

ESSAYS ON UNCONVENTIONAL MONETARY POLICY

Han Chen

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Supervisor of Dissertation

Frank Schorfheide,

Professor of Economics

Graduate Group Chairperson

George J. Mailath, Professor of Economics

Dissertation Committee:

Harold L. Cole, Professor of Economics

Frank X. Diebold, Professor of Economics

For Brandon.

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ABSTRACT

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Han Chen

Frank Schorfheide

This dissertation studies the Federal Reserve’s unconventional monetary policy tools: the large-scale asset purchases (LSAPs) and the extended period of a near-zero interest rate policy (ZIRP).

In the first chapter, we simulate the Federal Reserve second LSAPs program in a dynamic stochastic general equilibrium (DSGE) model with bond market segmentation estimated on U.S. data. GDP growth increases by less than a third of a percentage point and inflation barely changes relative to the absence of intervention. The key reasons behind our findings are small estimates for both the elasticity of the risk premium to the quantity of long-term debt and the degree of financial market segmentation. Absent the commitment to keep the nominal interest rate at its lower bound for an extended period, the effects of asset purchase programs would be even smaller.

The second chapter studies the effects of the LSAPs and ZIRP in DSGE models from a broader and deeper perspective. LSAPs are ineffective (neutral operations) in standard DSGE models, and standard DSGE models forecast an increase in interest rates immediately after the recent recession, contradictory to the ZIRP conducted by the Federal Reserve. I study two mechanisms for breaking LSAPs’ neutrality as in Chen, Cúrdia, and Ferrero (2012) and Harrison (2010) and two methods of modeling the ZIRP: the perfect foresight rational expectations model and the Markov regime-switching model which I develop. In this regime-switching model, in one regime, the policy follows a Taylor rule, while, in the other regime, it involves a zero interest rate.

I also construct the optimal filter to estimate this regime-switching DSGE model with Bayesian methods. I simulate the U.S. economy and compare the predicted paths of the macro variables with and without the policy intervention. I find that the sole LSAPs intervention has an insignificant effect. Both regime-switching model and the perfect foresight model imply a substantial stimulative effect of ZIRP. However, the actual path is closer to the predicted path of the regime-switching model.

The third chapter further uses VARs that relax the DSGE model restrictions to examine the reason for the small effects of LSAPs measured in the DSGE models.

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Preface

In response to the 2008-2009 financial crisis, economic recession, and the weak recovery that followed, the Federal Reserve has been giving the economy unprecedented support: the federal funds rate has been kept close to zero since late 2008, and the Federal Reserve has launched four rounds of large-scale asset purchases (LSAPs) (also known as “Quantitative Easing” (QE) by the financial community and financial media). The Federal Reserve purchased a total of \$1.75 trillion in agency debt, mortgage-backed securities, and Treasury notes starting in December 2008, followed by a second \$600 billion Treasury-only program in the fall of 2010. An additional \$400 billion “Operation Twist” program was announced in September of 2011. This program was a pure swap between short-term and long-term assets, and it did not create additional reserves. “QE3” was announced on September 13, 2012. The Federal Reserve has pledged to purchase \$40 billion monthly of agency mortgage-backed securities in an open-ended commitment in hopes of lowering the unemployment rate while maintaining extraordinarily low rate policy, which I refer to as zero interest rate policy (ZIRP), until “at least mid-2015.” “QE4” was announced on December 12, 2012. The Federal Reserve is going to continue buying \$40 billion monthly of agency-backed mortgage securities while using \$45 billion monthly created reserves to purchase intermediate and long term Treasury notes until unemployment falls to 6.5%.¹ Bernanke

¹The Bank of England also set up an asset purchases facility in early 2009, and has bought £375 billion assets (\$600 billion) at the time of writing. The European Central Bank purchased €60

and Reinhart (2004) refer to both the asset purchases and the commitment to keep interests low (forward guidance) as “unconventional monetary policy,” because conventional monetary policy refers to the manipulation by the central bank of the policy rate, which is the federal funds rate in the United States. Standard DSGE models designed to analyze monetary policy and match the macro data well before the crises must address the challenge of evaluating the Federal Reserve’s unconventional policy. There are two main issues.

The first issue is that asset purchases are completely ineffective (neutral operations) in the baseline New Keynesian model of Eggertsson and Woodford (2003). Market participants take full advantage of arbitrage opportunities, thus LSAPs should have no effect on real economic outcomes. The LSAPs’ neutrality result only depends on two postulates: All investors can sell and buy the same assets at the same market prices, and assets are only valued for their pecuniary returns. In order for LSAPs to have a real effect, a natural starting point is to break either one of these postulates. Chen, Cúrdia, and Ferrero (2012) introduce financial market segmentation to break the first postulate, which implies that the long-term interest rate matters for aggregate demand distinctly from the expectation of short-term rates. Some households are constrained in the sense that they can only invest the long-term bonds. In this world, asset purchases that successfully reduce the yield on long-term bonds should tilt the consumption profile of the constrained households towards the present and stimulate investment. This will have a positive consequence for both output and inflation. Harrison (2010)² assumes bonds-in-utility to break the second postulate. Since bonds directly enter agents’ Euler equation, central banks’ asset purchases program affects agents’ consumption choice, and thus aggregate output and inflation, by

billion (\$80 billion) of the Euro area covered bonds (a form of corporate bonds). The bank of Japan has expanded its asset purchases program to a total of ¥55 trillion (\$696 billion).

²Both Chen, Cúrdia, and Ferrero (2012) and Harrison (2010) are some variation of Andrés, López-Salido, and Nelson (2004).

affecting the quantity of outstanding long-term bonds.

The second issue is that since December of 2008, the U.S. federal funds rate has been effectively zero. Standard DSGE models assume a Taylor rule, which often predicts a quick rise of interest rates immediately after a recession.³ When analyzing the effects of the policy of keeping the interest rates extremely low for an extended period, the standard approach is to estimate a *stochastic* model and then conduct a counterfactual analysis using the perfect foresight rational expectations (PFRE) model (Cúrdia and Woodford (2011)).⁴ This method assumes that agents have *perfect foresight* of the path of future shocks and the interest rates, and rational expectations equilibrium can be solved backwards. The policy analysis (assuming *perfect foresight*) inherently conflicts with the assumption of the *stochastic* model that is used to fit the data. Furthermore, the PFRE model predicts an unrealistic path of macro variables. For example, this model predicts a spurious rise in inflation⁵.

In this work, I study two types of DSGE models that break the neutrality of LSAPs as in Chen, Cúrdia, and Ferrero (2012) and Harrison (2010) and two methods of modeling the ZIRP in DSGE models: the PFRE model and the regime-switching model I develop in chapter 2 in order to better predict the distribution of macroeconomic variables. I found that the effects of the LSAPs alone are insignificant measured in the DSGE models, while the ZIRP has a substantial effect and crucially depends on the models. I argue that the regime-switching model is more appropriate to analyze the effects of ZIRP because it generates more realistic predicted path of macro variables than PFRE model. The rest of the thesis proceeds as follows: Chapter 1, Chen, Cúrdia, and Ferrero (2012), studies the effects of the Federal Reserve's second round

³Reifschneider and Williams (2000), Chung, Laforte, Reifschneider, and Williams (2011), and Del Negro and Schorfheide (2012).

⁴A detailed description can be found in the AppendixA.

⁵Carlstrom, Fuerst, and Paustian (2012) interpret the explosive dynamics as a failure of New Keynesian monetary DSGE models, and Blake (2012) shares this sentiment.

large-scale asset purchase program by assuming market segmentation and a transient zero-interest-rate peg. Chapter 2 further studies the macroeconomic effects of asset purchases with the Harrison (2010) specification, develops a Markov regime-switching monetary policy rule to study the effects of an extended period of zero interest rates, and constructs the optimal filter to estimate this regime-switching DSGE model with Bayesian methods. I fit those modified DSGE models to the U.S. data and simulate the U.S. economy with and without the policy interventions. I compare the predictive paths of the macro variables through cross-assessment of the different models. Chapter 3 uses vector autoregressions (VARs) that relax the DSGE model restrictions to examine the reason for the small effects of LSAPs measured in the DSGE models.

Chapter 1

The Macroeconomic Effects of Large-Scale Asset Purchase Programs

This chapter was prepared for the conference “Learning the Lessons from QE and Other Unconventional Monetary Policies,” that took place at the Bank of England on November 17-18, 2011. This chapter was coauthored with Vasco Cúrdia (Federal Reserve Bank of New York) and Andrea Ferrero (Federal Reserve Bank of New York) and published in the Economic Journal November 2012 .

1.1 Introduction

The objective of the various LSAP programs, often referred to as “Quantitative Easing” (or QE), is to support aggregate economic activity in periods when the traditional instrument of monetary policy (the short-term nominal interest rate) is not available due to the zero bound constraint. The general idea is that asset purchases operate

directly on different segments of the yield curve, reducing rates at different maturities while the short-term rate is at zero.

Several papers find evidence that LSAP programs have indeed been effective in reducing long-term rates. For example, Gagnon, Raskin, Remache, and Sack (2011) estimate that the first round of asset purchases by the Federal Reserve lowered the ten-year Treasury yield by 58 basis points (bp).⁶

Yet, agreement on the effectiveness of LSAP programs in supporting the macroeconomy is far from universal. From a theoretical perspective, LSAP programs were criticized before their implementation, based on some version of the irrelevance result in Wallace (1981). Quantitative easing of this type is also completely ineffective in the baseline New Keynesian model of Eggertsson and Woodford (2003). In this framework, injecting reserves in exchange for longer term securities is a neutral operation. To the extent that market participants take full advantage of arbitrage opportunities, LSAP programs should have no effect on real economic outcomes. Cúrdia and Woodford (2011) extend this result to a New Keynesian model with credit frictions. If households perceive the assets purchased (such as short-term government bonds) as equivalent to reserves, again LSAP programs have no effect on the macroeconomy.⁷ Ex-post, the criticism has continued due to the difficulty of identifying empirically the effects of asset purchases from other macroeconomic forces (e.g. Cochrane (2011)).

In this paper, we estimate the effects of LSAP on macroeconomic variables in a dynamic stochastic general equilibrium (DSGE) model with segmented asset markets.

⁶A selected sample of other estimates include 13 bp in Hamilton and Wu (2010), 39 bp in Doh (2010), 45 bp in D’Amico and King (2010), and 107 bp in Neely (2010). Krishnamurthy and Vissing-Jorgensen (2011) find that LSAP II reduced the ten-year yield by about 16 bp. See more details in Table 1.1.

⁷Asset purchase programs may be an effective tool to boost the economy if the government buys securities that are not equivalent to reserves, either because not all households can invest in those assets or because financial frictions impair investment. Recent research along these lines, such as Cúrdia and Woodford (2011), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) and Gertler and Karadi (2011), has mostly focused on private credit markets. Here, instead, we study frictions that rationalize a role for government purchases of long-term bonds.

General equilibrium effects are at the heart of Wallace’s irrelevance theorem. By going beyond the effects of asset purchases on interest rates, we can evaluate the extent of the criticisms against this type of programs. At the same time, we want to give LSAP programs a chance. We introduce limits to arbitrage and market segmentation in a simple form that encompasses frictionless financial markets. Therefore, our strategy is to identify the degree of segmentation—and ultimately the effectiveness of asset purchases on macroeconomic activity—directly from the data, without assuming *a priori* that LSAP programs are bound to fail.

To implement this approach, we augment a standard DSGE model with nominal and real rigidities, along the lines of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), with segmented bond markets. In particular, we follow Andrés, López-Salido, and Nelson (2004)(henceforth ALSN) and assume that investors have heterogeneous preferences for assets of different maturities (a “preferred habitat” motive, similar to Vayanos and Vila (2009)). We do not model the details of why assets of different maturities are imperfect substitutes. Rather, we postulate that this type of market segmentation exists and estimate the importance of this friction for the transmission mechanism of monetary policy.

The form of asset market segmentation that we use in this paper implies that the long-term interest rate matters for aggregate demand distinctly from the expectation of short-term rates. In this world, even if the short-term rate is constrained by the zero lower bound (ZLB) for a long period of time, monetary policy can still be effective by directly influencing current long-term rates. In addition, we assume that the risk premium that arises in the model as a consequence of transaction costs is a positive function of the supply of long-term Treasury securities. This assumption captures, in reduced form, the notion that asset purchase programs are most effective in flattening the yield curve by reducing the risk premium (Gagnon, Raskin, Remache, and Sack

(2011)).

We estimate the model on U.S. data with standard Bayesian methods for the post-war sample, including the recent years. Our main experiment is a counterfactual evaluation of what would have happened to output, inflation and the other macroeconomic variables in the absence of LSAP programs.⁸

First, we calibrate the size of the asset purchase program to match a \$600 billion reduction of long-term debt in the hands of the private sector, as announced in the U.S. at the time of LSAP II. At the same time, we consider that the central bank announces the commitment to hold the interest rate at the ZLB for four quarters.⁹ The posterior median effect on GDP growth is an increase of 0.13% (annualized), while the posterior median inflation increase is 3 bp (annualized). The corresponding effect on the *level* of GDP is estimated to be very long lasting—six years after the start of the program the level of GDP is still 0.07% above the path that would have prevailed in the absence of the LSAP program.

Counterfactual simulations suggest that the commitment to hold the short-term nominal interest rate at the ZLB increases the response of real activity and inflation roughly by factors of three and two, respectively, and introduces upward skewness in the uncertainty surrounding the median estimates. Furthermore, the boost from the commitment to the ZLB is increasingly larger with the length of such a commitment. Overall, in our model, the effects of LSAP II are slightly smaller—and considerably more uncertain—than a 25 bp cut in the short-term rate.

These results suggest that the effects of LSAP programs on macroeconomic variables, such as GDP and inflation, are likely to be modest. In the technical appendix,

⁸These simulations present us with the key challenge of incorporating the zero bound of nominal interest rates. We deal with this problem using the techniques developed in Cúrdia and Woodford (2011).

⁹This assumption is consistent with the “extended period” language in the FOMC statements at the time of LSAP II and the market expectations as implied in surveys of private forecasters.

we consider several robustness exercises and find that the effects on GDP growth are unlikely to exceed a third of a percentage point. The inflationary consequences of asset purchase programs are consistently very small throughout all scenarios considered. As a comparison, using the FRB/US model, Chung, Laforde, Reifschneider, and Williams (2011) find that LSAP II induced a reduction in the risk premium of only 20 bp but increased the level of GDP by about 0.6% and the inflation rate by 0.1%. Baumeister and Benati (2010), using a VAR with time-varying coefficients, consider a change in the term premium of 60 bp and estimate a median increase in GDP growth of 3% and on inflation of 1%. Our results are therefore more moderate than in the existing literature, especially compared to the VAR methodology. Importantly, our results only touch upon the positive dimension of LSAP programs. Harrison (2010) evaluates the macroeconomic consequences of the optimal amount of asset purchases in a version of this model without capital. His findings are consistent with ours in the sense that asset purchases can improve aggregate welfare, but their quantitative relevance appears to be limited.

Our results do not depend on whether asset purchases are financed via reserves or sales of short-term debt, to the extent that these two assets are close to perfect substitutes. Therefore, according to our model, the effects of the Federal Reserve’s last round of asset purchases (also known as “Operation Twist Again”) should be in line with the estimates from LSAP II after controlling for the scale factor and for any differences in the duration of the commitment to the zero interest rate.

The rest of the paper proceeds as follows. The next section presents the model. Section 1.3 discusses the data, the estimation of the model, some basic analysis of parameter estimates, and an evaluation of how the model explains the level and slope of the term structure of interest rates. We discuss the LSAP simulation in Section 1.4. Finally, section 1.5 concludes. The companion technical appendix presents additional

details on the model equations and steady state, data, implementation of the zero lower bound commitment, additional robustness exercises, some diagnostics on shock, and variance analysis with respect to the components of the yield curve.

1.2 Model

Two types of households, *unrestricted* (denoted by u) and *restricted* (denoted by r), populate the economy and supply differentiated labor inputs. Competitive labor agencies combine these inputs into a homogeneous composite. Competitive capital producers transform the consumption good into capital. Monopolistic competitive firms hire the labor composite and rent capital to produce intermediate goods. Competitive final goods producing firms package intermediate goods into a homogeneous consumption good. Finally, the government sets monetary and fiscal policy.

1.2.1 Households

The key modification relative to a standard medium-scale DSGE model (Christiano, Eichenbaum, and Evans (2005); Smets and Wouters (2007)) is the introduction of segmentation and transaction costs in bond markets, as in ALSN.

A continuum of measure one of households populate the economy. Household $j = u, r$ enjoys consumption C_t^j (relative to productivity Z_t , as in An and Schorfheide (2007a) and dislikes hours worked L_t^j .¹⁰ Households supply differentiated labor inputs indexed by i but perfectly share consumption risk within each group. The life-time

¹⁰We express utility as a function of de-trended consumption to ensure the existence of a balanced growth path with constant relative risk aversion preferences. Imposing log-utility of consumption may be an excessively restrictive assumption in our model which is mainly concerned about asset pricing.

utility function for a generic households j is

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_j^s b_{t+s}^j \left[\frac{1}{1 - \sigma_j} \left(\frac{C_{t+s}^j}{Z_{t+s}} - h \frac{C_{t+s-1}^j}{Z_{t+s-1}} \right)^{1 - \sigma_j} - \frac{\varphi_{t+s}^j (L_{t+s}^j(i))^{1 + \nu}}{1 + \nu} \right], \quad (1.1)$$

where $\beta_j \in (0, 1)$ is the individual discount factor, b_t^j is a preference shock, $\sigma_j > 0$ is the coefficient of relative risk aversion, $h \in (0, 1)$ is the habit parameter, $\nu \geq 0$ is the inverse elasticity of labor supply and φ_t^j is a labor supply shock.¹¹ The preference and labor supply shocks both follow stationary AR(1) processes in logs.

Two types of bonds exist. Short-term bonds B_t are one-period securities purchased at time t that pay a nominal return R_t at time $t + 1$. Following Woodford (2001), long-term bonds are perpetuities that cost $P_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t + s + 1$, for $\kappa \in (0, 1]$.¹² We abstract from money and consider the limit of a cashless economy as in Woodford (1998).

The fraction ω_u of unrestricted households trade in both short-term and long-term government bonds. Unrestricted households, however, pay a transaction cost ζ_t per-unit of long-term bond purchased. This transaction cost is paid to a financial intermediary as a fee for its service. The financial intermediary distributes its profits, whose per-capita nominal value is $\mathcal{P}_t^{fi}$, as dividends to all shareholders (regardless of type). The remaining fraction of the population $\omega_r = 1 - \omega_u$ consists of restricted households who only trade in long-term bonds but pay no transaction costs.¹³

The flow budget constraint differs depending on whether the household belongs to the unrestricted or restricted group. For an unrestricted household that can trade

¹¹We allow for heterogeneity in preference shocks, discount factors and coefficient of relative risk aversions because these factors affect the household's consumption-saving decisions and financial market segmentation directly influences these optimality conditions. As such, this heterogeneity can potentially influence the simulation results in a substantial way.

¹²If $\kappa = 1$, this security is a consol.

¹³We discuss in more details the implications of transaction costs and bond market segmentation in sections 1.2.6 and 1.2.6 below.

both short and long-term bonds, we have

$$P_t C_t^u + B_t^u + (1 + \zeta_t) P_{L,t} B_t^{L,u} \leq R_{t-1} B_{t-1}^u + \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,u} + W_t^u(i) L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u. \quad (1.2)$$

For a restricted household that can only trade in long-term securities but does not pay transaction costs, we have

$$P_t C_t^r + P_{L,t} B_t^{L,r} \leq \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,r} + W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r. \quad (1.3)$$

In equations (1.2) and (1.3), P_t is the price of the final consumption good, $W_t^j(i)$ is the wage set by a household of type $j = \{u, r\}$ who supplies labor of type i , $\mathcal{P}_t$ and $\mathcal{P}_t^{cp}$ are profits from ownership of intermediate goods producers and capital producers respectively, and T_t^j are lump-sum taxes.¹⁴

One advantage of assuming that the entire stock of long-term government bonds consists of perpetuities is that the price in period t of a bond issued s periods ago $P_{L-s,t}$ is a function of the coupon and the current price, $P_{L,t}$. In the technical appendix, we show how we can write the budget constraints for the two types of households recursively as a function of the price of the bond in period t and the yield to maturity of the bond, $R_{L,t}$.

Household j consumption-saving decisions are the result of the maximization of (1.1) subject to (1.2) if $j = u$ or (1.3) if $j = r$. See the technical appendix for details and section 1.2.6 for some discussion.

¹⁴Each household receives the same dividend from intermediate goods and capital producers and pays the same amount of lump-sum taxes.

Labor Agencies and Wage Setting Decision

Perfectly competitive labor agencies combine differentiated labor inputs into a homogeneous labor composite L_t according to the technology

$$L_t = \left[\int_0^1 L_t(i)^{\frac{1}{1+\lambda_w}} di \right]^{1+\lambda_w},$$

where $\lambda_w \geq 0$ is the steady state wage markup.

Profit maximization gives the demand for the i^{th} labor input

$$L_t(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\frac{1+\lambda_w}{\lambda_w}} L_t. \quad (1.4)$$

From the zero profit condition for labour agencies, we obtain an expression for the aggregate wage index W_t as a function of the wage set by the i^{th} household

$$W_t = \left[\int_0^1 W_t(i)^{-\frac{1}{\lambda_w}} di \right]^{-\lambda_w}.$$

Households are monopolistic suppliers of differentiated labor inputs $L_t(i)$ and set wages on a staggered basis (Calvo (1983)) taking the demand for their input as given. In each period, the probability of resetting the wage is $1 - \zeta_w$, while with the complementary probability the wage is automatically increased by the steady state rates of inflation (Π) and of productivity growth (e^γ),

$$W_{t+s}^j(i) = (\Pi e^\gamma)^s \tilde{W}_t^j(i), \quad (1.5)$$

for $s > 0$, where $\tilde{W}_t^j(i)$ is the wage chosen at time t in the event of an adjustment. A

household of type j that can reset the wage at time t chooses $\tilde{W}_t^j(i)$ to maximize

$$\mathbb{E}_t \sum_{t=0}^{\infty} (\beta_j \zeta_w)^s \left[\Xi_{t+s}^{j,p} (\Pi e^\gamma)^s \tilde{W}_t^j(i) L_{t+s}^j(i) - \frac{\varphi_{t+s}^j (L_{t+s}^j(i))^{1+\nu}}{1+\nu} \right],$$

where $\Xi_t^{j,p}$ is the marginal utility of consumption in nominal terms, subject to (1.4) and (1.5). The technical appendix presents the first order condition for this problem.

1.2.2 Capital Producers

Competitive capital producers make investment decisions, choose the utilization rate and rent capital to intermediate good producing firms. By choosing the utilization rate u_t , capital producers end up renting in each period t an amount of “effective” capital equal to

$$K_t = u_t \bar{K}_{t-1},$$

Capital producers accumulate capital according to

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t, \quad (1.6)$$

where $\delta \in (0, 1)$ is the depreciation rate, μ_t is an investment-specific technology shock that follows a stationary AR(1) process in logs and $S(\cdot)$ is the cost of adjusting investment (with $S'(\cdot) \geq 0$ and $S''(\cdot) > 0$).¹⁵

Capital producers discount future profits at the marginal utility of the average shareholder

$$\Xi_{t+s}^p \equiv \omega_u \beta_u^s \Xi_{t+s}^{u,p} + \omega_r \beta_r^s \Xi_{t+s}^{r,p}.$$

This variable is the appropriate discount factor of future dividends because ownership

¹⁵Furthermore, we assume that $S(e^\gamma) = S'(e^\gamma) = 0$.

of capital producing firms is equally distributed among all households.¹⁶ Capital producers maximize the expected discounted stream of dividends to their shareholders

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Xi_{t+s}^p \left[R_{t+s}^k u_{t+s} \bar{K}_{t+s-1} - P_{t+s} a(u_{t+s}) \bar{K}_{t+s-1} - P_{t+s} I_{t+s} \right],$$

subject to the law of motion of capital (1.6), where R_t^k is the return per unit of effective capital. Note that we assume that utilization subtracts real resources measured in terms of the consumption good, $a(u_t) \bar{K}_{t-1}$.¹⁷

1.2.3 Final Goods Producers

Perfectly competitive final goods producers combine differentiated intermediate goods $Y_t(f)$, supplied by a continuum of firms f of measure 1, into a homogeneous good Y_t according to the technology

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\lambda_f}} df \right]^{1+\lambda_f},$$

where $\lambda_f \geq 0$ is the steady state price markup. The resulting demand for the f^{th} intermediate good is

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t. \quad (1.7)$$

From the zero profit condition for intermediate goods producers, we obtain an expression for the aggregate price index P_t as a function of the price set by the f^{th} intermediate good producer

$$P_t = \left[\int_0^1 P_t(f)^{-\frac{1}{\lambda_f}} df \right]^{-\lambda_f}.$$

¹⁶The same consideration applies below to intermediate goods producers.

¹⁷As in Christiano, Eichenbaum, and Rebelo (2011), we choose an implicit functional form for $a(u_t)$ such that $u = 1$ in steady state and $a(1) = 0$.

1.2.4 Intermediate Goods Producers

A continuum of measure one of monopolistic competitive firms combine rented capital and hired labor to produce intermediate goods according to a standard Cobb-Douglas technology

$$Y_t(f) = K_t(f)^\alpha (Z_t L_t(f))^{1-\alpha}, \quad (1.8)$$

where Z_t is a labor-augmenting technology process which evolves according to

$$\log \left(\frac{Z_t}{Z_{t-1}} \right) = (1 - \rho_z)\gamma + \rho_z \log \left(\frac{Z_{t-1}}{Z_{t-2}} \right) + \epsilon_{z,t}.$$

Cost minimization yields an expression for the marginal cost which only depends on aggregate variables

$$MC(f)_t = MC_t = \frac{(R_t^k)^\alpha W_t^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha} Z_t^{1-\alpha}}. \quad (1.9)$$

Intermediate goods producers set prices on a staggered basis (Calvo (1983)). In each period, a firm can readjust prices with probability $1 - \zeta_p$ independently of previous adjustments. We depart from the basic formulation of staggered price setting in assuming the firms that cannot adjust in the current period index their price to the steady state inflation rate Π . The problem for a firm that can adjust at time t is to choose the price $\tilde{P}_t(f)$ that maximizes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p \left[\tilde{P}_t(f) \Pi^s - \lambda_{f,t+s} MC_{t+s} \right] Y_{t+s}(f),$$

subject to (1.7) conditional on no further adjustments after t , where $\lambda_{f,t}$ is a goods markup shock that follows a stationary AR(1) process in logs.

1.2.5 Government Policies

The central bank follows a conventional feedback interest rate rule similar to Taylor (1993), amended to include interest rate smoothing (Clarida, Galí, and Gertler (2000)) and using the growth rate of output instead of the output gap (Justiniano, Primiceri, and Tambalotti (2011))

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_m} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}} \right)^{\phi_y} \right]^{1-\rho_m} e^{\epsilon_{m,t}},$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the inflation rate, $\rho_m \in (0, 1)$, $\phi_\pi > 1$, $\phi_y \geq 0$ and $\epsilon_{m,t}$ is an i.i.d. innovation.¹⁸

The presence of long-term bonds modifies the standard government budget constraint

$$B_t + P_{L,t} B_t^L = R_{t-1,t} B_{t-1} + (1 + \kappa P_{L,t}) B_{t-1}^L + P_t G_t - T_t. \quad (1.10)$$

The left-hand side of expression (1.10) is the market value, in nominal terms, of the total amount of bonds (short-term and long-term) issued by the government at time t . The right-hand side is the total deficit at time t , that is, the cost of servicing bonds maturing in that period plus spending G_t net of taxes.

We assume that the government controls the supply of long-term bonds following a simple autoregressive rule for their de-trended market value in real terms

$$\frac{P_{L,t} B_t^L}{P_t Z_t} = \left(\frac{P_{L,t-1} B_{t-1}^L}{P_{t-1} Z_{t-1}} \right)^{\rho_B} e^{\epsilon_{B,t}}, \quad (1.11)$$

where $\rho_B \in (0, 1)$ and $\epsilon_{B,t}$ is an i.i.d. exogenous shock. We interpret LSAP programs

¹⁸The presence of output growth, instead of the output gap, in the interest rate rule avoids the complication of solving for and estimating the system of equations that characterize the flexible price equilibrium of the model. In practice, GDP growth relative to trend is often cited as one of the main indicators of real activity for the conduct of monetary policy.

as shocks to the composition of outstanding government liabilities compared to the historical behavior of these series.

Finally, the government adjusts the real primary fiscal surplus in response to the lagged real value of long-term debt, as in Davig and Leeper (2006) and Eusepi and Preston (2011),

$$\frac{T_t}{P_t Z_t} - \frac{G_t}{Z_t} = \Phi \left(\frac{P_{L,t-1} B_{t-1}^L}{P_{t-1} Z_{t-1}} \right)^{\phi_T} e^{\epsilon_{T,t}}, \quad (1.12)$$

where $\phi_T > 0$ and $\epsilon_{T,t}$ follows a stationary AR(1) process. All fiscal variables in rule (1.12) are cyclically adjusted (i.e. expressed relative to the level of productivity) and the constant Φ is such that in steady state the fiscal rule is just an identity. Note that the presence of asset market segmentation breaks Ricardian equivalence in this model. Therefore, fiscal financing decisions have real consequences on the allocation. Given a strong enough feedback (a high enough value of the coefficient ϕ_T), rule (1.12) ensures that the primary surplus adjusts to satisfy the government intertemporal budget constraint.

1.2.6 Equilibrium and Solution Strategy

In equilibrium, households and firms maximize their objectives subject to their constraints and all markets clear. In particular, the resource constraint is

$$Y_t = \omega_u C_t^u + \omega_r C_t^r + I_t + G_t + a(u_t) \bar{K}_{t-1}.$$

We solve the model by taking a first-order log-linear approximation around a steady state in which quantities are normalized by the level of productivity Z_t and relative prices are expressed as function of P_t . The technical appendix shows the full set of non-linear normalized equilibrium relations, characterizes the steady state solution, and presents the full set of log-linearized equations that constitute the basis

for the estimation.

These conditions are standard in modern DSGE models (Christiano, Eichenbaum, and Evans (2005); Smets and Wouters (2007)) with the exception of the households' consumption-saving decisions. Here, we focus on these Euler equations to sharpen the intuition about the effects of segmentation in the bond market. This discussion should also clarify the channels through which asset purchase programs can support macroeconomic outcomes.

Since only unrestricted households trade in short-term bonds, the pricing equation for these securities is

$$1 = \beta_u \mathbb{E}_t \left[e^{-\gamma - z_{t+1}} \frac{\Xi_{t+1}^u}{\Xi_t^u} \frac{R_t}{\Pi_{t+1}} \right], \quad (1.13)$$

where Ξ_t^u is the marginal utility of de-trended consumption in real terms for an unrestricted household and $e^{-\gamma - z_{t+1}}$ is the correction factor due to productivity growth.

Both unrestricted and restricted households trade long-term bonds. For unrestricted households, the pricing equation of these securities is

$$(1 + \zeta_t) = \beta_u \mathbb{E}_t \left[e^{-\gamma - z_{t+1}} \frac{\Xi_{t+1}^u}{\Xi_t^u} \frac{P_{L,t+1}}{P_{L,t}} \frac{R_{L,t+1}}{\Pi_{t+1}} \right]. \quad (1.14)$$

For constrained households, the pricing condition is

$$1 = \beta_r \mathbb{E}_t \left[e^{-\gamma - z_{t+1}} \frac{\Xi_{t+1}^r}{\Xi_t^r} \frac{P_{L,t+1}}{P_{L,t}} \frac{R_{L,t+1}}{\Pi_{t+1}} \right]. \quad (1.15)$$

Restricted households have a different marginal utility of consumption and do not pay the transaction cost.

Transaction Costs and the Risk Premium

The presence of transaction costs for unrestricted households in the market for long-term bonds gives rise to a risk premium. Using equation (1.14), we define $R_{L,t}^{EH}$ as the counterfactual yield to maturity on a long-term bond at time t in the absence of transaction costs, given the same path for the marginal utility of consumption of unrestricted households. No arbitrage implies that this fictitious bond should have the same risk-adjusted return as the long-term security actually traded. We measure the risk premium as the difference between these two yields to maturity, up to a first order approximation

$$\hat{R}_{L,t} - \hat{R}_{L,t}^{EH} = \frac{1}{D_L} \sum_{s=0}^{\infty} \left(\frac{D_L - 1}{D_L} \right)^s \mathbb{E}_t \zeta_{t+s}, \quad (1.16)$$

where D_L is the steady state duration of the two securities.¹⁹ Expression (1.16) shows that the risk premium in this economy equals the present discounted value of current and expected future transaction costs.

In ALSN, the risk premium has two components, one endogenous and one exogenous. The endogenous component arises because households face a portfolio adjustment cost, function of the relative quantity of money relative to long-term assets. The idea is that long-term bonds entail a loss of liquidity that households hedge by increasing the amount of money in their portfolio. The transaction costs in the market for long-term bonds are treated as purely exogenous.

We retain the distinction between endogenous and exogenous component of the risk premium while abstracting from the portfolio adjustment cost component. Instead, we directly assume that transaction costs are function of the ratio of market

¹⁹The details of the derivation are in the technical appendix. In defining the yield to maturity of a bond in the absence of transaction costs, we adjust the parameter κ to guarantee that the fictitious security has the same steady state duration as the actual long-term bond.

value of long-term debt to short-term debt in the hands of the public, plus an error

$$\zeta_t \equiv \zeta \left(\frac{P_{L,t} B_{z,t}^L}{B_{z,t}}, \epsilon_{\zeta,t} \right), \quad (1.17)$$

where $B_{z,t}^L \equiv B_t^L / (P_t Z_t)$, and $B_{z,t} \equiv B_t / (P_t Z_t)$. We do not take a stand on the explicit functional form of $\zeta(\cdot)$. We only require the function and its first derivative to be positive when evaluated in steady state (i.e. $\zeta(P_L B_z^L / B_z, 0) > 0$ and $\zeta'(P_L B_z^L / B_z, 0) > 0$). The first assumption ensures the presence of a positive steady state risk premium, as in the data. The second assumption guarantees that the yield on long-term bonds drops following a reduction in their outstanding amount. This element gives LSAP programs a chance to work through the mechanism identified in the reduced form estimates (Gagnon, Raskin, Remache, and Sack (2011)).

Up to a log-linear approximation, our parsimonious formulation of transaction costs is observationally equivalent to the two frictions in ALSN. The idea that transaction costs depend directly on the aggregate stock of bonds captures the same intuition (i.e. a liquidity cost) of the adjustment cost function in the original formulation.²⁰

Limits to Arbitrage

The assumption of market segmentation captures, in reduced form, the observation that in reality some fraction of the population mostly saves through pension funds and other types of long-term institutional investors. These financial intermediaries are specialists in certain segments of the market and their transaction costs are likely to be small. Conversely, households who invest in long-term bonds mostly for diversification motives may face higher transaction costs.²¹ The parameter ω_u measures

²⁰An alternative approach to study the effects of LSAP programs on risk and term premia would be to use higher order approximation methods and estimate the model with the particle filter (van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2010)).

²¹See ALSN for a more detailed discussion of this interpretation.

this segmentation and is one of the key objects of interest in our estimation results.

The key implication of bond market segmentation is that not all agents in the model can take full advantage of arbitrage opportunities. Unrestricted households can arbitrage away, up to some transaction cost, differences in risk-adjusted expected returns between short and long-term bonds (equations 1.13 and 1.14) but restricted households do not have this possibility. Equation (1.15) fully characterizes the savings behavior of restricted households.

This friction provides a rationale for asset purchase programs to influence macroeconomic outcomes, thus breaking the irrelevance result in Wallace (1981). In particular, in our model, a program targeted to purchases of long-term securities reduces the risk premium (equation 1.17), changing their expected return. Absent segmentation, this program would affect the yield to maturity of the long-term bond (equation 1.14) but would have no effects on the real allocation. Because unrestricted households can invest in both securities, their portfolios would adjust until the two expected returns are equated again, implying a different yield to maturity on the long-term bond. In equilibrium, expected returns, inclusive of transaction costs, would be unchanged, hence avoiding any change of the stochastic discount factor. Thus, no real variable in this economy is affected.

Conversely, with segmented bond markets, LSAP programs do affect the real economy. The change in long-term yields induces a change in the expected return of the restricted households, which are not subject to the transaction costs. Because the expected return is different from the restricted households' perspective, their stochastic discount factor has to adjust. This change alters their intertemporal profile of consumption (equation 1.15) and indirectly influences both the pricing decisions of intermediate producing firms and the investment decisions of capital producers. Ultimately, general equilibrium forces imply that consumption for both types of agents,

investment and production respond as well.²² The simulations in section 1.4 illustrate the magnitude of the LSAP stimulus on aggregate demand and inflation.

1.3 Empirical Analysis

We estimate the model with Bayesian methods, as surveyed for example by An and Schorfheide (2007a). Bayesian estimation combines prior information on the parameters with the likelihood function of the model to form the posterior distribution. We construct the likelihood using the Kalman filter based on the state space representation of the rational expectations solution of the model.²³ In the remainder of this section, we first describe the data used and then present parameter prior and posterior distributions.

1.3.1 Data

We use quarterly data for the United States from the third quarter of 1987 (1987q3) to the third quarter of 2009 (2009q3) for the following seven series: real GDP per capita, hours worked, real wages, core personal consumption expenditures deflator, nominal effective Federal Funds rate, the 10-year Treasury constant maturity yield, and the ratio between long-term and short-term U.S. Treasury debt.²⁴ All data are extracted from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve Bank of St. Louis. In the technical appendix we describe more precisely how the data is constructed and how it maps to the state variables in the model.

²²In practice, another effect of asset purchase programs could be the incentive for households to shift their portfolios toward riskier assets, such as equity and corporate bonds. In the model, this mechanism is absent as the equity shares are non-tradable.

²³We impose a zero posterior density for parameter values that imply indeterminacy, which is equivalent to a truncation of the joint prior distribution.

²⁴We use an extended sample, starting in 1959q3, to initialize the Kalman filter, but the likelihood function itself is evaluated only for the period starting in 1987q3, conditional on the previous sample.

1.3.2 Prior Choice

Tables 1.2 and 1.3 (columns two to five) summarize the prior distributions of each parameter. We use a Gamma distribution for the parameters that economic theory suggests should be positive to constrain their support on the interval $[0, \infty]$. For those parameters that span only the unit interval, we use the Beta distribution. For the standard deviation of shock innovations, we use the Inverse-Gamma distribution.

The steady state value for inflation is centered at 2%, in line with the mandate-consistent level of inflation commonly assigned to the Federal Open Market Committee (FOMC). The steady state growth rate is centered at 2.5% (annualized). The discount factor has a prior that implies a real interest rate of about 2% (annualized). The steady state spread between the 10-year treasury yield and the federal funds rate has a prior centered at 0.75% (annualized), similar to the average in the data.

We follow Del Negro and Schorfheide (2008) for the priors of standard parameters. The investment adjustment cost convexity parameter S'' has prior mean of 4 and standard deviation of 1. The utilization cost elasticity parameter a'' has prior mean 0.2 and standard deviation 0.1, implying that in response to a 1% increase in the return to capital, utilization rates rise by about 0.17%. We calibrate the share of capital in production α to 0.33, and the capital depreciation rate δ to 2.5% per quarter.

The habit formation coefficient for both types of agents has prior mean of 0.6 and standard deviation 0.1, also fairly common in the literature. The parameter controlling the labor supply elasticity ν has a prior centered at 2. Similarly to Smets and Wouters (2007), we estimate the intertemporal elasticity of substitution of consumption for households, except that in our model we have two types of agents. The prior on σ_u and σ_r is relatively flat (centered at 2 with standard deviation of 1) and equal for both types, so that the data can be informative about their value.

The fraction of unrestricted agents ω_u is the crucial parameter to identify the degree of bond market segmentation in the model. At the mean, our prior implies that 70% of the households are unrestricted. As we show below, this degree of segmentation, conditional on the rest of the priors, is consistent with substantial effects of LSAP in the model. A standard deviation of 0.2, however, makes the distribution flat enough that the 90% prior interval is (0.32,0.96) and encompasses very large to minimal effects. The other key parameter is the elasticity of the risk premium to changes in the market value of long debt ζ' . The prior for this parameter has a mean 1.5/100 and a standard deviation big enough to match the range of estimates shown in Table 1.1 (see discussion above). The cash-flow parameter that controls the duration of long-term bonds (given the yield to maturity) is calibrated to imply a duration of 30 quarters, similar to the average duration in the secondary market for 10-year U.S. Treasury bills. We consider short-term debt to include both government bonds with less than one year to maturity as well as central bank liabilities in the form of reserves, vault cash and deposits. In the U.S., the average for this quantity since 1974 is about 16% of annual GDP. For long-term bonds, we consider all government bonds with maturity greater than one year, which in the U.S. is also about 16% of annual GDP since 1974.

Table 1.2 contains three non-standard parameters (Ξ^u/Ξ^r , C^u/C^r , and χ_{wu}) which refer to steady state ratios hard to pin down directly from the data. We decided not to calibrate these ratios to avoid biasing the estimation and the simulations in either direction. The posterior distribution for these three parameters turns out to deviate negligibly from our prior. Furthermore, the uncertainty in these ratios translates into uncertainty in the dynamics of the model and in the effects of asset purchases on macroeconomic variables.

The priors for the wage and price rigidity parameters ζ_w and ζ_p are centered at

0.5 with a standard deviation of 0.1, as in Smets and Wouters (2007). The fiscal rule parameter, ϕ_T is centered at 1.5 and its posterior does not differ too much from the prior. For the monetary policy rule, we consider fairly standard parameter priors. The interest rate smoothing parameter ρ_r is centered at 0.7. The response to output growth ϕ_y is centered at 0.4. The prior mean for the response to inflation ϕ_π , centered at 1.75, is slightly higher than the usual value of 1.5 in Taylor (1993). The 90% prior interval, however, is completely above one, consistent with the Taylor principle.

The shocks follow AR(1) processes, with autocorrelation coefficient ρ_i centered at 0.75, except for the autocorrelations of productivity shocks (equal to 0.4 so that the growth rate shock is not too persistent) and of the risk premium and debt shocks (equal to 0.8). The prior mean of the innovations have standard deviations σ_i centered at 0.5, except for the innovation to the monetary policy shock and the risk premium shock whose standard deviation is smaller because these variables refer to quarterly changes in interest rates.

1.3.3 Parameter Posterior Distribution

In order to obtain the posterior distribution, we first obtain the posterior mode.²⁵ We then use a normal approximation around the mode to form a jump distribution to generate a sample of parameter vector draws representative of the posterior based on the Metropolis random walk Markov Chain Monte Carlo (MCMC) simulation method.²⁶ The last five columns of tables 1.2 and 1.3 report the posterior distribution

²⁵We extract the mode by maximizing the posterior density function, which can be a very challenging task with a high-dimensional problem like the one in this paper. In order to reduce the chances of extracting a local maximum, we perform 50 maximizations of the mode starting at different guesses of the parameter vector. For each solution, we further test at least ten times whether a new maximization with a guess parameter vector in a small neighborhood of that solution can achieve a higher level of the posterior density function.

²⁶After obtaining four separate chains of 100,000 draws, we compute the covariance matrix (with a 25% burn-in) and generate four new chains of 100,000 draws. We repeat this step two more times with 200,000 and 500,000 draws, respectively. At this stage, we use these last four chains to extract

of each parameter.

The first result that emerges from these tables is that the measure of market segmentation is very small—the posterior 90% interval for ω_u is (0.824,0.993) with a median of 0.947 and a mode of 0.983. Given our prior, the data strongly pushes against a model with a significant degree of market segmentation. *Ceteris paribus*, we should expect small macroeconomic effects of asset purchases. In order to check the stability of the estimate of market segmentation (and, in general, of other parameters), we re-estimated the model with alternative samples. While in our baseline estimation the sample ends in 2009q3 (just before the first U.S. LSAP program), we considered three alternative endings: 2007q2 (before the recent financial turbulence), 2008q3 (before the federal funds rate reached the ZLB) and 2011q2 (the most recent available data). The parameter estimates always remain very comparable.²⁷

The other key parameter is the elasticity of the risk premium to asset purchases ζ' . If this elasticity were zero, asset purchases would affect neither the risk premium nor the real economy. The posterior distribution turns out to be concentrated at low levels, although different from zero, with a median of 0.327/100 and a 90% interval of (0.086,0.826), suggesting a fairly small impact of the quantity of debt on the risk premium and the 10-year yield. This finding collocates our estimate of the elasticity of the risk premium to the quantity of debt at the lower end of the spectrum in the literature.

The sensitivity of consumption to the interest rate is estimated to be 3.4 for the unrestricted type and 2.1 for the restricted type at the posterior median. These numbers suggest a specification of utility far enough from the usual log-utility assumption

the parameter posterior distribution properties and to simulate the effects of asset purchases.

²⁷One caveat is that most of our sample corresponds to a period of relative macroeconomic and financial stability in the U.S.. Because the recent crisis may have exacerbated financial frictions, we subject our main experiment to a robustness check where we allow for an (exogenous) increase in the degree of segmentation.

but also significant heterogeneity in the sensitivity to the interest rate for the two types. Finally, the posterior moments for the nominal rigidity parameters and policy rule coefficients are consistent with several contributions in the DSGE literature (e.g. Del Negro and Schorfheide (2008)). Importantly, price rigidities are estimated to be quite high relative to the micro-evidence. These parameters may significantly influence the simulations. Therefore, in the robustness analysis, we repeat our baseline experiment with ζ_p set at the prior mean.

1.3.4 Interest Rate Diagnostics

This section briefly discusses a number of interest rate diagnostics (variance, variance decomposition, historical shock decomposition). More details, including tables and plots, are available in the appendix.

As we use data on both short and long-term interest rates, our model can match the exact path of these variables through Kalman filtering and smoothing. Nonetheless, we can also compute the model-based unconditional moments for each variable. In particular, we focus on the variance of interest rates that the model is able to produce given the posterior distributions of the parameters.²⁸ Our DSGE model captures more than half of the variance of the FFR in the data (0.44 versus 0.81) and about one quarter of the ten-year yield (0.12 versus 0.47).²⁹ While our DSGE model, like most, fails to completely explain the term structure, we nevertheless provide a theory of how changes in long-term rates affect the real economy.

