

Theory and potentials of multi-layered plasmonic covers for multi-frequency cloaking

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Abstract. We have recently suggested that suitably designed plasmonic layers may cloak a given object simultaneously at multiple frequencies (Alù and Engheta 2008 *Phys. Rev. Lett.* **100** 113901). Here, we extend our theory and fully analyze this possibility, highlighting the potentials of this plasmonic cloaking technique and its fundamental limitations dictated by the passivity and causality of the materials involved. The cloaking mechanism relies on the scattering cancellation properties of plasmonic materials. By exploiting their inherent frequency dispersion, it is possible to reduce the ‘visibility’ of a given object by several orders of magnitude simultaneously at multiple frequencies, such that any of the particular layers composing the cloak is responsible for noticeable reduction of scattering at each frequency of operation.

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1. Introduction

Cloaking applications of metamaterials and artificial structures have become particularly popular in recent years [1]–[12] for a variety of reasons. Different techniques have been proposed to suppress the scattering from a given object, involving coordinate-transformation-based metamaterials [7]–[11], anomalous localized resonances [12] and plasmonic covers based on scattering cancellation [1]–[6]. This latter idea has been shown to rely on an inherently non-resonant scattering cancellation mechanism, which is fairly robust to geometry and frequency variations and it may be envisioned at optical frequencies by using conventional plasmonic materials [13] or at lower frequencies by utilizing metamaterial technology [5].

One of the main distinct features of plasmonic cloaking, when compared to other cloaking mechanisms, includes the possibility of the wave penetrating into the cloak and the cloaked region, while still maintaining the absence of scattering and disturbance in the surrounding region. This feature, which may be advantageous for several different applications, has allowed us to extend this cloaking technique to multi-layered geometries, which, by effectively utilizing multiple-layered cloaks, may operate simultaneously at several distinct frequencies [6]. If the wave can penetrate into the outer cloaking layer, nothing indeed forbids us, at least in principle, to design in cascade a second inner layer with different properties, which may cloak at a different frequency of operation. What is interesting, as we have preliminarily shown in [6], is that this operation may be performed by simple passive plasmonic materials, exploiting their natural frequency dispersions.

In the following, we analyze in detail to what extent it may be possible to realize a multi-layered passive cloak with plasmonic materials and what limitations their natural frequency dispersions imply on the bandwidth and frequency response of these multi-layered cloaks. An $e^{-i\omega t}$ time convention is assumed in the following.

2. Theoretical formulation

The scattering-cancellation-based plasmonic cloaking theory introduced in [1] may be extended to a multi-layered scenario by applying the well-known Mie theory to a multi-layered spherical object [6]. Following an analytical approach similar to [14], the TM Mie scattering coefficients of generic order n , relative to the amplitude of the n th multipolar expansion of the incident wave, may be symbolically written as:

$$c_n^{\text{TM}} = -\frac{U_n^{\text{TM}}}{U_n^{\text{TM}} + iV_n^{\text{TM}}}, \quad c_n^{\text{TE}} = -\frac{U_n^{\text{TE}}}{U_n^{\text{TE}} + iV_n^{\text{TE}}}. \quad (1)$$

The coefficients U_n and V_n are real in the limit of negligible material losses and they may be evaluated, by extending the results of [1, 14] to an N -layer cover, as the determinants of the following $(2N+2) \times (2N+2)$ matrices resulting from fulfilling the boundary conditions of a

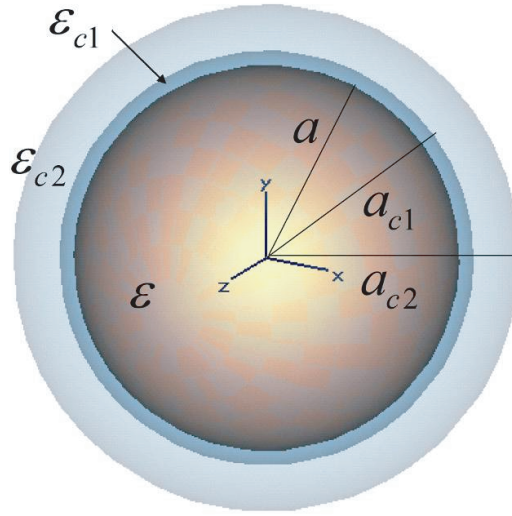


Figure 1. Geometry of a two-layered cloak made of different plasmonic materials for the shell.

