Essays on Disclosure*

Robert E. Verrecchia
The Wharton School
2400 Steinberg Hall - Dietrich Hall
University of Pennsylvania
Philadelphia, PA 19104-6365
telephone: (215) 898-6976
e-mail: verrecchia@wharton.upenn.edu

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Abstract

In this paper I attempt a taxonomy of the extant accounting literature on disclosure and suggest as categories: "association-based disclosure," work that studies the effects of disclosure on asset equilibrium prices and trading volume; "discretionary-based disclosure," work that examines managers' discretion in the disclosure of information about which

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they may have knowledge; and "efficiency-based disclosure," work that discusses which disclosure arrangements are preferred in the absence of prior knowledge of the information. In addition, in the final section of the paper, I discuss information asymmetry reduction as a starting point for a comprehensive theory of disclosure.

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1 Introduction

This document results from an assignment from the editors of the *Journal of Accounting and Economics* (*JAE*) to survey the extant literature on disclosure and write a paper that could be titled appropriately "Models of the Role of Disclosure in Capital Markets." The motivation for this assignment, I presume, is that over the past two decades disclosure research in accounting has burgeoned from a handful of papers on the topic to a substantial, and well-recognized, body of work. In addition, the *JAE* has been at the forefront of promoting economics-based research in accounting, and many papers commonly cited in the disclosure literature can be traced back to it. Finally, while some might debate where disclosure falls as a topic in the pantheon of all economics-based research, arguably, its role in accounting is central. Economics-based models of disclosure establish a link between financial reporting and the economic consequences of that activity. Without such a link, research in financial accounting is open to the charge that it studies bookkeeping rules and opinion promulgations, but in the absence of any economic motivation.

All this having been said, executing a task of this nature is less straightforward than it appears. Using the assigned title as a resource, one might suggest surveying comprehensively *models* that were employed to discuss *disclosure* in the context of *capital markets*, which, in addition, seem to have gained some prominence in the *accounting* literature (this, after all, being an assignment from an accounting journal). There are two considerations, however, that militate against a comprehensive survey: one practical, the other personal. The practical issue is that there is no comprehensive, or unifying, *theory* of disclosure, or at least none that I felt comfortable identifying as such. In the disclosure literature, there is no central paradigm, no

single compelling notion that gives rise to all subsequent research, no well-integrated "theory," however one interprets that term. Indeed, in its current composition the disclosure literature could probably best be characterized as an eclectic commingling of highly idiosyncratic (and highly stylized), economics-based models, each of which attempts to examine some small piece of the overall disclosure puzzle. Eclecticism is exacerbated by the fact that disclosure, as a topic, spans three literatures, accounting, finance, and economics, and thus inevitably takes on features of those literatures.

Acknowledging that a comprehensive theory of disclosure is a worthwhile goal, the objective of this paper is more modest. As a small, preliminary step toward a comprehensive theory, in this document first I consider the full panoply of theory-based, disclosure-related research in accounting and attempt a taxonomy of the literature: that is, a categorization of the various disclosure models into well-integrated topics. Then, in the final section of the paper, I recommend one starting point for a comprehensive theory.

With regard to the taxonomy, I suggest three broad categories of disclosure research in accounting. The first category of research is work whose primary concern is how exogenous disclosure is associated with, or related to, the change or disruption in the activities of investors who compete in capital market settings as individual, welfare-maximizing agents. I dub this research "association-based disclosure." The distinguishing feature of work in this category is that it studies the effects of exogenous disclosure on the aggregate or cumulative change in investors' actions, primarily through the behavior of asset equilibrium prices and trading volume. The second category is work that examines how managers and/or firms exercise discretion with regard to the disclosure of information about which they may have knowledge. I dub this research "discretionary-based disclosure." The distinguishing feature of work in this

category is that it treats disclosure as endogenous by considering managers' and/or firms' incentives to disclose information known to them; typically this is done in the context of a capital market setting in which the market is characterized as (simply) a single, representative consumer of disclosed information. The third category is work that discusses which disclosure arrangements are preferred in the absence of prior knowledge of the information, that is, *ex ante*. I dub this research "efficiency-based disclosure." The distinguishing feature of work in this category is that it examines unconditional disclosure choices; typically this is done in the context of a capital market setting in which the actions of individual, welfare-maximizing agents are endogenous. As with any taxonomy, there is an element of discretion in the categorization of some papers; I make no claim that my choice is definitive.

In this paper, I devote an essay to each category of research in my taxonomy. That is, the first essay concerns association-based disclosure research, the second examines discretionary-based disclosure research, and the third reviews efficiency-based disclosure research. This sequencing of topics has advantages and disadvantages. The main advantage is pedagogy. For example, association-based disclosure is discussed first because it is perhaps the most straightforward topic: it studies relations between disclosure and capital market phenomena under the assumption that the incentives and/or efficiency of disclosure arrangements are fixed or exogenous. Discretionary-based disclosure then introduces the incentives for disclosure activity (but typically in the absence of *ex ante* considerations). Finally, efficiency-based disclosure examines unconditional disclosure choice. The main disadvantage of discussing association-based disclosure first is that it requires that I discuss *how* disclosure affects capital market phenomena without first offering a rationale for *why* disclosure exists in the first place (e.g., efficiency-based disclosure). Suffice it to say that each of the three

essays is a self-contained discussion and there is no harm in reading the essays out of my suggested order.

In each essay I attempt to document the historical evolution of the topic, examine the role of maintained assumptions, and briefly review the overall strengths and weaknesses of individual contributions. In addition, in each survey I attempt to illustrate the analysis underlying individual models through a device that I refer to as a "modeling vignette." As a pedagogical device, modeling vignettes have three goals. First, they represent an attempt at distilling a complex analysis into its central feature, while at the same time being sufficiently robust to sustain that feature. Second, they represent an attempt at offering a series of fully integrated examples in which a reader can trace the evolution of a topic with the minimal amount of modeling dislocation. (If the ultimate goal is a comprehensive theory of disclosure, then perhaps the penultimate goal is a series of fully integrated modeling vignettes.) Finally, I am of the conviction that one cannot appreciate fully a paper's contribution without "getting one's hands dirty," which is to say actually working through simple examples as an exercise. Consequently, my intent in offering these vignettes is to suggest a series of exercises that an interested reader can work through, in the same fashion that textbooks offer problems at the end of each chapter.

It should go without saying, but I will state it anyway, that a truly comprehensive theory of disclosure would integrate simultaneously into its analysis all three elements of my taxonomy. That is, a comprehensive theory would recognize appropriately the roles of efficiency, incentives, and the endogeneity of the market process as it involves the interactions among individual, and diverse, welfare-maximizing investor agents. But this is a challenge for future research, and my goal here is limited to laying out what I regard as the building blocks of a comprehensive theory (in sections 2-4) and

recommending one starting point for a comprehensive theory (in section 5).

The personal issue that militates against a comprehensive survey is that I have some reservations about this type of assignment. At best, a survey is a poor substitute for reading the original source documents; at worst, it is a bland and uninspired regurgitation of the literature. To assist a reader interested in comprehensively surveying the literature, throughout this document I sprinkle footnote references to a large (but I make no claim exhaustive) list of disclosure-related research in the literatures of accounting, finance, and economics.[1] In lieu of a comprehensive survey, I offer a personal account, or *mémoire*, of work in which I have participated and continue to have a keen interest. I make no apology for this. I believe that a reader profits most from the personal reflections and commentary of someone who has participated in the evolution of a research paradigm. I leave it to others to write about research in which they are keenly interested.

With regard to the last point, one final caveat is appropriate. The *JAE* editors have assigned others the task of surveying two topics that deal with issues that are germane to my discussion: contract theory in accounting and disclosure in the empirical accounting literature.[2] In this document every attempt is made to eschew these topics so as to minimize the overlap among surveys.

A brief summary of this paper is as follows. Essays on association-based, discretionary-based, and efficiency-based disclosure are offered in sections 2, 3, and 4, respectively. In the final section of the paper, section 5, I summarize my observations and briefly discuss suggestions for future research.

[1] There exists a veritable cornucopia of research on this topic; thus, as a triage in preparing the references I did not include working papers (including my own).
[2] For the former, see Lambert [2000]; for the latter, see Healy and Palepu [2000] and Core [2000].

2 Association-Based Disclosure

How is disclosure associated with, or related to, the change or disruption in the activities of investors who are diverse and compete in capital market settings to maximize their individual welfares? Association-based disclosure research attempts to examine this problem by characterizing the effects of disclosure on the cumulative actions of individual, investor agents at the time of a disclosure event. Two characterizations of aggregate or cumulative behavior that are of particular interest in association-based studies concern the relations between disclosure and price changes, and disclosure and trading volume. In offering characterizations of this nature, association-based studies attempt to extend the literature on economics-based representations of financial markets with diverse investor agents that go back at least as far as Lintner.[3]

The motivation for this essay is two-fold. First, I offer a straightforward historical account of the evolution of the association-based literature. Second, to the extent to which a comprehensive theory is required to incorporate the effects of disclosure on the behavior of individual, welfare-maximizing agents who interact in capital market settings, I discuss general issues related to this topic. The historical account itself is done through a series of modeling vignettes. The role of the vignettes is to show how the literature developed, with increasingly more sophisticated models subsuming earlier, simpler models as deficiencies in prior work were identified. In addition, the modeling-vignette presentation format allows me to comment on the variety of maintained assumptions employed in this literature, and to point the interested reader in the direction of work that discusses the role of these assumptions in greater depth.

[3] Specifically, Lintner [1968]. See also Karpoff [1987], who surveys the literature on the relation between price changes and trading volume in capital markets through 1987, and points to the deficiency of most of the theory-based literature to explain price-volume relations up to that point in time (i.e., 1987).

A brief summary of the vignettes is as follows. Model #1 introduces a very stylized representation of disclosure and price change; its primary purpose is to motivate subsequent discussion. Model #2 introduces disclosure in a Walrasian setting. Models #3-6 extend that discussion to settings in which market agents condition their expectations over market clearing prices (i.e., so-called "rational expectations" models of trade): first in one-period settings and then in two-period settings. In model #7 disclosure is examined in conjunction with heuristic behavior and perfectly competitive markets. In models #8 and #9, disclosure is considered in the context of models of imperfect competition; first with market agents who are exclusively Bayesian and then with agents who are Bayesian and heuristic. Finally, in model #10, disclosure is discussed in a setting in which market expectations are conditioned over both contemporaneous demand and trading volume information.

A Simple Model of Disclosure Association (model #1).[4] To illustrate the evolution of research on disclosure association and other ideas, I begin by suggesting a very stylized model of disclosure. To start, I assume that there exists some asset (e.g., a firm) whose value is uncertain, and about which some information is disclosed. Uncertainty can be represented by a random variable of any variety, but the normal distribution is well behaved mathematically and understood at an intuitive level by most researchers. Consequently, I assume that uncertain firm value is represented by a variable $\tilde{u}$, which has a normal distribution with mean m and precision (i.e., the reciprocal of variance) h. The precision h can be interpreted as the market's prevailing level of common knowledge about the firm's uncertain value, $\tilde{u}$. Similarly, I assume that the disclosure is information about firm value, but information that is less than perfect. For example, let disclosure be represented by $\tilde{y} = \tilde{u} + \tilde{\eta}$, where $\tilde{\eta}$ is

[4] In conjunction with model #1, see Holthausen and Verrecchia [1988] and Subramanyam [1996].

also a normal distribution with mean 0 and precision n. I interpret, and refer to, n as the precision of the information content of the disclosure, $\tilde{y}$. Finally, I assume that all relevant issues related to disclosure can be characterized by assuming an economy with two periods: time $T-1$ is the period immediately before disclosure occurs, and time T is the time immediately after (i.e., the disclosure period). Let the prices for the assets at times $T-1$ and T be represented by P_{T-1} and P_T, respectively.

Before proceeding, let me comment on the role of a number of maintained assumptions. First, as alluded to in the introduction, disclosure, i.e., $\tilde{y} = \tilde{u} + \tilde{\eta}$, is an exogenous feature of the economy I describe. By virtue of this, no where have I established a rationale, or demand, for the disclosure in the first place. Presumably, however, an interested reader can look to the two subsequent essays for guidance as to why disclosure may be either supplied or demanded in the context of the economy I describe. For reasons of pure pedagogy, it is convenient to start out with the assumption that disclosure simply "exists," and deal with the rationale for that disclosure later. Second, I describe an economy that is comprised of only a single risky asset (and, starting with model #2, a risk-free numeraire commodity). Unlike the assumption of exogenous disclosure, this assumption is maintained throughout my essays. Where the role of multiple risky assets has been considered in settings similar to the ones that I am about to discuss, one typically finds that claims or results that arise in a single-asset economy can be reversed.[5] Reversal may occur in multiple-asset economies because of interactions among assets. Consequently, a maintained assumption here is that one can control for multiple-asset effects. Finally, my analysis is *ceteris paribus*: that is, my analysis is premised on the notion that all elements other than the ones I study are fixed, or constant. For example, consider a

[5] See, for example, Admati [1985] and Holthausen and Verrecchia [1988].

multiple-asset economy in which firms in one industry manufacture apples and firms in another manufacture oranges. In addition, suppose that disclosure has different effects on the valuation of, and/or the production or coordination-related activities within, the apple manufacturing industry versus the orange manufacturing industry. Then a comparison of the effects of disclosure on apple versus orange manufacturing firms is flawed because other features of the comparison are not fixed. As with multiple-asset effects, I abstract from this problem.

Continuing with the discussion of association-based research, as manifestations of the disruption in the cumulative actions of investors, there are a variety of phenomena one could study. A very incomplete list might include: the functional relation between disclosure and price change; the functional relation between disclosure and trading volume; the extent to which disclosure changes the collective uncertainty about the asset's value at the time of the disclosure event; the extent to which disclosure makes markets more liquid; etc. Many of these phenomena are discussed in the original source documents. To provide some appreciation for an analysis of that type, I start with two. First, I consider the functional relationship between an exogenous disclosure, $\tilde{y}$, and the change in an asset's price at time T, $P_T - P_{T-\mathbf{1}}$. Second, I assess the percentage of the variability in price change at time T explained exclusively by the disclosure (controlling for certain key factors). To illustrate these vehicles for studying the effects of disclosure, suppose for a moment that the change in price at time T has the following functional form

$$P_T - P_{T-\mathbf{1}} = \alpha + \beta\,(\tilde{y} - m) + \gamma\tilde{\Omega} + \tilde{\xi},$$

where α, β, and γ are (fixed) parameters, $\tilde{\Omega}$ represents variables other than $\tilde{y}$ that are related to firm value and the change in price, and $\tilde{\xi}$ represents variables unrelated

to firm value (e.g., noise). Here, one could interpret the coefficient on $\tilde{y}$, β, as that element of the functional relation in the change in price that results directly from disclosure, as opposed to other factors. When in the discussion below the change in price assumes a linear functional form like the one above, for convenience I refer to β as the disclosure response coefficient (DRC) in the change in price.

A DRC tells us something about how the change in price relies on, or is governed by, disclosure, as distinct from other factors. For example, intuition suggests that as the models become increasingly more complicated, the DRC will decline because other factors, such as the existence of private information as a substitute for public disclosure, will reduce the reliance of prices on disclosure. But to confirm this intuition, and perhaps also to highlight where it fails, I consider also the percentage of the variability in price change explained exclusively by disclosure. In computing this percentage, I control for certain key factors. Which factors one controls for is somewhat arbitrary, but here I suggest controlling for the price at time $T-1$, $\tilde{P}_{T-1}$, and noise, $\tilde{\xi}$. The reason for controlling for the price at time $T-1$ is that I want to eliminate from the variability of price change that part of the variability that arises from activities prior to the disclosure, as captured by $\tilde{P}_{T-1}$. In addition, I want to control for noise because its contribution to the variance in price change is not economically relevant. Let $\Delta\%$ represent the percentage of the variability of price change explained exclusively by disclosure at time T. When one controls for both price at time $T-1$ and noise, the percentage of the variability explained by the disclosure at time T is defined by

$$\Delta\% = 1 - \frac{VAR[\tilde{P}_T - \tilde{P}_{T-1} | \tilde{y} = y, \tilde{P}_{T-1} = P_{T-1}, \tilde{\xi} = \xi]}{VAR[\tilde{P}_T - \tilde{P}_{T-1} | \tilde{P}_{T-1} = P_{T-1}, \tilde{\xi} = \xi]}.$$

When in the discussion below I discuss the percentage of variability explained by

disclosure, for convenience I refer to it the "Δ%-statistic."

Now I return to developing a very stylized model of price change and disclosure. In this model I assume that all investor agents who participate in the market are risk-neutral, can assume unlimited liability for realizations of firm value, and have no information (private or public) about firm value at time $T-1$. Because of the absence of information, at time $T-1$ all expectations are based on the unconditional expectation of $\tilde{u}$, which is m. Furthermore, because investor agents are risk-neutral, the price of the asset at time $T-1$ is $P_{T-1} = m$. At time T disclosure occurs (i.e., $\tilde{y} = y$ is disclosed); I assume that it is either the only information about firm value, or, if there is other information about firm value revealed at the same time (e.g., private information), it is subsumed in $\tilde{y} = y$. That is, with regard to valuing the firm, $\tilde{y}$ is a sufficient statistic for $\tilde{y}$ and all other information. If $\tilde{y} = y$ is a sufficient statistic for all information and investors are risk-neutral, then $P_T = E[\tilde{u}|\tilde{y} = y] = m + \frac{n}{h+n}(y - m)$. This implies

$$P_T - P_{T-1} = \frac{n}{h+n}(\tilde{y} - m),$$

where the expression $\tilde{y} - m$ can be interpreted as the "disclosure surprise" in that it represents the extent to which $\tilde{y} = y$ deviates from its expected value of m, which is also the expected value of $\tilde{u}$. Here, the DRC is $\frac{n}{h+n}$; it can be described as the precision of the disclosure, n, relative to the total precision of firm value conditional on the disclosure, $h+n$. In other words, the DRC is the information content of the disclosure relative to all that is known about firm value subsequent to the disclosure. Finally, note that in this simple model all the variability in price change is explained by disclosure at time T. For example, $VAR[\tilde{P}_T - \tilde{P}_{T-1}|\tilde{y} = y] = 0$. Consequently, this model's Δ%-statistic is 1.

Before proceeding, let me briefly mention the role of two more assumptions that I

maintain throughout these essays. Unlimited liability ensures that price change characterizations remain facile and transparent. This virtue not withstanding, in models of the type that I discuss below researchers have long recognized that unlimited liability is an artifact and therefore have studied its role.[6] For example, unlimited liability is probably a poor assumption if one intends to study the role of equity in conjunction with debt as vehicles for financing a firm's activities. As for the assumption that $\tilde{y} = y$ is a sufficient statistic, a conventional interpretation of sufficiency is that any information in existence prior to period T is a forecast of $\tilde{y}$, which the actual disclosure of $\tilde{y}$ in period T subsumes.[7]

Continuing with the discussion, in the evolution of analyses that purport to associate disclosure with price change, the characterization offered so far is transparent and facile.[8] Nonetheless, the model's elegance is achieved at the expense of an extreme stylization of how markets function. In this model, for example, there is no information about firm value that has any relevance other than the information that arises directly from disclosure. Perhaps more significant, the model describes a world in which no trade occurs. The reason for this is that beliefs are homogeneous in both periods $T-1$ and T, and hence there is no rationale for trade based on information. So, if a minimum condition for "model robustness" is that *some* trading volume arises at the time of disclosure, more work remains.

To achieve trading volume, it is likely that we will need to appeal to some elements of investor-agent diversity, because trade evolves primarily from differences across

[6] See, for example, Fischer and Verrecchia [1997].
[7] See, for example, Abarbanell, *et al.* [1995].
[8] Some might even argue that it captures well the spirit, if perhaps not the detail, of an empirical investigation of the relation between disclosure and price change. Note, however, that this characterization implies that price changes are normally distributed, whereas empirical studies typically assume that *returns* are normally distributed. While this dislocation between theory and empirical work is fairly innocuous, it points to the fact that some caution should be exercised in interpreting too literally any statements that I make in the context of empirical-based research.

investors: for example, differences of opinion, differences in endowments, differences in how investors use information, etc. Consequently, let me first put forth a list of attributes of investor-agent rationality and diversity that it would seem important for a model to incorporate, or at least address, in any theory-based characterization of the interactions of individual, welfare-maximizing agents who compete in capital market settings. Having done that, in the subsequent discussion I successively fold into the model each attribute, as a way of understanding how the attribute affects assumptions and conclusions of prior work. The list is as follows.

1. Investors are diversely informed.

2. Investors make rational inferences from market prices.

3. Investors rationally anticipate disclosure.

4. Investors, in addition to being diversely informed, also have information of diverse or heterogeneous quality.

5. Investors interpret disclosure in diverse ways.

6. Investors incorporate disclosure into their beliefs in diverse ways: that is, some agents depart from (narrowly) Bayesian behavior in how they incorporate disclosure into their posterior expectations.

7. Investors condition their beliefs over diverse economic stimuli: specifically, they make rational inferences from both market prices and trading volume.

Diversely informed investors (model #2).[9] I start with the following expanded story of a market with trade. There are a large number of investor agents, say, N, who exchange shares in the asset whose value is uncertain by comparing its value relative to a numeraire commodity, whose value is fixed at 1 (e.g., a government bond). Each investor i holds an amount x_i of the uncertain-valued asset, and an

[9] In conjunction with model #2, see Lintner [1968].

amount b_i of the certain-valued asset. For convenience, let x represent the per-capita supply of the uncertain-valued asset, where x is defined by $x = \sum_i (x_i/N)$. As this analysis evolves, it will be useful to allow for the fact that the per-capita supply of the uncertain-valued asset, $\tilde{x}$, is also a normally distributed random variable with mean 0 and precision t. When, below, I examine the variability of price changes arising from disclosure, my intentions are to interpret $\tilde{x}$ as noise and t as the precision of the noise. Finally, while there are two asset-types in this economy, for all intents and purposes only the uncertain-valued asset will concern us. Consequently, for convenience and wherever it creates no confusion, henceforth I will refer to the uncertain-valued asset as simply "the asset."

Before proceeding, note that another assumption maintained throughout this analysis is that the first moment (i.e., mean) of all other random variables is zero (with the exception of the mean of uncertain firm value, $\tilde{u}$). In particular, the error term around the disclosure of firm value, $\tilde{\eta}$, has a mean of 0. If we treat disclosure as exogenous, which is a feature of the association-based disclosure literature, the assumption that all means are zero would appear to be without loss of generality. When disclosure is treated as endogenous, however, one would need to recognize that disclosure preparers and disseminators may not have incentives that are perfectly aligned with the goal of providing unbiased assessments of firm value. The existence of disclosure, or reporting, bias is an important issue in financial reporting, where often data are produced and disseminated in conjunction with achieving some objective.[10] Nonetheless, I abstract from this issue here in an attempt to facilitate the discussion.

[10]On specifically the topic of reporting bias, see Fischer and Verrecchia [2000]. More generally, this issue touches on concerns related to the truthfulness or credibility of the disclosure. This is a topic reserved for the second essay.

As in our previous model, I assume that at time $T-1$ there is no information about the asset (i.e., the uncertain-valued asset). Consequently, as in our previous model $P_{T-1} = m$. Before trade takes place at time T, however, each investor i obtains different private information about the value of $\tilde{u}$, where this information is represented by $\tilde{z}_i = \tilde{u} + \tilde{\varepsilon}_i$ and $\tilde{\varepsilon}_i$ also has a normal distribution with mean 0 and precision s, say. The parameter $\tilde{\varepsilon}_i$ is also a "noise" term. As such, it captures the extent to which each investor's information about the uncertain value of the asset is accurate. For example, a high s denotes very accurate private information, and a low s denotes very inaccurate information. For convenience, henceforth I assume that the covariance across any pair of error terms is zero: for example, $E[\tilde{\eta}\tilde{\varepsilon}_i] = E[\tilde{\varepsilon}_i\tilde{\varepsilon}_j] = 0$. This implies that $\tilde{u}, \tilde{y}$, and $\tilde{z}_i$ have a trivariate normal distribution with means of (m, m, m) and a covariance matrix given by

$$\begin{bmatrix} h^{-1} & h^{-1} & h^{-1} \\ h^{-1} & h^{-1}+n^{-1} & h^{-1} \\ h^{-1} & h^{-1} & h^{-1}+s^{-1} \end{bmatrix}.$$

Consequently, when investors condition their expectations over the public disclosure and their private information, their expectations are

$$E[\tilde{u}|y,\ z_i] = \frac{hm + ny + sz_i}{h+n+s},$$

and the precision of their expectations, $(VAR[\tilde{u}|y, z_i])^{-1}$, is

$$(VAR[\tilde{u}|y, z_i])^{-1} = h + n + s.$$

Finally, I assume that the $\tilde{\varepsilon}_i$'s have finite variance; because of this, $\lim_{N\to\infty} \frac{1}{N}\sum_i \varepsilon_i \to 0$ for any realizations of ε_i's by the law of large numbers. Note that this implies for any realizations of the z_i's, $\lim_{N\to\infty} \frac{1}{N}\sum_i z_i \to u$.

Note that another maintained assumption is that errors are uncorrelated. As with the introduction of multiple risky assets, correlated errors may reverse claims and results found in uncorrelated error settings.[11] Nonetheless, I abstract from this issue.

If the motivation for extending our model is to ensure trading volume, then it is useful to relax our assumption that all investors are risk-neutral. When all investors are risk-neutral and have different private information, trading volume can arise. It will be, however, of a very stylized nature. In effect, that investor with the highest conditional expectation for the value of the asset at time T (i.e., the investor with the highest $E[\tilde{u}|y,\ z_i]$) will acquire at a minimum the total supply of the asset. Indeed, the extent to which the investor with the highest conditional expectation will go long in the asset is only limited by the ability and/or cost of other traders to sell the asset short.

To ensure that trade is less stylized, I assume that investor agents are risk averse, with a utility for an amount of a consumable good g given by $U(g) = -\exp[-\frac{g}{r}]$, where r measures an investor's tolerance for risk. This utility function is the (negative) exponential, and has reasonably desirable properties for a utility function: it is increasing and concave in g, implying that an investor prefers more of a consumable good to less, but to a decreasing degree. The true appeal of the negative exponential, however, is that when it is used in conjunction with the normal distribution, it results in a facile analysis. Finally, note that homogeneous risk tolerances across risk-averse investor agents is a maintained assumption throughout the analysis. This is a very innocuous assumption, however, in that it is a straightforward exercise to generalize all the models that I discuss below to allow for heterogeneous risk tolerances (which

[11]See, for example, Lundholm [1988].

is commonly done in the original source documents).

Our next step is to determine the P_T. To do this, I appeal to the notion that the large number of investors inhabiting our market ensures that P_T evolves from perfect competition. To dwell on the notion of perfect competition briefly, note that perfect competition assumes that each investor agent in the market behaves as if his or her actions or behaviors have no effect on price, and in equilibrium this conjecture is true. In theory-based characterizations of trade, perfect competition is achieved by assuming that the number of investor agents in the market is large (typically, countably infinite). This ensures that while prices reflect the combined decisions of all market participants at an aggregate level, the actions of each individual market agent are sufficiently atomistic as to have no appreciable effect on price. By all accounts perfect competition seems to be a reasonable assumption about markets that are deep and/or assets that are widely traded. In addition, suffice it to say here that one reason why perfect competition is a favored vehicle for disclosure association studies is that it simplifies considerably the "game" that market agents play towards determining the market equilibrium price. That is, by virtue of the fact that each individual investor agent can ignore the effect of his or her action on price, determining an equilibrium price is simplified considerably, but especially in circumstances in which trade is assumed to occur over multiple periods (which I discuss below).