Different shocks explain the variance of short and long-term interest rates. For the

²⁸To compute the model-based unconditional variance of a certain variable, we draw a vector of parameters from the joint posterior distribution, compute the unconditional variance of the variable of interest, repeat the procedure 1000 times and then take the median.

²⁹The performance for long-term rates is not much worse than three-factors affine models of the term structure of interest rates. For example, the R^2 of the regression for the ten-year yield in Balduzzi, Das, Foresi, and Sundaram (1996) is 31%.

FFR, the shock to the marginal efficiency of investment is the single most important driver of the variance at business cycle frequencies (periodic components with cycles between 6 and 32 quarters), consistent with the findings in Justiniano, Primiceri, and Tambalotti (2010). In the shorter run, shocks to the monetary policy rule play a non-trivial role while at longer horizons preference shocks become relevant. The volatility of long-term rates is mostly accounted for (roughly 60%) by shocks to the risk premium, with preference and marginal utility of investment shocks splitting the remaining 40% of the variance more or less equally.

The historical shock decomposition is related to the variance decomposition but is conditional on the actual path of the data. The product of the historical shock decomposition is the marginal contribution of each shock to the path of each variable in the model. As with the variance, we report results for the median across 1000 draws. Shocks to the risk premium pushed down the FFR since 1994, between 2 and 3 percentage points. Since 2007, monetary policy shocks have been exerting the opposite pressure. Our model hence suggests that economic conditions, not discretionary policy decisions, account for the low policy rate during the recent financial crisis.³⁰ The shock to the marginal efficiency of investment—the key factor in the variance decomposition—captures fairly well the cyclical movements in the FFR with two notable exceptions: the early 1990s, when μ_t exercises downward pressures while the FFR is actually going up, and the end of the sample, when μ_t shock is pushing the FFR up. The same risk premium shocks that put downward pressure on the FFR since 1994 exert the opposite force on long-term rates, although other shocks (in particular, preference and productivity) partly offset this dynamics, especially at the end of the sample.

³⁰Because the FFR eventually hit the zero lower bound in the Fall of 2008, one way to recast this result is that the interest rate rule in the model calls for negative nominal interest rates.

1.4 Simulating LSAP II

Our baseline experiment corresponds to a simulation of the U.S. LSAP II program, announced with the FOMC statement of November 3, 2010. The central bank buys long-term bonds (in exchange for short-term bonds) over the course of four quarters, holds its balance sheet constant for the following two years and progressively shrinks its holdings of long-term securities over the final two years of the simulation. We calibrate the size of the asset purchase program to match a \$600 billion reduction of long-term debt in the hands of the private sector. Figure 1.1 illustrates the path of the market value of long-term bonds in the hands of the private sector (in deviations from trend) following the central bank purchases.³¹

We also impose that the FFR stays at the ZLB for the first four quarters after the beginning of the asset purchase program (the “extended period” language), consistent with the survey evidence from Blue Chip.³² In the technical appendix, we explain the exact details of how we implement this commitment to the zero lower bound.

We begin by showing our main simulation of LSAP II at the prior distribution and then repeat the same experiment at the posterior. The following two subsections discuss the role of the commitment to the zero lower bound and how LSAP compares to interest rate policy shocks. In the technical appendix we present several robustness results.

³¹To be precise, we perform the simulation by feeding a series of shocks to the rule controlling the level of long-term bonds in the hands of the public that is announced to all agents in the economy. As such, the private sector is aware of the whole path when forming expectations about the future.

³²Blue Chip has been asking the survey participants about the expected duration of the ZLB since the end of 2008. Until the recent (FOMC statement of August 9, 2011) change in the Federal Open Market Committee language that introduced a specific date for the expected liftoff, market participants had always maintained the expectation that the FFR would remain at the ZLB for the four/five quarters after the question was asked.

1.4.1 Simulation at the Prior Distribution

This section illustrates how the choice of the priors constrains the macroeconomic effects of asset purchase programs via Monte Carlo simulations. Specifically, we obtain 1000 random draws for the parameter vector using the prior distribution. We then use each of these draws to solve the model and extract the path of the state variables in response to the LSAP experiment described above. Finally, we compute moments and percentiles of this sample of responses for the variables of interest.³³

Figure 1.2 shows the response of output growth, output level, inflation, FFR, 10-year yield and risk premium to the simulated LSAP II experiment at the prior distribution, all in annualized percentage rates. The level of output corresponds to percentage deviations from trend, as opposed to a rate of change, and thus is not annualized. These plots represent the marginal contribution of LSAP II, i.e. the deviations of each variable relative to the path that would have prevailed absent the policy intervention. The red continuous line is the prior median response while the grey shaded area corresponds to the 50th, 60th, 70th, and 80th prior probability intervals, from darker to lighter shading respectively.

The prior for the elasticity of the risk premium to the quantity of long-term debt implies a median response at the peak of about 30 bp, consistent with the estimates in Gagnon, Raskin, Remache, and Sack (2011). The uncertainty bands cover pretty much the whole range of estimates in the empirical literature discussed in the introduction and summarized in Table 1.1. As a consequence of the change in the risk premium, output growth, the output level and inflation are higher than in the absence of asset purchases. By construction, the asset purchase program achieves the desired effect in the model. The key question is how big these effects are. Our prior is fairly generous, encompassing very large effects, but also relative agnostic, as

³³The results are not sensitive to increasing the number of draws.

to extract as much information as possible from the data without imposing too many ex-ante restrictions. To be more precise, the median prior response of output growth is 2.7% and the median response of inflation is 1.08%, roughly in line with the results in Baumeister and Benati (2010). Using a vector auto regression (VAR) model with time-varying coefficients, these authors find that a 60 bp reduction in long-term rates increases GDP growth by 3% and the inflation rate of the GDP deflator by 1% at the posterior median.

Our prior may be seen as too generous to the extent that we allow the effects of LSAP to be potentially quite extreme (for example the 80th percentile is above 15% for GDP growth and inflation). The literature, however, does not rule out these extreme outcomes. For example, Kapetanios, Mumtaz, Stevens, and Theodoris (2011) present VAR evidence for the effects of similar policies on GDP growth in the United Kingdom that can be as high as 5% at the mean, depending on the estimation method. Our choice of fairly uninformative priors gives the model a chance to generate such large effects.

In response to higher output and inflation, the central bank eventually increases the interest rate in accordance with the policy rule, but only after the end of the commitment to the ZLB. The evolution of the 10-year yield reflects the combined effect of the responses of the risk premium and the expected future short-term interest rate (expectations hypothesis). The former puts negative pressure on the long yield while the latter exerts the opposite pressure. The outcome depends on how effective asset purchases ultimately are in boosting the economy. If LSAP programs have a significant effect on output and inflation, the policy rule dictates a strong response of the federal funds rate which can potentially dominate over the negative impact on the risk premium and lead to an equilibrium increase in the 10-year yield.

1.4.2 Simulation at the Posterior Distribution

In the previous subsection, we concluded that, according to this model and our choice of the priors for the parameters, LSAP programs can boost output and inflation while the effect on the 10-year yield is somewhat ambiguous, depending on the interplay between the risk premium and the expectation hypothesis. In this section, we combine the prior with the data for the past twenty years or so to form a posterior distribution of the parameters. We then use the posterior to revisit our simulations of the effects of LSAP II. Figure 1.3 shows the same variables and simulation as Figure 1.2, but now using parameter draws from the posterior distribution.

The policy intervention reduces the risk premium by 11 bp on impact at the posterior median, reflecting the small elasticity of this variable to the quantity of debt discussed earlier. Combined with a small estimated degree of segmentation, not surprisingly the effects of LSAP II on aggregate activity are modest. On impact, GDP growth increases by 0.13% at the posterior median. The uncertainty is skewed on the upside to about 0.6%, partly due to the ZLB. After three quarters, the effect on output growth is less than a half of its peak (which occurs on impact) and completely vanishes after eight quarters. The effects on the level of output are modest too. The peak in this case occurs after 6 quarters at about 0.1% (posterior median), but now the effects persist longer—after 24 quarters, the output level is still more than 0.05% higher than without asset purchases. The reason for the high level of persistence of the level of real economic activity is that the asset purchase program induces small but long-lasting movements in real interest rates.³⁴ This modest but persistent effect on GDP level is likely to be important from a welfare perspective—even more so if we consider that the 90th probability interval allows for an increase in the level of GDP

³⁴The persistence of both inflation and the nominal interest rate after exiting the ZLB is quite evident from Figure 1.3.

as high as 0.5%. The effect on inflation is very small, 3 annualized bp at the median, and skewed upward, but even the 95th percentile is only about 15 bp.

In spite of the small magnitudes, the positive boost of asset purchases to GDP growth and inflation puts upward pressure on the interest rate. After the four quarters at the ZLB which correspond to the commitment period, the FFR becomes positive but the median increase is only 4 bp. Because asset purchases introduce little stimulus, the central bank does not raise interest rates by much upon exiting the ZLB. The upward skewness in the FFR reflects the skewness in the effects on GDP growth and inflation. Later, we disentangle the effects of the non-linearity introduced by the ZLB from the pure estimation uncertainty.

The drop in the 10-year yield almost coincides with the reduction in the risk premium. Because the FFR only increases few basis points upon exiting the ZLB, the expectation hypothesis component of long-term rates plays a minor role.

Importantly, the duration of the ZLB commitment interacts with the LSAP program. Recently, the Federal Reserve has extended its commitment to keep the nominal interest rate at zero for a longer period.³⁵ The marginal effect of increasing the ZLB commitment by one extra quarter almost doubles the effects on GDP growth and GDP level while the effect on inflation is 50% bigger.³⁶ These results show very clearly that in this model the ZLB commitment is very powerful in stimulating the economy, due to the strongly forward looking behavior of the agents in the economy, and its effects increase non-linearly with the number of quarters of the commitment.

To summarize, the effects of LSAP II on GDP and inflation are modest, especially compared to the simulation at the prior, although the effects on the output level are quite persistent. The main reason for this result is that the two crucial parameters that control the effects of asset purchases on real activity—the degree of segmentation

³⁵At the time we are writing this paper, the commitment is “at least through late 2014.”

³⁶More details are available in the appendix.

and the elasticity of the risk premium to the quantity of debt in the hands of the private sector—are estimated to be small. Yet, the posterior distributions for both these parameters are skewed so that we cannot completely discard the possibility of larger effects. Together with the ZLB, the long tails of the posterior estimates for ω_u and ζ' contribute to the upward skewness of the response of GDP growth and inflation in the baseline simulation.

Our results are at the lower end of the spectrum in the existing literature. Beside the estimates in Baumeister and Benati (2010) mentioned earlier, Chung, Laforde, Reifschneider, and Williams (2011), using the FRB/US model, assume that LSAP II induces a reduction in the risk premium of only 20 bp but increases the level of GDP by about 0.6% and inflation rate by 0.1%.

The results in this section are subject to the caveat, discussed for example in D’Amico, English, López-Salido, and Nelson (2011), that the use of aggregate data on debt may bias the results, weakening the effects of asset purchases on yields. One possible rationalization of this bias is that our treatment of the sample as homogeneous may have overlooked a structural change in the underlying structure of financial markets caused by the recent crisis. Alternatively, active debt management policy by the Treasury, in an attempt to minimize the financing costs of debt issuance, may make the elasticity of the risk premium to the quantity of debt hard to estimate. Under this hypothesis, the Treasury internalizes the asset market friction to minimize the interest rate cost of marginal funding so that, ex-post, the data display very little relationship between yield spreads and relative supply of assets at different maturities.³⁷

In the working paper version of this study (Chen, Cúrdia, and Ferrero (2011)), we estimate the model without using observations on the quantity of debt. In that

³⁷We thank the referee for suggesting this interpretation.

case, our prior on the elasticity of the risk premium to the quantity of debt encompasses most of the estimates in the empirical literature. Because the elasticity is not well-identified in this case, the posterior median coincides with the prior, which corresponds to a cumulative effect of -30 bp on the risk premium in response to LSAP II (e.g. Gagnon, Raskin, Remache, and Sack (2011)). Under this specification, LSAP II increases GDP growth by 0.4% at the posterior median, while the impact on inflation is very similar to the case in which the quantity of debt is used in the estimation.³⁸

1.4.3 The Role of the ZLB

In this subsection, we show that the commitment of the central bank to keep the short-term nominal interest rate at the ZLB for an “extended period” amplifies the effects of LSAP II. According to our simulations, asset purchases boost GDP growth and increase inflation, thus leading the central bank to increase the FFR. This endogenous interest rate response mitigates the macroeconomic stimulus of asset purchase programs through the conventional monetary policy channel. A commitment to keep the short-term nominal interest rate at the ZLB for an “extended” period of time prevents the endogenous response of the monetary authority and magnifies the contribution of asset purchases on macroeconomic outcomes. Here, we quantify the magnitude of such a mitigation effect.

Figure 1.4 shows the responses in the case in which we do not impose the commitment to the zero lower bound. For reference, the dashed blue line corresponds to the baseline simulation with the commitment to the ZLB imposed. Quantitatively, the ZLB commitment more than triples the effects of asset purchases on GDP growth. Absent this commitment, output growth increases by 0.04%, compared to 0.13% in

³⁸Interestingly, the estimated degree of segmentation does not change appreciably between the two cases. Hence, the different effect on GDP growth is to attribute entirely to the elasticity of the risk premium to the quantity of debt.

the baseline experiment. Inflation increases by less than 2 bp (0.018%), compared to 3 bp in the baseline case.

Interestingly, while the profile for the FFR differs from the baseline, the 10-year yield is almost identical. The cumulative effect of the increase in short rates on long rates via the expectation-hypothesis component is the same. Without ZLB, the increase in nominal rates occurs earlier but is smoother.

Importantly, the responses to LSAP II remain skewed upward, regardless of whether the ZLB is imposed or not. This observation suggests that the role of the skewness in the posterior distribution of the degree of segmentation and of the semi-elasticity of the risk premium to the quantity of debt play a central role in explaining the upside uncertainty of the response of macroeconomic variables.

1.4.4 Comparison with a Standard Monetary Policy Shock

One of the motivations for central banks to engage in asset purchases is to support output and inflation at times in which the ZLB constrains conventional interest rate setting. To give a sense of the relative effectiveness of these two policies, this section compares the effects of asset purchase programs discussed so far with a standard monetary policy shock, that is, an unexpected reduction of the short-term nominal interest rate.

Figure 1.5 shows the response of the key macroeconomic variables to an unexpected reduction of 25 bp in the short-term interest rate. The median effect on GDP growth is somewhat stronger than in the baseline simulation previously discussed while the median effect on inflation is very much comparable.³⁹ Furthermore, the ef-

³⁹The median effects of the monetary policy shock on output and inflation in our model are slightly smaller than in standard estimated DSGE models, such as Smets and Wouters (2007). The key parameters that determine this result are the higher estimated degree of price rigidity and the lower sensitivity of demand to the interest rate (higher coefficient of risk aversion).

fects of the interest rate shock on the output level are not only stronger but also more persistent than those of LSAP. The implied decrease in long-term rates, however, is much smaller, only 1 bp.⁴⁰ Moreover, the long-term rate quickly turns positive. This result is not surprising given that the risk premium does not change. Therefore, the expectation hypothesis component completely pins down the long-term interest rate in this case.

Another significant difference is the smaller uncertainty about the effects in the case of an interest rate shock. The absence of the ZLB constraint may in part explain why the posterior bands are more symmetric. Yet, as discussed in the previous subsection, even in the absence of a commitment to the ZLB, uncertainty remains skewed upward, mostly due to the skewness of the posterior estimates of the degree segmentation and of the semi-elasticity of the term premium to the quantity of debt. Because asset market frictions play a smaller role in case of a shock to the short-term interest rate, the uncertainty in the response of GDP growth and inflation becomes smaller and much more symmetric.

Overall, in this model, the effects of LSAP II on output and inflation are slightly smaller than those of a surprise reduction of the FFR by 25 bp, and much more uncertain. This conclusion stands in contrast with Furher and Moore (1995), who find that output is four times more sensitive to long-term than short-term rates. According to this metric, the 11 bp reduction in the risk premium triggered by LSAP II should be equivalent to a reduction of the FFR of about 44 bp. Our results are thus much less generous to changes in the risk premium, confirming our previous finding that the model simulations yield weaker effects of LSAP II on output and inflation than what the VAR literature suggests.

⁴⁰Our estimates thus imply a much smaller sensitivity of long-term rates to shocks to the short-term rate. As a point of comparison, Gurkaynak, Sack, and Swanson (2005) estimate that the typical response of long rates to a cut of 100 bp in the FFR is 15 bp.

1.5 Conclusions

Using an estimated medium-scale DSGE model, we find that the effects of recent asset purchase programs on macroeconomic variables, such as GDP growth and inflation, are likely to be modest, although with a lasting impact on the level of GDP. Asset purchase programs are in principle effective at stimulating the economy because of limits to arbitrage and market segmentation between short-term and long-term government bonds. The data, however, provide little support for these frictions to be pervasive.

In the appendix, we consider several robustness exercises and find that the effects on GDP growth are not very likely to exceed a third of a percentage point. The inflationary consequences of asset purchase programs are consistently small. Combining LSAP programs with a commitment to keep interest rates low for some period of time allows these programs to be more effective in boosting GDP growth and inflation.

Our results do not depend on whether asset purchases are financed via reserves or sales of short-term debt, to the extent that money and short-term bonds are close to perfect substitutes. Therefore, according to our model, the effects of the Federal Reserve’s last round of asset purchases (also known as “Operation Twist Again”) should be in line with the estimates from LSAP II after controlling for the scale factor.

Table 1.1: Estimated Impact of LSAPs on the 10-Year Treasury Yield in the Literature.

Papers	Total Impact	Impact per \$100 Bil
Hamilton and Wu (2010)	-13 bp	-3 bp
Doh (2010)	-39 bp	-4 bp
D’Amico and King (2010)	-45 bp	-15 bp
Bomfim and Meyer (2010)	-60 bp	-3 bp
Gagnon, Raskin, Remache, and Sack (2011)	-58 bp to -91 bp	-3 bp to -5 bp
Neely (2010)	-107 bp	-6 bp
Krishnamurthy and Vissing-Jorgensen (2011)	-33 bp (LSAP2)	-5 bp
D’Amico, English, López-Salido, and Nelson (2011)	-55 bp (LSAP2)	-9 bp
Swanson (2011)	-15 bp (Twist)	

Table 1.2: Parameter Prior and Posterior Distribution: Structural Parameters.

	Dist	Prior			Posterior				
		5%	Median	95%	Mode	Mean	5%	Median	95%
400γ	G	1.7382	2.4667	3.3752	1.9922	1.9867	1.5413	1.9805	2.4515
400π	G	1.2545	1.9585	2.8871	2.2404	2.1477	1.5075	2.1376	2.8237
$400(\beta_u^{-1} - 1)$	G	0.6272	0.9792	1.4436	0.4943	0.4890	0.3282	0.4828	0.6706
400ζ	G	0.3913	0.7224	1.2029	0.4761	0.5127	0.2735	0.4968	0.8082
B^{LMV}/B	G	0.6953	0.9867	1.3501	0.8222	0.8502	0.7164	0.8533	0.9712
S''	G	2.5090	3.9170	5.7743	4.4277	4.8371	3.3764	4.7815	6.5330
a''	G	0.0683	0.1836	0.3877	0.2093	0.2322	0.0994	0.2159	0.4189
h	B	0.4302	0.6029	0.7597	0.8370	0.7898	0.6421	0.8020	0.9007
σ_u	G	0.6832	1.8360	3.8768	3.0151	3.4958	1.9891	3.3548	5.4795
σ_r	G	0.6832	1.8360	3.8768	1.5635	2.2370	0.8692	2.0825	4.1268
$100\zeta'$	G	0.3067	1.2846	3.4294	0.2420	0.3763	0.0862	0.3274	0.8257
ω_u	B	0.3214	0.7334	0.9646	0.9832	0.9322	0.8237	0.9468	0.9934
Ξ^u/Ξ^r	G	0.3416	0.9180	1.9384	0.7917	1.1403	0.4537	1.0730	2.0683
C^u/C^r	G	0.3416	0.9180	1.9384	0.7641	1.0533	0.3921	0.9760	1.9747
χ_{wu}	B	0.2486	0.6143	0.9024	0.5170	0.5566	0.2825	0.5611	0.8187
ν	G	1.2545	1.9585	2.8871	1.6814	1.9658	1.2518	1.9295	2.7996
ζ_w	B	0.3351	0.5000	0.6649	0.6860	0.7309	0.6292	0.7342	0.8205
ζ_p	B	0.3351	0.5000	0.6649	0.9260	0.9287	0.9116	0.9288	0.9452
ϕ_T	G	0.7825	1.4448	2.4058	1.2660	1.3147	0.6890	1.2543	2.1216
ρ_r	B	0.5242	0.7068	0.8525	0.8547	0.8556	0.8182	0.8565	0.8903
ϕ_π	G	1.0164	1.7026	2.6453	1.5644	1.6090	1.3706	1.5979	1.8866
ϕ_y	G	0.1366	0.3672	0.7754	0.2975	0.3295	0.2487	0.3251	0.4247

Table 1.3: Parameter Prior and Posterior Distribution: Shock Process Parameters.

	Dist	Prior			Posterior				
		5%	Median	95%	Mode	Mean	5%	Median	95%
ρ_z	B	0.0976	0.3857	0.7514	0.1286	0.1486	0.0448	0.1430	0.2706
ρ_μ	B	0.5701	0.7595	0.8971	0.8293	0.8477	0.7906	0.8489	0.8998
ρ_b	B	0.5701	0.7595	0.8971	0.9603	0.9597	0.9388	0.9614	0.9764
ρ_ϕ	B	0.5701	0.7595	0.8971	0.5200	0.5063	0.3767	0.5074	0.6345
ρ_B	B	0.6146	0.8135	0.9389	0.9773	0.9652	0.9396	0.9659	0.9880
ρ_ζ	B	0.6146	0.8135	0.9389	0.9614	0.9402	0.9012	0.9426	0.9700
ρ_g	B	0.5701	0.7595	0.8971	0.7581	0.7397	0.5542	0.7491	0.8943
σ_z	IG1	0.1663	0.3433	1.2367	0.7426	0.7624	0.6742	0.7588	0.8634
σ_{λ_f}	IG1	0.1663	0.3433	1.2367	1.6306	1.9235	1.1103	1.7930	3.1600
σ_μ	IG1	0.1663	0.3433	1.2367	2.8430	3.0102	2.2984	2.9725	3.8662
σ_b	IG1	0.1663	0.3433	1.2367	3.0703	3.7799	2.2957	3.5860	5.8901
σ_ϕ	IG1	0.1663	0.3433	1.2367	0.4518	0.9048	0.3100	0.7628	1.9982
σ_B	IG1	0.1663	0.3433	1.2367	0.3686	0.3656	0.2930	0.3578	0.4660
σ_T	IG1	0.1663	0.3433	1.2367	0.2348	0.5521	0.1670	0.3457	1.3682
σ_m	IG1	0.0819	0.1700	0.6217	0.1130	0.1167	0.1019	0.1161	0.1335
σ_ζ	IG1	0.0819	0.1700	0.6217	0.2137	0.2667	0.1900	0.2592	0.3694
σ_g	IG1	0.1663	0.3433	1.2367	0.2439	0.3894	0.1688	0.3429	0.7619

Figure 1.1: Simulated Path of the Market Value of Long-Term Debt.

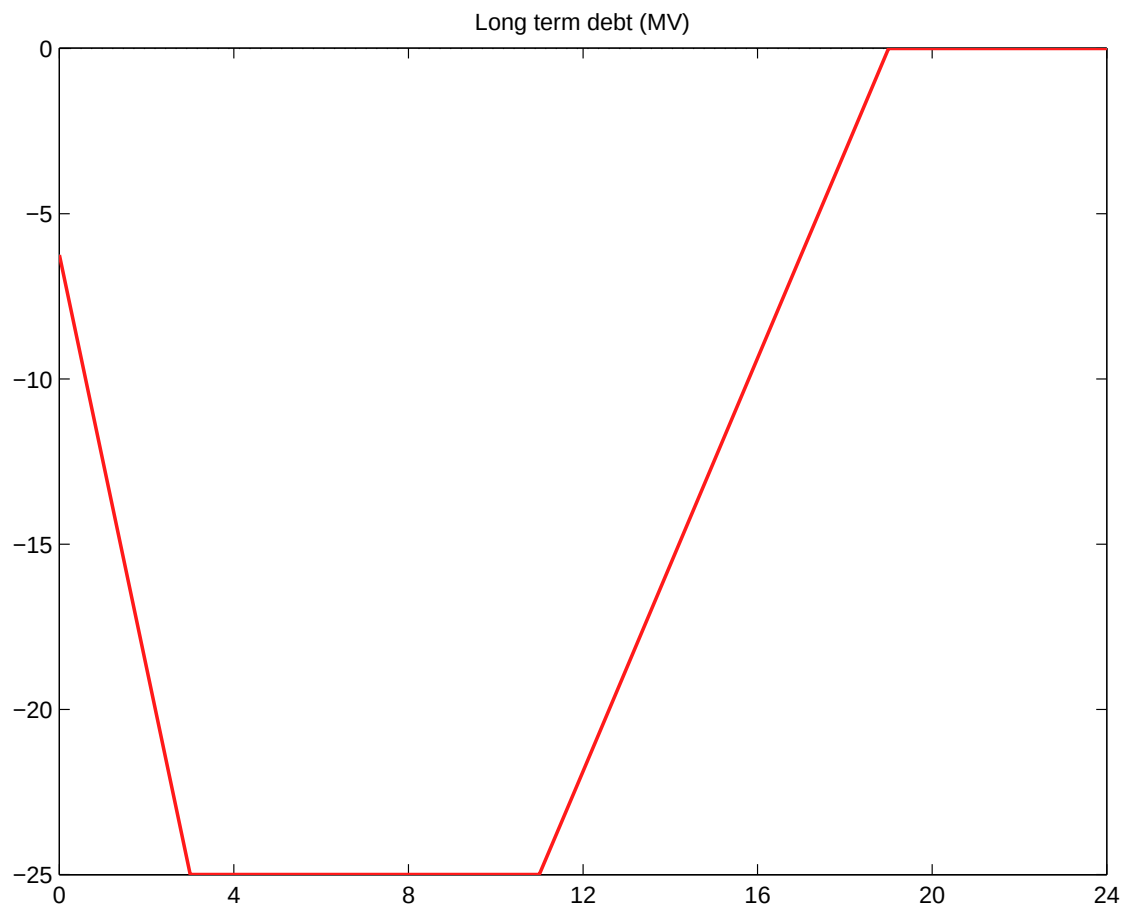


Figure 1.2: Responses to Calibrated LSAP II Experiment at the Prior Distribution.

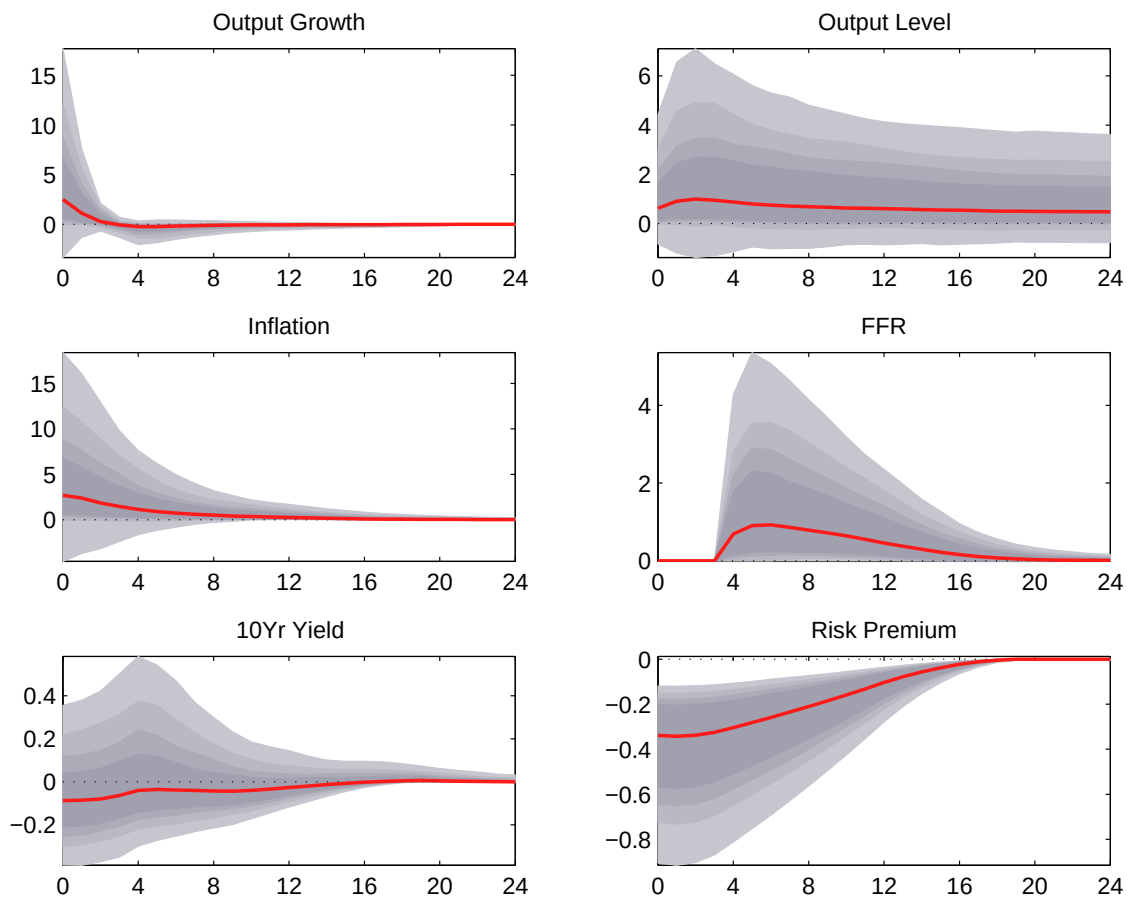


Figure 1.3: Responses to Calibrated LSAP II Experiment at the Posterior Distribution.

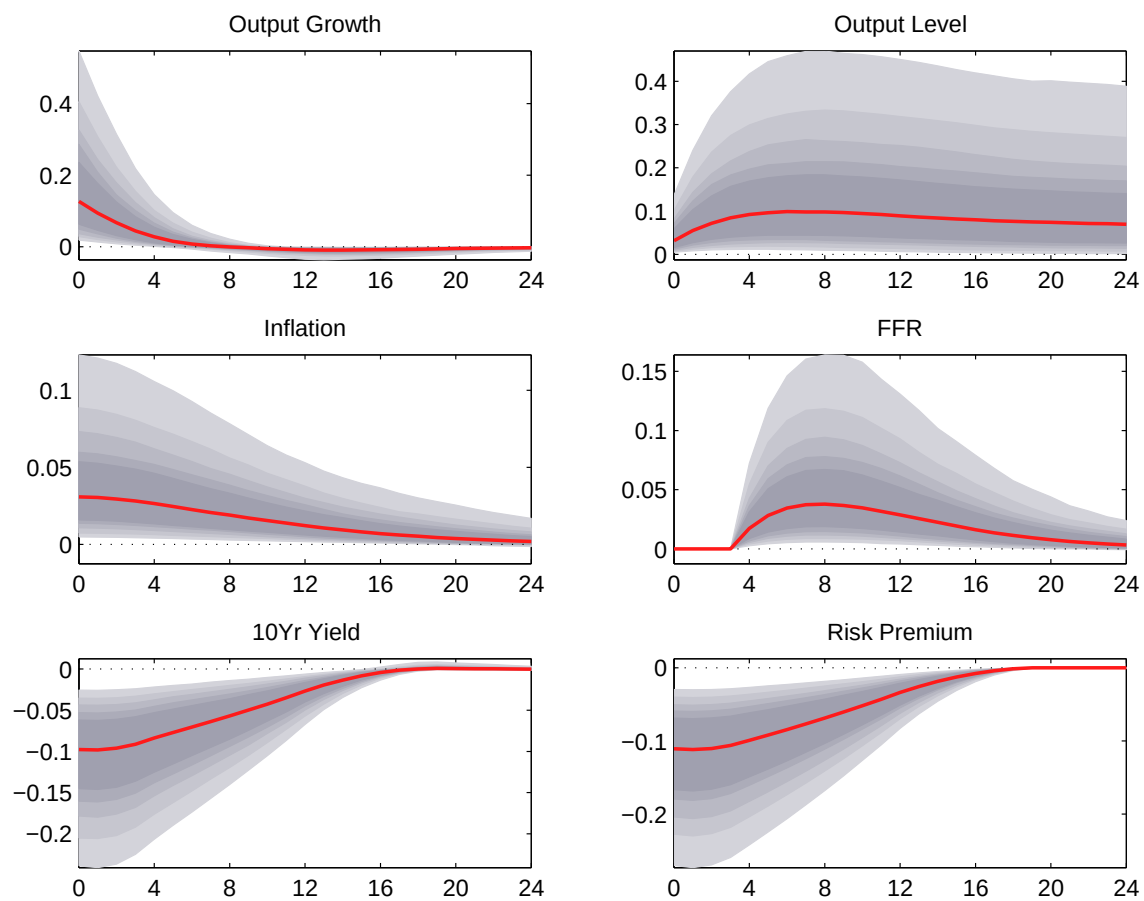


Figure 1.4: Responses to LSAP II Experiment with (dashed blue line) and without (continuous red line) ZLB Commitment.

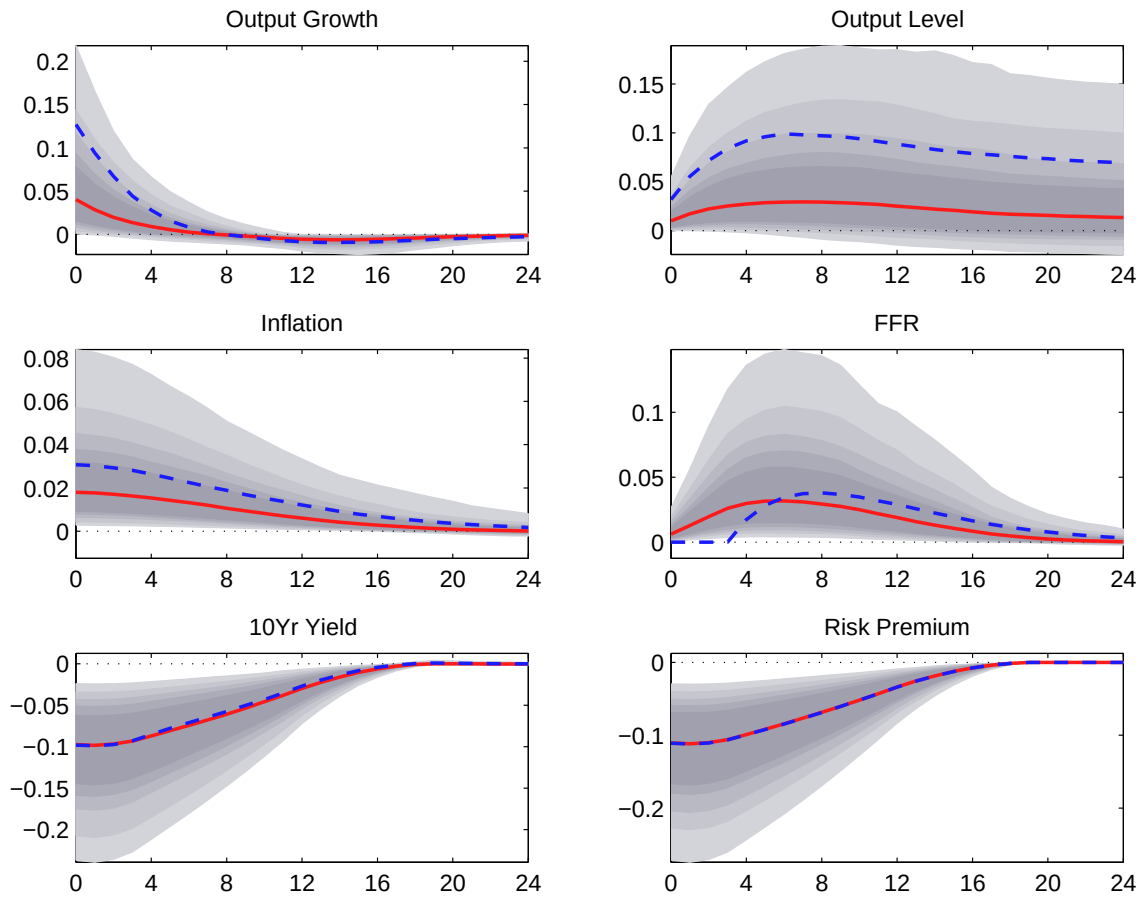
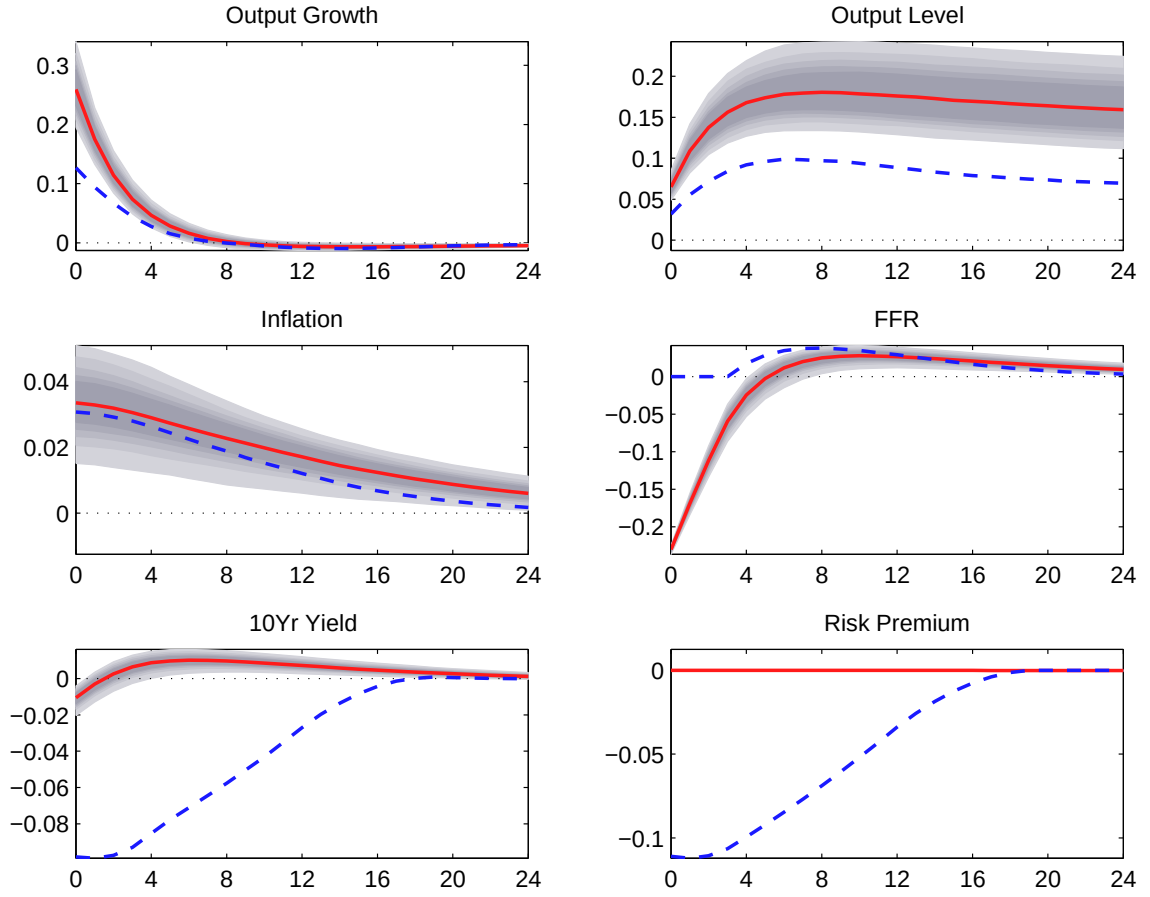


Figure 1.5: Responses to an Annualised 25 bp Innovation to the FFR (continuous red line), Compared to the Baseline LSAP II Experiment (dashed blue line).



Chapter 2

Assessing the Effects of Large-Scale Asset Purchases in a Zero-Interest-Rate Environment through the Lens of DSGE Models

2.1 Introduction

In this chapter, I study two types of DSGE models that break the neutrality of LSAPs as in chapter 1 and Harrison (2010) and two methods of modeling the ZIRP in DSGE models: the PFRE model and the Markov regime-switching model I develop in this chapter in order to better predict the distribution of macroeconomic variables. I fit those DSGE models to the U.S. data from the third quarter of 1987 to the second quarter of 2010, and then, starting from the third quarter of 2010, I simulate the U.S. economy forward under four scenarios: the counterfactual scenario when there is no policy intervention, only LSAP intervention, only ZIRP for an extended period, and

the combination of LSAPs and ZIRP. In order to assess the effectiveness of the asset purchase policy and the policy of an extended period of near-zero interest rates, I compare the predicted path of the macro variables (output and inflation) under the policy intervention with the predicted path of the macro variables absent of both asset purchase and ZIRP (the counterfactual scenario when there is no policy intervention). I found that the effects of the LSAPs alone are insignificant measured in the DSGE models, while the ZIRP has a substantial effect.

In chapter 1 the ZIRP is modeled by the PFRE model. This chapter proposes to model the ZIRP by a Markov regime-switching monetary policy rule where, in one regime, the policy rates follow a typical Taylor rule, and, in the other regime, it involves a policy of zero interest rates. I solve this regime-switching DSGE model by using the Farmer, Waggoner, and Zha (2011) minimum state variable solution. I construct the optimal filters in order to estimate this regime-switching DSGE model with Bayesian methods. I compare this method of modeling the ZIRP in DSGE models with the PFRE. The simulation of the Federal Reserve’s ZIRP reveals that the effects of ZIRP on macro variables crucially depend on the models: the regime-switching model implies a substantial effect of ZIRP. PFRE implies a five-fold stronger stimulus of ZIRP to inflation. The fundamental difference between these two types of models is how agents’ expectations are formulated. In the Markov regime-switching model, at each period agents attach certain probability of exiting the ZIRP regime in the next period despite the Federal Reserve’s “extended period” language, because, for example, the simple announcement would be subject to the time inconsistency problem, and is thus incredible. The PFRE assumes that agents believe the Federal Reserve’s announcement and have perfect foresight of future interest rates. The predicted path of macro variables generated by the regime-switching model is closer to the actual path.

Here, I am looking at this extended period of zero interest rates as a policy choice⁴¹ because the central bank could raise the interest rates when the output starts growing, and the economy is improving as advised by the Taylor rule. Carlstrom, Fuerst, and Paustian (2012), Cúrdia and Woodford (2010), and chapter 1 also study the effects of a *transient* interest rate peg. Under the assumption of either a deterministic exit or a stochastic exit of the interest rate peg in the previous studies, the policy rate will follow a Taylor rule after the exit and the interest rate peg will never occur again. In my regime-switching model, however, zero interest rate policy regime is a recurring event. Even at the normal interest rate regime, agents expect to enter zero interest rate regime in the future with certain probability. Expectations play an important role in the regime-switching model. An alternative angle to look at this persistent period of low interest rates is the zero lower bound (ZLB) problem. A persistent shock⁴² drives interest rates below zero if the central bank keeps following a Taylor rule. A rapidly growing literature on ZLB considers the zero interest rates as a modeling constraint that has to be considered. Global methods include Judd, Maliar, and Maliar (2011), Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012), and Aruoba and Schorfheide (2012). There are also a few short cuts for modeling ZLB: such as Braun and Korber (2011), Adam and Billi (2007), Eggertsson and Woodford (2003), and Christiano, Eichenbaum, and Rebelo (2011). Del Negro and Schorfheide (2012) describe how to impose zero interest rates via unanticipated or anticipated monetary policy shocks in a DSGE model.

The rest of the chapter proceeds as follows. Since chapter 1 and Appendix A describe the market segmentation model extensively, the next section only presents the bonds-in-utility model where the LSAPs' neutrality result can also be broken in

⁴¹Here the regime-switching is exogenous while ideally it should be endogenous and depend on the macroeconomic condition.

⁴²For example a preference shock or a technology shock.

the DSGE models. Section 2.3 discusses how to model ZIRP with a regime-switching monetary policy and with the PFRE. Section 2.4 describes the estimation of the regime-switching model, some basic analysis of parameter estimates, an evaluation of the effects of the LSAPs and the ZIRP, and the comparison between the regime-switching model and the PFRE model. Finally, section 2.5 concludes.

2.2 Models

In the households sector, I will explain how the typical no-arbitrage condition for short-term and long-term bonds can be broken in order for LSAPII⁴³ to have a real effect. I will describe a variation of Harrison (2010). The rest of the sectors are standard in medium-scale DSGE models (Christiano, Eichenbaum, and Evans (2005); Smets and Wouters (2007)) and the detailed description can be found in Appendix A: Monopolistic competitive firms hire the labor to produce intermediate goods; competitive final goods producing firms package intermediate goods into a homogeneous consumption good. Finally, the government sets monetary and fiscal policy. To simplify the analysis, I abstract from capital and wage stickiness.

2.2.1 Households

A common means by which the asset purchases can be effective is that if the central bank changes its portfolio composition in equilibrium, private investor must also change their portfolio choices, and, in order to induce them to do so, the equilibrium asset prices must also change accordingly. However, a mere difference in state-contingent returns on different assets is not enough for central bank portfolio changes

⁴³Notice that through out the thesis I only concentrate on the second round of LSAPs whose purpose is to bring down long-term interest rates and boost economic growth.

to have an effect because the private investors will fully take advantage of the arbitrage opportunities and hedge against the central bank's operation. Cúrdia and Woodford (2011) present a detailed explanation for this. This neutrality result only depends on two postulates: All investors can buy or sell the same assets at the same market prices, and all assets are valued only for their pecuniary returns. Chapter 1 proposes market segmentation to break the first postulate while Harrison (2010) targets the second postulate. Both approaches are based on Andrés, López-Salido, and Nelson (2004). Throughout the chapter, I will refer to the first approach as “market segmentation” approach and the second as the “BIU” (bonds-in-utility) approach.

