming matched with a VC, first consider the problem of a private firm with a VC that decided not to go public. This post-IPO-decision value is the one that a private firm without VC obtains when it is matched with a VC firm, because the process of searching for a VC follows the IPO decision. It is defined as

$$V_t^{pr}\left(q_{jt},\theta^H,\mathbb{I}_j^{vc}=1\right)=\max_{i_{jt}}\left\{\begin{array}{c}\pi q_{jt}-h\left(i_{jt}\right)/\theta^H\cdot q_{jt}+\frac{1}{1+r_{t+1}}\times\\ \left[\begin{array}{c}i_{jt}W_t^{pr}\left(q_{jt}+\lambda\bar{q}_t,\theta^H,1\right)+\\ \left(1-i_{jt}\right)W_t^{pr}\left(q_{jt},\theta^H,1\right)\end{array}\right]\end{array}\right\}. \tag{3.8}$$

where $\mathbb{I}_j^{vc}$ is an indicator function that takes the value 1 if the firm is matched to a VC. This problem is very similar to the one of the public firm, with three exceptions. First,

[17] Notice the low level of profits because public firms do not have VC support.

as the firm gets the expertise of VC, its efficiency increases to level $\theta^H > \theta^L$, and this is the *operational knowledge* channel through which a VC adds value to the firm. Second, due to its financial support, VC increases the profits of the firm to $\pi > \pi^L$ for a given quality level. The third difference reflects the fact that many relationships between the VC and the firm end up unsuccessfully, i.e. they do not lead to any IPO or acquisition by another firm where VC can have a profitable exit. To capture this, I assume that the VC and the firm can separate with an exogenous probability σ^{vc} in which case the firm exits the market. The continuation value $W_t^{pr}(\cdot, 1)$ incorporates these differences.

A private firm meets with VCs in a random matching environment.[18] The endogenous probability that a private firm matches with a VC firm is defined by

$$m_f(\Lambda) = \rho \frac{\Lambda}{1+\Lambda}$$

where n_{vc} and n_f denote the number of VC firms and available private firms, respectively, $\Lambda \equiv n_{vc}/n_f$ is the market tightness, and ρ refers to the efficiency parameter (Shi, 2009).[19] Then the value of a private firm without a VC, after deciding not to issue an IPO, becomes:[20]

$$\begin{aligned} V_t^{pr}\left(q_{jt}, \theta^L, \mathbb{I}_j^{vc} = 0\right) = & \; m_f\left(\frac{n_{vc}}{n_f}\right) \mathbb{I}_{s^{vc}>0} \times V_t^{pr}\left(q_{jt}, \theta^H, 1\right) + \\ & \left[\left(1 - m_f\left(\frac{n_{vc}}{n_f}\right)\right) + m_f\left(\frac{n_{vc}}{n_f}\right)(1 - \mathbb{I}_{s^{vc}>0})\right] \times \\ & \max_{i_{jt}} \left\{ \begin{array}{c} \pi^L q_{jt} - h(i_{jt})/\theta^L \cdot q_{jt} + \frac{1}{1+r_{t+1}} \times \\ \left[i_{jt} W_t^{pr}\left(q_{jt} + \lambda \bar{q}_t, \theta^L, 0\right) + (1 - i_{jt}) W_t^{pr}\left(q_{jt}, \theta^L, 0\right)\right] \end{array} \right\}. \end{aligned} \quad (3.9)$$

[18]The main friction in this market is the process of evaluation by the VC. It is a cumbersome process in which only one out of a hundred applicants gets funded on average, according to NVCA figures. A directed search on the firm side is also unlikely given the low probability of acceptance. Moreover, this would require the applicant to gain information about other companies in the portfolio the VC firm of interest, their financing stages, the human capital constraints of the VC firm, etc., which is probably not the case with most of the applications in reality.

[19]Correspondingly, the total number of matches is given by the so-called telegraph matching function.

[20]No search costs are assumed in this setting.

Here, $m_f(\cdot)$ denotes the probability that the firm will meet with a VC. The share of the firm to be left to VC, s^{vc}, is determined endogenously via Nash bargaining. The first line of this value simply tells that, if there is a match that generates a positive surplus the firm, matches with a VC. Otherwise, it moves on to make production, R&D and exit decisions where its R&D efficiency remains constant.

Private firms and IPO decision. Any private firm, with or without a VC, can issue an IPO in any period. The upside of IPO is an increase in the size of operations. Moreover, it enables the VC firm sell its share in the company and collect the return.[21] Let $\mathcal{V}_t^{pr}$ refer to the value of a private firm that considers going public or remaining private. Then the IPO decision is determined by the following maximization

$$\mathcal{V}_t^{pr}\left(q_{jt}, \theta_j, \mathbb{I}_j^{vc}\right) = \max\left\{V_t^{pr}\left(q_{jt}, \theta_j, \mathbb{I}_j^{vc}\right), (1-\Delta)\, V_t^{pb}\left(\kappa q_{jt}, \theta_j^{pb}\right) - \chi^{ipo}\bar{q}_t\right\}.$$

The first part of this maximization is the value of the firm if it remains private. In the second part, $V_t^{pb}\left(\kappa q_{jt}, \theta_j^{pb}\right)$ denotes the value of the public firm with larger size of operations, where $\kappa > 1$ denotes the increase. Firms that issue an IPO without any previous relationship with a VC do not experience any change in the efficiency of product development.

At last, the firm incurs various costs of issuing an IPO, which are captured by $\chi^{ipo}\bar{q}_t$. Moreover, a Δ share of the firm value is sold at IPO. The firm finances its investment in improving its product quality with the proceeds from this transaction. In addition, if the firm that goes public is matched with a VC, IPO allows the VC to liquidate its stocks in the company in order to obtain the return on its initial investment, s_j^{vc}.

[21] IPO is considered as the most profitable exit option and a measure of success for VC funds (Brander et al., 2002; Sørensen, 2007). According to the 2013 Yearbook of National Venture Capital Association, 16% of portfolio companies end up going public.

3.2.4 Venture Capital Firms

Venture capital firms are agents that provide operational knowledge and finance to private firms. There is an outside pool of VC firms. To enter the matching market, a VC firm has to pay an entry cost. The entry cost is given by $\chi_t^{vc} \equiv \chi^{vc}\bar{q}_t$, and is proportional to the average productivity in the economy. At any point in time, a VC firm can be matched with only one firm. When the VC exits its investment, it is assumed to exit the economy. The value of a VC that is not matched to a private firm is

$$A_t = m_{vc}\left(\frac{n_{vc}}{n_f}\right)\int s_t^{vc}\left(q,\theta^L;\cdot\right)\Psi_q\left(dq\right) + \left[1 - m_{vc}\left(\frac{n_{vc}}{n_f}\right)\right]\frac{A_{t+1}}{1+r_{t+1}} \tag{3.10}$$

where Ψ_q denotes the distributions over q of the private firms that are in the matching market. The first part of equation (3.10) explains that, with probability $m_{vc}\left(\cdot\right)$, the VC meets a private firm and gets a share s^{vc}. Otherwise it continues to search next period.

The share of the firm that the VC gains, $s_t^{vc}\left(q,\theta\right)$, is the solution of the following Nash bargaining problem

$$s_t^{vc}\left(q,\theta\right) = \arg\max_s \left[V_t^{pr}\left(q,\theta^H,1\right) - V_t^{pr}\left(q,\theta^L,0\right) - s\right]^{1-\phi}\left[s - \frac{A_{t+1}\left(\cdot\right)}{1+r_{t+1}}\right]^{\phi} \tag{3.11}$$

where ϕ is the bargaining power of the VC. Notice that, for a match to form between a VC firm and a private company, the payment to the VC firm needs to be a positive amount because the VC firm is subject to an entry cost. This implies that the VC firms invest in companies only if the expected surplus is larger than zero. This selection margin is integral for the identification purposes.[22]

[22] See Section 3.3.2 for details.

3.2.5 Equilibrium

Throughout this paper, I will focus on the Markov Perfect Equilibrium. In particular, the analysis will be based on the balanced growth equilibrium where aggregate variables grow at a constant rate. To this end, it will be necessary to transform the economy into a stationary one by normalizing the growing variables by the aggregate productivity $\bar{q}_t$. First, I denote $\hat{q}_{jt} \equiv q_{jt}/\bar{q}_t$ as the normalized quality. Next, I define the Markov Perfect Equilibrium where the asterisk refers to equilibrium values.

Definition 6 (Equilibrium). *Let $\boldsymbol{\xi}_j^d \in \{0,1\}$, $d \in$ {exit,vc,ipo}, denote the decisions of firm j regarding exit, VC search, and going public, respectively. A Markov Perfect Equilibrium consists of aggregate prices $\{r_t^*, w_t^*\}$; aggregate output, consumption, R&D expenditure, and intermediate input expenditure $\{Y_t^*, C_t^*, Z_t^*, K_t^*\}$; intermediate prices and quantities $\{k_{jt}^*, p_{jt}^*\}$; R&D, exit, search, and floating decisions $\{i_{jt}^*, \boldsymbol{\xi}_j^{d^*}\}_{d\in\{exit,vc,ipo\}}$; firm value functions $\{V_t^{f^*}, W_t^{f^*}\}_{f\in\{pb,pr\}}$ and VC value A_t^*; VC pricing function $\mathbf{s_t^{vc}}^*$; the normalized quality distribution and the mass of firms $\{\hat{\Upsilon}_t^*(\hat{q}), J_t^*\}$ where $t \in [0,\infty)$, $j \in J_t^*$ such that*

1. *$\{k_{jt}^*, p_{jt}^*\}$ are given by* (3.5) *and* (3.4)*, and maximize the operating profits,*
2. *$\{V_t^{f^*}, W_t^{f^*}\}_{f\in\{pb,pr\}}$ satisfy* (3.6), (3.7), (3.9), *and* (3.8),
3. *i_{jt}^* maximize the expected profits, and $\{\boldsymbol{\xi}_j^{d^*}\}_{d\in\{exit,vc,ipo\}}$ solves the value functions,*
4. *$\hat{\Upsilon}_t^*(\hat{q})$ is consistent with R&D, entry, exit, VC, and IPO decisions of the firms,*
5. *J_t^* supports the free entry condition to hold with equality,*
6. *A_t^* is given by* (3.10),
7. *$\mathbf{s_t^{vc}}^*$ as in* (3.11) *is determined by Nash bargaining;*
8. *$\{Y_t^*, C_t^*\}$ are given by* (3.2) *and* (3.1),
9. *and aggregate prices $\{r_t^*, w_t^*\}$ clear the market.*

Accordingly, a balanced growth equilibrium is defined as follows:

Definition 7 (Balanced Growth Path)**.** *A Balanced Growth Path (BGP) is an equilibrium where* $\hat{\Upsilon}^*(\hat{q})$ *defines an invariant distribution, the measure of firms,* J^**, has a fixed value, and the average quality* $\bar{q}$ *and the aggregate variables grow at a constant rate* $\mathbf{g}$*.*

Given the invariant distribution of normalized quality levels and the stationary R&D decisions, I can now derive the constant growth rate of the economy in a BGP:[23]

$$\mathbf{g} = \int_{j \in J^*} \left(1 - \boldsymbol{\xi}_j^{exit}\right) \left\{ i_j \lambda + \boldsymbol{\xi}_j^{ipo} (\kappa - 1) \left(\hat{q}_j + i_j \lambda\right) \right\} dj$$

$$- \int_{j \in J^*} \left\{ \boldsymbol{\xi}_j^{exit} + \mathbb{I}_j^{vc} \sigma^{vc} \left(1 - \boldsymbol{\xi}_j^{exit}\right) \right\} \hat{q}_j dj + \int_{j \in J^{entry}} \hat{q}_j dj. \tag{3.12}$$

There are several factors that contribute to the balanced growth rate. The first integral on the right-hand side of equation (3.12) captures the effect of surviving firms. Conditional on remaining in the business, intermediate firm j adds the step size λ if it generates an innovation, which happens at rate i_j. Moreover, if firm j issues public equity in the beginning of the next period, its quality increases by a factor $\kappa - 1$. The second integral captures the loss due to exiting firms. Notice that exit happens due to both the optimal decision of the firm and the attrition rate if the firm is matched with a VC. The last component of equation (3.12) denotes the contribution of entry.[24]

Finally, the following condition holds for the representative household.

Proposition 8 (Euler Equation)**.** *In BGP, the household maximization implies the equilibrium interest rate* $\mathbf{r} = (1 + \mathbf{g})^{\varepsilon} / \beta - 1$.

[23] See Appendix C.1 for the derivation.

[24] Note that the entrant firms do not contribute through IPO because the support of the distribution from which they draw the initial product quality does not extend over values that lead to IPO.

3.3 Estimation

In order to measure the specific effects of different channels through which VC financing affects firm-level innovation and aggregate growth, I estimate the parameters of the model via the simulated method of moments (SMM). In this section, I first describe the identification and computation procedures. Then, I present the estimation results and discuss the goodness of fit. As a brief overview, the model successfully captures the duration of firm-VC matches and the firm age at the time of initial public offering as well as the aggregate patterns of R&D and growth. The model also replicates the difference in firm growth patterns between VC-backed and non-VC-backed firms observed in the data. I start by describing the parameters that are determined outside the model.

3.3.1 Pre-determined Parameters

Because the model at hand is a fairly rich one with a large number of parameters, assigning some of them a priori mitigates the burden of estimation. There are 10 parameters that are chosen externally. The time period in the model corresponds to 1 year in the data. On the household side, the period utility function is assumed to have logarithmic form such that the curvature of the CRRA utility function, ε, equals 1, the midpoint between various estimates surveyed in Mehra and Prescott (1985). The discount rate, β, is picked to imply a reasonable long run interest rate level, given the targeted rate of growth of 2%. Setting $\beta = 0.98$ implies approximately a 4% real interest rate. On the final good production, the share of intermediate goods, α, is set to 0.825. This is in the ballpark of Akcigit et al. (2014), who find a calibrated share of 0.9 for tangible factors of production using US data on firm profitability. Akcigit et al. (2013) also assign a value of 0.85 to physical factors in their final good production function. Without loss of generality, the marginal cost of producing intermediate goods is normalized to $(1 - \alpha)$ for private firms that do not have VC support.[25]

The function $h(\cdot)$ that defines the cost of doing R&D is assumed to have the form $\gamma_0 x^{\gamma_1}$. The curvature parameter, γ_1, is set to 2 so that the function has quadratic shape. This in turn implies that the R&D elasticity in the innovation production function is 0.5, a value in line with the empirical literature.[26] The lowest product development efficiency, θ^L, is normalized to 1.

The parameter that governs the exogenous separation of matches between firms and VC funds, σ^{vc}, is set as follows. In NVCA (2013), the National Venture Capital Association (NVCA) reports that, among VC-backed firms that received their first round of funding between 1991-2000, about 16% made it to the IPO stage. Another 18% are reported to fail. The rest of the matches end in ways that I do not include in my model.[27] The exogenous separation parameter, σ^{vc}, captures the yearly attrition rate due to these external reasons.[28] For the average share sold at IPO, Δ, Ritter (1998) reports a range of 20%-40%. The telegraph matching function introduces a single scale parameter that is normalized to 1. Lastly, the bargaining power of the VC, ϕ, is assumed to be 0.5. Table (8) summarizes the predetermined values.

[25] This normalization simplifies the derivation of the profit function. The corresponding value for VC-backed and public firms, η, is determined in the estimation.

[26] Measuring innovations by patents, the empirical literature on patents and R&D provides estimates for this elasticity. Griliches (1990) gives a range from 0.3 to 0.6 while Blundell et al. (2002) find 0.5.

[27] Among these remaining matches, half of them resulted in acquisition of the private firm by another one. The other half is counted as "still private or not known", and most of them are believed to have failed. Because the success of a VC firm is generally measured by its IPO performance, I focus on IPOs.

[28] The total attrition rate is assumed to be the cumulative hazard rate over 7.5 years. This length of time represents the median tenure of VC investments, which is estimated to vary from 7 to 10 years in the data.

Value	Description	Source
$\beta = 0.98$	Discount Rate	Real Interest Rate
$\varepsilon = 1$	CRRA curvature	Mehra and Prescott (1985)
$\alpha = 0.825$	Share of physical factor	Akcigit et al. (2014)
$\eta^H = 1 - \alpha$	Cost of capital, high	normalized
$\gamma_1 = 2$	R&D cost elasticity	Blundell et al. (2002)
$\theta^L = 1$	Product development efficiency, low	normalized
$\rho = 1$	Scale of telegraph matching	imposed
$\phi = 0.5$	Bargaining power	imposed
$\sigma^{vc} = 8.7\%$	Attrition rate of VC-firm matches	Unsuccessful separations
$\Delta = .28$	Share sold at IPO	Ritter (1998)

Table 8: Parameters Fixed A Priori

3.3.2 Identification of the Estimated Parameters

There are 10 parameters to be estimated. Perhaps the most crucial parameters are $\{\theta^L, \theta^H\}$ because they determine the magnitude of the impact on firm growth of the operational knowledge provided by VCs. Having normalized θ^L to 1, I make use of Puri and Zarutskie (2012) in estimating θ^H. Puri and Zarutskie (2012) make a fundamental contribution to the empirical literature that investigates the effect of venture capital on firm dynamics by employing survey data on firms. In particular, they combine the VentureXpert and Longitudinal Business Database of US Census Bureau so that they are able to determine the firms that received VC financing. Controlling for the number of employees, age, geographical location, and the industry at four-digit SIC level, they create a matched sample of non-VC-backed firms and firms that are at the first round of getting VC funding.[29] The authors observe the firms in these two categories until they exit in

[29] It should be emphasized that Puri and Zarutskie (2012) do not control for the amount of VC investment received, and do not uncover particular mechanisms through which VC affects firm dynamics. One

some form (exit the data, become public, etc.) for a maximum of 10 years. Then, using these samples, they regress the logarithm of firm sales on a number of covariates and, in particular, provide the OLS estimate on the interaction term between a dummy for VC use and the time elapsed after matching.[30] This estimate determines the differential impact of VC financing on firm growth. To determine θ^H, I create analogous samples from the stationary distribution of my model. I simulate firms in these samples for 10 years, and conduct the same regression analysis. The estimation procedure tries to match the model counterpart of the OLS estimate with the one provided Puri and Zarutskie (2012).

The size of VC firms' financial impact is governed by the difference between η and η^H, the marginal costs of production for private firms with and without VC backing, respectively. To discipline this difference, I assume that the decline in the cost of capital due to VC investment reflects all the pecuniary support of VC companies. Then, including the ratio of VC investment to GDP as one of the data moments determines the size of this financial support in my model.[31]

In order to complete the estimation of the VC market, the entry cost for VC firms needs to be determined. The entry cost of the venture capitalist, χ^{vc}, creates a threshold for the intermediate good qualities above which VC firms would not agree to form a match with a firm, because they could not generate a great deal of improvement on already high quality levels due to decreasing returns to innovations. Moreover, this entry cost determines the ex-ante value of a venture capitalist before entering the market. Therefore, once the other parameters that describe the matching function and Nash bargaining are fixed, χ^{vc} is closely correlated with the probability of firms obtaining VC financing. Hence, to discipline χ^{vc}, I include as a target the NVCA (2013) estimate that, roughly, only one out of a hundred applications succeeds in securing VC financing. One

contribution of my paper is to establish this.