In conjunction with perfect competition, I also appeal to Walras.[12] Walras' notion of how market clearing prices are determined in markets where divisible assets (e.g., firm shares) are exchanged could be described somewhat colloquially as follows. First, investors submit their demand curves for an asset to a beneficent and altruistic market maker (commonly referred to as a "Walrasian auctioneer"). Investors' demand curves

[12]See Walras [1881]; see also Wald [1951].

represent their demands for the asset as a function of the price of the asset. Armed with this information, the Walrasian auctioneer determines the price that equates the aggregate demand for the asset (i.e., the aggregation of individual demand curves) to the total aggregate supply. This price "clears the market," and hence represents the equilibrium.

Now consider investor i's demand for the asset whose value is uncertain versus the asset whose value remains fixed at 1, conditional on his private information z_i. Let D_i represent the demand for uncertain-valued asset and B_i represent the demand for the asset whose value is fixed at 1. The price at which the former trades is P_T, and the price at which the latter trades is 1. Thus, the value of investor i's endowed portfolio is $x_i P_T + b_i$. The cost of holding a portfolio represented by D_i and B_i is $D_i P_T + B_i$, and the return on holding that portfolio is $D_i u + B_i$. Taken all together, this implies that the net return for holding a portfolio represented by D_i and B_i (and net of the proceeds from the value of i's endowed portfolio) is $D_i(u - P_T) + x_i P_T + b_i$. The expected value of this portfolio to investor i, based on his private information z_i and y, is $E[U\left(D_i(\tilde{u} - P_T) + x_i P_T + b_i\right) | y, z_i]$.

To determine a value for P_T, first I must compute each investor's demand for D_i and B_i. When the negative exponential utility function is used in combination with the normal distribution function, it yields a result that is linear in the argument of the exponential: that is,

$$\begin{aligned} & E[U\left(D_i(\tilde{u} - P_T) + x_i P_T + b_i\right) | y, z_i] \\ = \; & -\exp[-\frac{1}{r} D_i E[\tilde{u}|y, z_i] + \frac{1}{2r^2} D_i^2 VAR[\tilde{u}|y, z_i] + \frac{1}{r} D_i P_T - \frac{1}{r} x_i P_T - \frac{1}{r} b_i]. \end{aligned}$$

In determining his optimal portfolio, each investor chooses D_i so as to maximize the

above. This yields

$$D_i = r\frac{E[\tilde{u}|y, z_i] - P_T}{VAR[\tilde{u}|y, z_i]}.$$

This is a standard demand equation resulting from the negative exponential in conjunction with the normal distribution. It suggests that the demand for the asset is equal to: an investor's expectation of the value of the asset conditional on his private information and the disclosure, minus the price of the asset; an adjustment for his tolerance for risk (i.e., r); and an adjustment for (in the denominator) the confidence he has in his posterior expectations (i.e., $VAR[\tilde{u}|y, z_i]$). Straightforward results from multivariate normality imply that $E[\tilde{u}|y, z_i] = m + \frac{n}{h+n+s}(y - m) + \frac{s}{h+n+s}(z_i - m)$ and $VAR[\tilde{u}|y, z_i] = \frac{1}{h+n+s}$. Consequently, D_i can be rewritten as

$$D_i = r\left(hm + ny + sz_i - \{h + n + s\}P_T\right).$$

Now our stated goal remains to endogenize P_T. P_T is determined by equating the per-capita supply of the asset (i.e., the uncertain-valued asset) with per-capita demand; the P_T that achieves this, i.e., $x = \sum_i (x_i/N) = \sum_i (D_i/N)$, is

$$\begin{aligned} P_T &= \frac{1}{h+n+s}\left(hm + n\tilde{y} + s\lim_{N\to\infty}\frac{1}{N}\sum_i z_i - \frac{1}{r}\tilde{x}\right) \\ &= \frac{1}{h+n+s}\left(hm + n\tilde{y} + s\tilde{u} - \frac{1}{r}\tilde{x}\right). \end{aligned}$$

Hence,

$$P_T - P_{T-1} = \frac{1}{h+n+s}\left(n\left(\tilde{y} - m\right) + s\left(\tilde{u} - m\right) - \frac{1}{r}\tilde{x}\right).$$

Note that $E[\tilde{P}_T] = m$ and $E[\tilde{P}_T - \tilde{P}_{T-1}] = 0$. An interpretation of $\tilde{P}_T - \tilde{P}_{T-1}$ is that it represents: the change in the expectation of $\tilde{u}$ averaged across all investors, where the change is adjusted for the posterior precision of their expectations based on their knowledge of y and z_i; and adjusted further by the per-capita supply of the

uncertain-valued asset (which is also adjusted for investors' tolerance for risk, r). The "supply adjustment," $-\frac{1}{r(h+n+s)}\tilde{x}$, can be thought of as the extent to which the price of the asset at time T, P_T, must be reduced below variables whose expected value is m, the expected value of the asset (i.e., $E\left[\frac{1}{h+n+s}\left(hm+n\tilde{y}+s\tilde{u}\right)\right]=m$), to attract investors who are risk-averse (assuming that the realization of the per-capita supply of the asset, i.e., x, is positive). If, for example, investors' tolerance for risk is very large, which implies that they are approximately risk-neutral, then $r\to\infty$ and the adjustment is 0. Similarly, if the precision of their posterior expectations is very large, which implies that they are almost certain of the asset's value, then $h+n+s\to\infty$ and once again the adjustment is 0.

To digress briefly, another maintained assumption is that there exists a continuum of traders. Consequently, one cannot talk meaningfully about how increasing the investor base (i.e., the number of people who participate in the market) affects prices or price changes. Note, however, that r is per-capita risk tolerance. As such, one could interpret r as a proxy for investor base: that is, as the investor base increases, r increases. Allowing this interpretation and assuming for the moment that the realization of the per-capita supply of the asset is positive (i.e., x is positive), this implies that an increase in investor base (i.e., r) results in an increase in the change in price: effectively, an increase in returns.[13]

The salient feature of this model is that the DRC declines to $\frac{n}{h+n+s}$. As discussed previously, the reason for its decline is that there now exists in the economy private information, in the form of the z_i's, and this lessens the reliance of price on disclosure.

[13]See, for example, Merton [1987]. Note that if there were only a finite number of market participants, it would be more transparent that as their number increased (i.e., as the investor base increased), the supply adjustment would decrease, and hence the change in price would increase. Making the number of market participants finite, however, creates problems in conjunction with assuming perfect competition.

With regard to this model's $\Delta\%$-statistic, one interpretation of per-capita supply is that it represents a variable unrelated to the asset's true, economic value, but which nonetheless affects price change through the supply of the asset (and risk aversion). As such, in the context of this discussion I interpret per-capita supply, $\tilde{x}$, as a proxy for the noise term, $\tilde{\xi}$, discussed previously. Using x as a proxy for ξ, $VAR[\tilde{P}_T - \tilde{P}_{T-\mathbf{1}}|x] = \left(\frac{\mathbf{1}}{h+n+s}\right)^2 \left(n^2\left(\frac{h+n}{hn}\right) + 2ns\frac{\mathbf{1}}{h} + s^2\frac{\mathbf{1}}{h}\right)$ and $VAR[\tilde{P}_T - \tilde{P}_{T-\mathbf{1}}|y,x] = \left(\frac{\mathbf{1}}{h+n+s}\right)^2 \left(\frac{s^2}{h+n}\right)$: note that it is not necessary to control for price at time $T-1$ in these expressions because $P_{T-\mathbf{1}}$ is fixed at m. Consequently, here the $\Delta\%$-statistic reduces to

$$\begin{aligned}\Delta\% &= 1 - \frac{\frac{s^2}{h+n}}{n^2\left(\frac{h+n}{hn}\right) + 2ns\frac{\mathbf{1}}{h} + s^2\frac{\mathbf{1}}{h}} \\ &= \frac{n^2\left(\frac{h+n}{hn}\right) + 2ns\frac{\mathbf{1}}{h} + s^2\frac{n}{h(h+n)}}{n^2\left(\frac{h+n}{hn}\right) + 2ns\frac{\mathbf{1}}{h} + s^2\frac{\mathbf{1}}{h}},\end{aligned}$$

which is clearly less than 1 because $\frac{n}{h(h+n)} < \frac{\mathbf{1}}{h}$. In other words, consistent with the decline in the DRC, in this model disclosure explains less than 100% of the variability in price change. The reason for this should be clear: in this variation there exists private, as well as public, information about the value of the asset. Had, for example, there been no private information (i.e., $s = 0$), then here once again the $\Delta\%$-statistic would be 1.

To summarize the analysis to this point, as a characterization of the association between disclosure and price change, a Walrasian model has many appealing features. But it is not without controversy, which is the motivation for our next section.

Rational Inferences from Market Prices (model #3).[14] While Walras' notion of perfect competition offers many insights into the price setting process, it

[14]In conjunction with model #3, see Hellwig [1980], Diamond and Verrecchia [1981], and Lundholm [1988]. With regard to the last paper, note that Lundholm's chief concern is the role of correlated errors, whereas a maintained assumption throughout this analysis is that all error terms are uncorrelated.

can nonetheless be argued that it is conceptually flawed. Implicit in a Walrasian equilibrium is the notion that investors' beliefs about what the asset is worth are fixed, or invariant, to the price at which the market cleared. This was sometimes referred to as an "exogenous beliefs" model. The conceptual flaw in an exogenous beliefs model is that if investors are able to submit an entire demand curve to an auctioneer, then they should also be able to submit demand curves based on their expectations of what an asset is worth *as a function of* the market-clearing price. In other words, if their demands are a function of price, then their beliefs can also be a function of price, which, in turn, can affect their demands.[15] Market equilibria in which investors condition their expectations over the price at which markets clear are dubbed "rational expectations" models of trade.

An intuitive way to distinguish between a Walrasian and a rational expectations model of trade is to imagine first a price setting process under Walras. Here, investors determine their demand for an asset based on their tolerance for risk, information about what the asset is worth, and other preference characteristics. Then they submit their demand curves to an auctioneer, who determines the price at which the supply of the asset equilibrates against the aggregate demand. Now suppose that the auctioneer calls out the market-clearing price she determines. In Walras, nothing more would happen - this price would be the price at which trades are executed. In a rational expectations equilibrium, however, investors would start grumbling "well, had I known in advance that the market-clearing price was to be the one that was ultimately called out, then I would have changed my beliefs accordingly, and submitted a different demand curve." Presumably this grumbling would abrogate the equilibrium, and the auctioneer would be compelled to allow investors to submit a

[15]See Grossman [1976, 1978].

second round of demand curves, based on their revised beliefs. Now imagine a second round of trade in which a different market clearing price is called out, and once again investors grumble that had they known that this revised price was to be the market clearing price, they would have submitted yet another set of demand curves. And so on and so forth, until eventually the auctioneer calls out a price, and at that price no investor has any desire to recontract (e.g., they would cease grumbling). The price at which investors have no further interest in recontacting would be the *rational expectations* market clearing price. Stated somewhat differently, in Walras' setting the market-clearing price of an asset is a function of investors' expectations, but not the reverse, whereas in a rational expectations equilibrium, price is a function of expectations and expectations are a function of price. Note that the expression "rational expectations" to describe models in this genre of literature is somewhat misleading in that these models simply introduce as a modeling innovation the requirement that investor agents condition their expectations on market-clearing prices. Perhaps as an alternative one should dub this research "price-conditioned" models of trade.

"Trading-up" from a Walrasian to a rational expectations model of trade requires some additional analysis. In particular, the key feature of a rational expectations equilibrium is that investors conjecture that the market-clearing price of the asset contains information about what the asset is worth. Consequently, when investors condition their expectations over price, in addition to their private information, they glean more insight into the asset's uncertain value than had they ignored price. Here, I continue with all the assumptions introduced previously, but, in addition, assume that investors conjecture that the market equilibrium price at time T is of the form

$$\tilde{P}_T = a + b\tilde{u} + c\tilde{y} - d\tilde{x},$$

where a, b, and c are fixed parameters. Define $\tilde{q}$ as

$$\tilde{q} = \frac{\tilde{P}_T - a - c\tilde{y}}{b} = \tilde{u} - \frac{d}{b}\tilde{x}.$$

The variable $\tilde{q}$ represents the additional information investors glean from price by manipulating it to yield the essential information about $\tilde{u}$. When investors use $\tilde{q}$ in conjunction with $\tilde{u}$, $\tilde{y}$, and $\tilde{z}_i$, a quarto-variate normal distribution results with means of (m, m, m, m) and a covariance matrix given by

$$\begin{bmatrix} h^{-1} & h^{-1} & h^{-1} & h^{-1} \\ h^{-1} & h^{-1}+n^{-1} & h^{-1} & h^{-1} \\ h^{-1} & h^{-1} & h^{-1}+s^{-1} & h^{-1} \\ h^{-1} & h^{-1} & h^{-1} & h^{-1}+\left(\frac{d}{b}\right)^2 t^{-1} \end{bmatrix}.$$

Consequently, when investors condition their expectations over disclosure, their private information, and price (through q) as an additional source of information, their expectations are

$$E[\tilde{u}|y,\ z_i,\ q] = \frac{hm + ny + sz_i + \left(\frac{b}{d}\right)^2 tq}{h + n + s + \left(\frac{b}{d}\right)^2 t},$$

and the precision of their expectations, $(VAR[\tilde{u}|y, z_i, q])^{-1}$, is

$$(VAR[\tilde{u}|y, z_i, q])^{-1} = h + n + s + \left(\frac{b}{d}\right)^2 t.$$

To determine a value for P_T, once again first I must compute each investor's demand for D_i. As before, the negative exponential utility function yields a result that is linear in the argument of the exponential: that is,

$$\begin{aligned} & E[U\left(D_i(u - P_T) + x_i P_T + b_i\right)|y, z_i, q] \\ = \ & -\exp[-\frac{1}{r}D_i E[u|y, z_i, q] + \frac{1}{2r^2}D_i^2 VAR[u|y, z_i, q] + \frac{1}{r}D_i P_T - \frac{1}{r}x_i P_T - \frac{1}{r}b_i]. \end{aligned}$$

In determining his optimal portfolio, each investor chooses D_i so as to maximize the above. This yields

$$D_i = r\frac{E[u|y, z_i, q] - P_T}{VAR[u|y, z_i, q]},$$

which is the same expression as before except for the fact that now investors are conditioning their expectations on price (through q) in addition to y and z_i. Consequently, D_i can be rewritten as

$$D_i = r\left(hm + ny + sz_i + \left(\frac{b}{d}\right)^2 tq - \{h + n + s + \left(\frac{b}{d}\right)^2 t\}P_T\right).$$

As before, I endogenize P_T by equating the per-capita supply of the uncertain-valued asset with the per-capital demand: in other words, by setting $x = \sum_i (x_i/N) = \sum_i (D_i/N)$. When one does this, the value of P_T that results is

$$\begin{aligned} P_T &= \frac{1}{h + n + s + \left(\frac{b}{d}\right)^2 t}\left(hm + n\tilde{y} + \left(\frac{b}{d}\right)^2 t\tilde{q} + s\lim_{N\to\infty}\frac{1}{N}\sum_i \tilde{z}_i - \frac{1}{r}\tilde{x}\right) \\ &= \frac{1}{h + n + s + \left(\frac{b}{d}\right)^2 t}\left(hm + n\tilde{y} + \left(s + \left(\frac{b}{d}\right)^2 t\right)\tilde{u} - \left(\frac{1}{r} + \frac{b}{d}t\right)\tilde{x}\right). \end{aligned}$$

Note that for investors' original conjecture that $P_T = a + b\tilde{u} + c\tilde{y} - d\tilde{x}$ to be self-fulfilling (i.e., rational), it must be that

$$\frac{b}{d} = \frac{s + \left(\frac{b}{d}\right)^2 t}{\frac{1}{r} + \frac{b}{d}t},$$

which implies $\frac{b}{d} = rs$. Hence, a self-fulfilling equilibrium can be characterized by the coefficients a, b, c, and d in the expression $P_T = a + b\tilde{u} + c\tilde{y} - d\tilde{x}$ assuming the following forms: $a = \frac{hm}{h+n+s+(rs)^2 t}$; $b = \frac{s+(rs)^2 t}{h+n+s+(rs)^2 t}$; $c = \frac{n}{h+n+s+(rs)^2 t}$; and $d = \frac{\frac{1}{r}+rst}{h+n+s+(rs)^2 t}$. This, in turn, implies

$$P_T - P_{T-1} = \frac{1}{h + n + s + r^2 s^2 t}\left(n(\tilde{y} - m) + \left(s + r^2 s^2 t\right)(\tilde{u} - m) - \left(\frac{1}{r} + rst\right)\tilde{x}\right).$$

Note that this expression for the change in price is identical to the previous case, except for the additional information related to conditioning expectations over price. In effect, conditioning expectations over price creates an additional "information kick" that results in more precise beliefs in the rational expectations model than in the Walrasian model. Specifically, the precision of expectations in the former is $h+n+s+r^2s^2t$, and in the latter $h+n+s$. This implies that the "information kick" is r^2s^2t.

Here, the DRC also reflects the additional information gleaned from price: specifically, the DRC is $\frac{n}{h+n+s+r^2s^2t}$. The DRC is lower than in the Walrasian case because investors rely in part on price in a rational expectations model, and hence rely correspondingly less on disclosure. In addition, note that in this model $VAR[\tilde{P}_T - \tilde{P}_{T-1}|x] = \left(\frac{1}{h+n+s+r^2s^2t}\right)^2 \left(n^2\left(\frac{h+n}{hn}\right) + 2n\left(s+r^2s^2t\right)\frac{1}{h} + \left(s+r^2s^2t\right)^2\frac{1}{h}\right)$ and $VAR[\tilde{P}_T - \tilde{P}_{T-1}|y,x] = \left(\frac{s+r^2s^2t}{h+n+s+r^2s^2t}\right)^2\left(\frac{1}{h+n}\right)$. Consequently, here the $\Delta\%$-statistic reduces to

$$\begin{aligned}\Delta\% &= 1 - \frac{\frac{\left(s+r^2s^2t\right)^2}{h+n}}{n^2\left(\frac{h+n}{hn}\right) + 2n\left(s+r^2s^2t\right)\frac{1}{h} + \left(s+r^2s^2t\right)^2\frac{1}{h}} \\ &= \frac{n^2\left(\frac{h+n}{hn}\right) + 2n\left(s+r^2s^2t\right)\frac{1}{h} + \left(s+r^2s^2t\right)^2\frac{n}{h(h+n)}}{n^2\left(\frac{h+n}{hn}\right) + 2n\left(s+r^2s^2t\right)\frac{1}{h} + \left(s+r^2s^2t\right)^2\frac{1}{h}}.\end{aligned}$$

As in the Walrasian case, the $\Delta\%$-statistic in the rational expectations model is less than 1. The $\Delta\%$-statistic in the rational expectations case, however, is lower than in the Walrasian case (I leave this as an exercise for the interested reader). This suggests that the additional information about the asset gleaned from price in a rational expectations model implies less reliance on disclosure, and hence less variability in price change at time T explained by disclosure at time T.

Before proceeding to the next model, let me mention the role of two more maintained assumptions. Among the various conjectures that investors could make about the market equilibrium price at time T, a maintained assumption in the "rational ex-

pectations" literature is that investors make linear conjectures about the functional form of the market clearing price: that is, $\tilde{P}_T = a + b\tilde{u} + c\tilde{y} - d\tilde{x}$. This in no way precludes, or rules out, the possibility that there exist other, nonlinear conjectures that also lead to self-fulfilling equilibria. These alternative conjectures are simply not studied. Note that this restriction to linear conjectures is not unique to this literature. Models of imperfect competition, which are discussed below, are also premised on linear conjectures about the functional form of price.[16]

Another maintained assumption in model #3 is that investors have diverse private information. A competing model to the one discussed here is one in which investors are only one of two types: informed, and uninformed who glean some knowledge by conditioning their beliefs on price.[17] In the latter model price is only a *communicator* of information from the informed to the uninformed. Alternatively, in model #3 price is both an *aggregator* of information in that price aggregates the diverse beliefs of many investors (as manifest in the z_i), and a communicator of this aggregated data.

Rational Anticipation of Disclosure (model #4).[18] While allowing rational inferences from prices appears to be a clear improvement over the Walrasian model at relatively little cost in tractability, arguably there is yet another flaw. The flaw is that as the market setting was described above, there is no prior round of trade that allows market participants to resolve their differences (e.g., differences in risk preferences, differences in endowments, differences in private beliefs) prior to disclosure. Resolving differences through a prior round of trade is crucial to an association study, because without it a host of other factors unrelated to disclosure are commingled into the

[16]See, for example, the discussion of model #8 below.
[17]See, for example, Grossman and Stiglitz [1980] and Demski and Feltham [1994].
[18]In conjunction with model #4, see Grundy and McNichols [1989] and Brown and Jennings [1989]. Note that in Grundy and McNichols investors' preannouncement information structure consists of a common prior, private information with common error, and idiosyncratic errors with identical precision, whereas here the common error term is ignored.

change in price at the time of disclosure. For example, imagine a setting in which market participants enter a market at the beginning of the period to exchange shares of an asset and, based on their demands, a market-clearing price is established at the end of the period. In addition, imagine that as they enter the market, there is some public disclosure about the asset's value. In this scenario the end-of-period market price commingles different risk preferences, different endowments of the risky asset, and different private beliefs, along with disclosure. Consequently, here it is difficult to infer conclusively the effect of disclosure on price, separate and apart from all the other reasons why participants trade.

The way around this problem is to first allow market participants some prior round of trade before disclosure occurs, and then have a second round when disclosure occurs. It is important to require, however, that in the first round of trade market participants anticipate disclosure in the second round. The advantage of this approach is that any price change that results from the second round of trade represents conclusively the effect of disclosure on prices and price changes. The problem is that it is technically very difficult to allow for two rounds of trade and also satisfy all the other rationality criteria discussed above.

To illustrate some of these issues, consider the following proposal. Let us imagine that a prior round of trade in the asset takes place at time $T-1$, and the disclosure of $\tilde{y} = y$ occurs at time T. In a rational expectations model, investors are expected to learn from prices: that is, condition their expectations over prices. In two rounds of trade, in principle investors at time T should be able to condition their expectations over prices at both times $T-1$ and T. But in a rational expectations model of trade, prices at times $T-1$ and T could be described as being of the form $\tilde{P}_{T-1} = a_{T-1} + b_{T-1}\tilde{u} - d_{T-1}\tilde{x}$ and $\tilde{P}_T = a_T + b_T\tilde{u} + c_T\tilde{y} - d_T\tilde{x}$. In addition, as all fixed

parameters are assumed to be common knowledge in rational trading models (i.e., a_{T-1}, a_T, b_{T-1}, etc.), $\tilde{P}_{T-1}$ and $\tilde{P}_T$ represent a system of two simultaneous equations in two unknowns, $\tilde{u}$ and $\tilde{x}$. Consequently, if per-capita supply is the same at both times $T-1$ and T (i.e., $\tilde{x} = x$ at both times $T-1$ and T), then either $\tilde{P}_{T-1}$ and $\tilde{P}_T$ fully reveal $\tilde{u}$ and $\tilde{x}$ or $\tilde{P}_{T-1}$ and $\tilde{P}_T$ are redundant. The former occurs if $\tilde{P}_{T-1}$ and $\tilde{P}_T$ are independent equations, while the latter occurs if $\tilde{P}_{T-1}$ and $\tilde{P}_T$ are dependent equations (i.e., $a_{T-1} = a_T$, $b_{T-1} = b_T$, etc.). For example, $\frac{\tilde{P}_{T-1} - a_{T-1}}{b_{T-1}} = \tilde{u} - \frac{d_{T-1}}{b_{T-1}}\tilde{x}$ and $\frac{\tilde{P}_T - a_T - c_T\tilde{y}}{b_T} = \tilde{u} - \frac{d_T}{b_T}\tilde{x}$. Thus, if $\frac{d_{T-1}}{b_{T-1}} \neq \frac{d_T}{b_T}$ then $\tilde{P}_{T-1}$ and $\tilde{P}_T$ fully reveal $\tilde{u}$ and $\tilde{x}$. Alternatively, if $\frac{d_{T-1}}{b_{T-1}} = \frac{d_T}{b_T}$ then $\tilde{P}_{T-1}$ and $\tilde{P}_T$ are redundant.

While both a fully revealing and price-redundant equilibria are possible, depending upon investors' conjectures, the advantage of focusing exclusively on the latter is that there is little evidence that prices "fully reveal" an asset's value in real institutional settings. More significantly, the price-redundant equilibrium can be shown to be the generic equilibrium.[19]

In the context of our assumptions, one can show that allowing investors to trade in a prior period yields the following expression for price at time $T-1$

$$P_{T-1} = \frac{1}{h + s + r^2 s^2 t}\left(hm + \left(s + r^2 s^2 t\right)\tilde{u} - \left(\frac{1}{r} + rst\right)\tilde{x}\right).$$

To digress briefly, this expression for P_{T-1} is highly reminiscent of the one for price at time T in the previous model (model #3), except for the fact that it does not include disclosure (i.e., $\tilde{y}$). In other words, except for disclosure, P_{T-1} in this model is the same expression as P_T in model #3. Despite the similarity, note that P_T in model #3 results from investors behaving myopically in the sense of failing to anticipate disclosure at time T; alternatively, P_{T-1} in this model evolves endogenously, and *is*

[19]See the discussion in appendix A1 of Kim and Verrecchia [1991b].

based on investors rationally anticipating disclosure at time T. Continuing, one can show that the price of the asset at time T is

$$P_T = \frac{1}{h+n+s+r^2s^2t}\left(hm + n\tilde{y} + \left(s+r^2s^2t\right)\tilde{u} - \left(\frac{1}{r}+rst\right)\tilde{x}\right).$$

Consequently, after some algebraic manipulation, I get

$$P_T - P_{T-1} = \frac{n}{h+n+s+r^2s^2t}\left(y - \frac{hm + (s+r^2s^2t)\,\tilde{u} - \left(\frac{1}{r}+rst\right)\tilde{x}}{h+s+r^2s^2t}\right).$$

An interpretation of the expression

$$y - \frac{hm + (s+r^2s^2t)\,\tilde{u} - rst\tilde{x}}{h+s+r^2s^2t}$$

is that it is the disclosure "surprise" in price change and the expression

$$\frac{\frac{1}{r}\tilde{x}}{h+s+r^2s^2t}$$

is the "noise."

To digress briefly, the significance of two periods is that it allows one to study the change in the behavior of price coincident with a disclosure (e.g., an earnings announcement). As for the assumption that the level of noise is the same in both periods (i.e., $\tilde{x} = x$ in periods $T-1$ and T), recall that $\tilde{x}$ represents liquidity and/or asset supply shocks. Hence, one could interpret this assumption as suggesting that there is a sustained level of liquidity and/or supply shock activity surrounding an earnings announcement (i.e., immediately before and after). The primary role of this assumption, however, is convenience and transparency; generalizations to more complex settings are straightforward.[20]

Continuing with our discussion of this model, note that the DRC is the same in both the rational anticipation and non-anticipation-of-disclosure models, specifically,

[20]See, for example, He and Wang [1995].