Bonds-in-Utility

The representative household's objective function is a slight modification of Harrison (2010):

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[\frac{\left(\frac{C_{t+s}}{Z_{t+s}} \right)^{1-\sigma}}{1-\sigma} - \frac{\varphi_{t+s} L_{t+s}^{1+\nu}}{1+\nu} - \frac{\tilde{\nu}}{2} \left(\delta \frac{B_{t+s}}{P_{L,t+s} B_{L,t+s}} - 1 \right)^2 \right],$$

where in the last term $\frac{B_{t+s}}{P_{L,t+s} B_{L,t+s}}$ represents the ratio of the market value of short-term bonds to that of long-term bonds. δ is the inverse of the steady state of this ratio so that at steady state, the last term is zero. $\tilde{\nu}$ controls the elasticity of the households' portfolio choice in response to the long-term bond rate. The intuition of bonds-in-utility is similar to money-in-utility. Because long-term bonds are not as liquid as short-term bonds, holding a non-optimal portfolio composition induces a utility cost.

The time t budget constraint for a household is

$$P_t C_t + B_t + (1 + \zeta_t) P_{L,t} B_t^L \leq R_{t-1} B_{t-1} + P_{L,t} R_{L,t} B_{t-1}^L + W_t L_t + \mathcal{P}_t + \mathcal{P}_t^{fi} - T_t, \quad (2.1)$$

where, ζ_t , is also a transaction cost (but *not* a function of the bonds) with a nonzero steady state. This is to capture that, at steady state, the yield of the long-term bonds is higher than that of the short-term bonds, as observed in the data. The definitions of the rest of the variables are the same as the market segmentation model described in the previous chapter.

Let Ξ_t^P represent the Lagrange multiplier for (2.1). The loglinearized Euler equation for the short-term bonds is

$$\frac{\tilde{\nu}}{\Xi_{B_z}} \widehat{BLMV} B_t - \hat{\Xi}_t + \hat{R}_t + \hat{\Xi}_{t+1} - \hat{z}_{t+1} - \hat{\Pi}_{t+1} = 0,$$

where $\widehat{BLMV} B_t = \frac{\frac{B_{z,t}^L}{(R_{L,t}-\kappa)}}{B_{z,t}} 44$, and $\widehat{BLMV} B_t = \hat{B}_{z,t}^L - \hat{B}_{z,t} - \frac{R_L}{(R_L-\kappa)} \hat{R}_{L,t}$. And the loglinearized Euler equation for the long-term bonds is

$$\frac{\tilde{\nu}}{\delta(1+\zeta)\Xi_{B_z}} \widehat{BLMV} B_t + \hat{\Xi}_t + \hat{\zeta}_t - \frac{R_L}{R_L - \kappa} \hat{R}_{L,t} + \mathbb{E}_t \left[\frac{\kappa}{R_L - \kappa} \hat{R}_{L,t+1} - \hat{\Xi}_{t+1} + \hat{z}_{t+1} + \hat{\Pi}_{t+1} \right] = 0.$$

The BIU specification distinguishably differs from the market segmentation approach by allowing the portfolio choice to directly affect the households' consumption choice. This, in turn, will affect the stochastic discount factor and thus the price of the long-term bond. Again, LSAPs are designed to have a real effect. The advantage of this specification is its simplicity. Household heterogeneity dramatically increases the scale of the market segmentation model, and thus estimating and drawing from the posterior of the market segmentation model are challenging, while the BIU specification is a lot more manageable.

⁴⁴ $B_{z,t}^L = \frac{B_t^L}{P_t Z_t}$, and $B_{z,t} = \frac{B_t}{P_t Z_t}$.

2.2.2 Government Policies

The monetary policy is taken from the chapter 1. The central bank follows a conventional feedback interest rate rule similar to Taylor (1993), modified to include the interest rate smoothing (Clarida, Galí, and Gertler (2000)) and to use the growth rate of output instead of the output gap (Justiniano, Primiceri, and Tambalotti (2011)):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_m} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}} \right)^{\phi_y} \right]^{1-\rho_m} e^{\epsilon_{m,t}}, \quad (2.2)$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the inflation rate, $\rho_m \in (0, 1)$, $\phi_\pi > 1$, $\phi_y \geq 0$, and $\epsilon_{m,t}$ is an i.i.d. innovation.⁴⁵ In the section (2.3.1), I will elaborate how to modify the monetary policy rule to assess ZIRP.

The presence of long-term bonds modifies the standard government budget constraint as follows:

$$B_t + P_{L,t} B_t^L = R_{t-1,t} B_{t-1} + (1 + \kappa P_{L,t}) B_{t-1}^L + P_t G_t - T_t. \quad (2.3)$$

The left-hand side of expression (2.3) is the market value, in nominal terms, of the total amount of bonds (short-term and long-term) issued by the government at time t . The right-hand side is the total deficit at time t , that is, market value plus interest payment of the bonds maturing in that period plus spending G_t net of taxes.

I assume that the supply of the government bonds is exogenous, and the ratio of the market value of long-term bonds to that of the short-term bonds follows a simple

⁴⁵Chapter 1 uses the output growth in the Taylor rule, instead of the output gap, to avoid the complication of solving and estimating the system characterizing the flexible price equilibrium. In practice, GDP growth relative to trend is often cited as one of the main indicators of real activity for the conduct of monetary policy.

autoregressive rule

$$\frac{P_{L,t}B_t^L}{B_t} = S \left(\frac{P_{L,t-1}B_{t-1}^L}{B_{t-1}} \right)^{\rho_B} e^{\epsilon_{B,t}}, \quad (2.4)$$

where $\rho_B \in (0, 1)$, and $\epsilon_{B,t}$ is an i.i.d. exogenous supply shock. S is whatever constant needed to make the above equation an identity at the steady state. I interpret LSAPs program as shocks to the ratio of outstanding government long-term liabilities to short-term liabilities compared to the historical behavior of these series.

2.2.3 Exogenous Processes

The model is supposed to be fitted to data on output, inflation, hours worked, wages, nominal interest rates, and market value of bonds. There are seven structural shocks in total. The logarithm of the technology follows a random walk with drift.

$$\ln Z_t = \gamma + \ln Z_{t-1} + z_t,$$

where the shock z_t follows a first order autoregressive process (AR(1)):

$$z_t = \rho_z z_t + \epsilon_{z,t}.$$

The preference shock to leisure follows an AR(1) process:

$$\ln \varphi_t = \rho_\varphi \ln \varphi_{t-1} + \epsilon_{\varphi,t}.$$

The shock to the discount factor β (intertemporal preference shifter) is also assumed to follow an AR(1) process:

$$\ln b_t = \rho_b \ln b_{t-1} + \epsilon_{b,t}.$$

The government spending is assumed to be an exogenous process:

$$\ln g_t = \rho_g \ln g_{t-1} + \epsilon_{g,t}.$$

The risk premium shock also follows an AR(1) process:

$$\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_{\zeta,t}.$$

The monetary policy shock $\epsilon_{m,t}$ and the bond supply shock $\epsilon_{B,t}$ are independent and identically distributed shocks.

2.3 Zero Interest Rate Policy

In this section, I describe two methods of studying the effects of ZIRP in DSGE models. Both solution methods take some shortcuts rather than solve fully a nonlinear New Keynesian model incorporating ZIRP. I am going to consider a regime-switching model where, in one regime, the policy rate follows a typical Taylor rule, and, in the other regime, it simply involves ZIRP. Although the regime switching is imposed to the monetary policy rule before loglinearizing the system, the model is a forward-looking Markov-switching linear rational expectations model. Ideally, I should apply the perturbation method for Markov-switching models proposed by Foerster, Rubio-Ramírez, Waggoner, and Zha (2012). Their method begins from first principles rather than add Markov switching after linearizing the model, and it also allows higher order solutions. Simplifying assumptions in my model may miss some nonlinear interactions between the zero interest rates and the policy functions of the agents, however, I substantially gain tractability. I also construct the optimal filter so that I can fit this model to the macro data including the recent time where the interest rates are

maintained near zero for an extended period. This regime-switching model can not only explain the interest rate data, but also provides a plausible explanation for exiting the zero interest rate policy. This regime-switching model offers a tool to conduct forecasts and counterfactual analysis. The other approach to assessing the ZIRP, PFRE, on the other hand, can not explain the recent episodes of near-zero interest rates. It only asks the counterfactual questions such as what are the effects to the macro variables if the interest rates are kept at zero for an extended period, and agents have perfect knowledge of this policy experiment? Now I define the regime-switching model more precisely .

2.3.1 Regime-Switching Policy Rule

In this section, I introduce a regime-switching monetary policy rule that will be incorporated into the DSGE models introduced in section 2.2. I will use the Farmer, Waggoner, and Zha (2011) minimum state variable solution method to solve this regime-switching model, and the estimation strategy will be described in section 2.4.

Consider a regime-switching policy rule where, in one regime, the federal funds rate follows a Taylor rule while, in the other regime, it simply involves the zero interest rates. The policy rule is

$$R_t = (R_t^*(K_t))^{1-\rho_R(K_t)} \left[\left(\frac{\pi_t r}{R_t^*(K_t)} \right)^{\varphi_\pi(K_t)} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}} \right)^{\varphi_y(K_t)} \right]^{(1-\rho_R(K_t))} R_{t-1}^{\rho_R(K_t)} \exp(\varepsilon_{R,t}), \quad (2.5)$$

where all the parameters denoted by (K_t) are regime dependent, and R_t^* are the desired regime-dependent target nominal interest rates. Let $K_t = 1$ denote the normal regime, and $K_t = 2$ denote the ZIRP regime. For example, I can set $R_t^*(K_t = 1) = R_1^* = 1.005$ which corresponds to a target 2% annual interest rate at the normal

regime, and set $R_t^* (K_t = 2) = R_2^* = 1.0005$ which corresponds to a target 20 basis points annual interest rate at the second regime. To study the ZIRP, I set

$$\begin{aligned} R_2^* &= 1, \\ \rho_R (K_t = 2) &= 0, \\ \varphi_\pi (K_t = 2) &= 0, \\ \varphi_y (K_t = 2) &= 0, \\ \sigma_{\varepsilon_{R,t}} (K_t = 2) &= 0. \end{aligned}$$

I define the ergodic mean of the logarithm of the steady state interest rates as

$$\log (R) = \bar{\lambda}_1 \log (R_1^*) + \bar{\lambda}_2 \log (R_2^*),$$

where $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are ergodic probabilities.

Divide 2.5 by its ergodic mean, R , and thus:

$$\frac{R_t}{R} = \left(\frac{R_t^*}{R} \right)^{(1-\rho_R(K_t))(1-\varphi_\pi(K_t))} \left[\left(\frac{\pi_t}{\pi} \right)^{\varphi_\pi(K_t)} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}} \right)^{\varphi_y(K_t)} \right]^{(1-\rho_R(K_t))} \left(\frac{R_{t-1}}{R} \right)^{\rho_R(K_t)} \exp \varepsilon_{R,t}. \quad (2.6)$$

Loglinearize 2.6 and thus:

$$\begin{aligned} \hat{R}_t &= \rho_R (K_t) \hat{R}_{t-1} + (1 - \rho_R (K_t)) \left[\varphi_\pi (K_t) \hat{\pi}_t + \varphi_y (K_t) \left(\hat{y}_t - \hat{y}_{t-4} + \sum_{i=0}^{i=3} \hat{z}_{t-i} \right) \right] \\ &\quad + \varepsilon_{R,t} + (1 - \rho_R (K_t)) (1 - \varphi_\pi (K_t)) \hat{R}_t^*, \end{aligned} \quad (2.7)$$

where the last term represents a regime-switching constant. The Farmer, Waggoner, and Zha (2011) minimum state variable solution method does not deal with a system with a constant. I am going to apply the trick by Liu, Waggoner, and Zha (2011). They solve a system where the only regime-switching coefficient is the constant. I

can rewrite 2.7 as

$$\begin{aligned}\hat{R}_t &= \rho_R(K_t) \hat{R}_{t-1} + (1 - \rho_R(K_t)) \left[\varphi_\pi(K_t) \hat{\pi}_t + \varphi_y(K_t) \left(\hat{y}_t - \hat{y}_{t-4} + \sum_{i=0}^{i=3} \hat{z}_{t-i} \right) \right] + \varepsilon_{R,t} \\ &\quad + (1 - \rho_R(K_t)) (1 - \varphi_\pi(K_t)) \left[\log\left(\frac{R_1}{R}\right), \log\left(\frac{R_2}{R}\right) \right] \hat{\mathbf{e}}_{s,t},\end{aligned}$$

where $\hat{\mathbf{e}}_{s,t} = \mathbf{e}_{s,t} - \bar{\mathbf{e}}_s$, and $\bar{\mathbf{e}}_s$ is the ergodic probability. $\mathbf{e}_{s,t}$ is defined as:

$$\mathbf{e}_{s,t} = \begin{bmatrix} \mathbf{1}_{S_t=1} \\ \mathbf{1}_{S_t=2} \end{bmatrix},$$

with $\mathbf{1}\{s_t = j\} = 1$ if $s_t = j$, and 0 otherwise. As shown in Hamilton (1994), the random vector $\mathbf{e}_{s,t}$ follows an AR(1) process:

$$\mathbf{e}_{s,t} = P\mathbf{e}_{s,t-1} + \nu_t, \tag{2.8}$$

where P is the transition matrix of the Markov switching process, and the innovation vector has the property that $\mathbb{E}_{t-1}\nu_t = 0$. In the steady state, $\nu_t = 0$ so that 2.8 defines the ergodic probabilities for the Markov process $\bar{\mathbf{e}}_s$. Schorfheide (2005) also proposes an algorithm to solve DSGE models with a regime-switching constant in the policy rule. One can prove that Schorfheide (2005) and Liu, Waggoner, and Zha (2011) give rise to the same solution⁴⁶.

By adding two extra variables $\mathbf{e}_{s,t}$, I can use the Farmer, Waggoner, and Zha (2011) minimum state variable solution to solve this regime-switching model. The

⁴⁶See the appendix for proof.

solution of the model can be represented by

$$\begin{aligned} Z_t &= G_t(K_t) Z_{t-1} + R_t(K_t) \varepsilon_t \\ \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} &= \begin{bmatrix} G_{11} & G_{12} \\ 0 & P \end{bmatrix} \begin{bmatrix} Z_{1,t-1} \\ Z_{2,t-1} \end{bmatrix} + \begin{bmatrix} R_{11} & R_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \end{aligned}$$

where I can partition the variables Z_t and the shocks ε_t into two parts: $Z_{2,t}$ is $[\hat{e}_t(1) \ \hat{e}_t(2)]'$, $\varepsilon_{2,t}$ is $[v_{1,t} \ v_{2,t}]'$, $Z_{1,t}$ are the rest of the states, and $\varepsilon_{1,t}$ are the structural shocks of the DSGE models. I define

$$C(K_t) = G_{12} [\hat{e}_{t-1}(1) \ \hat{e}_{t-1}(2)]' + R_{12} [v_{1,t} \ v_{2,t}]'.$$

Notice that $C(K_t)$ is a regime-dependent constant. Finally I can rewrite the system as follows with regime-switching coefficients:

$$Z_t = C(K_t) + G_t(K_t) Z_{t-1} + R_t(K_t) \varepsilon_t.$$

2.3.2 Model ZIRP by the PFRE

The solution method of the PFRE model was proposed by Cúrdia and Woodford (2011). For a detailed description of the algorithm and an application, please refer to section A.6 or online appendix⁴⁷. The basic idea is that agents have perfect foresight of the path of the future interest rates and of all shocks until an arbitrary time point. From this point forward all the shocks are zero, and the solution method is standard such as Sims (2002). The system can be solved backwards from this point. The following is a very simple example to illustrate the solution method. Consider the

⁴⁷The appendix can be found at <http://onlinelibrary.wiley.com/doi/10.1111/j.1468-0297.2012.02549.x/supinfo>

equilibrium system:

$$\hat{y}_t = \mathbb{E}_t [\hat{y}_{t+1}] - \sigma^{-1} (\hat{i}_t - \mathbb{E}_t [\hat{\pi}_{t+1}]),$$

$$\hat{\pi}_t = \beta \mathbb{E}_t [\hat{\pi}_{t+1}] + \kappa \hat{y}_t,$$

and

$$\begin{aligned} \hat{i}_t &= \phi_\pi \hat{\pi}_t + \nu_t, \quad \text{for } t > K, \nu_t = 0, \\ &= 0, \quad \text{for } t = 1, \dots, K-1, K. \end{aligned}$$

The solution for $t > K$ is

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \end{bmatrix} = \begin{bmatrix} \psi_{y\nu} \\ \psi_{\pi\nu} \\ \psi_{i\nu} \end{bmatrix} \nu_t.$$

The system can be broken into the forward-looking and the backward-looking parts.

The forward-looking part is

$$\begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \mathbb{E}_t [\hat{y}_{t+1}] \\ \mathbb{E}_t [\hat{\pi}_{t+1}] \end{bmatrix} = \begin{bmatrix} 1 & \sigma^{-1} & 0 \\ -\kappa & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{\pi}_t \end{bmatrix},$$

and the backward-looking part is

$$\begin{bmatrix} 0 & 1 & -\phi_\pi \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{\pi}_t \end{bmatrix} = \nu_t.$$

At $t = K$, plug in the solution to the forward looking part and thus:

$$\begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \psi_{y\nu} \mathbb{E}_t \nu_{t+1} \\ \psi_{\pi\nu} \mathbb{E}_t \nu_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \sigma^{-1} & 0 \\ -\kappa & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{\pi}_t \end{bmatrix}.$$

Combine this with the backward looking part and thus:

$$\begin{bmatrix} 0 & 1 & -\phi_\pi \\ 1 & \sigma^{-1} & 0 \\ -\kappa & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 \\ (\psi_{y\nu} + \sigma^{-1} \psi_{\pi\nu}) \mathbb{E}_t \nu_{t+1} \\ \beta \psi_{\pi\nu} \mathbb{E}_t \nu_{t+1} \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \\ 0 \end{bmatrix}.$$

We can solve this system by inverting a matrix. The solution is

$$\begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & -\phi_\pi \\ 1 & \sigma^{-1} & 0 \\ -\kappa & 0 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} 0 \\ (\psi_{y\nu} + \sigma^{-1} \psi_{\pi\nu}) \mathbb{E}_t \nu_{t+1} \\ \beta \psi_{\pi\nu} \mathbb{E}_t \nu_{t+1} \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \\ 0 \end{bmatrix} \right).$$

We can iterate backwards until the first period.

2.4 Empirical Analysis

In this section, I compare two methods of modeling LSAPs and two approaches to modeling ZIRP in DSGE models. Since chapter 1 studies the market segmentation model carefully, I will only briefly show results. Here, I estimate the bonds-in-utility DSGE model that either incorporates a regime-switching monetary policy as 2.5 or a typical Taylor rule as 2.2. I extract the filtered states of those estimated DSGE models, and then, starting from the third quarter of 2010, I simulate the U.S. economy forward under four scenarios: no intervention and no shocks, only LSAP intervention,

only ZIRP for an extended period, and the combination of the LSAPs and the ZIRP for an extended period. I compare the predicted path of macro variables generated from the different models. When I evaluate ZIRP in the DSGE model with the regular Taylor rule, the PFRE method is used to simulate the economy. I will only explicate the estimation strategy of the regime-switching DSGE model. The description of the estimation procedure of the other non regime-switching model was omitted here. The Bayesian estimation methods for a linearized DSGE model with constant coefficients can be found, for example by An and Schorfheide (2007a). Bayesian estimation combines prior information on the parameters with the likelihood function of the model to form the posterior distribution. In the regime-switching model, the optimal filter is no longer the Kalman Filter. I will first illustrate the optimal filter and the likelihood function for this regime-switching model, and then describe data, show estimation results, and make comparisons of simulation results.

2.4.1 Optimal Filter and Likelihood Function

Regime-switching model is complicated because usually we have to keep track of the long history of the distribution of the states, and the number of the states grows exponentially⁴⁸. Fortunately, in my application, the distribution of the states at each time is degenerated, because I observe the interest rates, and thus deduce whether or not the economy is at the ZIRP regime in that period.

In this New Keynesian economy, the states are denoted by $\mathbf{S}_t$ and the observables are denoted by $\mathbf{y}_t$. Let K_t denote the Markov regime-switching states and λ_t denote the probability at the ZIRP regime $K_t = 2$ at time t , thus $K_t = 1$, the normal regime, has probability $1 - \lambda_t$. Let $\hat{R}_t$ denote the log deviation of the regime-switching interest

⁴⁸Even with a 2-state Markov regime switching process, at time t , the number of states is 2^t .

rates from their ergodic mean. Its density function can be written as:

$$P\left(\hat{R}_t\right)=\lambda_t^{\mathbf{1}_{\{R_t=0\}}}\left((1-\lambda_t) f_t\left(\hat{R}_t\right)\right)^{\mathbf{1}_{\{R_t>0\}}},$$

where $f_t\left(\hat{R}_t\right)$ is the conditional density, conditional on at the normal state. That is

$$P\left(\hat{R}_t|R_t>0\right)=f_t\left(\hat{R}_t\right).$$

Define the Dirac function as

$$\delta_{\tilde{x}}(x)=\left\{\begin{array}{ll} 0 & \text{if } x \neq \tilde{x} \\ \infty & \text{if } x = \tilde{x} \end{array}\right. \quad \text{and} \quad \int \delta_{\tilde{x}}(x) dx = 1.$$

Using the Dirac function, I can express the density of the interest rates as

$$P\left(\hat{R}_t\right)=\lambda_t \delta_{\tilde{x}}(x)+(1-\lambda_t) f_t\left(\hat{R}_t\right).$$

The transition equations are

$$\mathbf{S}_t\left(K_t\right)=C\left(K_t\right)+G_t\left(K_t\right) \mathbf{S}_{t-1}\left(K_{t-1}\right)+R_t\left(K_t\right) \varepsilon_t.$$

where all the coefficients are regime-dependent and the measurement equations are (no measurement error):

$$\mathbf{y}_t\left(K_t\right)=T \mathbf{S}_t\left(K_t\right).$$

Let $\bar{\lambda}$ denote the ergodic probability of the Markov chain and Σ_k denote the state-dependent variance-covariance matrix of the structural shocks:

$$\Sigma_k=E\left[\varepsilon_t \varepsilon_t'|K_t=k\right].$$

The algorithm of the optimal filter is as follows:

- Initializing at time $t = 1$, the mean of the states:

$$\bar{\mathbf{S}}_1 = \bar{\lambda}_1 (I - G(K_t = 1))^{-1} C(K_t = 1) + (1 - \bar{\lambda}_1) (I - G(K_t = 2))^{-1} C(K_t = 2),$$

and the variance,

$$\bar{P}_1 = \bar{\lambda}_1 X_1 + (1 - \bar{\lambda}_1) X_2,$$

where X_1 and X_2 solve the discrete Lyapunov matrix equations:

$$G(K_t = 1) X_1 G(K_t = 1)' - X_1 + R(K_t = 1) \Sigma_1 R(K_t = 1) = 0$$

and

$$G(K_t = 2) X_2 G(K_t = 2)' - X_2 + R(K_t = 2) \Sigma_2 R(K_t = 2) = 0$$

respectively.

- Forecasting $t + 1$ given t

– Transition equation

$$\begin{aligned}
& P(S_{t+1}, K_{t+1} | Y^t, \theta) \\
&= \int P(S_{t+1}, K_{t+1} | S_t, K_t) P(S_t, K_t | Y^t, \theta) d(S_t, K_t) \\
&= \int P(S_{t+1, -\hat{R}_{t+1}}, K_{t+1} | \hat{R}_{t+1}, S_t, K_t) P(\hat{R}_{t+1}, K_{t+1} | S_t, K_t) P(S_t, K_t | Y^t, \theta) d(S_t, K_t) \\
&= \int P(S_{t+1, -\hat{R}_{t+1}} | K_{t+1}, S_t, K_t) P(\hat{R}_{t+1} | K_{t+1}, S_t, K_t) P(K_{t+1} | S_t, K_t) P(S_t, K_t | Y^t, \theta) d(S_t, K_t) \\
&= \int P(S_{t+1, -\hat{R}_{t+1}} | K_{t+1} = 2, S_t, K_t) \delta_0(\hat{R}_{t+1} = 0) P(K_{t+1} = 2 | S_t, K_t) P(S_t, K_t | Y^t, \theta) d(S_t, K_t) \\
&\quad + \int P(S_{t+1} | K_{t+1} = 1, S_t, K_t) P(K_{t+1} = 1 | S_t, K_t) P(S_t, K_t | Y^t, \theta) d(S_t, K_t),
\end{aligned}$$

where $S_{t+1, -\hat{R}_{t+1}}$ denotes all the states excluding the interest rates. Since the density of the regime K_{t+1} , conditional on the last period states and regime, $P(K_{t+1} | S_t, K_t)$, is discrete, I can break the integral into two parts when it is in a ZIRP regime, and when it is in the normal regime. Notice that when it is in the ZIRP regime, I do not need to track the distribution of interest rates, because it is degenerated.

- – Measurement equation $\implies$ likelihood function

$$\begin{aligned}
& P(y_{t+1} | Y^t, \theta) \\
&= \int P(y_{t+1} | S_{t+1}, K_{t+1}, Y^t, \theta) P(S_{t+1} | K_{t+1}, Y^t, \theta) P(K_{t+1} | Y^t, \theta) dS_{t+1} dK_{t+1} \\
&= P(K_{t+1} = 1 | Y^t, \theta) \int P(y_{t+1} | S_{t+1}, K_{t+1}, Y^t, \theta) P(S_{t+1} | K_{t+1}, Y^t, \theta) dS_{t+1} \\
&\quad + P(K_{t+1} = 2 | Y^t, \theta) \int P(y_{t+1, -\hat{R}_{t+1}} | S_{t+1}, K_{t+1}, Y^t, \theta) P(S_{t+1} | K_{t+1}, Y^t, \theta) dS_{t+1}.
\end{aligned}$$

- Updating

- Updating states

$$\begin{aligned}
& P(S_{t+1}, K_{t+1} | Y^{t+1}, \theta) \\
& \propto P(y_{t+1} | S_{t+1}, K_{t+1}, Y^t, \theta) P(S_{t+1}, K_{t+1} | Y^t, \theta) \\
& \propto P(y_{t+1} | S_{t+1}, K_{t+1}, Y^t, \theta) P(S_{t+1} | K_{t+1}, Y^t, \theta) P(K_{t+1} | Y^t, \theta) \\
& \propto P(y_{t+1} | S_{t+1}, K_{t+1}, Y^t, \theta) P(S_{t+1} | K_{t+1}, Y^t, \theta) P(K_{t+1} = 1 | Y^t, \theta) \\
& \quad + P(y_{t+1} - \hat{R}_{t+1} | S_{t+1} - \hat{R}_{t+1}, K_{t+1}, Y^t, \theta) P(S_{t+1} - \hat{R}_{t+1} | K_{t+1}, Y^t, \theta) P(K_{t+1} = 2 | Y^t, \theta).
\end{aligned}$$

- – Updating states probability

Since I observe the data y_{t+1} , I observe the interest rate. If $R_{t+1} = 0$, I deduce that

$$P(K_{t+1} = 1 | Y^{t+1}) = 0, \text{ and } P(K_{t+1} = 2 | Y^{t+1}) = 1$$

and *vice versa*. So I do not need to track the long history of the states, because when I know the history of Y^t , I know the history of the states for sure. The distribution of the states at each time is degenerated. In practice, any quarterly Federal Funds rate that is smaller than 40bp is treated as zero interest rate.

2.4.2 Data

Data are the same as those used in chapter 1. For a detailed description, please refer to section A.5.

2.4.3 Prior Choice

Tables 2.1 and 2.2 (columns two to four) summarize the prior distributions of each parameter in the regime-switching DSGE model. I fix the coefficient of relative risk

aversion σ at 2, and the steady state of the ratio of long-term bonds to short-term bonds at 1.01, which is consistent with the average of this series in the data. I use Gamma distributions for the prior distributions of the parameters that economic theory suggests must be positive. For those parameters that are defined over the interval $[0, 1]$, I use the Beta distribution. For the standard deviation of the structural shocks, I use the Inverse-Gamma distribution.

The ergodic mean for inflation is centered at 2%, consistent with the Federal Open Market Committee’s long-term inflation mandate. The steady state annualized growth rate of output is centered at 2.5%. The prior distribution of the discount factor implies the mean of the annualized real interest rate is 2%. The spread between the short-term rates and long-term rates has a mean of 0.75% (annualized) at its prior distribution.

I follow Del Negro and Schorfheide (2008) to choose the priors for the standard parameters in the DSGE models. As in chapter 1, the dividend payment parameter k for the long-term bonds is calibrated to imply a duration of 30 quarters, which is consistent with the average duration of the U.S. 10-year Treasury bonds in the secondary market.

Table 2.1 contains three non-standard parameters ($\tilde{\nu}$, P_{11} , and P_{22}) specific to this regime-switching bonds-in-utility model, which controls the elasticity of households’ portfolio mix in response to the long-term rate, the Markov switching probability of staying in the normal regime at time $t + 1$ when it is in the normal regime at time t , and the Markov switching probability of staying in the ZIRP regime at time $t + 1$ when it is in the ZIRP regime at time t . $\tilde{\nu}$ is centered at 0.1 at the prior. Harrison (2010) uses a parameter with a similar role, and he calibrates this parameter to be 0.09. Andrés, López-Salido, and Nelson (2004) estimate a similar parameter to be 0.045, which describes the elasticity of the risk premium to a change in the ratio of

long-term bonds to money. I do not have money in my model, but the short-term bonds fill a similar role as money because it is more liquid than long-term bonds. Bernanke, Reinhart, and Sack (2004) suggest that a 10% reduction in the stock of long-term bonds associated with the U.S. Treasury buy-backs reduces long yields by around 100 basis points. The second round large-scale asset purchases is equivalent to a 25% reduction in long-term bonds⁴⁹. This suggests a value for $\tilde{\nu}$ around 0.25. My prior mean lies in between those estimates. P_{11} is centered at 0.99, which implies an expected duration of staying in the normal regime is 25 years. P_{22} is centered at 0.85 at prior, which implies an expected duration of staying in the ZIRP regime is 6.7 quarters, consistent with what is observed in the data.

The prior for the price rigidity parameter, ζ_p , is centred at 0.5 with a standard deviation of 0.1, as in Smets and Wouters (2007). The interest rate smoothing parameter, ρ_r , is centered at 0.7. The interest rate feedback to output growth, ϕ_y , is centered at 0.4, and the feedback to inflation, ϕ_π , is centered at 1.5 at priors.

All the structural shocks follow AR(1) processes. Their autocorrelation coefficients are centred at 0.75 or 0.8, with the exception of productivity shocks whose autocorrelation coefficient is centered at 0.4, because this process characterizes the transitory shock to the growth rate of the technology process.

2.4.4 Parameter Posterior Distribution

In order to obtain the posterior distribution of the parameters, I first obtain the posterior mode by maximizing the likelihood function. The last column of tables 2.1 and 2.2 report the posterior mode of each parameter. I then use the random walk Metropolis Hastings algorithm to draw from the posterior distributions. I store those parameter draws and use them for simulation exercises discussed later.

⁴⁹It corresponds to roughly a 24% reduction in the ratio of long-term bonds to short-term bonds.

The Markov switching probabilities are well identified because, although the priors are concentrated at their mean, the posterior modes of the transition probabilities are very distinguishable from the prior means. The posterior distributions indicate that the expected duration of staying in the normal regime is 24.15 quarters, and the expected duration of staying in the ZIRP regime is 4.5 quarters. One may argue that data seem to suggest that we have been in the ZIRP regime for at least 17 quarters (from 2009Q1 to 2013Q1). There are two reasons why the estimated duration is substantially shorter than this period. First, the data in my estimation stops at the second quarter of 2010, by which there were only 6 quarters of zero interest rate policy. Second, I treat the 8 quarters from 2002Q4 to 2004Q3 as a ZIRP regime (quarterly FFR is less than 40 basis points) so that we have observations of exiting the ZIRP regime. The time of staying in the ZIRP regime is also short here.

2.4.5 The Efficacy of the LSAPs in DSGE models

Having estimated the DSGE models, I abstract the filtered states, and, starting from 2010Q3, I simulate U.S. economy forward for 20 quarters under two scenarios. Under the first scenario, there is no intervention from the central bank, and all the structural shocks are zero. So, output should gradually go back to its long-term trend, and inflation and interest rates should gradually go back to their steady states. Under the second scenario, the economy is under the intervention of asset purchases by the central bank simulated to mimic the Federal Reserve's second round LSAPs, a \$600 billion reduction of long-term debt in the hands of the private sector. The central bank buys long-term bonds (in exchange for the short-term bonds) over the course of the first four quarters, holds the ratio of the market value of the long-term bonds to that of the short-term bonds constant for the next two years, and gradually reverts the

LSAPs program over the final two years. Figure 2.1 illustrates the path of the ratio of the market value of long-term bonds to that of the short-term bonds in the hands of the private sector following the LSAPs by the central bank. In the regime-switching bonds-in-utility model, this simulation is achieved by feeding the unanticipated shocks to the bond supply rule 2.4. In the non-regime-switching bonds-in-utility model, with a regular Taylor rule, agents have perfect knowledge of the bond purchases path, and the equilibrium is solved by the PFRE solution method explained in section 2.3.2.

I simulate the LSAPs 500 times using the parameter draws from the posterior distributions and take the average of the predicted path. Figure 2.2 shows the predicted path generated by the non-regime-switching bonds-in-utility model, and Figure 2.3 shows the predicted path generated by the regime-switching bonds-in-utility model⁵⁰. The red lines in those two figures are the predicted path without intervention, the blue lines are the predicted path under the LSAPs, and the black dots are actual observations. Output is per capita level data, while the units of the other variables are percentage measured quarterly. It is clear from those figures that the effects of the LSAPs are unlikely to be significant no matter what model we use, and whether or not agents are taken by surprise. At each time point, I take the percentage difference of the macro variables between the path with and the path without the LSAPs intervention, and sum up the difference over the 20 quarters to measure the total effects. The non-regime-switching bonds-in-utility DSGE model suggests on average⁵¹ the LSAPs increase output level by 0.34% and inflation by 0.16% over the course of 20 quarters. The regime-switching model suggests a slightly bigger effect, on average the LSAPs increase output level by 1.03% and inflation by 0.25% over the course of

⁵⁰Another complication in the simulation in the regime-switching DSGE model is that agents have uncertainty over the future states. There are 2^t possible states at time t . To maintain tractability, I collide the states with similar history and only keep track of 16 states at each period (See Schorfheide (2005) for how this can be achieved.). The predicted path of the macro variables plotted is thus the probability weighted average of those 16 states.

⁵¹“On average” means average over parameter uncertainty.

20 quarters. This finding agrees with the results reported by chapter 1. Section 3.2 investigate further why the effects of the LSAPs are so small measured in the DSGE models and evaluate their effects with VARs.

2.4.6 The Efficacy of the ZIRP in DSGE models

Zero interest rate policy is effective in boosting output and inflation. Both of the models considered suggest substantial effects of the ZIRP. When I simulate the U.S. economy under the ZIRP for an extended period, I consider keeping interest rates at zero for four quarters at the regime-switching model and keeping interest rates at the 2010Q2 level for four quarters in the model where the ZIRP is implemented by the PFRE. In the regime-switching model, at each period, agents *ex ante* always attach certain probability of exiting the ZIRP regime in the next period, and the ZIRP regime is realized for four quarters *ex post*. In the PFRE model, agents know that the ZIRP will be kept for four quarters. I choose four quarters because although the Federal Reserve announced on September 13th, 2012 that the ZIRP will last to “at least mid-2015”, participants of the Blue Chip Survey, professionals and economists, expected the ZIRP to last four or five quarters at the end of 2010 when the LSAPs II were implemented. Figure 2.4 and Figure 2.5 show the predicted path under the ZIRP generated by the PFRE model and the regime-switching model. The red lines in those two figures are the predicted path without the ZIRP, the blue lines are the predicted path with the ZIRP, and the black dots are actual observations. The regime-switching bonds-in-utility DSGE model suggests on average the ZIRP increases output level by 12.83% and inflation by 2.08% over the course of 20 quarters. The non-regime-switching model where the ZIRP is implemented by the PFRE suggests a two fold stronger effect on output level and five fold stronger stimulus to inflation: On

average the ZIRP increases output level by 25.01% and inflation by 11.71% over the course of 20 quarters. As mentioned earlier, those two models are fundamentally different in how agents formulate expectations about the future monetary policy. The central bank’s “extended period” language is treated as completely credible by the agents in the PFRE model, while in the regime-switching model, agents ignore the central bank’s forward guidance. Figure 2.6 compares the predicted path of inflation generated by those two models. The red line is the predicted path from the regime-switching model and the green line is the predicted path from the PFRE model. The black dots are actual data. It demonstrates that actual path is a lot closer to the path from the regime-switching model.

Figure 2.9 summarizes the effects of the LSAPs and the ZIRP in the DSGE models. At each time of the simulated path, I take the percentage difference of the macro variables with and without intervention, and sum up over 20 quarters. This figure plots the total effects. The color green represents the bonds-in-utility model. The squares are mean responses and the circles reflects the parameter uncertainty. The blue square reports the mean effects measured in the market segmentation model reported by chapter 1. This figure clearly shows that the effects of LSAPs are very small, while the efficacy of ZIRP is substantial, and crucially depends on the models.

2.4.7 The Efficacy of the Combination of the LSAPs and the ZIRP

Since the effects of the LSAPs alone is very small, unsurprisingly, the effects of the combination of the LSAPs and the ZIRP are dominated by the effects of the ZIRP. Figure 2.7 (the PFRE model) and Figure 2.8 (the regime-switching model) shows that the predictive paths of the macro variables under the ZIRP (blue lines) and under the

combination of the LSAPs and the ZIRP (green lines) are almost indistinguishable from each other⁵². Chapter 1 also emphasize the importance of the Federal Reserve's commitment to keep the interest rates at zero for an extended period.

2.5 Conclusions

Given the unusual size and scope of the unconventional monetary policies, it is critical for economists to construct models capable of assessing their effectiveness and guiding policy. This chapter develops a new approach to modeling the ZIRP, which not only fits the macro data featuring a persistent period of extremely low interest rates, and generates a predicted path closer to the actual path, but also provides a plausible mechanism for modeling the exit of the zero interest rate policy. Also, by cross-evaluation of the different models of the LSAPs and the ZIRP, I find that the Federal Reserve's commitment to an extended period of low interest rates is likely to be effective in boosting the economy while the efficacy of LSAPs is uncertain.

⁵²Red lines are the predictive path under no intervention and no shocks.

Table 2.1: Parameter Prior and Posterior Distribution for chapter 2: Structural Parameters.

	Dist	Prior		Posterior
		Mean	Std	Mode
400γ	G	2.5	0.5	1.9263
400π	G	2.0	0.5	2.0711
$400(\beta^{-1} - 1)$	G	1.0	0.25	0.8210
400ζ	G	0.75	0.25	0.6633
ν	G	2.0	0.5	0.5682
ζ_p	B	0.5	0.1	0.9066
ρ_r	B	0.7	0.1	0.7958
ϕ_π	G	1.5	0.25	1.8069
ϕ_y	G	0.4	0.2	0.3261
P_{11}	B	0.99	0.05	0.9586
P_{22}	B	0.85	0.10	0.7793
$\tilde{\nu}$	G	0.10	0.05	0.0797

Table 2.2: Parameter Prior and Posterior Distribution for chapter 2: Shock Process Parameters.

	Dist	Prior		Posterior
		Mean	Std	Mode
ρ_z	B	0.4	0.2	0.1560
ρ_b	B	0.75	0.1	0.7624
ρ_ϕ	B	0.75	0.1	0.9694
ρ_B	B	0.8	0.1	0.9154
ρ_ζ	B	0.8	0.1	0.9307
ρ_g	B	0.75	0.1	0.9329
$100\sigma_z$	IG1	0.5	4	0.5542
$100\sigma_b$	IG1	0.5	4	1.5873
$100\sigma_\phi$	IG1	0.5	4	1.2965
$100\sigma_B$	IG1	0.5	4	0.1162
$100\sigma_m$	IG1	0.5	4	0.1646
$100\sigma_\zeta$	IG1	0.5	4	0.3201
$100\sigma_g$	IG1	0.5	4	4.2947

Figure 2.1: *Simulated path of the ratio of the market value of long term bonds to that of the short-term bonds*

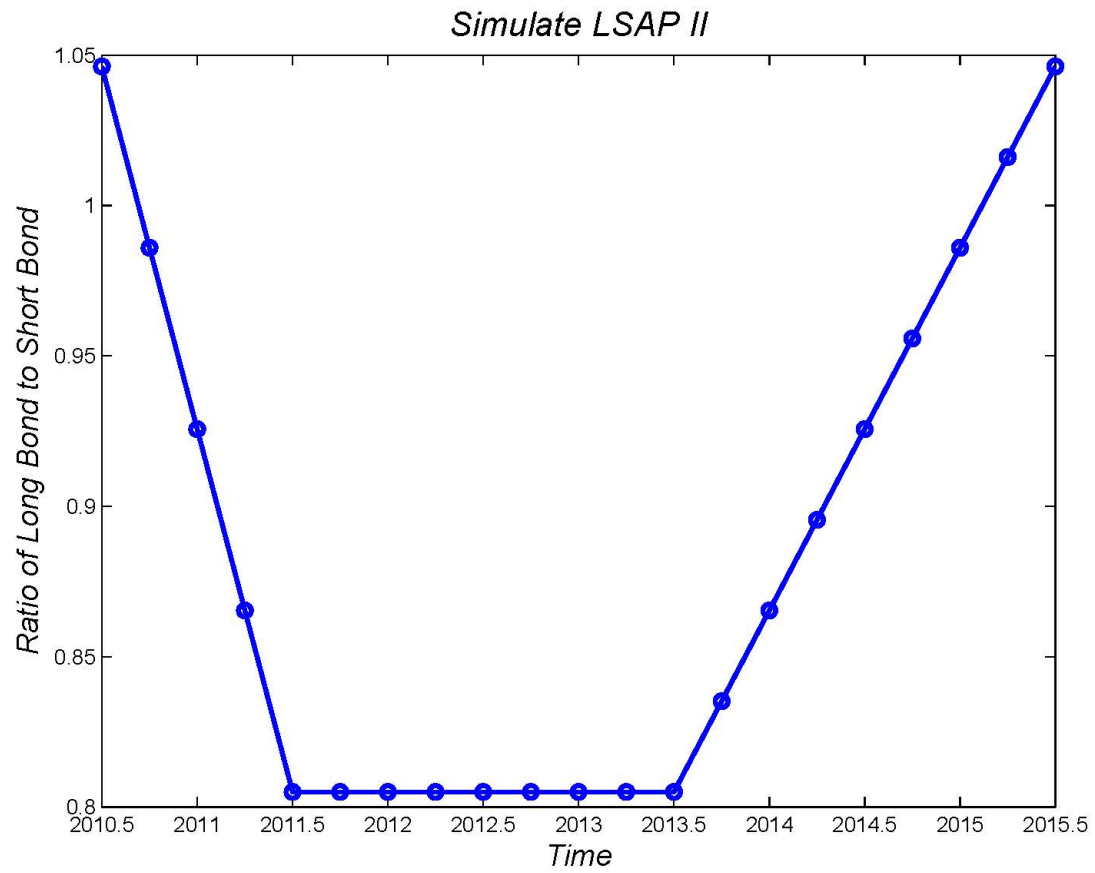


Figure 2.2: *Simulate the U.S. economy forward from 2010Q3 under the LSAPs II intervention in the NON-regime-switching bonds-in-utility DSGE model with standard Taylor rule.* The red lines show the mean of predicted paths of the macro variables without shocks and under no intervention. The blue lines show the mean of predicted paths of the macro variables under the LSAPs II intervention generated by the same model. The black dots are actual observations.