[30]The OLS regression results are presented in Appendix C.2.

[31]Venture capital investments do not only include funding of early- and growth-stage companies, but also buyouts, later-stage investments, etc. not relevant to the point of this paper. Therefore, when calculating the ratio of VC investment to GDP, I take into account only the early- and expansionary-stage investments by VC.

caveat: in my model, every meeting in the matching market results in a match. This happens both because there is no search cost for the firms, and because VC firms are identical. Any firm that knows that a match would create a positive surplus goes into the market, and the ones with the expectation of a negative surplus stay out. To map the NVCA statistic to my model, I interpret the 1% success on applications as the chance of meeting a VC company that would accept the firm. When solving the model, I fix the probability of matching with a VC at this level, and solve for the level of entry cost that supports the equilibrium by looping over χ^{vc}.

To complete the cost structure of an IPO, the fixed cost of IPO, χ^{IPO}, needs to be determined. This parameter maps to direct costs of IPO observed in the data, such as registration fee and underwriting costs. The statistics provided by Ritter (1998) indicate that, on average, these costs amount to 11% of the total proceeds raised by IPO. Using this figure, I can directly estimate χ^{IPO}.

The benefits of an IPO are determined by two parameters: κ, the abrupt increase in quality level, and θ^M, the permanent product development efficiency that VC-backed firms retain after becoming public. Determining the gains from IPO, these parameters are crucial for the decision of the optimal time to go public. To pin down κ and θ^M, I therefore use the median age across all private firms at the time of IPO, together with the median length of firm-VC matches that lead to an IPO. Because product development efficiency is assumed to remain fixed for non-VC-funded private firms after going public, κ is the only parameter that determines the gains from going public for these type of firms. Then, the median time to IPO for VC-backed companies is helpful primarily in identifying θ^M. Both κ and θ^M are negatively related to these age moments.

The rest of the parameters are λ, γ_0, χ^o, and χ^e. The first one determines the quality gain due to process innovations and is mostly tied to the average growth, for which the target value is the average US post-war annual growth rate of 2%. The scale parameter of R&D cost function γ_0 is used to match the R&D share of GDP. The outside option for intermediate firms, χ^o, is estimated by targeting a 5.5% equilibrium exit rate. I take this value from Lee and Mukoyama (2012), who calculate estimates using US plant-level data from 1972-1997. I set the entry cost, χ^e, that the potential entrepreneurs face, such that the equilibrium measure of intermediate good firms is equal to unity. As a result, the set of 11 parameters to be estimated within the feasible set Ω is

$$\omega \equiv \left[\eta,\ \gamma_0,\ \chi_{vc},\ \rho,\ \chi_{IPO},\ \kappa,\ \theta^M,\ \theta^H,\ \lambda,\ \chi_e,\ \chi_o\right]^T \in \Omega.$$

3.3.3 Algorithm

The computation of general equilibrium given a parameter set $\omega^{given} \in \Omega$ consists of two nested fixed point problems. The outer loop searches for convergence on the growth rate. Given the growth rate, the inner loop computes the value functions. Computation of the value function for non-VC-backed firms requires another nested fixed point solution in the sense that the equilibrium matching rate and the value functions needs to be solved jointly. At this point, I modify the problem so that I fix the matching rate at the targeted moment, and solve for the corresponding VC entry cost instead. This step requires calculation of the endogenous (normalized) quality distribution across firms. The reason is that, given the fixed matching rate, I use the value of the VC firm to update the guess for χ^{vc}, and the value function of the VC firm depends on the endogenous distribution of firms searching for VC. To yield a smooth distribution, I discretize the possible values of the normalized quality levels into 1200 points for each firm type. Once I obtain the general equilibrium, I simulate samples from the stationary distribution to calculate the moments regarding the age of IPO for private firms, median duration to IPO in firm-VC matches, and the regression statistic that determines χ^{vc}. Given a set of parameters ω^{guess} the solution routine continues as follows:

1. Guess a growth rate $\mathbf{g}^{guess}$.

2. Solve for the value functions of

    (a) Public firms

    (b) VC-backed private firms

3. Solve for the value function of non-VC-backed private firms.

    (a) Guess a candidate entry cost, χ^{guess}, for VC firms.

    (b) Compute the value function of non-VC-backed private firms.

    (c) With all value functions at hand, compute the stationary distribution.

    (d) Compute the implied χ^{new} using the problem of the VC firm. Update until $\|\chi^{guess} - \chi^{new}\| < \epsilon$.

4. Compute the implied $\mathbf{g}^{new}$. Update until $\|\mathbf{g}^{guess} - \mathbf{g}^{new}\| < \epsilon$.

3.3.4 Estimation Results

Parameter Estimates

Table 9 reports the values for the parameter estimates obtained via the computation algorithm introduced above.

A number of parameter estimates in Table 9 merit special attention. The first variable in the table, η, determines the magnitude of the financial help of VC firms. The estimated value implies that VC firms decrease the marginal cost of intermediate good production by 6%. The economic meaning of this estimate is better reflected in the resulting difference in operation profit levels. A back-of-the-envelope calculation shows that the estimated reduction in marginal costs translates into 30% higher operational profits for a VC-backed company compared to a non-VC-backed counterpart with the *same* product quality.

Value	Description	Identification
$\eta = 0.94 \cdot \eta^H$	Cost of capital ratio	VC investment/GDP
$\gamma_0 = 50$	R&D cost scale	R&D investment/GDP
$\chi^{vc} = 0.435$	VC entry cost	Success in due diligence
$\chi^{ipo} = 0.20$	IPO fixed cost	Direct cost of IPO
$\kappa = 1.60$	Quality jump, IPO	Median duration with VC
$\theta^M = 4$	Efficiency, after VC	Median age at IPO
$\theta^H = 5$	Efficiency, with VC	Puri&Zarutskie (2012)
$\lambda = 0.275$	Innovation size	Growth rate
$\chi^e = 2.43{*}10^{-4}$	Entry cost	Fixing measure to unity
$\chi^o = 6.06$	Outside option	Exit rate

Table 9: Estimated Parameters

Two other important parameters are θ^H and θ^M, which, respectively, measure the direct and permanent (post-IPO) efficiency gains in product development due to VC firms' operational knowledge. The former implies that a VC-backed firm is five times more efficient than its non-VC-backed counterpart in improving a certain quality level with innovation intensity. Moreover, the estimate for θ^M implies that the VC-backed firm retains 80% of this efficiency gain after going public. As the counterfactual experiments reveal below, this limited loss of efficiency even after separation from the VC firm has important implications for the effect of VC financing on long-run economic growth.

Goodness of Fit

Table 10 summarizes the moment targets and their counterparts in the model. First of all, the model is successful in matching the aggregate growth rate and the ratio of R&D investment to GDP. Because innovation and aggregate growth are integral parts of the analysis, it is critical that the model reflects these aspects of the data well.

Looking at the data moments that largely define the VC market, the first result is that the model accounts for a fair amount of VC investment in the data. Notice that the implied parameter estimate results in a sizable improvement in operational profits of VC-backed companies, as explained above. Thus, it is fair to conclude that the estimation allows the monetary aspect of VC financing to have a significant impact on firm dynamics. The other channel, operational knowledge, has both a direct and a permanent effect on the firms that receive VC support. The direct effect is disciplined by the regression statistic obtained from the analysis of Puri and Zarutskie (2012), and the model proves to be successful in matching this crucial target.[32] Moreover, the median duration of VC-backed firms until IPO in the model mirrors the data target very closely. Matching this target is important because it disciplines the permanent effect of VC's operational knowledge as well as the IPO cost for VC-backed firms in this regard.

Target	Data	Model
VC investment/GDP	0.17%	0.12%
IPO direct costs	11%	6.26%
Match probability	1%	1%
R&D investment/GDP	2.8%	2.89%
Median duration with VC	5.5*yrs*	6*yrs*
Median age at IPO	12*yrs*	11*yrs*
Regression statistic	0.212	0.242
Growth rate	2.0%	1.95%
Exit rate	5.5%	2.75%

[1] The regression statistic provided by Puri and Zarutskie (2012) is highly significant with a t-statistic 11.23.

Table 10: Model Fit

[32] I discuss the implications for firm growth below.

Non-targeted Moments

Table 11 reports statistics observed in the data and not targeted in the estimation of the model, together with their data counterparts.

	Data	Model
IPO probability of VC-backed firms	16%	20%
VC-backed IPOs	50%	51%
Output share of private firms	46%	48%
Relative firm growth	0.75	0.22

Table 11: Non-targeted Moments

First of all, the model captures the IPO patterns in the data accurately. The model simulations based on samples of 50,000 firms imply that about 16% of VC-firm matches end up with an IPO. This number is the ballpark of the value found in the 2013 report of the National Venture Capital Association (NVCA, 2013). For the private firms without VC support, the corresponding value is 1.7% in the model. This is well below the value for VC-backed companies, a pattern also observed in the data. Similarly, the fraction of IPOs involving VC-backed firms is also closely reflected in the model. The recent IPO report by WilmerHale (2014), a widely recognized law company in the US, documents that in 2013, VC-backed IPOs constituted half of all IPOs, whereas the data statistics in Ritter (2014) indicate that an average of 38% of IPOs were VC-backed between 2006-2013.

A closer look at the firm type composition of the model economy shows that 48% of output is produced by privately held firms. The most recent figures from the U.S. Small Business Administration data similarly show that a little less than 50% of the US GDP is produced by firms with fewer than 500 employees of which almost all are private firms (Kobe, 2012).[33,34] Regarding the growth rates of private firms, the estimated model predicts that the yearly average growth rate of VC-backed firms is 22% higher than the rate of the non-VC-backed sample.[35] The corresponding figure in the data is obtained from Puri and Zarutskie (2012). As explained in detail, Puri and Zarutskie (2012) explore growth rates of different samples of private firms with and without VC backing that are matched based on observable characteristics. They document that, over the first 10 years after the time of matching, the average growth rate of the VC-backed sample is 75% higher. Although at a smaller magnitude, the model captures this pattern qualitatively. This smaller magnitude indicates that the model provides a lower bound for the VC impact observed in the data.[36]

Figure 14 shows the impact of VC on firm distribution over a 10-year period. Following Puri and Zarutskie (2012) I create a model sample of private firms from the stationary distribution that defines new matches with VC firms. The thin solid line shows this initial distribution. I then simulate two versions of this sample across 10 years. In one version, firms are assumed to receive VC financing whereas in the other, firms continue without VCs and are observed until they obtain VC, issue public equity or exit the market. Starting the simulation with the identical group of firms replicates the matching exercise of firms in te data based on their sales, as done by Puri and Zarutskie (2012). The resulting difference between VC-backed and non-VC-backed samples after 10 years is illustrated by the dashed and thick solid lines, respectively. Among the VC-backed firms that remain after 10 years, there is a population of firms that survive with lower sales and profits. No-

[33] It is true that a small fraction of firms in the economy are public, and most of the large firms with more than 500 employees are private. However, given that my focus is on the dynamics of young and innovative firms as opposed to very large private firms, matching the output share of firms with less than 500 employees is a reasonable comparison.

[34] Asker et al. (2014) report that all private firms account for 59% of sales.

[35] In this exercise, firm growth is defined as sales growth, in line with Puri and Zarutskie (2012).

[36] This smaller magnitude can be partially attributed to the exogenous attrition process that hits every VC match with the same probability,i.e., it destroys successful matches at the same rate as it does relatively unsuccessful ones. However, in reality, an important share of the exits that the attrition rate accounts for in the model are unsuccessful firms. Therefore, the figure of 22% generated by the model can be considered as an attenuated value for the growth rate differential between VC-backed and non-VC-backed samples.

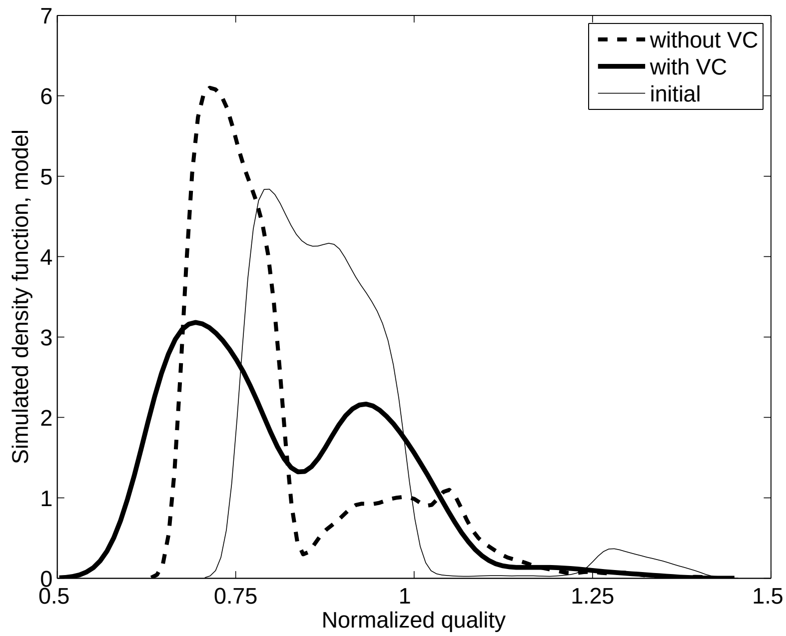


Figure 14: Evolution of Matched Samples in the Model

tice that, in the model, the value of the outside option, $\chi^o \bar{q}_t$, is the same for any firm type. This shows that financial support from VCs through lower intermediate production costs helps some firms with a lower productive capacity remain in the economy. However, as the last row of Table 11 shows, the yearly average growth rate of the VC-backed sample is 22% higher in the model. This impact is reflected by the fatter right tail of the resulting distribution of the VC-backed sample. VC firms' operational knowledge enables a larger subgroup of firms to achieve higher levels of production compared to the non-VC-backed sample. This outcome is in line with the reality that many portfolio companies of VC funds are relatively unsuccessful, while a few perform exceptionally.

Regarding VC impact on firm growth, one caveat is worth mentioning. Despite the fact that Puri and Zarutskie (2012) controlled for observable characteristics when creating matched samples, this procedure did not account for a possible selection of firms by VC companies according to unobservable features. Suppose that there was sorting of firms that are superior on some unobservable quality towards VC investment. If the matching procedure does not account for this type of sorting, and if that affects firm growth positively, then this would inflate the apparent effect of VC investment through operational knowledge in my model, since the contribution of this unobserved quality would be inaccurately assigned to that channel. It is fundamentally important to notice that this *would not* bias my estimation because it proceeds on the method of indirect inference, replicating the same empirical experiment as in Puri and Zarutskie (2012). Nonetheless, when interpreting the impact of operational knowledge in both the model and in the data, the potential effect of selection on unobservables can be included, using the findings of Sørensen (2007). Using data on IPO rates of VC-backed companies, Sørensen (2007) shows that the portfolio companies of more experienced VCs are more likely to go public. Then, he structurally estimates a two-sided matching model to find that sorting, defined as the fact that more experienced VCs invest in better firms, accounts for 50% to 60% of the higher IPO rate in companies backed by more experienced VCs. In other words, the direct influence of VC on the firm is 40-50%. This estimate, however, reflects the differential effect of VCs' expertise *only* across VC-backed firms, and does not account for its significance in comparison to firms that completely lack VC backing. This means that it attenuates the relevance of VCs' direct influence on firms. Nevertheless, if a conservative path were followed based on Sørensen (2007), the estimate of 22% that my model implies for VCs' impact on firm growth would still translate into 10%.

3.4 Quantitative Exploration

Having estimated the parameters of the model and analyzed the model fit, I use this framework for two purposes. First, I measure the significance of VCs' operational knowledge relative to the financing channel, in terms of the aggregate growth of the economy. To do so, I run counterfactual experiments in which I marginally increase the parameters that govern financing and the operational knowledge channels. I then compare the resulting changes in the growth rates of the economy. Next, I replicate a recent policy measure that the European Union has adopted to make the investment environment more hospitable for venture capital firms. In the model, I capture the essence of the policy by decreasing the entry cost of VC firms, and explore the impact on long-run economic growth.

3.4.1 Counterfactual Analysis: Strength of Operational Knowledge

To measure the relative impact of the operational knowledge channel in aggregate growth terms, I first consider a hypothetical economy in which I increase the parameters $\{\theta^M, \theta^H\}$ by 5% without changing θ^L.[37] I then run a similar experiment where I increase the size of the marginal cost reduction due to VC help by the same amount, keeping the other parameters at the estimated levels. These experiments allow me to compare the elasticity of the growth rate to the distinct channels through which VC firms affect firm dynamics.

	Benchmark	$\{\theta^M, \theta^H\}$ 5% higher	$(\eta^H - \eta)$ 5% higher	No VC	IPO fixed cost 15% higher	Subsidy: VC entry cost 3.5% lower
Growth	1.95%	2.01%	2.06%	1.39%	1.93%	2.02%
Measure of firms	1	1	1.16	0.99	1	1.01

Table 12: Counterfactual Experiments

The first three columns of Table 12 summarize the response of the economy to marginal changes in different aspects of VC support in comparison to the estimated economy. The table also reports the equilibrium measure of intermediate firms because the changes also affect the endogenously determined size of the economy. Comparing the growth rates in the second and third columns shows that the marginal increase in the strength of the operational channel leads to a 0.06% gain in the growth rate, whereas this

[37]One alternative approach to measure the impact of operational knowledge is to consider its absence by removing the increase in product development efficiency due to VC support. The reason I do not use this approach is that in this case no private firm searches for VC. Nevertheless, this endogenous shutting down of the market happens even when there are small efficiency gains from VC help. In that case, removing the efficiency gain completely does not measure the exact impact of operational knowledge on aggregate growth.

number is around 11% when the change in the financial channel is considered. Therefore, the main message of this comparison is that, in terms of long-run growth, the operational channel is about half as effective as the financial channel. In other words, the influence of the operational knowledge channel on growth through its impact on firms accounts for about 1/3 of VCs' total contribution to aggregate growth through its impact on firms. The increase in financial impact also expands the equilibrium measure of products by 16%, in addition to its effect on long-run growth.