$\frac{n}{h+n+s+r^2s^2t}$. Despite this, the $\Delta\%$-statistic for the rational anticipation model can be shown to be 1. Initially, this result may seem surprising, but good economic intuition suggests why. Recall that in calculating our $\Delta\%$-statistic, I hold constant the price of the asset at time $T-1$ (i.e., $\tilde{P}_{T-1}$). Consequently, if investors trade to an equilibrium at time $T-1$ in anticipation of disclosure at time T, then price at time $T-1$ accounts for all the variability in our model *except* for disclosure at time T. Thus, disclosure at time T explains *all* the variability of price. In other words, the fact that the $\Delta\%$-statistic is 1 points out one of the compelling features of the rational-anticipation-of-disclosure model: all the variability in price change arises exclusively from disclosure (controlling for the behavior of price at time $T-1$).

Before proceeding to the next model, note that a maintained assumption is that private information is information about the uncertain asset's value (i.e., $\tilde{z}_i = \tilde{u} + \tilde{\varepsilon}_i$) and not private information forecasts of the disclosure (e.g., $\tilde{z}_i = \tilde{y} + \tilde{\varepsilon}_i$). It is a straightforward exercise to adapt the model presented here to allow for private information forecasts of the disclosure.[21] To preserve continuity in our discussion, however, I stay with the former.

Private Information of Heterogeneous Quality (model #5).[22] The model developed to this point has many attractive features. Investors have rational expectations in the sense that they condition their expectations over prices and in the sense that they anticipate the disclosure by establishing an equilibrium in advance of its public dissemination. The problem is that there is no trading volume at time T, when disclosure occurs. Consequently, one could argue that by insisting on a "conceptually correct" model of trade, I have lost sight of the objective of the exercise.

[21]See Abarbanell *et al.* [1995] for a paper that incorporates private information forecasts of future disclosure in a model similar to the one discussed here.
[22]In conjunction with model #5, see Kim and Verrecchia [1991a, 1991b].

The reason for the absence of trade is that investors have what has been dubbed "concordant beliefs," in combination with the fact that the asset allocation achieved in the prior round of trade (at time $T-1$) is *ex ante* Pareto efficient. In models with these features, public disclosure generates no trade.[23] The intuition underlying this result is that at time $T-1$ investors achieve asset portfolios that align their beliefs to the price of those assets. Consequently, at time T, disclosure shifts price, but it preserves the alignment because price and investors' beliefs move in parallel. For example, if an investor's valuation of what an asset is worth relative to the price at which it sold is some value at time $T-1$, disclosure at time T shifts beliefs and prices, but in a fashion that preserves that value. Consequently, there is no incentive to trade at time T.

This returns us to the role of two of our maintained assumptions. Homogeneous precision of private information across investors ensures e*x ante* Pareto efficiency in the prior round of trade and the negative exponential utility function ensures concordant beliefs. Consequently, if we continue to maintain these assumptions, we have reached the proverbial end-of-the-road: all our efforts have led us to a world in which there is no role for disclosure. There is no role for disclosure because there is no incentive to trade at time T. If we are reluctant to abandon the negative exponential because of its obvious tractability, one device to ensure that disclosure has a role is to assume that allocations are not *ex ante* Pareto efficient at time $T-1$. This is achieved by assuming that the precisions of investors' private information are heterogeneous. For example, it is sufficient to assume that there exists some investors i and j such that the precisions of their private information, s_i and s_j, have the feature that $s_i \neq s_j$. Consequently, henceforth my maintained assumption is that for some

[23]See, for example, the discussion in Milgrom and Stokey [1982]; see also Wilson [1968].

investors i and j, $s_i \neq s_j$.

To digress briefly, one should be clear about what one is doing here. In general, heterogeneous precisions are not a requirement to achieve trade at time T: it simply happens that the constant risk tolerance feature of the negative exponential forces this requirement. But this means that *heterogeneous* precisions in conjunction with the negative exponential utility function should be interpreted properly as a proxy for utility preferences that are more general than the negative exponential, and not a strict requirement for trade, *per se*.

The shift from homogeneous to heterogeneous precisions does not affect the characterization of price change discussed above in model #4, provided that one now interpret the expression for s in the previous price change equation above as the *average* precision across investors: that is, $s = \lim_{N\to\infty} \frac{1}{N} \sum_i s_i$. What heterogeneous precisions do allow, however, is a characterization of trading volume at time T. Specifically, the (per-capita) trading volume that results when there exists some $s_i \neq s_j$ is

$$Volume = \frac{1}{2} \left(\lim_{N\to\infty} \frac{1}{N} \sum_i r|s_i - s| \right) |P_T - P_{T-1}|,$$

where, once again, $s = \lim_{N\to\infty} \frac{1}{N} \sum_i s_i$. For example, to see the effect of heterogeneous precisions, note that volume is 0 when $s_i = s_j$ for all i and j.[24]

In effect, the compelling feature of a model of trade with private information of heterogeneous quality is that it results in an expression for trading volume that

[24]This also points up the major difference between Kim and Verrecchia [1991a] and Grundy and McNichols [1989], KV and GM, respectively. In GM investors' preannouncement information structure consists of a common prior and private information with common error and idiosyncratic errors with identical precision: that is, $s_i = s_j$ for all i and j. Alternatively, in KV there is no common error, but the idiosyncratic errors have *heterogeneous* precisions. This explains why in GM there is no trade in the partially revealing equilibrium: the precisions of all investors are homogeneous. For this reason GM focus on the fully-revealing equilibrium (not the price redundant equilibrium discussed here) in which investors observe the market price and correct their idiosyncratic errors; this, in turn, results in trade.

is the product of the average, absolute-value difference in the quality of investors' private information, i.e., $\frac{1}{2}\left(\lim_{N\to\infty}\frac{1}{N}\sum_i r|s_i - s|\right)$, and absolute-value price change, i.e., $|P_T - P_{T-1}|$. Among other things, this relation explains the positive association between trading volume and absolute value price change commonly cited in the literature.[25] The relation itself is very intuitive in that it suggests that trading volume is the product of the extent to which investors hold diverse opinions at an idiosyncratic level through their heterogeneous, private precision s_i weights, and the extent to which these opinions change *on average* at the time of disclosure through $P_T - P_{T-1}$. But, as I discuss below, a problem remains.

Heterogeneous Interpretations of a Common Disclosure (model #6).[26] A maintained assumption throughout the analysis has been that investors interpret disclosure in some common fashion. One artifact of common interpretations of disclosure is that the characterization for trading volume implied by the previous equation suggests that volume is related to absolute value price change through the coefficient $\frac{1}{2}\left(\frac{1}{N}\sum_i r|s_i - s|\right)$, but with a zero intercept. For example, $Volume = \alpha + \beta\,|P_T - P_{T-1}|$, where $\beta = \frac{1}{2}\left(\frac{1}{N}\sum_i r|s_i - s|\right)$ and $\alpha = 0$.

A zero intercept implies that trading volume cannot arise in the absence of price change. But *this* relation has been criticized by those who claim that empirically volume arises even in the absence of price change.[27] So the question now is: how might one extend the model further to address this concern? In other words, how might it be possible to characterize trading volume in the absence of price changes?

One way to extend the model to incorporate the possibility of volume even in the absence of price changes is to allow investors to interpret disclosure diversely.

[25] See, for example, Karpoff [1987].
[26] In conjunction with model #6, see Dontoh and Ronen [1993], Harris and Raviv [1995], Kandel and Pearson [1995], and Kim and Verrecchia [1997].
[27] See Kandel and Pearson [1995].

Accounting research has long debated the extent to which disclosure is interpreted similarly versus dissimilarly by market participants, and in the accounting literature there exists many characterizations of a common disclosure being interpreted diversely.[28] To incorporate that possibility of diverse interpretations, first recall that $\tilde{y} = \tilde{u} + \tilde{\eta}$. Suppose that in addition to private information about $\tilde{u}$ directly through $\tilde{z}_i$, investors also possess private information about $\tilde{\eta}$, in the form of $\tilde{O}_i = \tilde{\eta} - \tilde{\omega}_i$, where the $\tilde{\omega}_i$'s have a normal distribution with mean 0 and precisions w_i. Institutionally, $\tilde{O}_i$ can be thought of as the information an investor gleans by studying the error in disclosures, where the error arises from the application of random, liberal, or conservative accrual-based accounting practices and estimates. When there is disclosure, this information can then be used to partially correct for the error.

When diverse interpretations of a common disclosure are added to our previous assumptions, expected trading volume can be represented now by

$$E[Volume|u, P_T, P_{T-\mathbf{1}}] = \frac{1}{2} \lim_{N\to\infty} \frac{1}{N} \sum_i r \left|(w_i - w)(u - P_T) + (s_i - s)(P_{T-\mathbf{1}} - P_T)\right| + R,$$

where R is a (positive) residual term.[29] For example, consider the case where there is no price change, that is, $P_T = P_{T-\mathbf{1}}$. For this model specification expected trading volume arises despite the absence of price changes. Specifically, when $P_T = P_{T-\mathbf{1}}$ expected volume reduces to

$$E[Volume|u, P_T, P_{T-\mathbf{1}}] = \frac{1}{2} \lim_{N\to\infty} \frac{1}{N} \sum_i r \left|(w_i - w)(u - P_T)\right| + R,$$

an expression that is always positive. In short, this model suggests how volume may arise in the absence of price changes. It must be acknowledged, of course, that while this model characterizes trading volume in the absence of price changes, it lacks

[28]See, for example, discussions in Holthausen and Verrecchia [1990], Indjejikian [1991], and Kim and Verrecchia [1994].
[29]See the discussion on pp. 408-413 of Kim and Verrecchia [1997].

the more transparent and elegant relation between price change and trading volume suggested in model #5, $Volume = \frac{1}{2}\left(\lim_{N\to\infty}\frac{1}{N}\sum_i r|s_i - s|\right)|P_T - P_{T-1}|$. Suffice it to say that at this juncture the specification for the relation between price change and volume in model #5 is a benchmark routinely used in empirical studies.[30]

To summarize our efforts to this point, so far many elements of rationality and investor diversity have been incorporated into the analysis. But it could be argued also that whatever has been accomplished has only been at the expense of a very parochial view of "investor diversity." Therefore, I explore this issue in the next section.

Heuristic Behavior (model #7).[31] A maintained assumption throughout our analysis has been that all investor agents who participate in the market use whatever information is at their disposal, either private or public, in accordance with Bayes rule. But is this reasonable? Does this superimpose onto the analysis an element of rationality that no one would expect each and every investor agent to achieve in all cases? In theory-based, economic analyses, reliance on Bayes rule is so routinized an assumption as rarely to warrant any justification. The compelling feature of Bayes rule is that it implies the most efficient use of information. Consequently, in market settings, investors who use information more efficiently (i.e., Bayesians) should be able to exploit and dominate their less efficient counterparts. In addition, even if this were not the case, it could be argued that Bayesian behavior captures well the behavior of market participants at an aggregate level, where individual, idiosyncratic departures from Bayes rule cancel out "on average." In other words, while strict reliance on Bayes rule by everyone may seem a little far-fetched, one might expect that behaviors averaged over many people approximate Bayesian behavior.

[30]See, for example, Atiase and Bamber [1994].
[31]In conjunction with model #7, see DeLong, *et al.* [1990].

Recently, however, a fashionable element of contemporary research in finance has allied itself with studies in the psychology literature, and called into question the degree to which markets participants adhere to Bayes rule in real market settings.[32] The Bayes-rule-doubters point to the wealth of empirical evidence that market prices sometimes appear to overreact to events, and sometimes underreact (e.g., the post-announcement drift phenomenon). While there may be a host of reasons why markets behave in ways that defy rational economic analysis, investors' inability to apply correctly Bayes rule explains all manner of anomalous behavior. Consequently, this may be an opportune time to assess the role of this assumption.

The major difficulty with substituting some heuristic use of information for Bayes rule is that potentially it explains everything, which, in turn, suggests that it explains nothing. For example, price underreactions are explained easily by a class of investors who are anchored to their prior beliefs. Alternatively, overreactions are explained easily by a class of investors who place more weight on the most recent information stimulus than can be justified under Bayes Rule. In this environment, what "ground rules" should we require in exploring the possibility of heuristic behaviors? I argue that one rule should be that a heuristic behavior be survivable. There are market settings where this can happen. That is, in some market settings there may be advantages to heuristic behaviors that offset the fact that failure to adhere to Bayes rule means that heuristic investors use information less efficiently than their Bayesian counterparts (on average). But in the absence of demonstrating conclusively that a heuristic behavior can survive in competition against Bayes rule, the safest course may be to continue to assume that market participants use information in accordance with Bayes rule.

[32]See, for example, Thaler [1993].

To illustrate some of these points in the context of our discussion, let me return to model #4 to incorporate the possibility of some measurable set of investors behaving heuristically. As the development of the following model is somewhat longer than those already discussed, let me briefly point out its motivation. First, it demonstrates that it is difficult to reconcile heuristic behavior with survivability in models of perfect competition. Second, it is useful for simply illustrating issues related to the introduction of heuristic behavior into models of (otherwise rational) trade.

To start, imagine an economy in which a fraction π of investors are heuristic and a fraction $1-\pi$ are Bayesians, where $0 < \pi < 1$. To keep the discussion simple, I assume that neither type possesses any private information and each is equivalently endowed: that is, $s = 0$, and $x_i \equiv x$ and $b_i \equiv b$ for all investors i. As purely a modeling element, the introduction of heuristic behavior introduces subtle issues concerning the extent to which heuristic investors are rational (and/or Bayesian) versus the extent to which they are heuristic. Specifically, in the context of rational models of trade, heuristic behavior presupposes some element of schizophrenia on the part of heuristic investors in that it requires that they combine some elements of rational (and/or Bayesian) behavior along with some elements of heuristic behavior. To address these issues and for the sake of simplicity, I assume that heuristic investors are rational/Bayesian in all regards *except* for the fact that based on a disclosure y, the Bayesian investor's expectation of firm value is $E[\tilde{u}|y] = m + \frac{n}{h+n}(y - m)$, which is the correct statistical valuation, whereas the heuristic investor's expectation of firm value is $E_H[\tilde{u}|y] = m + \frac{n}{h+n}(y - m)$. This characterization of heuristic behavior suggests that when $\theta > 1$ heuristic investors "overreact" to the disclosure relative to the unconditional mean of $\tilde{u}$, which is m, whereas $\theta < 1$ suggests that they "underreact." Despite the potential over- or underreaction on the part of heuristic investors, I assume that

both types continue to assess posterior variances correctly: that is, for both investor types $VAR[\tilde{u}|y] = (h+n)^{-1}$. Let me emphasize that this characterization of heuristic behavior is only one of many possible ways to illustrate non-Bayesian behavior.

Using model #4 as a benchmark and assuming $s = 0$, one can show that the price for the asset at time $T-1$ is $P_{T-1} = m - \frac{1}{rh}\tilde{x}$ and at time T is $P_T = \frac{1}{h+n}\left(hm + n\tilde{y} + \pi n\left[\tilde{y} - m\right]\left[\theta - 1\right] - \frac{1}{r}\tilde{x}\right)$, and, hence, the expression for price change is

$$P_T - P_{T-1} = \frac{n}{h+n}\left(\left(\pi\theta + 1 - \pi\right)\left(\tilde{y} - m\right) + \frac{1}{rh}\tilde{x}\right).$$

Here the DRC is $\frac{n}{h+n}\left(\pi\theta + 1 - \pi\right)$. Moreover, the DRC is greater than (less than) the coefficient with exclusively Bayesian investors when $\theta > (<) 1$. In other words, if heuristic traders "overreact" ("underreact") to the disclosure, price change will be more (less) reliant on disclosure.

The only problem with *this* model as a characterization of heuristic behavior is that because the market is perfectly competitive, heuristic investors will always do worse than Bayesian investors. First I show this and then I discuss the intuition underlying this observation. Using the analysis introduced previously, a heuristic investor's demand for the asset is

$$D_H = r\frac{E_H[\tilde{u}|y] - P_T}{VAR[\tilde{u}|y]} = -rn\left(1-\pi\right)\left(1-\theta\right)\left(y-m\right) + x.$$

In comparing this expression for demand to the one derived above in model #4, note that one implication of the assumption that $s = 0$ is that investors no longer benefit from conditioning their expectations over price because price does not aggregate private information. Alternatively, a Bayesian investor's demand for the asset is

$$D_B = r\frac{E[\tilde{u}|y] - P_T}{VAR[\tilde{u}|y]} = rn\pi\left(1-\theta\right)\left(y-m\right) + x.$$

As a check that these demand characterizations are correct, note that $\pi D_H + (1-\pi) D_B = x$, which is what one would expect: total per-capita demand equals total per-capita supply.

Now consider the respective expected utilities of the heuristic and Bayesian investors at time T. Regardless of how the heuristic investor evaluates the disclosure y, the correct statistical valuation of $\tilde{u}$ conditional on y is $E[\tilde{u}|y]$. This implies that based on a disclosure of y, the heuristic investor's expected utility (correctly evaluated) is

$$\begin{aligned} & E[U\left(D_H(\tilde{u}-P_T)+xP_T+b\right)|y] \\ = & -\exp[-\frac{1}{r}D_H E[\tilde{u}|y]+\frac{1}{2r^2}D_H^2 VAR[\tilde{u}|y]+\frac{1}{r}D_H P_T-\frac{1}{r}xP_T-\frac{1}{r}b], \end{aligned}$$

whereas a Bayesian investor's expected utility is

$$\begin{aligned} & E[U\left(D_B(\tilde{u}-P_T)+xP_T+b\right)|y] \\ = & -\exp[-\frac{1}{r}D_B E[\tilde{u}|y]+\frac{1}{2r^2}D_B^2 VAR[\tilde{u}|y]+\frac{1}{r}D_B P_T-\frac{1}{r}xP_T-\frac{1}{r}b]. \end{aligned}$$

A Bayesian investor's expected utility is higher than that of a heuristic investor if the argument in the exponential of a Bayesian investor's expected utility is lower than that of a heuristic investor, which happens if

$$-\frac{1}{r}D_H E[\tilde{u}|y]+\frac{1}{2r^2}D_H^2\left(h+n\right)^{-1}+\frac{1}{r}D_H P_T > -\frac{1}{r}D_B E[\tilde{u}|y]+\frac{1}{2r^2}D_B^2\left(h+n\right)^{-1}+\frac{1}{r}D_B P_T;$$

this inequality can be reexpressed as

$$\left(E[\tilde{u}|y]-P_T\right)\left(D_B-D_H\right)+\frac{1}{2r}\left(D_H^2-D_B^2\right)\left(h+n\right)^{-1}>0.$$

Note, however, that

$$\left(E[\tilde{u}|y]-P_T\right)\left(D_B-D_H\right)+\frac{1}{2r}\left(D_H^2-D_B^2\right)\left(h+n\right)^{-1}=\frac{1}{2}r\frac{n^2}{h+n}\left(1-\theta\right)^2\left(y-m\right)^2,$$

and the expression $\frac{1}{2} r \frac{n^2}{h+n} (1-\theta)^2 (y-m)^2$ is positive for all $y \neq m$ and $\theta \neq 1$. But *this* implies that heuristic investors always do worse than Bayesian investors, and hence are unlikely to survive.

The intuition underlying this result is that in a perfectly competitive market no single investor's actions or demands affect price. In addition, Bayesian investors make statistically correct portfolio rebalancing decisions (on average) in the presence of disclosure, whereas heuristic investors make inferior portfolio rebalancing decisions. Consequently, over time Bayesian behavior should outperform heuristic behaviors, and, for this reason, presumably drive heuristic behaviors from the market. Of course, one device to ensure the survival of heuristic traders is to assume that these investors have private information that is superior to the information available to Bayesian investors. In this case the inferior use of information by heuristic traders is offset by their superior information. But endowing heuristic investors with superior private information is a bit of a dodge. The interesting question is: can they survive when they are as well informed as Bayesians?

Imperfect Competition (model #8).[33] While the result that heuristic behavior will not survive is certainly nice and tidy, it may be that the failure to demonstrate survivability is not a consequence of heuristic behavior, *per se*, but rather the fact that markets are assumed to be perfectly competitive. To explore this issue, first I digress and consider the alternative of *im*perfect competition.

A maintained assumption throughout the analysis is that markets are perfectly competitive. Markets may not be perfectly competitive, however, when the actions of some investors do indeed affect the price at which their trades are executed. One way to rationalize the possibility of an investor's actions affecting price institutionally is

[33] In conjunction with model #8, see Kyle [1985], Admati and Pfleiderer [1988a], Kyle [1989], Kim and Verrecchia [1994], Trueman and McNichols [1994], and Marzano [1999].

to imagine that the investor agents are large institutional traders whose actions drive markets. For example, imagine that the market is comprised of a large institutional investor and "the market," which represents, in effect, everyone else. For convenience, henceforth I assume that both the investor and "the market" are risk-neutral, with a utility for an amount of a consumable good g given by $U(g) = g$.[34] I continue to represent disclosure as $\tilde{y} = \tilde{u} + \tilde{\eta}$, but now assume that the large institutional investor (the investor, henceforth) knows $\tilde{\eta} = \eta$. As discussed in the context of model #6, a justification for knowing η is that the investor studies the firm's accounting practices and procedures well enough to understand the errors in disclosures that arise from the application of random, liberal, or conservative accrual-based accounting. Knowledge of $\tilde{\eta} = \eta$ in combination with $\tilde{y}$ implies that the investor knows $\tilde{u}$, the value of the firm. Alternatively, I assume that "the market" is not as astute about accounting practices and procedures as the investor, and consequently only knows $\tilde{y}$.

Imperfect competition implies that the investor knows that his actions will have an effect on the market price at which his trades are executed, and takes that into consideration in submitting demand orders. Consequently, the investor and "the market" play the following game. First, the investor determines the demand order he wants executed based on his knowledge of $\tilde{u}$. Then this demand order gets "batched" with the demand orders generated from random shocks in the supply of the asset, $\tilde{x}$. Finally, "the market" executes this combined demand order at a single price.

Let d represent the demand order of the investor, $\tilde{D} = d + \tilde{x}$ the total or combined demand orders of the investor and random supply shocks, and P the price set by "the

[34]While it would be a straightforward exercise to preserve the assumption that all agents have a utility for a consumable good represented by the negative exponential utility function and offer a discussion consistent with prior models, risk neutrality is a common assumption in the literature that this modeling vignette characterizes. Consequently, henceforth my maintained assumption about utility preferences is that all market agents are risk neutral.

market" for executing orders.[35] I assume that competition to execute demand orders forces demand orders to be executed at a price that reflects the expected value of the asset conditional on what "the market" knows at the time the order is executed. At the time the order is executed, "the market" knows $\tilde{y}$ and the total demand order $\tilde{D}$. This implies that $P = E[\tilde{u}|\tilde{y}, \tilde{D}]$. The investor moves first in this game, and therefore must make some assumption about how "the market" will interpret a demand order of a particular size. I assume that the investor conjectures that the price set by "the market" based on a disclosure y and the submission of a total demand order D is

$$P = m + \beta(y - m) + \lambda D.$$

In effect, the price is a linear function of y and D. Once again, the coefficient β is the DRC, while λ is commonly interpreted as market depth.

The play of the trading game can be summarized through a series of chronological steps.

1) Firm value is realized; this is represented by $\tilde{u} = u$.

2) The variable $\tilde{y} = y$ is disclosed and the investor observes $\tilde{\eta} = \eta$.

3) The investor submits a demand order to "the market," which is combined with random supply shocks represented by $\tilde{x} = x$.

4) Based on the total demand order, "the market" sets the price at which trades are executed (i.e., "the market" picks P equal to the firm's expected value conditional on disclosure and total demand). All trades are then executed at that price.

5) The firm is liquidated, paying out a return to shareholders of u.

The equilibrium to this game could be thought to arise from steps 3) and 4), each of which is self-serving on the part of the individual who executes the step. For

[35]For notational convenience, henceforth I drop the "T" subscript in making reference to price: in effect, all subsequent models are treated as exclusively one-period models of trade.

example, in step 3) the investor determines his demand order d by solving

$$\max_d d \cdot E[u - \tilde{P} | \tilde{u} = u, \tilde{y} = y],$$

where he conjectures that $\tilde{P} = m + \beta(\tilde{y} - m) + \lambda \tilde{D}$. This implies that he solves

$$\max_d d \cdot E[u - m - \beta(y - m) - \lambda(d + \tilde{x}) | \tilde{u} = u, \tilde{y} = y],$$

which, in turn, implies

$$d = \frac{1}{2\lambda}(u - m - \beta[y - m]).$$

A consequence of the investor's choice of d is that $\tilde{u}, \tilde{y}$, and $\tilde{D} = \tilde{d} + \tilde{x}$ have a trivariate normal distribution with means of $(m, m, 0)$ and a covariance matrix given by

$$\begin{bmatrix} h^{-1} & h^{-1} & \frac{1}{2\lambda} h^{-1}(1-\beta) \\ h^{-1} & h^{-1} + n^{-1} & \frac{1}{2\lambda}(h^{-1} - \beta[h^{-1} + n^{-1}]) \\ \frac{1}{2\lambda} h^{-1}(1-\beta) & \frac{1}{2\lambda}(h^{-1} - \beta[h^{-1} + n^{-1}]) & \frac{1}{4\lambda^2}\left(h^{-1} - 2\beta h^{-1} + \beta^2[h^{-1} + n^{-1}]\right) + t^{-1} \end{bmatrix}.$$

In step 4) "the market" sets P conditional on the disclosure and the total demand order received. The covariance matrix given above implies that this results in the following relation

$$E[\tilde{u} | y, D] = m + \frac{4\lambda^2 n + t\beta}{4\lambda^2 n + t + 4\lambda^2 h}(y - m) + \frac{2\lambda t}{4\lambda^2 n + t + 4\lambda^2 h} D.$$

Note, however, that for the investor's original conjecture about β and λ to be fulfilled, it must be the case that $\beta = \frac{4\lambda^2 n + t\beta}{4\lambda^2 n + t + 4\lambda^2 h}$, and $\lambda = \frac{2\lambda t}{4\lambda^2 n + t + 4\lambda^2 h}$. This, in turn, implies $\beta = \frac{n}{h+n}$ and $\lambda = \frac{1}{2}\sqrt{\frac{t}{h+n}}$. In short, a self-fulfilling equilibrium is one in which the price at which demand orders are executed is given by

$$P = m + \frac{n}{h+n}(y - m) + \frac{1}{2}\sqrt{\frac{t}{h+n}} D,$$

where $\frac{n}{h+n}$ is the DRC and $\frac{1}{2}\sqrt{\frac{t}{h+n}}$ is market depth, respectively.

From a disclosure perspective, this equilibrium has a number of interesting features. First, unlike a model of perfect competition, the investor does not go infinitely long or infinitely short in the asset, even though he knows the asset's value (i.e., the investor knows $\tilde{u} = u$). The reason for this is that he must take into consideration the effect of his demand order on the price at which his demand order will be executed. The larger his demand order (i.e., the larger d), the more he expects it will cost to execute that order (i.e., the higher $E[\tilde{P}]$). For example, because $\lambda > 0$, when total demand is positive (i.e., $D > 0$) the investor has his trades executed at a higher price than that implied by "the market" knowing exclusively the disclosure y (i.e., $P = m + \frac{n}{h+n}(y - m)$). Another feature of his demand order, as well as the total demand order, is that it is uncorrelated with the disclosure: that is, $E[(\tilde{y} - m)\,\tilde{d}] = E[(\tilde{y} - m)\,\tilde{D}] = 0$. The intuition underlying this result is that the investor knows $\tilde{y}$ when he submits his demand order $\tilde{d}$, and knows that the information content of $\tilde{y}$ will be fully priced in $\tilde{P}$ when his demand order is executed because $\tilde{y}$ is public information. Consequently, he adjusts his demand order to take into account the effect of the disclosure on price, which is tantamount to ensuring that his demand order and the disclosure are uncorrelated. Finally, note that the DRC in this model is identical to the one that arose in the context of our model #1, a situation in which the only information in the economy was the information that arose directly from disclosure. The intuition for this is the DRC captures the effect of disclosure, while the coefficient on the total demand, λ, captures the incremental knowledge that arises from observing total demand, D, in addition to the disclosure.