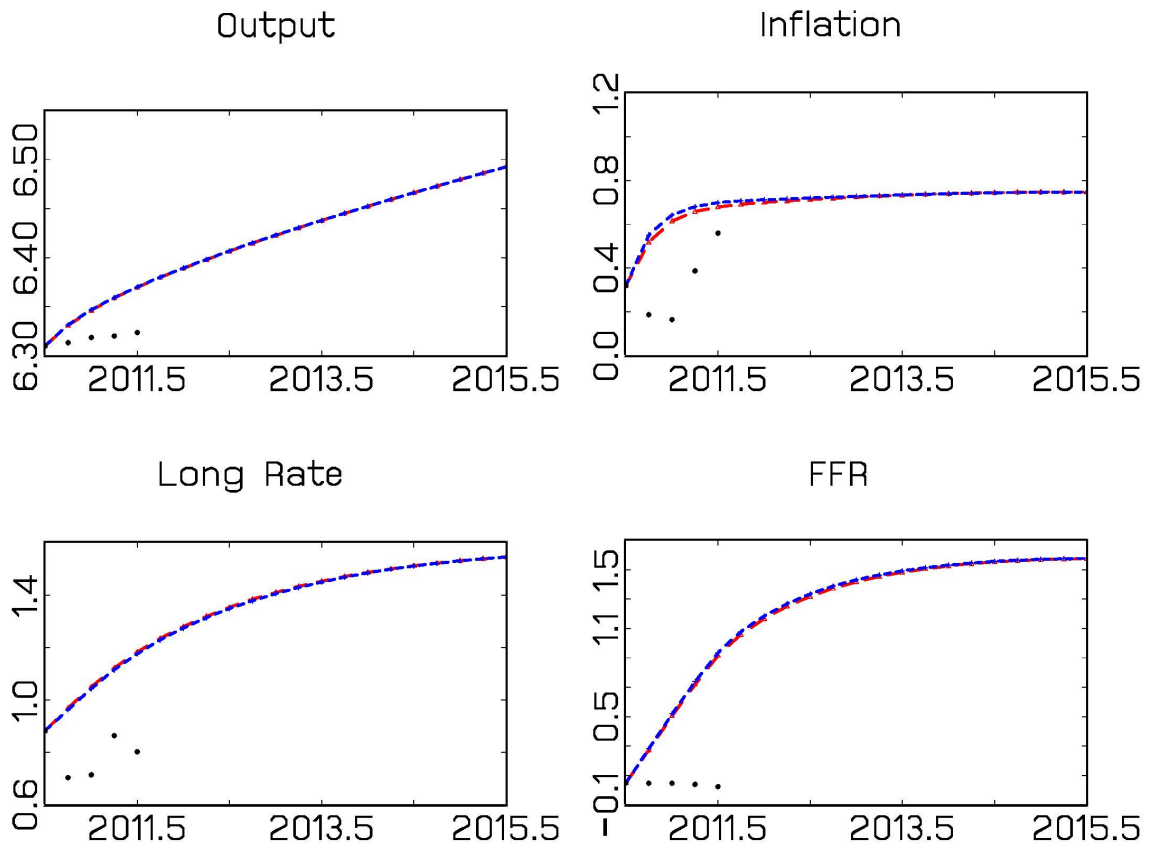


Figure 2.3: *Simulate the U.S. economy forward from 2010Q3 under the LSAPs II intervention in the regime switching Bonds-in-utility DSGE model. The red lines show the mean of predicted paths of the macro variables without shocks and under no intervention. The blue lines show the mean of predicted paths of the macro variables under the LSAPs II intervention generated by the same model. The black dots are actual observations.*

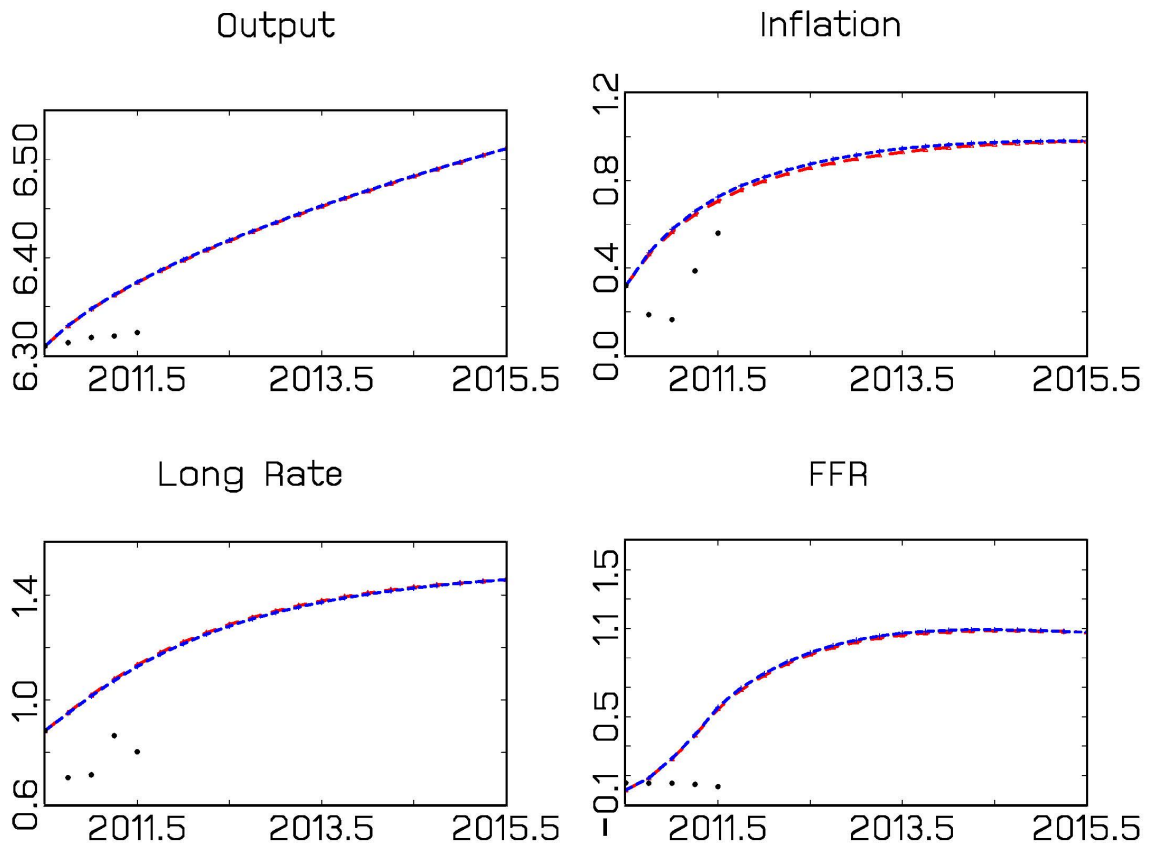


Figure 2.4: *Simulate the U.S. economy forward from 2010Q3 under the ZIRP intervention implemented by the PFRE.* The red lines show the mean of predicted paths of the macro variables without shocks and under no intervention generated by the bonds-in-utility DSGE models. The blue lines show the mean of predicted paths of the macro variables under the ZIRP for four quarters generated by the same model. The black dots are actual observations.

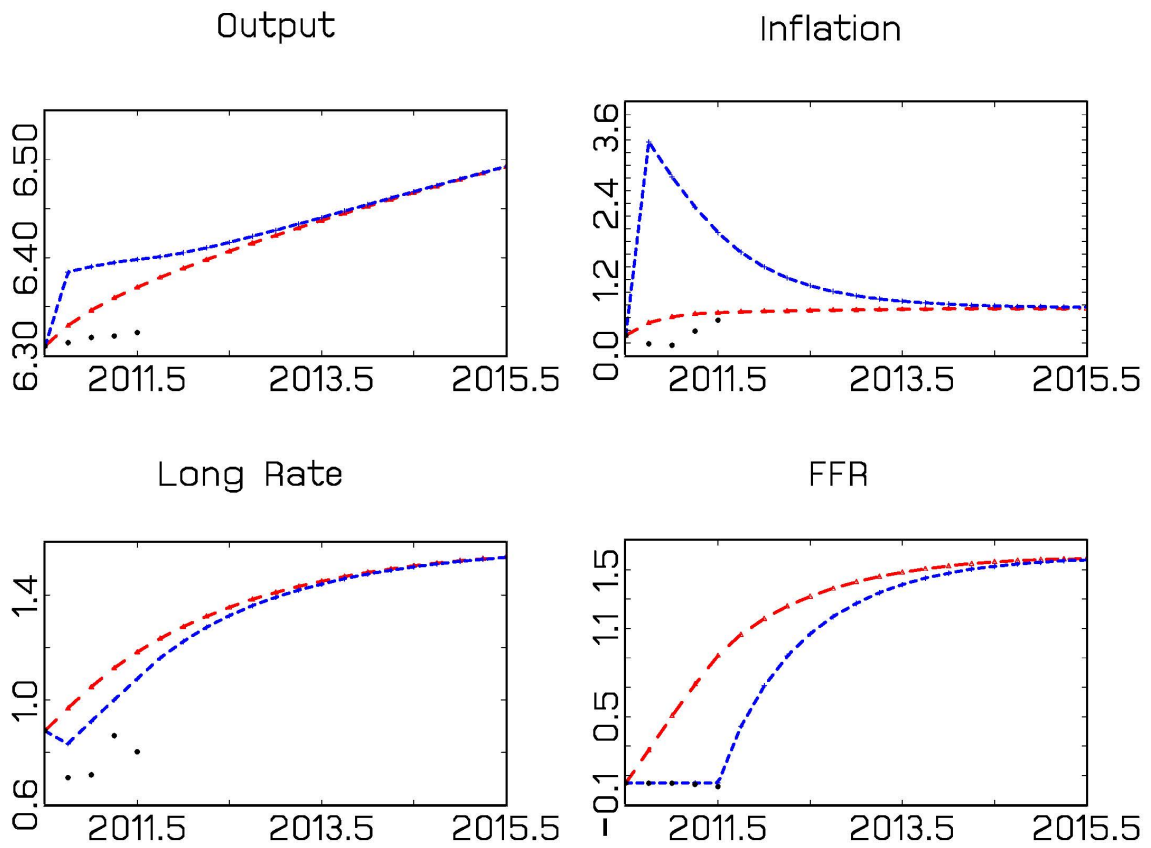


Figure 2.5: *Simulate the U.S. economy forward from 2010Q3 under the ZIRP intervention in the regime switching bonds-in-utility DSGE model.* The red lines show the mean of predicted paths of the macro variables without shocks and under no intervention. The blue lines show the mean of predicted paths of the macro variables under the ZIRP for four quarters generated by the same model. The black dots are actual observations.

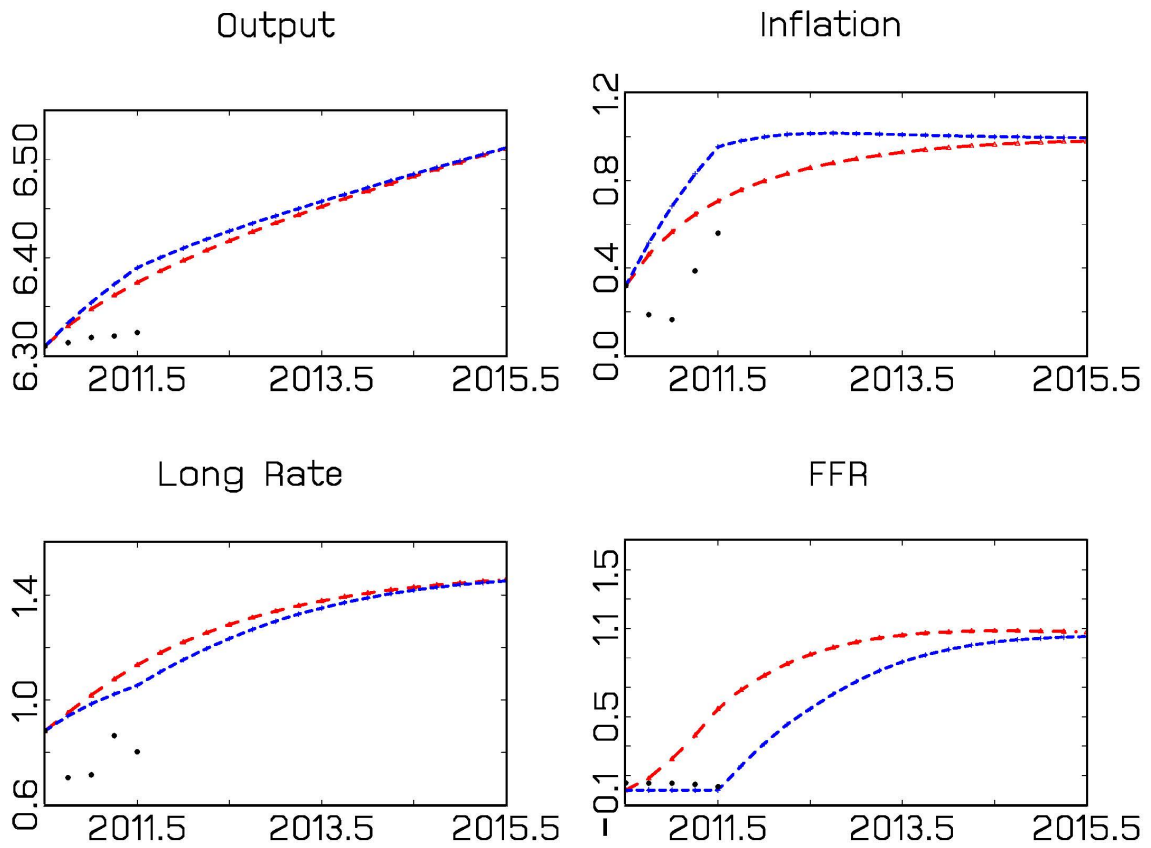


Figure 2.6: Compare the predicted path of inflation generated by two different models of ZIRP. Red represents regime switching model while green stands for PFRE model. The black dots are actual observations.

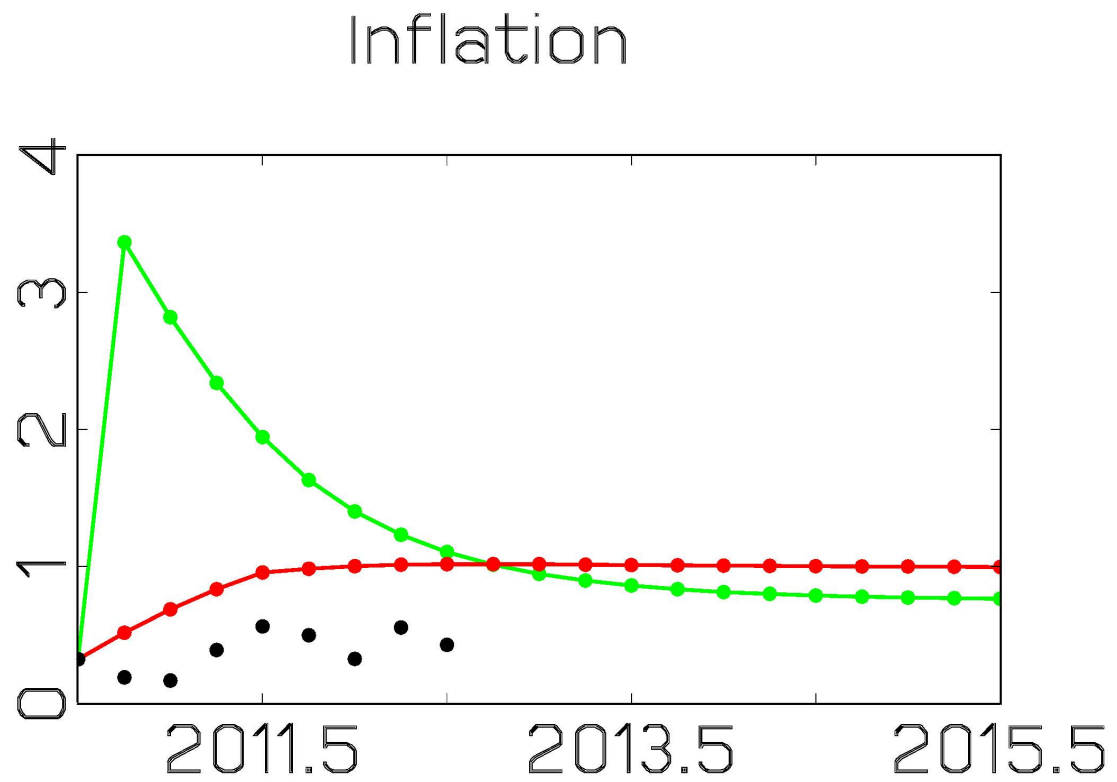


Figure 2.7: *Simulate the U.S. economy forward from 2010Q3 under the ZIRP intervention implemented by the PFRE and under the combination of the LSAPs and the ZIRP.* The red lines show the mean of predicted paths of the macro variables without shocks and under no intervention generated by the bonds-in-utility DSGE models. The blue lines show the mean of predicted paths of the macro variables under the ZIRP for four quarters generated by the same model, and the green lines show the predictive paths under the intervention of the combination of the LSAPs and the ZIRP for four quarters. The black dots are actual observations.

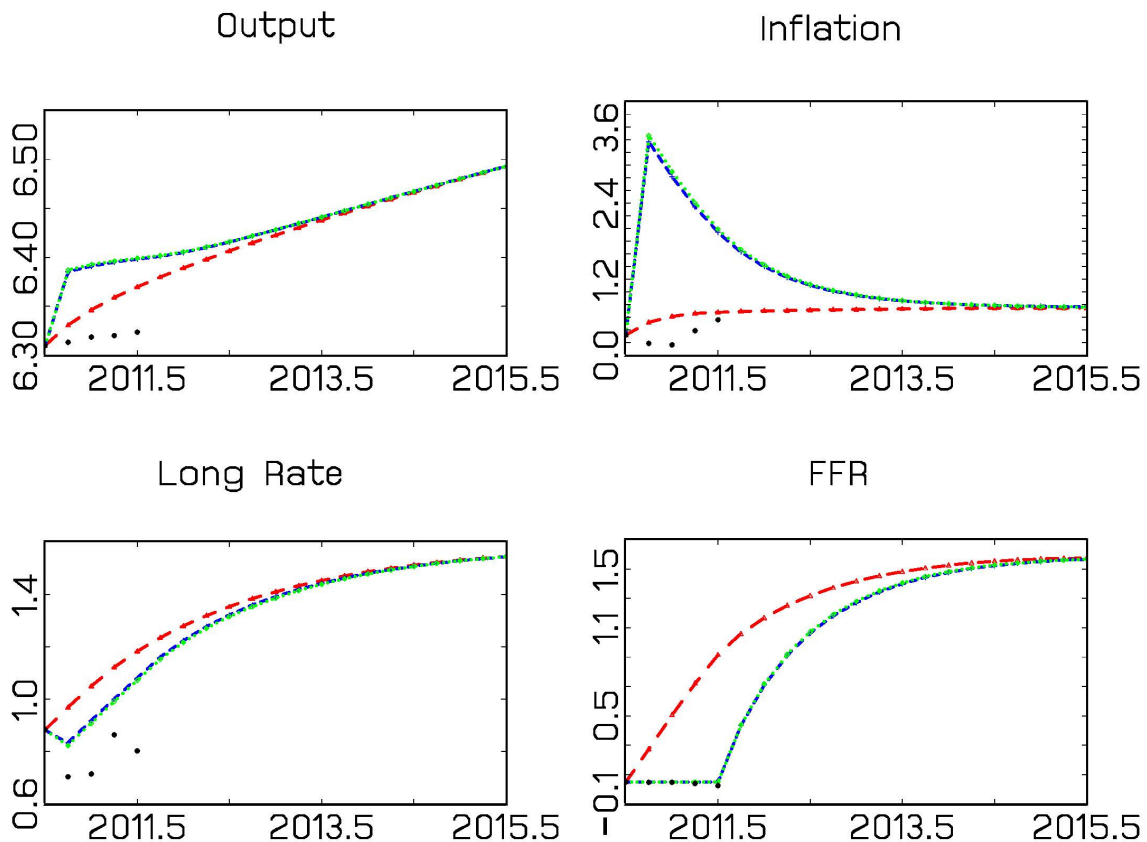


Figure 2.8: *Simulate the U.S. economy forward from 2010Q3 under the ZIRP intervention implemented by the regime-switching bonds-in-utility model and under the combination of the LSAPs and the ZIRP. The red lines show the mean of predicted paths of the macro variables without shocks and under no intervention, the blue lines show the mean of predicted paths of the macro variables under the ZIRP for four quarters, and the green lines show the predictive paths under the intervention of the combination of the LSAPs and the ZIRP for four quarters. The black dots are actual observations.*

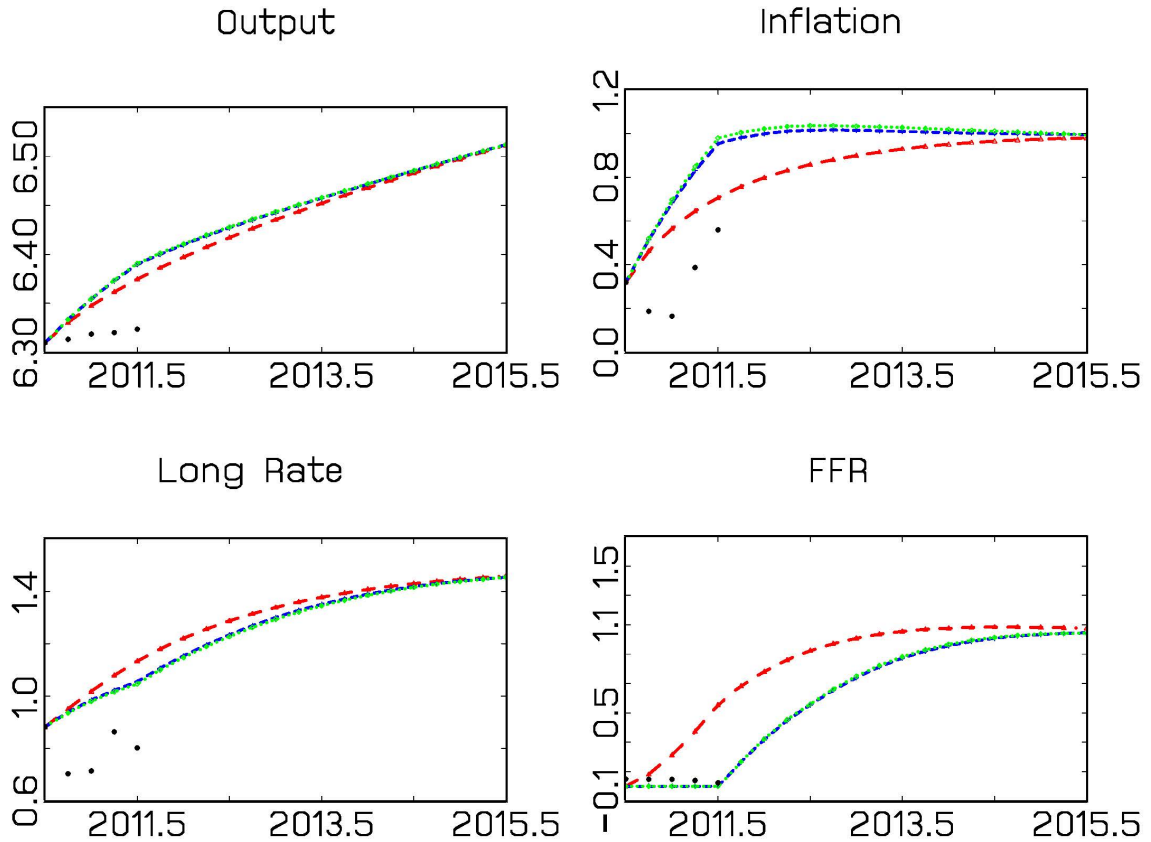
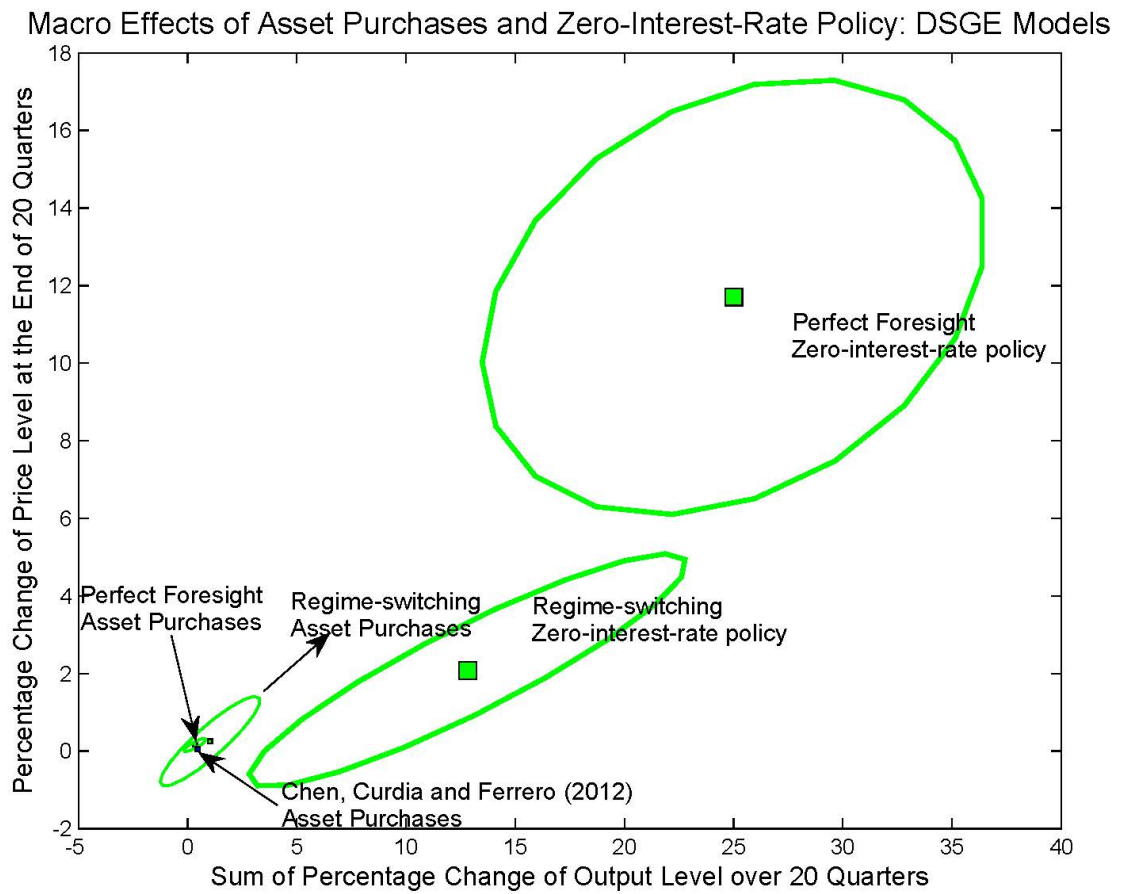


Figure 2.9: *Summary of effects of LSAPs and ZIRP in DSGE models.* The squares stand for mean effects and the circles reflect the uncertainty. Green represents bonds-in-utility model and blue represents the results reported by chapter 1.



Chapter 3

Assessing the Effects of Large-Scale Asset Purchases in a Zero-Interest-Rate Environment through the Lens of VAR Models

3.1 Introduction

DSGE models impose strict cross-equation restrictions. I use VARs that relax the DSGE model restrictions to further examine the reason for the small effects of LSAPs measured in the DSGE models. I investigate how the effects of LSAPII are empirically identified in the DSGE models that break the neutrality of the LSAP operation such as chapter 1 and Harrison (2010). I ask the questions: What happens when you relax some of the DSGE model restrictions? How do DSGE models compare to VAR studies? Using the exogenous restrictions implied by the DSGE models, the estimated VAR model suggests no evidence of positive effects of LSAP on output and inflation.

An estimated VAR with a further relaxation of DSGE restrictions can generate a sizable effect of LSAPs but with considerable uncertainty.

3.2 The Efficacy of the LSAPs in VAR models

The DSGE models considered in this work impose a strong assumption on how LSAPs are identified: Equation 2.4 shows that the bond supply follows an AR(1) process exogenously, and other structural shocks do not affect the dynamics of bonds. LSAPs were never implemented before in U.S. history until the recent recession; however, DSGE models use the covariance relationship between bonds and other macro variables in the historical data to “identify” the effects of the assets purchases to macro variables. In the data, the variation of bonds in the past could be due to an entirely different reason. It could be a demand shock. For example, by preferred habitat theory, long-term interest rates could experience a large and long-lasting drop because of a demand shock of a long-maturity clientele such as pension funds, which in turn would stimulate private borrowing and investment. This implies a positive covariance between long-term bond quantity in the hands of the private sector and macrovariables: opposite of the covariance relationship the LSAPs assume. Although by construction the LSAPs should have a positive effect in DSGE models, the insignificant effects found in the DSGE models are probably due to the identification strategy of those models: the covariances between bonds and macro variables in the historical data are not informative about the effectiveness of the LSAPs. To further investigate how much of the finding that the effects of the LSAPs are small is due to the strict restrictions imposed by the DSGE models, I compare the DSGE models with the VARs. I ask the question, what are the effects of the LSAPs in an estimated VAR using the identification restrictions imposed by the DSGE models? What happens if

I further relax those restrictions?

3.2.1 VAR with Exogenous Restrictions

The assumption of the DSGE models that the bond supply follows an AR(1) process exogenously, and other structural shocks do not affect the dynamics of bonds provides an exogenous restriction to identify a bond supply shock in a VAR model. I estimate the following VAR:

$$\mathbf{y}_{1,t} = \Phi_0 + \Phi_1 \mathbf{y}_{t-1} + \Phi_2 \mathbf{y}_{t-2} + \Phi_3 \mathbf{y}_{t-3} + \Phi_4 \mathbf{y}_{t-4} + \Psi (y_{2,t} - C - \rho_B y_{2,t-1}) + \mathbf{u}_{1,t}$$

$$y_{2,t} = C + \rho_B y_{2,t-1} + \sigma_B \varepsilon_{B,t}$$

where $\mathbf{y}_{1,t}$ are the growth rate of output, inflation, long rates, and short rates, and $y_{2,t}$ is the ratio of the market value of the long bonds to that of the short bonds. The definitions of those variables are described in section 2.4.2. $\mathbf{u}_{1,t}$ are measurement errors. $\varepsilon_{B,t}$ is the bond supply shock. $\mathbf{y}_{1,t}$ are affected by the bond supply shock, but the bond supply is exogenous and unaffected by other macro variables. To simulate the Federal Reserve's second round LSAPs, I calibrate the bond shocks as described in section 2.4.5. In order to assess the effects of ZIRP, I also identify a monetary policy structural shock and impose ZIRP by unanticipated monetary policy shocks. I identify this monetary policy shock by short-run restriction, that is, monetary authority shocks do not affect the private sector's activity on impact. Suppose the first two elements of $\mathbf{y}_{1,t}$ are the growth rate of output and inflation. Let $\Sigma_{\mathbf{u}}$ denote the variance and covariance matrix of $\mathbf{u}_1$, and let Σ_{tr} denote the Cholesky decomposition of $\Sigma_{\mathbf{u}}$. I draw a unit length vector q , the first two elements of which equal zero. $\Sigma_{tr} \cdot q$ identifies the impact of the monetary shock to the observables $\mathbf{y}_{1,t}$. Finally, I simulate the economy forward with the estimated VAR model. Figure 3.1 shows the mean of the predicted

path under no intervention or shocks, under the LSAPs, and under the ZIRP for four quarters. The red line shows the predicted path of the macro variables under no intervention and no shocks, where output is the per capita output level, inflation is the quarterly percentage change of the core PCE, short rate is the quarterly federal funds rate, and long rates are the quarterly rates for the 10-year Treasury constant maturity bonds. The blue and green lines are the corresponding paths under the LSAPs and the ZIRP. Figure 3.2 plots the mean and 90% Bayesian credible intervals of the predicted path of macro variables under no intervention (red lines) and under the asset purchase policy intervention (blue lines). Figure 3.3 plots the mean and 90% Bayesian credible intervals of the predicted path of macro variables under no intervention (red lines) and under the policy of keeping interest rates at the 2010Q2 level (0.048%) for four quarters (blue lines). A comparison between the red and the blue lines shows no evidence of a positive effect of the LSAPs, while ZIRP has a stimulative effect (difference between the green line and the red line). This explains why the effects of asset purchases measured in DSGE models are small. Asset purchases should have positive effects in DSGE models by construction; however, data provide no evidence of such restrictions imposed by DSGE models: Bond supply is an exogenous process and asset purchases are identified as a supply shock. Figure 3.4 adds another grey line on each panel of the Figure 3.1. This grey line on each panel represents the mean of the predictive path of the corresponding macro variable under the intervention of the combination of the LSAPs and the ZIRP. Figure 3.5 adds Bayesian credible intervals to Figure 3.4. The red lines show the mean and the 90% Bayesian credible intervals of the predicted macro variables under no policy intervention, and the magenta-colored lines plot the mean and the 90% Bayesian credible intervals of the predicted path of the same macro variables under the policy intervention of both asset purchases and keeping interest rates at the 2010Q2 level for four quarters. Unlike the case in DSGE

models, the combination effects seem to be dominated by the effects of LSAPs since in Figure 3.4 the grey line is very close to the blue line which is the predictive path from the intervention of LSAPs only.

3.2.2 VAR with Sign Restrictions

The exogenous restriction is a very strong assumption. Whether or not it is valid is subject to debate. The DSGE model also implies certain directional restrictions of the responses of the macro variables to the LSAPs. The DSGE models imply that the LSAPs reduce long-term rates, stimulate output and inflation. Those directional restrictions provide the sign restrictions to identify a risk premium shock of the following VAR.

$$\mathbf{y}_t = \Phi_0 + \Phi_1 \mathbf{y}_{t-1} + \Phi_2 \mathbf{y}_{t-2} + \Phi_3 \mathbf{y}_{t-3} + \Phi_4 \mathbf{y}_{t-4} + \mathbf{u}_t,$$

where $\mathbf{y}_t$ is a collection of the growth rate of output, inflation, short rates, and long rates. I assume that the risk premium shock has zero impact on short-term rates, reduces the long-term bond rates, and increases output and inflation on impact⁵³. I also calibrate the size of this shock so that the mean reduction of the long-term bond rates on impact is 30 basis point, which lies in the mid-range of the values reported by empirical studies of the effects of LSAPs. The monetary policy shock is identified by sign restrictions. The monetary policy shock increases short and long rates on impact, but decreases output growth rate and inflation on impact. This identification scheme is very similar to Baumeister and Benati (2010) and Chen, Cúrdia, and Ferrero (2011) working paper. Baumeister and Benati (2010) use zero and sign restrictions to identify a risk premium shock that decreases long rates by 1 percent, and Chen,

⁵³The DSGE models suggest those sign restrictions. The empirical question is then, how big are the effects of the policies.

Cúrdia, and Ferrero (2011) calibrate whatever size of the bond supply shock necessary to decrease the long-term bond rates by 30 basis point on impact. Figure 3.6 shows the simulation results of the same experiment: I simulate the economy forward under no intervention and no shock, under the LSAPs, and under the ZIRP⁵⁴. The red line is the predicted path of the macro variables under no intervention, averaged over different parameter draws from the posterior distributions. The blue line is the predicted path of the macro variables under the LSAPs intervention, and the green line is the predictive path under the ZIRP. The ZIRP has a substantial effect as measured in the VAR model. There is potentially a positive effect of the LSAPs, but it is considerably uncertain⁵⁵. Figure 3.7 plots the estimate of the identified set of the effects of the LSAPs. The red lines are the counterfactual scenario when there is no policy intervention, while the blue lines are the mean and identified set of the predicted path of the macro variables under the LSAPs II intervention. The effects of LSAPs could be potentially substantial, but it is considerably uncertain. The green lines in Figure 3.8 plot the mean and the identified set of the effects of ZIRP, while the red lines are the mean and identified set of the predicted path of macro variables absent of any policy intervention. Figure 3.9 adds another grey line on each panel of the Figure 3.6. This grey line on each panel represents the predictive path of the corresponding macro variable under the intervention of the combination of the LSAPs and the ZIRP. It is interesting to notice that in the VAR model with sign restrictions the effects of the combination of those two policies seem to be a weighted average of the LSAPs and the ZIRP. The effects of ZIRP to output dominates the effects of LSAPs, while the effects of the LSAPs to inflation dominated ZIRP. Figure 3.10 summarizes the effects of the LSAPs and the ZIRP aggregate over 20 quarters. I take the log-difference of the predicted macro variables with and without intervention at each time point and sum

⁵⁴Here, ZIRP means keeping interest rates at the 2010Q2 (0.048%) level for four quarters

⁵⁵See Figure 3.10 where the uncertainty is reflected by the ellipse in red.

up over 20 quarters to reflect the total effects. The squares are the mean effects, and the circles reflect the uncertainty of the parameter draws. The pink color represents the results generated by the VAR with the exogenous restriction, while the red color represents the results generated by the VAR with the sign restrictions. One reason why the effects of the LSAPs and the ZIRP are considerably uncertain is the partial identification of the sign restrictions.

3.3 Conclusions and Future Research

The identification assumption of the asset purchases in the DSGE models is particularly strong. Estimated VAR using the exogenous restriction imposed by the DSGE models suggests no evidence of a positive effect of asset purchases. This explains that the small effects of asset purchases found by the DSGE models may be due to the usage of the historical data to identify LSAPs as a supply shock. Estimated VAR with further relaxed DSGE restrictions (using only sign restrictions implied by the DSGE models) shows that asset purchases could potentially have a large effect on economy, but the identification scheme adopted by this VAR prevents further sharpening of the bounds of the effects.

Constructing DSGE models that are capable of correctly identifying the macro effects of the unconventional monetary policy from macro data is critical not only to assess the effectiveness of the policy but also to guide future exit strategies. Without understanding the transmission mechanism of those unconventional monetary policy to the macro economy, it is impossible to forecast how a future reversal in asset purchases or a raise in policy rate can impact the economy, and thus advise when the Federal Reserve should exit and how fast the pace of the sales of the assets should be. Neither large-scale asset purchase nor near zero-interest-rate policy is a

new experience. Japan has experienced near zero-interest-rate policy since 1999⁵⁶ and has adopted the “quantitative easing” policy between March 2001 and March 2006. Japan’s experience can provide economists valuable lessons to identify the transmission mechanism of those unconventional monetary policy. This points to my future research: using Japanese data to better identify the effect of asset purchases and better estimate the Markov-switching probability of the policy regime in the DSGE model. Japan also has experienced the exit of ZIRP twice, the abolishment of the quantitative easing, and the removal of the excess bank reserves. Like Japan before, the Federal Reserve now is running up an enormous balance sheet⁵⁷ and facing the uncertainty surrounding an exit strategy. Possibly, Japan before and the United States now have something critical in common. Japan’s lesson will help forecast the evolution of the the Federal Reserve’s balance sheet going forward and the economic outlook.

Technically, the next step is to apply the perturbation method for Markov-switching models proposed by Foerster, Rubio-Ramírez, Waggoner, and Zha (2012). This method begins from the first principles and allows higher order approximation which may be important when taking into account the risk of the long-term bonds.

⁵⁶Japan has experienced three ZIRP episodes: 1999Q2-2000Q2, 2001Q1-2006Q1, and 2010Q4-present.

⁵⁷The Federal Reserve’s balance sheet stood at a record-large \$3.189 trillion on March 20, 2013.

Figure 3.1: *VAR identified by the exogenous restriction.* The red lines show the mean of predicted path of the macro variables without shocks and under no intervention generated by the estimated VAR model using the DSGE exogenous restriction identification. The blue lines show the mean of the predicted path of the macro variables under the LSAPs II generated by the same VAR model. The green lines show the mean of the predicted path of the macro variables under the ZIRP for four quarters generated by the same VAR model.

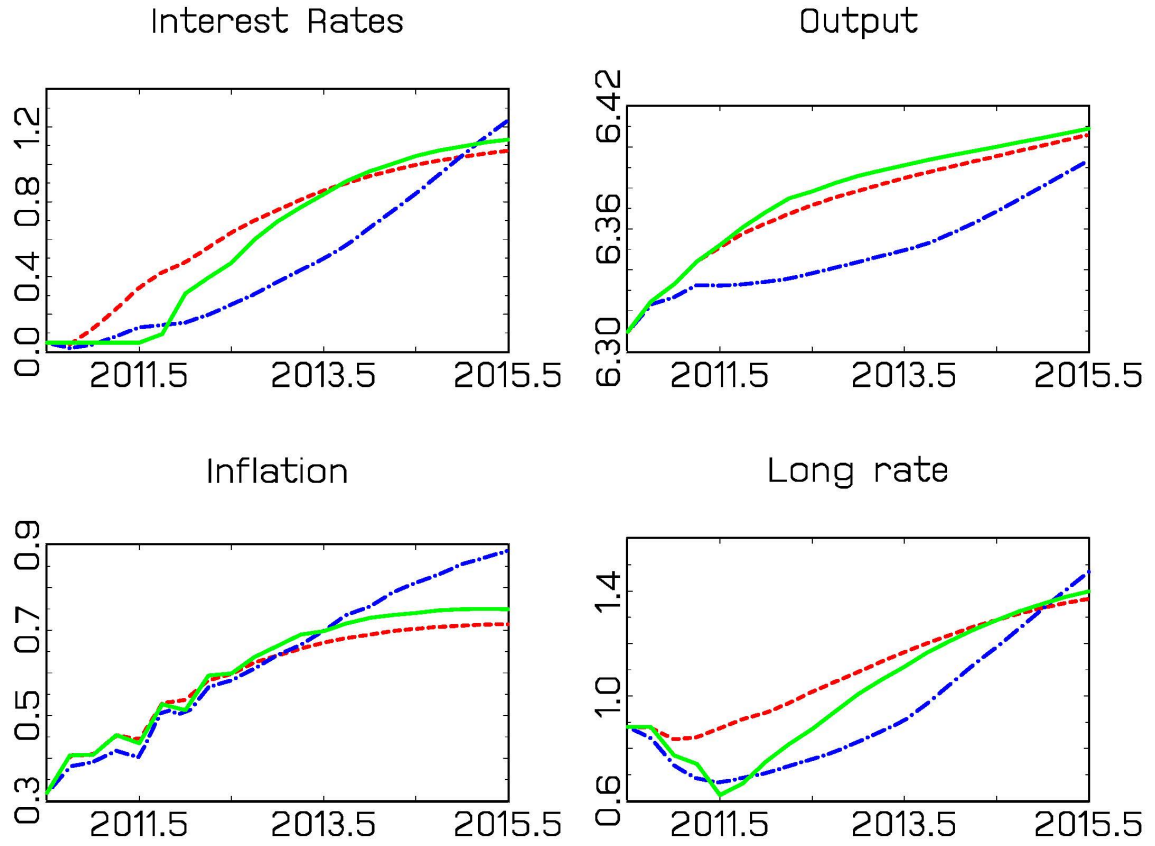


Figure 3.2: *VAR identified by the exogenous restriction: effects of LSAPs.* The red lines show the mean and the 90% Bayesian credible intervals of predicted path of the macro variables without shocks and under no intervention generated by the estimated VAR model using the DSGE exogenous restriction identification. The blue lines show the mean and the 90% Bayesian credible intervals of the predicted path of the macro variables under the LSAPs II generated by the same VAR model.

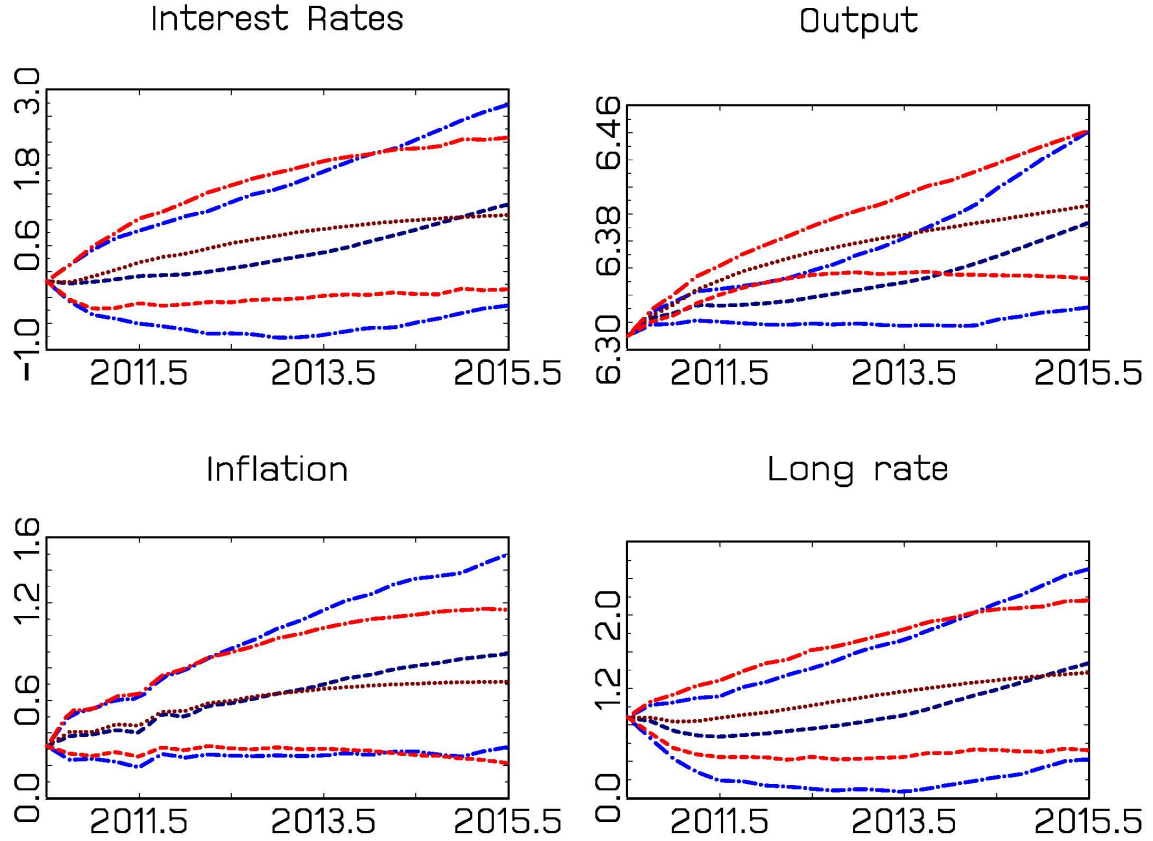


Figure 3.3: *VAR identified by the exogenous restriction: effects of ZIRP.* The red lines show the mean and the 90% Bayesian credible intervals of predicted path of the macro variables without shocks and under no intervention generated by the estimated VAR model using the DSGE exogenous restriction identification. The green lines show the mean and the 90% Bayesian credible intervals of the predicted path of the macro variables under the policy of keep interest rates at the 2010Q2 level (0.048%) for 4 quarters generated by the same VAR model.

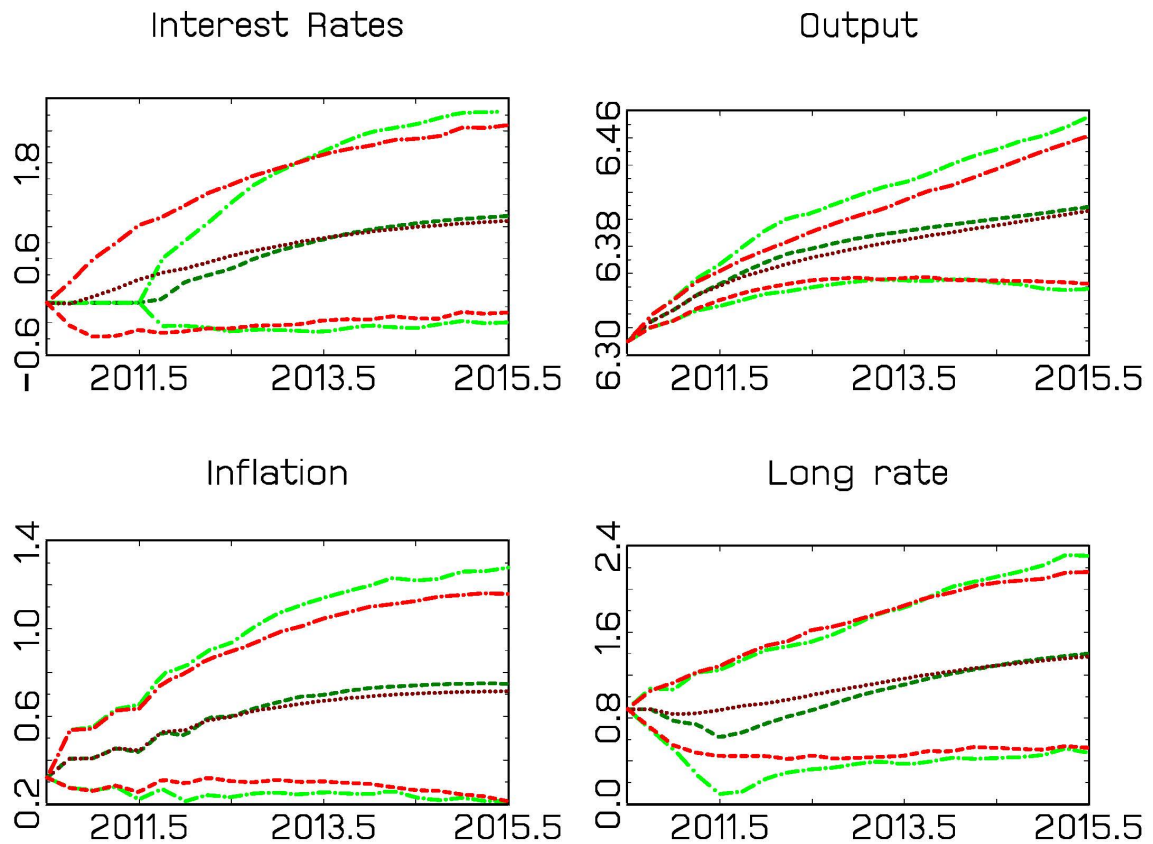


Figure 3.4: *VAR identified by exogenous restrictions.* The red lines show the mean of predicted paths of the macro variables without shocks and under no intervention generated by the estimated VAR model using the DSGE exogenous restriction identification. The blue lines show the mean of the predicted paths of the macro variables under the LSAPs II generated by the same VAR model. The green lines show the mean of the predicted path of the macro variables under the ZIRP for four quarters generated by the same VAR model. The grey lines are the predictive paths under the combination of the LSAPs and the ZIRP.