multi-layered spherical geometry [15]:

$$U_n^{\text{TM}} =$$

$$\begin{vmatrix} j_n(ka) & j_n(k_{c1}a) & y_n(k_{c1}a) & 0 & 0 & 0 \\ [ka j_n(ka)]' / \varepsilon & [k_{c1} a j_n(k_{c1}a)]' / \varepsilon_{c1} & [k_{c1} a y_n(k_{c1}a)]' / \varepsilon_{c1} & 0 & 0 & 0 \\ 0 & j_n(k_{c1}a_{c1}) & y_n(k_{c1}a_{c1}) & j_n(k_{c2}a_{c1}) & y_n(k_{c2}a_{c1}) & 0 \\ 0 & [k_{c1} a j_n(k_{c1}a_{c1})]' / \varepsilon_{c1} & [k_{c1} a y_n(k_{c1}a_{c1})]' / \varepsilon_{c1} & [k_{c2} a j_n(k_{c2}a_{c1})]' / \varepsilon_{c2} & [k_{c2} a y_n(k_{c2}a_{c1})]' / \varepsilon_{c2} & 0 \\ 0 & 0 & 0 & j_n(k_{c2}a_{c2}) & y_n(k_{c2}a_{c2}) & j_n(k_0a_{c2}) \\ 0 & 0 & 0 & [k_{c2} a j_n(k_{c2}a_{c2})]' / \varepsilon_{c2} & [k_{c2} a y_n(k_{c2}a_{c2})]' / \varepsilon_{c2} & [k_0 a j_n(k_0a_{c2})]' / \varepsilon_0 \end{vmatrix} \quad (2)$$

$$V_n^{\text{TM}} =$$

$$\begin{vmatrix} j_n(ka) & j_n(k_{c1}a) & y_n(k_{c1}a) & 0 & 0 & 0 \\ [ka j_n(ka)]' / \varepsilon & [k_{c1} a j_n(k_{c1}a)]' / \varepsilon_{c1} & [k_{c1} a y_n(k_{c1}a)]' / \varepsilon_{c1} & 0 & 0 & 0 \\ 0 & j_n(k_{c1}a_{c1}) & y_n(k_{c1}a_{c1}) & j_n(k_{c2}a_{c1}) & y_n(k_{c2}a_{c1}) & 0 \\ 0 & [k_{c1} a j_n(k_{c1}a_{c1})]' / \varepsilon_{c1} & [k_{c1} a y_n(k_{c1}a_{c1})]' / \varepsilon_{c1} & [k_{c2} a j_n(k_{c2}a_{c1})]' / \varepsilon_{c2} & [k_{c2} a y_n(k_{c2}a_{c1})]' / \varepsilon_{c2} & 0 \\ 0 & 0 & 0 & j_n(k_{c2}a_{c2}) & y_n(k_{c2}a_{c2}) & y_n(k_0a_{c2}) \\ 0 & 0 & 0 & [k_{c2} a j_n(k_{c2}a_{c2})]' / \varepsilon_{c2} & [k_{c2} a y_n(k_{c2}a_{c2})]' / \varepsilon_{c2} & [k_0 a y_n(k_0a_{c2})]' / \varepsilon_0 \end{vmatrix}.$$

In particular, these formulae refer to the specific case of $N = 2$, corresponding to the geometry of figure 1, but they may be easily generalized to any value of N , as described in [15].

The TE scattering coefficients may be obtained by duality from equation (2). In the previous formulae, j_n and y_n are the spherical Bessel functions, k are the wave numbers in the different regions, ε are the corresponding permittivities. The geometrical parameters are described in figure 1. The total scattering cross section from the cloaked system, denoted as Q_s which determines its overall visibility, is determined in terms of all the coefficients given in equation (1) for the different multipolar orders, with proper weights, as follows:

$$Q_s = \frac{2\pi}{|k_0|^2} \sum_{n=1}^{\infty} (2n+1) \left(|c_n^{\text{TE}}|^2 + |c_n^{\text{TM}}|^2 \right). \quad (3)$$

In the limit of relatively small objects, consistent with [1, 2], the condition $U_n^{\text{TM}} = 0$, which ensures $c_n^{\text{TM}} = 0$, i.e. a zero scattering contribution from the n th spherical harmonic may be obtained under the generalized small-radii approximate condition on the outer cover thickness [6], as shown below:

$$\eta_{c2} \simeq \eta_{c1}^{2n+1} \sqrt{\frac{[(n+1)\varepsilon_{c2} + n\varepsilon_0][\eta_{c1}^{2n+1}(\varepsilon_{c2} - \varepsilon_{c1})[(n+1)\varepsilon_{c1} + n\varepsilon] + (\varepsilon_{c1} - \varepsilon)[(n+1)\varepsilon_{c1} + n\varepsilon_{c2}]]}{(\varepsilon_{c2} - \varepsilon_0)[n(n+1)(\varepsilon_{c1} - \varepsilon)(\varepsilon_{c2} - \varepsilon_{c1}) + \eta_{c1}^{2n+1}[(n+1)\varepsilon_{c1} + n\varepsilon][(n+1)\varepsilon_{c2} + n\varepsilon_{c1}]]}}, \quad (4)$$

where $\eta_{c1} \equiv a_{c1}/a$ and $\eta_{c2} \equiv a_{c2}/a$.