A deeper look into the hypothetical model economies reveals that, in the case of stronger financial support, most of the impact is generated through the changing composition of private firms. In the hypothetical world with increased financial impact, the ratio of output produced by VC-backed firms to the amount produced by all private firms is about 50%, whereas this number is about 4% in the estimated actual economy.[38] One reason for this result is that, due to the higher aggregate growth rate, the fixed cost of IPO, which is proportional to aggregate productivity, increases faster. This leads to longer durations of VC matches before VC-backed firms go public.[39] In the case of stronger operational knowledge influence, however, these fluctuations are much more limited. Instead, the impact on long-run growth of the economy stems from the increased efficiency of development, both for the VC-backed firms and for the public firms that received VC support.

Lastly, the fourth column in Table 12 implies that, in a hypothetical world without a market for venture capital, the growth rate would go down to 1.40%. Here, I assume that all firms operate with low efficiency, and there is no means to affect it. This lower efficiency, in turn, leads to a drastic fall in the growth rate. Regarding this experiment, one caveat is that all firms operate at the higher marginal cost of production because the

[38]Notice that the 5% additional reduction in marginal cost translates into more than 25% additional profits per unit of product quality. The huge responses in the hypothetical economy stem from this fact. Thus, it is plausible to think that these counterfactual comparisons provide lower bounds for the relative importance of the operational knowledge channel.

[39]The resulting median age is around 9 years, whereas in the benchmark economy this was 6 years.

only financial intermediary in the model is removed. This has an indirect growth impact because higher profits due to VCs' financial support create an indirect incentive for innovation due to a larger return per unit of product quality. The drastic fall in the aggregate growth rate would potentially be smaller if there were an alternative intermediary with a similar financial impact. Therefore, this result should be interpreted cautiously.

3.4.2 Counterfactual Analysis: Higher Cost of IPO

A widely held belief is that there are strong complementarities between VC finance and an active public equity market (Black and Gilson, 1998; Michelacci and Suarez, 2004).[40] The intuition is the following: on the one hand, VC firms accelerate new ventures towards issuing IPO, through the aforementioned influences. On the other hand, liquid stock markets provide an attractive and affordable IPO option for private firms, and a profitable way for VC firms to separate from portfolio companies.

To analyze the linkages between public equity issuance and VC financing in my model, I now consider an economy in which the fixed cost of IPO is 15% higher. I obtain this value from Kim et al. (2003). In their study of equity and debt issues from 1970s to 2000s, Kim et al. (2003) document that average underwriting spreads for IPOs in the US were 8.5% and 7.4% over the periods 1976-1985 and 1996-2005, respectively.[41] Having used data from the latter period in my estimation, I now analyze the counterfactual setting where I set the fixed cost of IPO to its earlier value. As shown in the fifth column of Table 12, the growth rate falls by about 1.5 basis points.

Figure 15 shows the resulting changes in the IPO decisions. The increase in the fixed cost affects the IPO threshold on the firm size negligibly for VC-backed firms whereas the threshold for non-VC-backed firms rises discernibly. The difference stems from the

[40] Bottazzi and Rin (2002) report supporting evidence on the positive impact of Euro.nm, the European counterpart of Nasdaq, on European VC activity in 1990s. Euro.nm, opened in 1997, is the alliance of new European stock markets that focuses on growth companies.

[41] The underwriting costs are one of the main determinants of the fixed costs associated with IPOs.

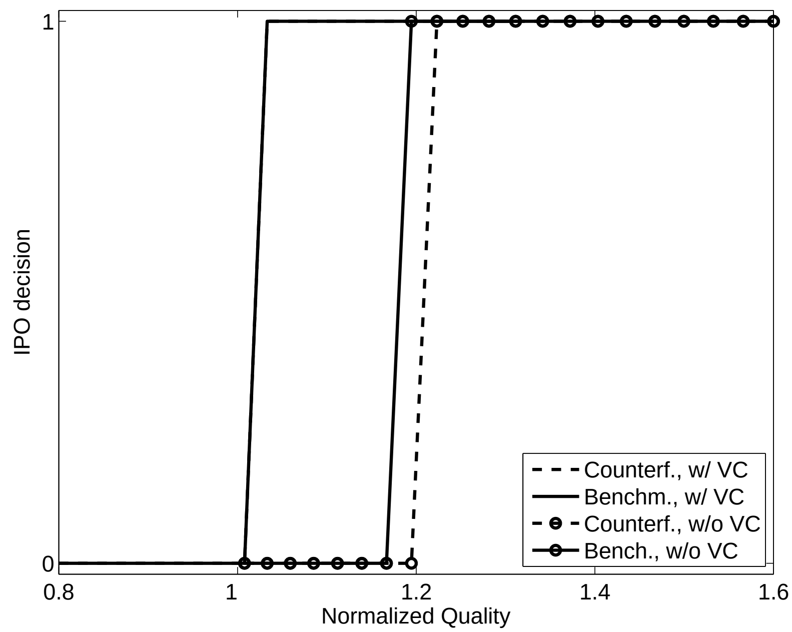


Figure 15: IPO decisions

higher profit levels that VC-backed firms generate with the same quality level due to VCs' financing support. The higher threshold in turn implies a 3% fall in the non-VC-backed equity issuances. Therefore, it expands the group of private firms available to match with a VC. Together with the VC firms' willingness to search longer due to the higher equilibrium discount factor, this change means a 10% lower probability that a private firm will match with a VC..[42] In turn, the share of VC-backed firms in the economy decreases by 11%, which also lowers the share of public firms that had received VC support when they were private. In combination, all these responses result in a 1.5 basis point loss in the long-run growth rate of the economy.

3.4.3 Policy Analysis: VC Entry Cost

In 2013, the European Union adopted a new regulation on venture capital funds to enhance funding to small and medium businesses through venture capital financing.[43]

[42] A lower growth rate implies a lower interest rate; therefore, VC firms discount the future value less.

[43] The legislative act by the European Commission (2013) explicitly recognizes that "venture capital funds provide undertakings with valuable expertise and knowledge, business contacts, brand equity and strategic

As a main obstacle to adequate VC funding, the European Commission recognized the lack of a harmonized VC market across the Union. According to the Commission, the fragmented structure of the VC market across national borders increases VC firms' costs due to changing national regulatory environments, especially regarding the raising of capital. To help VC firms expand their operations by easing fundraising, the EU passed a new regulation that introduces a designation called the "European Venture Capital Fund". VC managers whose funds meet certain requirements, such as high concentration on investment in young and innovative companies, can raise capital under this rubric and be subject to a single rulebook across all EU countries.[44]

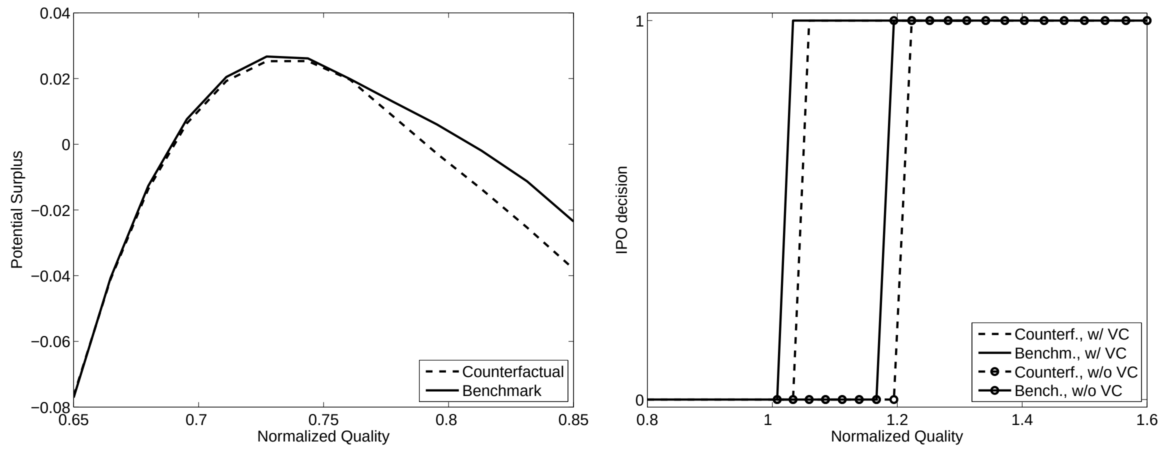
To analyze the potential effects of this regulation, I interpret the lower fundraising cost for VC firms through the lens of my model as lower entry costs. In the experiment, I assume that 3.5% of the entry cost is subsidized through a lump-sum tax on the representative household.[45] As the last column of Table 12 shows, this subsidy increases the long-run growth rate by around 7 basis points. A back-of-the-envelope calculation illustrates that, in equilibrium, the cost of this policy is 0.09‰ of output. Correspondingly, the subsidized amount is about 8% of the total investment made by VCs into portfolio companies in the benchmark economy.

The details of how the economy responds to the subsidy show similarities to the counterfactual experiment in which the financial support of VC was expanded. Through the general equilibrium effects, there is a reallocation of private firms towards being VC-backed companies, and these stay with the VC firm for a greater period of time. This longer match duration amplifies VCs' effect on the aggregate growth rate through the influence of superior operational knowledge. This happens due to two factors. First, as illustrated in Figure 17b for both VC-backed and non-VC-baked firms, there is an

advice".

[44]The law requires qualifying funds to channel at least 70% of their capital to small and medium enterprises (SME).

[45]This value has been picked such that, in the new equilibrium, a reasonably low share of output is used to finance the subsidy.



(a) Match surplus

(b) IPO decisions

Figure 16: Response to Lower VC Entry Cost

increase in the (normalized) quality thresholds above which the two types private firms issue IPO. This shift emerges, again, due to a combination of the proportionality of the fixed IPO cost to average productivity, and the higher resulting equilibrium aggregate growth rate. As a result, firms need to develop their product quality further to afford IPO issuance costs. The second factor for increased match durations is that VC firms match with companies of smaller quality levels, which have larger potential for growth. For a given IPO threshold, this implies that, on average VC-backed firms have to innovate more to reach the IPO stage. Figure 17a delineates this point. The curves show the present discounted value of the surplus of a potential match between a VC firm and private firms with quality $\hat{q}_j$. In this economy with subsidies, this curve shifts towards the left so that a positive surplus, and thus a profitable match, is possible with firms of smaller size. These combined changes result in a 7 basis point higher long-run growth rate of the economy.[46]

[46] In this economy, all private firms available for VC matches have a certain identical efficiency level. A heterogeneity in this margin could dampen the effect of the policy change because some of the new VC firms had to meet with firms that already have higher efficiency. A parallel impact could arise if there were heterogeneity across VC firms in their potential to affect product development efficiency (Hsu, 2004; Bottazzi et al., 2008). A similar concept of heterogeneous firm entry and its aggregate productivity implications is investigated by Ateş and Saffie (2014). Incorporating these margins of heterogeneity by deriving the relevant empirical distributions in the data and deploying them in the estimation procedure is an attractive area for future research.

3.5 Conclusion

Motivated by the disproportionate investment of venture capital finance in young and innovative businesses, I study in this paper the quantitative impact of VC financing on firm dynamics and economic growth. I propose a new dynamic general equilibrium model of innovation with heterogeneous firms by introducing an explicit venture capital (VC) market. The model allows me to conduct counterfactual experiments which I use to quantify the impact of VC financing and examine relevant policies. I pay particular attention to a unique feature of VC firms that is largely overlooked by current macroeconomic analysis: the operational knowledge that VC firms bundle with their cash investment. In the model, technologically heterogeneous firms engage in innovative activities to improve their product quality and increase their profits. The efficiency of this product development process can be enhanced through the operational knowledge of VC firms. The model also includes an endogenous search and matching setting where VC companies and firms meet. In this way, the model accounts for the selection aspect of the VC market in addition to the cash investment and the operational knowledge. This is crucial to capture general equilibrium effects.

I structurally estimate this model using US data on VC finance, public equity issuances, and research and development expenditures. I identify the operational knowledge channel through its distinct impact on firm growth. Out-of-sample tests demonstrate that the estimated model successfully captures the non-targeted data moments such as IPO frequency of VC-backed firms, and the differences in growth rates of VC-backed and non-VC-backed firms, among others.

I then use the estimated model to conduct counterfactual and policy analyses. First, I measure the impact of the operational knowledge channel in terms of aggregate economic growth. The analysis indicates that a sizeable fraction, 1/3, of VCs' impact on economic growth is generated through operational knowledge channel. This result implies that, for financing innovation, VC has significant value beyond capital investment alone. Next, I evaluate a recent policy adopted by the European Union. The regulation aims to decrease fundraising costs for VC firms and expand VC investment across borders by harmonizing the relevant regulatory environment throughout the Union. I examine effects of a subsidy on VC entry cost that simulates the policy and find that this change can generate a 7 basis point gain in the long-run growth rate of the economy.

This paper provides fruitful ground for several directions of future research. One immediate step could be to explore the implications of heterogeneity across VC firms and the sensitivity of VC impact on firm growth to this aspect. A broader research question would be how the VC market arises endogenously. The optimal provision of operational expertise by VC requires managers who possess both sufficient operational knowledge and financial wealth. Explaining the reasons why and how venture capitalists emerge could help us understand the vast differences in the size of VC markets across different regions, such as the US and continental Europe.

Appendix A

Appendix to Chapter 1

A.1 Proposition 1

Proof. First note that, for any $\bar{\theta} \in [0,1]$, the probability of a randomly drawn project $e \in [0,1]$ having a probability $\theta(e) \leq \bar{\theta}$ is given by:

$$F(\bar{\theta}) = \left(\bar{\theta}\right)^{\frac{1}{\nu}}$$

Then, $F(\theta)$ is the cumulative density function of θ, and we can use it to find its probability density function:

$$f(\theta) = \frac{\partial F(\theta)}{\partial \theta} = \frac{1}{\nu} \left(\theta\right)^{\frac{1}{\nu}-1}$$

More algebra delivers:

$$
\begin{aligned}
E\left[\theta\right] &= \int_0^1 \frac{\theta}{\nu}\left(\theta\right)^{\frac{1}{\nu}-1} d\theta = \frac{1}{\nu+1} \\
V\left[\theta\right] &= E\left[\left(\theta - E\left[\theta\right]\right)^2\right] = \frac{\nu^2}{\left(\nu+1\right)^2\left(2\nu+1\right)} \\
S\left[\theta\right] &= \frac{E\left[\left(\theta - E\left[\theta\right]\right)^3\right]}{\left(E\left[\left(\theta - E\left[\theta\right]\right)^2\right]\right)^{\frac{3}{2}}} = \frac{2(\nu-1)\sqrt{1+2\nu}}{1+3\nu}
\end{aligned}
$$

□

Note that $\nu = 1$ corresponds to a uniform distribution. For $\nu \geq 1$ this distribution resembles a Truncated Pareto distribution, but it behaves better on the neighborhood of 0.

A.2 Proposition 2

Proof. Denote by $P(H|\tilde{e})$ the expected probability of a project generating a drastic innovation conditional on delivering a signal $\tilde{e}$. Then:

$$P(H|\tilde{e}) = \rho\tilde{e}^{\nu} + (1-\rho)\frac{1}{\nu+1}$$

$P(H|\tilde{e})$ is increasing in the signal $\tilde{e}$. Then if $V_t^H > V_t^L$, the expected benefits of enacting a project is also increasing in $\tilde{e}$. As the cost of enacting a project is independent of the signal, the optimal strategy is to pick the desired mass M of projects with the highest signal. Finally, in order to get a mass M, the cut-off $\bar{e}$ must satisfy:

$$\int_0^{\bar{e}} \left(1-\rho\right)\left(1-\bar{e}\right) de + \int_{\bar{e}}^1 \left\{\left(1-\rho\right)\left(1-\bar{e}\right) + \rho\right\} de = M \Leftrightarrow \bar{e} = 1 - M$$

□

A.3 Proposition 4

Proof. We start solving for the profits of the intermediate good sector. Given (1.7), (1.9), and (1.16) the profits of a type d firm are given by

$$\pi_{j,t}^{d} = l_{j,t}^{d} q_{j,t} \left(\frac{w_t}{\tilde{q}_{j,t}} - \frac{w_t}{q_{j,t}} \right) = l_{j,t}^{d} w_t \sigma^{d} = \frac{\sigma^{d}}{(1+\sigma^{d})} Y_t. \tag{A.1}$$

Thus, $\forall j \in [0,1]$, $\pi_{j,t}^{d} = \pi_t^{d}$. Then, by (1.10), we have $\forall j \in [0,1]$, $V_{j,t}^{d} = V_t^{d}$. Also, as $\sigma^{H} > \sigma^{L}$, we have $\pi_t^{H} > \pi_t^{L}$, and then $V_t^{H} > V_t^{L}$. This rationalizes the equilibrium cut-off strategy of the financial intermediary. Moreover, σ^{d} determines the constant markup of type d leader in any product line.