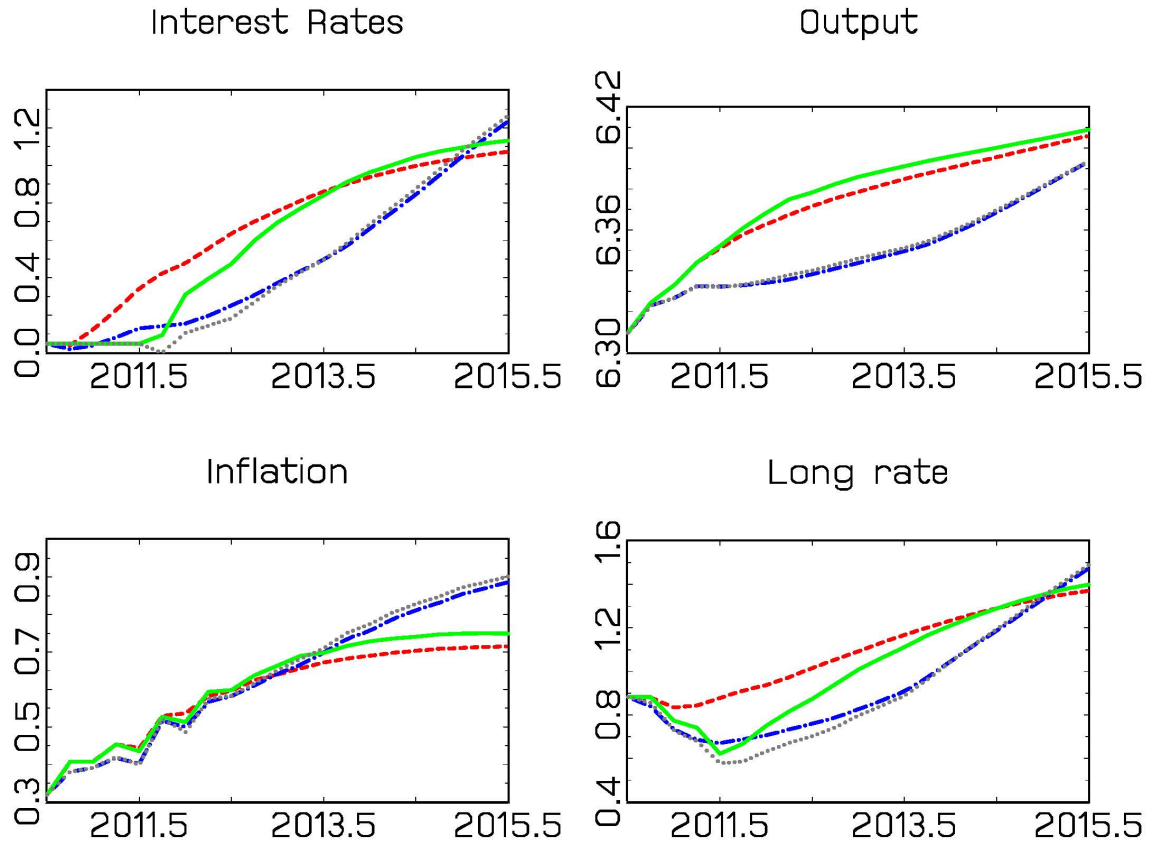


Figure 3.5: *VAR identified by exogenous restrictions: combination of LSAPs and ZIRP.* The red lines show the mean and 90% Bayesian credible intervals of predicted paths of the macro variables without shocks and under no intervention generated by the estimated VAR model using the DSGE exogenous restriction identification. The magenta lines are the mean and 90% Bayesian credible intervals of the predictive paths under the combination of the LSAPs and the ZIRP.

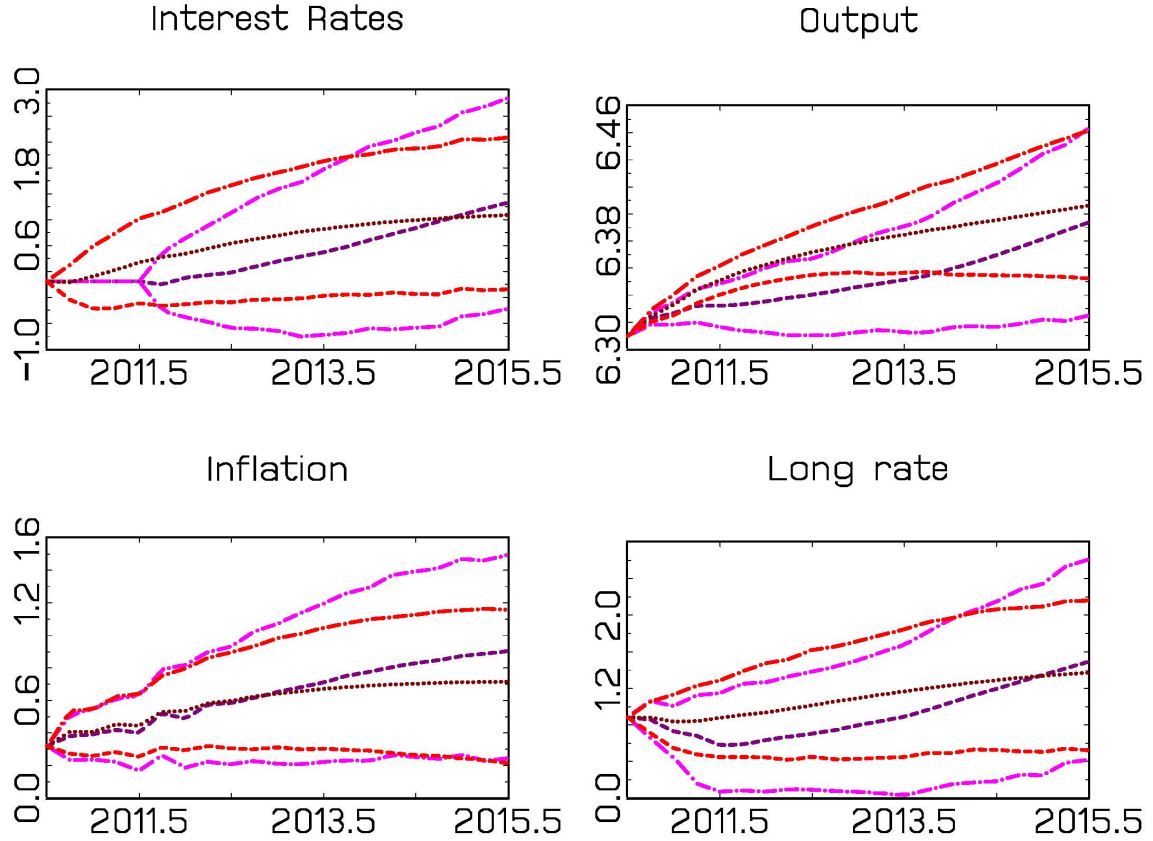


Figure 3.6: *VAR identified by sign restrictions.* The red line shows the mean of predicted path of macro variables without shocks and under no intervention generated by the estimated VAR model using the sign restriction identification. The blue line shows the mean of the predicted path of macro variables under the LSAPs II generated by the same VAR model. The green line shows the mean of the predicted path of macro variables under the ZIRP for four quarters generated by the same VAR model.

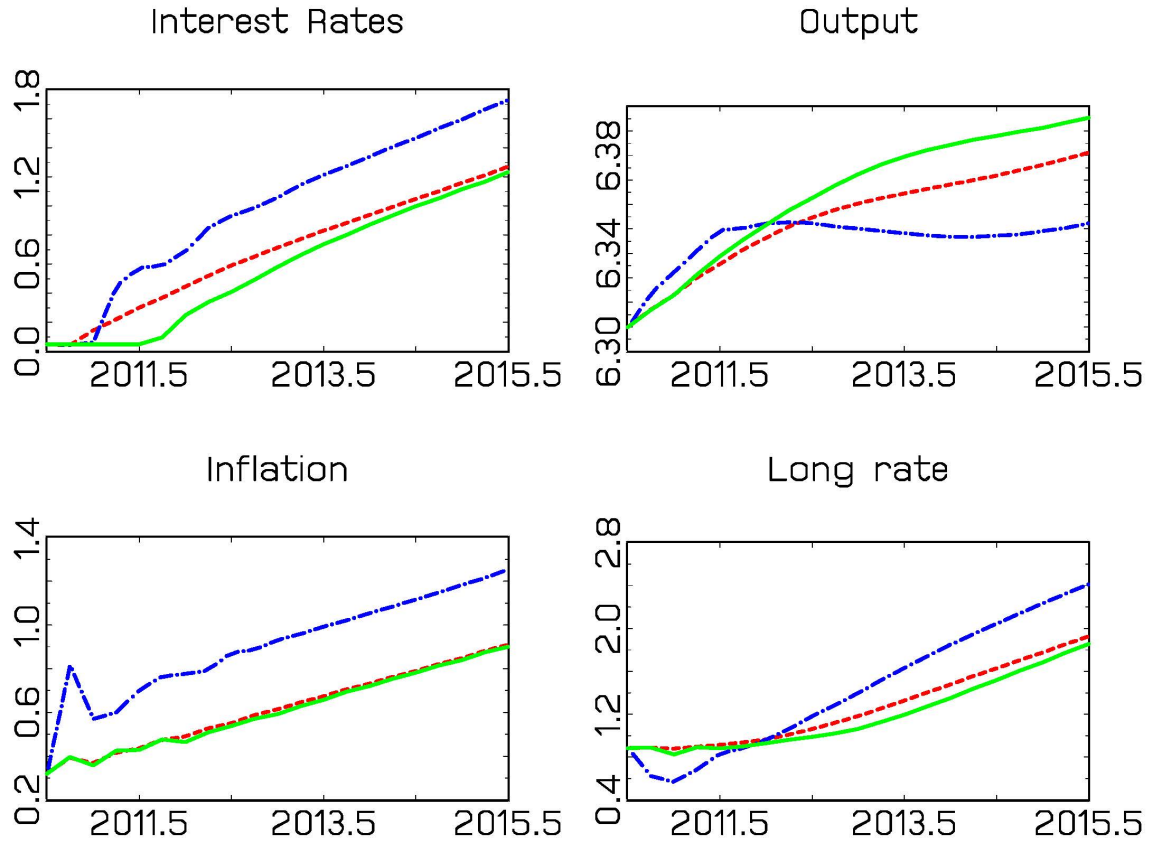


Figure 3.7: *VAR identified by sign restrictions with identified set: effects of LSAPs.* The red lines show the mean and the identified set of predicted path of macro variables without shocks and under no intervention generated by the estimated VAR model using the sign restriction identification. The blue lines show the mean and the identified set of the predicted path of macro variables under the LSAPs II generated by the same VAR model.

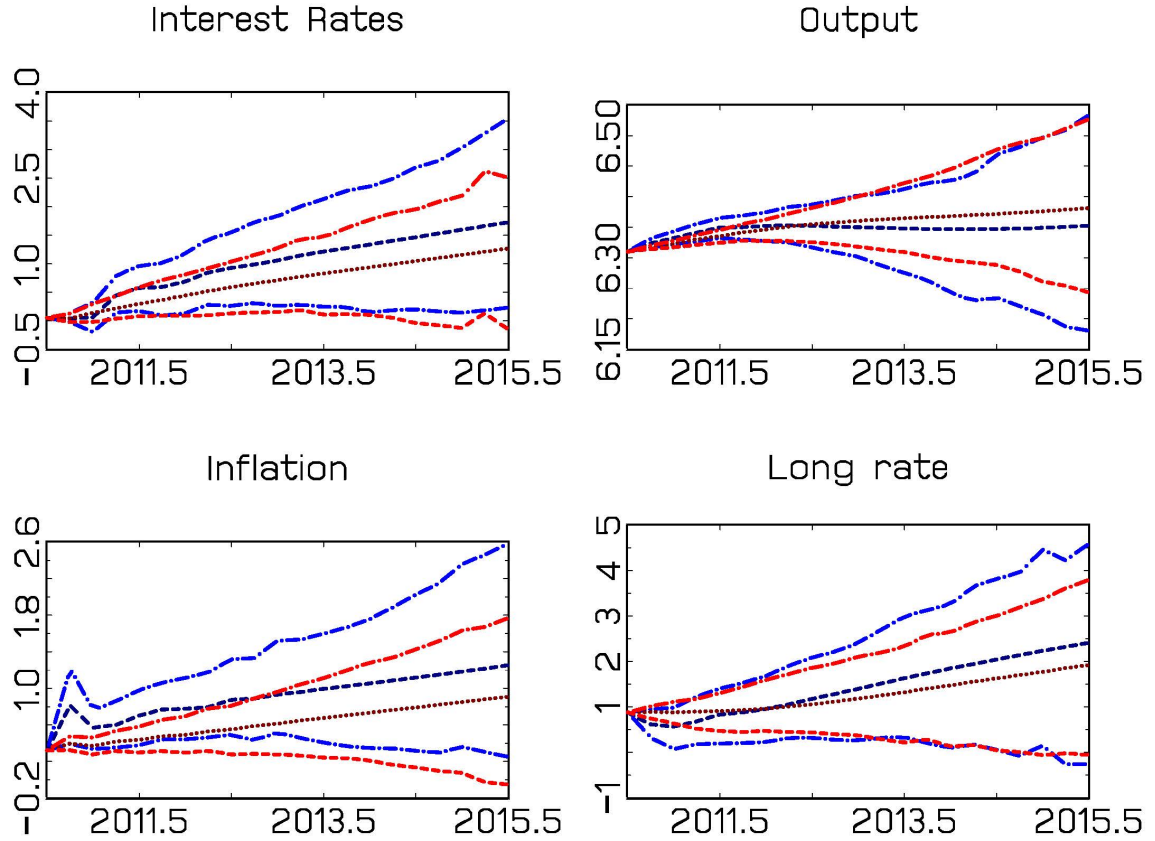


Figure 3.8: *VAR identified by sign restrictions with identified set: effects of ZIRP.* The red lines show the mean and the identified set of predicted path of macro variables without shocks and under no intervention generated by the estimated VAR model using the sign restriction identification. The green lines show the mean and identified set of the predicted path of macro variables under the LSAPs II generated by the same VAR model.

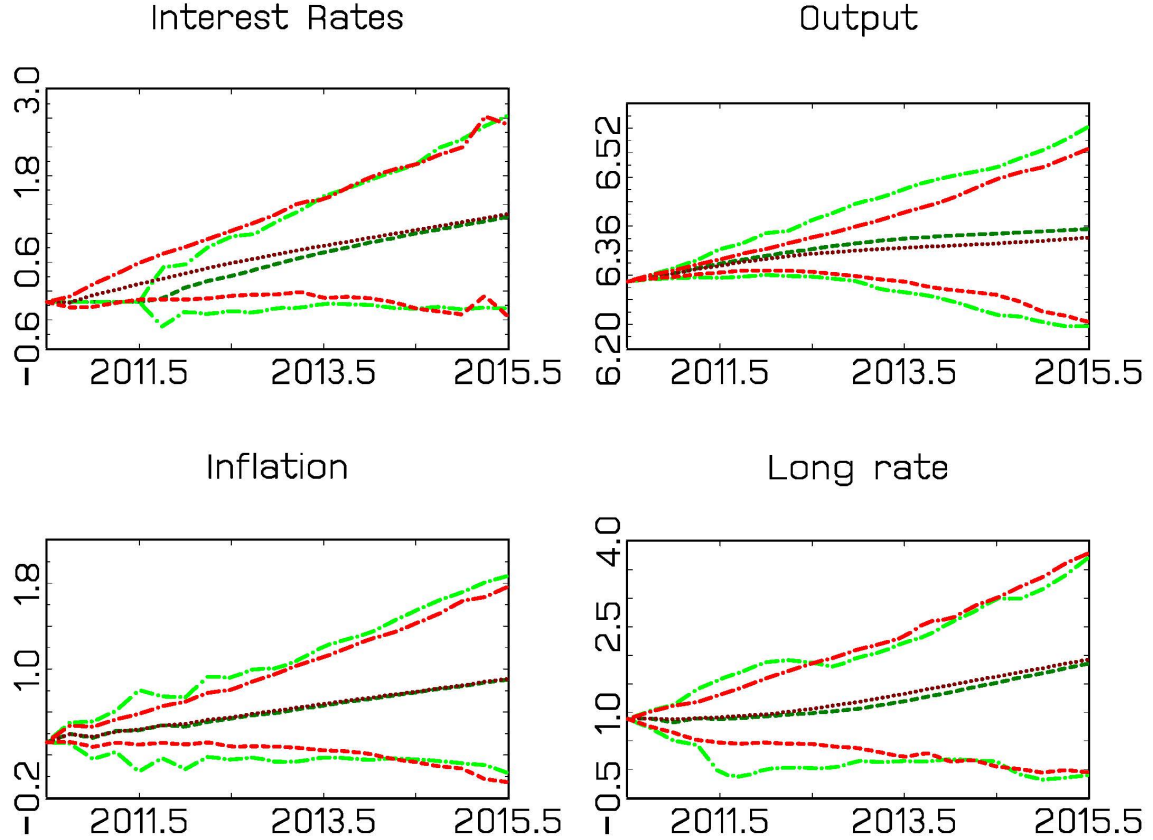


Figure 3.9: *VAR identified by sign restrictions*. The red lines show the mean of predicted paths of macro variables without shocks and under no intervention generated by the estimated VAR model using the sign restriction identification. The blue lines show the mean of the predicted paths of the macro variables under the LSAPs II generated by the same VAR model. The green lines show the mean of the predicted paths of the macro variables under the ZIRP for four quarters generated by the same VAR model. The grey lines are the predictive paths under the combination of the LSAPs and the ZIRP.

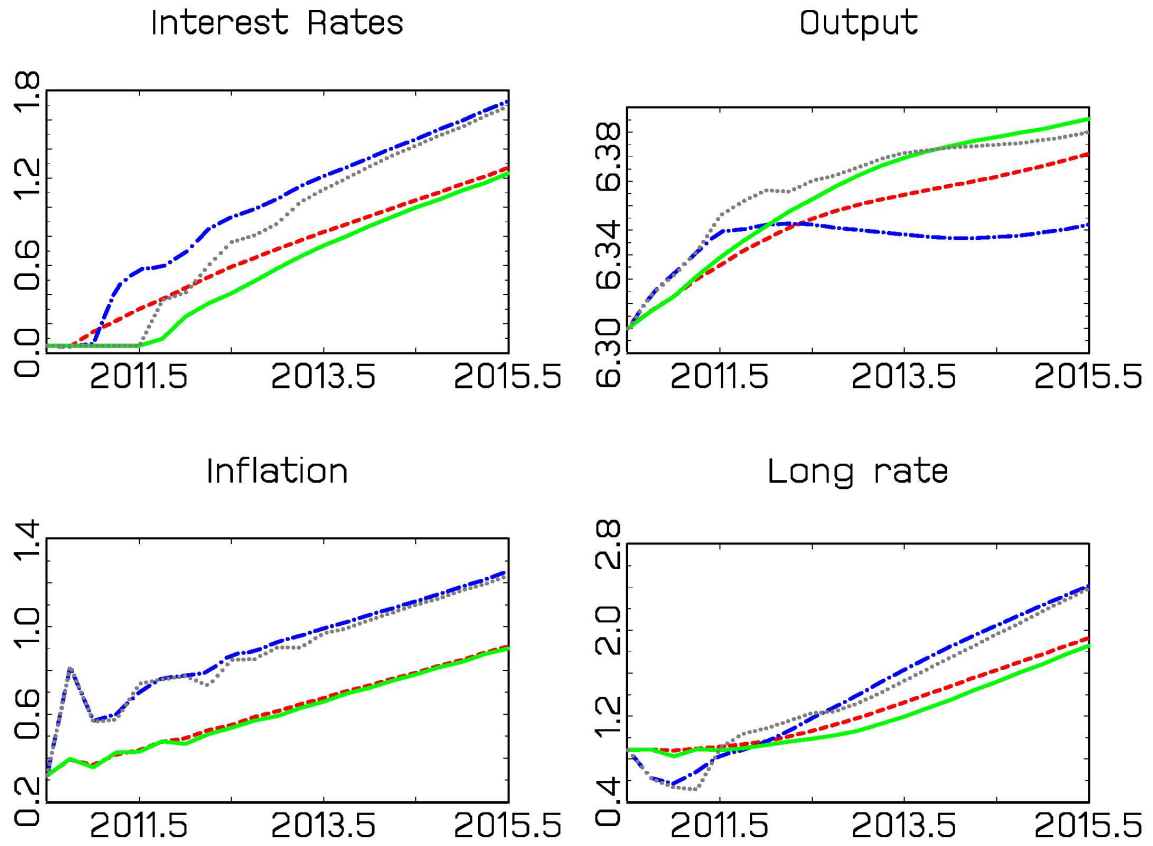
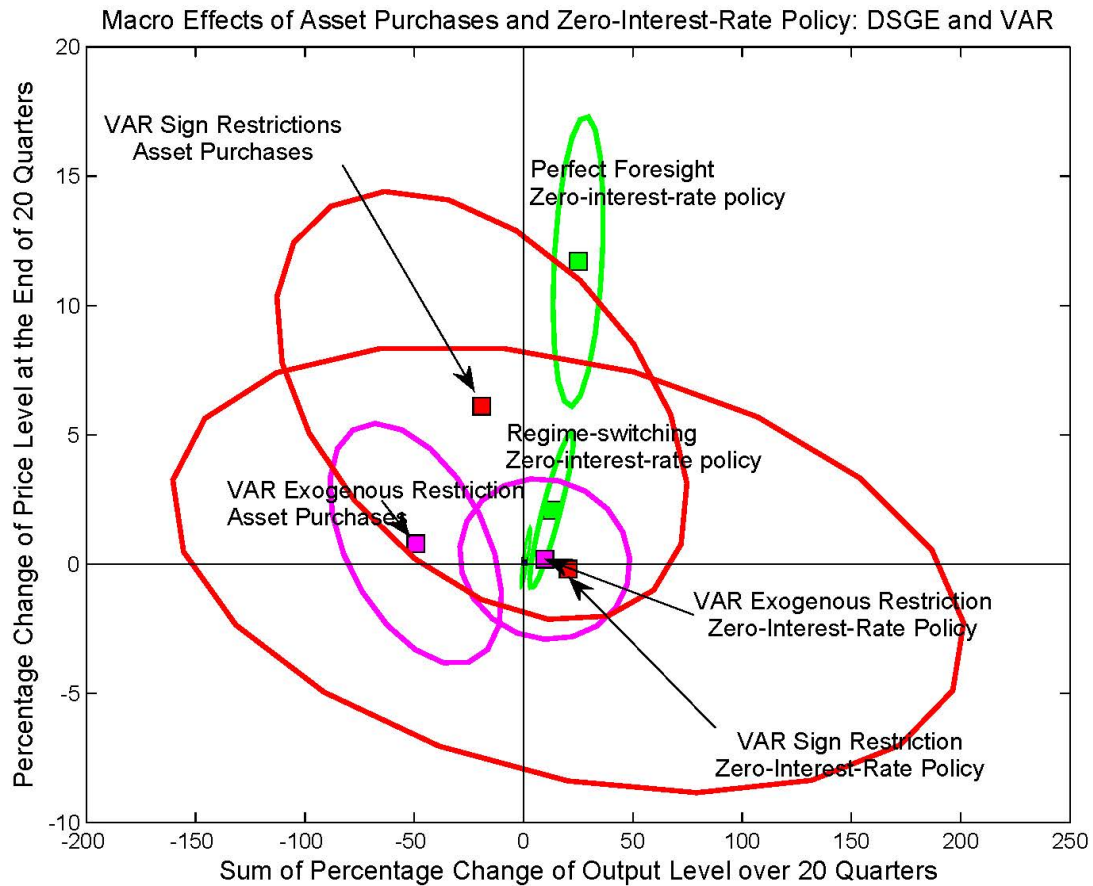


Figure 3.10: *Summary of effects of LSAPs and ZIRP in DSGE models and VAR models.* The squares stand for mean effects and the circles reflect the uncertainty. Green represents bonds-in-utility model, blue represents the results reported by chapter 1, pink represents the VAR with exogenous restrictions, and red represents the VAR with sign restrictions



Appendix A

Appendix to Chapter 1

A.1 Model

A.1.1 Final goods producers

The final good Y_t is a composite made of a continuum of goods indexed by $i \in (0, 1)$

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}. \quad (\text{A.1})$$

The **final goods producers** buy the intermediate goods on the market, package Y_t , and resell it to consumers. These firms maximize profits in a perfectly competitive environment. Their problem is

$$\begin{aligned} \max_{Y_t, Y_t(i)} \quad & P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \\ \text{s.t.} \quad & Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f} (\mu_{f,t}). \end{aligned} \quad (\text{A.2})$$

The FOCs are

$$[\partial Y_t] : P_t = \mu_{f,t}, \quad (\text{A.3})$$

$$[\partial Y_t(i)] : -P_t(i) + \mu_{f,t} \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{\lambda_f} Y_t(i)^{-\frac{\lambda_f}{1+\lambda_f}} = 0. \quad (\text{A.4})$$

Note that

$$\left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{\lambda_f} = Y_t^{\frac{\lambda_f}{1+\lambda_f}}.$$

From the FOCs one obtains

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t.$$

Combining this condition with the zero profit condition (because these firms operate in a perfectly competitive market) one obtains the expression for the price of the composite good

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_f}} di \right]^{-\lambda_f}. \quad (\text{A.5})$$

Note that the elasticity is $\frac{1+\lambda_f}{\lambda_f}$. $\lambda_f = 0$ corresponds to the linear case. $\lambda_f \rightarrow \infty$ corresponds to the Cobb-Douglas case. We will constrain $\lambda_f \in (0, \infty)$.

A.1.2 Intermediate goods producers

Intermediate goods producer i uses the following technology:

$$Y_t(i) = Z_t^{1-\alpha} K_t(i)^\alpha L_t(i)^{1-\alpha}. \quad (\text{A.6})$$

The log of the growth rate of productivity $z_t = \log \left(\frac{Z_t/Z_{t-1}}{1+\gamma} \right)$ follows the process

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim \mathbb{N}(0, \sigma_{\epsilon_z}^2), \quad (\text{A.7})$$

The firm's profit is given by:

$$P_t(i)Y_t(i) - W_tL_t(i) - R_t^kK_t(i).$$

Cost minimization subject to (A.6) yields the conditions:

$$\begin{aligned} [\partial L_t(i)] &: \mathcal{V}_t(i)(1-\alpha)Z_t^{1-\alpha}K_t(i)^\alpha L_t(i)^{-\alpha} = W_t. \\ [\partial K_t(i)] &: \mathcal{V}_t(i)\alpha Z_t^{1-\alpha}K_t(i)^{\alpha-1}L_t(i)^{1-\alpha} = R_t^k. \end{aligned}$$

where $\mathcal{V}_t(i)$ is the Lagrange multiplier associated with A.6. In turn, these conditions imply

$$\frac{K_t(i)}{L_t(i)} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}.$$

Note that if we integrate both sides of the equation with respect to i and define $K_t = \int K_t(i)di$ and $L_t = \int L_t(i)di$, we obtain a relationship between aggregate labor and capital:

$$K_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} L_t. \quad (\text{A.8})$$

The marginal cost MC_t is the same for all firms and equal to

$$\begin{aligned} MC_t &= \left[W_t + R_t^k \frac{K_t(i)}{L_t(i)} \right] Z_t^{-(1-\alpha)} \left(\frac{K_t(i)}{L_t(i)} \right)^{-\alpha} \\ &= \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} (R_t^k)^\alpha [(1+\gamma)e^{z_t}]^{-(1-\alpha)}. \end{aligned} \quad (\text{A.9})$$

Profits can then be expressed as $(P_t(i) - \lambda_{f,t} MC_t) Y_t(i)$, where $\lambda_{f,t}$ is a shock to the time-varying price markup and is assumed to follow the exogenous process:

$$\ln \lambda_{f,t} = \rho_{\lambda_f} \ln \lambda_{f,t-1} + \epsilon_{\lambda,t}, \quad \epsilon_{\lambda,t} \sim \mathbb{N}(0, \sigma_{\epsilon_\lambda}^2). \quad (\text{A.10})$$

Prices are sticky as in Calvo (1983). Specifically, each firm can readjust prices with probability $1 - \zeta_p$ in each period. We depart from Calvo (1983) in assuming that for those firms that cannot adjust prices, $P_t(i)$ will increase at the steady state rate of inflation π . For those firms that can adjust prices, the problem is to choose a price level $\tilde{P}_t(i)$ that maximizes the expected present discounted value of profits in all states of nature where the firm is stuck with that price in the future:

$$\begin{aligned} \max_{\tilde{P}_t(i)} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p \left[\tilde{P}_t(i) \Pi^s - \lambda_{f,t+s} MC_{t+s} \right] Y_{t+s}(i) \\ \text{s.t.} \quad & Y_{t+s}(i) = \left[\frac{\tilde{P}_t(i) \Pi^s}{P_{t+s}} \right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_{t+s}, \end{aligned} \quad (\text{A.11})$$

where $\Pi \equiv 1 + \pi$, and Ξ_{t+s}^p is today's value of a future dollar for the average shareholder. This variable is the appropriate discount factor of future dividends because we assume that ownership of intermediate goods producing firms is equally distributed among all households. The definition of average marginal utility is

$$\Xi_{t+s}^p \equiv \sum_j \omega_j \beta_j^s \Xi_{t+s}^{j,p},$$

where ω_j represents the measure of type j in the population.

The FOC for the firm is:

$$\begin{aligned} 0 = \quad & \tilde{P}_t(i) \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p \frac{1}{\lambda_f} \Pi^{s \left(1 - \frac{1+\lambda_f}{\lambda_f}\right)} P_{t+s}^{\frac{1+\lambda_f}{\lambda_f}} Y_{t+s} \\ & - \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p \frac{1 + \lambda_f}{\lambda_f} \Pi^{-s \frac{1+\lambda_f}{\lambda_f}} P_{t+s}^{\frac{1+\lambda_f}{\lambda_f}} Y_{t+s} \lambda_{f,t+s} MC_{t+s}. \end{aligned} \quad (\text{A.12})$$

Note that all firms readjusting prices face an identical problem. We will consider only the symmetric equilibrium in which all firms that can readjust prices will choose

the same $\tilde{P}_t(i)$, so we can drop the i index from now on. From A.5 it follows that:

$$P_t = \left[(1 - \zeta_p) \tilde{P}_t^{-\frac{1}{\lambda_f}} + \zeta_p [\Pi P_{t-1}]^{-\frac{1}{\lambda_f}} \right]^{-\lambda_f}. \quad (\text{A.13})$$

A.1.3 Capital producers

There is a representative firm, owned by all households, that operates under perfect competition, invests in capital, chooses utilization and rents it to intermediate firms. By choosing the utilization rate u_t , capital producers end up renting in each period t an amount of “effective” capital equal to

$$K_t = u_t \bar{K}_{t-1}, \quad (\text{A.14})$$

and R_t^k is the return per unit of effective capital. Utilization, however, subtracts real resources measured in terms of the consumption good

$$a(u_t) \bar{K}_{t-1}.$$

The law of motion of capital is

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t, \quad (\text{A.15})$$

where $\delta \in (0, 1)$ is the depreciation rate and $S(\cdot)$ is the cost of adjusting investment, with $S'(\cdot) > 0$ and $S''(\cdot) > 0$.

Capital producers maximize expected discounted stream of dividends to their

shareholders:

$$\max_{\bar{K}_t, u_t, I_t} \mathbb{E}_t \sum_{s=0}^{\infty} (\omega_u \beta_u^s \Xi_{t+s}^{p,u} + \omega_r \beta_r^s \Xi_{t+s}^{p,r}) [R_{t+s}^k u_{t+s} \bar{K}_{t+s-1} - P_{t+s} a(u_{t+s}) \bar{K}_{t+s-1} - P_{t+s} I_{t+s}]$$

subject to the LOM of capital (A.15), with Q_t the lagrange multiplier associated with the constraint, and consider that the multiplier for time $t + s$ constraint is premultiplied by $(\omega_u \beta_u^s \Xi_{t+s}^{p,u} + \omega_r \beta_r^s \Xi_{t+s}^{p,r})$. FOC are:

$$[\partial u_t] : 0 = R_t^k - P_t a'(u_t) \quad (\text{A.16})$$

$$[\partial \bar{K}_t] : Q_t = \mathbb{E}_t \left\{ \frac{\omega_u \beta_u \Xi_{t+1}^{p,u} + \omega_r \beta_r \Xi_{t+1}^{p,r}}{\omega_u \Xi_t^{p,u} + \omega_r \Xi_t^{p,r}} [R_{t+1}^k u_{t+1} - P_{t+1} a(u_{t+1}) + (1 - \delta) Q_{t+1}] \right\} \quad (\text{A.17})$$

$$[\partial I_t] : 0 = -1 + \frac{Q_t}{P_t} \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] - \frac{Q_t}{P_t} \mu_t S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \quad (\text{A.18})$$

$$+ \mathbb{E}_t \left[\frac{(\omega_u \beta_u \Xi_{t+1}^{p,u} + \omega_r \beta_r \Xi_{t+1}^{p,r}) P_{t+1}}{(\omega_u \Xi_t^{p,u} + \omega_r \Xi_t^{p,r}) P_t} \frac{Q_{t+1}}{P_{t+1}} \mu_{t+1} S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]$$

A.1.4 Households

The key modification relative to the standard model is the introduction of long-term bonds and segmentation. We follow the formulation in Woodford (2001) and consider long-term bonds with coupon equal to κ^s paid at time $t + 1 + s$, for $s \geq 0$. This implies that the gross yield to maturity is given by

$$R_{L,t} = \frac{1}{P_{L,t}} + \kappa \quad (\text{A.19})$$

or, equivalently, the price of such bond is given by

$$P_{L,t} = \frac{1}{R_{L,t} - \kappa}. \quad (\text{A.20})$$

The duration of this bond is $R_{L,t} / (R_{L,t} - \kappa)$, which we will match to the average duration of ten-year Treasury Bills. Notice also that the price of a bond issued s

periods before is given by $P_{L,t}(s) = \kappa^s P_{L,t}$, which will be used to write the flow budget constraint as a function of the stock of total long term debt, B_t^L , instead of the current period's purchases of long-term debt. As in standard models, short-term assets B_t are one-period bonds, purchased at time t , which pay a nominal return R_t at time $t + 1$.

Households are ordered on a continuum of measure 1. A fraction ω_u of households (unrestricted, or u) trade in both short-term (one-period) and long-term (L-period) bonds. The remaining fraction $\omega_r = 1 - \omega_u$ (restricted, or r) only trade in long-term bonds. Additionally, unrestricted households pay a transaction cost ζ_t per-unit of long-term bond purchased while restricted households do not.

The flow budget constraint differs depending on whether the household is unrestricted or restricted. For an unrestricted household who can trade both short and long-term bonds, we have

$$P_t C_t^u + B_t^u + (1 + \zeta_t) P_{L,t} B_t^{L,u} \leq R_{t-1} B_{t-1}^u + \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,u} + W_t^u(i) L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u, \quad (\text{A.21})$$

where $\zeta_t P_{L,t} B_t^{L,u}$ is paid to the financial institution who redistributes the proceeds $\mathcal{P}_t^{fi}$ to the household. For a restricted household who can only trade in long-term securities but does not pay transaction costs, we have

$$P_t C_t^r + P_{L,t} B_t^{L,r} \leq \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,r} + W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r. \quad (\text{A.22})$$

In equations (A.21) and (A.22), P_t is the price of the final consumption good, $W_t^j(i)$ is the wage set by a household of type $j = \{u, r\}$ who supplies labor of type i , $\mathcal{P}_t$ and $\mathcal{P}_t^{cp}$ are profits from ownership of intermediate goods producers and capital producers respectively, and T_t^j are lump-sum taxes.

One advantage of assuming that the entire stock of long-term government bonds

consists of perpetuities of this type is that the price in period t of a bond issued s periods ago, $P_{L,t}(s)$, is a function of the coupon and the current price:

$$P_{L,t}(s) = \kappa^s P_{L,t}.$$

This relation allows us to rewrite the household budget constraint in a more convenient recursive formulation. One bond of this type that has been issued $s - 1$ periods ago is equivalent to κ^{s-1} new bonds. By no arbitrage at time $t - 1$

$$\begin{aligned} P_{L,t-1} B_{t-1}^L &= \sum_{s=1}^{\infty} P_{L,t}(s) B_{t-s}^L \\ P_{L,t-1} B_{t-1}^L &= \sum_{s=1}^{\infty} P_{L,t-1} \kappa^{s-1} B_{t-s}^L \\ B_{t-1}^L &= \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^L \end{aligned}$$

at time t , B_{t-1}^L is worth $B_{t-1}^L (1 + \kappa P_{L,t}) = B_{t-1}^L \left(1 + \frac{\kappa}{R_{L,t} - \kappa}\right) = P_{L,t} R_{L,t} B_{t-1}^L$.

The budget constraint of an unrestricted household becomes

$$P_t C_t^u + B_t^u + (1 + \zeta_t) P_{L,t} B_t^{L,u} \leq R_{t-1} B_{t-1}^u + P_{L,t} R_{L,t} B_{t-1}^{L,u} + W_t^u(i) L_t^u(i) + \mathcal{P}_t^u + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u. \quad (\text{A.23})$$

For a restricted household we have

$$P_t C_t^r + P_{L,t} B_t^{L,r} \leq P_{L,t} R_{L,t} B_{t-1}^{L,r} + W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r, \quad (\text{A.24})$$

where $R_{L,t}$ is the gross yield to maturity at time t on the long-term bond⁵⁸ and we have

$$R_{L,t} = \frac{1}{P_{L,t}} + \kappa.$$

⁵⁸We match the duration of this bond ($R_{L,t}/(R_{L,t} - \kappa)$) to the average duration of ten-year U.S. Treasury Bills.

Household j consumption-saving decisions are then the result of the maximization of (A.25) subject to (A.23) if $j = u$ or (A.24) if $j = r$.

Households enjoy consumption $C_{j,t}$ and dislike hours worked $L_{j,t}$. The objective function for all households is

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_j^s b_{j,t+s} \left[\frac{\left(\frac{C_{t+s}^j}{Z_{t+s}} - h \frac{C_{t+s-1}^j}{Z_{t+s-1}} \right)^{1-\sigma_j}}{1-\sigma_j} - \frac{\varphi_{t+s}^j L_{t+s}^j (i)^{1+\nu}}{1+\nu} \right], \quad (\text{A.25})$$

where $j = \{u, r\}$, $\beta_j \in (0, 1)$ is the individual discount factor (which may differ between restricted and unrestricted households), $\sigma_j > 0$ is the individual coefficient of relative risk aversion (which may also differ between the different types of households), $\nu \geq 0$ is the inverse elasticity of labor supply, b_t^j is a preference shock to individual j , and ψ_t is a labor supply shock.

Define $\Xi_t^{p,u}$ as the Lagrange multiplier associated with the budget constraint (A.23) and $\Xi_t^{p,r}$ the Lagrange multiplier associated with the budget constraint (A.24). Households perfectly share their consumption within their groups (restricted and unrestricted). This assumption implies that the multipliers $\Xi_t^{p,u}$ and $\Xi_t^{p,r}$ are the same for all households of a certain type in all periods and across all states of nature.

The first order conditions for consumption and bond holdings for an unrestricted

household are

$$[\partial C_t^u] : \frac{1}{P_t} \left\{ \frac{b_t^u}{Z_t} \left(\frac{C_t^u}{Z_t} - h \frac{C_{t-1}^u}{Z_{t-1}} \right)^{-\sigma_u} - \beta_u h \mathbb{E}_t \left[\frac{b_{t+1}^u}{Z_t} \left(\frac{C_{t+1}^u}{Z_{t+1}} - h \frac{C_t^u}{Z_t} \right)^{-\sigma_u} \right] \right\} = \Xi_t^{p,u}, \quad (\text{A.26})$$

$$[\partial B_t(u)] : \Xi_t^{p,u} = \beta_u R_t \mathbb{E}_t[\Xi_{t+1}^{p,u}], \quad (\text{A.27})$$

$$[\partial B_t^L(u)] : \frac{1 + \zeta_t}{R_{L,t} - \kappa} \Xi_t^{p,u} = \beta_u \mathbb{E}_t \left[\frac{R_{L,t+1}}{R_{L,t+1} - \kappa} \Xi_{t+1}^{p,u} \right]. \quad (\text{A.28})$$

The first order conditions for consumption and bond holdings for a restricted household are

$$[\partial C_t(r)] : \frac{1}{P_t} \left\{ \frac{b_t^r}{Z_t} \left(\frac{C_t^r}{Z_t} - h \frac{C_{t-1}^r}{Z_{t-1}} \right)^{-\sigma_r} - \beta_r h \mathbb{E}_t \left[\frac{b_{t+1}^r}{Z_t} \left(\frac{C_{t+1}^r}{Z_{t+1}} - h \frac{C_t^r}{Z_t} \right)^{-\sigma_r} \right] \right\} = \Xi_t^{p,r}, \quad (\text{A.29})$$

$$[\partial B_t^L(r)] : \frac{1}{R_{L,t} - \kappa} \Xi_t^{p,r} = \beta_r \mathbb{E}_t \left[\frac{R_{L,t+1}}{R_{L,t+1} - \kappa} \Xi_{t+1}^{p,r} \right]. \quad (\text{A.30})$$

Households are monopolistic suppliers of labor inputs $L_t(i)$, which perfectly competitive labor agencies aggregate into a homogenous labor composite L_t according to the technology

$$L_t = \left[\int_0^1 L_t(i)^{\frac{1}{1+\lambda_w}} di \right]^{1+\lambda_w}, \quad (\text{A.31})$$

where $\lambda_w \geq 0$ is the steady state wage markup. The first order condition for the demand of labor input i is

$$L_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t. \quad (\text{A.32})$$

Combining this condition with the zero profit condition for labor agencies we obtain

an expression for the aggregate wage index W_t as a function of the wage specific to the i^{th} labor input

$$W_t = \left[\int_0^1 W_t(i)^{-\frac{1}{\lambda_w}} di \right]^{-\lambda_w}. \quad (\text{A.33})$$

Household members set wages on a staggered basis (Calvo (1983)) subject to the demand for their specific labor input (A.32). The wage gets reset with probability $1 - \zeta_w$ in each period, while with the complementary probability the wage grows at the steady state rate of inflation and productivity. Formally, the problem for a household member i of type j who can reset her wage at time t is

$$\min_{\tilde{W}_t^j(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s b_{t+s}^j \frac{\varphi_{t+s}^j}{1 + \nu_j} L_{t+s}^j(i)^{1+\nu_j} \quad (\text{A.34})$$

subject to the budget constraint (A.23) or (A.24), the demand for labor (A.32) and the wage updating scheme

$$W_{t+s}^j(i) = (\Pi e^\gamma)^s \tilde{W}_t^j(i). \quad (\text{A.35})$$

The first order condition for this problem is

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s \Xi_{t+s}^{p,j} L_{t+s}^j(i) \left[(\Pi e^\gamma)^s \tilde{W}_t^j(i) - (1 + \lambda_w) \frac{b_{t+s}^j \varphi_{t+s}^j L_{t+s}^j(i)^{\nu_j}}{\Xi_{t+s}^{p,j}} \right] = 0. \quad (\text{A.36})$$

In the absence of nominal rigidities, this condition would amount to setting the real wage as a markup over the marginal rate of substitution between consumption and leisure.

All agents of type $j = u, r$ resetting their wage face an identical problem. We focus on the symmetric equilibrium in which all agents of type j that can readjust

their wage choose the same $\tilde{W}_t^j$, in which case we get

$$\left(\tilde{W}_t^j\right)^{1+\frac{1+\lambda_w}{\lambda_w}\nu_j} = (1+\lambda_w) \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s b_{t+s}^j \varphi_{t+s}^j (\Pi e^\gamma)^{-s\frac{1+\lambda_w}{\lambda_w}(1+\nu_j)} W_{t+s}^{\frac{1+\lambda_w}{\lambda_w}(1+\nu_j)} L_{t+s}^{1+\nu_j}}{\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta_j)^s \Xi_{t+s}^{p,j} (\Pi e^\gamma)^{s(1-\frac{1+\lambda_w}{\lambda_w})} W_{t+s}^{\frac{1+\lambda_w}{\lambda_w}} L_{t+s}} \quad (\text{A.37})$$

for $j = u, r$.

Therefore, the aggregate wage index (A.33) can be written as

$$W_t = \left[(1 - \zeta_w) \left(\omega_u \left(\tilde{W}_t^u \right)^{-\frac{1}{\lambda_w}} + \omega_r \left(\tilde{W}_t^r \right)^{-\frac{1}{\lambda_w}} \right) + \zeta_w (\Pi e^\gamma W_{t-1})^{-\frac{1}{\lambda_w}} \right]^{-\lambda_w}. \quad (\text{A.38})$$

A.1.5 Government Policies

The central bank follows a conventional feedback interest rate rule (Taylor (1993)) with smoothing:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_m} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}} \right)^{\phi_y} \right]^{1-\rho_m} e^{\epsilon_{m,t}},$$

where $\rho_m \in (0, 1)$, $\phi_\pi > 1$ and $\phi_y \geq 0$.