In the special case of perfectly conducting objects, in which the wave cannot penetrate and which may be of interest for several microwave applications, equation (4) simplifies into:

$$\eta_{c2} \simeq \eta_{c1}^{2n+1} \sqrt{\frac{[n(\eta_{c1}^{2n+1} - 1)\varepsilon_{c2} - (n\eta_{c1}^{2n+1} + n+1)\varepsilon_{c1}][(n+1)\varepsilon_{c2} + n\varepsilon_0]}{n(\varepsilon_{c2} - \varepsilon_0)[(n+1)(\eta_{c1}^{2n+1} - 1)\varepsilon_{c2} + (n\eta_{c1}^{2n+1} + n+1)\varepsilon_{c1}]}}, \quad (5)$$

These formulae may be extended to the case of the cloak with more layers, which would provide several degrees of freedom in the design of the cloak for which drastic scattering reduction may occur simultaneously at different frequencies. Moreover, they may be considered for TE coefficients by substituting the permittivities by the corresponding permeabilities of the materials involved, implying that in this small-radii limit the electric and magnetic problems are effectively and approximately uncoupled. In particular, in this limit the dipolar scattering order is dominant, and therefore equations (4) and (5) with $n = 1$ would ensure a generalized cloaking condition for small three-layered spherical objects. For relatively larger objects, full-wave formulae like equation (2) with $U_n = 0$ should be considered for the dominant scattering orders in the specific problem at hand.

It is interesting to note that the dual condition $V_n^{\text{TM}} = 0$ would ensure resonant scattering from the system, since the corresponding scattering coefficient $c_n^{\text{TM}} = -1$ would be maximized. The corresponding small-radii condition on the cover thickness for this resonant scattering becomes:

$$\eta_{c2} \simeq \eta_{c1}^{2n+1} \sqrt{\frac{n(n+1)(\varepsilon_{c2} - \varepsilon_0)[\eta_{c1}^{2n+1}(\varepsilon_{c2} - \varepsilon_{c1})[(n+1)\varepsilon_{c1} + n\varepsilon] + (\varepsilon_{c1} - \varepsilon)[(n+1)\varepsilon_{c1} + n\varepsilon_{c2}]]}{[n\varepsilon_{c2} + (n+1)\varepsilon_0][n(n+1)(\varepsilon_{c1} - \varepsilon)(\varepsilon_{c2} - \varepsilon_{c1}) + \eta_{c1}^{2n+1}[(n+1)\varepsilon_{c1} + n\varepsilon][(n+1)\varepsilon_{c2} + n\varepsilon_{c1}]]}}, \quad (6)$$

and for conducting cores:

$$\eta_{c2} \simeq \eta_{c1}^{2n+1} \sqrt{\frac{(n+1)\eta_{c1}^{2n+1}[n(\eta_{c1}^{2n+1} - 1)\varepsilon_{c2} - (n\eta_{c1}^{2n+1} + n+1)\varepsilon_{c1}][\varepsilon_{c2} - \varepsilon_0]}{[n\varepsilon_{c2} + (n+1)\varepsilon_0][(n+1)(\eta_{c1}^{2n+1} - 1)\varepsilon_{c2} + (n\eta_{c1}^{2n+1} + n+1)\varepsilon_{c1}]}}, \quad (7)$$

It is noticed that equations (4)–(7) converge to the small-radii formulae derived in [1, 14] for the case $N = 1$ when $\eta_{c1} = 1$ or $\varepsilon_{c1} = \varepsilon_{c2}$, as expected.

The transparency conditions $U_n = 0$ and the resonance conditions $V_n = 0$ are very distinct physical phenomena associated with the anomalous properties of plasmonic materials. The former, on which plasmonic cloaking is based, takes place due to the negative polarizability of plasmonic materials, which may produce an anomalous scattering cancellation, that makes the overall cloaked particle approximately ‘invisible’ to external observers. The resonance condition, on the other hand, is associated with classic Mie resonances, which the plasmonic features of the cover may allow to happen even in the small-radii limit, consistent with [14].

Despite the drastic difference in the nature of these two phenomena—one is a non-resonant integral effect, the other is strongly resonant and may be weakened by material absorption—these two dual conditions are actually intermingled by the natural frequency dispersion of plasmonic materials. As we show in the next section, material dispersion indeed leads to some conditions on the occurrence of the invisibility and the resonance conditions as a function of frequency.

3. Cloaking dispersion

The extra degrees of freedom provided by the addition of extra layers in the cloak, when compared with the single-layered cloak described in [1]–[5], may allow a cloak to be designed that operates simultaneously at different frequencies of operation [6]: the natural frequency dispersion of the materials composing the cloak may indeed provide the potential to design a multilayered cover that satisfies equation (4) simultaneously at different frequencies. In [6], we have discussed a specific example of this possibility, as applied to an optical nanoparticle. Here, we consider how the material dispersion may facilitate or limit this multifrequency cloaking mechanism.