The last part of equation (A.1) reveals that $l_{j,t}^{d} = l_t^{d}$ for all industries. Using (1.14) and (A.1) we can find the expression for the labor demand that only depends on the type of the leader, d:

$$l_t^{L} = \frac{(1+\sigma^{H})\left[L-(1-\bar{e}_t)\kappa\right]}{1+\sigma^{H}-\mu_t^{H}(\sigma^{H}-\sigma^{L})}; \qquad l_t^{H} = \frac{(1+\sigma^{L})\left[L-(1-\bar{e}_t)\kappa\right]}{1+\sigma^{H}-\mu_t^{H}(\sigma^{H}-\sigma^{L})}. \tag{A.2}$$

Note that $l_t^{L} > l_t^{H}$. □

A.4 Dynamic System

From (A.1) and (A.2) we get the following expression for wages:

$$w_t = \frac{\left[1+\sigma^{H}-\mu_t^{H}(\sigma^{H}-\sigma^{L})\right]}{(1+\sigma^{L})(1+\sigma^{H})\left[L-(1-\bar{e}_t)\kappa\right]} Y_t. \tag{A.3}$$

Now, we are able to characterize the output growth in the model:

$$(1+g_t) \quad = \quad \frac{Y_{t+1}}{Y_t} = e^{\left(\int_0^1 \left\{ \ln \frac{l_{j,t+1}}{l_{j,t}} \right\} dj + \int_0^1 \left\{ \ln \frac{q_{j,t+1}}{q_{j,t}} \right\} dj \right)}. \tag{A.4}$$

Recall that $Q_t \equiv \exp(\int_0^1 \ln q_{j,t} dj)$. Then:

$$\begin{aligned}
\ln(Q_{t+1}) &= \lambda M_{t+1} \left\{ \tilde{\mu}^H_{t+1} \int \ln[q_{jt}(1+\sigma^H)]\, dj + \left(1 - \tilde{\mu}^H_{t+1}\right) \int \ln[q_{jt}(1+\sigma^L)]\, dj \right\} + \\
& \quad (1 - \lambda M_{t+1}) \int \ln q_{jt}\, dj \\
\Rightarrow \quad & \ln\left(\frac{Q_{t+1}}{Q_t}\right) = \lambda M_{t+1} \left\{ \tilde{\mu}^H_{t+1} \ln(1+\sigma^H) + \left(1 - \tilde{\mu}^H_{t+1}\right) \ln(1+\sigma^L) \right\} \qquad \text{(A.5)}
\end{aligned}$$

We also have:

$$\int_0^1 \ln\left(l_{j,t}\right) dj = \mu^H_t \ln\left(l^H_t\right) + (1 - \mu^H_t) \ln\left(l^L_t\right) \qquad \text{(A.6)}$$

Using (A.5) and (A.6) on (A.4) we get:

$$(1 + g_{t+1}) = \left(\frac{(l^H_{t+1})^{\mu^H_{t+1}} (l^L_{t+1})^{1-\mu^H_{t+1}}}{(l^H_t)^{\mu^H_t} (l^L_t)^{1-\mu^H_t}} \right) \left(\left[(1+\sigma^H)^{\tilde{\mu}^H_{t+1}} (1+\sigma^L)^{1-\tilde{\mu}^H_{t+1}} \right]^{\lambda(1-\bar{e}_{t+1})} \right) \qquad \text{(A.7)}$$

Finally, combining equations (1.4) and 1.17 we get the following quilibrium relationship between output growth and interest rate:

$$\frac{(1 + g_{t+1})^\gamma}{\beta} = 1 + r_{t+1} \qquad \text{(A.8)}$$

The following nine equation dynamic system fully characterizes the equilibrium of this economy. The system is written in its stationary form.

$$
\begin{aligned}
1+r_{t+1} &= \frac{(1+g_{t+1})^{\gamma}}{\beta} && \text{(A.9)}\\
\mu_t^H &= \mu_{t-1}^H + \lambda(1-\bar{e}_t)\left[\frac{1}{\nu+1}\left(1-\rho+\frac{\rho}{1-\bar{e}_t}\left(1-\bar{e}_t^{\nu+1}\right)\right)-\mu_{t-1}^H\right] && \text{(A.10)}\\
l_t^H &= \frac{(1+\sigma^L)(L-(1-\bar{e}_t)\kappa)}{1+\sigma^H-\mu_t^H(\sigma^H-\sigma^L)} && \text{(A.11)}\\
l_t^L &= \frac{(1+\sigma^H)(L-(1-\bar{e}_t)\kappa)}{1+\sigma^H-\mu_t^H(\sigma^H-\sigma^L)} && \text{(A.12)}\\
1+g_{t+1} &= \left[\left(1+\sigma^H\right)^{\mu_{t+1}^H-\mu_t^H(1-\lambda(1-\bar{e}_{t+1}))}\left(1+\sigma^L\right)^{\lambda(1-\bar{e}_{t+1})-\left(\mu_{t+1}^H-\mu_t^H(1-\lambda(1-\bar{e}_{t+1}))\right)}\right]\\
&\quad \left[\frac{(l_{t+1}^H)^{\mu_{t+1}^H}(l_{t+1}^L)^{1-\mu_{t+1}^H}}{(l_t^H)^{\mu_t^H}(l_t^L)^{1-\mu_t^H}}\right] && \text{(A.13)}\\
\frac{w_t}{Y_t} &= \frac{(1+\sigma^H-\mu_t^H(\sigma^H-\sigma^L))}{(1+\sigma^L)(1+\sigma^H)(L-(1-\bar{e}_t)\kappa)} && \text{(A.14)}\\
\frac{V_t^H}{Y_t} &= \frac{(1-\tau)\sigma^H}{1+\sigma^H}+\frac{1-\lambda(1-\bar{e}_{t+1})}{1+r_{t+1}}(1+g_{t+1})\left(\frac{V_{t+1}^H}{Y_{t+1}}\right) && \text{(A.15)}\\
\frac{V_t^L}{Y_t} &= \frac{(1-\tau)\sigma^L}{1+\sigma^L}+\frac{1-\lambda(1-\bar{e}_{t+1})}{1+r_{t+1}}(1+g_{t+1})\left(\frac{V_{t+1}^L}{Y_{t+1}}\right) && \text{(A.16)}\\
\bar{e}_t &= \left[\frac{\frac{\kappa}{\lambda}\frac{w_t}{Y_t}(1+r_t)-\frac{V_t^L}{Y_t}}{\rho\left(\frac{V_t^H}{Y_t}-\frac{V_t^L}{Y_t}\right)}-\frac{1-\rho}{\rho(\nu+1)}\right]^{\frac{1}{\nu}} && \text{(A.17)}
\end{aligned}
$$

Note that, since the model has no capital, the composition μ_t^H drives all the dynamics.

A.5 Lemma 1

Proof. First we characterize the system of two equations that defines an interior BGP.

A.5.1 The System on BGP

Note that, (A.8) implies that the interest rate is constant along the BGP. Then, as $\gamma \geq 1$, we can collapse (1.10) using (A.1) and (A.8):

$$V_t^d = \frac{(1-\tau)\sigma^d}{\beta\left[\left(\lambda(1-\bar{e}_t)-1\right)(1+g)^{1-\gamma}+\frac{1}{\beta}\right](1+\sigma^d)} Y_t. \tag{A.18}$$

In an interior BGP (1.13) must hold, so, using (A.3) and (A.18), we obtain the following relationship:

$$\begin{aligned} \rho\bar{e}_t^{\nu} \quad = \quad & \frac{(1+g)^{\gamma}\left[1+\sigma^H-\Delta\tilde{\mu}^H\right]\left[\left(1-\bar{e}-\frac{1}{\lambda}\right)(1+g)^{1-\gamma}+\frac{1}{\lambda\beta}\right]}{\Gamma_0\left[\frac{L}{\kappa}-(1-\bar{e})\right]} - \\ & \frac{(1+\sigma^H)(1-\tau)\sigma^L}{\Gamma_0} - \frac{1-\rho}{(\nu+1)} \end{aligned} \tag{A.19}$$

where $\Gamma_0 = (1-\tau)\Delta$ and $\Delta = \sigma^H - \sigma^L$. The last formula proves that indeed, $\bar{e}_t$ is constant on BGP, and so is $\tilde{\mu}_t^H$, hence, $\tilde{\mu}^H = \mu^H$. Then, from (A.2), it follows that l_t^d is also constant. Hence, (A.7) becomes

$$1+g = \left[\left(1+\sigma^H\right)^{\mu^H}\left(1+\sigma^L\right)^{1-\mu^H}\right]^{\lambda(1-\bar{e})}. \tag{A.20}$$

Then, the system is characterized by (A.19), (A.20), and

$$\mu^H(\bar{e}) \quad = \quad \frac{1}{\nu+1}\left[1-\rho+\frac{\rho}{1-\bar{e}}\left(1-\bar{e}^{\nu+1}\right)\right].$$

Now we find sufficient conditions for existence and uniqueness of a solution to that system.

A.5.2 Existence and Uniqueness

Preliminary Derivations

$$\begin{aligned}
\frac{\partial[1+g(\bar{e})]}{\partial\bar{e}} &= \lambda[1+g(\bar{e})]\times \\
&\left[\left[\ln(1+\sigma^H)-\ln(1+\sigma^L)\right]\left[(1-\bar{e})\frac{\partial\mu^H(\bar{e})}{\partial\bar{e}}-\mu^H(\bar{e})\right]-\ln(1+\sigma^L)\right] \\
\frac{\partial\mu^H(\bar{e})}{\partial\bar{e}} &= \frac{\rho}{\nu+1}\left[\frac{1-\bar{e}^{\nu+1}-(\nu+1)(1-\bar{e})\bar{e}^{\nu}}{(1-\bar{e})^2}\right]>0.
\end{aligned}$$

This implies:

$$\frac{\partial[1+g(\bar{e})]}{\partial\bar{e}}=-\lambda[1+g(\bar{e})]\left[\left[\ln(1+\sigma^H)-\ln(1+\sigma^L)\right]\left(\rho\bar{e}^{\nu}+\frac{1-\rho}{\nu+1}\right)+\ln(1+\sigma^L)\right]<0.$$

Uniqueness

Define the following function of $\bar{e}$:

$$A(\bar{e})=\frac{(1+g)^{\gamma}\left[1+\sigma^H-\Delta\mu^H\right]\left[\left(1-\bar{e}-\frac{1}{\lambda}\right)(1+g)^{1-\gamma}+\frac{1}{\lambda\beta}\right]}{\left[\frac{L}{\kappa}-(1-\bar{e})\right]}$$

Then we can rewrite (A.19) as:

$$\rho\bar{e}^{\nu}=\frac{1}{\Gamma_0}\left(A(\bar{e})-(1+\sigma^H)(1-\tau)\sigma^L\right)-\frac{1-\rho}{(\nu+1)} \tag{A.21}$$

Note that, the left hand side of (A.21) is increasing in $\bar{e}$. Then, if the right hand side of (A.21) is decreasing in $\bar{e}$ any interior solution must be unique. The right hand side of (A.21) is decreasing if and only if $A(\bar{e})$ is decreasing.

Note that, as $\gamma\geq 1$ and as equation (A.8), we have $\forall e\in[0,1]$ all the multiplicative terms are positive. So, we can study the derivative of $\ln(A(\bar{e}))$:

$$
\begin{aligned}
\ln(A(\bar{e})) &= \gamma \ln[1+g(\bar{e})] + \ln[1+\sigma^H - \Delta\mu^H(\bar{e})] + \\
&\ln\left[(1-\bar{e}-\frac{1}{\lambda})\,(1+g)^{1-\gamma} + \frac{1}{\lambda\beta}\right] - \ln\left[L-(1-\bar{e})\kappa\right]
\end{aligned}
$$

Differentiating we get:

$$
\begin{aligned}
\frac{\partial \ln(A(\bar{e}))}{\partial \bar{e}} &= \gamma\frac{\partial \ln[1+g(\bar{e})]}{\partial \bar{e}} - \frac{\frac{\partial \mu^H(\bar{e})}{\partial \bar{e}}\Delta}{1+\sigma^H-\mu^H(\bar{e})(\sigma^H-\sigma^L)} \\
&- \frac{(1+g)^{1-\gamma}-(1-\bar{e}-\frac{1}{\lambda})(1-\gamma)(1+g)^{-\gamma}\frac{\partial(1+g(\bar{e}))}{\partial \bar{e}}}{(1-\bar{e}-\frac{1}{\lambda})\,(1+g)+\frac{1}{\lambda\beta}} - \frac{\kappa}{L-(1-\bar{e})\kappa}
\end{aligned}
$$

As $0 \leq \lambda \leq 1$ and $\gamma \geq 1$ we have $\frac{\partial \ln(A(\bar{e}))}{\partial \bar{e}} < 0$. Then if the system composed by (A.19) and (A.20) has an interior solution, it is unique.

Existence

Now we need to find sufficient conditions for the existence of $\bar{e} \in [0,1]$ that solves (A.21). Note that (A.21) is continuous in $\bar{e}$, then if the right hand side of (A.19) is smaller than ρ when $\bar{e} \to 1$, and positive at $\bar{e} = 0$, the existence of an interior solution is guaranteed.

The first condition will hold if:

$$
\rho \quad > \quad -\frac{1-\rho}{(\nu+1)} + \frac{1}{\Gamma_0}\left[A(1) - (1+\sigma^H)(1-\tau)\sigma^L\right]
$$

Note that, $\lim_{\bar{e}\to 1} \mu^H(\bar{e}) = \bar{\mu}^H = \frac{1+\nu\rho}{\nu+1}$, and $g(1) = 0$. Then:

$$
A(1) = \left[1+\sigma^H - \frac{1+\nu\rho}{\nu+1}\Delta\right]\left[\frac{1-\beta}{\lambda\beta}\right]\frac{\kappa}{L}
$$

We can then find the following condition on $\frac{\kappa}{L}$, the percentage of the labor force needed to enact all the projects of the economy:

$$b = \frac{\lambda\beta}{1-\beta}\left[\frac{\Gamma_0\left(\rho + \frac{1-\rho}{(\nu+1)}\right) + (1+\sigma^H)(1-\tau)\sigma^L}{1+\sigma^H - \frac{(1+\nu\rho)\Delta}{\nu+1}}\right] \quad > \quad \frac{\kappa}{L}$$

Let's study now the case where $\bar{e} = 0$. We need:

$$\frac{1-\rho}{(\nu+1)}\Gamma_0 \quad < \quad A(0) - (1+\sigma^H)(1-\tau)\sigma^L$$

Note that, $\mu^H(0) = \underline{\mu}^H = \frac{1}{\nu+1}$, and $1 + g(0) = \left[(1+\sigma^H)^{\underline{\mu}^H}\left(1+\sigma^L\right)^{1-\underline{\mu}^H}\right]^{\lambda}$. Then:

$$A(0) = \frac{\left[1+\sigma^H - \frac{\Delta}{1+\nu}\right]\left[\left(1-\frac{1}{\lambda}\right)(1+g(0)) + \frac{(1+g(0))^\gamma}{\lambda\beta}\right]}{\left[\frac{L}{\kappa}-1\right]}$$

We can then find the following condition on $\frac{\kappa}{L}$:

$$a = \frac{\kappa}{L} > \frac{\frac{1-\rho}{(\nu+1)}\Gamma_0 + (1+\sigma^H)(1-\tau)\sigma^L}{\left[1+\sigma^H - \frac{\Delta}{1+\nu}\right]\left[\left(1-\frac{1}{\lambda}\right)(1+g(0)) + \frac{(1+g(0))^\gamma}{\lambda\beta}\right] + \frac{1-\rho}{(\nu+1)}\Gamma_0 + (1+\sigma^H)(1-\tau)\sigma^L}.$$

Then $\forall \frac{\kappa}{L} \in [a, b]$ we have existence and uniqueness of an interior solution. Finally, after solving for $\{e, g\}$ in equations (A.19) and (A.20), all the other variables can be recovered.

A.5.3 Recovering all Variables

$$
\begin{aligned}
\left(\mu_t^H\right)_{bgp} &= \mu^H = \frac{1}{\nu+1}\left[1-\rho+\frac{\rho}{1-\bar{e}}\left(1-\bar{e}^{\nu+1}\right)\right] \\
(r_{t+1})_{bgp} &= r = \frac{(1+g)^\gamma}{\beta}-1 \\
\left(l_t^H\right)_{bgp} &= l^H = \frac{(1+\sigma^L)\left[L-(1-\bar{e})\kappa\right]}{1+\sigma^H-\mu_t^H(\sigma^H-\sigma^L)} \\
\left(l_t^L\right)_{bgp} &= l^L = \frac{(1+\sigma^H)\left[L-(1-\bar{e})\kappa\right]}{1+\sigma^H-\mu^H(\sigma^H-\sigma^L)} \\
\left(\frac{V_t^H}{Y_t}\right)_{bgp} &= v^H = \frac{(1-\tau)\sigma^H}{\beta\left[\lambda(1-\bar{e})+\frac{1}{\beta}-1\right](1+\sigma^H)} \\
\left(\frac{V_t^L}{Y_t}\right)_{bgp} &= v^L = \frac{(1-\tau)\sigma^L}{\beta\left[\lambda(1-\bar{e})+\frac{1}{\beta}-1\right](1+\sigma^L)} \\
\left(\frac{w_t}{Y_t}\right)_{bgp} &= w = \frac{\left[1+\sigma^H-\mu^H(\sigma^H-\sigma^L)\right]}{(1+\sigma^L)(1+\sigma^H)\left[L-(1-\bar{e})\kappa\right]} \\
\left(\frac{C_t}{Y_t}\right)_{bgp} &= c = 1
\end{aligned}
$$

$\square$

A.6 Proposition 5 and Proposition 6

A.6.1 Entry

Preliminairies

Define the parameter set of the model as $\Omega \equiv \left\{\rho, \tau, \sigma^H, \sigma^L, \gamma, \nu, \beta, \lambda, \kappa, L\right\}$. We can rewrite equation (A.21) as:

$$A(\bar{e}, \Omega) = C(\bar{e}, \Omega) \tag{A.22}$$

Where $A(\bar{e},\Omega)$ is $A(\bar{e})$ from Appendix A.5 and:

$$C(\bar{e},\Omega) = (1-\tau)\left[\left(\rho\bar{e}^{\nu} + \frac{1-\rho}{\nu+1}\right)\Delta + (1+\sigma^H)\sigma^L\right]$$

Denoting the partial derivatives by sub indexes we have, for any fixed plausible set Ω satisfying the condition of Lemma 1, $\forall\bar{e} \in (0,1)$:

$$\begin{array}{ccc} A(\bar{e},\Omega) > 0 & ; & A_{\bar{e}}(\bar{e},\Omega) < 0 \\ C(\bar{e},\Omega) > 0 & ; & C_{\bar{e}}(\bar{e},\Omega) > 0 \end{array}$$

Then, using implicit derivative on equation A.22 for $\bar{e}$ and any parameter $p \in \Omega$ we get:

$$\frac{\partial\bar{e}}{\partial p} = \frac{A_p(\bar{e},\Omega) - C_p(\bar{e},\Omega)}{C_{\bar{e}}(\bar{e},\Omega) - A_{\bar{e}}(\bar{e},\Omega)} \Rightarrow sign\left(\frac{\partial\bar{e}}{\partial p}\right) \quad = \quad sign\left(A_p(\bar{e},\Omega) - C_p(\bar{e},\Omega)\right)$$

Enacting cost κ

$$\begin{aligned} sign\left(\frac{\partial\bar{e}}{\partial\kappa}\right) &= sign\left(A_\kappa(\bar{e},\Omega) - C_\kappa(\bar{e},\Omega)\right) = sign\left(A_\kappa(\bar{e},\Omega)\right) \\ &= sign\left(\frac{\partial\ln\left(A(\bar{e},\Omega)\right)}{\partial\kappa}\right) = sign\left(\frac{1-\bar{e}}{L-(1-\bar{e})\kappa}\right) \end{aligned}$$

We know by labor market clearing condition that $L-(1-\bar{e})\kappa > 0$. Hence, we have $\frac{d\bar{e}}{d\kappa} > 0$, and entry decreases in the enacting cost κ.

Discount factor β

$$
\begin{aligned}
sign\left(\frac{\partial \bar{e}}{\partial \beta}\right) &= sign\left(A_\beta(\bar{e},\Omega) - C_\beta(\bar{e},\Omega)\right) = sign\left(A_\beta(\bar{e},\Omega)\right) \\
&= sign\left(\frac{\partial \ln\left(A(\bar{e},\Omega)\right)}{\partial \beta}\right) = sign\left(\frac{-\frac{1}{\lambda\beta^2}}{\left(1-\bar{e}-\frac{1}{\lambda}\right)(1+g)^{1-\gamma}+\frac{1}{\lambda\beta}}\right)
\end{aligned}
$$

As $\gamma \geq 1$ and given equation (A.8) we have: $\left(1-\bar{e}-\frac{1}{\lambda}\right)(1+g)^{1-\gamma}+\frac{1}{\lambda\beta} > 0$. Hence, we have $\frac{d\bar{e}}{d\beta} < 0$, and entry increases in the discount factor β.