The presence of long-term bonds modifies the standard government budget constraint

$$B_t + P_{L,t} B_t^L = R_{t-1,t} B_{t-1} + (1 + \kappa P_{L,t}) B_{t-1}^L + P_t G_t - T_t. \quad (\text{A.39})$$

The left-hand side of expression (A.39) is the market value, in nominal terms, of the total amount of bonds (short-term and long-term) issued by the government at time t . The right-hand side features the cost of servicing bonds maturing at time t as well as spending G_t and taxes T_t .

We assume that the government controls the supply of long-term bond following

a simple autoregressive rule

$$\frac{P_{L,t}B_t^L}{P_tZ_t} = S \left(\frac{P_{L,t-1}B_{t-1}^L}{P_{t-1}Z_{t-1}} \right)^{\rho_B} e^{\epsilon_{B,t}}, \quad (\text{A.40})$$

where $\rho_B \in (0, 1)$ and $\epsilon_{B,t}$ is an i.i.d. exogenous shock. S is whatever constant needed to make the above equation an identity at the steady state. We interpret LSAP programs as shocks to the outstanding government long-term liabilities compared to the historical behavior of these series.

Finally, we set taxes according to the feedback rule

$$\frac{T_t}{P_tZ_t} - \frac{G_t}{Z_t} \equiv \Phi_{z,t} = \Phi \left(\frac{\frac{1}{R_{L,t-1}-\kappa}B_{Z,t-1}^L + B_{Z,t-1}}{\frac{1}{R_L-\kappa}B_Z^L + B_Z} \right)^{\phi_T} e^{\epsilon_{T,t}}, \quad (\text{A.41})$$

where $\epsilon_{T,t}$ follows a stationary AR(1) process and the term in parenthesis on the right hand side is the ratio of total debt value in period t to its steady state value.

A.1.6 Term Premium and Preferred Habitat

Our baseline formulation of the relation between transaction costs and the quantity of debt is

$$\zeta_t = \left(\frac{P_{L,t}B_t^L}{P_tZ_t} \right)^{\rho_\zeta} \exp(\epsilon_{\zeta,t}).$$

The Euler Equation of an *unrestricted* household for investing in long-term bonds is

$$(1 + \zeta_t)P_{L,t}\Xi_t^{p,u} = \beta_u \mathbb{E}_t(P_{L,t+1}R_{L,t+1}\Xi_{t+1}^{p,u}). \quad (\text{A.42})$$

Define $P_{L,t}^{EH}$ and $R_{L,t}^{EH}$ the price and yield to maturity of the long-term bond that would arise in the absence of transaction costs, holding constant the path for the marginal utility of consumption. In defining $R_{L,t}^{EH}$, we also adjust the parameter κ so

that in steady state the counterfactual long-term bond has the same maturity of the bond in the model with transaction costs, that is

$$D_L = \frac{R_L}{R_L - \kappa} = \frac{R_L^{EH}}{R_L^{EH} - \kappa^{EH}} = D_L^{EH}. \quad (\text{A.43})$$

The counterpart of equation (A.42) in this counterfactual world is

$$P_{L,t}^{EH} \Xi_t^{p,u} = \beta_u \mathbb{E}_t (P_{L,t+1}^{EH} R_{L,t+1}^{EH} \Xi_{t+1}^{p,u}). \quad (\text{A.44})$$

No arbitrage implies that the counterfactual long-term bond should have the same risk-adjusted return as the long-term bond in the actual economy with transaction costs. Rearranging (A.42) and (A.44) and taking the difference yields

$$\mathbb{E}_t \left\{ \frac{\Xi_{t+1}^{p,u}}{\Xi_t^{p,u}} \left[\frac{P_{L,t+1}}{(1 + \zeta_t) P_{L,t}} R_{L,t+1} - \frac{P_{L,t+1}^{EH}}{P_{L,t}^{EH}} R_{L,t+1}^{EH} \right] \right\} = 0.$$

Up to a first order approximation, the previous equation becomes

$$\mathbb{E}_t \left[\hat{P}_{L,t+1} - \hat{P}_{L,t} - \zeta_t + \hat{R}_{L,t+1} - (\hat{P}_{L,t+1}^{EH} - \hat{P}_{L,t}^{EH} + \hat{R}_{L,t+1}^{EH}) \right] = 0.$$

Also up to the first order, from equation (A.43) the relation between price and yields is

$$\hat{P}_{L,t} = -D_L \hat{R}_{L,t}.$$

We define the risk premium as the difference, in log-deviations from steady state, of the yield to maturity with and without transaction costs

$$\widehat{RP}_t \equiv \hat{R}_{L,t} - \hat{R}_{L,t}^{EH}.$$

We can then combine the approximation of the no arbitrage condition and the relation between price and yield to obtain a first-order forward looking difference equation in the risk premium

$$(D_L - 1)\mathbb{E}_t \widehat{RP}_{t+1} - D_L \widehat{RP}_t + \zeta_t = 0.$$

Because $D_L > 1$, the previous equation can be solved forward to obtain

$$\widehat{RP}_t = \frac{1}{D_L} \sum_{s=0}^{\infty} \left(\frac{D_L - 1}{D_L} \right)^s \mathbb{E}_t \zeta_{t+s},$$

which corresponds to the equation in the text.

A.1.7 Aggregation

Resource constraints

Budget constraint for the unconstrained household is

$$P_t C_t^u + B_t^u + \frac{1 + \zeta_t}{R_{L,t} - \kappa} B_t^{L,u} = R_{t-1} B_{t-1}^u + \frac{R_{L,t}}{R_{L,t} - \kappa} B_{t-1}^{L,u} + \int W_t^u(i) L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^u$$

Budget constraint for the constrained household is

$$P_t C_t^r + \frac{1}{R_{L,t} - \kappa} B_t^{L,r} = \frac{R_{L,t}}{R_{L,t} - \kappa} B_{t-1}^{L,r} + \int W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^{fi} - T_t^r$$

Government's budget constraint is

$$B_t + \frac{1}{R_{L,t} - \kappa} B_t^L = R_{t-1} B_{t-1} + \frac{R_{L,t}}{R_{L,t} - \kappa} B_{t-1}^L + P_t G_t - T_t$$

Next, realize that

$$\mathcal{P} = \int_t \mathcal{P}(i) di = \int P_t(i) Y_t(i) di - W_t L_t - R_t^K K_t,$$

where $L_t = \int L_t(i) di$ is total labor supplied by the labor packers and demanded by the firms. $K_t = \int K_t(i) di$. We plug the definition of Π_t into household's budget constraints and realize that the profit of labor packer and good packer is zero.

It must be the case that

$$W_t L_t = \int W_t^u(i) L_t^u(i) di + \int W_t^r(i) L_t^r(i) di$$

and

$$P_t Y_t = \int P_t(i) Y_t(i) di.$$

The capital producer's profit is

$$R_t^K K_t - P_t a(u_t) \bar{K}_{t-1} - P_t I_t.$$

The financial institution's profit is

$$\mathcal{P}_t^{fi} = \varpi_u \frac{\zeta_t}{R_{L,t} - \kappa} B_t^{L,u}.$$

Finally the budget constraint is

$$\varpi_u C_t^u + \varpi_r C_t^r + G_t + a(u_t) \bar{K}_{t-1} + I_t = Y_t. \quad (\text{A.45})$$

Exogenous Processes

The model is supposed to be fitted to data on output, consumption, investment, employment, wages, nominal interest rates and market value of bonds.

- Technology process: let $z_t = \ln(e^{-\gamma} Z_t / Z_{t-1})$

$$z_t = \rho_z z_t + \epsilon_{z,t} \quad (\text{A.46})$$

- Preference for leisure:

$$\ln \varphi_t = \rho_\varphi \ln \varphi_{t-1} + \epsilon_{\varphi,t} \quad (\text{A.47})$$

- Price Mark-up shock:

$$\ln \lambda_{f,t} = \epsilon_{\lambda,t} \quad (\text{A.48})$$

- Capital adjustment cost process:

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \epsilon_{\mu,t} \quad (\text{A.49})$$

- Intertemporal preference shifter:

$$\ln b_t = \rho_b \ln b_{t-1} + \epsilon_{b,t} \quad (\text{A.50})$$

- Government spending process:

$$\ln g_t = \rho_g \ln g_{t-1} + \epsilon_{g,t} \quad (\text{A.51})$$

- Monetary Policy Shock $\epsilon_{m,t}$.

- Exogenous risk premium shock:

$$\epsilon_{\zeta,t} = \rho_\zeta \epsilon_{\zeta,t-1} + \epsilon_{\zeta,t} \quad (\text{A.52})$$

- Fiscal shock $\epsilon_{T,t}$
- Long-term bond supply shock $\epsilon_{B,t}$

A.2 Normalized Equations

Consider the following normalizations:

- $r_t^k \equiv \frac{R_t^k}{P_t}$; $w_{z,t} \equiv \frac{W_t}{Z_t P_t}$; $mc_t \equiv \frac{MC_t}{P_t}$; $q_t \equiv \frac{Q_t}{P_t}$
- $\Xi_t^j \equiv \Xi_t^p(j) Z_t P_t, \forall j$
- $x_{z,t} \equiv x_t/Z_t, \forall x_t$, except for the cases below
- $B_{z,t} \equiv \frac{B_t}{P_t Z_t}$; $B_{z,t}^L \equiv \frac{B_t^L}{P_t Z_t}$; $G_{z,t} \equiv \frac{G_t}{Z_t}$; $T_{z,t} \equiv \frac{T_t}{P_t Z_t}$

Real marginal cost

$$mc_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} (r_t^k)^\alpha w_{z,t}^{1-\alpha} \quad (\text{A.53})$$

Capital demand

$$K_{z,t} = \frac{\alpha}{1 - \alpha} \frac{w_{z,t}}{r_t^k} L_t \quad (\text{A.54})$$

Technology

$$Y_{z,t} = K_{z,t}^\alpha L_t^{1-\alpha} \quad (\text{A.55})$$

price setting

$$\tilde{p}_t = \frac{\omega_u X_t^{n,u} + \omega_r X_t^{n,r}}{\omega_u X_t^{d,u} + \omega_r X_t^{d,r}} \quad (\text{A.56})$$

with

$$X_t^{pn,j} = \Xi_t^j Y_{z,t} (1 + \lambda_f) \lambda_{f,t} m c_t + \beta_j \zeta_p \mathbb{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{1+\lambda_f}{\lambda_f}} X_{t+1}^{pn,j} \right], j = u, r \quad (\text{A.57})$$

$$X_t^{pd,j} = \Xi_t^j Y_{z,t} + \beta_j \zeta_p \mathbb{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{1}{\lambda_f}} X_{t+1}^{pd,j} \right], j = u, r \quad (\text{A.58})$$

LOM prices

$$1 = (1 - \zeta_p) \left(\frac{\omega_u X_t^{pn,u} + \omega_r X_t^{pn,r}}{\omega_u X_t^{pd,u} + \omega_r X_t^{pd,r}} \right)^{-\frac{1}{\lambda_f}} + \zeta_p \left(\frac{\Pi}{\Pi_t} \right)^{-\frac{1}{\lambda_f}} \quad (\text{A.59})$$

Effective capital

$$K_{z,t} = e^{-\gamma - z_t} u_t \bar{K}_{z,t-1} \quad (\text{A.60})$$

Law of motion of capital

$$\bar{K}_{z,t} = (1 - \delta) e^{-\gamma - z_t} \bar{K}_{z,t-1} + \mu_t \left[1 - S \left(e^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \right) \right] I_{z,t} \quad (\text{A.61})$$

capital utilization

$$r_t^k = a'(u_t) \quad (\text{A.62})$$

Law of motion of Q

$$q_t = \mathbb{E}_t \left\{ \frac{\omega_u \beta_u \Xi_{t+1}^u + \omega_r \beta_r \Xi_{t+1}^r}{\omega_u \Xi_t^u + \omega_r \Xi_t^r} e^{-\gamma - z_{t+1}} [r_{t+1}^k u_{t+1} - a(u_{t+1}) + (1 - \delta) q_{t+1}] \right\} \quad (\text{A.63})$$

Investment decision

$$\begin{aligned} 0 = & -1 + q_t \mu_t \left[1 - S \left(e^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \right) \right] - q_t \mu_t S' \left(e^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \right) e^{\gamma + z_t} \frac{I_{z,t}}{I_{z,t-1}} \\ & + \mathbb{E}_t \left[\frac{\omega_u \beta_u \Xi_{t+1}^u + \omega_r \beta_r \Xi_{t+1}^r}{\omega_u \Xi_t^u + \omega_r \Xi_t^r} e^{-\gamma - z_{t+1}} q_{t+1} \mu_{t+1} S' \left(e^{\gamma + z_{t+1}} \frac{I_{z,t+1}}{I_{z,t}} \right) \left(e^{\gamma + z_{t+1}} \frac{I_{z,t+1}}{I_{z,t}} \right)^2 \right] \end{aligned} \quad (\text{A.64})$$

Marginal Utilities for each type

$$\Xi_t^j = b_t^j (C_{z,t}^j - hC_{z,t-1}^j)^{-\sigma_j} - \beta_j h \mathbb{E}_t \left[b_{t+1}^j (C_{z,t+1}^j - hC_{z,t}^j)^{-\sigma_j} \right], j = u, r \quad (\text{A.65})$$

Euler equation: Unconstrained, short

$$\Xi_t^u = \beta_u R_t \mathbb{E}_t [e^{-\gamma - z_{t+1}} \Xi_{t+1}^u \Pi_{t+1}^{-1}] \quad (\text{A.66})$$

Euler equation: Unconstrained, long

$$(1 + \zeta_t) \Xi_t^u = \beta_u \mathbb{E}_t \left[\Xi_{t+1}^u e^{-\gamma - z_{t+1}} \Pi_{t+1}^{-1} \frac{R_{L,t} - \kappa}{R_{L,t+1} - \kappa} R_{L,t+1} \right] \quad (\text{A.67})$$

Euler equation: Constrained, long

$$\Xi_t^r = \beta_r \mathbb{E}_t \left[\Xi_{t+1}^r e^{-\gamma - z_{t+1}} \Pi_{t+1}^{-1} \frac{R_{L,t} - \kappa}{R_{L,t+1} - \kappa} R_{L,t+1} \right] \quad (\text{A.68})$$

Wage setting

$$\left(\tilde{w}_{z,t}^j \right)^{1 + \frac{1+\lambda_w}{\lambda_w} \nu_j} = \frac{X_t^{wn,j}}{X_t^{wd,j}}, j = u, r \quad (\text{A.69})$$

$$X_t^{wn,j} = (1 + \lambda_w) b_t^j \varphi_t^j L_t^{1+\nu_j} w_{z,t}^{\frac{1+\lambda_w}{\lambda_w} (1+\nu_j)} + \zeta_w \beta_j \mathbb{E}_t \left[\left(\frac{\Pi_{t+1} e^{z_{t+1}}}{\Pi} \right)^{\frac{1+\lambda_w}{\lambda_w} (1+\nu_j)} X_{t+1}^{wn,j} \right], j = u, r \quad (\text{A.70})$$

$$X_t^{wd,j} = \Xi_t^j L_t w_{z,t}^{\frac{1+\lambda_w}{\lambda_w}} + \zeta_w \beta_j \mathbb{E}_t \left[\left(\frac{\Pi_{t+1} e^{z_{t+1}}}{\Pi} \right)^{\frac{1}{\lambda_w}} X_{t+1}^{wd,j} \right], j = u, r \quad (\text{A.71})$$

Law of motion of real wages

$$w_{z,t} = \left[(1 - \zeta_w) \left(\omega_u \left(\frac{X_t^{wn,u}}{X_t^{wd,u}} \right)^{-\frac{1}{\lambda_w} \frac{1}{1 + \frac{1+\lambda_w}{\lambda_w} \nu_u}} + \omega_r \left(\frac{X_t^{wn,r}}{X_t^{wd,r}} \right)^{-\frac{1}{\lambda_w} \frac{1}{1 + \frac{1+\lambda_w}{\lambda_w} \nu_r}} \right) + \zeta_w \left(\frac{\Pi w_{z,t-1}}{\Pi_t e^{z_t}} \right)^{-\frac{1}{\lambda_w}} \right]^{-\lambda_w} \quad (\text{A.72})$$

Budget constraint

$$B_{z,t} + \frac{1}{R_{L,t} - \kappa} B_{z,t}^L = \frac{R_{t-1}}{e^{\gamma+z_t} \Pi_t} B_{z,t-1} + \frac{R_{L,t}}{R_{L,t} - \kappa} \frac{1}{e^{\gamma+z_t} \Pi_t} B_{z,t-1}^L + G_{z,t} - T_{z,t} \quad (\text{A.73})$$

Long term bond policy

$$P_{L,t} B_{z,t}^L = S (P_{L,t-1} B_{z,t-1}^L)^{\rho_B} e^{\epsilon_{B,t}} \quad (\text{A.74})$$

Transfers feedback rule

$$T_{z,t} - G_{z,t} \equiv \Phi_{z,t} = \Phi \left(\frac{\frac{1}{R_{L,t-1} - \kappa} B_{Z,t-1}^L + B_{Z,t-1}}{\frac{1}{R_L - \kappa} B_Z^L + B_Z} \right)^{\phi_T} \exp^{\epsilon_{T,t}} \quad (\text{A.75})$$

Monetary policy

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_m} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_{z,t}}{Y_{z,t-4}} e^{z_{t-3} + \dots + z_t} \right)^{\phi_y} \right]^{1-\rho_m} e^{\epsilon_{m,t}} \quad (\text{A.76})$$

Term premium

$$\zeta_t \equiv \zeta(P_{L,t} B_{z,t}^L, \epsilon_{\zeta,t}) \quad (\text{A.77})$$

Aggregate resources constraint

$$\omega_u C_{z,t}^u + \omega_r C_{z,t}^r + I_{z,t} + G_{z,t} + e^{-\gamma-z_t} a(u_t) \bar{K}_{z,t-1} = Y_{z,t} \quad (\text{A.78})$$

A.3 Model Steady State

In steady state, the log of productivity grows at the constant rate γ and inflation is constant and equal to Π .

We choose a functional form for $a(u_t)$ such that $u = 1$ in steady state and $a(1) = 0$

(Christiano, Motto, and Rostagno (2009)). Furthermore we consider

$$Y_z = 1,$$

$$\nu_u = \nu_r = \nu,$$

$$S(e^\gamma) = S'(e^\gamma) = 0,$$

and estimate

$$\frac{C^u}{C^r}, \frac{\Xi^u}{\Xi^r}$$

and let the levels of b^u and b^r be whatever they need to be to allow these ratios to be consistent with each other and the resources constraint in levels.

Euler equations imply

$$1 = \beta_u R e^{-\gamma} \Pi^{-1}, \quad (\text{A.79})$$

$$(1 + \zeta) = \frac{R_L}{R}, \quad (\text{A.80})$$

$$\beta_u = \beta_r (1 + \zeta). \quad (\text{A.81})$$

Risk premium relation determines level of long debt

$$B_z^{LMV} = \zeta^{-1}(\zeta). \quad (\text{A.82})$$

Govt BC determines taxes

$$T_z = G_z - (1 - \beta_u^{-1}) B_z - \left(\frac{1}{R_L^L - \kappa} - \frac{R_L^L}{R_L^L - \kappa} \frac{1}{e^\gamma \Pi} \right) B_z^L. \quad (\text{A.83})$$

Unit MEI shock implies

$$1 = q. \quad (\text{A.84})$$

Unit utilization implies

$$r^k = a'(1), \quad (\text{A.85})$$

which pins down $a'(1)$ given r^k .

FOC for investment implies

$$r^k = \bar{\beta}^{-1} e^\gamma - (1 - \delta), \quad (\text{A.86})$$

with

$$\bar{\beta} \equiv \frac{\omega_u \beta_u \Xi^u + \omega_r \beta_r \Xi^r}{\omega_u \Xi^u + \omega_r \Xi^r} = \frac{\omega_u \beta_u \frac{\Xi^u}{\Xi^r} + \omega_r \beta_r}{\omega_u \frac{\Xi^u}{\Xi^r} + \omega_r},$$

which is a function of Ξ^u/Ξ^r . Hence r^k is also known given the estimate/calibration of Ξ^u/Ξ^r .

Price setting implies

$$mc = \frac{1}{1 + \lambda_f}. \quad (\text{A.87})$$

Definition of marginal cost implies

$$w_z = \tilde{w}_z (r^k)^{-\frac{\alpha}{1-\alpha}}, \quad (\text{A.88})$$

with

$$\tilde{w}_z \equiv (1 + \lambda_f)^{-\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha).$$

Technology function implies

$$L = K_z^{-\frac{\alpha}{1-\alpha}}, \quad (\text{A.89})$$

and plugging A.89 into capital demand implies

$$K_z = \tilde{K}_z (r^k)^{-1}, \quad (\text{A.90})$$

with

$$\tilde{K}_z = \frac{\alpha}{1 + \lambda_f},$$

which then implies that

$$L = \tilde{L} (r^k)^{\frac{\alpha}{1-\alpha}}, \quad (\text{A.91})$$

with

$$\tilde{L} \equiv \left(\frac{\alpha}{1 + \lambda_f} \right)^{-\frac{\alpha}{1-\alpha}}.$$

Effective capital is

$$\bar{K}_z = e^\gamma \tilde{K}_z (r^k)^{-1}. \quad (\text{A.92})$$

Investment is

$$I_z = [e^\gamma - (1 - \delta)] \tilde{K}_z (r^k)^{-1} \quad (\text{A.93})$$

Resources constraint is

$$\omega_u C_z^u + \omega_r C_z^r = 1 - I_z - G_z \quad (\text{A.94})$$

and given the ratio of consumptions, we get

$$C_z^r = \frac{1 - I_z - G_z}{\omega_u \frac{C_z^u}{C_z^r} + \omega_r}, \quad (\text{A.95})$$

$$C_z^u = \frac{C_z^r}{C_z^r} C_z^r. \quad (\text{A.96})$$

Further notice that

$$\frac{X^{pn,u}}{X^{pn,r}} = \frac{X^{pd,u}}{X^{pd,r}} = \frac{\Xi^u}{\Xi^r} \frac{1 - \beta_r \zeta_p}{1 - \beta_u \zeta_p}, \quad (\text{A.97})$$

which is known.

For the wages, we have

$$\frac{X^{wn,j}}{X^{wd,j}} = (1 + \lambda_w) L^\nu w_z^{\frac{1+\lambda_w}{\lambda_w} \nu} \frac{b^j \varphi^j}{\Xi^j}$$

and in the log-linearization, we will need the ratio

$$\frac{\frac{X^{wn,u}}{X^{wd,u}}}{\frac{X^{wn,r}}{X^{wd,r}}} = \frac{b^u \varphi^u \Xi^r}{b^r \varphi^r \Xi^u} \quad (\text{A.98})$$

or

$$\chi_{wu} = \frac{\omega_u}{\omega_u + \omega_r \left(\frac{b^u \varphi^u \Xi^r}{b^r \varphi^r \Xi^u} \right)^{\frac{1}{\lambda_w} \frac{1}{1 + \frac{1+\lambda_w}{\lambda_w} \nu}}}$$

which, given b^u/b^r and Ξ^u/Ξ^r is given by φ^u/φ^r . Let us then estimate/calibrate this ratio, χ_{wu} , which has to be between 0 and 1.

Rest of steady state relations (not explicitly needed for the numerical analysis) are

$$X^{pn,j} = \frac{\Xi^j Y_z (1 + \lambda_f) m c}{1 - \beta_j \zeta_p}, \quad j = u, r, \quad (\text{A.99})$$

$$X^{pd,j} = \frac{\Xi^j Y_z}{1 - \beta_j \zeta_p}, \quad j = u, r, \quad (\text{A.100})$$

$$\Xi^j = b^j (1 - \beta_j h) (1 - h)^{-\sigma_j} (C_z^j)^{-\sigma_j}, \quad j = u, r, \quad (\text{A.101})$$

$$X^{wn,j} = (1 + \lambda_w) \frac{b^j \varphi^j L^{1+\nu_j} w_z^{\frac{1+\lambda_w}{\lambda_w} (1+\nu)}}{1 - \zeta_w \beta_j}, \quad j = u, r, \quad (\text{A.102})$$

$$X^{wd,j} = \frac{\Xi^j L w_z^{\frac{1+\lambda_w}{\lambda_w}}}{1 - \zeta_w \beta_j}, \quad j = u, r, \quad (\text{A.103})$$

$$w_z = \left[\omega_u \left(\frac{X^{wn,u}}{X^{wd,u}} \right)^{-\frac{1}{\lambda_w} \frac{1}{1 + \frac{1+\lambda_w}{\lambda_w} \nu}} + \omega_r \left(\frac{X^{wn,r}}{X^{wd,r}} \right)^{-\frac{1}{\lambda_w} \frac{1}{1 + \frac{1+\lambda_w}{\lambda_w} \nu}} \right]^{-\lambda_w}. \quad (\text{A.104})$$

A.4 Log-linear Approximation

Consider in general that

$$\hat{x}_t \equiv \ln(x_t/x)$$

for any variable x , except for $\hat{\zeta}_t \equiv \ln\left(\frac{1+\zeta_t}{1+\zeta}\right)$, $r_t \equiv \ln(R_t/R)$, and $r_{L,t} \equiv \ln(R_{L,t}/R_L)$

Real marginal cost

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_{z,t} \quad (\text{A.105})$$

Capital demand

$$\hat{K}_{z,t} = \hat{w}_{z,t} - \hat{r}_t^k + \hat{L}_t \quad (\text{A.106})$$

Technology

$$\hat{Y}_{z,t} = \alpha \hat{K}_{z,t} + (1 - \alpha) \hat{L}_t \quad (\text{A.107})$$

price setting

$$\begin{aligned} \hat{X}_t^{pn,j} &= (1 - \beta_j \zeta_p) \left(\hat{\Xi}_t^j + \hat{Y}_{z,t} + \hat{\lambda}_{f,t} + \widehat{mc}_t \right) + \beta_j \zeta_p E_t \left[\frac{1 + \lambda_f}{\lambda_f} \pi_{t+1} + \hat{X}_{t+1}^{pn,j} \right], \\ \hat{X}_t^{pd,j} &= (1 - \beta_j \zeta_p) \left(\hat{\Xi}_t^j + \hat{Y}_{z,t} \right) + \beta_j \zeta_p E_t \left[\frac{1}{\lambda_f} \pi_{t+1} + \hat{X}_{t+1}^{pd,j} \right], \quad j = u, r. \end{aligned}$$

LOM prices

$$\pi_t = \frac{1 - \zeta_p}{\zeta_p} \left[\chi_{pu} \hat{X}_t^{pn,u} + (1 - \chi_{pu}) \hat{X}_t^{pn,r} - \chi_{pu} \hat{X}_t^{pd,u} - (1 - \chi_{pu}) \hat{X}_t^{pd,r} \right], \quad (\text{A.108})$$

with

$$\chi_{pu} \equiv \frac{\omega_u}{\omega_u + \omega_r \frac{1 - \beta_u \zeta_p}{1 - \beta_r \zeta_p} \left(\frac{\Xi^u}{\Xi^r} \right)^{-1}}.$$

Effective capital

$$\hat{K}_{z,t} = -z_t + \hat{u}_t + \hat{K}_{z,t-1}. \quad (\text{A.109})$$

Law of motion of capital

$$\hat{K}_{z,t} = (1 - \delta)e^{-\gamma}(\hat{K}_{z,t-1} - z_t) + [1 - (1 - \delta)e^{-\gamma}](\hat{\mu}_t + \hat{I}_{z,t}). \quad (\text{A.110})$$

Capital utilization

$$\hat{r}_t^k = \frac{a''(1)}{r^k} \hat{u}_t. \quad (\text{A.111})$$

Law of motion of Q

$$\begin{aligned} \hat{q}_t = & \bar{\beta}e^{-\gamma}E_t \left[r^k \hat{r}_{t+1}^k + (1 - \delta) \hat{q}_{t+1} \right] - E_t \hat{z}_{t+1} \\ & + E_t \left[q_u \left(\frac{1 + \zeta}{1 + q_u \zeta} \hat{\Xi}_{t+1}^u - \hat{\Xi}_t^u \right) + (1 - q_u) \left(\frac{1}{1 + q_u \zeta} \hat{\Xi}_{t+1}^r - \hat{\Xi}_t^r \right) \right], \end{aligned} \quad (\text{A.112})$$

with

$$q_u \equiv \frac{\omega_u \Xi^u}{\omega_u \Xi^u + \omega_r \Xi^r} = \left(\frac{\bar{\beta}}{\beta_r} - 1 \right) \zeta^{-1}.$$

Investment decisions

$$0 = \hat{q}_t + \hat{\mu}_t - e^{2\gamma} S'' \left(\hat{z}_t + \hat{I}_{z,t} - \hat{I}_{z,t-1} \right) + \bar{\beta} e^{2\gamma} S'' \mathbb{E}_t \left[z_{t+1} + \hat{I}_{z,t+1} - \hat{I}_{z,t} \right]. \quad (\text{A.113})$$

Marginal Utilities for each type:

$$\hat{\Xi}_t^j = \frac{1}{1 - \beta_j h} \left[\left(\hat{b}_t^j - \beta_j h E_t \hat{b}_{t+1}^j \right) - \frac{\sigma_j}{1 - h} \left\{ (1 + \beta_j h^2) \hat{C}_{z,t}^j - \beta_j h E_t \hat{C}_{z,t+1}^j - h \hat{C}_{z,t-1}^j \right\} \right], \quad j = u, r. \quad (\text{A.114})$$

Euler equation: Unconstrained, short

$$\hat{\Xi}_t^u = r_t + \mathbb{E}_t(\hat{\Xi}_{t+1}^u - z_{t+1} - \pi_{t+1}). \quad (\text{A.115})$$

Euler equation: Unconstrained, long

$$\hat{\zeta}_t + \hat{\Xi}_t^u = \frac{R_L}{R_L - \kappa} r_{L,t} + \mathbb{E}_t \left[\hat{\Xi}_{t+1}^u - z_{t+1} - \pi_{t+1} - \frac{\kappa}{R_L - \kappa} r_{L,t+1} \right]. \quad (\text{A.116})$$

Euler equation: Constrained, long

$$\hat{\Xi}_t^r = \frac{R_L}{R_L - \kappa} r_{L,t} + \mathbb{E}_t \left[\hat{\Xi}_{t+1}^r - z_{t+1} - \pi_{t+1} - \frac{\kappa}{R_L - \kappa} r_{L,t+1} \right]. \quad (\text{A.117})$$

Wage setting

$$\begin{aligned} \hat{X}_t^{wn,j} &= (1 - \zeta_w \beta_j) \left[\hat{b}_t^j + \hat{\varphi}_t^j + (1 + \nu) \hat{L}_t + \left(\frac{1 + \lambda_w}{\lambda_w} \right) (1 + \nu) \hat{w}_{z,t} \right] \\ &+ \zeta_w \beta_j \mathbb{E}_t \left[\frac{1 + \lambda_w}{\lambda_w} (1 + \nu) (\pi_{t+1} + z_{t+1}) + \hat{X}_{t+1}^{wn,j} \right], \quad j = u, r, \end{aligned} \quad (\text{A.118})$$

$$\begin{aligned} \hat{X}_t^{wd,j} &= (1 - \zeta_w \beta_j) \left[\hat{\Xi}_t^j + \hat{L}_t + \frac{1 + \lambda_w}{\lambda_w} \hat{w}_{z,t} \right] \\ &+ \zeta_w \beta_j \mathbb{E}_t \left[\frac{1}{\lambda_w} (\pi_{t+1} + z_{t+1}) + \hat{X}_{t+1}^{wd,j} \right], \quad j = u, r. \end{aligned} \quad (\text{A.119})$$

Law of motion of real wages

$$\begin{aligned} \hat{w}_{z,t} &= (1 - \zeta_w) \frac{1}{1 + \frac{1 + \lambda_w}{\lambda_w} \nu} \left[\chi_{wu} \left(\hat{X}_t^{wn,u} - \hat{X}_t^{wd,u} \right) + (1 - \chi_{wu}) \left(\hat{X}_t^{wn,r} - \hat{X}_t^{wd,r} \right) \right] \\ &+ \zeta_w (\hat{w}_{z,t-1} - \pi_t - z_t), \end{aligned} \quad (\text{A.120})$$

with

$$\chi_{wu} = \frac{\omega_u}{\omega_u + \omega_r w_{ur}^{\frac{1}{\lambda_w + (1 + \lambda_w) \nu}}}.$$

Budget constraint:

$$\begin{aligned} \hat{B}_{z,t} + \frac{B_z^L/B_z}{R_L - \kappa} \hat{B}_{z,t}^L &= \beta_u^{-1} \left(\hat{B}_{z,t-1} + r_{t-1} \right) + \frac{B_z^L/B_z}{R_L - \kappa} \beta_r^{-1} \hat{B}_{z,t-1}^L \\ &+ \frac{(1 - e^{-\gamma} \Pi^{-1} \kappa) R_L}{R_L - \kappa} \frac{B_z^L/B_z}{R_L - \kappa} r_{L,t} \\ &+ \frac{G_z}{B_z} \hat{G}_{z,t} - \frac{Y_z}{B_z} \hat{T}_{z,t} - \left(\beta_u^{-1} + \frac{B_z^L/B_z}{R_L - \kappa} \beta_r^{-1} \right) (z_t + \pi_t), \end{aligned} \quad (\text{A.121})$$

with

$$T_{z,t} \equiv T_z + Y_z \hat{T}_{z,t} \Rightarrow \hat{T}_{z,t} = \frac{T_{z,t}}{Y_z} - \frac{T_z}{Y_z}.$$

Long term bond policy

$$-\frac{R_L}{R_L - \kappa} r_{L,t} + \hat{B}_{z,t}^L = \rho_B \left(-\frac{R_L}{R_L - \kappa} r_{L,t-1} + \hat{B}_{z,t-1}^L \right) + \epsilon_{B,t}. \quad (\text{A.122})$$

Transfers feedback rule

$$\frac{\hat{T}_{z,t} - G_z \hat{G}_{z,t}}{T_z - G_z} = \phi_T \left[\frac{\hat{B}_{z,t-1} + \frac{1}{R_L - \kappa} (B_z^L/B_z) \hat{B}_{z,t-1}^L - \frac{R_L}{(R_L - \kappa)^2} (B_z^L/B_z) r_{L,t-1}}{1 + \frac{1}{R_L - \kappa} (B_z^L/B_z)} \right] + \epsilon_{T,t}. \quad (\text{A.123})$$

Monetary policy:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \left[\phi_\pi \pi_t + \phi_y \left(\hat{Y}_{z,t} - \hat{Y}_{z,t-4} + \sum_{i=0}^3 z_{t-1} \right) \right] + \epsilon_{m,t}. \quad (\text{A.124})$$

Term premium:

$$\hat{\zeta}_t = \zeta' \hat{B}_{z,t}^L + \epsilon_{\zeta,t}. \quad (\text{A.125})$$

Aggregate resources constraint

$$\hat{Y}_{z,t} = \frac{\omega_u C_z^u}{Y_z} \hat{C}_{z,t}^u + \frac{\omega_r C_z^r}{Y_z} \hat{C}_{z,t}^r + \frac{I_z}{Y_z} \hat{I}_{z,t} + \frac{G_z}{Y_z} \hat{G}_{z,t} + e^{-\gamma} r^k \frac{\bar{K}_z}{Y_z} \hat{u}_t. \quad (\text{A.126})$$

A.5 Data

We use quarterly data for the United States from the third quarter of 1987 (1987q3) to the third quarter of 2009 (2009q3) for the following seven series: real GDP per capita, hours worked, real wages, core personal consumption expenditures deflator, nominal effective Federal Funds rate, the 10-year Treasury constant maturity yield, and the ratio between long-term and short-term U.S. Treasury debt. All data are extracted from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve Bank of St. Louis. The mapping of these variables to the states is

$$\begin{aligned}
\Delta Y_t^{obs} &= 100(\gamma + \hat{Y}_{z,t} - \hat{Y}_{z,t-1} + \hat{z}_t), \\
L_t^{obs} &= 100 \left(L + \hat{L}_t \right), \\
\Delta w_t^{obs} &= 100(\gamma + \hat{w}_{z,t} - \hat{w}_{z,t-1} + \hat{z}_t), \\
\pi_t^{obs} &= 100(\pi + \hat{\pi}_t), \\
r_t^{obs} &= 100(r + \hat{r}_t), \\
r_{L,t}^{obs} &= 100(r_L + \hat{r}_{L,t}), \text{ and} \\
B_t^{ratio,obs} &= \frac{P_L B_z^L}{B_z} (1 + \hat{P}_{L,t} + \hat{B}_{z,t}^L - \hat{B}_{z,t}),
\end{aligned}$$

where all state variables are in deviations from their steady state values, $\pi \equiv \ln(\Pi)$, $r \equiv \ln(R)$, and $r_L \equiv \ln(R_L)$.

We construct real GDP by dividing the nominal GDP series by population and the GDP deflator. The observable ΔY_t^{obs} corresponds to the first difference in logs of this series, multiplied by 100. We measure the labor input by the log of hours of all persons in the non-farm business sector divided by population. Real wages correspond to nominal compensation per hour in the non-farm business sector, divided by the GDP deflator. As for GDP, Δw_t^{obs} is the first difference in logs of this series, multiplied

by 100. The quarterly log-difference in the personal consumption expenditures (PCE) core price index is our measure of inflation. We use the effective Federal Funds Rate as our measure of nominal short-term rate and the 10-year Treasury constant maturity rate as our measure of nominal long-term interest rate. Finally, we identify long-term bonds as U.S. Treasury securities with maturity greater than one year, consistent with the announcement of LSAP II, and construct the ratio to short-term bonds as our measure for the quantity of debt.

A.6 Implementing the Commitment to the Zero Lower Bound

In this section we describe how we implement the commitment to the zero lower bound. This same approach is also used to guarantee that none of simulation paths violates the non-negative interest rate constraint.

A.6.1 Canonical model

Consider the economic model in its canonical form, as in Sims (2002):

$$\Gamma_4^{s(t)}(\theta) z_t = \bar{\Gamma}_0^{s(t)} + \Gamma_1^{s(t)}(\theta) z_{t-1} + \Gamma_2^{s(t)}(\theta) \varepsilon_t + \Gamma_3^{s(t)}(\theta) \eta_t \quad (\text{A.127})$$

where $s(t) \in \{n, zlb\}$ refers to the state of the economy, with n referring to normal times and zlb for times in which the zero lower bound is binding; z_t is the vector of state variables, whether they are endogenous or exogenous; ε_t is a vector of exogenous i.i.d. innovations; η_t is a vector of endogenous expectational errors; and $\left\{ \Gamma_\iota^{s(t)}(\theta) \right\}_{\iota=0,1,2,3,4}$ are matrices defining the state space for any given vector of parameters θ .

For simplification of notation, below I will omit the reference to the vector of parameters when writing the matrices.

With some restrictions it is possible to break the system in (A.127) into two blocks: a forward looking one and a backward looking one. So for each equation we can write:

$$j \in FL : \Gamma_4^{s(t)}(j) E_t z_{t+1} = \Gamma_0^{s(t)}(j) + \Gamma_1^{s(t)}(j) z_t \quad (\text{A.128})$$

$$i \in BL : \Gamma_4^{s(t)}(i) z_t = \Gamma_0^{s(t)}(i) + \Gamma_1^{s(t)}(i) z_{t-1} + \Gamma_2^{s(t)}(i) \varepsilon_t \quad (\text{A.129})$$

where i denotes BL equations and j the FL ones.

A.6.2 Perfect Foresight Solution Method

Consider a sequence of periods $\{s(t)\}_{t=0}^K$ such that for $t > K$ we have $s(t) = n$ and $\varepsilon_t = 0$ — *i.e.* n eventually becomes an absorbing state and no additional innovations are expected beyond K . In this case we can solve for the REE solution backwards.

Absorbing state

In normal times, for $t > K$, the REE solution can be represented by

$$z_t = \Phi_0^n + \Phi_1^n z_{t-1} + \Phi_2^n \varepsilon_t \quad (\text{A.130})$$

Before the absorbing state

We need to solve for the REE matrices recursively.

Notice first that for the last period before the absorbing state kicks in, and using (A.130), we can write the forward looking component of the system as

$$\Gamma_4^{s(t)}(j) E_t [\Phi_0^n + \Phi_1^n z_t + \Phi_2^n \varepsilon_{t+1}] = \Gamma_0^{s(t)}(j) + \Gamma_1^{s(t)}(j) z_t$$

which we can rewrite as

$$\left[\Gamma_4^{s(t)}(j) \Phi_1^n - \Gamma_1^{s(t)}(j) \right] z_t = \Gamma_0^{s(t)}(j) - \Gamma_4^{s(t)}(j) [\Phi_0^n + \Phi_2^n \hat{\varepsilon}_{t+1}]$$

and combine this with the backward looking to get the full system written as

$$\tilde{\Gamma}_4(t) z_t = \tilde{\Gamma}_0(t) + \tilde{\Gamma}_1(t) z_{t-1} + \tilde{\Gamma}_2(t) \varepsilon_t \quad (\text{A.131})$$

with

$$\tilde{\Gamma}_4(t) \equiv \begin{bmatrix} \Gamma_4^{s(t)}(j) \Phi_1(t+1) - \Gamma_1^{s(t)}(j) \\ \Gamma_4^{s(t)}(i) \end{bmatrix}, \quad (\text{A.132})$$

$$\tilde{\Gamma}_0(t) \equiv \begin{bmatrix} \Gamma_0^{s(t)}(j) - \Gamma_4^{s(t)}(j) [\Phi_0(t+1) + \Phi_2^n(t+1) \hat{\varepsilon}_{t+1}] \\ \Gamma_0^{s(t)}(i) \end{bmatrix}, \quad (\text{A.133})$$

$$\tilde{\Gamma}_\iota(t) \equiv \begin{bmatrix} 0 \\ \Gamma_\iota^{s(t)}(i) \end{bmatrix}, \text{ for } \iota = 1, 2, \quad (\text{A.134})$$

and

$$\Phi_\iota(t+1) = \Phi_\iota^n, \text{ for } t = K \text{ and } \iota = 0, 1. \quad (\text{A.135})$$

Now we can solve this system for z_t and write

$$z_t = \Phi_0(t) + \Phi_1(t) z_{t-1} + \Phi_2(t) \varepsilon_t \quad (\text{A.136})$$

with

$$\Phi_0(t) \equiv \left[\tilde{\Gamma}_4(t) \right]^{-1} \tilde{\Gamma}_0(t), \quad (\text{A.137})$$

$$\Phi_1(t) \equiv \left[\tilde{\Gamma}_4(t) \right]^{-1} \tilde{\Gamma}_1(t), \quad (\text{A.138})$$

$$\Phi_2(t) \equiv \left[\tilde{\Gamma}_4(t) \right]^{-1} \tilde{\Gamma}_2(t) \quad (\text{A.139})$$

and notice that we need to use a pseudo inverse, to account for the fact that $\tilde{\Gamma}_4(t)$ might not be invertible.

Notice that (A.136) is in the exact same form of (A.130). So, iterating backwards, the system (A.131) and the REE solution (A.136) are valid for $\forall t \leq K$.

A.6.3 Implementing the ZLB commitment

We use the convention in our simulations that period $t = 0$ is the period in which LSAP is announced and implementation started, and the commitment to the zero lower bound applies to the first four periods, including period $t = 0$. Given the framework just described, then implementing the commitment to the ZLB implies setting a sequence of states $\{s(t)\}_{t=0}^K$ such that $s_t = zlb$ for $t = 0, 1, 2, 3$ and $s_t = n$ for $t > 3$. Then iterate backwards, starting in period 3 towards the initial period to find the REE solution matrices for periods $t = 0, 1, 2, 3$. For periods $t > 3$ the solution is the usual one in the absence of policy regime change.

For the *zlb* regime we have exactly the same equations as in regime *n*, but replace the interest rate rule equation with one setting the interest rate to zero.

A.6.4 Enforcing Non-Negative Interest Rate

We can also use this same framework to enforce the non-negative interest rate constraint after the commitment to the zero lower bound is lifted. This is relevant because

for some parameter draws we get this constraint to be violated. In order to accomplish this we use a guess and verify approach.

In the first step we make the simulation under the assumption that the sequence of states $\{s(t)\}_{t=0}^K$ is the one described above. Then we check for any violations of the non-negative interest rate constraint and switch the regime for those periods from n to zlb , and solve again for the solution. We keep doing this until there are no violations.

A.7 Robustness

This section considers four robustness exercises. First, we consider the implications of extending the duration of the LSAP program. Second, we consider a longer commitment to the zero lower bound by the monetary authority. Third, we ask how sensitive the model is to the degree of market segmentation. Fourth, we study the role of nominal rigidities.

The first two robustness check has obvious policy interest and implications. The motivation for the other two exercises is that the financial crisis may have introduced a (possibly temporary) change in regime, both in the financial market structure and in the price setting mechanism. Ideally, we could capture these phenomena with a regime-switching model. Beside the technical complications, the main limitation of this approach is that the change in regime is probably one of a kind and occurred at the very end of the sample. As such, regime-switching techniques may not have enough data to identify the change in the economic environment. The less formal robustness analysis presented here is still informative to document this point, while research on this is left for future work.

A.7.1 The Role of the Length of LSAP Programs

In our baseline simulation, the central bank accumulates assets over four quarters and holds the balance sheet constant for the next two years, before gradually winding down the program over two additional years. This assumption is fairly arbitrary. Depending on economic conditions, policy makers may change the length of the programs, as the recent U.S. and U.K. experience suggests. Without undertaking an exhaustive analysis, this subsection considers one alternative path: the central bank still accumulates assets over the first year (as per the FOMC announcement in November 2010) but then holds the balance-sheet constant for four years, instead of two, before gradually exiting. Figure A.1 shows the corresponding responses, in the same format as the figures show in the paper, with red continuous line for the this simulation, with grey shaded regions representing the uncertainty and the dashed blue line showing the baseline simulation effects for easy comparison.