In order to highlight these features, we report in figure 2 a series of contour maps that show the variation of η_{c2} , as derived from equation (4), for different values of η_{c1} , when varying ε_{c1} and ε_{c2} in the case of a dielectric object with $\varepsilon = 10\varepsilon_0$. The light (blue in color) regions are those for which the corresponding values of ε_{c1} , ε_{c2} and η_{c1} do not provide any physically reasonable value of η_{c2} in equation (4), i.e. for which cloaking is not permissible. Darker regions in the plot correspond to larger values of η_{c2} . The top left panel of figure 2 refers to the case for which $\eta_{c1} = 1$, i.e. for which the cloak is constituted only by a homogeneous layer with permittivity ε_{c2} . In this scenario, the contour regions in which the cloaking conditions are possible are obviously independent of ε_{c1} , and they are included in the regions between $0 < \varepsilon_{c2} < \varepsilon_0$ and $\varepsilon_{c2} < -5\varepsilon_0$, as expected from the results reported in [1]. Allowing an extra layer in the cloak, i.e. in the other panels for which $\eta_{c1} > 1$, the cloaking regions widen up and there are several combinations of the design parameters that may satisfy the small-radii cloaking condition given in (4). As expected, the only region that may not support the cloaking condition in all the panels of figure 2 is the one for which both ε_{c1} and ε_{c2} are larger than ε_0 , since in this case both cloak layers would have a positive polarizability and cannot ensure scattering cancellation. However, provided that at least one of the two permittivities is less than that of the background, a proper choice of thicknesses may satisfy the small-radii cloaking condition (4).

Figure 3 reports analogous plots for the special case of a conducting object i.e. for the case $\varepsilon \rightarrow i\infty$. Similar features are seen, even though in this second case for a homogeneous cloak (top left panel, $\eta_{c1} = 1$) the only allowed permittivities lie in the range $0 < \varepsilon_{c2} < \varepsilon_0$. Adding a second inner layer to the cloak, as in the other panels of figure 3, allows a wider range of combinations of design parameters for the cloak that may satisfy the invisibility condition (5). It is noticed that in this scenario it is not possible to cloak a conducting particle with two materials both having a negative value of permittivity.

Figures 4 and 5 report similar plots for the dual resonance conditions described by equations (6) and (7). The plots highlight similar possibilities in terms of the range of values that may admit a plasmonic resonance in the two scenarios of dielectric and conducting particles. As expected, these conditions require that at least one of the two parameters must be negative in order to ensure a plasmonic response from the shell that may produce a small-radii resonance.

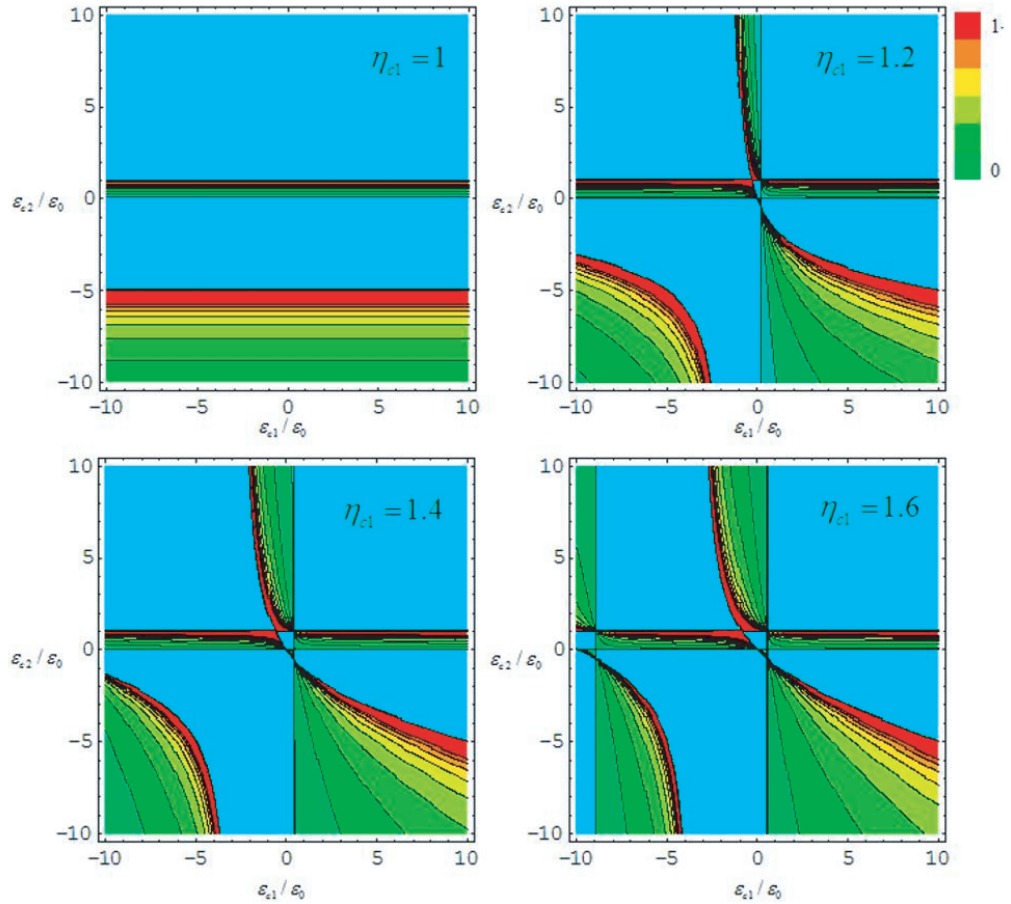


Figure 2. Contour regions for the ‘invisibility’ condition (4) in the case of $n = 1$, for a dielectric particle with $\varepsilon = 10\varepsilon_0$. The light (blue) regions represent the range of parameters for which the condition (4) cannot be satisfied, whereas the colored regions show different values of η_{c2} that fulfill the condition (4). Darker regions correspond to larger values for η_{c2} .