Heuristic Behavior Revisited (model #9).[36] Having laid out the notion of imperfect competition, now I revisit heuristic behavior in the context of a model of

[36]In conjunction with model #9, see Palamino [1996], Kyle and Wang [1997], and Fischer and Verrecchia [1999].

this type. What I hope to show here is that when heuristic behavior is combined with imperfect competition, there is a possibility of a heuristic trader surviving in competition with a (rational) Bayesian investor. The rationale for this is that when an investor's actions affect price, placing more weight on disclosure than that implied by Bayes rule will drive prices in the direction of the disclosure. For example, in the presence of "good news," a heuristic investor who places more weight on disclosure than that implied by Bayes rule will drive prices up further on "good news" than could otherwise be justified by "correct" (i.e., Bayesian) valuation. But a (rational) Bayesian investor, knowing this, will reduce *his* demand because he seeks to avoid paying a price for the asset beyond the one implied by correct valuation. But in "backing off" part of his demand, a Bayesian, in turn, *reduces* price through his actions, and this accommodation to heuristic behavior allows heuristic traders to pay *less* for their greater share of the asset than they would otherwise be able to achieve. Consequently, even though heuristic traders use disclosure less capably than Bayesians, Bayesians may accommodate heuristic traders in a fashion that results in their heuristic motivated transactions being executed at more favorable prices. Here, it may be possible for heuristic and Bayesian investors to both survive, provided that the accommodation afforded by the latter perfectly offsets the decreased capabilities of the former as users of information.

To illustrate this point, consider an economy inhabited by two "large" institutional investors, one of whom is Bayesian and the other of whom is heuristic. By "large," I mean that both investors have tolerances for risk, financial resources, and/or reputations that enable them to take very significant positions in the market. In addition, as a by-product of being "large," I assume that both investor types are risk-neutral. The very significant positions these large investors take will, in turn, affect the price

at which they transact. For example, based on a disclosure of y, I assume that the price at which either large investor will have his or her trades for the asset executed is

$$P = m + \beta\,(y - m) + \lambda\,(d_H + d_B)\,,$$

where β and λ are fixed, positive coefficients, and d_H and d_B represent the demands of the heuristic and Bayesian investors, respectively. The β coefficient can be interpreted as the (exogenously specified) DRC and the λ coefficient can be interpreted as the sensitivity of the demands of the two large traders on the price at which all transactions are executed: in effect, market depth. For example, the latter implies that as λ increases, the price at which trades for the asset are executed increases when "large investor" net demand for the asset is positive (i.e., $d_H + d_B > 0$), and decreases when "large investor" net demand for the asset is negative.

Recall that based on a disclosure y, the Bayesian investor's expectation of firm value is $E[\tilde{u}|y] = m + \frac{n}{h+n}\,(y - m)$ and the heuristic investor's expectation of firm value is $E_H[\tilde{u}|y] = m + \frac{n}{h+n}\theta\,(y - m)$. As the heuristic investor values the firm at $E_H[\tilde{u}|y]$ based on the disclosure and must pay a price P, he chooses d_H to maximize the following objective function

$$d_H(E_H[\tilde{u}|y] - P) = d_H(E_H[\tilde{u}|y] - (m + \beta\,(y - m) + \lambda\,(d_H + d_B))).$$

This function is concave and maximized at

$$d_H = \frac{1}{2\lambda}(E_H[\tilde{u}|y] - m - \beta\,(y - m) - \lambda d_B).$$

Similarly, the Bayesian investor's choice of d_B is optimal at

$$d_B = \frac{1}{2\lambda}(E[\tilde{u}|y] - m - \beta\,(y - m) - \lambda d_H).$$

Solving for d_H and d_B, in equilibrium the heuristic and Bayesian investors choose, respectively,

$$\begin{aligned} d_H &= \frac{1}{3\lambda}\left(2E_H[\tilde{u}|y] - E[\tilde{u}|y] - m - \beta\left(y-m\right)\right), \text{ and} \\ d_B &= \frac{1}{3\lambda}\left(2E[\tilde{u}|y] - E_H[\tilde{u}|y] - m - \beta\left(y-m\right)\right). \end{aligned}$$

This, in turn, implies that in equilibrium

$$\begin{aligned} P &= m + \beta\left(y-m\right) + \lambda\left(d_H + d_B\right) \\ &= \frac{1}{3}\left(m + \beta\left(y-m\right) + E_H[\tilde{u}|y] + E[\tilde{u}|y]\right). \end{aligned}$$

Now consider the respective expected utilities of the heuristic and Bayesian investors. Regardless of how the heuristic investor evaluates the disclosure y, once again the correct statistical valuation of $\tilde{u}$ conditional on y is $E[\tilde{u}|y]$. This implies that based on a disclosure of y, the heuristic investor's expected utility (correctly evaluated) for having his trade executed is $EU_H\left(y\right) = d_H\left(E[\tilde{u}|y] - P\right)$, whereas the Bayesian investor's expected utility is $EU_B\left(y\right) = d_B\left(E[\tilde{u}|y] - P\right)$. Consequently, here the difference in the respective expected utilities of the heuristic and Bayesian investors can be shown to be

$$\begin{aligned} & EU_H\left(\tilde{y}\right) - EU_B\left(\tilde{y}\right) \\ = & \frac{1}{3\lambda}\left(E_H[\tilde{u}|y] - E[\tilde{u}|y]\right)\left(2E[\tilde{u}|y] - E_H[\tilde{u}|y] - m - \beta\left(y-m\right)\right) \\ = & \frac{1}{3\lambda}\left(\frac{n}{h+n}\right)\left(\theta-1\right)\left(\frac{n}{h+n}\left(2-\theta\right) - \beta\right)\left(y-m\right)^2. \end{aligned}$$

This expression is concave in θ and equals 0 at two points: $\theta = 1$ and $\theta = 2 - \left(\frac{h+n}{n}\right)\beta$. Assume that $2 - \left(\frac{h+n}{n}\right)\beta > 1$. This implies that for any θ in the interval $[1, 2 - \left(\frac{h+n}{n}\right)\beta]$ the heuristic investor does better than the Bayesian investor. Similarly, assume $2 - \left(\frac{h+n}{n}\right)\beta < 1$. This implies that for any θ in the interval $[2 - \left(\frac{h+n}{n}\right)\beta, 1]$ the heuristic

investor also does better. In other words, as long as there is some interval between 1 and $2 - \left(\frac{h+n}{n}\right)\beta$, and θ is in that interval, the heuristic investor will outperform the Bayesian on average. In particular, note that when $\theta = 2 - \left(\frac{h+n}{n}\right)\beta \neq 1$, the heuristic investor is not Bayesian and yet both do equally well.

In addition, note that a θ above 1 implies that the heuristic investor "overreacts" to the disclosure, while a θ below 1 implies that a heuristic investor "underreacts" to the disclosure. Consequently, there exist "overreacting" and "underreacting" behaviors for which the heuristic investor does better than the Bayesian investor, *despite* the fact that the heuristic investor's valuation of the asset (i.e., $\tilde{u}$) conditional on disclosure (i.e., y) is inferior to that of the Bayesian (on average), *and* both pay an identical price for buying and selling the asset.

Before I conclude, however, note the role of the DRC. In the discussion of the previous model (model #8) I showed that the DRC on any public disclosure was $\beta = \frac{n}{h+n}$. But $\beta = \frac{n}{h+n}$, implies $2 - \left(\frac{h+n}{n}\right)\beta = 1$, which, in turn, implies that the only value for θ at which the heuristic investor does no worse than the Bayesian is at $\theta = 1$. What is the significance of this? Well, one could interpret a DRC of $\frac{n}{h+n}$ as one in which "the market" is Bayesian *on average*, because from our discussion of model #8 we know that $\frac{n}{h+n}$ is the correct (i.e., Bayesian) coefficient on price. And when "the market" as a whole is Bayesian, the Bayesian investor always outperforms the heuristic investor, absent the case in which $\theta = 1$, where the heuristic investor does equally well. But $\theta = 1$ implies that the heuristic investor is actually Bayesian! So the moral of the story is simple. When "the market" is Bayesian on average, a heuristic (i.e., non-Bayesian) investor will always be outperformed by a Bayesian investor.

In short, the provocative feature of this model, in conjunction with model #7, is

that it suggests that heuristic behavior is not survivable in either a perfectly competitive or imperfectly competitive market, provided that, in the case of the latter, "the market" is Bayesian on average.

Conditioning Beliefs over Trading Volume (model #10).[37] To conclude this essay, I consider the role of one last maintained assumption. In all the models discussed up to this point, investor agents who participate in the market, either as investors or market makers, condition their expectations exclusively over total net demand, either indirectly through the market price (see, for example, models #3-6) or directly as in "the market" conditioning its expectations over total net demand (see, for example, model #8). This raises the question as to whether investors and/or "the market" would benefit from conditioning their expectations over other variables, like trading volume, and how this would change various market characterizations.[38]

To understand some of the issues involved in including trading volume as a conditioning variable, I offer a very simple model in which this is achieved. In an attempt to maintain as facile an exposition as possible, I emphasize that I chose the simplest set of assumptions that are still sufficiently robust to capture the problem.[39] Extensions of this simple model to more general settings should be straightforward.

[37]In conjunction with model #10, see Kim and Verrecchia [2001]; see also Glosten and Milgrom [1985] and Kyle [1985].

[38]Prior work on trading volume as a source of information includes Blume, *et al.* [1994], who propose a model of perfect competition in which market participants condition their expectations over demand and trading volume information from the prior period (as well as their own current period's private information) and Campbell, *et al.* [1993], who propose a model in which trading volume communicates changes in the demand for an asset by noninformational traders. Note that in the case of the latter, the trading activity of noninformational traders arises from transitory shifts in their tastes and preferences for an asset, and not information that is in any way superior or dissimilar to public knowledge.

[39]Specifically, the assumptions underlying this model are a commingling of ideas from Glosten and Milgrom [1985] (GM) and Kyle [1985], but is unique in the way it combines those elements. For example, as in GM, I assume that trade in the asset is limited to a single unit (i.e., buy one unit or sell one unit), but, as in subsequent extensions of their work (i.e, Diamond and Verrecchia [1987]) also allow for the possibility that trade is deferred (i.e., neither buy nor sell). In addition, as in both

To start, recall that the uncertain firm value is represented by $\tilde{u}$ and has a normal distribution with mean 0 and precision h (i.e., the reciprocal of variance). Let $F(u)$ represent the cumulative probability distribution of realizations of $\tilde{u}$ and $f(u)$ its density function. This implies that $f(u) = \frac{h^{\frac{1}{2}}}{\sqrt{2\pi}} \exp[-\frac{1}{2}h(u)^2]$. Now imagine an economy with N market participants, one of whom is a "large," risk neutral, informed investor. The informed investor observes $\tilde{u} = u$ (i.e., as in model #8 he knows firm value) and attempts to trade on the information. Specifically, conditional on u, the informed investor submits a demand order of either 1, 0, or -1. In the context of our model, a demand order of 0 implies that an investor defers trade. There are also $N-1$ uninformed or liquidity traders, each of whom independently submits a demand order of either 1, 0, or -1. The probability of an individual liquidity trader submitting a demand order of 0 is $x \in (0,1)$, and the probability of submitting an order of either 1 or -1 is $\frac{1}{2}(1-x)$. With the exception of knowledge of u and the identity of the informed investor, every feature of the economy is common knowledge.

As in model #8, the existence of a large informed investor suggests a model of imperfect competition in which the following game is played between the informed investor and "the market." Let D and V represent total net demand and total trading volume, respectively, and P the price at which trades are executed by "the market maker." Play can be summarized through a series of chronological steps.

1) Firm value is realized; this is represented by $\tilde{u} = u$.

2) The informed investor observes $\tilde{u} = u$.

GM and Kyle, I assume that all trades are executed by a risk neutral market maker who operates in an environment of perfect competition. Unlike, however, either GM, who assumes that trades are executed sequentially, or Kyle, who assumes that trades are executed based on total net demand, I assume that trades, and prices at which those trades are executed, are based on order flow over some interval of time. In this context, order-flow information is equivalent to information on both total net demand and trading volume. Through this device, asset returns in this model depend on trading volume information (among other things).

3) The (risk-neutral) informed investor and uninformed traders submit demand orders to "the market."

4) Based on knowledge of total net demand *and* total trading volume that results from these demand orders, "the (risk-neutral) market" sets the price at which trades are executed (i.e., "the market" picks P equal to the firm's expected value conditional on D and V). All trades are then executed at that price.

5) The firm is liquidated, paying out a return to shareholders of u.

As described above, this game is obviously reminiscent of model #8. It differs, however, in one important way: here "the market" conditions over *two* sources of information. Specifically, "the market" also conditions over total trading volume in addition to total net demand. More broadly, one can interpret "the market" in our analysis as a stylization of a market process in which the market price of firm shares reacts to contemporaneous demand and volume information.[40] To explain briefly those inferences that result from conditioning expectations over demand and volume. Suppose that there are M informed investors. In addition, let N denote the total number of market participants: that is, $N = M + L$. Finally, let N^+, N^0, and N^- denote the exact numbers of each of the submitted demand orders 1, 0, and -1. The informational benefit of observing V in addition to D is that "the market" can infer the exact numbers of each of the submitted demand orders 1, 0, and -1. For example, it is straightforward exercise to verify that N^+, N^0, and N^- can be

[40]The salient feature of this model is that "the market" conditions its expectations over *contemporaneous* trading volume information, in conjunction with contemporaneous information on total net demand. An alternative approach to the one suggested here is Blume, *et al.* [1994], which is based on Hellwig's [1982] model of perfect competion in which information about total net demand (through price) is learned in a subsequent period. In effect, in Blume, *et al.* market participants condition their expectations over the *prior* period's demand and volume information.

determined through knowledge of D and V as follows

$$N^+ = \frac{V+D}{2}, \qquad N^- = \frac{V-D}{2}, \qquad N^0 = N - V.$$

Alternatively, D alone only reveals the *difference* between the number of buy orders and the number of sell orders. In short, knowledge of demand and volume is a finer partition of information than demand alone, and hence should result in more precise inferences.

Returning to the game, the informed investor submits a demand order without knowing the price at which his trade will be executed. Let d represent the informed investor's demand order and P the price at which trades are executed. The informed investor chooses d so as to maximize his expected profit based on the effect that his action has on the expected price at which trades are executed

$$d = Arg\max\ d(u - E[\tilde{P}|d]).$$

Let λ be some element on R^+, the positive half-real line. Note that the informed investor's trading rule or strategy can be completely characterized by λ, in the following fashion. The informed investor chooses $d = 1$ for all $u \geq \lambda$ such that $\lambda = E[\tilde{P}|d = 1]$; he chooses $d = -1$ for all $u \leq -\lambda$ such that $-\lambda = E[\tilde{P}|d = -1]$; and he chooses $d = 0$ for all $u \in (-\lambda, \lambda)$. This implies that based on this strategy, the probabilities that the informed investor submits demand orders of $d = 1$, $d = -1$, and $d = 0$ are $1 - F(\lambda)$, $F(-\lambda)$, and $F(\lambda) - F(-\lambda)$, respectively.

Our search for an equilibrium to this trading game is limited to one that fulfills the following conjecture on the part of "the market": there exists a $\hat{\lambda} \in R^+$ such that when the informed investor observes a value of $u \geq \hat{\lambda}$, then he submits a demand order of 1; when the informed investor observes a value of u between $-\hat{\lambda}$ and $\hat{\lambda}$, then he submits an order of 0; and, finally, when the informed investor observes a value of

$u \leq -\hat{\lambda}$, then he submits an order of -1. Because of symmetry, all the results with λ are true with $-\lambda$, with appropriate changes in sign. For this reason, we only show and prove our results with $\lambda \geq 0$.

An equilibrium to this game is characterized as follows:

i) "the market" chooses P after observing D and V based on its conjecture that the informed investor uses $\hat{\lambda}$ in choosing his trading rule;

ii) anticipating "the market's" behavior, the informed investor chooses a trading rule characterized by λ;

iii) in equilibrium, $\hat{\lambda} = \lambda$ ("the market" correctly anticipates the informed investor's trading rule).

Now let $M = 1$, which implies $N = 1 + L$. When "the market" observes net demand, D, and volume, V, it can infer the number of each of the 1, 0, and -1 demand orders. The joint probabilities of the informed demand being 1, 0, and -1 and a $\{D, V\}$ pair are, respectively,

$$\Pr(d = 1, D, V) = (1 - F(\lambda)) \cdot \frac{(N-1)!}{(\frac{V+D}{2} - 1)!(N-V)!(\frac{V-D}{2})!}(\frac{1-x}{2})^{V-1} x^{N-V}$$

for $V > -D$, and $\Pr(d = 1) = 0$ for $V = -D$;

$$\Pr(d = 0, D, V) = (F(\lambda) - F(-\lambda)) \cdot \frac{(N-1)!}{(\frac{V+D}{2})!(N-V-1)!(\frac{V-D}{2})!}(\frac{1-x}{2})^{V} x^{N-V-1}$$

for $V < N$, and $\Pr(d = 0) = 0$ for $V = N$;

$$\Pr(d = -1, D, V) = F(-\lambda) \cdot \frac{(N-1)!}{(\frac{V+D}{2})!(N-V)!(\frac{V-D}{2} - 1)!}(\frac{1-x}{2})^{V-1} x^{N-V}$$

for $V > D$, and $\Pr(d = -1) = 0$ for $V = D$.

Let $P(D, V : x, \lambda)$, or more simply $P(D, V)$, be the market maker's expectation of u after observing D and V, given x and a conjecture λ.[41] From the above relations,

[41]

one can show that the price chosen by "the market" on the basis of a $\{D,V\}$ pair can be characterized as

$$\begin{aligned}
P(D,V) &= E[\tilde{u}|D,V] \\
&= E[\tilde{u}|u \geq \lambda] \\
&\quad \times \frac{\Pr(d=1,D,V)\cdot(1)+\Pr(d=0,D,V)\cdot(0)+\Pr(d=-1,D,V)\cdot(-1)}{\Pr(d=1,D,V)+\Pr(d=0,D,V)+\Pr(d=-1,D,V)} \\
&= \frac{E[\tilde{u}|u \geq \lambda]\cdot(1-F(\lambda))x(\frac{V+D}{2}-\frac{V-D}{2})}{(1-F(\lambda))x(\frac{V+D}{2}+\frac{V-D}{2})+(F(\lambda)-F(-\lambda))(\frac{1-x}{2})(N-V)} \\
&= E[\tilde{u}|u \geq \lambda]\cdot\left(\frac{D}{N}\right) \\
&\qquad \times\left[1+\frac{(N-V)\{x+1-2F(\lambda)\}}{V\{x+1-2F(\lambda)\}+N\{2F(\lambda)-1\}(1-x)}\right].
\end{aligned}$$

The informed investor's optimal trading rule is to choose $d = 1$ whenever $u \geq \lambda$. Therefore, "the market's" conjecture about λ is fulfilled if and only if $\lambda = E[P(D,V)|d=1]$. Finally, one can show that $\lambda = E[P(D,V)|d=1]$ is equivalent to determining a λ that satisfies the following relation

$$\begin{aligned}
\lambda &= \frac{E[\tilde{u}|u \geq \lambda]}{N}(1+\sum_{V=1}^{N}\sum_{\theta=1}^{V}\left(\frac{(N-V)\{x+1-2F(\lambda)\}}{V\{x+1-2F(\lambda)\}+N\{2F(\lambda)-1\}(1-x)}\right. \\
&\qquad \left.\times\frac{(N-1)!(\frac{1-x}{2})^{V-1}x^{N-V}}{(N-V)!(V-\theta)!(\theta-1)!}\right)).
\end{aligned}$$

Now consider again the expression above for price as a function of the $\{D,V\}$ pair

$$P(D,V) = E[\tilde{u}|u \geq \lambda]\left(\frac{D}{N}\right)\left[1+\frac{(N-V)\{x+1-2F(\lambda)\}}{V\{x+1-2F(\lambda)\}+N\{2F(\lambda)-1\}(1-x)}\right].$$

Note that this expression is linear in total net demand, D, but not trading volume, V. To be exclusively linear in D, it must be that $x+1-2F(\lambda)=0$; the latter

Formally, any expression, such as $P(D,V:x,\lambda)$, that results from the market maker's beliefs about λ should have a carot over the λ (i.e., $\hat{\lambda}$), as $\hat{\lambda}$ represents the market maker's conjecture about the behavior of λ. To simplify the notion, however, I suppress the carot in my discussion.

forces the second expression inside the square brackets to zero. But the expression $x+1-2F(\lambda)=0$ is equivalent to $2(1-F(\lambda))-(1-x)=0$. This, in turn, requires that the probability that the informed investor participate (i.e., not defer trade), which is $2(1-F(\lambda))$, be equal to the probability that an uninformed trader participate, which is $1-x$. In other words, when the probability of trade is *independent of type* (i.e., $2(1-F(\lambda))-(1-x)=0$), trading volume plays no role! Otherwise, it does play a role and price depends upon both total net demand and trading volume in some nonlinear fashion.

Unlike previous models, note that I have not included an explicit term for disclosure (primarily to ease the notational burden). As an alternative, consider a concept that is very similar to disclosure: the amount of common knowledge, or *à priori* information, available about the asset. In the context of the model under discussion, this is represented by h, the precision of $\tilde{u}$. In effect, increasing h is tantamount to more *à priori* information about the asset.[42] Alternatively, λ represents the inverse of market depth: as λ declines, market depth increases. For the model proposed here, one can show that the derivative of λ with respect to h, λ_h, is negative: that is, λ decreases as the precision of the firm's uncertain value, h increases. Specifically, one can show: $\lambda_h = -\frac{1}{2}h^{-1}\lambda$. In other words, more *à priori* information implies an increase in market depth. There are a variety of additional insights arising from the relation between h and λ. For example, the likelihood of an informed investor participating in the market is $2(1-F(\lambda))$. Note, however, that this likelihood is invariant with

[42]As such, it may be inappropriate to interpret h as disclosure, *per se*, because changing h changes the underlying nature of the asset. For example, higher (lower) h also implies a less (more) risky asset. Perhaps a better characterization of disclosure would be one in which the amount of information available about the asset's realization increases without changing its return behavior: see Kim and Verrecchia [2001].

respect to changes in h

$$\begin{aligned}\frac{d}{dh}2\left(1-F\left(\lambda\right)\right) &= -2(\frac{d}{dh}F\left(\lambda\right)+f\left(\lambda\right)\lambda_h)\\ &= -(h^{-1}\lambda f\left(\lambda\right)-h^{-1}\lambda f\left(\lambda\right))\\ &= 0.\end{aligned}$$

In effect, for the normal distribution that I assume, any change in λ brought about by a change in h offsets any change in the distribution function itself, so as to ensure that $\frac{d}{dh}F\left(\lambda\right)=0$. Nonetheless, the informed investor's expected trading profits fall as precision increases. To see this, first note that the informed investor's expected trading profits are given by

$$\begin{aligned}\int_{\lambda}^{\infty}[t-E[P|d &= 1]]f\left(t\right)dt+\int_{-\infty}^{-\lambda}-[t-E[P|d=-1]]f\left(t\right)dt\\ &= 2\left(\int_{\lambda}^{\infty}tf\left(t\right)dt-\lambda\left[1-F\left(\lambda\right)\right]\right)\\ &= 2\int_{\lambda}^{\infty}\left(1-F\left(t\right)\right)dt,\end{aligned}$$

where this computation relies on the equilibrium relation $E[P|d=1]=\lambda$. Furthermore, the derivative of this expression with respect to h is negative: that is,

$$\begin{aligned}\frac{d}{dh}2\int_{\lambda}^{\infty}\left(1-F\left(t\right)\right)dt &= -2\left(1-F\left(\lambda\right)\right)\lambda_h+2\int_{\lambda}^{\infty}\frac{d}{dh}\left(1-F\left(t\right)\right)dt\\ &= h^{-1}\lambda\left(1-F\left(\lambda\right)\right)-\int_{\lambda}^{\infty}h^{-1}tf\left(t\right)dt\\ &= -h^{-1}\int_{\lambda}^{\infty}\left(1-F\left(t\right)\right)dt.\end{aligned}$$

In short, an increase in common knowledge about the asset (i.e., an increase in h) does not change the informed investor's expected participation, but it does reduce his expected trading profits. The latter seems consistent with the sensible notion that more *à priori* information about $\tilde{u}$ makes the informed investor worse off by reducing his information advantage (if not his actual participation).

Let me summarize the implications of this model as follows. To the extent to which one believes, or has reason to believe, that trading volume information is an important conditioning variable, this model lays out how expectations could be conditioned over information about both demand and volume. To the extent to which one believes that trading volume information is only very incremental in the presence of demand information, this model suggests that there is a maintained assumption that achieves this. Specifically, if one maintains that the likelihood of trade is independent of type, trading volume is a "wash" and plays no informational role. With regard to these two beliefs, I profess that I am an agnostic. That having been said, in the more complex model I present here in which expectations are conditioned over trading volume, it should be acknowledged that the relation between common knowledge (i.e., h) and market depth (i.e., λ) is unaffected. That is, as in models in which agents condition exclusively over net demand, more common knowledge results in more market depth. Consequently, to the extent to which one wants to avoid the charge that one is responding to an "imagined" problem, some motivation for including trading volume as a conditioning variable may be required.

By way of summary let me say the following. As with all maintained assumptions, whether or not including volume as a conditioning variable is useful depends upon the nature of the problem one is studying. For example, if a study is premised on the notion that volume is a useful source of information for determining firm value in the presence of an already rich disclosure environment, then obviously its omission from a model of trade is a serious oversight. In the absence of that premise, the seriousness of the oversight is unclear.