Corporate tax rate τ

$$
\begin{aligned}
sign\left(\frac{\partial \bar{e}}{\partial \tau}\right) &= sign\left(A_\tau(\bar{e},\Omega) - C_\tau(\bar{e},\Omega)\right) = sign\left(-C_\tau(\bar{e},\Omega)\right) \\
&= sign\left(-\frac{\partial \ln\left(C(\bar{e},\Omega)\right)}{\partial \tau}\right) = sign\left(\frac{1}{1-\tau}\right) > 0
\end{aligned}
$$

Hence, we have $\frac{d\bar{e}}{d\tau} > 0$, and entry decreases in the corporate tax rate τ.

Accuracy ρ

$$
sign\left(\frac{\partial \bar{e}}{\partial \rho}\right) = sign\left(A_\rho(\bar{e},\Omega) - C_\rho(\bar{e},\Omega)\right)
$$

Note first the following auxiliary results:

$$
\begin{aligned}
\frac{\partial \mu^H}{\partial \rho} &= \frac{1}{\nu+1}\left[\frac{1-\bar{e}^{\nu+1}}{1-\bar{e}}-1\right] > 0 \\
\frac{\partial g}{\partial \rho} &= \frac{\partial g}{\partial \mu^H}\frac{\partial \mu^H}{\partial \rho} = (1+g)\lambda(1-\bar{e})\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\frac{\partial \mu^H}{\partial \rho} > 0.
\end{aligned}
$$

Now, we have:

$$
\begin{aligned}
A_\rho(\bar{e},\Omega) &= \frac{(1+g)\frac{\partial \mu^H}{\partial \rho}}{\left[\frac{L}{\kappa}-(1-\bar{e})\right]}\left(\left(1-\bar{e}-\frac{1}{\lambda}\right)\left(\lambda(1-\bar{e})\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\left[1+\sigma^H-\Delta\mu^H\right]-\Delta\right)\right. \\
&+ \left.\left(\frac{(1+g)^{\gamma-1}}{\lambda\beta}\left(\gamma\lambda(1-\bar{e})\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\right)\left[1+\sigma^H-\Delta\mu^H\right]-\Delta\right)\right) \\
&= \frac{(1+g)\frac{\partial \mu^H}{\partial \rho}}{\left[\frac{L}{\kappa}-(1-\bar{e})\right]}B(\bar{e},\Omega))
\end{aligned}
$$

Then $sign\left(A(\bar{e},\Omega))\right) = sign\left(B(\bar{e},\Omega))\right)$.

$$
\begin{aligned}
B_\rho(\bar{e},\Omega) &= \left(1-\bar{e}-\frac{1}{\lambda}+\frac{\gamma(1+g)^{\gamma-1}}{\lambda\beta}\right)\lambda(1-\bar{e})\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\left[1+\sigma^H-\Delta\mu^H\right] \\
&- \left(1-\bar{e}-\frac{1}{\lambda}+\frac{(1+g)^{\gamma-1}}{\lambda\beta}\right)\Delta
\end{aligned}
$$

Note that $f(x) = x - \ln(1+x)$ is increasing in x. This means that $\Delta > \ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)$. Hence, a sufficient condition for $A_\rho(\bar{e},\Omega) < 0$ is:

$$
\bar{e} \geq \bar{e}_A = 1 - \frac{1}{\lambda\gamma\left[1+\frac{\sigma^L+\nu\sigma^H}{\nu+1}\right]}
$$

Also note that:

$$
C_\rho(\bar{e},\Omega) = (1-\tau)\Delta\left(\bar{e}^\nu - \frac{1}{\nu+1}\right)
$$

$C_\rho(\bar{e},\Omega)$ is positive for $\bar{e} \geq \bar{e}_C = \left(\frac{1}{\nu+1}\right)^{\frac{1}{\nu}}$. Then we know that

$$\bar{e}(\rho) \geq \min\left\{\max\left(\bar{e}_A, \bar{e}_C\right), 1\right\} \equiv \bar{s} \Rightarrow \frac{\partial \bar{e}}{\partial \rho} < 0.$$

For $\bar{e} < \max\left(\bar{e}_A, \bar{e}_C\right)$, the sign of $\frac{\partial \bar{e}}{\partial \rho}$ is not clear. For example, for $\bar{e}(\rho) = 0$ we have $\frac{\partial \mu^H}{\partial \rho} = 0$, and hence $\frac{\partial \bar{e}}{\partial \rho} > 0$. This is quite intuitive, in fact, an economy performing no selection will have increasing incentives to select when they gain access to better screening technology. Nevertheless, we can also find a sufficient condition for $\frac{\partial \bar{e}}{\partial \rho} > 0$. First, a sufficient condition for $B_\rho(\bar{e},\Omega) > 0$ is given by:

$$\bar{e} \leq \underline{e}_A = 1 - \frac{\Delta}{\lambda\gamma \ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\left[1 + \frac{\nu\sigma^H + \sigma^L - \Delta\rho\nu}{\nu+1}\right]}$$

Note that $\underline{e}_A < \bar{e}_A$. Then we know that

$$\bar{e}(\rho) \leq \max\left\{0, \min\left(\underline{e}_A, \bar{e}_C\right) \equiv \underline{s}\right\} \Rightarrow \frac{\partial \bar{e}}{\partial \rho} > 0.$$

Note that κ does not enter in $\bar{s}$ or $\underline{s}$ but it affects $\bar{e}$ monotonically. So, economies with high κ, characterized by a high $\bar{e}$ and a low entry rate, are likely to increase entry when ρ increases, but economies with low κ do just the opposite. We explore this margin on the quantitative illustration of the mechanism.

A.6.2 Growth

1. Given the former results and that $\frac{\partial g}{\partial \bar{e}} < 0$, we can easily show:

$$\begin{aligned}
\frac{\partial g}{\partial \kappa} &= \frac{\partial g}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \kappa} < 0 \\
\frac{\partial g}{\partial \beta} &= \frac{\partial g}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \beta} > 0, \\
\frac{\partial g}{\partial \tau} &= \frac{\partial g}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \tau} < 0
\end{aligned}$$

2. We can also study:

$$\frac{\partial g}{\partial \rho} = \underbrace{\frac{\partial g}{\partial \bar{e}}}_{<0}\underbrace{\frac{\partial \bar{e}}{\partial \rho}}_{?} + \underbrace{\frac{\partial g}{\partial \mu^H}\frac{\partial \mu^H}{\partial \rho}}_{>0}.$$

Note that $\frac{\partial \bar{e}}{\partial \rho} < 0 \Rightarrow \frac{\partial g}{\partial \rho} > 0$.

A.6.3 Composition

1. From previous results:

$$\begin{aligned}
\frac{\partial \mu^H}{\partial \kappa} &= \frac{\partial \mu^H}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \kappa} > 0 \\
\frac{\partial \mu^H}{\partial \beta} &= \frac{\partial \mu^H}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \beta} < 0, \\
\frac{\partial \mu^H}{\partial \tau} &= \frac{\partial \mu^H}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \tau} > 0
\end{aligned}$$

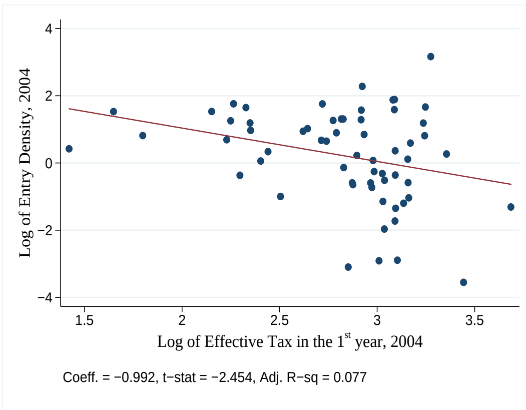
2. We can also study:

$$\frac{\partial \mu^H}{\partial \rho} = \underbrace{\frac{\partial \bar{e}}{\partial \rho}}_{?}\underbrace{\frac{\partial \mu^H}{\partial \bar{e}}}_{>0} + \underbrace{\frac{1-\bar{e}^{\nu+1}}{1-\bar{e}} - 1}_{>0}$$

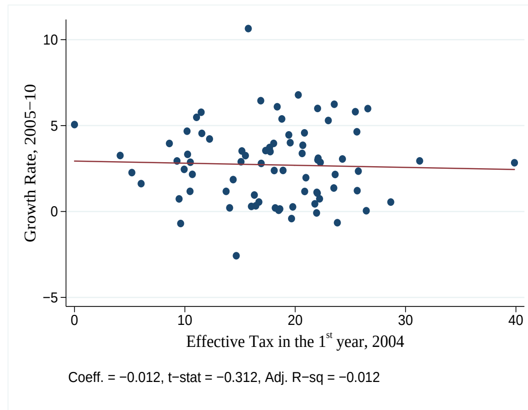
Note that $\frac{\partial \bar{e}}{\partial \rho} > 0 \Rightarrow \frac{\partial \mu^H}{\partial \rho} > 0$

A.7 Corporate Tax, Entry, and Growth

As argued on the main text, empirical research points to a strong and significant effect of taxation in firm entry, nevertheless, the effect of taxation in long-run growth is practically insignificant. Figure (17) uses cross country data to illustrate this puzzle: Left



(a) Entry



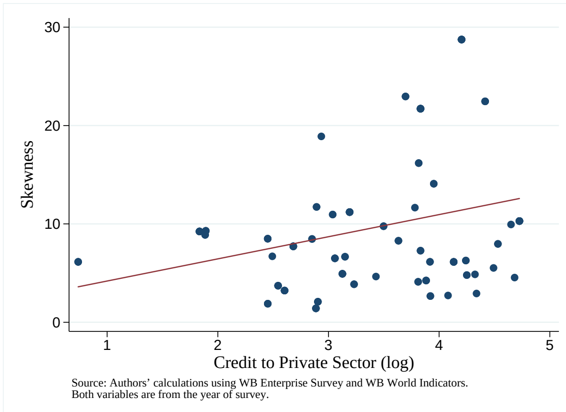
(b) Growth

Figure 17: Corporate Taxes, Entry and Growth

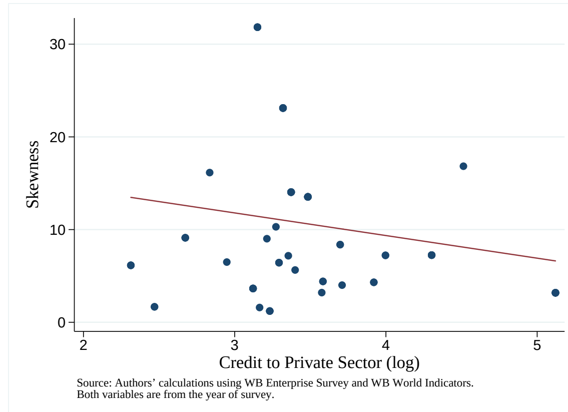
panel of figure (17) plots the natural logarithm of entry density against the logarithm of effective first year corporate tax rates in 2004 for a set of 60 countries.[1] The right panel shows the relationship between the average growth rates of the next five years and effective first year corporate tax rates. It is easily discernible that higher corporate tax rates are associated with lower entry rates whereas there is no clear effect on the 5-year average of growth rates. According to our model, the explanation lies on project heterogeneity and financial selection: higher taxation induces stronger selection which reduces entry significantly decreasing the direct effect of a larger cohort, nevertheless, tighter selection also implies a better composition of the incoming cohort which might offset an important part of the negative effect on growth.

[1]The data for effective rates of corporate taxes in the first year of a firm is available in Djankov et al. (2010).

A.8 Skewness



(a) Above median entry



(b) Below median entry

Figure 18: Financial Development and Private Credit

Appendix B

Appendix to Chapter 2

B.1 Model Solution

In this section, we derive the system of equations that characterizes the normalized model. We follow the same order as in the main text, but here we report only the main equations. Then we derive the system that characterizes the balanced growth path, and finally we prove the Lemma that is shown in the main text.

B.1.1 Normalized Model: System of Equations

Final Good Producer

$$y(s^t) = \left((L^H(s^t))^{\mu(s^t)}(L^L(s^t))^{1-\mu(s^t)}\right)^{\alpha}\left(\frac{k(s^{t-1})}{1+a(s^{t-1},s_t)}\right)^{1-\alpha} \tag{B.1}$$

$$x_j(s^t) = \frac{\alpha y(s^t)}{p_j(s^t)\,(1+\eta(R(s^t)-1))}. \tag{B.2}$$

$$k(s^{t-1}) = \frac{(1-\alpha)y(s^t)}{r(s^t)}\left(1+a(s^{t-1},s_t)\right) \tag{B.3}$$

Intermediate Good Producer

$$
\begin{aligned}
L^d(s^t) &= \frac{\alpha y(s^t)}{w(s^t)(1+\sigma^d)\left(1+\eta\left(R(s^t)-1\right)\right)} \Rightarrow \frac{L^H(s^t)}{L^L(s^t)} = \frac{1+\sigma^L}{1+\sigma^H} && \text{(B.4)}\\
\pi_j^d(s^t) &= \frac{\alpha\sigma^d}{\left(1+\sigma^d\right)\left(1+\eta(R(s^t)-1)\right)} y(s^t) && \text{(B.5)}\\
v^d(s^t) &= (1-\tau)\pi^d(s^t) + E\left[m(s^t,s_{t+1})\left(1-\lambda M(s^t,s_{t+1})\right) v^d(s^t,s_{t+1})|s^t\right] && \text{(B.6)}
\end{aligned}
$$

Financial Intermediary and Composition

$$
\begin{aligned}
\tilde{\mu}(\bar{z}(s^t)) &= \tilde{\mu}^H(\bar{z})(s^t) = \frac{1}{\nu+1}\left[1-\rho+\rho\frac{1-\left(\bar{z}(s^t)\right)^{\nu+1}}{1-\bar{z}(s^t)}\right] && \text{(B.7)}\\
\rho(\bar{z}_t(s^t))^{\nu} &= \frac{\frac{w(s^t)\kappa}{\lambda}(R(s^t)) - v^L(s^t)}{(v^H(s^t)-v^L(s^t))} - \frac{1-\rho}{(\nu+1)} && \text{(B.8)}\\
\mu(s^t) &= \mu(s^{t-1}) + \lambda(1-\bar{z}(s^t))\left(\tilde{\mu}(\bar{z}(s^t)) - \mu(s^{t-1})\right) && \text{(B.9)}
\end{aligned}
$$

Representative Household

$$
\begin{aligned}
1 &= E\left[m(s^t, s_{t+1})|s^t\right] R(s^t) - \psi\left(\frac{b(s^t)}{y(s^t)} - \bar{b}\right) && \text{(B.10)}\\
& E\left[m(s^t, s_{t+1})\frac{r(s^t, s_{t+1}) + (1-\delta) - \frac{\phi}{2}\left(\left[1+g_{bgp}\right]^2 - \left[\frac{k(s^t, s_{t+1})}{k(s^t)}(1+a(s^t, s_{t+1}))\right]^2\right)}{1+\phi\left[\frac{k(s^t)}{k(s^{t-1})}(1+a(s^{t-1}, s_t)) - \left(1+g_{bgp}\right)\right]}|s^t\right] \\
&= 1 && \text{(B.11)}\\
L(s^t) &= \left(\frac{w(s^t)}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}} && \text{(B.12)}
\end{aligned}
$$

with:

$$
m(s^{t+1}) = \frac{E\left[\frac{\beta}{(1+a(s^t, s_{t+1}))^{\gamma}}\left\{\left(c(s^{t+1}) - \Theta_l\left(L(s^{t+1})\right)^{\chi}\right)^{-\gamma}\right\}|s^t\right]}{\left(c(s^t) - \Theta_l\left(L(s^t)\right)^{\chi}\right)^{-\gamma}}
$$

Open Economy Variables

$$
\begin{aligned}
\ln\left(\frac{R(s^t)}{\bar{R}}\right) &= \rho_r \ln\left(\frac{R(s^{t-1})}{\bar{R}}\right) + \sigma_r \epsilon_t \quad \text{where} \quad \epsilon_t \overset{iid}{\sim} N(0,1) && \text{(B.13)}\\
nx(s^t) &= y(s^t) - c(s^t) - i(s^t) - \frac{\psi}{2} y(s^t)\left(\frac{b(s^t)}{y(s^t)} - \bar{b}\right)^2 && \text{(B.14)}\\
d(s^t) &= \frac{b(s^{t-1})}{1+a(s^{t-1}, s_t)} - \eta\frac{\alpha y(s^t)}{1+\eta(R(s^t)-1)} - (1-\bar{z}(s^t))\kappa w(s^t) && \text{(B.15)}
\end{aligned}
$$

Labor Market Clearing

$$
\left(\frac{w(s^t)}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}} = \frac{\alpha y(s^t)\left(\mu(s^t) + (1-\mu(s^t))\frac{1+\sigma^H}{1+\sigma^L}\right)}{w(s^t)(1+\sigma^H)\left(1+\eta\left(R(s^t)-1\right)\right)} + (1-\bar{z}(s^t))\kappa \qquad \text{(B.16)}
$$

Output Growth

$$
\begin{aligned}
\ln(1+g(s^{t-1},s_t)) \quad &= \quad \alpha \int_0^1 \ln\left(\frac{L_j(s^t)}{L_j(s^{t-1})}\right) + \ln\left(\frac{q_j(s^t)}{q_j(s^{t-1})}\right) dj + \\
&\quad (1-\alpha)\ln\left(\frac{K(s^{t-1})}{K(s^{t-2})}\right) \qquad \text{(B.17)}
\end{aligned}
$$

Let's work term by term:

$$
\begin{aligned}
\int_0^1 \ln\left(\frac{L_j(s^t)}{L_j(s^{t-1})}\right) dj \quad &= \quad \mu(s^t)\ln\left(\frac{L^H(s^t)}{L^L(s^t)}\right) - \mu(s^{t-1})\ln\left(\frac{L^H(s^{t-1})}{L^L(s^{t-1})}\right) + \ln\left(\frac{L^L(s^t)}{L^L(s^{t-1})}\right) \\
&= \quad (\mu(s^t)-\mu(s^{t-1}))\ln\left(\frac{1+\sigma^L}{1+\sigma^H}\right) + \ln\left(\frac{L^L(s^t)}{L^L(s^{t-1})}\right)
\end{aligned}
$$

Second term:

$$
\int_0^1 \ln\left(\frac{q_j(s^t)}{q_j(s^{t-1})}\right) dj \quad = \quad \lambda(1-\bar{z}(s^t))\left(\tilde{\mu}(s^t)\ln(1+\sigma^H) + (1-\tilde{\mu}(s^t))\ln(1+\sigma^L)\right)
$$

Third term:

$$
\ln\left(\frac{K(s^{t-1})}{K(s^{t-2})}\right) = \ln\left(\frac{k(s^{t-1})}{k(s^{t-2})}(1+a(s^{t-2},s_{t-1}))\right)
$$