This is consistent with our results for a homogeneous layer reported in [14]. In the case of a conducting core, i.e. figure 5, the resonance condition is more stringent, also requiring that at least one of the two permittivities must be larger than $-2\varepsilon_0$.

The contour plots reported in this section may provide useful design charts for an approximate design of a two-layered cloak that may yield invisibility or resonance conditions as described in the previous section. Moreover, these plots are very informative in understanding the physics and limitations of the cloaking effect when dealing with multilayered cloaks, as we discuss in the following.

Considering the contour plots of figures 2 and 3, one may indeed notice that a given multilayered cover (i.e. with the same values of η_{c1} and η_{c2}) may provide invisibility and resonance conditions for multiple choices of ε_{c1} and ε_{c2} . By exploiting the natural dispersion of plasmonic materials one may be able to ‘hit’ different combinations of cloaking parameters at different frequencies, as we have shown for a cloak operating simultaneously at two optical frequencies in [6]. However, figures 2–5 show a major limitation in obtaining an arbitrary number of

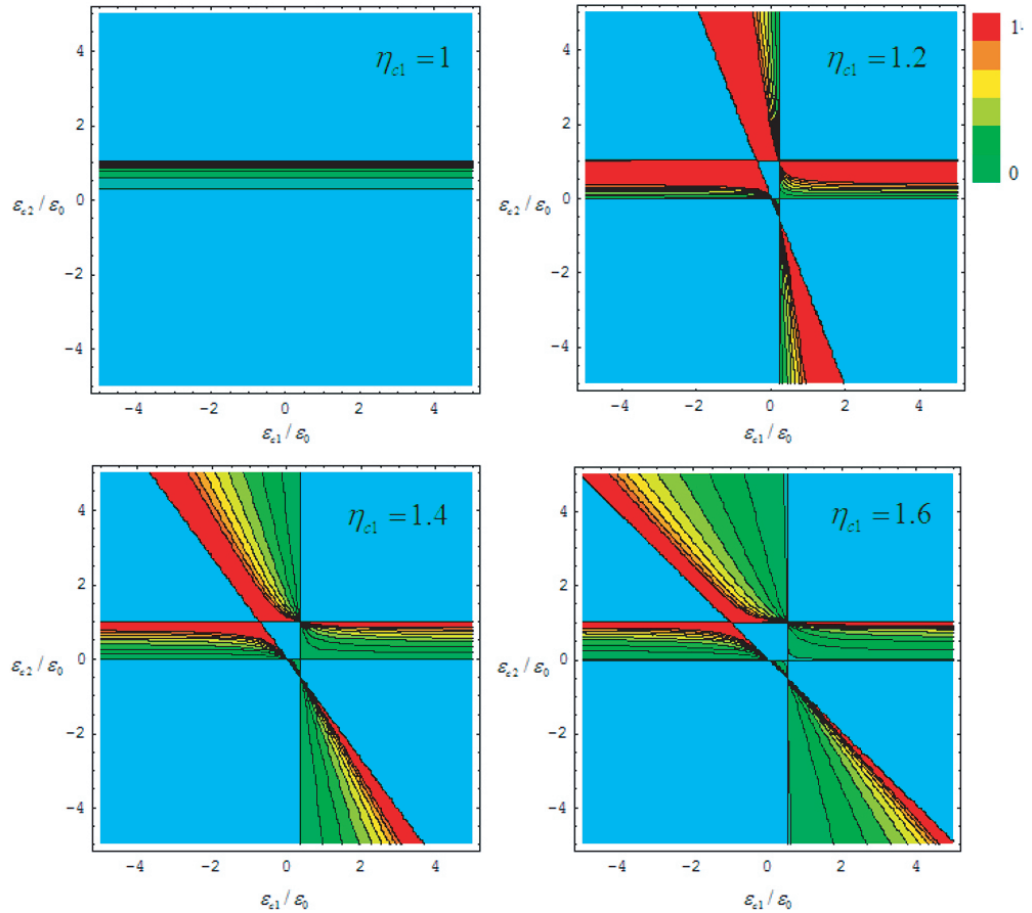


Figure 3. Similar to figure 2, but for a conducting particle, i.e. $\varepsilon = i\infty$.

invisibility or resonance conditions from the same two-layered cloak: the contours that provide equi- η_{c2} lines on the same panel (corresponding therefore to a fixed geometry for the cloak) always have a negative slope in all the panels. This implies that, as long as we lie on one specific region of the plot, to move along the same contour line we are required to reduce one of the permittivities and simultaneously increase the other. However, as long as passive low-loss materials are employed (required to apply the previous small-radii formulae), the derivative $\partial\varepsilon/\partial\omega > 0$ [16] in all layers, and therefore it may not be possible to follow a contour line in the plots of figures 2 and 3 by simply changing the frequency of operation (unless using active materials). This is physically expected because had we been able to follow such contour lines by simply detuning the frequency of operation, we might have been able to achieve ultra-wideband resonance and/or cloaking conditions, beating Chu and causality limitations on electrically small resonators and/or cloaks. This is of course not possible with passive materials. In fact, the negative slope of the contour lines of figures 2–5, a general feature of these families of plots, implies that, despite the increased degrees of design freedom, there is a fundamental limitation on the overall bandwidth achievable by a passive cloak, whatever the number N may be.