Summary. Before offering a summary of the association-based disclosure, let me first briefly list some additional assumptions that were maintained in this exer-

cise. For example, I have ignored the role of diverse analyst and/or management forecasts in advance of disclosure.[43] Forecasts may represent an additional element of disclosure that alters all the relations discussed above. In addition, I have ignored the role of asymmetric tax effects in conjunction with disclosure. Differences in short- and long-term capital gains tax rates can result in a "lock-in" effect at the time of disclosure. The lock-in effect, in turn, may dampen price changes and trading volume at the time of disclosure (if the marginal investor is subject to tax).[44] Also, I have endowed exogenously investor agents with private information. When private information acquisition is endogenous, however, relations among disclosure, price changes, and trading volume can be altered. This is because anticipated public disclosure changes the incentives of investor agents to become privately informed; this, in turn, affects price changes and trading volume at the time of disclosure.[45] This problem is exacerbated when, in addition, the cost of acquiring private information is not homogeneous across investors. Finally, I have ignored the incentives to sell and/or distribute information.[46]

By way of summary, let me submit that the association-based disclosure studies have been very successful. They offer detailed characterizations of the relations, or associations, among disclosure, price changes, trading volume, and other market phenomena (e.g., market depth) for a broad class of investor-agent diversity. For example, in this essay I discussed investor agents who were diversely informed, in-

[43]For a discussion of the effects of analysts' forecasts, see Abarbanell, *et al.* [1995], Barron, *et al.* [1998], and Trueman [1996] (see also Verrecchia [1996b]).
[44]For suggestions as to how tax effects may affect the associations among disclosure, price changes, and trading volume, see Shevlin and Shackelford [2000].
[45]See, for example, Verrecchia [1982a], Kim and Verrecchia [1991b], Demski and Feltham [1994], McNichols and Trueman [1994], Fischer and Verrecchia [1998], and Barth, *et al.* [1999]. For a recent paper in the economics literature that reviews briefly prior work on information acquisition in financial markets, see Barlevy and Veronesi [2000].
[46]See, for example, Bushman and Indjejikian [1995]; see also Admati and Phleiderer [1986, 1988b].

terpret disclosure in diverse ways, incorporate disclosure into their beliefs in diverse ways, etc. As for the models themselves, they are remarkably facile and robust, easy to work with, and lead to a variety of interesting characterizations. It must also be acknowledged, however, that a critical maintained assumption in these models is that disclosure is exogenous. To understand the role of endogenous disclosure, let us proceed to the next essay.

3 Discretionary-Based Disclosure

What discretion does a manager or firm exercise with regard to the disclosure of information that may be useful for valuing the firm, and about which they may have knowledge? Economists have long argued in a variety of venues that the adverse-selection problem inherent in a seller simultaneously offering an asset for sale to a potential buyer and withholding information about the asset's quality, propels the seller to fully disclose to the buyer.[47] The rationale underlying this result is that a rational buyer interprets withheld information as information that is unfavorable about the asset's value or quality. Consequently, the buyer discounts the asset's value until the point at which it is in the seller's best interests to reveal the information, however unfavorable it may be. The notion that withheld information can be "unraveled" by the behavior of rational buyers is a seminal result that forms the basis for nearly all of the subsequent research on this topic.

Extending this idea into the realm of financial reporting is not difficult. While a considerable amount of financial reporting is mandatory (e.g., quarterly statements, annual reports, proxy statements, etc.), managers may still posses additional information whose disclosure is not required - information that is nonetheless useful in

[47]See Grossman and Hart [1980], Grossman [1981], and Milgrom [1981].

valuing the firm's future prospects. Consequently, under what circumstances will a manager disclose or withhold this information?

In the accounting literature, early work on this question suggested the following.[48] If a manager's objective is to maximize the current market capitalization of the firm and there are costs associated with the information's disclosure, equilibria exist in which information that favorably enhances the firm's current market capitalization is disclosed, and information that unfavorably enhances market capitalization is withheld. In other words, there exist equilibria in which not all information is disclosed. Note, in particular, that information is withheld despite the fact that market agents (e.g., investors) have "rational expectations" about its content: that is, they presume that withheld information is less favorable information. While there are a variety of costs that can support the withholding of information in equilibrium, arguably the most compelling is the cost associated with disclosing information that is proprietary in nature.

Features integral to this early work spawned a host of competing models of voluntary disclosure. For example, some suggested that this work offered exclusively a theory of information that is permanently withheld, and hence failed to explain why managers exercise discretion (through the timing of disclosure or forecasts) over information whose release was inevitable, like earnings announcements.[49] Others explored the sensitivity of the results to multiple signals.[50] Still others examined the effect of disclosure on the expected contribution to a public good (e.g., "free-riding") and the

[48]See Verrecchia [1983]; see also Jovanovic [1982] and Lanen and Verrecchia [1987].
[49]See, for example, Trueman [1986]. The analysis in Trueman [1986] is more a signaling story than a discretionary disclosure analysis in that the manager unambiguously discloses information (as opposed to exercising discretion) as soon as he receives new information, so as to "signal" his competence; see also Hughes [1986]. Note that Verrecchia [1983] does offer a rationale for exercising discretion (through the timing of disclosure) over information whose release is inevitable by appealing to the idea that proprietary costs dissipate through time.
[50]See Kirschenheiter [1997].

likelihood that cooperative efforts collapse.[51] Finally, some pointed out the failure to consider the interaction between either: voluntary disclosure and mandated disclosure, where the requirement for more of the latter may increase the incentives for more of the former; or costly disclosure and costless disclosure, where the existence of the former may inhibit the latter because of possible interdependencies between the two.[52]

With regard to early work in the accounting literature, three issues seemed to be of particular concern: 1) the reliance on an exogenous proprietary cost to explain the withholding of information; 2) the reliance on truthful reporting; and 3) the reliance on the manager's objective as one of boosting the firm's current capitalization level, even in the event that this practice jeopardizes firm value in the future. With regard to the reliance on costs, in the disclosure literature "uncertainty" offers an alternative rationale for the withholding of information in the absence of an exogenous proprietary cost. For example, there is the possibility that information is withheld because uncertainty exists about whether the manager is informed or, equivalently, whether the information in question has yet to arrive.[53] To digress briefly, uncertain information existence or arrival works like a disclosure cost in that it creates doubt in the minds of the uninformed, thereby ameliorating the adverse-selection problem. Hence it supports the withholding of information. In addition to withholding arising from uncertainty about the existence of information, there is also the possibility that information is withheld because of uncertainty about "types": for example, the

[51]See Teoh [1997].
[52]See Dye [1986]: on the topic of voluntary versus mandated disclosure, see also Dye [1985b,1990]. See also Gigler and Hemmer [1999], who suggest that one role of mandatory disclosure may be as a vehicle useful in creating an environment in which managers can credibly communicate their more value-relevant voluntary disclosures. They refer to this as the "confirmatory role" of mandatory disclosure.
[53]See, for example, Dye [1985a], Jung and Kwon [1988], and Dye and Sridhar [1995].

"type" of manager or the "type" of firm. In the case of the former, information may be (rationally) withheld because it can be used to value the human capital of the manager, as well as the firm.[54] In the case of the latter, information may be withheld because the immediate benefit (cost) of a favorable (unfavorable) report has to be weighed against the credibility gain or loss at a subsequent date when more information is forthcoming.[55]

With regard to the reliance on the exogenous restriction to truthful reporting, some have questioned the assumption that if the manager chooses to release her private information, then she does so truthfully. Truthful reporting is typically justified by appealing to the potential litigation and human capital erosion costs associated with dissembling. While this restriction seems descriptive of many settings in accounting where audited financial statements may corroborate the manager's disclosure, there are instances, such as the provision of forward-looking information, where it is more difficult to assess the integrity of the manager's disclosure.[56] Persuasion and cheap-talk games relax the restriction that the manager is obliged to disclose truthfully, if she discloses at all. In these games, the credibility of the manager's disclosure becomes a key issue. For example, persuasion games have the feature that while the manager need not fully reveal his private information, he may not misrepresent it: for example, the manager may vaguely claim that the firm is expected to have earnings of *at least* \$1 per share when in fact he expects earnings to be *exactly* \$1 per

[54]See Nagar [1999]; see also Kim [1999].
[55]See Teoh and Hwang [1991].
[56]While on the topic of credible disclosure, by prior agreement it was decided that contracting issues in general, and the Revelation Principle in particular, would fall under the auspices of a companion survey paper, Lambert [2000]. Specifically, Lambert [2000] discusses private information and communication (in section 4) and earnings management and the Revelation Principle (in section 5). While these topics have some bearing on issues discussed in this essay, we decided to allocate them in this manner so as to avoid overlap in the respective surveys.

share.[57] Other work in this area examines the effect of rules governing disclosure.[58] Cheap-talk games are those where the players' payoffs are determined by the action that the manager's disclosure induces and not directly by his costless disclosure. In these games, the manager's disclosure may be false: for example, a manager may claim that the firm is expected to have earnings of $2 per share when in fact he expects earnings to be exactly $1 per share. Because the manager is free to offer any self-serving report irrespective of his privately observed, non-verifiable information, this modeling choice is particularly useful for examining the amount of information a manager can communicate when the credibility of his disclosure is a key feature of the environment.[59] Finally, as an alternative to cheap-talk games where disclosure distortions are costless, the notion of costly distortions has also been considered.[60]

[57]For example, interpreting this work in a "market" context, one could suggest that Milgrom and Roberts [1986] consider a game where a manager attempts to increase the stock price by convincing investors that the firm has favorable earnings prospects. Investors assume the worst in the sense that they believe that the firm's earnings prospects equal the lowest level consistent with the claim being truthful. These beliefs support an equilibrium characterized by full revelation. Shin [1994] extends this model and analyzes the pricing of a firm's stock when a manager is exogenously endowed with information and investors are uncertain about its quality. He shows that the responsiveness of the stock price to the firm's disclosure reveals investors' beliefs about the credibility of that disclosure.

[58]Once again interpreting this work in a "market" context, one could suggest that Matthews and Postlewaite [1985] assume that the manager is not exogenously informed about the firm's earnings prospects, but must decide whether to voluntarily gather information that will perfectly reveal the firm's earnings. They examine the effect on the manager's incentives to gather information if he is required to report it. On the other hand, Fishman and Hagerty [1990] consider the effect of restricting the vagueness with which a manager may reveal his information. They note that limits on the manager's discretion may increase the amount of information communicated.

[59]Crawford and Sobel [1982] established that disclosure may be partially informative, provided the manager's and investor's incentives are not too misaligned. Subsequent work has further examined the credibility problem that is central to cheap-talk games. For instance, Farrell and Gibbons [1989], Newman and Sansing [1993], and Gigler [1994] focus on the impact of different users (e.g., investors and competitors) on a manager's incentives to disclose his information. They show that the presence of two audiences who respond to the information differently may enhance the credibility of the manager's disclosure. Nevertheless, within these single period settings, full revelation of the manager's information does not occur. In contrast, within a multi-period setting, Stocken [2000] shows that reputational considerations may be sufficient to support full revelation of a manager's information.

[60]See Fischer and Verrecchia [2000].

But in reading over the accounting literature, it would appear that the thorniest problem with the early work was its reliance on the assumption that the manager's objective in exercising discretion in disclosure was to boost the firm's current capitalization level, even in the event that this practice jeopardized future returns. For example, if there are costs associated with disclosing information whose dissemination is not required, and the sole effect of disclosure is to provide an immediate and ephemeral boost to the firm's current market price, perhaps firm shareholders should contract with the manager to never disclose voluntarily. In other words, to what extent does maximizing current capitalization eschew totally efficiency and/or agency problems implicit in voluntary disclosure?[61]

Throughout accounting, one often-stated rationale for a manager to be concerned with the firm's current capitalization level, as opposed to the firm's future value, is that contracts are incomplete. For example, there may be no way for the manager to be rewarded on the basis of the future value of the firm, if for no other reason than the fact that he may not be around when the future arrives. In addition, there may be compelling reasons why current market valuations are important, such as the possibility that the firm intends to issue additional equity to finance future operations, or as currency in stock-swaps. Yet another interesting rationale to consider is that maximizing current market capitalization may simply be a heuristic behavior on the manager's part. For example, perhaps the manager maximizes current market capi-

[61]As pointed out by Dontoh [1989], another problem with maximizing current capitalization, as opposed to future returns, is that it offers no possibility for the voluntary disclosure of bad news, which seems inconsistent with the extant empirical literature. Of course, one way around *this* problem is to assume that managers voluntarily disclose so as to *minimize* current firm price, perhaps in part to ensure positive price reactions to subsequent mandated announcements (as well as to minimize the liability associated with withholding information that negatively impacts current firm valuations). See Teoh and Hwang [1991] for a model in which a separating equilibrium exists in which "good news" is withheld and "bad news" disclosed.

talization because he has been conditioned to believe that he is *truly* being evaluated based on this benchmark, regardless of his contract. Anecdotal evidence in support of this idea is the fact that business media articles about top managers commonly allude to the level of market capitalization increase or decrease during that manager's tenure with the firm.

But in view of this problem, an alternative model for motivating voluntary disclosure is to follow the general outline of the original story, which is based on the notion of proprietary costs, and then show how these proprietary costs arise endogenously in a duopoly game played between two firms that seek to maximize future returns (as opposed to current market value).[62] By couching the disclosure problem in the context of a duopoly game played between two firms, the decision to disclose by one firm assists the other firm's production decisions, and/or whether to enter a particular market for the manufacture of some good in the first place. Because duopoly games typically lead to quadratic optimization problems, they are very facile. Consequently, it is little wonder that there are a cornucopia of papers in the accounting literature that exploit this technology to study discretionary disclosure.[63]

Duopoly game papers have two important features. First, in their voluntary disclosure decisions, managers can be made to be concerned with future firm value, which resolves the problem of assuming that managers seek to maximize current value.[64] Second, a duopoly setting characterizes well how the release of information

[62]This is essentially the central feature of papers like Darrough and Stoughton [1988], Feltham and Xie [1992], and Wagenhofer [1990], among others.
[63]To cite a few more papers, see Feltham, *et al.* [1992], Darrough [1993], and Gigler, *et al.* [1994].
[64]Of course, maximizing future value is not a requirement of a duopoly game. In the context of a duopoly game managers can still be concerned exclusively with current value: see, for example, Hayes and Lundholm [1996], as well as model #2 in the subsequent subsection. In addition, it is also possible that managers' motivation to disclose is governed by neither current *nor* future firm values. For example, Bushman and Indjejikian [1995] assume that a manager discloses voluntarily to boost insider trading profits by reducing the trading aggressiveness of other privately informed agents.

creates proprietary costs endogenously. While all of this is to the good, there are criticisms of this approach. First, it could be argued that once the nature of the cost has been identified (i.e., the fact that it is proprietary), there is little additional insight associated with showing how it evolves endogenously. Second, duopoly games, *per se*, may not thwart the "unraveling" of withheld information in the absence of some additional modeling feature. The reason for this is that if two firms compete in the same (or similar) product market(s), the act of withholding information by one firm may be interpreted by his competitor as information that favors boosting output. Once boosted beyond a certain level by a competitor, however, output negatively impacts on the informed firm's ability to generate revenues in his product market, thereby propelling the latter to fully disclose to the former.

Putting aside the narrow issue of which collection of stylized assumptions best characterizes the discretionary nature of disclosure, a broader criticism of discretionary disclosure models in the accounting literature *in toto* should focus on two problems: the results offered are highly sensitive to specific modeling assumptions, and the discretionary disclosure arrangements, *per se*, are typically inefficient. With regard to the first issue, the literature documents that results in duopoly games depend upon whether: the competition is Cournot (quantity setting) or Bertrand (price setting), the private information is cost or demand information, and whether the decision to disclose is made *ex post* or *ex ante*.[65] Another area in which results depend upon model nuances is in the relation between voluntary disclosure and competition. For example, some have modeled competition in the context of an entry game (i.e., a game in which one firm contemplates producing a good already produced by some other firm) and claimed that greater competition encourages more disclosure.

[65]See specifically Darrough [1993], who does an excellent job of delineating the sensitivity of these assumptions on results.

Alternatively, others have modeled competition in the context of a post-entry game (i.e., a game in which both firms are currently producing) and claimed that greater competition inhibits more disclosure.[66] Yet another area of controversy brought on by seemingly innocuous differences in modeling assumptions is the relation between voluntary disclosure and the *ex ante* differential information quality between the manager and the market. Here, some have claimed that higher differential information quality leads to more voluntary disclosure while others have claimed the reverse.[67]

With regard to the efficiency of discretionary disclosure arrangements, it is useful to introduce the following semantic distinctions. By a discretionary disclosure arrangement, I mean a situation in which managers or firms exercise discretion with respect to the disclosure of information about which they may have knowledge (i.e., *ex post*). Alternatively, by a precommitment arrangement or mechanism, I mean a situation in which managers or firms establish a preferred disclosure policy in the absence of any prior knowledge of the information (i.e., *ex ante*). My point about efficiency is that often arrangements in which managers are granted the discretion to disclose *ex post* are inefficient in comparison to arrangements in which the firm (or managers) precommit *ex ante*. For example, typically one can show that precommitting to a policy of either no disclosure or full disclosure before the information arrives, or perhaps electing to never become informed in the first place, dominates alternatives in which the manager exercises discretion after receiving the information. For example, some have shown that precommitments to nondisclosure in the Cournot/demand and Bertrand/cost cases, and precommitments to full disclosure in the Cournot/cost and Bertrand/demand cases, dominate alternative disclosure arrangements.[68] Similarly,

[66]For the former see Darrough and Stoughton [1990] (see also Verrecchia [1990b]), and for the latter see Clinch and Verrecchia [1997].
[67]See Verrecchia [1990a] for the former and Penno [1997] and Dye [1998] for the reverse.
[68]See Darrough [1993].

others have shown that it is optimal for the manager to precommit never to become informed (or otherwise proscribe this behavior), lest the manager be tempted to engage in costly disclosure activities *ex post*.[69] Consequently, if, in the face of all this, a manager continues to exercise discretion in the disclosure of information *ex post*, there must be some unstated, unmodeled, and/or unresolved agency problem or efficiency consideration that lurks in the background. Of course, one commonly stated rationale for why managers are allowed the discretion to disclose *ex post* is that precommitment mechanisms do not exist. And while this rationale is indeed true, it has little to recommend it on economic grounds other than expediency.

As in the previous essay, below I present a series of increasingly more sophisticated, discretionary disclosure modeling vignettes in an attempt to illustrate the evolution of the literature. Specially, in model #1 I discuss how the existence of either a fixed proprietary cost or uncertainty about the existence of withheld information leads to equilibria in which information is some times disclosed and some times withheld, assuming that a firm seeks to maximize its current value. In model #2 I relax the assumption of fixed costs to allow for endogenous, variable proprietary costs, but continue to assume that a firm seeks to maximize its current value. The provocative feature of model #2 is that while it suggests that the optimal disclosure policy *ex post* is one of full disclosure, it also suggests that the optimal disclosure policy *ex ante* is one of no disclosure; this points out the potential inefficiency of discretionary disclosure arrangements. In model #3, I extend the analysis to a duopoly situation in which the firm adopts a disclosure policy to maximize expected profits: that is,

[69]See Verrecchia [1990a]. See also Pae [1999], who demonstrates in a clever model of discretionary disclosure that in the absence of prohibitions on becoming informed, two types of potential efficiency losses may arise. First, there is the (potential) efficiency loss that results from a manager acquiring costly information so as to be able to disclose favorable news at his discretion. Second, there is the (potential) efficiency loss that results from the manager overinvesting in effort (relative to the first best level) because doing so reduces information acquisition costs.

the firm maximizes future value and not current value. Here, as well, I point out that there may exist *ex ante* precommitment arrangements that dominate allowing the firm the discretion to disclose *ex post*. Finally, in model #4 I extend the duopoly setting further to one in which there is no requirement that firms disclose truthfully.

Constant Proprietary Costs (Model #1).[70] Consider a firm that produces a good this period based on demand for the product next period. Next period's demand is characterized by a price P, where P is represented by

$$P = \alpha + \beta\tilde{Y} - x,$$

and α, and β are fixed, positive constants (i.e., $\alpha > 0$, and $\beta > 0$), $\tilde{Y}$ is some proprietary information about next period's price that is known only to the firm, and x is the quantity produced by the firm this period. In other words, the firm produces x this period to achieve revenue of xP next period. Because realizations of $\tilde{Y}$ are proprietary, they are only known to the market if they are disclosed by the firm. In the absence of disclosure, the market treats $\tilde{Y}$ as an unknown random variable that is distributed uniformly between $-k$ and k. For reasons that will become clearer in subsequent extensions of this model, at this stage of the discussion I want to limit the interpretation of model #1 to one in which there is (exclusively) a positive association between realizations of $\tilde{Y} = Y$ along the continuum between $-k$ and k and the firm's revenue next period, xP. The benefit of a positive association is that one can interpret increasingly higher realizations of $\tilde{Y} = Y$ as increasingly "better news" because they indicate higher revenue next period. As I show below, a sufficient condition to achieve a positive association is to assume that $\alpha \geq \beta k$. Consequently, I assume $\alpha \geq \beta k$ in both models #1 and #2, and then relax this assumption in model

[70] In conjunction with model #1, see Jovanovic [1982] and Verrecchia [1983].

#3.[71]

Also in models #1 and #2, I assume that the firm's discretionary disclosure policy is to maximize the firm's current value (for whatever reason). Because increasingly higher realizations of $\tilde{Y} = Y$ imply increasingly "better news," the firm is naturally predisposed toward disclosing high realizations of $\tilde{Y} = Y$ as an indication of high revenue next period. The dilemma for the firm is that knowledge of $\tilde{Y} = Y$ is proprietary, perhaps because it can be used by other, competing firms to set their production schedules for the good, or goods that are close substitutes. Here, I assume that the proprietary cost associated with disclosing any realization of $\tilde{Y} = Y$ is c, where $c > 0$. Note that this implies that the proprietary cost associated with disclosing any information is fixed and invariant, independent of the information.

To determine whether the firm discloses its proprietary information, consider its investment decision when $\tilde{Y} = Y$ is disclosed. In this situation the firm produces the amount x so as to maximize

$$\max \; xE[\tilde{P}|\tilde{Y} = Y] = x(\alpha + \beta Y - x).$$

Note that this function is concave in x and otherwise well behaved. This implies producing a quantity

$$x = \frac{1}{2}(\alpha + \beta Y),$$

and selling this quantity for

$$P = \alpha + \beta Y - x = \frac{1}{2}(\alpha + \beta Y)$$

in the next period. Note that one consequence of assuming $\alpha \geq \beta k$ is that the quantity of the good produced this period and the price at which the good sells next

[71]As I show below, another feature of assuming $\alpha \geq \beta k$ is that it ensures that in this model the quantity of the good produced this period and the price at which the good sells next period are both nonnegative in equilibrium.

period, i.e., $x = P = \frac{1}{2}(\alpha+\beta Y)$, are both nonnegative for all $Y \in [-k, k]$. In addition, regardless of whether the firm discloses or withholds $\tilde{Y} = Y$, its revenue next period (exclusive of any proprietary costs) is $xP = \frac{1}{4}(\alpha + \beta Y)^2$. Finally, note that because $\frac{d}{dY}xP = \frac{1}{2}\beta(\alpha + \beta Y) \geq 0$ for all $Y \in [-k, k]$ when $\alpha \geq \beta k$, realizations of $\tilde{Y}$ and revenue are positively associated.

The market values the firm based on its knowledge of the firm's revenue next period (if $\tilde{Y} = Y$ is disclosed) or its expectation of revenue (if $\tilde{Y} = Y$ is withheld). When $\tilde{Y} = Y$ is disclosed, the market knows that the firm's revenue next period is (including the proprietary cost)

$$E[\tilde{x}\tilde{P}|\tilde{Y} = Y] = \frac{1}{4}(\alpha + \beta Y)^2 - c.$$

Alternatively, consider what occurs when $\tilde{Y} = Y$ is not disclosed. Because realizations of $\tilde{Y} = Y$ and revenue are positively associated, the market conjectures that realizations of $\tilde{Y}$ that are withheld must be below some threshold $\hat{Y}$ that does not warrant incurring the proprietary cost c. Consequently, when $\tilde{Y} = Y$ is not disclosed, the market assesses the firm's revenue next period as

$$\begin{aligned} E\left[\tilde{x}\tilde{P}|\tilde{Y} = Y \leq \hat{Y}\right] &= E[\frac{1}{4}(\alpha + \beta\tilde{Y})^2|\tilde{Y} = Y \leq \hat{Y}] \\ &= \frac{1}{12}\left(3\alpha^2 + 3\alpha\beta(\hat{Y} - k) + \beta^2(\hat{Y}^2 - \hat{Y}k + k^2)\right). \end{aligned}$$

This implies that based on a realization of $\tilde{Y} = Y$, the difference between disclosing and withholding this information from the market on the firm's current value is

$$\begin{aligned} & E\left[\tilde{x}\tilde{P}|\tilde{Y} = Y\right] - c - E\left[\tilde{x}\tilde{P}|\tilde{Y} \leq Y\right] \\ = \ & \frac{1}{4}(\alpha + \beta Y)^2 - c - \frac{1}{12}\left(3\alpha^2 + 3\alpha\beta(\hat{Y} - k) + \beta^2(\hat{Y}^2 - \hat{Y}k + k^2)\right). \end{aligned}$$

Consequently, the firm is motivated to disclose $\tilde{Y} = Y$ when this expression is positive and withhold it when the expression is negative, because this arrangement maximizes

the market's *expectation* of the firm's revenue next period, and hence the firm's current value. That value of $\tilde{Y}$ that leaves the firm indifferent between disclosing and withholding is the threshold level of disclosure. Specifically, the threshold level $\hat{Y}$ is defined such that $E[\tilde{x}\tilde{P}|\tilde{Y} = Y] - c - E[\tilde{x}\tilde{P}|\tilde{Y} \leq Y]$ is nonnegative for all $Y \geq \hat{Y}$ and negative for all $Y < \hat{Y}$. One can show that here a unique threshold level of disclosure occurs at

$$\hat{Y} = -\frac{1}{4\beta}\left(3\alpha + \beta k - \sqrt{9(\alpha - \beta k)^2 + 96c}\right).$$

In particular, this threshold has the feature that $\hat{Y} > -k$, provided $c > 0$.

The economic interpretation of $\hat{Y} = -\frac{1}{4\beta}\left(3\alpha + \beta k - \sqrt{9(\alpha - \beta k)^2 + 96c}\right)$ is that it is the level of "news" that leaves the firm indifferent between disclosing $\tilde{Y} = Y$ at a cost c and withholding the realization $\tilde{Y} = Y$. Because values of $\tilde{Y} = Y$ above $\hat{Y}$ indicate high demand for the good, the firm is willing to provide this information to the market for valuation purposes despite the proprietary costs associated with this decision. Alternatively, because values of $\tilde{Y} = Y$ below $\hat{Y}$ indicate average or low demand for the good, the firm is justified in withholding knowledge of $\tilde{Y} = Y$ because this information does not enhance valuation *and* its disclosure entails a proprietary cost. Note that at $c = 0$, $\hat{Y} = -k$. In other words, in the absence of proprietary costs, the only equilibrium threshold is one that implies full disclosure. In addition, as c increases, $\hat{Y}$ increases. In other words, disclosure thresholds rise as proprietary costs rise.

To digress briefly, a variation on this model is to assume that there are no proprietary costs, but, instead, the firm is only known to be informed with probability q and uninformed with probability $1 - q$.[72] Note that when the firm is uninformed, it produces $x = \frac{1}{2}\alpha$, and at that quantity the goods sell for a price $P = \frac{1}{2}\alpha + \beta Y$. This

[72]See, for example, Dye [1985a] and Jung and Kwon [1988].

implies that $E[\tilde{x}\tilde{P}] = \frac{1}{4}\alpha^2$. Consequently, here the threshold level of disclosure, $\hat{Y}$, is determined by finding the Y that solves

$$E[\tilde{x}\tilde{P}|\tilde{Y} = Y] - qE[\tilde{x}\tilde{P}|\tilde{Y} \leq Y] - (1-q)\,E[\tilde{x}\tilde{P}] = 0,$$

which is equivalent to finding the Y that solves

$$\frac{1}{4}(\alpha + \beta Y)^2 - q\frac{1}{12}\left(3\alpha^2 + 3\alpha\beta(Y-k) + \beta^2(Y^2 - Yk + k^2)\right) - (1-q)\,\frac{1}{4}\alpha^2 = 0.$$

Note, for example, that if the firm is known to be informed with certainty (i.e., $q = 1$), then the threshold level of disclosure is $-k$, which implies full disclosure. In other words, a firm known to be informed with certainty is tantamount to a firm with no proprietary costs.