B.1.2 Balanced Growth Path

First note that the three components of equation (B.17) imply that the long-run growth rate is given by:

$$
1+g(\bar{z}) = \quad \left((1+\sigma^H)^{\mu(\bar{z})}(1+\sigma^L)^{1-\mu(\bar{z})}\right)^{\lambda(1-\bar{z})} = 1+a(\bar{z})
$$

From equation (B.11), we get:

$$\frac{(1+a(\bar{z}))^{\gamma}}{\beta} \quad = \quad 1+r-\delta \tag{B.18}$$

From equation (B.4), we get:

$$L^d(y,w) \quad = \quad \frac{\alpha y}{w(1+\sigma^d)\left(1+\eta\left(\bar{R}-1\right)\right)} \tag{B.19}$$

And we characterize $k(y,\bar{z})$ using (B.3) and (B.18):

$$k(y,\bar{z}) = \frac{(1-\alpha)\left(1+a(\bar{z})\right)}{\frac{(1+a(\bar{z}))^{\gamma}}{\beta}-1+\delta}y \tag{B.20}$$

Replacing equations (B.20), and (B.19) in equation (B.1), we write $w(\bar{z})$ as:

$$w(\bar{z}) \quad = \quad \left(\frac{\alpha\left(1+a(\bar{z})\right)^{\frac{1}{\lambda(\bar{z}-1)}}}{\left(1+\eta\left(\bar{R}-1\right)\right)}\right)\left(\frac{(1-\alpha)}{\frac{(1+a(\bar{z}))^{\gamma}}{\beta}-1+\delta}\right)^{\frac{1-\alpha}{\alpha}} \tag{B.21}$$

We characterize $y(\bar{z})$ using (B.16):

$$y(\bar{z}) = \frac{(1+\sigma^H)\left(1+\eta\left(\bar{R}-1\right)\right)\left(\left(w(\bar{z})\right)^{\frac{\chi}{\chi-1}}\left(\Theta_l\chi\right)^{\frac{1}{1-\chi}}-(1-\bar{z})\kappa w(\bar{z})\right)}{\alpha\left(\frac{1+\sigma^H}{1+\sigma^L}-\mu(\bar{z})\frac{\sigma^H-\sigma^L}{1+\sigma^L}\right)}$$

Given $y(\bar{z})$, we write $L^d(\bar{z})$ and $k(\bar{z})$ using equations (B.20) and (B.19). Moreover, as normalized profits are constant over the BGP, we write $v^d(\bar{z})$ as:

$$v^d(\bar{z}) \quad = \quad \frac{\alpha(1-\tau)\sigma^d}{(1+\sigma^d)\left(1+\eta(\bar{R}-1)\right)\left(1-\left(1-\lambda(1-\bar{z})\right)\beta(1+a(\bar{z}))^{1-\gamma}\right)}y(\bar{z})$$

Finally, $\bar{z}$ must also be the unique solution to the Financial Intermediary problem:

$$\rho(\bar{z})^{\nu} = \frac{\frac{w(\bar{z})\kappa}{\lambda}(\bar{R})-v^L(\bar{z})}{(v^H(\bar{z})-v^L(\bar{z}))}-\frac{1-\rho}{(\nu+1)} \tag{B.22}$$

The former equation pins down $\bar{z}$, and hence the complete balanced growth path of this open economy model. The long-run level of bond holding $b(\bar{z})$ is characterized by equation (B.10):

$$\frac{\bar{R}}{1+\psi\left(\frac{b(\bar{z})}{y(\bar{z})}-\bar{b}\right)} = \frac{(1+a(\bar{z}))^{\gamma}}{\beta} \Rightarrow b(\bar{z}) = \left(\frac{\frac{\beta\bar{R}}{(1+a(\bar{z}))^{\gamma}}-1}{\psi}+\bar{b}\right) y(\bar{z}) \tag{B.23}$$

This is the only level of debt consistent with the exogenous interest rate and the endogenous growth rate of the economy. Hence, it uniquely pins down household consumption, as the budget constraint holds with equality. Also note that setting $\bar{b} = \frac{b(\bar{z})}{y(\bar{z})}$, so that no cost is paid along the BGP, implies $\beta\bar{R} = (1+a(\bar{z}))^{\gamma}$.

B.1.3 Existence and Uniqueness

Uniqueness of an Interior Solution

Recall that $\chi > 1$ and $\gamma > 1$. Let's first find an expression for the right hand side of (B.22). Let's work term by term, first noting that:

$$v^H(\bar{z}) - v^L(\bar{z}) = \frac{(1-\tau)\left((w(\bar{z}))^{\frac{\chi}{\chi-1}}(\Theta_l\chi)^{\frac{1}{1-\chi}} - (1-\bar{z})\kappa w(\bar{z})\right)}{\left(1-(1-\lambda(1-\bar{z}))\,\beta(1+a(\bar{z}))^{1-\gamma}\right)\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L}-\mu(\bar{z})\right)}$$

Then we get:

$$\frac{v^L(\bar{z})}{v^H(\bar{z})-v^L(\bar{z})} = \frac{1}{\frac{v^H(\bar{z})}{v^L(\bar{z})}-1} = \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}$$

Note that:

$$\frac{\frac{w(\bar{z})\kappa}{\lambda}}{v^H(\bar{z})-v^L(\bar{z})} = \frac{\kappa}{\lambda(1-\tau)}\frac{\left(1-(1-\lambda(1-\bar{z}))\,\beta(1+a(\bar{z}))^{1-\gamma}\right)\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L}-\mu(\bar{z})\right)}{\left(\left(\frac{w(\bar{z})}{\Theta_l\chi}\right)^{\frac{1}{\chi-1}}-(1-\bar{z})\kappa\right)} \tag{B.24}$$

Then, the right hand side of equation (B.22) is decreasing in $\bar{z}$ if and only if equation (B.24) also decreases in $\bar{z}$. Taking the natural logarithm of equation (B.24) and dropping the constant, we define the following function:

$$\begin{aligned} S(\bar{z}) &= \ln\left(1-(1-\lambda(1-\bar{z}))\,\beta(1+a(\bar{z}))^{1-\gamma}\right)+\ln\left(\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L}-\mu(\bar{z})\right)\right)- \\ &\quad \ln\left(\left(\frac{w(\bar{z})}{\Theta_l\chi}\right)^{\frac{1}{\chi-1}}-(1-\bar{z})\kappa\right) \end{aligned}$$

Some preliminary derivatives are given by:

$$\begin{aligned} \mu(\bar{z}) &= \frac{1}{\nu+1}\left[1-\rho+\frac{\rho}{1-\bar{z}}\left(1-\bar{z}^{1+\nu}\right)\right] \\ \frac{d(\mu(\bar{z}))}{d\bar{z}} &= \frac{\rho}{1+\nu}\frac{1+\bar{z}^{\nu}\left(\nu\bar{z}-(\nu+1)\right)}{(1-\bar{z})^2}>0 \quad \text{and} \quad \lim_{\bar{z}\to 1}\frac{d(\mu(\bar{z}))}{d\bar{z}}=\frac{\rho\nu}{2} \\ \frac{d(1+a(\bar{z}))}{d\bar{z}} &= -(1+a(\bar{z}))\lambda\left[\left(\frac{1-\rho}{\nu+1}+\rho\bar{z}^{\nu}\right)\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)+\ln(1+\sigma^L)\right]<0 \\ \frac{d(w(\bar{z}))}{dz} &= \left(\frac{\gamma\lambda(1-\alpha)}{\alpha}\frac{\left[\left(\frac{1-\rho}{\nu+1}+\rho\bar{z}^{\nu}\right)\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)+\ln(1+\sigma^L)\right]}{1-\frac{(1-\delta)\beta}{((1+a(\bar{z}))^{\gamma}}}-\right. \\ &\quad \left.\frac{d\mu(\bar{z})}{d\bar{z}}\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\right)w(\bar{z})\equiv\Gamma_0 w(\bar{z}) \end{aligned}$$

It is easy to show that the first two components of $S(\bar{z})$ are decreasing in $\bar{z}$. Now we find a condition that guarantees that the third component is also decreasing in $\bar{z}$.

$$sign\left(\frac{d\ln\left(\left(\frac{w(\bar{z})}{\Theta_l\chi}\right)^{\frac{1}{\chi-1}}-(1-\bar{z})\kappa\right)}{d\bar{z}}\right)=sign\left(\frac{\Gamma_0}{\chi-1}\left(\frac{w(\bar{z})}{\Theta_l\chi}\right)^{\frac{1}{\chi-1}}+\kappa\right)$$

Let's focus on the problematic region where $\Gamma_0\leq 0$. Note that:

$$\Gamma_0\geq\left(\frac{\gamma\lambda(1-\alpha)}{\alpha}\frac{\left[\left(\frac{1-\rho}{\nu+1}\right)\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)+\ln(1+\sigma^L)\right]}{1-(1-\delta)\beta}-\frac{\nu\rho}{2}\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\right)\equiv\Gamma_1\leq 0$$

So, a sufficient condition is given by:

$$\left(\frac{w(\bar{z})}{\chi \left(\frac{\kappa(1-\chi)}{\Gamma_1} \right)^{\chi - 1}} \right) \leq \Theta_l$$

Note also that:

$$w(\bar{z}) \leq \left(\frac{\alpha}{\left(1 + \eta \left(\frac{1}{\beta} - 1\right)\right)} \right) \left(\frac{(1-\alpha)}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1-\alpha}{\alpha}} = \Gamma_3$$

So, a sufficient condition for the existence of a unique solution to the above problem is given by:

$$\frac{\Gamma_3}{\chi \left(\frac{\kappa(1-\chi)}{\Gamma_1} \right)^{\chi - 1}} \leq \Theta_l$$

Note that the third term of $S(\bar{z})$ is the labor used in intermediate production. Moreover, in the region where $\Gamma_0 < 0$ wages decrease in $\bar{z}$, given GHH preferences, this implies that the supply of labor decreases in $\bar{z}$. Hence, a higher level of Θ_l decreases the response of labor supply to wages, so that part of the labor released by the decrease in project enactment is absorbed by intermediate producers. This translates into higher $y(\bar{z})$, increasing the value of each product line, and hence, increasing the incentives to enact projects.

Existence and Uniqueness of an Interior Solution

We need to find conditions such that equation (B.22) for $\bar{z} = 0$ becomes:

$$\begin{aligned}
\rho(0)^{\nu} &< \frac{\frac{w(0)\kappa}{\lambda}(\bar{R}) - v^L(0)}{(v^H(0) - v^L(0))} - \frac{1-\rho}{(\nu+1)} \\
\frac{w(0)}{v^H(0) - v^L(0)} &> \lambda \frac{\frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}}{\kappa \bar{R}} \\
\frac{1-(1-\lambda)\,\beta(1+a(0))^{1-\gamma}}{\left(\left(\frac{w(0)}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}} - \kappa\right)} &> (1-\tau)\lambda \frac{\frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}}{\kappa \bar{R}\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L} - \frac{1}{\nu+1}\right)}
\end{aligned}$$

A sufficient condition for this to hold is given by:

$$\begin{aligned}
\frac{1-(1-\lambda)\,\beta}{\left(\left(\frac{\Gamma_3}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}} - \kappa\right)} &> (1-\tau)\lambda \frac{\frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}}{\kappa \bar{R}\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L} - \frac{1}{\nu+1}\right)} \\
\kappa &> \frac{\left(\frac{\Gamma_3}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}}(1-\tau)\left[\lambda \frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}\right]}{(1-(1-\lambda)\,\beta)\,\bar{R}\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L} - \frac{1}{\nu+1}\right) + (1-\tau)\lambda\left[\frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}\right]}
\end{aligned}$$

For the $\bar{z} = 1$ case, we have:

$$\begin{aligned}
\rho(1)^{\nu} &> \frac{\frac{w(1)\kappa}{\lambda}(\bar{R}) - v^L(1)}{(v^H(1) - v^L(1))} - \frac{1-\rho}{(\nu+1)} \\
\frac{\lambda}{\bar{R}}\left(\rho + \frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}\right) &> \kappa \frac{w(1)}{v^H(1) - v^L(1)} \\
\frac{\lambda}{\bar{R}}\left(\rho + \frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}\right) &> \kappa \frac{\left(1-\beta(1+a(1))^{1-\gamma}\right)\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L} - \mu(1)\right)}{\left(\frac{w(1)}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}}(1-\tau)} \\
\kappa &< \frac{\frac{\lambda}{\bar{R}}\left(\rho + \frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}\right)\left(\frac{w(1)}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}}(1-\tau)}{\left(1-\beta(1+a(1))^{1-\gamma}\right)\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L} - \mu(1)\right)}
\end{aligned}$$

We can state a sufficient condition as:

$$\kappa \quad < \quad \frac{\frac{\lambda}{\bar{R}}\left(\rho + \frac{1-\rho}{(\nu+1)} + \frac{\sigma^L(1+\sigma^H)}{\sigma^H-\sigma^L}\right)\left(\frac{\left(\frac{\alpha}{(1+\sigma^H)(1+\eta(\bar{R}-1))}\right)\left(\frac{(1-\alpha)}{\frac{1}{\beta}-1+\delta}\right)^{\frac{1-\alpha}{\alpha}}}{\Theta_l \chi}\right)^{\frac{1}{\chi-1}}(1-\tau)}{(1-\beta)\left(\frac{1+\sigma^H}{\sigma^H-\sigma^L} - \frac{1+\nu\rho}{\nu+1}\right)}$$

Intuitively, there is a lower and an upper bound on the enactment cost κ that guarantees an interior solution. In fact, when the cost is too low, every project is enacted; when it is too high, no project is realized.

B.2 Macroeconomic Data

In this section, we present the sources of the macroeconomic data used in this paper and the behavior of the aggregated time series during the crisis. We first present a general description of the Chilean economy from the *World Bank Database*, in Table 13.

To start, note that Chile is a small economy, both in terms of population and aggregate output. It has also experienced spectacular growth, which led it to be the first OECD member in South America (2010). Its trade and debt ratio justify the small open economy framework adopted in this paper. In particular, while its trade to GDP ratio is quite high, according to the *World Trade Organization* database, in 2011 Chile had 0.45% of the world's exports and 0.41% of the world's imports. Chile is also the 7^{th} freest economy in the world (2013 *International Economic Freedom Ranking*).

The main source of data for the macroeconomic analysis in Section 2.4 is the International Financial Statistics (IFS) database from the International Monetary Fund (IMF). From that source, we use the following series between 1996:I and 2011:II: GDP volume index (22899BVPZF...), nominal GDP (22899B..ZF...), gross fixed capital formation (22893E..ZF...), changes in inventory (22893I..ZF...), exchange rate (228..RF.ZF...), exports (22890C..ZF...), imports (22898C..ZF...), financial accounts (22878BJ DZF...), direct investment abroad (22878BDDZF...), direct investment in Chile (22878BEDZF...), net errors and omissions (22878CADZF...), household consumption (22896F..ZF...), and government consumption (22891F..ZF...). We use employment data from the Instituto Nacional de Estadística (INE, National institute of Statistics) of Chile and hours worked per week from the *Encuesta de Ocupación y Desocupación* from the Economics Department of *Universidad de Chile*. We also use the average interest rate charged by commercial banks for one to three month loans from the Chilean Central Bank database. All the data is seasonally adjusted with the X-12 procedure of the US Census. We follow the procedure of Bergoeing et al.

(2002) to build real aggregate macroeconomic variables.[1]

Figure 19 shows the evolution of the annualized real lending interest rate between 1996 and 2005, where the grey area spanning the period between 1998:II and 2000:III highlights the crisis period. Figure 20 explores some of the macroeconomic consequences of the Russian default in the Chilean economy.[2]Figure 20a shows a drop of more than 30% in real investment over just one quarter. In that same period, Figure 20b points to a drop of more than 6% in hours worked. Figures 20c and 20d show that both output and consumption decreased by 5% and took more than a year to return to the pre-crisis level.

	1995	2012
Population	14,440,103	17,464,814
GDP per capita	7,400.8	22,362.5
Trade to GDP	56.4%	66.6%
Gross capital formation to GDP	26.2%	25.6%
External debt to GNI	32.1%	41.0%

Table 13: Chilean Economy

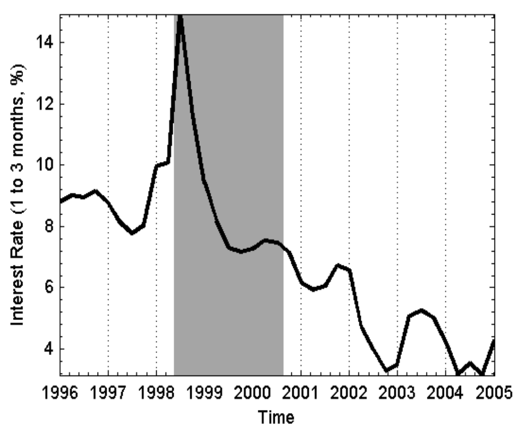
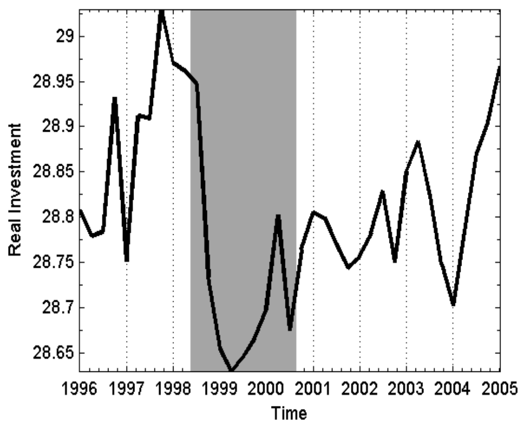


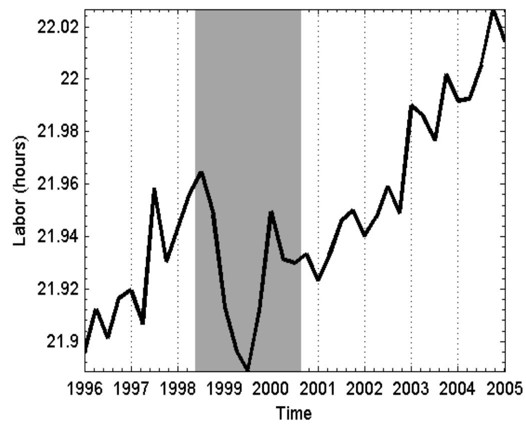
Figure 19: The Chilean Sudden Stop

[1]We build capital series using the perpetual inventory method; we assume an annual depreciation rate of 8%, and we solve for the initial stock that delivers an average annual capital to output ratio of 1.96.