The only possibility to achieve more than one cloaking and/or resonance condition with the same cloak design, exploiting the frequency dispersion of the passive materials, is to work on different regions of the panels of figures 2 and 3. Consider, for instance, the bottom left

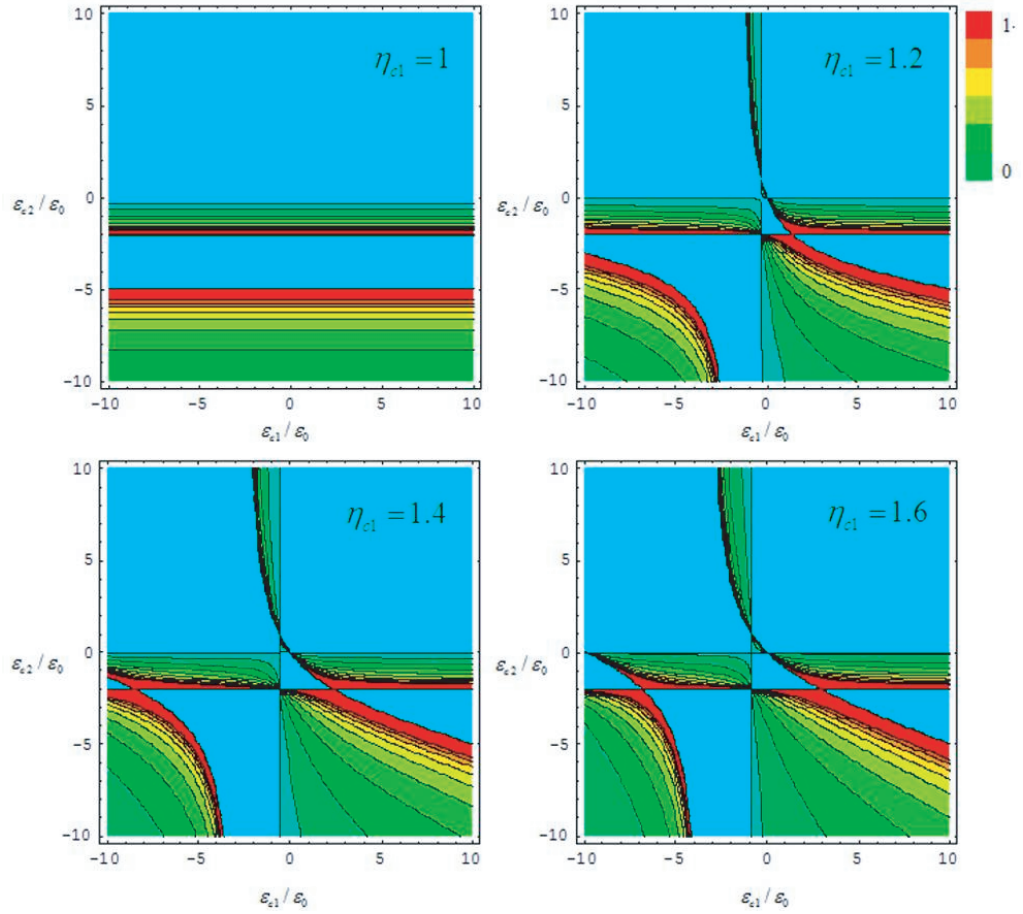


Figure 4. Contour regions for the resonance condition (6) in the case of $n = 1$, for a dielectric particle with $\varepsilon = 10\varepsilon_0$. Similar description as in figure 2, but for the resonance condition.

panel of figure 2, corresponding to the case for which $\eta_{c1} = 1.4$ and $\varepsilon = 10\varepsilon_0$, reproduced for convenience in figure 6. We have drawn in the plot a curve with positive slope that may describe the dispersion of ε_{c1} and ε_{c2} with frequency. In this example, this curve has been obtained by choosing the two permittivities to follow a Drude model $\varepsilon_{ci} = 1 - (\omega_{pi}^2/\omega^2)$ with plasma frequencies $\omega_{p2} = 0.8 \omega_{p1}$. The black dots show the intersections of this curve with contour lines with $\eta_{c2} = 1.6$. It is evident that in this case the same cloak may have three different intersections, ensuring in principle the possibilities of simultaneous cloaking at three different frequencies by just employing a two-layered cloak. For this specific example, the intersections arise at frequencies $\omega_{c1} = 0.31 \omega_{p1}$, $\omega_{c2} = 0.92 \omega_{p1}$ and $\omega_{c3} = 1.46 \omega_{p1}$.