Returning to my original model, while much of the discretionary disclosure literature has focused primarily on threshold levels of disclosure, arguably it is not threshold levels *per se* that are of interest, but rather the unconditional probability, or likelihood, of disclosure. One reason for this is that from an empirical perspective, threshold levels of disclosure are likely unobservable, whereas the probability of disclosure is potentially knowable through observations on repeated plays of a discretionary disclosure game. Recall that if $\tilde{Y}$ is a random variable distributed uniformly between k and $-k$, then the probability of disclosure is

$$\begin{aligned}\max[\frac{1}{2k}\int_{\hat{Y}}^{k} dY, 0] &= \max[\frac{1}{2k}\left(k - \hat{Y}\right), 0] \\ &= \max[\frac{1}{8\beta k}\left(5\beta k + 3\alpha - \sqrt{9(\alpha - \beta k)^2 + 96c}\right), 0].\end{aligned}$$

Note that the probability of disclosure is less than 1 provided that $c > 0$, and greater than 0 provided that c is not too large.

Now consider the relation between disclosure and information quality. In our analysis, note that the manager is assumed to know the determinant of price perfectly

(i.e., he knows Y), while *à priori* the market only knows $\tilde{Y}$ to be uniformly distributed between k and $-k$. The variance of a uniformly distributed random variable is $\frac{1}{3}k^2$. In effect, the variance increases as the support of the uniform, k, widens. Consequently, k can be thought of as a measure of information asymmetry, or *ex ante* differential information quality, between the market and the manager. The higher k, the less the market knows relative to the manager *à priori*. In this context an interesting question here is how the threshold level of disclosure shifts as information asymmetry between the market and the manager grows. In the model currently under discussion, one can show that the probability of disclosure generally increases as k increases.[73] This implies that as information asymmetry between the market and the manager grows, in equilibrium the manager discloses more often. One could argue that this result is eminently sensible: greater disparity between what the manager knows and what the market knows exacerbates the adverse-selection problem. Thus, amelioration of this adverse-selection problem requires more disclosure. The problem is that some have argued that models with the opposite prediction are equally sensible.[74] All this points to a fragile relation between assumptions and predictions.

The rationale for the next model is that proprietary costs may not be constant: specifically, they may depend upon realizations of $\tilde{Y}$. For example, higher realizations of $\tilde{Y}$ may imply greater costs and lower realizations may imply lower costs - or perhaps the reverse! But in any event, there may be *some* relation between information and the costs of disclosing that information. To see the effect of proprietary costs that vary as a function of the manager's private knowledge, consider the next model.

[73]If $\alpha \geq 2\beta k$, then the probability of disclosure is always increasing as k increases. If $\alpha < 2\beta k$, then, in addition, one needs $c > \alpha\frac{3}{32}(2\beta k - \alpha)$: in other words, proprietary costs cannot be insignificant.
[74]See specifically Penno [1997] and Dye [1998]. Of course, as the previous footnote suggests, one can also get this prediction by assuming that proprietary costs are insignificant.

Endogenous and Variable Proprietary Costs (Model #2).[75] In this variation on the prior model, all previous assumptions are maintained except for the one concerning a constant proprietary cost. Specifically, here two firms are introduced who compete in a Cournot (quantity setting) duopoly in which one firm is informed and the other uninformed. In effect, each firm invests in the current period to produce some good, in anticipation of the fact that the good will sell for a price P in a future period, where P is represented by

$$P = \alpha + \beta\tilde{Y} - x_I - x_U,$$

where α and β are all fixed, positive constants (i.e., $\alpha > 0$, and $\beta > 0$), $\tilde{Y}$ is some proprietary information about the anticipated price that is known only to the informed firm, and x_I and x_U are the amounts produced by the informed and uninformed firms, respectively. Each firm's decision to produce is made without knowledge of the other firm's decision. Because $\tilde{Y}$ is unknown to either the uninformed firm or the market that values the future prospects of both firms, I continue to represent it by a random variable $\tilde{Y}$, where $\tilde{Y}$ has a uniform distribution between $-k$ and k. Also as before, note that a high value of $\tilde{Y} = Y$ along the continuum between $-k$ and k continues to be "good news" because it indicates that the price at which the good will sell will be high in a future period.

The dilemma faced by the informed firm is that if it discloses "good news," the uninformed firm will also come to know this fact and boost its production accordingly, thereby negatively impacting on the informed firm's revenue. Consequently, in this model disclosure of a "good news" value of $\tilde{Y} = Y$ entails a proprietary cost that varies with the disclosed information.

[75] In conjunction with model #2, see Hayes and Lundholm [1996].

To determine whether and when the informed firm discloses $\tilde{Y} = Y$, consider its investment decision when $\tilde{Y} = Y$ is disclosed. When $\tilde{Y} = Y$ is disclosed, let x_I^D and x_U^D represent the quantities of the good produced by the informed and uninformed firms, respectively. Here, the informed firm produces the quantity x_I^D so as to maximize

$$\max \; x_I^D E[\tilde{P}|\tilde{Y} = Y] = x_I^D(\alpha + \beta Y - x_I^D - x_U^D).$$

This implies that the informed firm produces an amount

$$x_I^D = \frac{1}{2}(\alpha + \beta Y - \hat{x}_U^D),$$

where $\hat{x}_U^D$ is the informed firm's conjecture about the uninformed firm's production decision. Similarly, when $\tilde{Y} = Y$ is disclosed, the uninformed firm produces a quantity

$$x_U^D = \frac{1}{2}(\alpha + \beta Y - \hat{x}_I^D),$$

where $\hat{x}_I^D$ is the uninformed firm's conjecture about the informed firm's production decision. Here, self-fulfilling conjectures on the part of both the informed firm (about $\hat{x}_U^D$) and the uninformed firm (about $\hat{x}_I^D$) that are self-fulfilling are $\hat{x}_I^D = \hat{x}_U^D = \frac{1}{3}(\alpha + \beta Y)$, which implies that in equilibrium both firms produce the same amount. In addition, here the price at which both firms sell their goods is

$$P^D = \alpha + \beta Y - x_I^D - x_U^D = \frac{1}{3}(\alpha + \beta Y),$$

which implies that the informed firm achieves revenues next period of $x_I^D P^D = \frac{1}{9}(\alpha + \beta Y)^2$.

Now consider the case in which the informed firm does not disclose $\tilde{Y} = Y$. When $\tilde{Y} = Y$ is not disclosed, let x_I^N and x_U^N represent the quantities of the good produced by the informed and uninformed firms, respectively. Here the informed firm continues to produce a quantity

$$x_I^N = \frac{1}{2}(\alpha + \beta Y - \hat{x}_U^N),$$

where, once again, $\hat{x}_U^D$ is the informed firm's conjecture about the uninformed firm's production decision. Here, the uninformed firm does not observe $\tilde{Y} = Y$. Despite this fact, being rational it conjectures that if $\tilde{Y}$ were above some threshold, the informed firm would disclose $\tilde{Y} = Y$. As in model #1, the basis for this conjecture is the presumption that the informed firm seeks to boost its current market value (for whatever reason), and disclosing high values of $\tilde{Y} = Y$ achieves this. Define V by $V = E[\tilde{Y}|\tilde{Y} = Y \leq \hat{Y}] = \frac{1}{2}\left(\hat{Y} - k\right)$. In the absence of disclosing $\tilde{Y} = Y$, the uninformed firm solves

$$\begin{aligned} \max \; x_U^N E\left[\tilde{P}|\tilde{Y} = Y \leq \hat{Y}\right] &= x_U^N E[\alpha + \beta Y - \hat{x}_I^N - x_U^N|\tilde{Y} = Y \leq \hat{Y}] \\ &= x_U^N(\alpha + \beta V - \hat{x}_I^N - x_U^N), \end{aligned}$$

where, once again, $\hat{x}_I^N$ is the uninformed firm's conjecture about the informed firm's production decision conditional on $\tilde{Y}$ being below some threshold $\hat{Y}$. This, in turn, implies that the uninformed firm produces an amount

$$x_U^N = \frac{1}{2}(\alpha + \beta V - \hat{x}_I^N).$$

Here, conjectures that are self-fulfilling are $\hat{x}_I^N = \frac{1}{6}(2\alpha + 3\beta Y - \beta V)$ and $\hat{x}_U^N = \frac{1}{3}(\alpha + \beta V)$, which implies that the price at which both firms sell their goods is

$$P^N = \alpha + \beta Y - x_I^N - x_U^N = \frac{1}{6}(2\alpha + 3\beta Y - \beta V).$$

Consequently, here the informed firm's revenue next period is $x_I^N P^N = \frac{1}{36}(2\alpha + 3\beta Y - \beta V)^2$.

Now define current market value as the market's valuation of the informed firm based on the market's *expectation* of the revenue likely to be achieved next period. When $\tilde{Y} = Y$ is disclosed, the market's expectation of the revenue of the informed

firm next period is

$$E[\tilde{x}_I^D \tilde{P}^D | \tilde{Y} = Y] = \frac{1}{9}(\alpha + \beta Y)^2 .$$

Alternatively, when $\tilde{Y} = Y$ is not disclosed, the market's expectation of the revenue of the informed firm next period is

$$\begin{aligned} E\left[\tilde{x}_I^N \tilde{P}^N | \tilde{Y} = Y \leq \hat{Y}\right] &= E[\frac{1}{36}(2\alpha + 3\beta Y - \beta V)^2 | \tilde{Y} = Y \leq \hat{Y}] \\ &= \frac{1}{36}\left(4\alpha^2 + 4\alpha\beta(\hat{Y} - k) + \frac{7}{4}\beta^2(\hat{Y}^2 - \frac{2}{7}\hat{Y}k + k^2)\right), \end{aligned}$$

recalling that $V = \frac{1}{2}\left(\hat{Y} - k\right)$.

To understand at an intuitive level the nature of an equilibrium to this problem, note that the difference between the revenue of the informed firm when disclosing $\tilde{Y} = Y$ versus not disclosing and establishing a threshold at $\hat{Y}$ is

$$\begin{aligned} & E\left[\tilde{x}_I^D \tilde{P}^D | \tilde{Y} = Y\right] - E\left[\tilde{x}_I^N \tilde{P}^N | \tilde{Y} \leq Y\right] \\ = \; & \frac{1}{9}(\alpha + \beta Y)^2 - \frac{1}{36}\left(4\alpha^2 + 4\alpha\beta(Y - k) + \frac{7}{4}\beta^2(Y^2 - \frac{2}{7}Yk + k^2)\right). \end{aligned}$$

Consequently, as in the previous model, the informed firm is motivated to disclose $\tilde{Y} = Y$ when this expression is positive and withhold it when the expression is negative. One can show that the expression $x_I^D E[\tilde{P}^D | \tilde{Y} = Y] - x_I^N E[\tilde{P}^N | \tilde{Y} \leq Y]$ reduces to $\frac{1}{144}\beta(Y + k)(16\alpha + 9\beta Y - 7\beta k)$ and, consequently, there exists a threshold level of disclosure at $\hat{Y} = \frac{1}{9\beta}(7\beta k - 16\alpha)$. That is, there exists a unique threshold level of disclosure, namely, $\hat{Y} = \frac{1}{9\beta}(7\beta k - 16\alpha)$, which has the feature that the informed firm discloses whenever $Y \geq \hat{Y} = \frac{1}{9\beta}(7\beta k - 16\alpha)$ and withholds otherwise. In addition, this threshold implies that the probability of disclosure is

$$\frac{1}{2k}\int_{\hat{Y}}^{k} dx = \frac{1}{2k}\left(k - \hat{Y}\right) = \frac{1}{9\beta k}(8\alpha + \beta k).$$

This all sounds good, but consider the following. When $\alpha \geq \beta k$, the threshold level of disclosure is at the lowest realization of $\tilde{Y}$ and the probability of disclosure

is always 1. That is, $\alpha \geq \beta k$ implies that $\hat{Y} = \frac{1}{9\beta}(7\beta k - 16\alpha) \leq -k$ and the probability of disclosure is $\frac{1}{9k}(8\alpha + \beta k) \geq 1$. In other words, in an attempt to ensure a positive association between realizations of $\tilde{Y}$ and revenue next period, our assumptions have led us unwittingly to a model of full disclosure![76] Why is it the case that attempts on the part of the informed firm to withhold proprietary information "unravel" here, but not in the previous model (i.e., model #1)? Because realizations of $\tilde{Y}$ and expected revenue next period are positively related, both the market and the uninformed firm interpret withheld information as unambiguously "bad news," just as they would have in model #1. Unlike model #1, however, here there is no fixed, or constant, proprietary cost to act as a discontinuity in the valuation of the informed firm depending upon whether the information is disclosed or withheld. (In addition, there is no uncertainty as to the existence of the withheld information.) Consequently, here, information is always disclosed to the market, despite its proprietary nature.

But the provocative feature of model #2 is that while it suggests that the inevitable disclosure policy *ex post* is one of full disclosure, it also suggests that the preferred disclosure policy *ex ante* is one of no disclosure. For example, from the discussion above we know that on the basis of establishing a threshold level of disclosure $\hat{Y}$, the informed firm's expected revenue *before* $\tilde{Y}$ is known or observed (i.e., *ex ante*) is

$$\begin{aligned} E[E[\tilde{x}_I^D \tilde{P}^D | \tilde{Y} &= Y] | \tilde{Y} > \hat{Y}] + E[E[\tilde{x}_I^N \tilde{P}^N | \tilde{Y} \leq \hat{Y}] | \tilde{Y} \leq \hat{Y}] \\ &= \frac{1}{2k} \int_{\hat{Y}}^{k} \frac{1}{9} (\alpha + \beta Y)^2 \, dY \end{aligned}$$

[76] To see that there is indeed a positive association, note that when $\alpha \geq \beta k$ and the threshold level of disclosure implies full disclosure (i.e., $\hat{Y} = -k$), then $\frac{d}{dY} x_I^D P^D = \frac{2}{9}\beta(\alpha + \beta Y) \geq 0$ and $\frac{d}{dY} x_I^N P^N = \frac{1}{6}\beta\left(2\alpha + 3\beta Y - \frac{1}{2}\beta\left(\hat{Y} - k\right)\right) = \frac{1}{6}\beta(2\alpha + 3\beta Y + \beta k) \geq 0$ for all $Y \in [-k, k]$. Thus, as in model #1, high realizations of $\tilde{Y} = Y$ can be interpreted unambiguously as "good news" because they indicate high revenue next period.

$$+\frac{1}{2k}\int_{-k}^{\hat{Y}}\frac{1}{36}\left(4\alpha^2+4\alpha\beta(\hat{Y}-k)+\frac{7}{4}\beta^2(\hat{Y}^2-\frac{2}{7}\hat{Y}k+k^2)\right)dY$$
$$= \frac{1}{864k}\left(96\alpha^2k+37\beta^2k^3+5\beta^2\hat{Y}^3+15\beta^2\hat{Y}^2k+15\hat{Y}\beta^2k^2\right).$$

Next, note that the derivative of this function with respect to $\hat{Y}$ is

$$\frac{15\beta^2}{864k}\left(\hat{Y}+k\right)^2>0.$$

This result implies that expected revenue is unambiguously increasing as the informed firm *increases* its threshold level of disclosure, $\hat{Y}$. In effect, expected revenue and shareholders' welfare are maximized when the firm precommits to a policy of no disclosure (or, alternatively, prohibits the manager from becoming informed in the first place).[77] In short, the (inevitable) *ex post* policy of full disclosure is in obvious conflict with the preferred *ex ante* policy of no disclosure.

The fact that this model suggests an *ex post* policy of full disclosure, despite the apparent inefficiency of this arrangement *ex ante*, leaves open the question as to whether there exists either some unstated or unmodeled benefit to exercising disclosure with some discretion. But a full discussion of this is left for the next essay. Before we get there, in our next voluntary disclosure model we need to address another concern. Both models #1 and #2 were premised on the assumption that managers and/or firms were concerned exclusively with the market's current valuation of the firm. As discussed previously, this assumption is controversial. How can we expand these models to restrict the concerns of firms and/or managers to the expected (or future or liquidating) value of the firm?

Maximizing Expected Firm Value (Model #3).[78] To incorporate opti-

[77]This result should be equally obvious in the context of a model with constant proprietary costs (e.g., model #1). For example, with constant proprietary costs, the only effect of disclosure is to reduce expected net revenues by c. See, for example, corollary 5 of Verrecchia [1990a].
[78]In conjunction with model #3, see Clinch and Verrecchia [1997].

mization over a firm's expected value, consider the following alternative characterization of price. Now I assume that there are two informed firms, the first of which sells its goods in a product market in which demand is characterized by a price $P_1 = \alpha + \beta\tilde{Y} - x_1 - \gamma x_2$, and the second of which sells its goods in a product market in which demand is characterized by $P_2 = \alpha + \beta\tilde{Y} - \gamma x_1 - x_2$, where γ can be thought to represent either the degree to which the products are substitutes or the competitiveness between the two product markets. For example, here $\gamma = 0$ indicates no competition between the two firms (i.e., each firm has a monopoly on the good that it produces), while $\gamma = 1$ indicates that both firms produce identical goods. Set $\tilde{Y} = \tilde{y}_1 + \tilde{y}_2$. Here I assume that the first firm observes (exclusively) $\tilde{y}_1 = y_1$, while the second firm observes (exclusively) $\tilde{y}_2 = y_2$, where $\tilde{y}_1$ and $\tilde{y}_2$ are both distributed uniformly between $-k$ and k. Unlike before, however, I make no assumption about the relation among α, β, and k.

As before, each firm faces a sequence of two decisions: whether to disclose its information concerning that element of total demand (i.e., $\tilde{Y}$) known to it (i.e., y_1 in the case of the first firm and y_2 in the case of the second), and, subsequently, what quantity of output to produce (i.e., x_1 and x_2). As is standard in a Cournot setting, I assume that each firm chooses its optimal quantity based on its own information plus the information voluntarily supplied by, perhaps, its competitor. In addition, I assume that each firm chooses an equilibrium reporting strategy based on rational inferences about withheld information. Unlike models #1 and #2, however, here I assume that each firm chooses the quantity that it produces *solely* to maximize expected revenue, and not current value. In other words, here market expectations of current value play no role.

Also in contrast with models #1 and #2, here I allow for negative prices and

negative quantities: that is, all of P_1, P_2, x_1, and x_2 can assume any values along the real line. The existence of negative prices and quantities requires special interpretation, and is not benign in the nature of the disclosure equilibrium I describe below. In effect, if $P_1 > 0$, then firm 1 can produce a positive quantity $x_1 > 0$ for positive revenue of $x_1 P_1 > 0$. Alternatively, if $P_1 < 0$, then firm 1 can produce a negative quantity $x_1 < 0$ for positive revenue of $x_1 P_1 > 0$. One way to interpret negative quantities that lead to positive revenue is that they characterize circumstances in which a firm is paid a fee for storing, or withdrawing from the market, a good with socially undesirable features (e.g., surplus grain, radioactive waste materials, etc.). One consequence of negative prices and negative quantities is that, unlike model #2, the equilibrium does not unravel. Specifically, in this model one can show that a unique equilibrium exists in which firm i discloses y_i when it is in the interval

$$[\frac{\gamma\beta\hat{y}_i - 4\alpha}{\beta(4+\gamma)}, \hat{y}_i],$$

where $\hat{y}_i$ solves

$$\hat{y}_i = E[\tilde{y}_i | y_i \notin [\frac{\gamma\beta\hat{y}_i - 4\alpha}{\beta(4+\gamma)}, \hat{y}_i]],$$

and withholds y_i otherwise. Note that a necessary condition for the existence of such an interval is that $0 < \hat{y}_i < 4\frac{\alpha}{\beta\gamma}$. In other words, any potential equilibrium disclosure policy must involve disclosing realizations of $\tilde{y}_i$ that form an interval that straddles 0, the unconditional expected value of $\tilde{y}_i$. Stated somewhat differently, a (potential) disclosure equilibrium consists of withholding "dramatic news" (i.e., realizations of $\tilde{y}_i$ in the tails of their distribution) and disclosing "anticipated news" (i.e., realizations of $\tilde{y}_i$ surrounding its unconditional mean). To understand this result, note that firm i is indifferent between disclosure and non-disclosure when $y_i = \hat{y}_i$ because j's expectations remain unchanged whether y_i is disclosed or not. Now if

firm i observes $y_i > \hat{y}_i$ and does not disclose, firm j will set its production levels based on it expectations (i.e., $\hat{y}_i$), whereas i will know that demand is greater and hence be able to exploit underproduction by its rival. This explains why i hides realizations of $\tilde{y}_i$ greater than $\hat{y}_i$. In contrast, as realizations move below $\hat{y}_i$, firm i initially suffers from non-disclosure for exactly the opposite reason: that is, firm j sets production too high relative to i's knowledge of demand conditions, pushing prices down for both firms. This results in firm i disclosing realizations of $\tilde{y}_i$ immediately below $\hat{y}_i$. But, as realizations get further below $\hat{y}_i$, firm i also reacts by reducing its production levels; this dissipates the relative negative impact of non-disclosure. Furthermore, when realizations of y_i are sufficiently below $\hat{y}_i$, firm i benefits from j's overproduction: in effect, i benefits from j's inclination to overproduce at a time of low demand because i can earn positive revenue by choosing a negative quantity x_i in conjunction with a negative price. As a result, firm i withholds y_i values substantially below j's expectations.

To digress briefly, one appealing notion of this model is that it promulgates the notion of a "U-shaped" disclosure region: information in the tails is withheld while "anticipated news" is disclosed. *Prima facie*, as a characterization of discretionary disclosure in real institutional settings, this has some appeal. More importantly, with this model we can address the relation between disclosure and competition. Specifically, one can show that both the size of the disclosure region and the (*ex ante*) probability of disclosure decrease as the degree of competition, as manifest in γ, increases.[79] In other words, at least in this model, more competition implies less disclosure. But remember that as appealing as these features may be, nonetheless they are artifacts of a particular model. Features contrary to these are just as likely

[79]See, for example, corollary 1 of Clinch and Verrecchia [1997].

to occur in models with different, but equally appealing, assumptions.

Furthermore, regardless of how we feel about this particular model, note that it is still the case that there exist *ex ante* precommitment arrangements (i.e., arrangements before the information arrives) that dominate allowing the managers and/or the firm the discretion to disclose *ex post* (i.e., after y_i and y_j are observed). For example, if competition between the two firms is not severe, both firms do better by precommitting to a policy of full disclosure in advance of receiving their information.[80] In other words, despite the advantages of model #3 over the two prior models as a characterization of discretionary disclosure, it does not reconcile completely discretionary disclosure arrangements with *ex ante* disclosure choices.

Before concluding this essay, it is useful to explore the role of one, final assumption maintained throughout models #1-3: if the manager and/or firm chooses to disseminate his private information, he does so truthfully. As alluded to in the survey of prior research on disclosure, there has been some attempt in the literatures of both accounting and economics to understand the implications of relaxing this assumption. These attempts fall under the broad auspices of "cheap-talk" games.

Disclosure in "Cheap-Talk" Settings (Model #4).[81] Consider a duopolistic setting where one firm observes some proprietary, non-verifiable information Y about the next period's price of a product that it produces, but its competitor does not. Assume that Y is a realization of a random variable $\tilde{Y}$ distributed uniformly between 0 and 1. The informed firm wishes to send a message, m, where m is some element on the real line, about Y to its competitor so that they can coordinate better their production levels. Despite an intent to coordinate production levels, I also

[80] By not "too severe," I mean specifically $\gamma < 2\left(\sqrt{2}-1\right)$: see corollary 2 of Clinch and Verrecchia [1997].

[81] In conjunction with model #4, see Crawford and Sobel [1982].

assume that because of some element of (unmodeled) competition between the firms, firms' interests are misaligned. Specifically, the informed firm chooses m to maximize $-\left(y-\left(\tilde{Y}+b\right)\right)^2$, whereas the competitor chooses y to maximize $-\left(y-\tilde{Y}\right)^2$, where y denotes the uninformed firm's expectation of $\tilde{Y}$ given the informed firm's message m, and an exogenous parameter b ($b \neq 0$) reflects the extent to which firms' interests are aligned.[82] In contrast to prior models, the salient feature of this game is that the message chosen by the informed firm, in the sense of a message m, and how the uninformed competitor chooses to implement this message, in the sense of choosing a y, are wholly endogenous: this is what is meant by the expression "cheap-talk." In other words, there is no requirement that the informed firm truthfully report; similarly, there is no requirement that an uninformed competitor accept the informed firm's message at face value (i.e., as a truthful disclosure).

As it happens, all equilibria in this communication game are partition equilibria. That is, in a partition equilibrium the informed firm partitions the support of the private information $\tilde{Y}=Y$ into N elements, $\{a_0(N)=0,...,a_i(N),...,a_N(N)=1\}$, where $1 \leq N \leq N(b)$, and sends a message revealing the interval containing $\tilde{Y}=Y$, say a message claiming that $Y \in (a_i(N), a_{i+1}(N))$. The firm does not communicate, however, its full knowledge of the realization of $\tilde{Y}=Y$. The uninformed firm, upon receiving this (noisy) message, interprets this message (correctly) as suggesting that the conditional expectation of $\tilde{Y}$ is $Y=\frac{a_i(N)+a_{i+1}(N)}{2}$. In other words, on the one hand the informed firm sends a message that is "truthful" in the sense that it does not misrepresent the interval in which Y lies, but the message is nonetheless "vague." On the other hand, the uninformed firm chooses to interpret the message "correctly"

[82]The informed firm (perfectly) observes $\tilde{Y}=Y$ in this setting. Fischer and Stocken [2000] consider a setting where the informed firm has imperfect information about $\tilde{Y}=Y$. They establish that the quality of the uninformed firm's information about $\tilde{Y}=Y$ is maximized when the informed firm has coarse or imperfect information.

in that the conditional expectation of $\tilde{Y}$ is indeed $\frac{a_i(N)+a_{i+1}(N)}{2}$ when $\tilde{Y}$ is uniformly distributed in the interval $(a_i(N), a_{i+1}(N)))$. Note the role misaligned interests play in determining equilibria. The maximum amount of information that can be communicated potentially in equilibrium, measured using the residual variance of $\tilde{Y}$ that the uninformed firm expects after hearing the equilibrium message, decreases as the misalignment of the competing firms' incentives, b, increases. For example, when $b = 0$ there is no misalignment and full revelation results (in equilibrium). Alternatively, when $b \to \infty$, misalignment is total and there is no possibility of communicating any information between the informed and uninformed.

In the context of the discussion above, the interesting feature of a "cheap-talk" equilibrium is that despite the fact that the informed firm's disclosure is non-verifiable and without cost, the informed firm sends a message, albeit noisy, to the uninformed firm for all realizations of $\tilde{Y}$. This result differs from the equilibria characterized for models #1-3 where for some realizations of $\tilde{Y}$ disclosure occurs, whereas for others it does not. In short, a "cheap-talk" equilibrium comports nicely with the notion that in practice managers and/or firms comment on everything, but in a fashion whereby proprietary information is always disclosed with some element of vagueness.