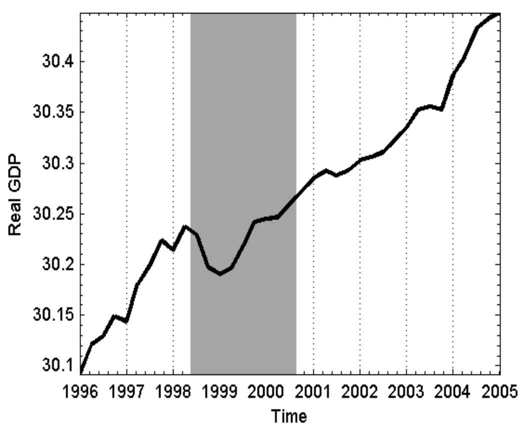
[2]The data of Figure 20 is seasonally adjusted, in real terms, and in logarithms.



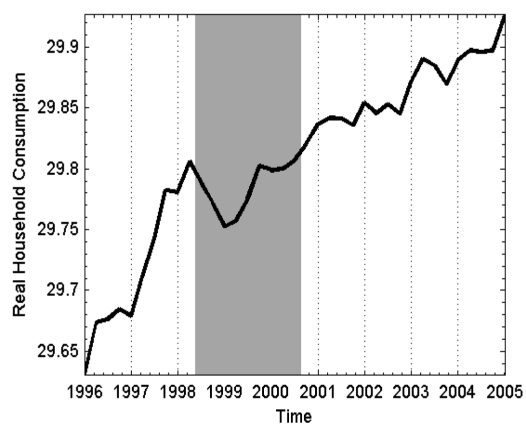
(a) Real Investment (↓ 30%)



(b) Total Hours (↓ 6%)



(c) Real GDP (↓ 5%)



(d) Household Consumption (↓ 5%)

Figure 20: Macroeconomic Impact of the Crisis.

B.3 ENIA and Empirical Analysis

The Encuesta Nacional Industrial Anual (ENIA, Annual National Industrial Survey) conducted the by the INE covers all manufacturing plants in Chile with more than 10 workers. Our version extends from 1995 to 2007.

B.3.1 Data Cleaning

We eliminate observations with one or more of the following inconsistencies, with original variable names provided in parenthesis: negative electricity consumption (*ele-cons*), worked days less than or equal to 0 (*diatra*), gross value of the production less than value added (*vpn*<*va*), value added less than 0 (*va*), remuneration of workers equal to 0 (*rempag*), size equal to 0 (*tamano*), ISIC code less than 3000 (bad coding in *sector*), and sales income less than income from exports (*ingtot*<*ingexp*). Finally, as mentioned in the text, we dropped industries 314 (Tobacco), 323 (Leather), 353 (Oil and Gas 1), 354 (Oil and Gas 2), 361 (Pottery), 362 (Glass), 371 (Metals 1), 372 (Metals 2) and 385 (other) due to an insufficient number of observations or inadequate entry dynamics. To minimize problems due to the 10 workers threshold, we count as the first observation of a firm the first time it appears in the data with 11 or more workers. The restricted sample still contains more than 90% of the original observations and total workers in the sample.

B.3.2 Variable Construction and Other Controls

We calculate entry rates at year t at the industry level for each cohort, dividing the number of new plants in year t by the average of the total plants in years t and $t-1$. The revenue (*ingtot-revval-reviva*) used to calculate the profitability measures and the Herfindahl-Hirschman concentration Index (HHI) excludes non-manufactured products (re-selling products and their tax shield); the costs include wages and exclude the costs and taxes associated to non-manufactured products (*costot-mrevval-mreviva+rempag*). The production used to build the labor productivity proxy used in Table 4 includes the changes in inventories as a fraction of the sales of manufactured products (*vpf-provap+provaf-acavap+acavaf*). We define capital as the end-of-period value of land, machinery, buildings and vehicles (*salter+saledi+salmaq+salveh*). We use the net increase in physical capital (*abaf*) to build the capital accumulation variable used in Table 4. We deflate monetary variables using the industry level deflators provided by the INE. The index of manufacturing production (22866EY.ZF...), the unemployment rate (22867R..ZF...), and the producer price and wholesale price index (PPI/WPI, 22863...ZF...) are taken from the IFS database. The labor cost index is from the Chilean Central Bank.

B.3.3 Descriptive Tables

The following table presents the mean, standard deviation, number of observations, and the 25^{th}, 50^{th}, and 75^{th} percentiles of the key variables used in the empirical analysis and for calibration purposes. For firm level observations, top and bottom 1% have been removed to control for outliers. Firms born prior to 1996 are excluded from the tables and regressions, because we cannot infer their cohort. Firms born in 2007 are also excluded because we observe them only at age 0. Note that the raw data reflects the main message of the empirical section. In fact, the simple average industry level entry rate is 11% before the crisis, 7% during the crisis, and 9% after the crisis. Moreover, the average lifetime profitability of the cohorts born during the crisis is also higher in the raw data.

Variable	Mean	Std. Dev.	N	P25	P50	P75
Profitability	0.231	0.209	17868	.126	.242	.359
Labor productivity proxy (log)	4.68	1.042	16945	4.092	4.613	5.237
Capital accumulation rate	0.128	0.243	17179	0	.021	.164
Electricity consumption (log)	-0.666	1.769	17874	-1.843	-.88	.32
Total workers	58.998	131.307	18234	16	24	49
Capital (log)	6.467	2.013	17347	5.199	6.439	7.715
Workers at entry	52.002	12.991	4089	42.817	48.031	57.319
Capital at entry (log)	6.29	0.259	4089	6.08	6.212	6.543
Average exit age	2.407	2.536	2241	0	2	4
HHI	0.057	0.103	220	.011	.021	.054
Average industry level entry	0.086	0.052	220	.051	.076	.112
Fraction dying at age= 0	0.172	0.15	220	.077	.15	.25
Cohort size	371.727	166.339	11	252	302	454
Fraction of the cohort not dying in the sample	0.474	0.163	11	.312	.434	.635
Unemployment rate	0.076	0.013	12	.072	.077	.082
PPI/WPI	84.113	17.932	12	66.015	84.065	97.445
Labor cost index	92.697	5.916	12	88.535	92.86	97.545
Manufacturing production (log)	4.465	0.121	12	4.38	4.409	4.579

Table 14: Summary Statistics: All Cohorts.

Variable	Mean	Std. Dev.	N	P25	P50	P75
Profitability	0.23	0.203	6794	.127	.243	.357
Labor productivity proxy (log)	4.521	0.986	6552	3.984	4.453	5.049
Capital accumulation rate	0.106	0.219	6497	0	.012	.13
Electricity consumption (log)	-0.783	1.684	6729	-1.918	-1	.125
Total workers	55.902	135.665	6863	16	23	45
Capital (log)	6.416	1.926	6577	5.21	6.423	7.563
Workers at entry	47.589	6.817	1170	42.817	42.817	57.319
Capital at entry (log)	6.203	0.177	1170	6.08	6.08	6.456
Average exit age	3.426	3.089	843	1	3	6
HHI	0.03	0.045	40	.009	.016	.034
Average industry level entry	0.113	0.069	40	.076	.102	.127
Fraction dying at age= 0	0.151	0.093	40	.096	.148	.186
Cohort size	585	282.843	2	385	585	785
Fraction of the cohort not dying in the sample	0.288	0.034	2	.264	.288	.312
Unemployment rate	0.054	0.001	2	.053	.054	.054
PPI/WPI	62.63	0.721	2	62.12	62.63	63.14
Labor cost index	83.895	1.478	2	82.85	83.895	84.94
Manufacturing production (log)	4.36	0.034	2	4.336	4.36	4.384

Table 15: Summary Statistics: Before Crisis Cohorts.

Variable	**Mean**	**Std. Dev.**	**N**	**P25**	**P50**	**P75**
Profitability	0.239	0.214	4169	.132	.247	.366
Labor productivity proxy (log)	4.768	1.044	4029	4.136	4.67	5.345
Capital accumulation rate	0.115	0.228	3976	0	.011	.142
Electricity consumption (log)	-0.698	1.83	4188	-1.91	-.878	.36
Total workers	58.532	129.278	4306	15	24	48
Capital (log)	6.388	2.12	4021	5.121	6.341	7.772
Workers at entry	48.737	2.733	839	44.667	49.798	51.211
Capital at entry (log)	6.03	0.167	839	5.782	6.089	6.181
Average exit age	2.647	2.333	529	0	2	5
HHI	0.064	0.11	60	.012	.023	.062
Average industry level entry	0.067	0.035	60	.041	.061	.078
Fraction dying at age= 0	0.152	0.123	60	.063	.162	.223
Cohort size	279.667	25.423	3	252	285	302
Fraction of the cohort not dying in the sample	0.366	0.064	3			
Unemployment rate	0.081	0.009	3	.072	.083	.089
PPI/WPI	69.147	5.677	3	64.34	67.69	75.41
Labor cost index	89.207	1.726	3	87.26	89.81	90.55
Manufacturing production (log)	4.368	0.021	3	4.344	4.379	4.382

Table 16: Summary Statistics: Crisis Cohorts.

Variable	Mean	Std. Dev.	N	P25	P50	P75
Profitability	0.226	0.211	6905	.122	.237	.356
Labor productivity proxy (log)	4.787	1.076	6364	4.201	4.735	5.381
Capital accumulation rate	0.158	0.269	6706	0	.04	.22
Electricity consumption (log)	-0.534	1.803	6957	-1.72	-.762	.485
Total workers	62.289	128.134	7065	17	27	55
Capital (log)	6.563	2.027	6749	5.241	6.505	7.856
Workers at entry	55.802	16.522	2080	43.55	48.031	69.957
Capital at entry (log)	6.443	0.22	2080	6.219	6.543	6.598
Average exit age	1.274	1.304	869	0	1	2
HHI	0.082	0.158	120	.011	.026	.079
Average industry level entry	0.086	0.048	120	.054	.076	.114
Fraction dying at age= 0	0.189	0.174	120	.064	.146	.286
Cohort size	346.667	122.662	6	221	352.5	454
Fraction of the cohort not dying in the sample	0.589	0.12	6	.497	.597	.678
Unemployment rate	0.08	0.009	7	.074	.078	.08
PPI/WPI	96.666	11.359	7	86.84	94.89	106.97
Labor cost index	96.707	3.483	7	93.88	96.61	100.26
Manufacturing production (log)	4.537	0.11	7	4.418	4.552	4.637

Table 17: Summary Statistics: Post Crisis Cohorts.

B.3.4 Industry Level Entry Rates

Cohorts	311	312	313	321	322	324	331	332	341	342
96 – 97	11.6%	13.4%	11.3%	9.6%	15.5%	7.1%	10.1%	18.2%	12.7%	7.0%
98 – 99	5.3%	3.6%	7.7%	3.6%	6.1%	5.1%	6.8%	7.7%	5.3%	5.2%
00 – 01	4.2%	6.3%	9.1%	4.1%	5.5%	4.1%	6.3%	11.2%	6.9%	8.1%
02 – 03	10.2%	10.0%	12.6%	6.9%	12.9%	5.7%	13.7%	13.1%	9.0%	20.2%
04 – 05	7.3%	8.5%	19.7%	6.3%	5.7%	3.0%	6.9%	12.3%	8.2%	6.4%

Cohorts	351	352	355	356	369	381	382	383	384	390
96 – 97	9.7%	10.3%	5.5%	8.0%	11.7%	13.1%	10.8%	9.2%	9.5%	22.2%
98 – 99	8.9%	5.0%	7.0%	4.9%	13.1%	4.4%	5.3%	6.0%	4.5%	4.7%
00 – 01	7.1%	5.1%	2.5%	5.2%	9.9%	8.8%	7.8%	8.7%	3.6%	10.5%
02 – 03	7.4%	10.7%	8.9%	12.8%	7.1%	13.6%	17.3%	13.9%	9.7%	7.5%
04 – 05	14.8%	5.8%	8.8%	8.6%	8.4%	9.8%	11.4%	7.0%	10.4%	8.9%

Table 18: Two year average entry rates by industry.

B.3.5 Hausman and Taylor (1981)

The method can be summarized as a four-step procedure. First, a fixed effects regression delivers consistent estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ that are used to retrieve estimators $\hat{u}_{i,t}$ and $\hat{\sigma}_u$. The second step is an instrumental variables (IV) regression with $\hat{u}_{i,t}$ as dependent variable, Z^1 and Z^2 as independent variables, and Z^1 and X^1 as instruments; this delivers a consistent estimator for $\tilde{\sigma}$ (the dispersion of the residual). Third, an estimator for the variance of the unobserved fixed effect component can be built as $\hat{\sigma}^2_\mu = \tilde{\sigma}^2 - \frac{\hat{\sigma}^2_u}{T}$, in order to form the usual generalized least squares (GLS) correction. Finally, the GLS correction is used to transform the original equation and estimate all the coefficients simultaneously in equation (2.32), using an IV procedure where the instruments are given by Z^1, the mean of X^1 and the deviations from the mean of X^1 and X^2. After every estimation we perform the Sargan-Hansen test to assess the validity of the instrumental variables procedure.

B.4 Cox Estimation

This section shows that the higher profitability of the cohorts born during the sudden stop is not due to *ex-post* selection. In particular, we perform the following stratified proportional hazard estimation in order to show that firms born during the crisis are not more likely to die at any horizon.

$$\begin{aligned} h_{r,c}\left(t|\boldsymbol{X}_i\right) &= h_{0,r,c}\left(t\right)\exp\left[\beta_1\ln(L_{i,t})+\beta_2\ln(L_{i,0})+\beta_3\ln(elec_{i,t})+\beta_4\ln(elec_{i,0})\right. \\ &\quad \left. +\,\beta_5\ln(K_{i,t})+\beta_6\ln(K_{i,0})+\beta_7\bar{P}_{j,t}+\beta_8 HHI_{j,0}+\gamma_j\right] \end{aligned}$$

The two strata are geographical region (r) and time period (c). This means that the baseline hazard $h_{r,c}$ varies across these two dimensions. We divide Chile into five geographical regions. The time periods correspond to the *pre-crisis*, *crisis*, and *post-crisis* period of the second specification in the Hausman and Taylor estimation of Section 2.4. The Cox-Snell test cannot reject the proportional hazard structure with 95% confidence. Sub-index t refers to time, while i refers to a plant, and j to an industry. The following table shows the estimates of the common covariates.

Note that bigger plants have less probability of exiting (for both electricity consumption and number of workers), while the initial size increases the probability of exiting (for number of workers and electricity consumption). The specification controls for the industry cycle (using the average varying profitability of the industry $\bar{P}_{j,t}$) and industry specific effects. Figure 21 plots the survival rates at different horizons for cohorts born during the three different time periods in the central zone of Chile. We pick this zone because it concentrates most of the plants in the sample; the main message does not change when considering the other four regions.

	_t
$\ln\left(L_{i,t}\right)$	-0.547***
	(0.0708)
$\ln\left(L_{i,0}\right)$	0.445***
	(0.0709)
$\ln\left(elec_{i,t}\right)$	-0.0783***
	(0.0262)
$\ln\left(elec_{i,0}\right)$	0.0543**
	(0.0252)
$\ln\left(K_{i,t}\right)$	-0.0237
	(0.0246)
$\ln\left(K_{i,0}\right)$	-0.0373
	(0.0237)
$\bar{P}_{j,t}$	0.0403
	(0.187)
$\text{HHI}_{j,t}$	-0.0796
	(0.356)
Industry control	Yes
Observations	16554
Plants	3778
Exits	2024

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 19: Proportional Hazard

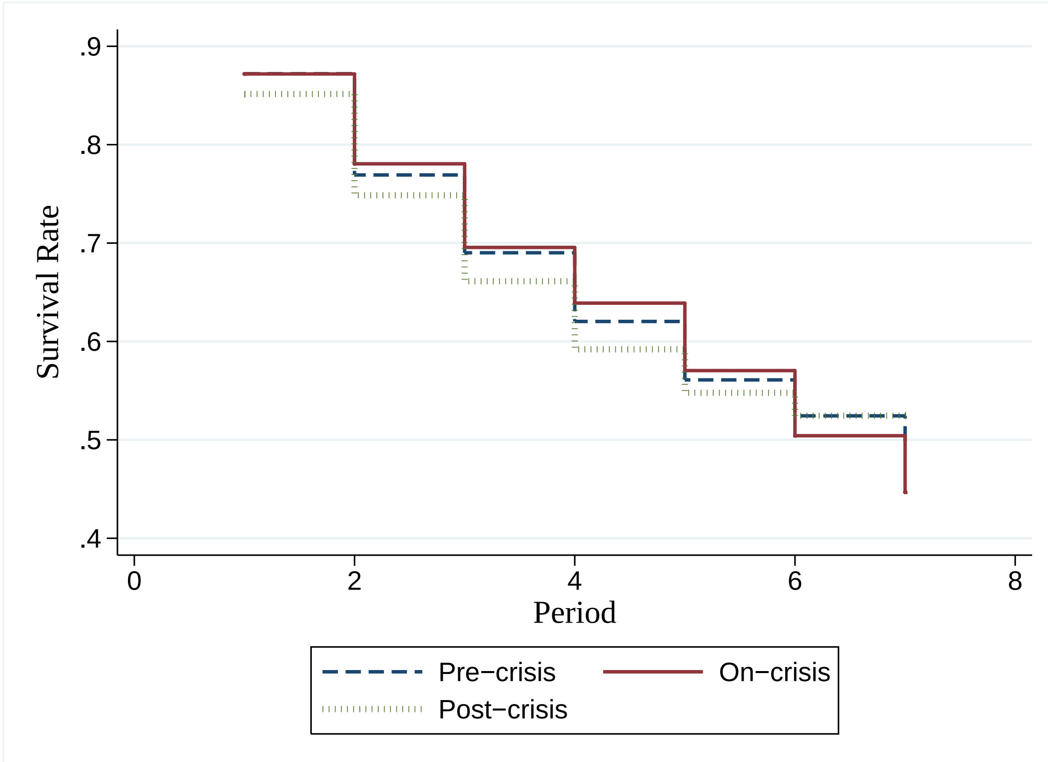
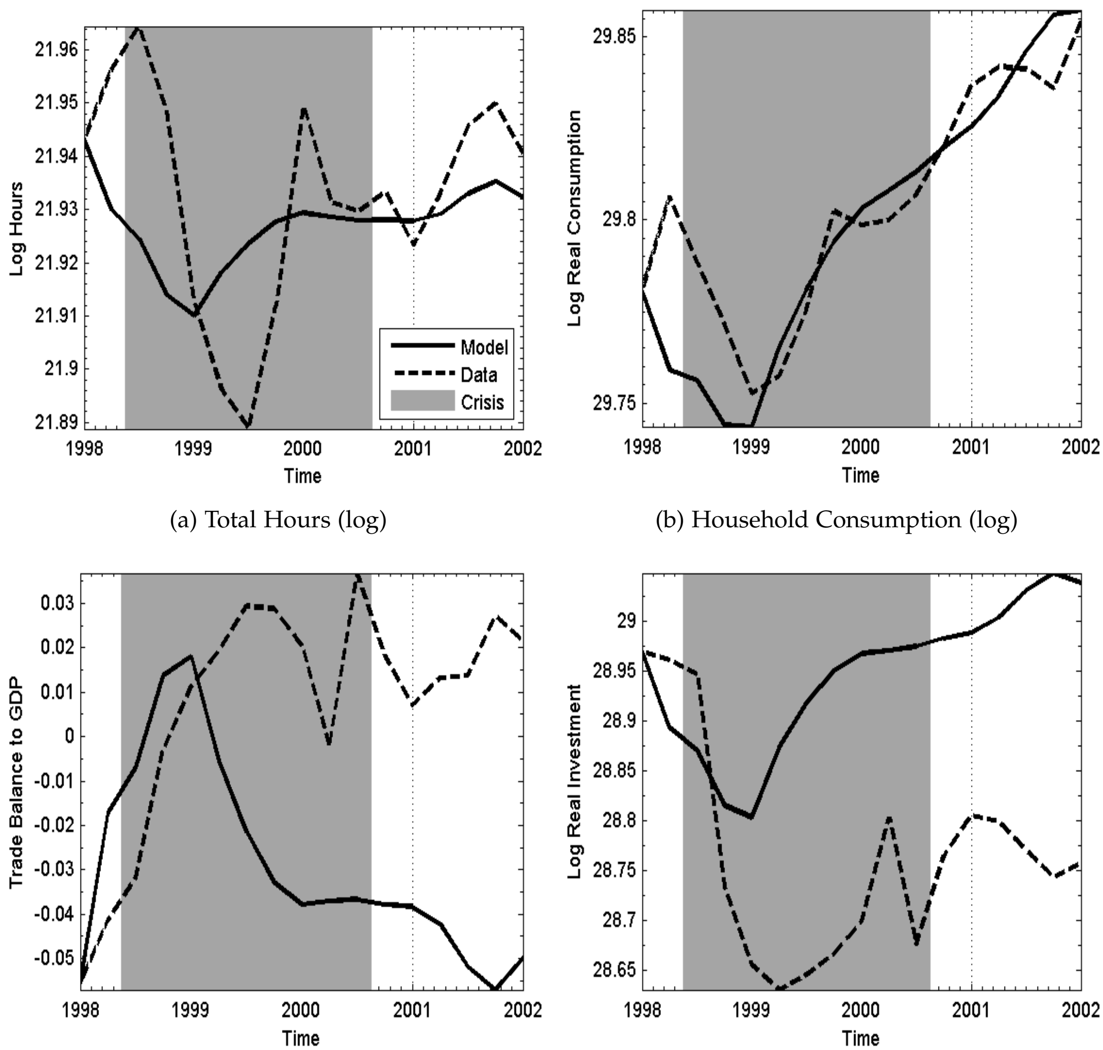