It is clear that each intersection necessarily lies on a separate region of the contour plot of interest, due to the negative slope of the curves. Another interesting feature comes from the corresponding plasmonic resonances of this same cloak. Figure 7 reports the corresponding case for the resonance condition (6), for the same geometry as in figure 6. It is evident how in this case we also have three possible intersections between contour lines with $\eta_{c2} = 1.6$ and the dispersion curve for the permittivities. In this case, the three frequencies correspond to: $\omega_{r1} = 0.27 \omega_{p1}$, $\omega_{r2} = 0.71 \omega_{p1}$ and $\omega_{r3} = 0.93 \omega_{p1}$. Comparing the values of cloaking and

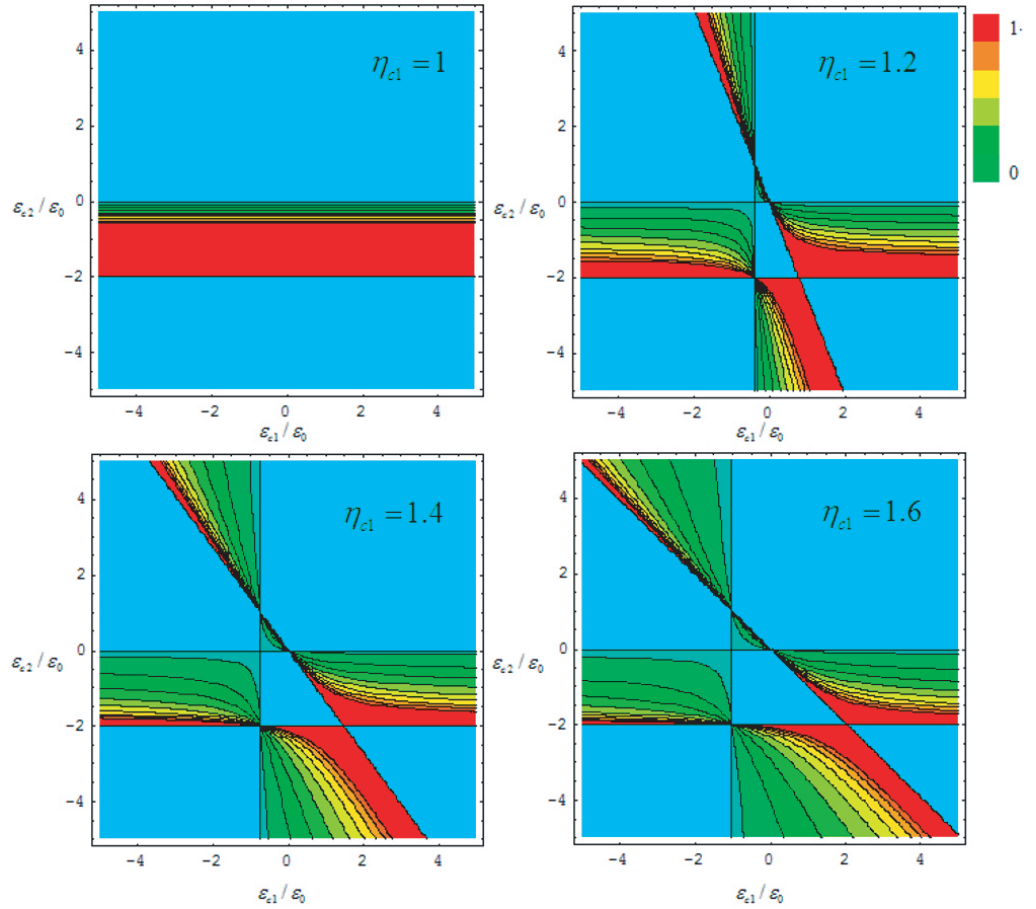


Figure 5. Similar to figure 4, but for a conducting particle and the resonance condition (7).

resonance frequencies, it is seen that the values are interleaved with each other: it may be shown that in general, when passive materials are employed, there is always a resonant peak between two neighboring cloaking frequencies, and vice versa. This property clearly limits the overall bandwidth over which cloaking may be achieved, which is related to the causality and passivity of the cloaks that we are considering here. If active cloaks are to be considered, one may properly engineer the dispersion of the active materials to follow one of the contour lines in the panels of figures 2 and 3. In this way, a broadband active cloak may conceivably be envisioned.

Similar properties may be described for conducting objects, corresponding to the panels in figures 3 and 5. We may also generalize this discussion to a larger number $N > 2$ of layers composing the cloak. In this case, more intersections may be realized between the dispersion space and the equi-geometry space, yielding the possibility of simultaneously cloaking at a larger number of distinct frequencies. However, for passive cloaks the limitation on the existence of a resonant peak between any two neighboring cloaking frequencies still holds. This, in many senses, is the equivalence of Foster's reactance theorem [17] for scattering systems.