Summary. While I have alluded to many deficiencies in the context of the discretionary-based disclosure literature, the provocative feature of this literature is that it has changed the way researchers in accounting think about disclosure, while at the same time offering conclusions that seem immediate. The main conclusion is particularly compelling: in the presence of costs and/or uncertainty, broadly defined, managers will elect to disclose or withhold information about firm value despite the fact that agents outside the firm interpret withheld information rationally. In other words, this literature tells a compelling economic story about the incentives on the

part of the manager or firm to disclose voluntarily. This strength seems to overcome weaknesses that include: a reliance on the assumptions in some models that managers seek to maximize current market value (as opposed to future value) and truthfully report; the fact that results in the literature are highly sensitive to assumptions; and that discretionary disclosure strategies, *per se*, are typically inefficient in that the firm does better by precommitting never to disclose. But having alluded to the potential inefficiency of discretionary-based disclosure models, let me use this as a segue to the next essay.

4 Efficiency-Based Disclosure

What disclosure arrangements or strategies are preferred unconditionally: that is, without prior knowledge of the information? As has been discussed previously, association-based research is premised on the notion that disclosure is exogenous. Discretionary-based research posits endogenous disclosure arrangements, but with no requirement that they be preferred *ex ante*. Therefore, having discussed those two topics in some detail, this is an opportune time to ask whether there exist disclosure arrangements that would also have this feature. In the context of this discussion, I refer to such arrangements as "efficient": that is, efficient disclosure arrangements are those that are preferred unconditionally. Notions of efficiency are central to economics. Therefore, if one objective of the disclosure literature is to forge a link between financial reporting and economics, failure to integrate efficiency into the discussion may be a fatal oversight.

This is not to suggest that discussions of the relation between disclosure and efficiency have never entered the domain of accounting research. Arguably, the earliest theory-based, economic analyses of disclosure in capital markets concerned how

disclosure affected an economy's social welfare in pure exchange economies.[83] In particular, early work examined the extent to which (unconditional) disclosure choice achieved (weak) Pareto improvements in markets of perfect competition and pure exchange: that is, circumstances in which the disclosure yielded no productive benefits on its own account. This early literature, and much of the controversy this work engendered, has been discussed previously.[84] Nonetheless, it is useful to review briefly some of the themes in this early literature as a segue into more contemporary thought.

Pareto improvement is a very strong welfare criterion in a pure exchange economy setting. In the context of disclosure and even in its weakest form (i.e., weak Pareto improvement), it requires that disclosure make no investor agent who participates in the market worse off even in the event that other market participants unambiguously benefit. From the very start, Pareto improvement, pure exchange, and disclosure seemed incompatible. One reason for this is that disclosure benefits the less well informed at the expense of the better informed. But another, and perhaps more subtle, reason is that the assumption of perfect competition in combination with pure exchange leaves little opportunity for disclosure to yield a benefit.

To understand this last point, recall from the first essay that perfect competition assumes that each investor agent behaves as if his or her actions or behaviors have no effect on price, and in equilibrium this conjecture is true. Perfect competition is achieved typically by assuming that the number of investor agents is large (say, countably infinite). This ensures that while prices reflect the combined deci-

[83]See, for example, the trio of papers Ng [1975, 1977] and Hakannson, *et al.* [1982]. Note that while these papers were published in finance journals, the authors themselves held accounting faculty appointments at the time of their publication, and hence these papers are representative of accounting thought at that time. This is also true of the Kunkel [1982], a paper cited in a subsequent footnote.
[84]See Verrecchia [1982b].

sions of all market participants at an aggregate level, the actions of each individual agent have no effect on price because of his or her atomistic feature. By all accounts perfect competition is a reasonable assumption about markets that are deep and/or assets that are widely traded. Its role in welfare analyses, however, is not benign. When markets are perfectly competitive, disclosure's primary effect is to redistribute wealth among market participants. For example, consider the consequences of a "good news" disclosure concerning the value of an asset. A "good news" disclosure makes individuals who are overweighted in that asset better off and individuals who are underweighted in that asset worse off (where the notions of overweight and underweight are relative to some norm, such as their per-capita share of the risky asset absent different beliefs or expectations about the asset's uncertain value). Similarly, "bad news" disclosures do the reverse. Thus, to the extent to which markets are populated by agents who are risk-averse, a consequence of an anticipated disclosure is that it makes market participants collectively worse off (in expectation). This is referred to as the adverse risk-sharing effect of increased disclosure. Of course, in complete markets where all anticipated events can be contracted over through trading in advance of the events, effectively market participants can insure themselves against adverse risk-sharing consequences. But this would only ensure that disclosure has no beneficial role in markets that are complete, in conjunction with a debilitating role in markets that are less than perfectly complete. Consequently, these results inclined much early research to conclude that the benefits of disclosure were at best illusive, and at worst harmful.[85]

It should come as no surprise to suggest that researchers were less than fully

[85]See, for example, the discussions in Hirshleifer [1971] and Marshall [1974]. For example, Marshall [1974, p.380] states: "If the impact of information is insured before its arrival, that insurance precludes further trade based on the news...In the contrary case when the news must arrive before its impact is insured in a preliminary market, the information is harmful."

satisfied with this conclusion, and interest in a welfare role for disclosure remained keen. At this juncture, however, the literature seems to have bifurcated down two distinct paths. The first path was to suggest that the problem lay with the maintained assumption of a pure exchange economy. That is, if one allowed for *production* and exchange, there existed conditions under which disclosure would be preferred because altered production plans lead to more efficient allocation of resources across time and firms.[86] In effect, this research path suggested sufficient conditions for disclosure to yield Pareto improvements when employed in conjunction with production. The second path was to suggest that the problem lay with the maintained assumption of *costless* private information acquisition. That is, a welfare role for disclosure could be posited in an exclusively pure exchange economy by suggesting that one potential benefit from costless public disclosure is that it may preclude costly private information acquisition.[87] In effect, this research path explored whether, by reducing or eliminating incentives to become privately informed at some cost, costless public disclosure made investors better off despite adverse risk-sharing effects.

The two papers most representative of these two research paths were published in the same (very prominent) journal within a few years of one another.[88] Despite this, the one that suggested that the problem lay with the maintained assumption of pure exchange seems to have fallen into obscurity, whereas the one that suggested that the problem lay with the maintained assumption of costless private information acquisition spawned considerable interest, especially among accounting researchers.[89] Why? If I were permitted to speculate (and here I am truly speculating and not

[86]See Kunkel [1982].
[87]See, Verrecchia [1982b; specificaly, pp. 29-37] and Diamond [1985].
[88]Kunkel [1982] and Diamond [1985], respectively: both were published in the *Journal of Finance*.
[89]Work in accounting inspired by Diamond's [1985] paper includes Bushman [1991], Indjejikian [1991], Lundholm [1991], and Alles and Lundholm [1993]. To give some indication of how quickly Kunkel's work fell into obscurity, note that Diamond [1985] does not even cite Kunkel [1982], despite

offering a critique of the papers), my explanation would be that researchers had long recognized that production militates against all potential debilitating effects of disclosure, including adverse risk sharing.[90] Consequently, the path that promoted disclosure as a device to yield social value in production and exchange economies was deemed insufficiently provocative. Alternatively, paths that promoted a utility for disclosure in (exclusively) pure exchange settings remained popular because they appeared to be addressing the "disclosure paradox": that is, explaining why it was *not* the case that more disclosure was bad, and not good.[91] Consequently, this remained the primary focus of research endeavors in accounting.[92]

Despite the popularity of work that promoted disclosure as a device to thwart the acquisition of costly private information, the research itself was not immune from criticism. Subsequent work questioned whether it actually solved the "paradox." Let me mention three concerns. First, if one makes market agents (e.g., investors, shareholders, etc.) sufficiently diverse, it is difficult, if not impossible, for disclosure to yield a positive benefit for everyone. Consequently, the results of this path only seem to have applied to settings in which investor agents were fairly homogeneous. For example, an important paper in this literature shows that better informed shareholders in a firm always prefer less firm disclosure than less well-informed shareholders.[93] This is due to heterogeneity in the adverse risk-sharing and beneficial cost-saving effects of

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the fact that Diamond's work is in the same journal and published only three years later.

[90]For example, more than a decade before Kunkel [1982], Hirshleifer [1971, p.567] had emphasized this point: "*Public information ... is indeed of social value in a regime of production and exchange*," [original emphasis].

[91]For example, Marshall [1974, p. 382] states: "The argument has been that public information is valueless and private information valuable, leading to inefficient allocation of resources by overspending on information. This might imply a policy of suppressing these kinds of information...the logic is compelling...but it seems a *paradox* that more information should be bad instead of good..." [emphasis added].

[92]Once again, see Bushman [1991], Indjejikian [1991], Lundholm [1991], and Alles and Lundholm [1993].

[93]See Kim [1993].

disclosure among shareholders with different risk tolerances and different information acquisition cost functions.[94] Second, one technical feature common to all the papers in this area is that they are single-period models. That is, the decision to publicly disclose is made in conjunction with the decision to acquire private information, the decision to rebalance one's portfolio, etc. As alluded to in the first essay, assessing the effects of disclosure in the context of a single period model of trade risks commingling a host of factors that may obfuscate or obscure disclosure's role.[95] Third, this work ignores the possibility that when public disclosure is costly and firms compete for shareholders' attention, firms may actually have an incentive to disclose too much information: that is, more than the socially optimal amount.[96]

The Information Asymmetry Component of the Cost of Capital. Allow me to summarize the discussion to this point. What started out originally as a literature that sought normative prescriptions for Pareto improvements among all investor agents who participate in the economy metamorphosed into one of individual firms making disclosure choices to maximize the expected utilities of exclusively their own shareholders. It remained the case, however, that the focus of the efficiency literature continued to be on markets that were perfectly competitive. Allowing for the fact that changing the maintained assumption from one of pure exchange to one of production and exchange may yield efficiency gains that offer a rationale for disclosure, for the remainder of this essay I explore an alternative way to link disclosure to efficiency. Specifically, I explore what happens when one changes the maintained assumption

[94]In effect, Kim [1993] shows that the results in Verrecchia [1982b] and Diamond's [1985] rely critically on investors having homogeneous economic features. Indjejikian [1991, p.294] acknowledges the role of homogeneity in his work: he states, "The high degree of investor homogeneity is an unfortunate limitation of this [i.e., his] study."

[95]See specifically my discussion of the motivation for model #4 in the first essay. This limitation was widely acknowledged: see, e.g., Bushman [1991]. Nonetheless, some papers, e.g., Alles and Lundholm [1993], also discuss why they do not believe it to be a fatal flaw.

[96]See, for example, Fishman and Hagerty [1989].

from one of perfect competition to one of *im*perfect competition.

In primary capital markets, equity shares of a firm are sold to investors to raise cash proceeds for investment. One disclosure-related cost that inhibits investment and hence makes firm equity sales more costly is a transaction cost that arises from the adverse-selection problem inherent in the exchange of assets among investor agents of varying degrees of informedness. I refer to this transaction cost as the "information asymmetry component of the cost of capital." The information asymmetry component of the cost of capital is the discount that firms provide as a means of accommodating the adverse-selection problem. As such, it does not manifest itself in perfectly competitive markets because there is no adverse-selection: the purchase and sale of firm equities by individual, investor agents has no effect on price. In other words, perfect competition ensures that a well informed investor will be able to exchange assets with a less well informed agent, without being penalized in any way by the fact that, on average, the former will always profit at the expense of the latter. Alternatively, in models of *im*perfect competition, the actions or behaviors of each investor are assumed to be sufficiently substantive in relation to the market as a whole as to guarantee that these actions will have an effect on the price at which trades are executed. In short, the salient feature of imperfect competition is that all investor agents may be required to pay or offer some "liquidity premium" when assets are exchanged, so as to protect those on the other side of the transaction against the adverse-selection problem inherent in the exchange of assets among different agents with varying degrees of informedness. All agents pay or offer the liquidity premium because it is assumed that market "type" (i.e., the extent of an agent's informedness) is unknown or unobservable. In effect, a liquidity premium is a transaction cost absorbed by all agent-types, independent of how well or poorly informed each is in

relation to the market as a whole.

If investors hold their shares until the firm liquidated, they should be unconcerned about how transaction costs that arise from the exchange of asset shares prior to liquidation will affect their proceeds. If, however, investors anticipate that they may sell some shares prior to liquidation, or buy additional shares (through, say, dividend reinvestment programs or otherwise), then they should factor these transaction costs into what they are willing to pay to hold shares initially. The higher the anticipated transaction costs, the less investors will pay initially, and hence the lower the proceeds a firm receives for investment and production when shares of the firm were sold in a primary capital market. Consequently, in the interests of efficiency it is to the firm's advantage to reduce information asymmetry so as to reduce the information asymmetry component of the cost of capital. One way to achieve information asymmetry reduction is for the firm to commit to the highest level of public disclosure at the time shares in the firm are first offered. Specifically, the firm could commit to preparing its financial statements using: the most transparent set of accounting standards (e.g., a multi-national firm electing the standards of the International Accounting Standards Committee versus some less transparent alternative); the most transparent procedures within a particular set of standards (e.g., purchase versus pooling, capital leases versus operating leases); or listing on exchanges that attract the greatest analyst or investor following (e.g., the New York Stock Exchange versus the American Stock Exchange).

A Modeling Vignette on Disclosure and the Cost of Capital.[97] To summarize the discussion so far, my proposal for linking disclosure to efficiency is through the information asymmetry component of the cost of capital. But what exactly does

[97]In conjunction with this vignette, see Diamond and Verrecchia [1991] and Baiman and Verrecchia [1996].

one mean by the expression "information asymmetry component of the cost of capital"? What *I* mean by this expression is the factor by which investors discount firm equity offerings in anticipation of transaction costs that may arise from adverse selection; these are transaction costs that original, equity-holder investors must bear in the event that they liquidate their equity holdings at some future date. The factor by which investors discount firm equity offerings to accommodate these transactions costs makes investment by the firm more costly. While I believe that my explanation of the "information asymmetry component of the cost of capital" is cogent, by the same token I am aware of the fact that this is a term that lends itself to disparate interpretations. Consequently, the purpose of the following vignette is to illustrate this concept in an example that retains some of the same spirit and flavor of vignettes from previous essays.

Of course, if a firm benefits from a commitment to greater disclosure through the reduction in its cost of capital, then why would there be an information asymmetry cost component? In other words, what would preclude a firm from choosing the corner solution of full disclosure, thereby eliminating any potential cost? Presumably, managers and/or firms do not choose the full-disclosure corner solution because there are costs that countervail against that choice. In the literature, examples of countervailing economic forces that lead to interior disclosure choices (i.e., less than full disclosure) include risk sharing and agency costs.[98] Interestingly, however, nowhere in the literature can one find a discussion predicated on what is perhaps the most obvious device to ensure an interior solution: proprietary costs. Consequently, an ancillary purpose of this example is to show how proprietary costs work in this con-

[98]See Diamond and Verrecchia [1991] for the former and Baiman and Verrecchia [1996] for the latter. Of course, in the absence of any countervailing force, nothing precludes full disclosure from becoming the corner solution. See, for example, Bushman, *et al.* [1996] (see also Verrecchia [1996a]). The issue of corner solutions is also discussed in Verrecchia [1999].

text. In short, the motivation for the following vignette is to illustrate the concept of the information asymmetry component of the cost of capital and to show how the existence of proprietary costs may lead to a disclosure policy in the interior (that is, disclose in some circumstances, and withhold in others).

Consider an entrepreneur who owns the process to sell a good in a market in which there is another competitor. To initiate the manufacturing process, however, in the first period the entrepreneur needs to raise C dollars of capital. To raise the capital, the entrepreneur offers to sell a percentage Q of the firm to a risk-neutral investor. The entrepreneur's objective is to maximize his return from owning part of the firm's revenue generating activities after selling Q percent of the firm to the investor in exchange for C dollars of capital. I interpret that part of the firm's revenue generating activities that the entrepreneur sells off as his cost of capital.

To begin, I reintroduce the Cournot duopoly product market game discussed in model #3 in the previous essay (with perfect competition, i.e., $\gamma = 1$). Each firm invests in the current period in producing some good, in anticipation of the fact that the good will sell for a price P in a future period (say, the second period), where P is represented by

$$P = \alpha + \beta\tilde{Y} - x_e - x_o,$$

where α and β are fixed, positive constants, $\tilde{Y}$ is some proprietary information about the anticipated price that is known only to the entrepreneur, and x_e and x_o are the amounts produced by the entrepreneur's firm and the other firm, respectively. Each firm makes a decision as to what to produce without knowledge of the amount the other firm produces. Also as before I represent $\tilde{Y}$ as a random variable that is uniformly distributed between $-k$ and k.

A factor that affects an investor's decision to invest with the entrepreneur is that

she anticipates a liquidity shock with probability t in the second period to either purchase or sell one share of the firm. After the C dollars of initial capital are raised, purchases or sales of shares in the entrepreneur's firm take place in a secondary market. The secondary market is populated by equal numbers of informed traders who also know $\tilde{Y} = Y$ and uninformed, or liquidity traders, who have no knowledge of $\tilde{Y} = Y$ unless this information is disclosed by the entrepreneur. And in all cases, trading is restricted to buying or selling one share in the firm.

Trades for firm shares in the secondary market are executed by a large number of market makers, each of whom has the responsibility of executing one demand order (to buy or sell one share) in the second period. Market makers also do not know $\tilde{Y} = Y$ unless it is disclosed, and I assume that with probability $\frac{1}{2}$ a market maker executes an informed trader's demand order, and with probability $\frac{1}{2}$ he executes an uninformed demand order. Consequently, market makers charge a fee to ensure that they break even in executing trades. This fee can be interpreted as the "liquidity premium" charged for executing trades in the presence of adverse selection.

To reduce this liquidity premium and make investment in his firm potentially more attractive, in the first period the entrepreneur commits to disclosing $\tilde{Y} = Y$ in the second period if $\tilde{Y} = Y \in [-q, q]$ and withholding it if $\tilde{Y} = Y \in [-k, -q] \cup [q, k]$. At an intuitive level, one can think of the entrepreneur's commitment as a decision to disclose "anticipated news" (i.e., $-q \leq Y \leq q$) but to withhold "dramatic news," that is, realizations of $\tilde{Y}$ in the tails of its distribution.[99] Consequently, the higher (lower) the value of q (keeping in mind that $0 \leq q \leq k$), the more (less) disclosure to which the entrepreneur commits. In the event that $\tilde{Y} = Y$ is disclosed, market makers do not charge a fee because there is no information asymmetry. In the event

[99]See, once again, Clinch and Verrecchia [1997].

that $\tilde{Y} = Y$ is withheld, however, a liquidity premium is charged.

The trade-off for the entrepreneur should be clear. In choosing a high q, he chooses greater disclosure, makes markets more liquid, and thus reduces the information asymmetry component of the cost of capital for a potential investor in the primary market for equity offerings. But also in choosing a high q, the entrepreneur increases proprietary costs by allowing his competitor to choose a more efficient production schedule.

As before, if the entrepreneur discloses $\tilde{Y} = Y$, both his firm and the other firm produce

$$x_e^D = x_o^D = \frac{1}{3}(\alpha + \beta Y),$$

and the price at which the goods sell is

$$P^D = \alpha + \beta Y - x_e - x_o = \frac{1}{3}(\alpha + \beta Y).$$

If the entrepreneur does not disclose $\tilde{Y} = Y$, then the other firm does not know this value. Consequently, because an undisclosed $\tilde{Y}$ is uniformly distributed between $[-k, -q]$ and $[q, k]$, $\tilde{Y}$ (undisclosed) can only be interpreted based on its conditional expectation (which is 0) . Therefore, here

$$\begin{aligned} x_e^N &= \frac{1}{3}\alpha + \frac{1}{2}\beta Y, \text{ and} \\ x_o^N &= \frac{1}{3}\alpha, \end{aligned}$$

and the price at which the goods sell is

$$P^N = \alpha + \beta Y - x_e - x_o = \frac{1}{3}\alpha + \frac{1}{2}\beta Y.$$

When the entrepreneur discloses, he earns revenue of

$$x_e^D P^D = \frac{1}{9}(\alpha + \beta Y)^2$$

when $\tilde{Y} = Y$. When the entrepreneur does not disclose, he earns revenue of

$$x_e^N P^N = \frac{1}{9}\left(\alpha + \frac{3}{2}\beta Y\right)^2$$

when $\tilde{Y} = Y$. To digress briefly, note that this implies that independent of other considerations, a strategy of never disclosing always dominates a strategy of always disclosing, because

$$\begin{aligned} E[\tilde{x}_e^N \tilde{P}^N] &= \frac{1}{9}\left(\alpha^2 + \frac{3}{4}\beta^2 k^2\right) \\ &> \frac{1}{9}\left(\alpha^2 + \frac{1}{3}\beta^2 k^2\right) \\ &= E[\tilde{x}_e^D \tilde{P}^D]. \end{aligned}$$

The policy of disclosing when $Y \in [-q, q]$ and not disclosing when $Y \in [-k, -q] \cup [q, k]$ yields expected revenue of

$$\begin{aligned} & E\left[\frac{1}{9}\left(\alpha + \beta\tilde{Y}\right)^2 | Y \in [-q, q]\right] \times \Pr\left(Y \in [-q, q]\right) \\ & + E\left[\frac{1}{9}\left(\alpha + \frac{3}{2}\beta\tilde{Y}\right)^2 | Y \in [-k, -q] \cup [q, k]\right] \times \Pr\left(Y \in [-k, -q] \cup [q, k]\right) \\ = \; & \frac{1}{9k}\left(\alpha^2 k + \beta^2\left(\frac{3}{4}k^3 - \frac{5}{12}q^3\right)\right). \end{aligned}$$

Henceforth define $R(q) = \frac{1}{9k}\left(\alpha^2 k + \beta^2\left(\frac{3}{4}k^3 - \frac{5}{12}q^3\right)\right)$ as the entrepreneur's expected revenue as a function of his disclosure choice q. Note that expected revenue function, $R(q)$, is decreasing in q. This is what one would expect: in the presence of proprietary costs expected revenue declines as the entrepreneur elects greater disclosure.

But another consequence of nondisclosure is that the market also does not know $\tilde{Y} = Y$ when $Y \in [-k, -q] \cup [q, k]$. Consequently, in the event of nondisclosure the market values the entrepreneur's expected revenue as

$$\begin{aligned} E\left[\tilde{x}_e^N \tilde{P}^N | Y \in [-k, -q] \cup [q, k]\right] &= E\left[\frac{1}{9}\left(\alpha + \frac{3}{2}\beta\tilde{Y}\right)^2 | Y \in [-k, -q] \cup [q, k]\right] \\ &= \frac{1}{9}\left(\frac{3}{4}\beta^2 q^2 + \frac{3}{4}q\beta^2 k + \alpha^2 + \frac{3}{4}\beta^2 k^2\right). \end{aligned}$$

But informed traders know $\tilde{Y} = Y$. This implies that whenever $\tilde{Y} = Y$ is not disclosed, an informed trader who executes a trade in the secondary market earns a return based on the difference between the market's *expectation* of the firm's revenue, $E[\tilde{x}_e^N \tilde{P}^N | Y \in [-k,-q] \cup [q,k]]$, and the *actual* revenue of $\frac{1}{9}\left(\alpha + \frac{3}{2}\beta Y\right)^2$. That is, in the event of nondisclosure, an informed trader expects to earn the following amount as a function of the entrepreneur's disclosure choice q

$$\lambda(q) = \frac{1}{2(k-q)} \int_{-k}^{-q} \left| \frac{1}{9}\left(\alpha + \frac{3}{2}\beta Y\right)^2 - \frac{1}{9}\left(\alpha^2 + \frac{3}{4}\beta^2 q^2 + \frac{3}{4}q\beta^2 k + \frac{3}{4}\beta^2 k^2\right) \right| dY$$
$$+ \frac{1}{2(k-q)} \int_{q}^{k} \left| \frac{1}{9}\left(\alpha + \frac{3}{2}\beta Y\right)^2 - \frac{1}{9}\left(\alpha^2 + \frac{3}{4}\beta^2 q^2 + \frac{3}{4}q\beta^2 k + \frac{3}{4}\beta^2 k^2\right) \right| dY.$$

This means that in the event of nondisclosure, market makers must charge each trader (equivalently, each transaction) a liquidity premium of $\frac{1}{2}\lambda(q)$ so as to break even in a market populated by a 50%-50% mix of informed and uninformed traders.

Now I return to the investor's problem. The investor contributes capital of C and, in return, she expects to receive a percentage $Q(q)$ of the entrepreneur's expected revenue, which is $R(q)$. In addition, with probability t the investor receives a liquidity shock to purchase or sell more of the firm. Note that in the event of a liquidity shock, the investor purchases or sells shares in the firm at the firm's expected value. Consequently, the only effect of a liquidity shock on the investor's expected return is that, in addition, she must pay the liquidity premium of $\frac{1}{2}\lambda(q)$. Assume that competition to invest in the entrepreneur's firm is perfect; hence, investors can only hope to break even when they invest C dollars of capital. Taken together, all of these imply that the investor's expected payout for investing with the entrepreneur is

$$Q(q)R(q) - C - \frac{k-q}{k} t \frac{1}{2}\lambda(q),$$

where $\frac{k-q}{k}t$ represents the probability that the investor receives a liquidity shock during a period in which the entrepreneur happens not to be disclosing. Therefore,

to "break even" in this arrangement, the investor must receive the percentage $Q(q)$, where $Q(q)$ is determined by

$$Q(q) = \frac{C + \frac{k-q}{2k} t\lambda(q)}{R(q)};$$

the entrepreneur receives the residual, $1 - Q(q)$. The key feature of this analysis is that potential investors rationally anticipate *all* the benefits and costs of investing before they purchase equity in the firm.[100]

What disclosure policy choice minimizes the entrepreneur's cost of capital? The entrepreneur's return from selling $Q(q)$ percent of the firm to the investor in exchange for C dollars of capital is

$$\begin{aligned}(1 - Q(q)) R(q) &= \left(1 - \frac{C + \frac{k-q}{2k} t\lambda(q)}{R(q)}\right) R(q) \\ &= R(q) - C - \frac{k-q}{2k} t\lambda(q),\end{aligned}$$

where the first equality results from the fact that the investor only breaks even. Consequently, the disclosure choice that minimizes the entrepreneur's cost of capital is the one that maximizes $R(q) - C - \frac{k-q}{2k} t\lambda(q)$. It is a straightforward exercise to show that when the investor is immune from liquidity shocks (i.e., $t = 0$), a policy of nondisclosure maximizes $R(q) - C - \frac{k-q}{2k} t\lambda(q)$ (i.e., $q = 0$). This results from the fact that $R(q)$ is decreasing in q. Alternatively, consider what happens when $t = 0.5$. To facilitate the solution for the q that minimizes $Q(q)$, assume that $\alpha = 0.5$, $\beta = 1$, $k = 1$, and $C = 0.05$. To illustrate the calculation of $\lambda(q)$ for these parameter values, define $F(Y, q)$ and $G(q)$ as

$$F(Y, q) = \frac{1}{6}Y + \frac{1}{4}Y^2 - \frac{1}{12}q^2 - \frac{1}{12}q - \frac{1}{12},$$

[100] This is in contrast with Huddart, *et al.* [1999], for example. There, equity holders in a firm are treated as exogenous, thereby giving them no opportunity to decide whether they want to invest in the first place.

$$G(q) = -\frac{1}{3} + \frac{1}{3}\sqrt{(3q^2 + 3q + 4)},$$

respectively. Together, these imply that $\lambda(q)$ is defined as

$$\lambda(q) = -\frac{1}{2(k-q)}\left(\int_{-1}^{-q} F(Y,q)\,dY + \int_{q}^{G(q)} F(Y,q)\,dY - \int_{G(q)}^{1} F(Y,q)\,dY\right)$$

for all $q \in [0, 0.5]$, whereas

$$\lambda(q) = -\frac{1}{2(k-q)}\left(\int_{-1}^{-q} F(Y,q)\,dY - \int_{q}^{1} F(Y,q)\,dY\right)$$

for all $q \in [0.5, 1]$. Using this expression for $\lambda(q)$, one can show that $R(q) - C - \frac{k-q}{2k}t\lambda(q)$ is maximized at $q = 0.37979$. In other words, the entrepreneur's return is maximized when the marginal benefit of disclosure equals the marginal cost (through proprietary costs); this occurs at $q = 0.37979$.[101] In addition, at this value the investor receives $Q(0.37979) = 62\%$ of the revenue generating activities of the firm, while the entrepreneur retains 38%. This implies that the entrepreneur's cost of capital is $Q(0.37979) \times R(0.37979) = 0.068$; it compares with a cost of $C = 0.05$ in the absence of an adverse selection problem (which occurs when $\lambda \equiv 0$). Consequently, in this example the information asymmetry component of the cost of capital is the difference, which is 0.018.