Figure 21: Survival Rates, Cox Proportional Hazard Model

Note that firms born during the crisis do not exit more than other cohorts. Moreover, they even seem stronger in this dimension, in that, until year 6, they have a higher predicted survival probability than firms born either before or after the episode. Hence, *ex-post* selection does not explain the higher profitability of cohorts born during the sudden stop.

B.5 Macroeconomic Aggregates: Model and Data

Figure 22 compares the model generated series for the logarithm of total hours, the logarithm of household consumption, trade balance divided by GDP, and the logarithm of investment with the actual series. The model is assumed to be on its BGP on 1998:I and the levels are adjusted so that model and data coincide at that date.



(a) Total Hours (log)

(b) Household Consumption (log)

(c) Trade Balance over GDP

(d) Investment (log)

Figure 22: Model Macro Performance

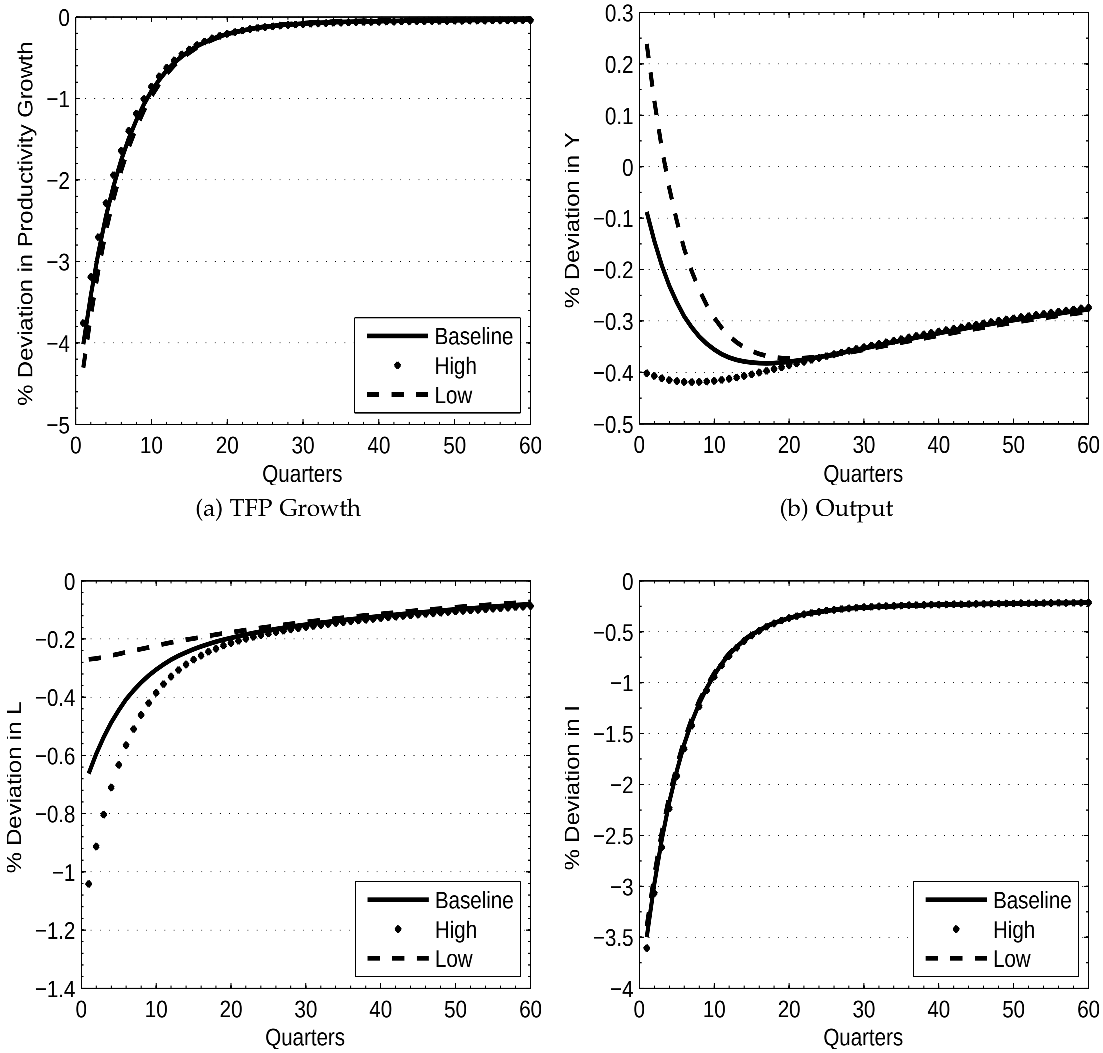
Abstracting from the timing, if we evaluate the model on the magnitude of the contemporaneous response we see that the model captures 60% of the decrease in hours, overshoots by 1% the decrease in consumption, captures almost 90% of the reversal on trade balance, and predicts 50% of the contraction in investment. The recovery of the model is significantly faster than in the data. In fact, as Figure 19 shows, the interest rate recovers quickly. This suggest that the financial conditions faced by the firms are not fully reflected by the interest rate data.

B.6 The Working Capital Channel

This section studies the role of working capital friction in the model. In particular, Figure 23 displays the responses of TFP growth, GDP, labor, and investment to a 100 basis point shock to the interest rate for three different levels of η, i.e., baseline ($\eta = 1$), low ($\eta = 0$), and high ($\eta = 2$).[3]

First, note that most of the impact of the working capital constraint takes place in the short run. In fact, a higher working capital constraint amplifies the effect on output through a labor channel. As shown in Figure 23c, labor decreases almost 50% more on impact when comparing the high η case with the baseline. Also note that Figure 23d shows no major differences in term of investment. Thus, η provides amplification in the short run by exacerbating the labor channel. Second, and more importantly for the main point of this paper, Figure 23a does not display strong differences in terms of TFP growth. Moreover, Figure 23b can be used to assess the long-run effect of η. Note that higher η reduces the demand for intermediate goods, and, hence, intermediate good producers scale down their production and reduce their labor demand. But η does not have a direct effect on the cost of enacting new projects; in fact, it affects the problem of the financial intermediary only through general equilibrium effects, i.e., reduction in wages and in the value of each product line type. In this sense, the higher η, the more the reduction in

[3]In order to avoid bond holding costs in the long run, we also re-calibrate β. The low value of η is associated with a higher β (0.9977). Higher η implies less long-run growth and therefore a lower β (0.9972).



(a) TFP Growth

(b) Output

(c) Labor

(d) Investment

Figure 23: The Role of Working Capital

labor is directed to intermediate good production, and the less is absorbed by the financial intermediary. Hence, the higher the working capital friction, the lower the effect on entry, and, thus, the lower the long run cost of the crisis. Quantitatively, the long-run loss changes are on the order of 0.001%, thus this parameter does not play a role in the main mechanism of the paper. The reason is simple: η affects the benefit of entry (decreases values) and the cost of entry (decreases wages) in virtually the same magnitude, so the entry margin is practically unaffected. As in Neumeyer and Perri (2005), this parameter is useful in matching the immediate impact of a crisis.

Appendix C

Appendix to Chapter 3

C.1 Derivation of the Growth Rate

The average growth rate in the economy is equal to the growth rate of the average quality level $\bar{q}_t$ whose value in $(t+1)$ becomes:

$$\begin{aligned}\bar{q}_{t+1} &= \int_{j \in J_{t+1}} q_{jt+1} dj \\ &= \int_{j \in J_t} \left(1 - \xi_j^{exit}\right) \left\{ \left(q_{jt} + i_j \lambda \bar{q}_{jt}\right) + \xi_j^{ipo} \left(\kappa - 1\right) \left(q_{jt} + i_j \lambda \bar{q}_{jt}\right) \right\} dj + \int_{j \in J^{entry}} q_{jt} dj\end{aligned}$$

The components of this expression take into account the changes due to innovation, IPO, exit, and entry, as explained in detail in Section 3.2.5. Dividing both sides of this expression by $\bar{q}_t$ and dropping time subscripts in BGP, we obtain

$$
\begin{aligned}
1+\mathbf{g} &= \int_{j\in J^*}\left(1-\boldsymbol{\xi}_j^{exit}\right)\left\{\left(\hat{q}_j+i_j\lambda\right)+\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)\left(\hat{q}_j+i_j\lambda\right)\right\}dj+\int_{j\in J^{entry}}\hat{q}_j dj \\
&= \int_{j\in J^*}\left(1-\boldsymbol{\xi}_j^{exit}\right)\left\{\hat{q}_j+\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)\hat{q}_j\right\}dj+ \\
&\int_{j\in J^*}\left(1-\boldsymbol{\xi}_j^{exit}\right)\left\{i_j\lambda+\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)i_j\lambda\right\}dj+\int_{j\in J^{entry}}\hat{q}_j dj \\
&= \int_{j\in J^*}\hat{q}_j dj+\int_{j\in J^*}\left\{\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)\hat{q}_j-\boldsymbol{\xi}_j^{exit}\left[\hat{q}_j+\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)\hat{q}_j\right]\right\}dj+ \\
&\int_{j\in J^*}\left(1-\boldsymbol{\xi}_j^{exit}\right)\left\{i_j\lambda+\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)i_j\lambda\right\}dj+\int_{j\in J^{entry}}\hat{q}_j dj.
\end{aligned}
$$

The second equality above collects the normalized quality levels into the first integral. The third equality separates $\int \hat{q}_j dj$ which, by definition of normalized quality, equals 1. Hence, we arrive at

$$
\begin{aligned}
\mathbf{g} &= \int_{j\in J^*}\left(1-\boldsymbol{\xi}_j^{exit}\right)\delta_j^{ipo}\left(\kappa-1\right)\hat{q}_j dj-\int_{j\in J^*}\delta_j^{exit}\hat{q}_j dj+ \\
&\int_{j\in J^*}\left(1-\boldsymbol{\xi}_j^{exit}\right)\left\{i_j\lambda+\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)i_j\lambda\right\}dj+\int_{j\in J^{entry}}\hat{q}_j dj \\
&= \int_{j\in J^*}\left(1-\boldsymbol{\xi}_j^{exit}\right)\left\{i_j\lambda+\boldsymbol{\xi}_j^{ipo}\left(\kappa-1\right)\left(\hat{q}_j+i_j\lambda\right)\right\}dj-\int_{j\in J^*}\boldsymbol{\xi}_j^{exit}\hat{q}_j dj+\int_{j\in J^{entry}}\hat{q}_j dj.
\end{aligned}
$$

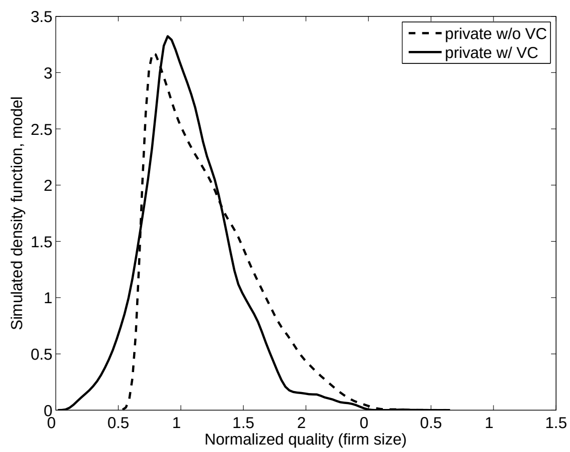
C.2 Firm Growth in Data

	Log(Sales)
VC	0.502^{***}
	(11.03)
VC*TimefromMatch	0.211^{***}
	(11.23)
VC*TimefromMatch^2	-0.012^{***}
	(-8.73)
TimefromMatch	0.121^{***}
	(10.11)
TimefromMatch^2	-0.003^{***}
	(-4.12)
Industry FE	Yes
Year FE	Yes
N	17,885
R^2	0.18

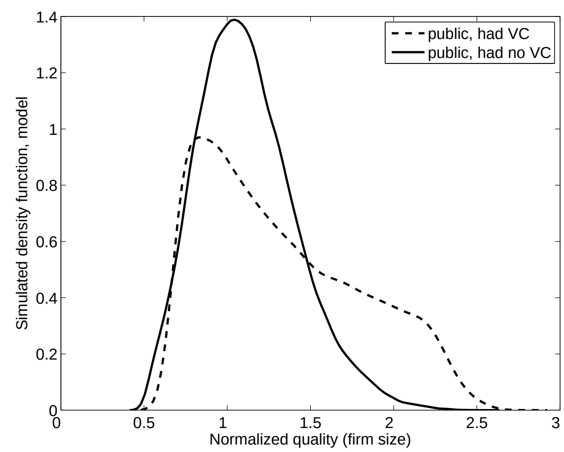
Table 20: Regression Results, Puri and Zarutskie (2012)

Table 20 summarizes the OLS regression results obtained Puri and Zarutskie (2012) with t-statistics given in parentheses. The logarithm of sales of matched VC-backed and non-VC-backed samples is regressed on a number of independent variables and controls. "VC*TimefromMatch" is the variable of interest that captures the effect of VC firms over time. It is highly significant with a t-statistics of 11.23.

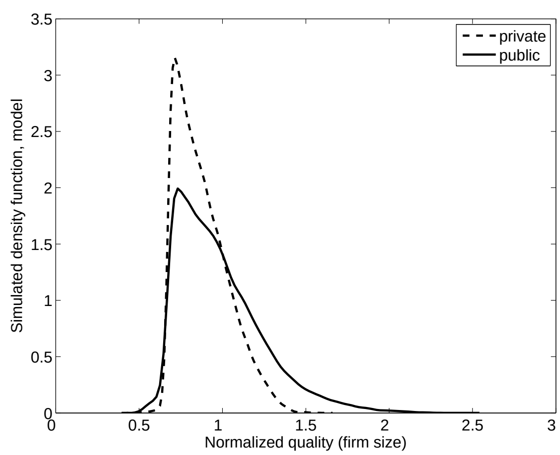
C.3 Firm Size Distributions



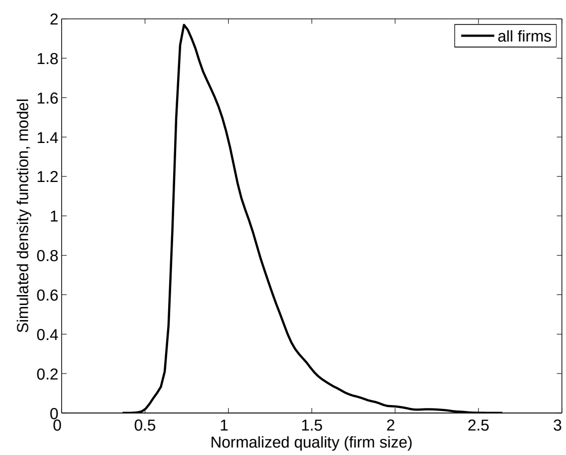
(a) Private Firms, decomposed



(b) Public Firms, decomposed



(c) Private vs. Public firms, aggregated



(d) All firms

Figure 24: Firm Size Distributions by Firm Types

Figure 25a illustrates the stationary distributions of VC- and non-VC-backed private firms. The distribution of VC-backed firms has a larger mass of smaller companies compared to non-VC-backed counterparts. Although it may look counter-intuitive at first sight, this is a natural result of three factors.[1] First, VC firms select smaller companies

[1]Notice that this is a static comparison. The comparison of growth rates in Section 3.3.4 has already explained the positive growth impact of VC financing on firms that they invest in.

to for their higher growth potential. Second, the increased profit level per unit of quality due to VC's financial support helps firms with smaller capacities survive in the business. And third, as demonstrated in Figure 17b, VC firms go public at smaller sizes as they can afford its cost due to higher profits. This implies that companies that are smaller than a relatively lower threshold remain in the VC-backed distribution.

The lower IPO threshold for VC-backed firms also implies that, every period, smaller firms enter the distribution of public firms via VC-backed IPOs. Therefore, as shown in Figure 25b, the distribution that defines public firms that had VC backing has a thinner tail compared to the stationary distribution of public firms that never received VC support. Figure 25c compares the stationary distributions of all private and public firms. As expected, the latter has a fatter right tail because larger firms issue an IPO. Lastly, Figure 25d shows the stationary distribution of all firms in the economy.

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