Not surprisingly, this change in the time profile of the asset holdings by the central bank induces a stronger response by the risk premium, with a median peak response of -16 bp (instead of -11 bp). As a result, output and inflation respond more strongly. However, while the inflation response roughly doubles compared to the baseline scenario (median response at the peak of 0.059%, compared to 0.031%), the response of output is only 50% stronger (median response of 0.19%, instead of 0.13%, for GDP growth). Not surprisingly, the uncertainty around the median is larger, with the 95th percentiles increasing proportionally.

In sum, if the central bank holds the purchased assets for longer, the stimulative impact on output and inflation increases and becomes more persistent. Moreover, the additional boost is stronger for inflation than for output. Nominal rigidities play an important role in this respect. Because the shock lasts longer, more firms and workers are expected to change their prices and wages over time, which in turn leads the firms

and workers who can change their prices and wages early to do so more aggressively.

A.7.2 The Role of the Length of the ZLB Commitment

In the paper we discuss how important is the commitment of the central bank to keep the interest rate at zero to boost the effects of the LSAP program. Here we take that analysis one step further by considering a longer commitment. Instead of four quarters, we consider five quarters of commitment. Figure A.4 shows the corresponding responses, in the same format as the previous figure.

The figure gives a strong message: adding just one more quarter to the commitment gives a powerful boost to the effects of LSAP. GDP growth increases on impact by 0.22% (instead of 0.13%) and inflation increases by 0.045% instead of 0.031. So the effects on the real economy are stronger by a magnitude between 50% and 70%, depending on the variable considered. This means that the effects of additional quarters of commitment to the ZLB are highly non-linear, due to the power of the expectations channel. As a result skewness also increases.

This result also confirms the importance of looking at the effects of interest rate policy and asset purchases in combination. They interact with each other and thus can and should be used in a coordinated fashion.

A.7.3 The Role of Market Segmentation

The baseline experiment suggests that the effects of LSAP II are fairly modest on GDP and quite small on inflation. One reason why our results may underestimate the effects of asset purchase programs is that the degree of financial market segmentation may have recently increased due to the financial crisis.⁵⁹ As discussed earlier, our reduced-

⁵⁹Baumeister and Benati (2010) estimate a VAR with time-varying coefficients and stochastic volatility to account for this type of effects, on top of other changes in the structural relations

form friction for market segmentation aims at capturing a combination of preferences for certain asset classes and institutional restrictions on the type of investments certain financial intermediaries undertake. By shifting the true and perceived distribution of risk, the financial crisis may have induced a fraction of investors previously active in multiple segments of financial markets to concentrate on one particular asset class.

For this purpose, we repeat the baseline experiment in the presence of a high degree of market segmentation. Figure A.2 shows the results of the same simulated LSAP II experiment as in the baseline case. The difference is that, in this case, we only draw from the lower half of the posterior distribution of the parameter ω_u . All other parameters are drawn from the same posterior distribution as before.⁶⁰

The median responses of GDP growth and inflation with the ZLB commitment are about 50% bigger than in the baseline case, at +0.21% and +0.044% respectively (compared to 0.13% and 0.031%). Upside posterior uncertainty is now more pronounced. The 95th percentile now nearly reaches 1% for GDP growth and 0.2% for inflation, compared to 0.6% and 0.15% before. The stronger response of macroeconomic variables requires the central bank to increase the short-term nominal interest rate by two additional basis points. As a consequence, given that the drop in the risk premium is the same, long-term rates decrease one basis point less.

The bottom line from this exercise is that allowing for a higher degree of segmentation does increase the response of GDP growth and inflation to the stimulus of asset purchase programs. However, unless segmentation becomes really extreme, the macroeconomic effects of LSAP remain quite small, especially for inflation.

among macroeconomic variables potentially triggered by the financial crisis.

⁶⁰To be precise, this is a counterfactual simulation. As for the main simulations, we draw a parameter vector from the MCMC posterior sample. However, we then perform a resample exercise for the ω_u parameter in which we extract the marginal sample for this parameter, perform an ascending ordering and keep only the lower half of it. Then, for each parameter vector used in the simulation, we draw independently the ω_u parameter value from this modified subsample.

A.7.4 The Role of Nominal Rigidities

One reason for the small response of inflation to LSAP II is that the estimated degree of nominal rigidities, especially for prices, is quite high. While our priors for the probability of holding prices and wages fixed in any given period (ζ_p and ζ_w) are both centered at 0.5, the posterior medians for the two parameters are 0.93 and 0.73, respectively.

A high degree of stickiness in prices and wages is not an uncommon finding in the DSGE literature, especially in the absence of real rigidities like in our case (Del Negro and Schorfheide (2008)). Additionally, Hall (2011) has recently emphasized how prices have failed to fall substantially in the last recession. As in the case of segmentation, the financial crisis may have caused a structural change in the price setting process that the model interprets as an increase in price rigidities (the same consideration applies to wages).⁶¹

Nevertheless, we want to quantify the sensitivity of our results to a lower degree of nominal rigidities, more in line with standard values from the empirical literature that uses micro data (e.g. Nakamura and Steinsson (2010)). Figure A.3 shows the results of the baseline LSAP II experiment when we fix ζ_p at 0.75, more in line with the recent empirical evidence. All other parameters are drawn from the same posterior distribution as before.

The figure shows that nominal rigidities play an important quantitative role in the response of inflation to asset purchases. When prices are more flexible, the median response of inflation to LSAP II on impact is more than twice bigger than when we use the estimated posterior distribution. The counterparts are a less persistent inflation process and a slightly smaller effect on GDP growth. Notice that the effects

⁶¹Indeed, if we consider a sample that ends before the recent crisis (second quarter of 2007), the posterior median for ζ_p is somewhat smaller

on the GDP level are now considerably smaller and less persistent. In equilibrium (i.e., taking into account the endogenous response of monetary policy), the two effects roughly compensate each other. The increase in the short-term interest rates is almost the same in the two cases. Therefore, also the behavior of long-term rates is very similar. The increase in upside uncertainty for inflation is roughly proportional to the changes in the median. The 95th percentile of the response in inflation is 0.4%, compared to just above 0.15% in the baseline experiment.

In sum, higher price flexibility shifts the adjustment in response to asset purchase programs from GDP growth to inflation, by making its process more front-loaded.

A.8 Diagnostics

This section provides more detailed analysis of the empirical diagnostics for the model. As stated in the main text of the paper, since we include the long-term bond as an observable, switching on all the shocks, we should match the short rate and long rate perfectly. In order to evaluate how the model performs empirically, we want to compare the model generated moments with those of the data. The rest of this section, we will analyze variance, variance decomposition and historical shock decomposition in turn.

A.8.1 Variance

In this section we compare the variance of each variable in the data with that predicted by our model. For the model variance we compute for each parameter draw the unconditional variance of the relevant state variable and then take the median across draws. We focus on the short-term interest rate and long-term interest rate for this model diagnostics exercise. The model's unconditional variance for the short

rate is 0.44, which is just above half of that observed in the data (0.81) while the model's unconditional variance for the long rate is 0.12, which is only one fourth of that observed in the data (0.47). This model does a decent job in terms of explaining the variance of the ratio of the long bond to the short bond. (the data variance is 0.08 and the unconditional model variance is 0.07) This suggests that this model has a limited ability to match properties of the yield curve in the data. However, our main purpose is not to explain yield curve shape or dynamics, rather, we are interested in analyzing how changes in the risk premium affect macroeconomy and the monetary transmission mechanism of the Fed's unconventional policy.

A.8.2 Variance Decomposition

Here we compare the relative importance of different shocks in determining some of the variables of interest, from an unconditional perspective. Table A.1 shows the median percentage contribution of each shock to the unconditional variance of some variables of interest. Figures A.5 through A.7 show the variance decomposition at different forecasting horizons for the FFR, long yield and slope of the yield curve. Yield curve slope is defined as the difference between the long and short rate $rL_t - r_t$. For the definition of risk premium and the long rate implied by the expectation hypothesis, see section A.1.6.

The marginal efficiency of the investment shock (μ) is the single most important factor in determining the short rate at the business cycle frequencies, which is consistent with the findings in Justiniano, Primiceri, and Tambalotti (2011). In the very short run (less than 2 years) the short rate is also somewhat influenced by the policy shocks. Preference shock (shock to the discount factor) becomes relatively more important to the short rate in the medium to long run, climbing to as much as 19%.

On the contrary, shock to the risk premium is the most important driver for the

long yield volatility, accounting for 64% in the short run and 39% in the long run. In second place come the shock to the discount factor (ranges between 13% and 24%) and the shock to the marginal efficiency of investment (ranges between 13% and 22%). Productivity shock accounts for very little in the short run (3%) but rises to 9% in the long run.

The largest contributor to the volatility of the slope of the yield curve is the monetary policy shock. On impact it accounts for 57% and decays to only 7% in the long run. The risk premium shock accounts for as much as 42% 2 to 3 quarters ahead, and keeps its important role throughout by fluctuating around 30%, depending on the horizon. The shock to the marginal efficiency of investment has a more limited role in the short run but becomes the single most important driver for the volatility of the yield curve slope over the medium and long run, accounting for more than 50% for horizons of 8 quarters and longer. Discount factor shock plays a more or less residual role, and contributes mostly in the long run, reaching eventually 6%.

If we look at the expectations hypothesis component then the contributions are similar to the long rate, but now the contribution of the shock to the risk premium is smaller, as expected. The only reason the risk premium shock even shows up here is due to the real effects and the endogenous response of the economy and monetary policy to the shock to the risk premium.

A.8.3 Historical Shock Decomposition

In this exercise, we use a disturbance smoother (as described in Carter and Kohn (1994)) to recover draws for the historical paths of the shocks. We then feed these shocks to the model, one at a time, to generate the counterfactual path of each variable, which gives us the marginal contribution of each shock to the evolution of each variable at each point in the sample. We show the median across parameter

draws. Figures A.9 through A.18 show select contributions of shocks to the yield curve related variables. The black line shows the median estimated path for the variable under consideration and the vertical bars show the marginal contribution of each shock in each period in time to that variable's path.

In terms of the short rate, Figure A.8 shows that shock to the risk premium has been pushing the FFR down since 1994 by 2 to 3 percentage points. Interestingly Figure A.9 demonstrates that monetary policy shock has been pushing the FFR up since 2007. This means that the recent low interest rates are more likely to be explained by the economic conditions, as opposed to being artificially low due to discretionary policy decisions. As Figure A.10 shows, the marginal efficiency of investment, the key factor in the unconditional analysis, captures fairly well the cyclical movements in the FFR except the early 90s and the recent period of time. Finally, the productivity shock has been pushing the FFR down since the beginning of 2000.

In the 1990s the risk premium shock (Figure A.11) contributes heavily to the movements in the long rate, and was compensated down by other shocks. In the most recent period leading to 2009, the risk premium contributes to increase in the long rate, with help from the increasing ratio of long term debt in the hands of the public. (Also see Figure A.12 for the long-term bond supply shock). On the other hand, Figure A.13 shows that the shock to the discount factor has been pushing the long rate down at the end of the sample. Similarly Figure A.14 demonstrates the productivity has been pushing down the long yield.

The volatility of the slope of the yield curve is mostly explained by the evolution of the risk premium shock. (Figure A.15) At the end of the sample the risk premium shock is pushing the slope up, helped a bit by the shock to the discount factor and the shock to the long term bond supply that increases the ratio of the long debt to the short debt in the hands of the public. However the effects are countered by the

negative contributions by the monetary policy shock and the shock to the marginal efficiency of investment. (See Figure A.16 Figure A.17 and Figure A.18)

Table A.1: Variance decomposition for short and long rates, slope of the yield curve, risk premium component of the slope and expectations hypothesis component of the long rate. For each variable the table shows the median marginal contribution of each shock to the unconditional variance of that variable, shown in percentage points.

	short rate	long rate	slope	risk premium	long rate (EH)
productivity (ε_z)	4.8	9.4	1.4	0.1	12.2
markup (ε_λ)	0.2	0	0.2	0	0
investment (ε_μ)	58.4	21.7	52.3	0.1	31.6
discount factor (ε_b)	1.8	0.5	1.5	0	0.7
labor supply (ε_ϕ)	18.8	24	5.6	0.1	30.8
long bond supply (ε_{BL})	0.1	0.7	0.4	0.6	0.2
tax (ε_T)	0.0	0	0	0	0
monetary policy (ε_m)	0.6	1.2	6.9	0	1.6
risk premium (ε_ζ)	5.7	38.6	29.1	98.8	15.7
government spending (ε_g)	0.1	0	0	0	0

Figure A.1: Responses to simulated shock to market value of long-term debt (as shown in Figure 1 in chapter 1) in the case in which the central bank keeps the purchased assets for four years (instead of two). All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). The continuous red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the posterior median response of the variables in the baseline simulation, shown in Figure 3 in the paper.

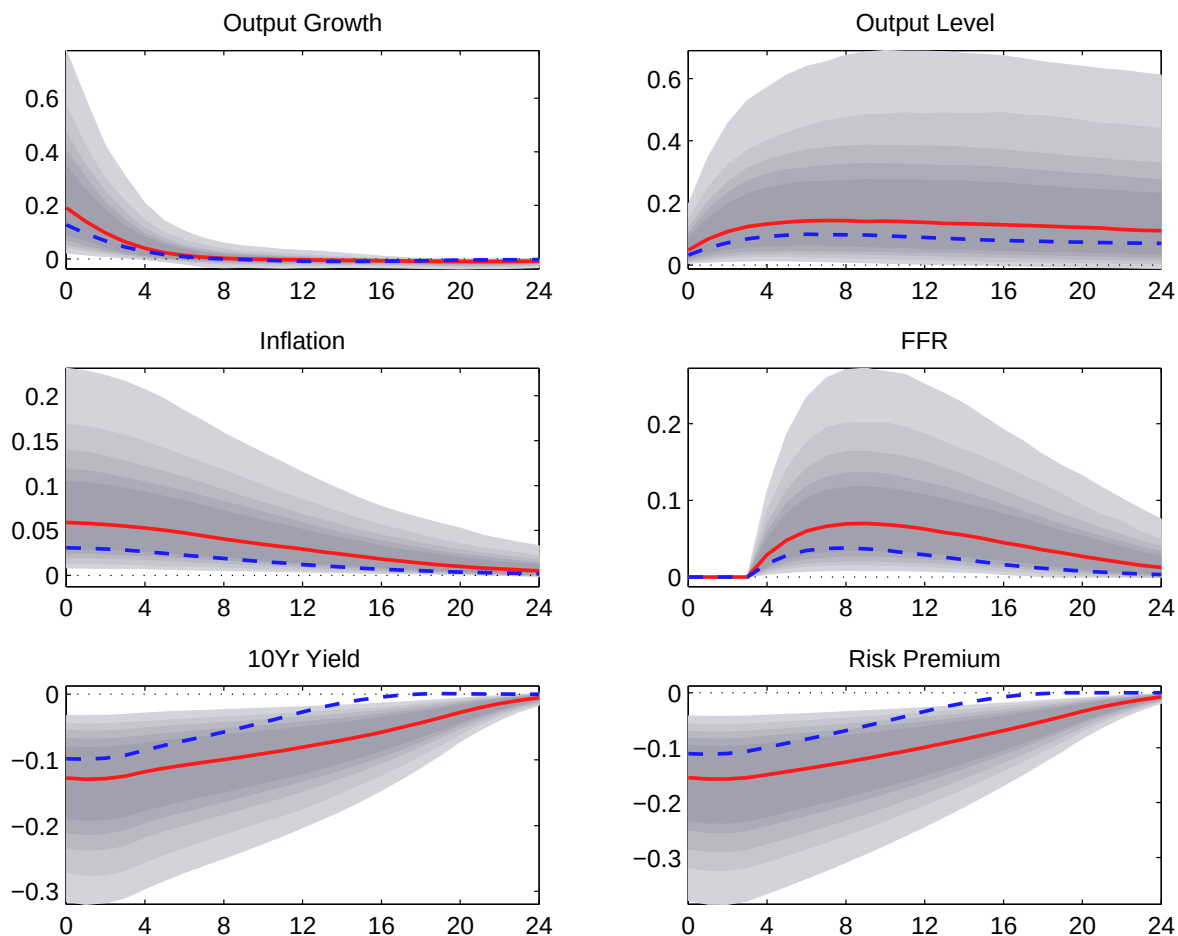


Figure A.2: Responses to simulated shock to market value of long-term debt (shown in Figure 1 in chapter 1) in the presence of a high degree of market segmentation (by considering the lower half of the distribution of ω_u). All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). The continuous red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the posterior median response of the variables in the baseline simulation, shown in Figure 3 in the paper.

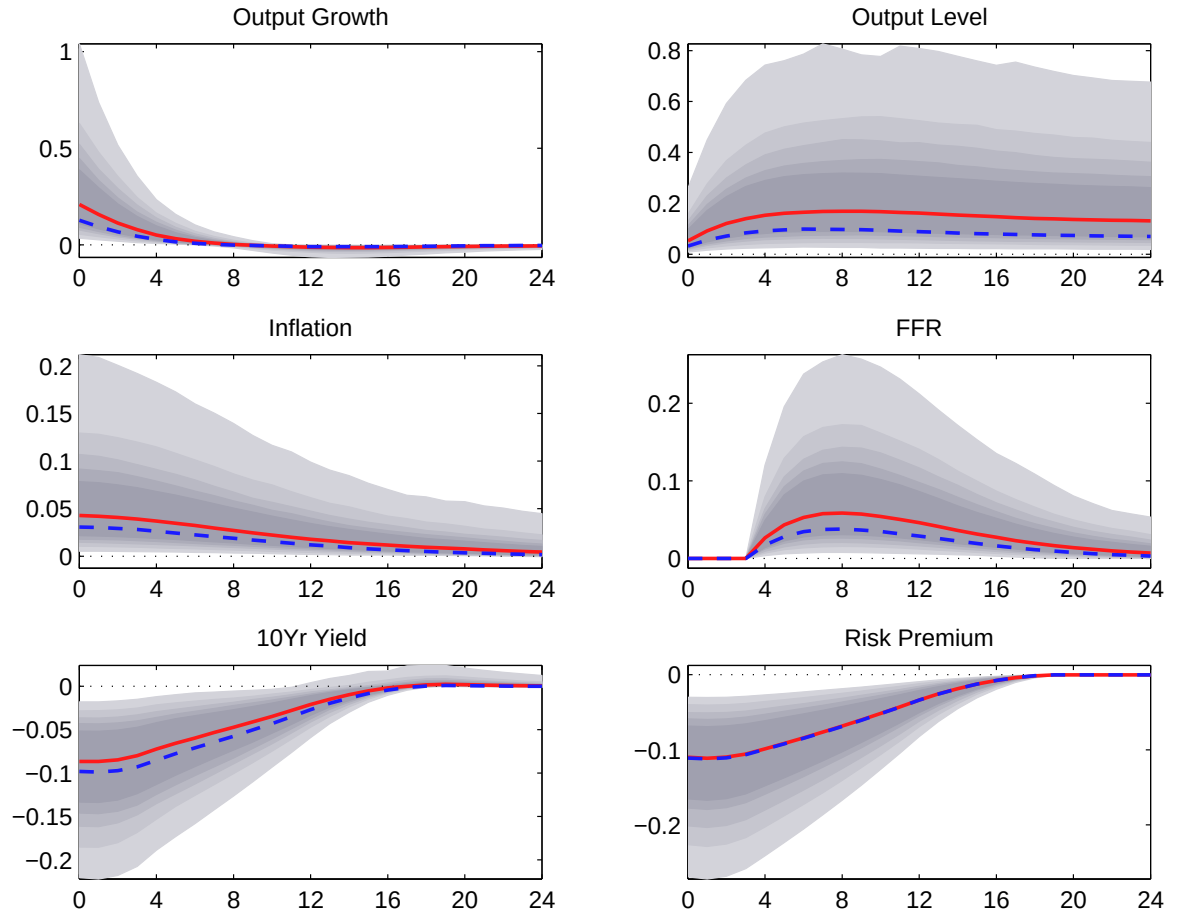


Figure A.3: Responses to simulated shock to market value of long-term debt (shown in Figure 1 in chapter 1) in the presence of lower price rigidities ($\zeta_p = 0.75$). All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). The continuous red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the posterior median response of the variables in the baseline simulation, shown in Figure 3 in the paper.

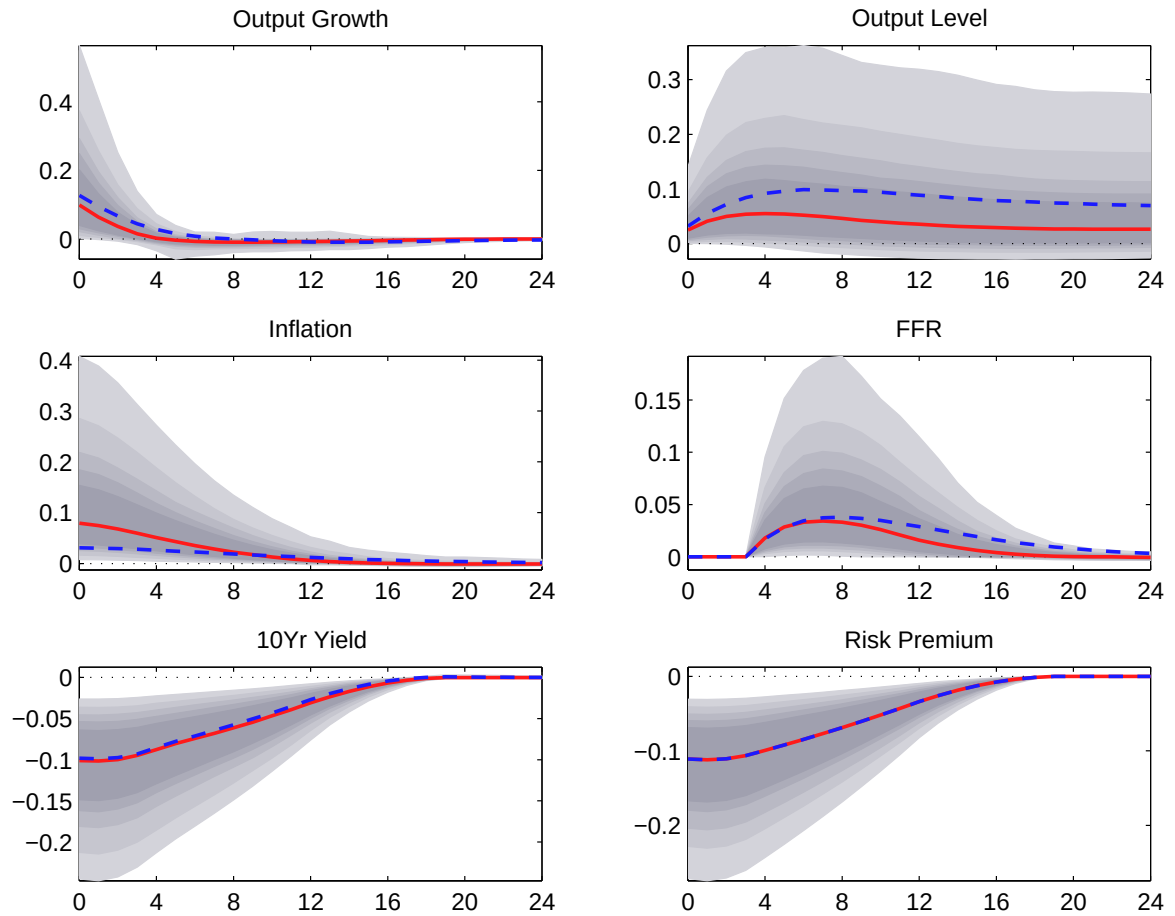


Figure A.4: Responses to simulated shock to market value of long-term debt (shown in Figure 1 in chapter 1) in the case in which the central bank keeps ZLB for five quarters (instead of four). All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). The continuous red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the posterior median response of the variables in the baseline simulation, shown in Figure 3 in the paper.

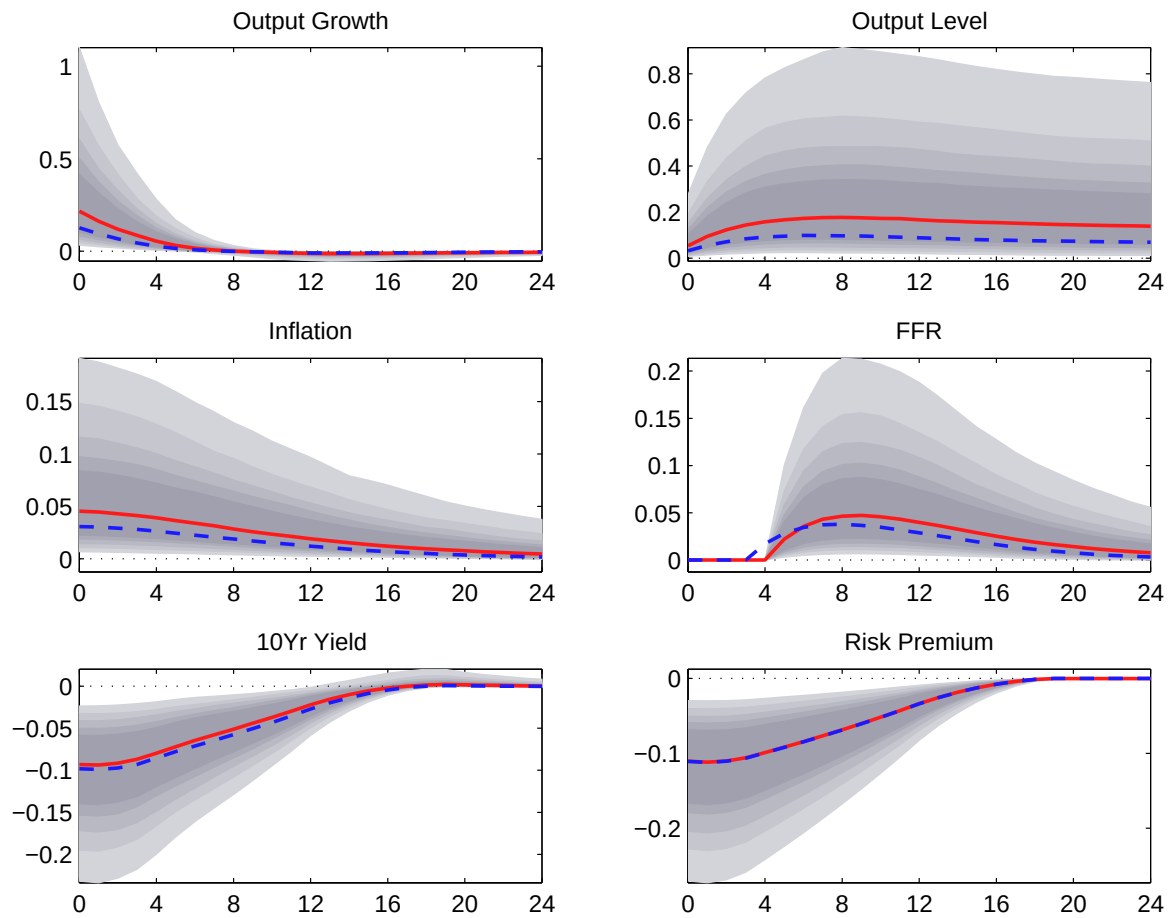


Figure A.5: Variance decomposition for the FFR at different horizons.

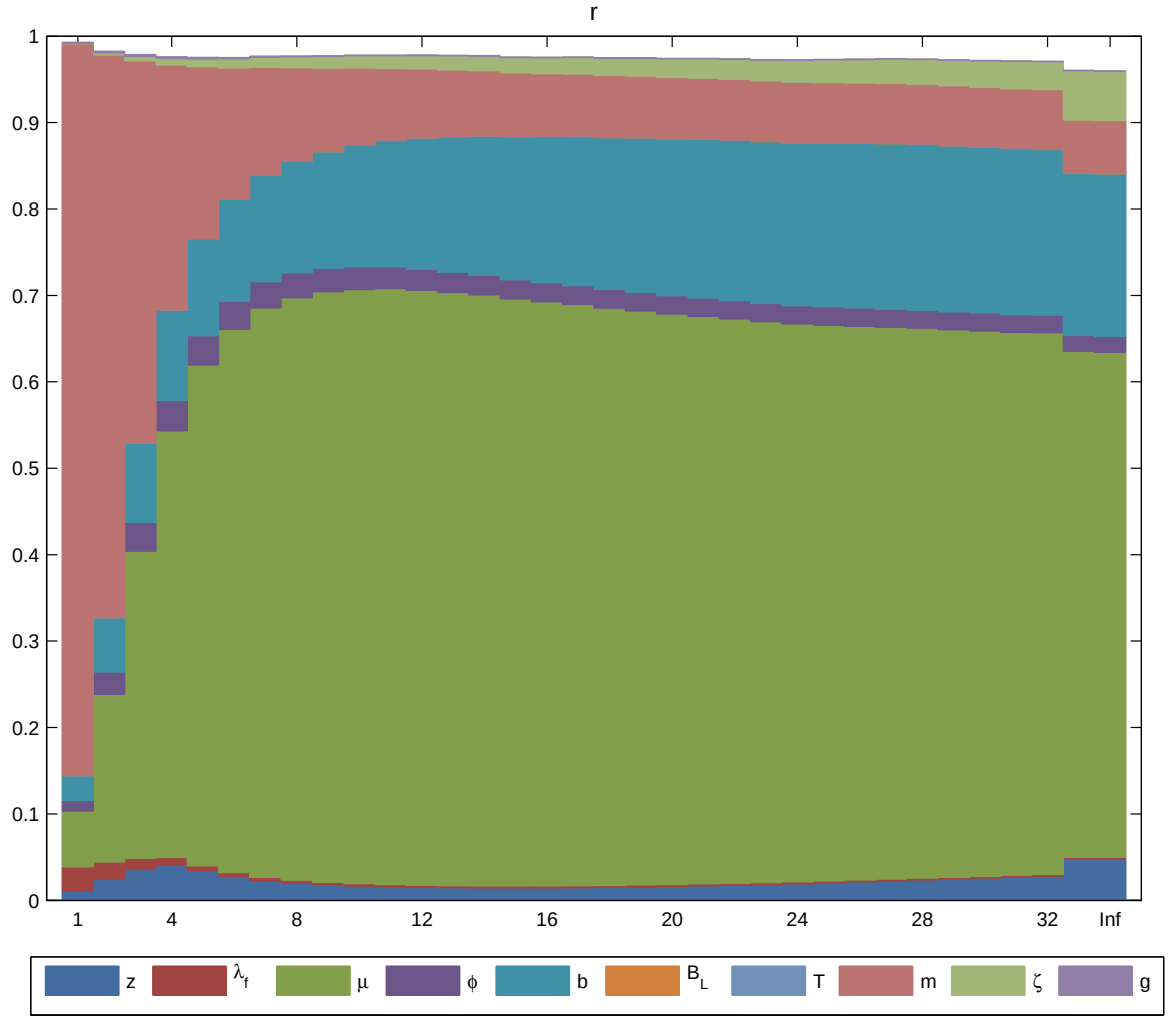


Figure A.6: Variance decomposition for the long yield at different horizons.

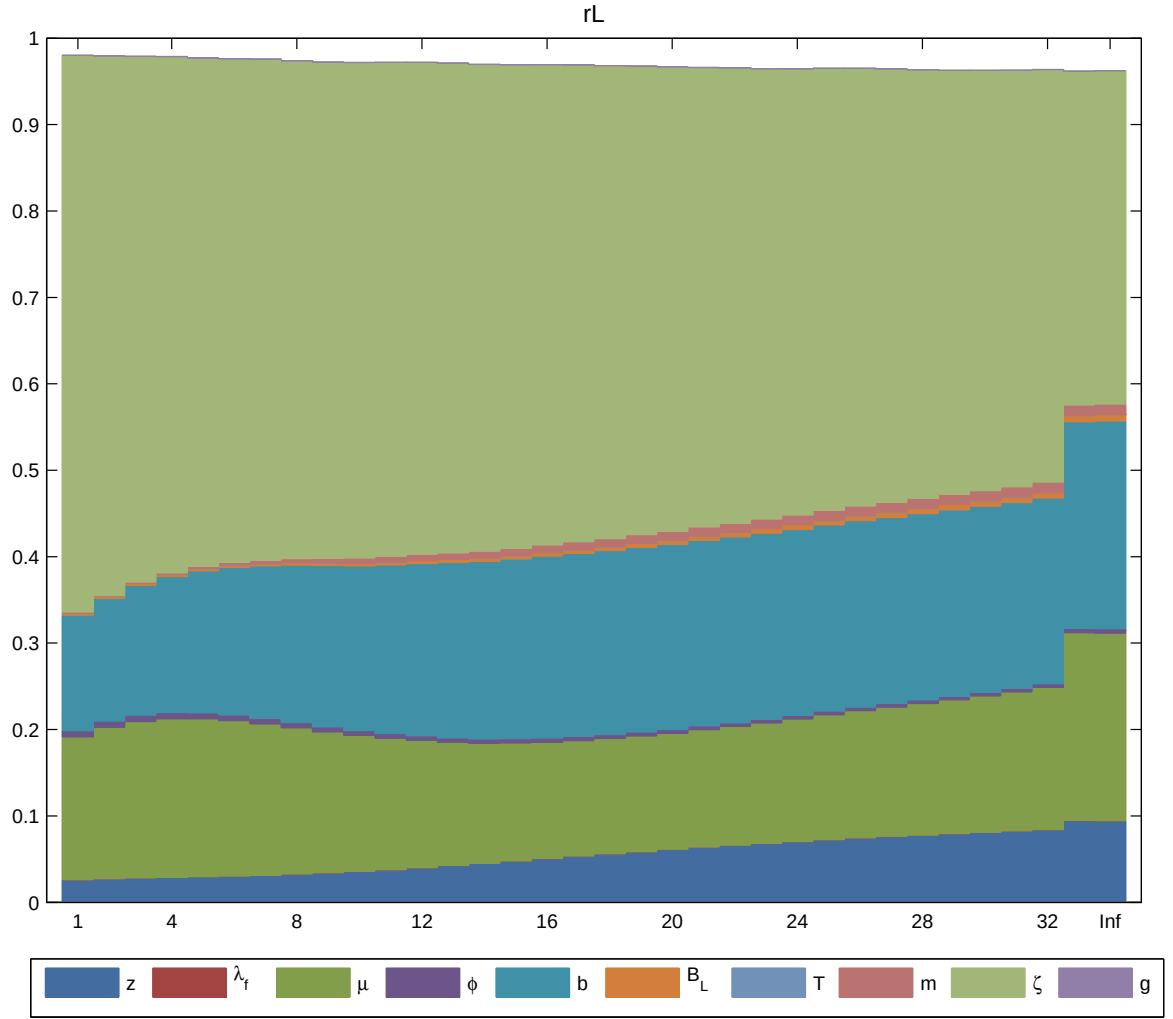


Figure A.7: Variance decomposition for the yield curve slope at different horizons.

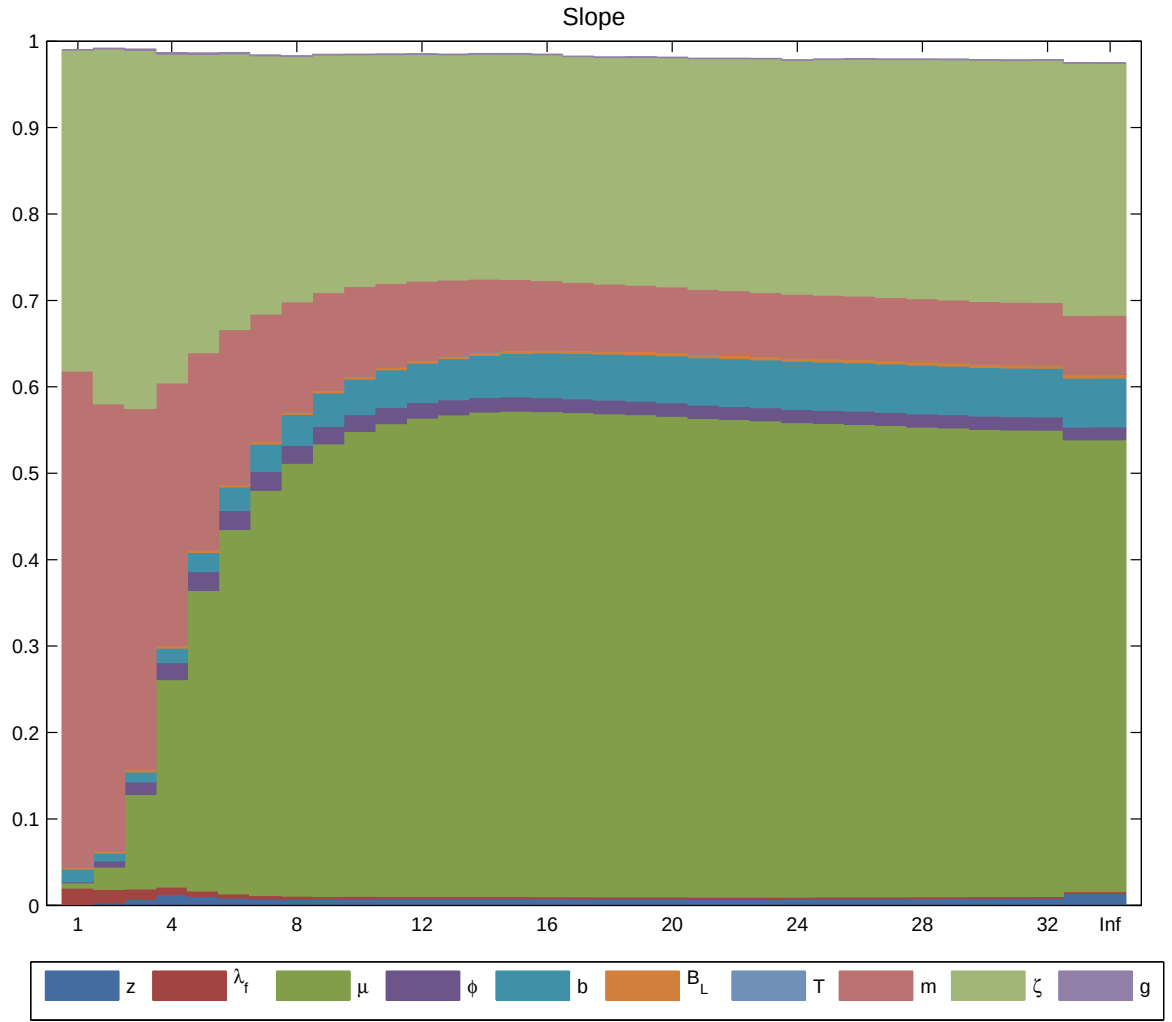


Figure A.8: Historical shock decomposition: contribution of the shock to the risk premium to path of the short rate.

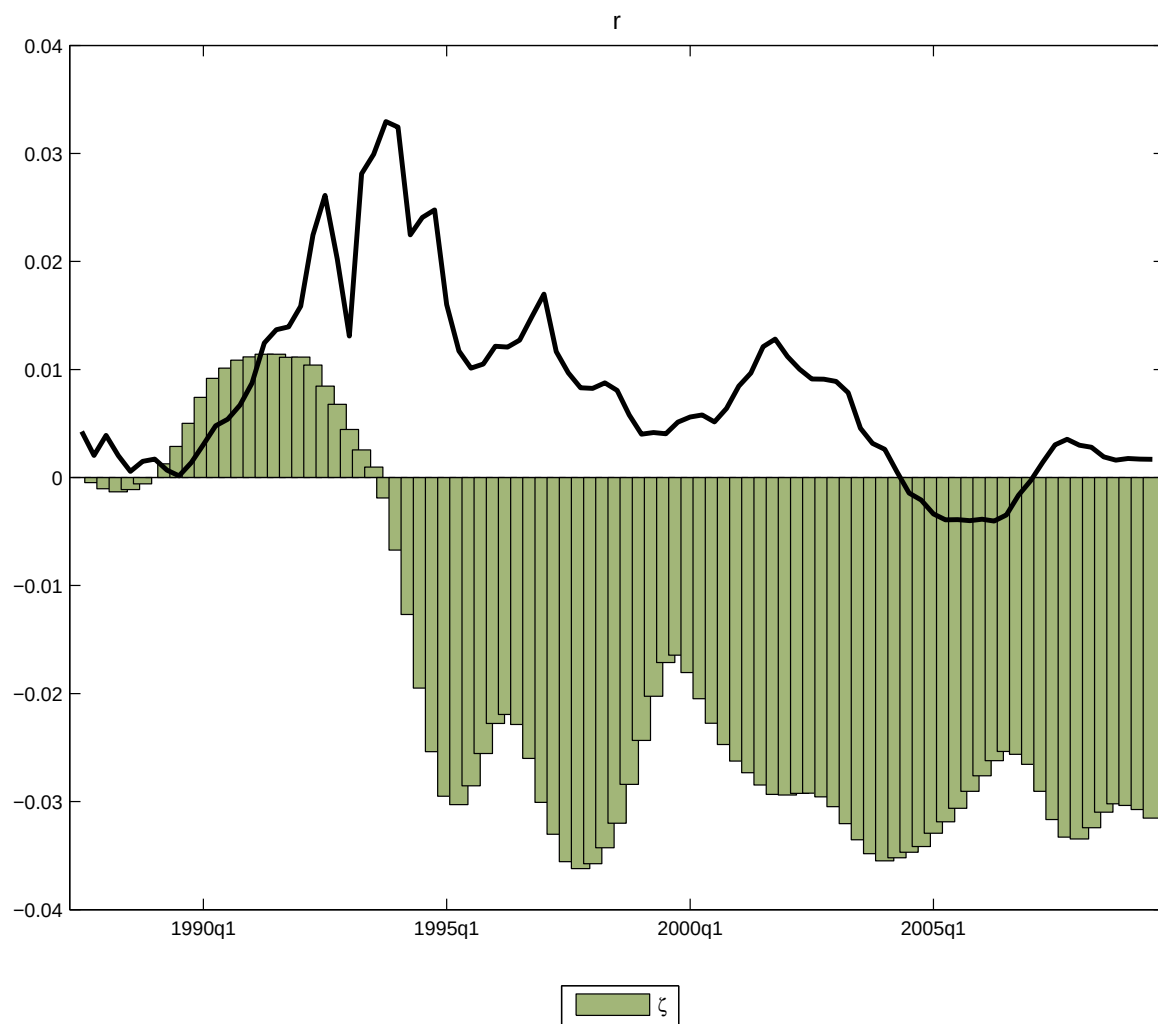


Figure A.9: Historical shock decomposition: contribution of the shock to monetary policy rule to the short rate.

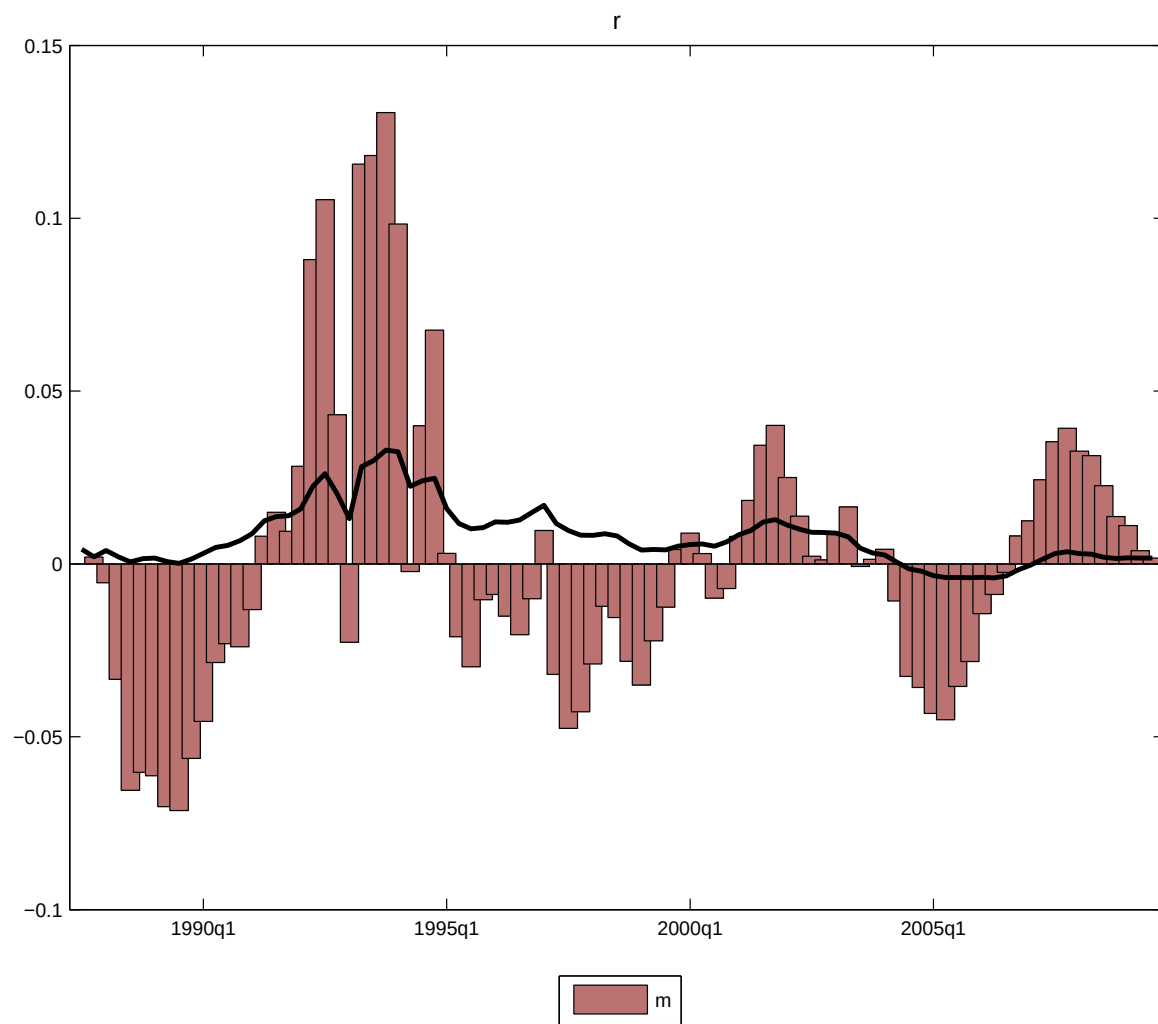


Figure A.10: Historical shock decomposition: contribution of the the shock to the marginal efficiency of investment to the path of the short rate.