To summarize this example, its purpose is to illustrate the concept of the information asymmetry component of the cost of capital. The cost of capital is the percentage of the firm an entrepreneur must sell to raise a fixed amount of capital. The information asymmetry component of the cost of capital is the difference in the

[101] I determined the value of q that maximizes $R(q) - C - \frac{k-q}{2k}t\lambda(q)$ by plotting this function using the parameters values $\alpha = 0.5$, $\beta = 1$, $\gamma = 1$, $k = 1$, $C = 0.05$ and $t = 0.5$, observing that it is concave over the range $q \in [0, 1]$ for these values, and then noting that the only value for which $\frac{d}{dq}\left(R(q) - C - \frac{k-q}{2k}t\lambda(q)\right) = 0$ over $q \in [0, 1]$ is $q = 0.37979$. A more complete proof requires showing that the function $R(q) - C - \frac{k-q}{2k}t\lambda(q)$ is concave over $q \in [0, k]$ for some general class of parameter values, and then determining the q that maximizes the function for those parameter values.

cost of capital in the presence versus absence of an adverse selection problem that arises from information asymmetry. In effect, it results as a consequence of the entrepreneur's inability to commit to a policy of full disclosure because of the presence of other, disclosure-related costs (e.g., proprietary costs). An ancillary purpose of the vignette is to show how liquidity premia in combination with proprietary costs may lead to efficient disclosure choices in which the firm neither fully discloses, nor totally withholds, information. Specifically, in the example above the entrepreneur commits to some disclosure so as to mitigate problems arising from illiquid markets. He does not, however, commit to full disclosure because the proprietary costs that arise from this action are too high. Nor, for that matter, does he suppress all disclosure, as this would drive up his cost of capital precipitously.

Before concluding, let me discuss the role of one last maintained assumption. This vignette, and indeed my entire discussion throughout this essay, presumes that a commitment to more disclosure leads to less information asymmetry, and this presumption is not without controversy. For example, recall model #6 in the first essay. In that model disclosure was represented by $\tilde{y} = \tilde{u} + \tilde{\eta}$; in addition it was assumed that investors also possessed private information about $\tilde{\eta}$, in the form of $\tilde{O}_i = \tilde{\eta} - \tilde{\omega}_i$, where the $\tilde{\omega}_i$'s have a normal distribution with mean 0 and precisions w_i. Here, greater disclosure exacerbates (as opposed to ameliorates) information asymmetry among investors.[102] Thought of somewhat loosely, this characterization treats disclosure and private information gathering, and hence information asymmetry, as complements, not substitutes. While models that posit a positive relation between disclosure and information asymmetry are no more or less valid than those that posit a negative relation, the former typically speak to a type of transitory behavior that may arise around

[102] See, for example, Kim and Verrecchia [1994] and Bushman, *et al.* [1997].

the brief window of an anticipated disclosure (e.g., an earnings announcement), and not to commitments to greater disclosure over longer windows. Alternatively, the discussion in this essay speaks specifically to commitments to greater disclosure over longer windows. In short, the discussion here maintains as an assumption the notion that a commitment to greater disclosure degrades the private benefits of information gathering, and hence reduces information asymmetry.

Summary. While this essay reviews a variety of work that has attempted to link efficiency to disclosure, either in the context of social welfare or single-firm efficiency, in my opinion the one with the greatest potential is the link between disclosure and the information asymmetry reduction. To date there is very little research on this topic, either theory- or empirical-based. One explanation for the paucity of research is that establishing a link is difficult, especially in empirical studies.[103] Even in the "simple" modeling vignette offered above (with all its stylized assumptions), the link is far from transparent. Of course, an alternative explanation is that researchers are simply not aware of the issue. To the extent to which this is the case, perhaps this document will serve as a rallying cry for more work on this topic. But in view of the fact that the discussion has turned to the topic of directions for future research, perhaps it is appropriate to make our way to the final section of this paper.

5 Directions for Future Research

What research activity do I hope this document will encourage in the future? Having alluded to the absence of a comprehensive theory in the introduction, this would certainly be a worthwhile outcome. To be truly comprehensive, however, a theory

[10]Some empirical-based work that has attempted to link disclosure and cost of capital includes Welker [1995], Botosan [1997], Healy, *et al.* [1999], and Leuz and Verrecchia [2000].

must embrace efficiency, incentives, and the endogeneity of the market process as it involves interactions among diverse investor agents. For example, I view research that examines incentives to disclose in markets comprised of a single, representative trader (e.g., discretionary-based disclosure studies) as no more or less "comprehensive" than those that endogenize the market and treat disclosure as exogenous (e.g., association-based disclosure). Both approaches only look at one piece of the overall disclosure puzzle.

My suggestion for linking disclosure to efficiency, incentives, and the endogeneity of the market process is through the reduction in the information asymmetry component of the cost of capital. Information asymmetry inhibits investment, thereby making it more costly for a firm to engage in those activities for which it has been incorporated. As discussed in the previous essay, a commitment to greater disclosure reduces information asymmetry; this, in turn, lowers that component of a firm's cost of capital that arises from information asymmetry. In short, information asymmetry reduction provides a rationale for efficient disclosure choice. In this sense it may be the natural progeny of early efficiency work in accounting that attempted to find sufficient conditions for disclosure. Whether or not one accepts the latter, the notion of increasing market liquidity through information asymmetry reduction seems *prima facie* consistent with the language regulators often use when they describe the role of accounting standards as one of "leveling the playing field" and increasing "investor confidence."[104] As Arthur Levitt states:

> "high quality accounting standards result in greater investor confidence, which improves liquidity, reduces capital costs, and makes market prices possible."[105]

[104] See Sutton [1997].
[105]

I interpret this statement as speaking to the notion that a commitment to higher quality disclosure is efficient in that it leads to a reduction in the information asymmetry component of the cost of capital.

I hasten to add, however, that none of my discussion is intended to suggest that no other vehicles exist to integrate the theory of disclosure comprehensively. May the proverbial "thousand flowers bloom": if there exist more successful approaches for linking disclosure to efficiency, I will not be displeased. If this document inspires such a treatise, I will have some claim to paternity.

But another potential research activity that I hope will result from this document is empirical work that forges a link between disclosure and its economic consequences. While I am interested in all such links, let me suggest again that the one with the greatest potential may be the link between disclosure and information asymmetry reduction. While it may strike a reader as unusual for the author of paper on theory-based models to be promulgating the idea of more empirical research on the economic consequences of disclosure, I would like to see more empirical resources committed to many of the issues discussed here. Theory-based work has attributed many associations, incentives, and efficiencies to disclosure. While there is extensive empirical work on associations and incentives, efficiency, as I define that term in the context of section 4, is less well studied. It would be of some interest to know the nature and type of efficiencies that exist in real institutional settings, and, if they do exist, whether they have any economic significance. In other words, as the theory of disclosure matures it seems reasonable to inquire whether the empirical literature can provide additional insights into the economic consequences of disclosure. These insights could be especially valuable if they were premised on the variety of issues

See Levitt [1998, p. 81].

discussed in these essays.

For example, for all my enthusiasm for the information asymmetry component of the cost of capital as a starting point for a comprehensive theory, I acknowledge the difficulty of ferreting it out in real market settings. Information asymmetry, like many of the economic consequences posited in these essays, is a "second-moment" effect (i.e., a variance effect), and second-moment effects may be very secondary or tertiary in nature when compared against "first-moment" effects (i.e., mean effects). For example, one would expect to be able to document that, as a first-moment effect, "good news" drives prices up and "bad news" drives prices down. Theory-based models, however, commonly characterize information asymmetry as a second-moment effect that is unrelated to means, or first moments. Information asymmetry is commonly characterized this way because variables are posited to have a normal distribution, which implies two independent moments; obviously, for other (i.e., non-normal) distributional forms, there may be higher moments and all moments may be related. The problem with second-moment effects is that they may be too subtle or obscure to manifest themselves in measurable ways. This is especially true when one uses data from firms publicly registered in the US because under current US Generally Accepted Accounting Principles (US-GAAP), the disclosure environment is already rich. In other words, commitments to increased (or reduced) levels of disclosure in the US may be primarily incremental, thereby leading to economic consequences that are difficult to document. One alternative is to suggest that researchers consider less developed capital markets than those found in the US.

To conclude, one issue that deserves greater attention in the accounting literature, both theory-based and empirical, is the relation between disclosure and information asymmetry reduction. Among other things, this relation links disclosure to efficiency,

and in this sense provides an economic rationale for the utility of financial reporting. But while the existing theory on this topic is compelling, demonstrating the link empirically has proved elusive. This may mean that we need better theories; it may also mean that we need better empirical methodologies. It is probably the case that we need a little more of both.

References

Abarbanell, J.S., W.N. Lanen, and R.E. Verrecchia, 1995, Analysts' forecasts as proxies for investor beliefs in empirical research, Journal of Accounting and Economics 20, 31-60.

Admati, A.R., 1985, A noisy rational expectations equilibrium for multi-asset securities markets, Econometrica 53, 629-657.

Admati, A.R. and P. Pfleiderer, 1986, A monopolistic market for information, Journal of Economic Theory 39, 400-438.

Admati, A.R. and P. Pfleiderer, 1988a, A theory of intraday patterns: volume and price variability, Review of Financial Studies 1, 3-40.

Admati, A.R. and P. Pfleiderer, 1988b, Selling and trading on information in financial markets, American Economic Review 78, 96-103.

Alles, M. and R.J. Lundholm, 1993, On the optimality of public signals in the presence of private information, The Accounting Review 68, 93-112.

Atiase, R.K. and L.S. Bamber, 1994, Trading volume reactions to annual accounting earnings announcements, Journal of Accounting and Economics 17, 309-329.

Baiman, S. and R.E. Verrecchia, 1996, The relation among capital markets, financial disclosure, production efficiency, and insider trading, Journal of Accounting Research 34, 1-22.

Barlevy, G. and P. Veronesi, 2000, Information acquisition in financial markets, Review of Economic Studies 67, 79-90.

Barron, O.E., O. Kim, S.C. Lim, and D.E. Stevens, 1998, Using analysts' forecasts to measure properties of analysts' information environment, The Accounting Review 73, 421-433.

Barth, M. E., G. Clinch, and T. Shibano, 1999, International accounting harmoniza-

tion and global equity markets. Journal of Accounting and Economics 26, 201-235.

Blume, L., D. Easley, and M. O'Hara, 1994, Market statistics and technical analysis: the role of volume, Journal of Finance 49, 153-181.

Botosan, C.A., 1997, Disclosure level and the cost of equity capital, The Accounting Review 72, 323-349.

Brown, D.P. and R.H. Jennings, 1989, On technical analysis, Review of Financial Studies 2, 527-551.

Bushman, R.M., 1991, Public disclosure and the structure of private information markets, Journal of Accounting Research 29, 261-276.

Bushman, R.M., F. Gigler, and R. Indjejikian, 1996, A model of two-tiered financial reporting, Journal of Accounting Research 34 (Supplement), 51-74.

Bushman, R.M. and R. Indjejikian, 1995, Voluntary disclosures and trading behavior of corporate insiders, Journal of Accounting Research 33, 293-316.

Bushman, R.M., D. Dutta, J. Hughes, and R. Indjejikian, 1997, Earnings announcements and market depth, Contemporary Accounting Research 14, 43-68.

Campbell, J., S. Grossman and J. Wang, 1993, Trading volume and serial correlation in stock returns, The Quarterly Journal of Economics 108, 905-939.

Clinch, G. and R.E. Verrecchia, 1997, Competitive Disadvantage and Discretionary Disclosure in Industries, Australian Journal of Management 22, 125-137.

Core, J.E., 2000, Firm's disclosure and their cost of capital: A discussion of "A Review of the empirical disclosure literature," Journal of Accounting and Economics, forthcoming.

Crawford, V. and J. Sobel, 1982, Strategic information transformation, Econometrica 50,1431-1451.

Darrough, M.N., 1993, Disclosure policy and competition: Cournot vs. Bertrand,

Accounting Review 68, 534-561.

Darrough, M.N. and N.M. Stoughton, 1988, Financial disclosure policy in an entry game, Journal of Accounting and Economics 12, 480-511.

De Long, J.B., A.S. Shleifer, L.H. Summers, and R.J. Waldmann, 1990, Noise trader risk in financial markets, Journal of Political Economy 98, 703-738.

Demski, J. and G. Feltham, 1994, Market response to financial reports, Journal of Accounting and Economics 17, 3-40.

Diamond, D.W., 1985, Optimal release of information by firms, Journal of Finance 40, 1071-1094.

Diamond, D.W., and R.E. Verrecchia, 1981, Information aggregation in a noisy rational expectations economy, Journal of Financial Economics 9, 221-235.

Diamond, D.W. and R.E. Verrecchia, 1987, Constraints on short-selling and asset price adjustment to private information, Journal of Financial Economics 18, 277-311.

Diamond, D.W. and R.E. Verrecchia, 1991, Disclosure, liquidity, and the cost of capital, Journal of Finance 46, 1325-1359.

Dontoh, A., 1989, Voluntary disclosure, Journal of Accounting, Auditing, and Finance 4, 480-511.

Dontoh, A. and J. Ronen, 1993, Information Content of Accounting Announcements, The Accounting Review 68, 857-869.

Dye, R.A., 1985a, Disclosure of nonproprietary information, Journal of Accounting Research 23, 123-145.

Dye, R.A., 1985b, Strategic accounting choice and the effects of alternative financial reporting requirements, Journal of Accounting Research 23, 544-574.

Dye, R.A., 1986, Proprietary and nonproprietary disclosures, Journal of Business 59, 331-336.

Dye, R.A., 1990, Mandatory versus voluntary disclosures: the case of financial and real externalities, The Accounting Review 65, 1-24.

Dye, R.A., 1998, Investor sophistication and voluntary disclosures, Review of Accounting Studies 3, 261-287.

Dye, R.A. and S.S. Sridhar, 1995, Industry-wide disclosure dynamics, Journal of Accounting Research 33, 157-174.

Farrell, J. and R. Gibbons, 1989, Cheap talk with two audiences, American Economic Review 79, 1214-1223.

Feltham, G.A., F. Gigler, and J.S. Hughes, 1992, The effects of line-of-business reporting on competition in oligopoly settings, Contemporary Accounting Research 9, 1-23.

Feltham, G.A. and J.Z. Xie, 1992, Voluntary financial disclosure in an entry game with continua of types, Contemporary Accounting Research 9, 46-80.

Fischer, P.E. and P.A. Stocken, 2000, Imperfect competition and credible communication, Journal of Accounting Research, forthcoming.

Fischer, P.E. and R.E. Verrecchia, 1997, The effect of limited liability on the market response to disclosure, Contemporary Accounting Research 14, 515-543.

Fischer, P.E. and R.E. Verrecchia, 1998, Correlated public forecasts, Journal of Accounting Research 36, 91-110.

Fischer, P.E. and R.E. Verrecchia, 1999, Public information and heuristic trade, Journal of Accounting and Economics 27, 89-124.

Fischer, P.E. and R.E. Verrecchia, 2000, Reporting bias, The Accounting Review 75, 229-245.

Fishman, M.J. and K.M. Hagerty, 1989, Disclosure decisions by firms and the competition for firm efficiency, Journal of Finance 44, 633-646.

Fishman, M.J. and K.M. Hagerty, 1990, The optimal amount of discretion to allow in disclosure, The Quarterly Journal of Economics. 105, 427-444.

Gigler, F., 1994, Self-enforcing voluntary disclosures, Journal of Accounting Research 32, 224-240.

Gigler, F. and T. Hemmer, 1999, On the Frequency, quality, and informational role of mandatory financial reports, Journal of Accounting Research 36 (Supplement), 117-147.

Gigler, F., J. Hughes, and J. Rayburn, 1994, International accounting standards for line-of-business reporting and oligopoly competition, Contemporary Accounting Research 11, 619-632.

Glosten, L. and P. Milgrom, 1985, Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders, Journal of Financial Economics 14, 71-100.

Grossman, S.J., 1976, On the efficiency of competitive stock markets where traders have diverse information, Journal of Finance 31, 573-585.

Grossman, S.J., 1978, Further results on the information efficiency of competitive stock markets, Journal of Economic Theory 18, 81-101.

Grossman, S.J., 1981, The role of warranties and private disclosure about product quality, Journal of Law and Economics 24, 461-483.

Grossman, S. J., and O.D. Hart, 1980, Disclosure laws and takeover bids, Journal of Finance 35, 323-334.

Grossman, S.J. and J. E. Stiglitz, 1980, On the impossibility of informationally efficient markets, American Economic Review 70, 393-408.

Grundy, B.D. and M. McNichols, 1989, Trade and the revelation of information through prices and direct disclosure, Review of Financial Studies 2, 495-526.

Hakannson, N.H., J.G. Kunkel, and J.A. Ohlson, 1982, Sufficient and necessary conditions for information to have social value in pure exchange, Journal of Finance 37, 1169-1181.

Harris, M. and R. Raviv, 1993, Differences of opinion make a horse race, Review of Financial Studies 6, 473-506.

Hayes, R.M. and R.J. Lundholm, 1996, Segment reporting to the capital market in the presence of a competitor, Journal of Accounting Research 34, 261-279.

He, H., and J. Wang, 1995, Differential information and dynamic behavior of stock trading volume, Review of Financial Studies 8, 919-972.

Healy, P.M., A.P. Hutton, and K.G. Palepu, 1999, Stock performance and intermediation changes surrounding sustained increases in disclosure, Contemporary Accounting Research 16, 485-520.

Healy, P.M. and K.G. Palepu, 2000, A review of the empirical disclosure literature, Journal of Accounting and Economics, forthcoming.

Hellwig, M.F., 1980, On the aggregation of information in competitive markets, Journal of Economic Theory 22, 477-498.

Hellwig, M., 1982, Rational expectations equilibrium with conditioning on past prices: a mean-variance example, Journal of Economic Theory 26, 279-312.

Hirshleifer, J., 1971, The private and social value of information and the reward to inventive activity, The American Economic Review 61, 561-574.

Holthausen, R.W. and R.E. Verrecchia, 1988, The effect of sequential information releases on the variance of price changes in an intertemporal multi-asset market, Journal of Accounting Research 26, 82-106.

Holthausen, R.W. and R.E. Verrecchia, 1990, The effect of informedness and consensus on price and volume behavior, The Accounting Review 65, 191-208.

Huddart, S., J. S. Hughes, and M. Brunnermeier, 1999, Disclosure requirements and stock exchange listing choice in an international context, Journal of Accounting and Economics, 237-269.

Hughes, P. J., 1986, Signaling by direct disclosure under asymmetric information, Journal of Accounting and Economics 8, 119-142.

Indjejikian, R., 1991, The impact of costly information interpretation on firm disclosure decisions, Journal of Accounting Research 29, 277-301.

Jovanovic, B., 1982, Truthful disclosure of information, Bell Journal of Economics 13, 36-44.

Jung, W.O. and Y.K. Kwon, 1988, Disclosure when the market is unsure of information endowment of managers, Journal of Accounting Research 26, 146-153.

Kandel, E. and N. Pearson, 1995, Differential interpretation of public signals and trade in speculative markets, Journal of Political Economy 103, 831-872.

Karpoff, J.M., 1987, The relation between price changes and trading volume: a survey, Journal of Financial & Quantitative Analysis 22, 109-126.

Kim, O., 1993, Disagreements among shareholders over a firm's disclosure policy, Journal of Finance 48, 747-760.

Kim, O., 1999, Discussion of the role of the manager's human capital in discretionary disclosure, Journal of Accounting Research 37 (Supplement), 183-185.

Kim, O. and R.E. Verrecchia, 1991a, Trading volume and price reactions to public announcements, Journal of Accounting Research 29, 302-321.

Kim, O. and R.E. Verrecchia, 1991b, Market reactions to anticipated announcements, Journal of Financial Economics, 30, 273-309.

Kim, O. and R.E. Verrecchia, 1994, Market liquidity and volume around earnings announcements, Journal of Accounting and Economics 17, 41-67.

Kim, O. and R.E. Verrecchia, 1997, Pre-announcement and event-period private information, Journal of Accounting and Economics 24, 395-419.

Kim, O. and R.E. Verrecchia, 2001, The relation among disclosure, returns, and trading volume information, The Accounting Review, forthcoming.

Kirschenheiter, M., 1997, Information quality and correlated signals, Journal of Accounting Research 35, 43-59.

Kunkel, G., 1982, Sufficient conditions for public information to have social value in a production and exchange economy, Journal of Finance 37, 1005-1013.

Kyle, A.S., 1985, Continuous auctions and insider trading, Econometrica 53, 1315-1335.

Kyle, A.S., and F.A. Wang, 1997, Speculation duopoly with agreement to disagree: can overconfidence survive the market test?, Journal of Finance 52, 2073-2090.

Kyle, A.S., 1989, Informed speculation with imperfect competition, Review of Economic Studies 56, 317-356.

Lambert, R.A., 2000, Contract theory and accounting, Journal of Accounting and Economics, forthcoming.

Lanen, W.N., and R.E. Verrecchia, 1987, Operating decisions and the disclosure of management accounting information, Journal of Accounting Research 25 (Supplement), 165-189.

Leuz, C. and R.E. Verrecchia, 2000, The economic consequences of increased disclosure, Journal of Accounting Research, forthcoming.

Levitt, A., 1998, The importance of high quality accounting standards, Accounting Horizons 12, 79-82.

Lintner, J., 1969, The aggregation of investors diverse judgements and preferences in purely competitive security markets, Journal of Financial and Quantitative Analysis

4, 347-400.

Lundholm, R.J., 1988, Price-signal relations in the presence of correlated public and private information, Journal of Accounting Research 26, 107-118.

Lundholm, R.J., 1991, Public signals and the equilibrium allocation of private information, Journal of Accounting Research 29, 322-349.

Manzano, C., 1999, Price signal relations in an imperfectly competitive financial market with public and private information, Journal of Accounting Research 37, 451-463.

Marshall, J.M., 1974, Private incentives and public information, The American Economic Review 64, 373-390.

Matthews, S. and A. Postlewaite, 1985, Quality testing and disclosure, Rand Journal of Economics 16, 328-340.

Merton, R.C., 1987, A simple model of capital market equilibrium with incomplete information, Journal of Finance 42, 483-510.

Milgrom, P., 1981, Good news and bad news: representation theorems and applications, Bell Journal of Economics 12, 380-391.

Milgrom, P. and N. Stokey, 1982, Information, trade, and common knowledge, Journal of Economic Theory 26, 17-27.

Milgrom, P. and J. Roberts, 1986, Relying on the information of interested parties, Rand Journal of Economics 17, 18-32.

McNichols, M. and B. Trueman, 1994, Public disclosure, private information collection, and short-term trading, Journal of Accounting and Economics 17, 69-94.

Nagar, V., 1999, The role of the manager's human capital in discretionary disclosure, Journal of Accounting Research 37 (Supplement), 167-181.

Newman, P. and R. Sansing, 1993, Disclosure policies with multiple users, Journal of

Accounting Research 31, 92-112.

Ng, D., 1975, Information accuracy and social welfare under homogeneous beliefs, Journal of Financial Economics 2:53-70.

Ng, D., 1977, Pareto-optimality of authentic information, Journal of Finance 32, 1717-1728.

Pae, S., 1999, Acquisition and discretionary disclosure of private information and its implications for firms' productive activities, Journal of Accounting Research 37, 465-474.

Palomino, F., 1996, Noise trading in small markets, The Journal of Finance 51, 1537-1550.

Penno, M.C., 1997, Information quality and voluntary disclosure, Accounting Review 72, 275-284.

Shin, H.S., 1994, News management and the value of firms, Rand Journal of Economics 25, 58-71.

Shevlin, T. and D.A. Shackelford, 2000, Empirical research in tax accounting, Journal of Accounting and Economics, forthcoming.

Stocken, P.C., 2000, Credibility of voluntary disclosure, Rand Journal of Economics 31, 359-374.

Subramanyam, K.R., 1996, Uncertain precision and price reactions to information, Accounting Review 71, 207-220. Sutton, M., 1997, Financial reporting in U.S. capital markets: international dimensions, Accounting Horizons 11, 96-102.

Teoh, S.H., 1997, Information disclosure and voluntary contributions to public goods, Rand Journal of Economics 28, 385-406.

Teoh, S.H., and C.Y. Hwang, 1991, Nondisclosure and adverse disclosure as signals of firm value, The Review of Financial Studies 4, 283-313.

Thaler, R.H., 1993, Advances in Behavioral Finance, New York, Russell Sage Foundation, 1993.

Trueman, B., 1986, Why do managers voluntarily release earnings forecasts?, Journal of Accounting and Economics 8, 53-71.

Trueman, B., 1996, The impact of analyst following on stock prices and the implications for firms' disclosure policies, Journal of Accounting, Auditing & Finance 11, 333-354.

Verrecchia, R.E., 1982a, Information acquisition in a noisy rational expectations economy, Econometrica 50, 1415-1430.

Verrecchia, R.E., 1982b, The use of mathematical models in financial accounting, Journal of Accounting Research 20 (Supplement), 1-42.

Verrecchia, R.E., 1983, Discretionary disclosure, Journal of Accounting and Economics 5, 365–380.

Verrecchia, R.E., 1990a, Discretionary disclosure and information quality, Journal of Accounting and Economics 12, 179-194.

Verrecchia, R.E., 1990b, Endogenous proprietary costs through firm interdependence, Journal of Accounting and Economics 12, 245-250.

Verrecchia, R.E., 1996a, Discussion of a model of two-tiered financial reporting, Journal of Accounting Research 34 (Supplement), 75-82.

Verrecchia, R.E., 1996b, Discussion:'Impact of analyst following on stock prices and the implications for firms' disclosure policies', Journal of Accounting, Auditing & Finance 11, 1996, 355-359.

Verrecchia, R.E., 1999, Disclosure and the cost of capital: a discussion, Journal of Accounting and Economics 26, 271-283.

Wagenhofer, A., 1990, Voluntary disclosure with a strategic opponent, Journal of

Accounting and Economics 12, 341-363.

Wald, A., 1951, On some systems of equations of mathematical economics, Econometrica 19, 368-403.

Walras, L., 1881, Mathematische Theorie der Preisbestimmung der wirtschaftlichen Güter, Stuttgart, Ferdinand Enke.

Welker, M., 1995, Disclosure policy, information asymmetry, and liquidity in equity markets, Contemporary Accounting Research 11, 801-827.

Wilson, R., 1968, Theory of syndicates, Econometrica 36, 119-132.