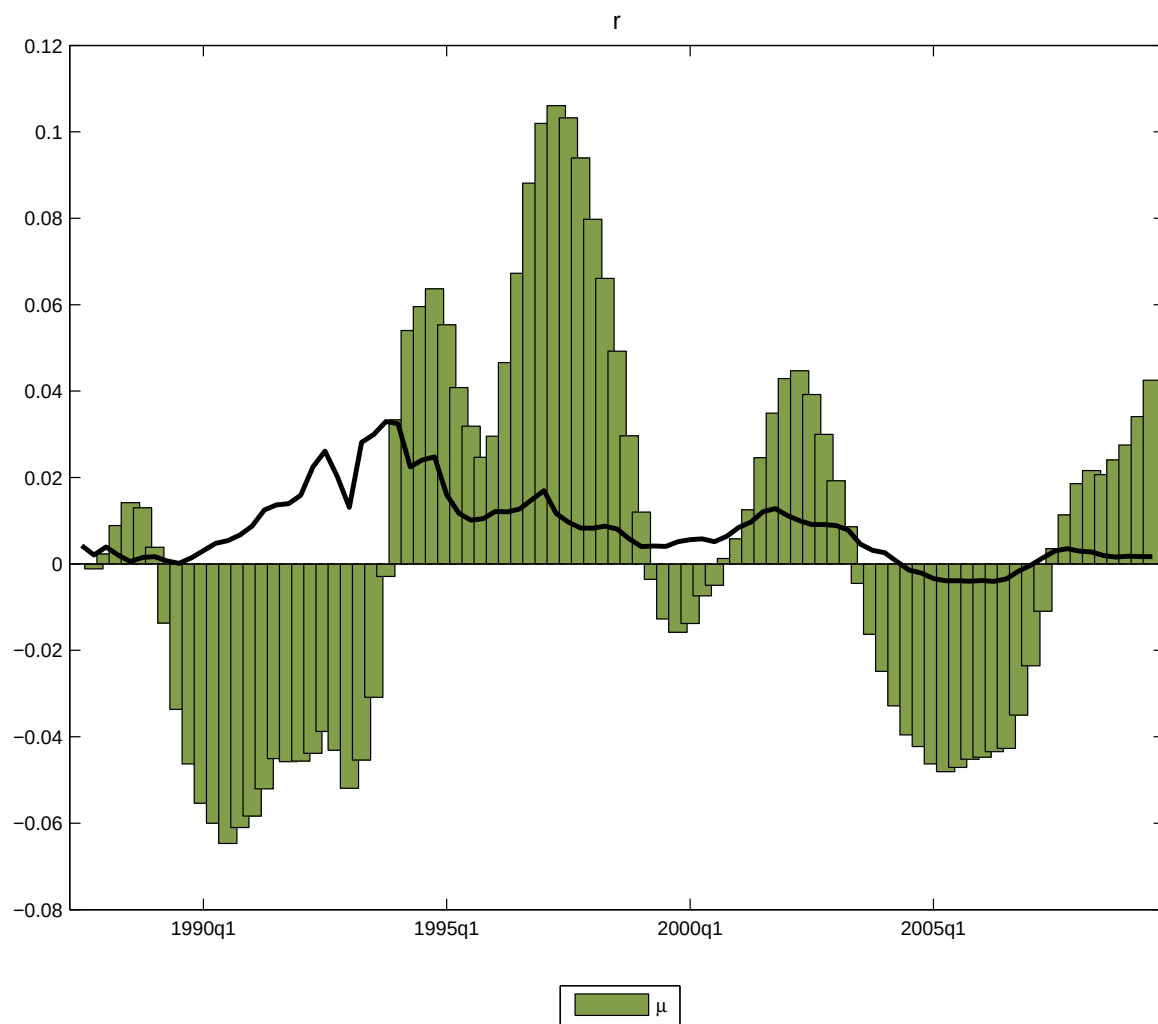


Figure A.11: Historical shock decomposition: contribution of the shock to the risk premium to the long rate.

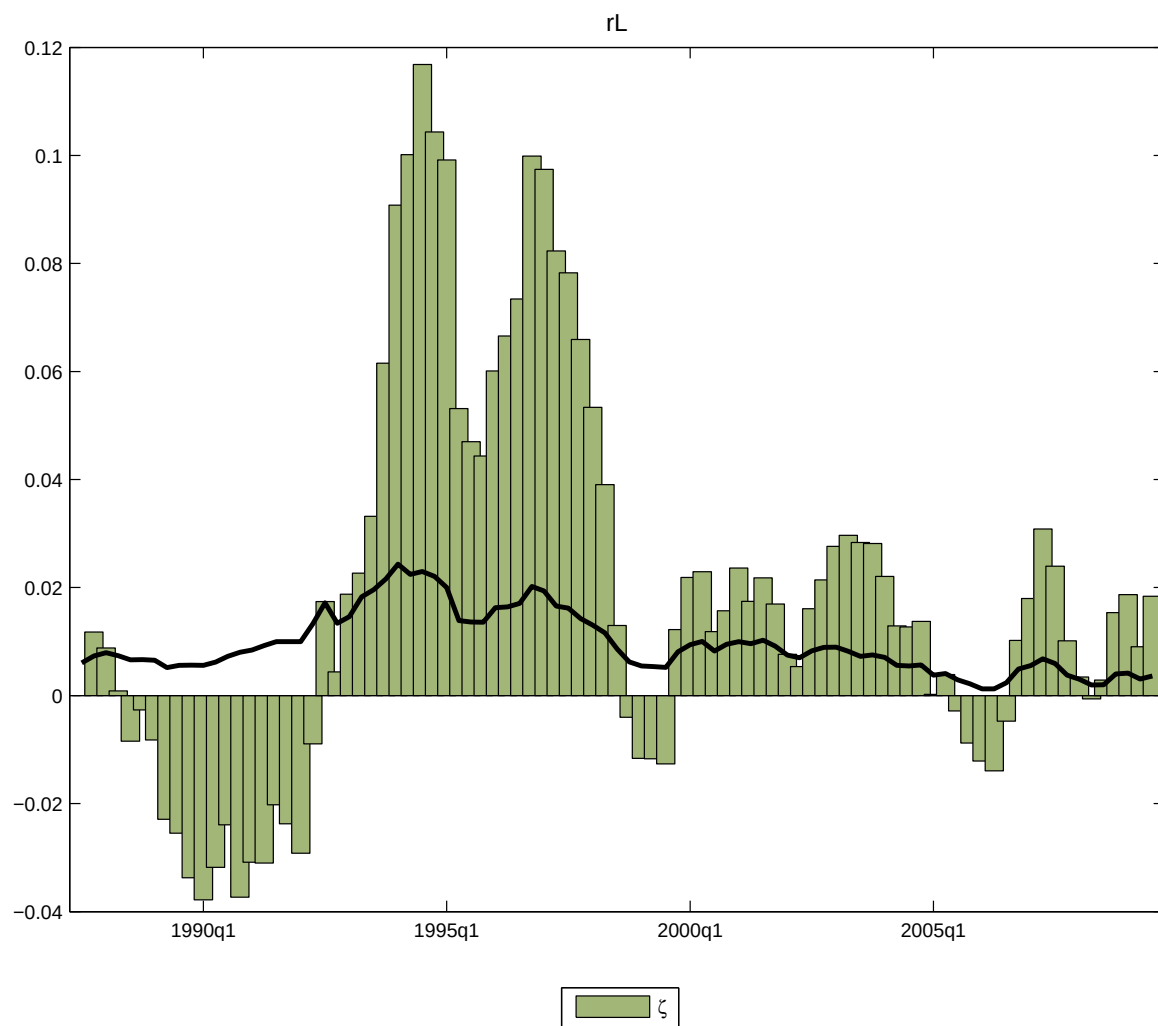


Figure A.12: Historical shock decomposition: contribution of the shock to long bond supply to the path of the long rate

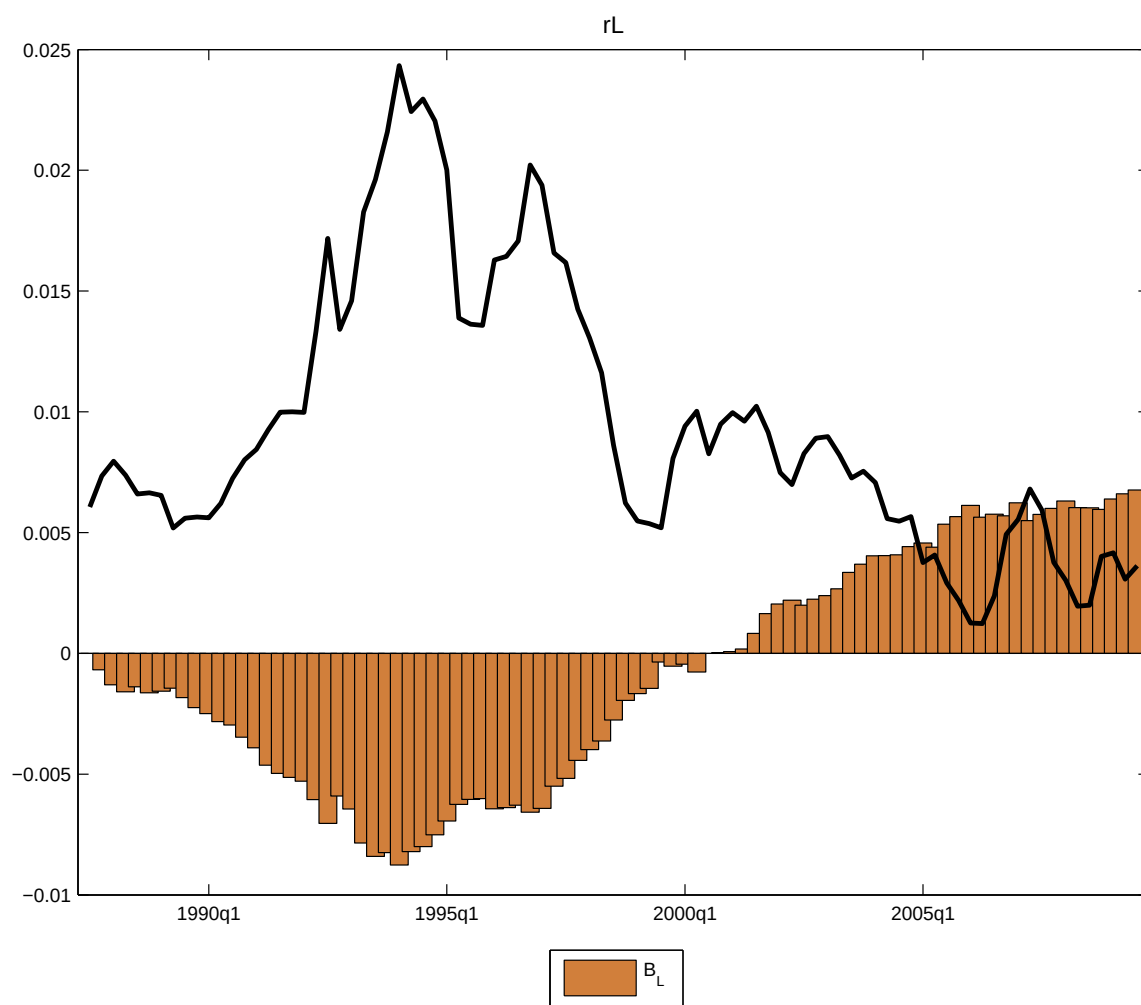


Figure A.13: Historical shock decomposition: contribution of the shock to the discount factor to the path of the long rate.

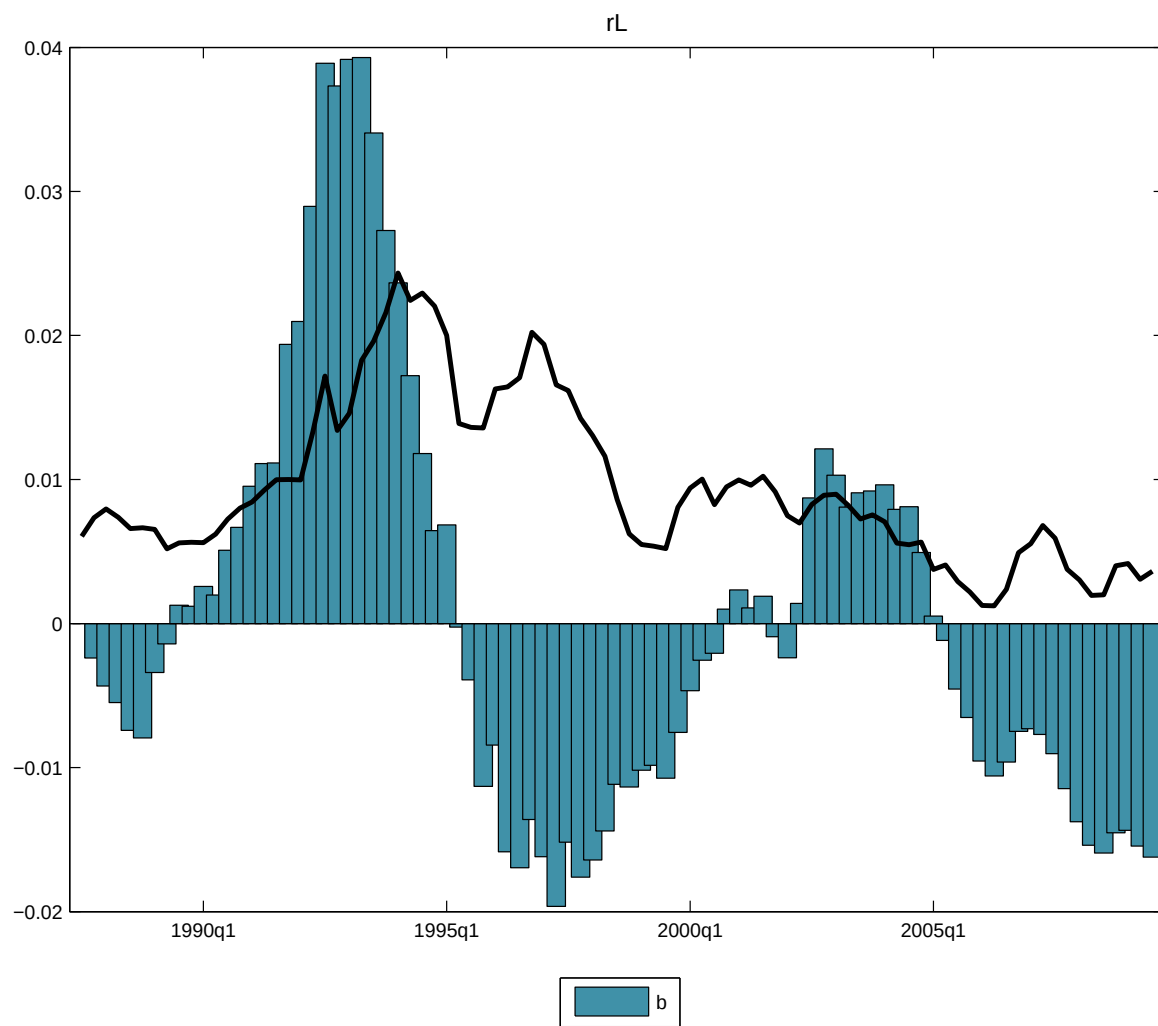


Figure A.14: Historical shock decomposition: contribution of the productivity shock to the path of the long rate.

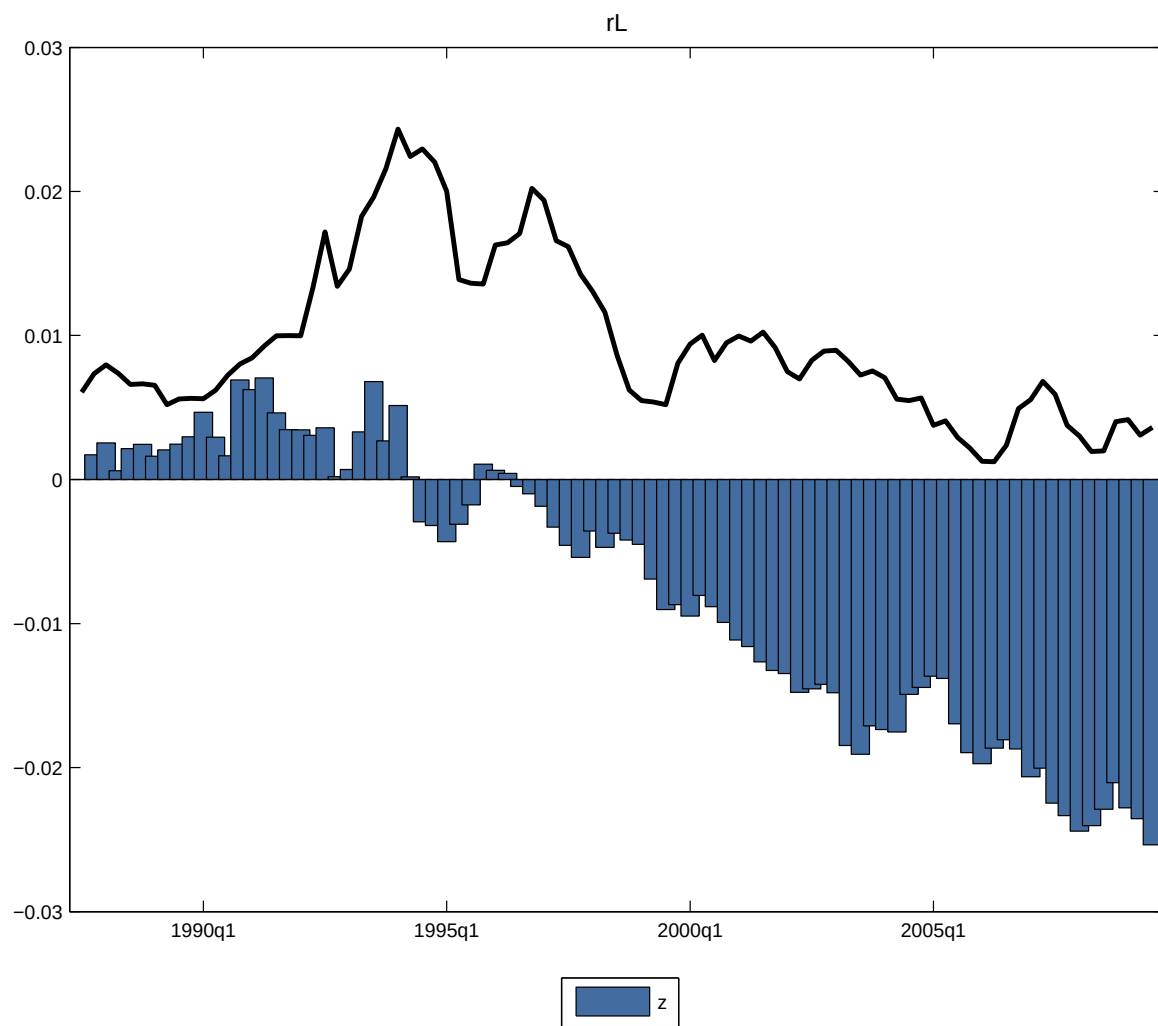


Figure A.15: Historical shock decomposition: contribution of the shock to risk premium to the path of the slope of the yield curve.

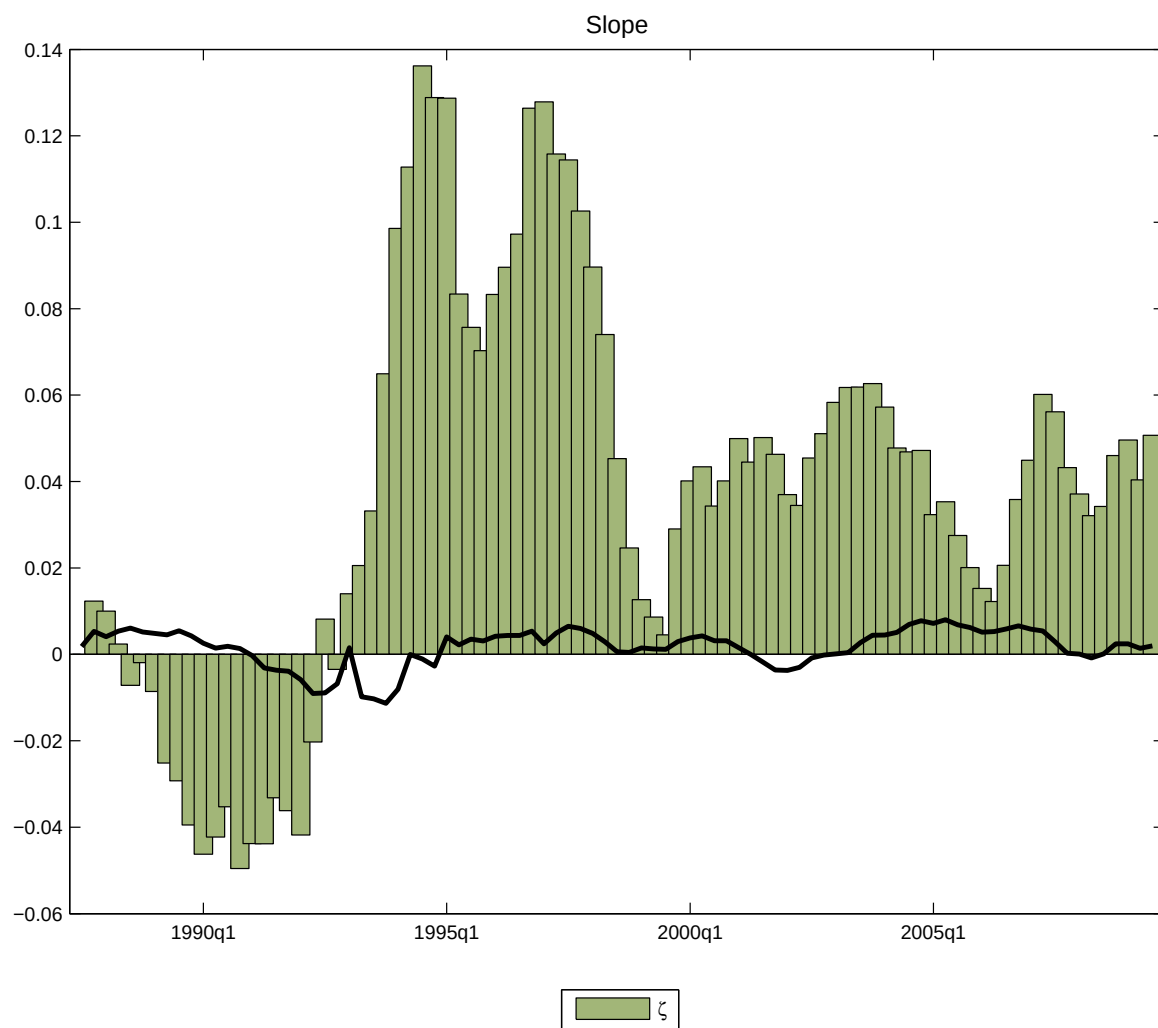


Figure A.16: Historical shock decomposition: contribution of the shock to the discount factor to the path of the slope of the yield curve.

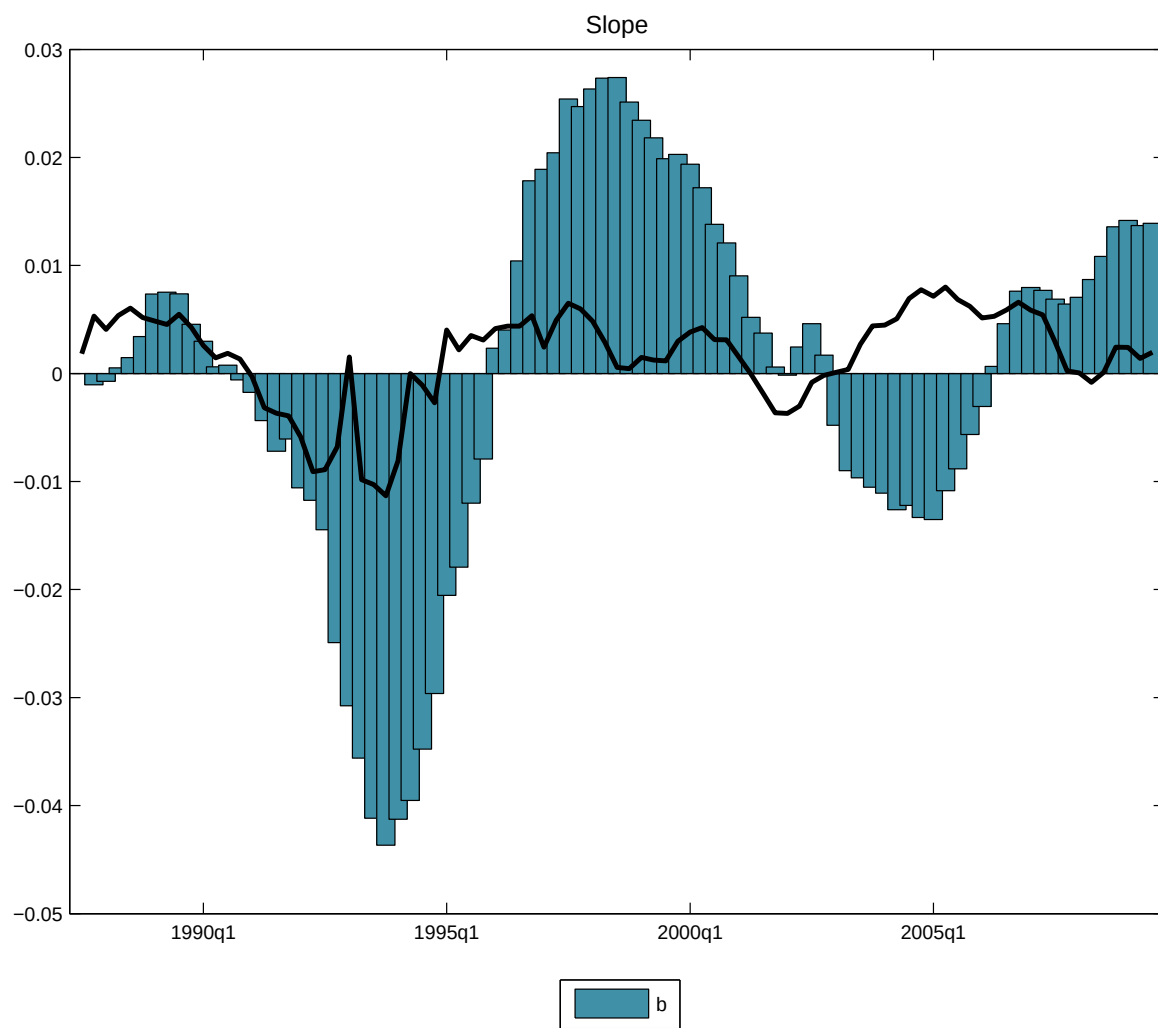


Figure A.17: Historical shock decomposition: contribution of the shock to monetary policy to the path of the slope of the yield curve.

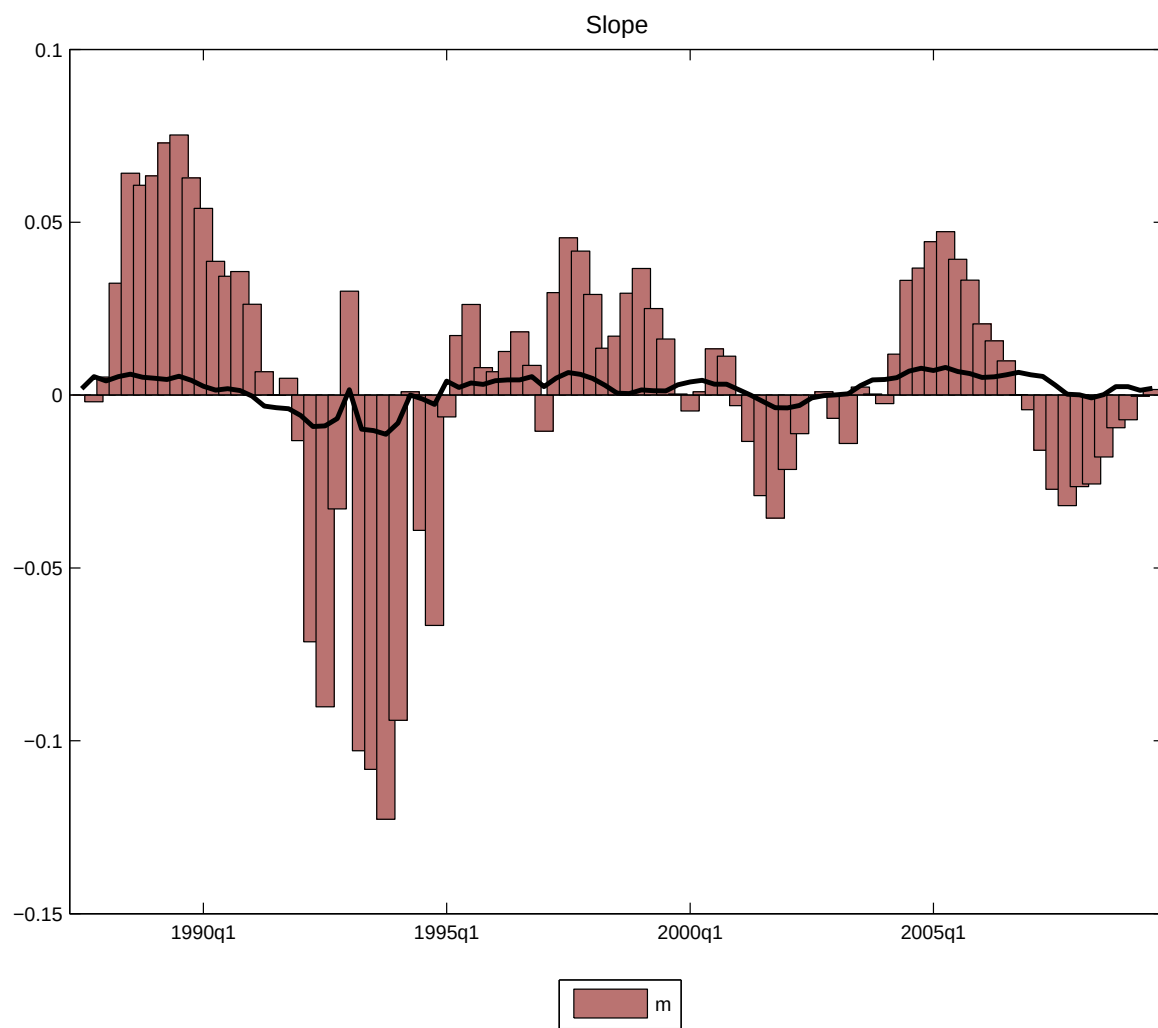
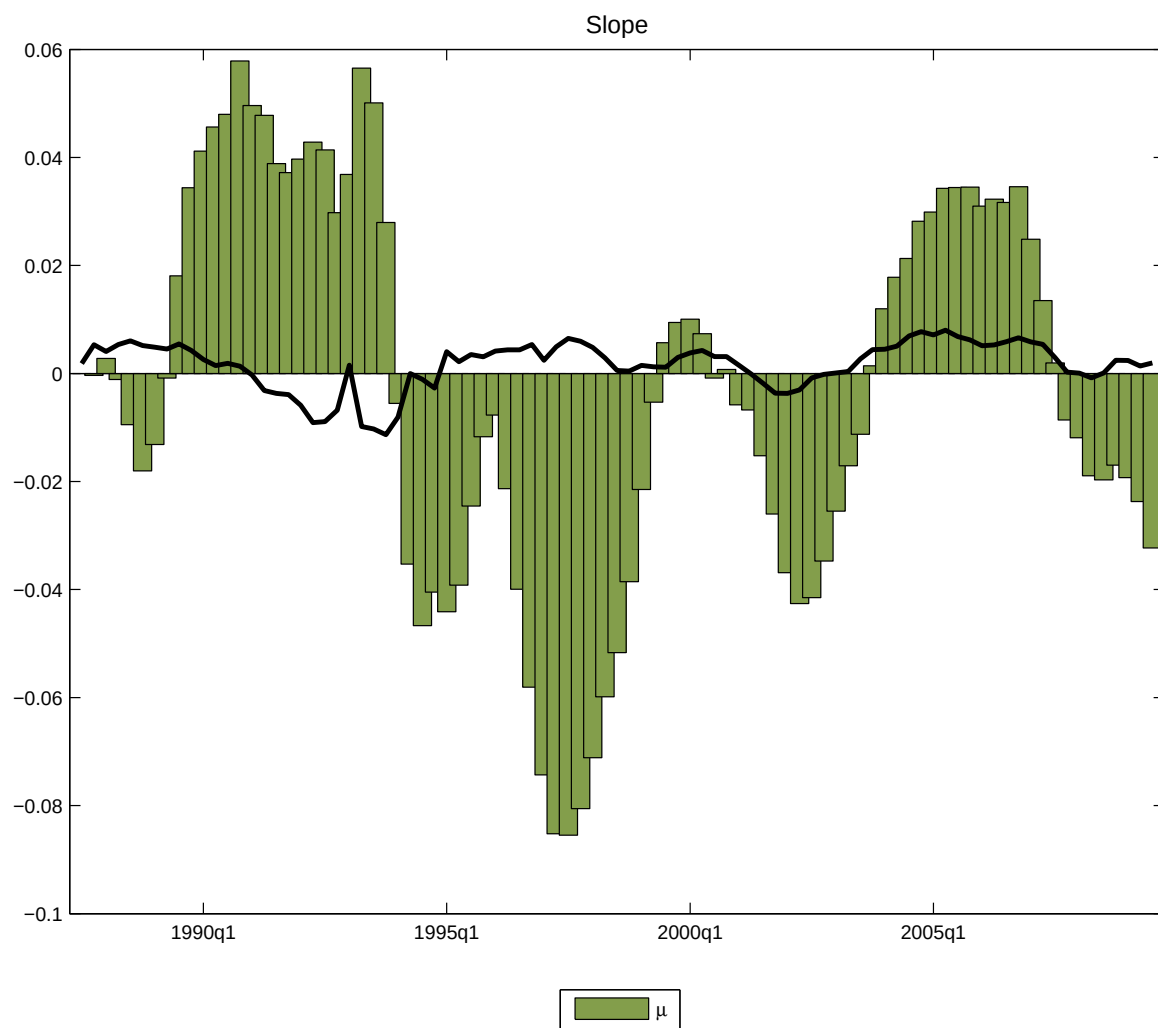


Figure A.18: Historical shock decomposition: contribution of the shock to marginal efficiency of investment the slope of the yield curve.



Appendix B

Appendix to Chapter 2: Proof that Schorfheide (2005) and Liu, Waggoner, and Zha (2011) give rise to the same solution

This section assumes that the only regime-switching parameter is the target steady state interest rate. Schorfheide (2005) implies:

$$\begin{aligned}\hat{R}_t &= \rho_R \hat{R}_{t-1} + (1 - \rho_R) \varphi_\pi \hat{\pi}_t + (1 - \rho_R) \varphi_y \hat{y}_t + \varepsilon_{R,t} + (1 - \rho_R) (1 - \varphi_\pi) \hat{R}_t^* \\ &= \rho_R \hat{R}_{t-1} + (1 - \rho_R) \varphi_\pi \hat{\pi}_t + (1 - \rho_R) \varphi_y \hat{y}_t + \varepsilon_{R,t}^*,\end{aligned}$$

where

$$\varepsilon_{R,t}^* = \varepsilon_{R,t} + (1 - \rho_R) (1 - \varphi_\pi) \left[\log \left(\frac{R_1}{R} \right), \log \left(\frac{R_2}{R} \right) \right] \hat{\mathbf{e}}_{s,t}.$$

Solution by gensys can be written as below where I assume the first shock is $\varepsilon_{R,t}^*$:

$$\begin{aligned}
y_t &= \Theta_1 y_{t-1} + \Theta_0 z_t + \Theta_y \sum_{s=1}^{\infty} \Theta_f^s \Theta_z E_t z_{t+s} \\
&= \Theta_1 y_{t-1} + \Theta_0 z_t + (1 - \rho_R) (1 - \varphi_\pi) \Theta_y \sum_{s=1}^{\infty} \Theta_f^s \Theta_z \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \left[\log \left(\frac{R_1}{R} \right), \log \left(\frac{R_2}{R} \right) \right] P^s.
\end{aligned}$$

So the constant is

$$\begin{aligned}
\Theta_c(K_t) &= (1 - \rho_R) (1 - \varphi_\pi) \Theta_{0.1} \cdot \left[\log \left(\frac{R_1}{R} \right), \log \left(\frac{R_2}{R} \right) \right] \mathbf{e}_{s,t} \\
&= (1 - \rho_R) (1 - \varphi_\pi) \Theta_y \sum_{s=1}^{\infty} \Theta_f^s \Theta_z \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \left[\log \left(\frac{R_1}{R} \right), \log \left(\frac{R_2}{R} \right) \right] P^s \mathbf{e}_{s,t}.
\end{aligned}$$

Now I will prove that Liu, Waggoner, and Zha (2011) give rise to the same solution.

Assuming the first row of the equilibrium conditions is for the Federal Funds Rate:

$$\begin{aligned}
&\begin{bmatrix} \Gamma_0, \begin{pmatrix} -(1 - \rho_R) (1 - \varphi_\pi) \left[\log \left(\frac{R_1}{R} \right), \log \left(\frac{R_2}{R} \right) \right] \\ 0 \\ \vdots \end{pmatrix} \\ \mathbf{0} \dots, I_2 \end{bmatrix} \begin{bmatrix} y_t \\ \hat{\mathbf{e}}_{s,t} \end{bmatrix} \\
&= \begin{bmatrix} \Gamma_1 & \mathbf{0} \\ \mathbf{0} & P \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \hat{\mathbf{e}}_{s,t-1} \end{bmatrix} + \begin{bmatrix} \Psi & \mathbf{0} \\ \mathbf{0} & I_2 \end{bmatrix} \begin{bmatrix} z_t \\ \nu_t \end{bmatrix} + \begin{bmatrix} \Pi \eta_t \\ 0 \end{bmatrix}.
\end{aligned}$$

Perform QZ decomposition on Γ_0 and Γ_1 and then premultiply both sides by

$$\begin{bmatrix} Q & \mathbf{0} \\ \mathbf{0} & I_2 \end{bmatrix} :$$

$$\begin{aligned} & \begin{bmatrix} Q_{n \times n} & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} Q' \Lambda Z', \begin{pmatrix} -(1 - \rho_R)(1 - \varphi_\pi) [\log(\frac{R_1}{R}), \log(\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \\ [0]_{2 \times n} \quad \dots, \quad I_2 \end{bmatrix}_{n \times 2} \begin{bmatrix} y_t \\ \hat{\mathbf{e}}_{s,t} \end{bmatrix} \\ &= \begin{bmatrix} Q_{n \times n} & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} Q' \Omega Z' & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \hat{\mathbf{e}}_{s,t-1} \end{bmatrix} + \begin{bmatrix} Q_{n \times n} & 0 \\ 0 & I_2 \end{bmatrix} \left(\begin{bmatrix} \Psi & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} z_t \\ \nu_t \end{bmatrix} + \begin{bmatrix} \Pi \eta_t \\ 0 \end{bmatrix} \right), \end{aligned}$$

and thus:

$$\begin{aligned} & \begin{bmatrix} \Lambda Z' & Q \begin{pmatrix} -(1 - \rho_R)(1 - \varphi_\pi) [\log(\frac{R_1}{R}), \log(\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} y_t \\ \hat{\mathbf{e}}_{s,t} \end{bmatrix} \quad (\text{B.1}) \\ &= \begin{bmatrix} \Omega Z' & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \hat{\mathbf{e}}_{s,t-1} \end{bmatrix} + \begin{bmatrix} Q \Psi & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} z_t \\ \nu_t \end{bmatrix} + \begin{bmatrix} Q \Pi \eta_t \\ 0 \end{bmatrix}. \end{aligned}$$

Let $w_t = Z' y_t$, and $w_{t-1} = Z' y_{t-1}$. B.1 becomes:

$$\begin{aligned} & \Lambda w_t + Q \begin{pmatrix} -(1 - \rho_R)(1 - \varphi_\pi) [\log(\frac{R_1}{R}), \log(\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \hat{\mathbf{e}}_{s,t} \\ &= \Omega w_{t-1} + Q \Psi z_t + Q \Pi \eta_t, \end{aligned}$$

and thus:

$$\begin{aligned}
& \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} - Q \underbrace{\left(\begin{pmatrix} (1-\rho_R)(1-\varphi_\pi) \left[\log\left(\frac{R_1}{R}\right), \log\left(\frac{R_2}{R}\right) \right] \\ 0 \\ \vdots \end{pmatrix} \right)}_{x_t} \hat{\mathbf{e}}_{s,t} + \Psi z_t + \Pi \eta_t \\
&= \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} w_1(t-1) \\ w_2(t-1) \end{bmatrix}.
\end{aligned}$$

Let $M = \Omega_{22}^{-1} \Lambda_{22}$ and solve forward:

$$\begin{aligned}
w_2(t) &= -E_t \left[\sum_{s=1}^{\infty} M^{s-1} \Omega_{22}^{-1} x_2(t+s) \right] \\
&= - \left[\sum_{s=1}^{\infty} M^{s-1} \Omega_{22}^{-1} x_2(t+s) \right].
\end{aligned}$$

Replace x_t with their definition and use the fact $E_t \eta_{t+s} = 0$:

$$\begin{aligned}
&= -E_t \left[\sum_{s=1}^{\infty} M^{s-1} \Omega_{22}^{-1} Q_2 \cdot \left(\Psi Z_{t+s} + \begin{pmatrix} (1-\rho_R)(1-\varphi_\pi) \left[\log\left(\frac{R_1}{R}\right), \log\left(\frac{R_2}{R}\right) \right] \\ 0 \\ \vdots \end{pmatrix} \hat{\mathbf{e}}_{s,t+s} \right) \right] \\
&= - \left[\sum_{s=1}^{\infty} M^{s-1} \Omega_{22}^{-1} Q_2 \cdot \left(\Psi Z_{t+s} + \begin{pmatrix} (1-\rho_R)(1-\varphi_\pi) \left[\log\left(\frac{R_1}{R}\right), \log\left(\frac{R_2}{R}\right) \right] \\ 0 \\ \vdots \end{pmatrix} \hat{\mathbf{e}}_{s,t+s} + \Pi \eta_{t+s} \right) \right],
\end{aligned}$$

and thus:

$$\begin{aligned}
Q_2.\Pi\eta_{t+1} &= \sum_{s=1}^{\infty} \Omega_{22} M^{s-1} \Omega_{22}^{-1} Q_2. (\Psi (E_{t+1} z_{t+s} - E_t z_{t+s})) \\
&+ \sum_{s=1}^{\infty} \Omega_{22} M^{s-1} \Omega_{22}^{-1} Q_2. \left(\begin{pmatrix} (1 - \rho_R) (1 - \varphi_\pi) [\log (\frac{R_1}{R}), \log (\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} (E_{t+1} \hat{\mathbf{e}}_{s,t+s} - E_t \hat{\mathbf{e}}_{s,t+s}) \right).
\end{aligned}$$

If the solution is unique:

$$Q_1.\Pi = \Phi Q_2.\Pi.$$

Premultiplying B.2 by $[I - \Phi]$:

$$\begin{aligned}
&\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi \Lambda_{22} \\ 0 & I \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \\
&- \begin{bmatrix} Q_1. - \Phi Q_2. \\ 0 \end{bmatrix} \left(\begin{pmatrix} (1 - \rho_R) (1 - \varphi_\pi) [\log (\frac{R_1}{R}), \log (\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \hat{\mathbf{e}}_{s,t} \right) \\
&= \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi \Omega_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_1(t-1) \\ w_2(t-1) \end{bmatrix} + \begin{bmatrix} Q_1. - \Phi Q_2. \\ 0 \end{bmatrix} \Psi z_t \\
&- \begin{bmatrix} 0 \\ E_t [\sum_{s=1}^{\infty} M^{s-1} \Omega_{22}^{-1} x_2(t+s)] \end{bmatrix}.
\end{aligned}$$

Finally,

$$\begin{aligned}
& y_t + Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ 0 \end{bmatrix} \begin{pmatrix} -(1 - \rho_R)(1 - \varphi_\pi) [\log(\frac{R_1}{R}), \log(\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \hat{\mathbf{e}}_{s,t} \\
&= Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi \Omega_{22} \\ 0 & 0 \end{bmatrix} Z' y_{t-1} \\
&\quad + Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ 0 \end{bmatrix} \Psi z_t \\
&\quad - Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix} \begin{bmatrix} 0 \\ E_t [\sum_{s=1}^{\infty} M^{s-1} \Omega_{22}^{-1} Q_{2\cdot} \Psi z_{t+s}] \end{bmatrix} \\
&\quad - Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix} \\
&\quad \left[E_t \begin{bmatrix} 0 \\ \sum_{s=1}^{\infty} M^{s-1} \Omega_{22}^{-1} Q_{2\cdot} \begin{pmatrix} (1 - \rho_R)(1 - \varphi_\pi) [\log(\frac{R_1}{R}), \log(\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \end{bmatrix} \hat{\mathbf{e}}_{s,t+s} \right].
\end{aligned}$$

By simplifying notation, I can rewrite the above equation as:

$$\begin{aligned}
y_t &= \Theta_1 y_{t-1} + \Theta_0 \left(\Psi z_t + \begin{pmatrix} (1 - \rho_R)(1 - \varphi_\pi) [\log(\frac{R_1}{R}), \log(\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \hat{\mathbf{e}}_{s,t} \right) \\
&+ \Theta_y \sum_{s=1}^{\infty} \Theta_f^{s-1} \Theta_z E_t \left(\Psi z_{t+s} + \begin{pmatrix} (1 - \rho_R)(1 - \varphi_\pi) [\log(\frac{R_1}{R}), \log(\frac{R_2}{R})] \\ 0 \\ \vdots \end{pmatrix} \hat{\mathbf{e}}_{s,t+s} \right),
\end{aligned}$$

where

$$\Theta_1 = Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi \Omega_{22} \\ 0 & 0 \end{bmatrix} Z',$$

$$\Theta_0 = Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ 0 \end{bmatrix},$$

$$\Theta_y = -Z \begin{bmatrix} \Lambda_{11}^{-1} & \Lambda_{11}^{-1} (\Lambda_{12} - \Phi \Lambda_{22}) \\ 0 & I \end{bmatrix},$$

$$\Theta_f = M,$$

and

$$\Theta_z = \Omega_{22}^{-1} Q_{2\cdot}.$$

This is exactly the same as treating $\hat{\mathbf{e}}_{s,t+s}$ as a shock as in Schorfheide (2005). ■

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