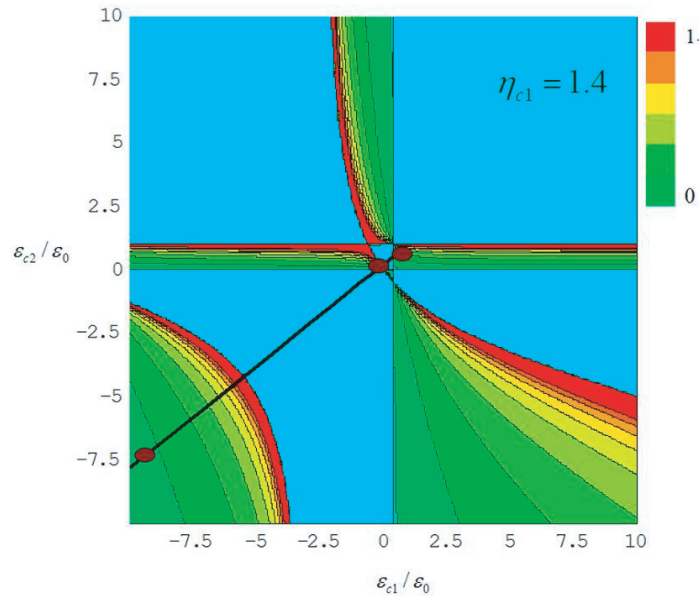


Figure 6. Same as the bottom left panel of figure 2, highlighting the intersection points that may provide cloaking for naturally dispersive plasmonic materials forming the cloak. In this scenario, with $\omega_{p2} = 0.8 \omega_{p1}$, $\varepsilon = 10\varepsilon_0$, $\eta_{c1} = 1.4$ and $\eta_{c2} = 1.6$, three different cloaking frequencies are expected (thick points in the plot).

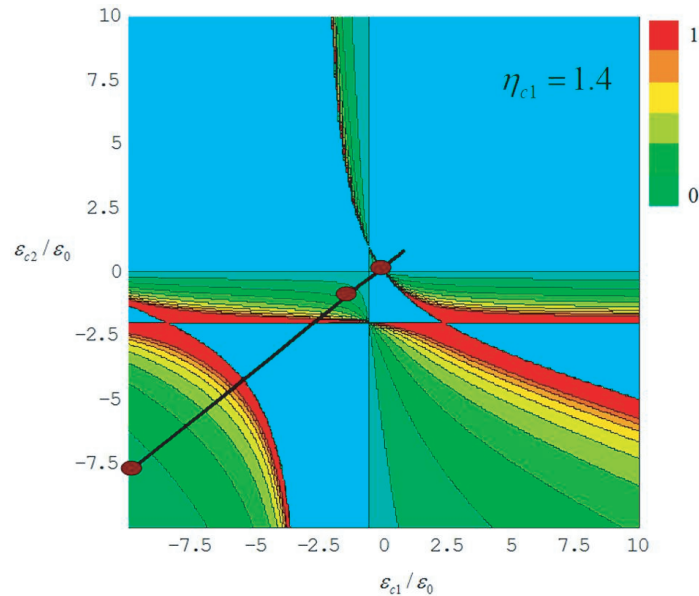


Figure 7. Similar to figure 6, but for the corresponding resonance condition. Same as the bottom left panel of figure 4.

4. Numerical example

In this section, we analyze a specific two-layered geometry to report the scattering dispersion and corresponding field distributions. In particular, here we consider the geometry of figure 1

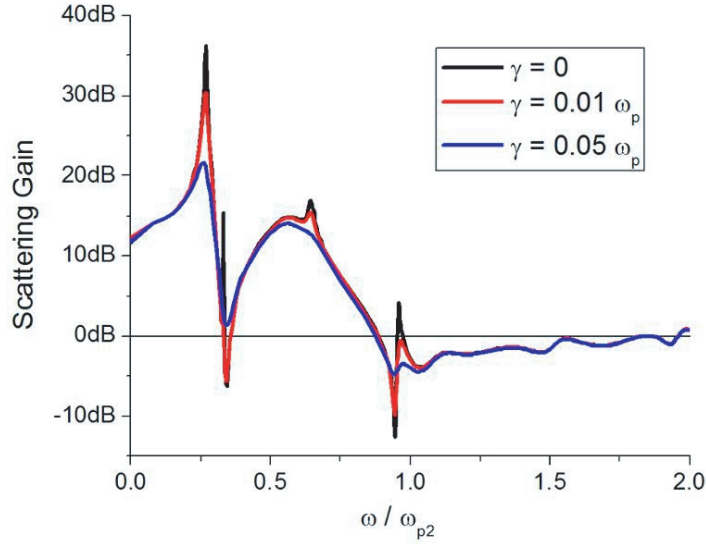


Figure 8. Scattering gain for a two-layered cloak with parameters $\eta_{c1} = 1.2$, $\eta_{c2} = 1.4$ and $a = \lambda_0/5$ at the plasma frequency ω_{p2} and $\omega_{p1} = 0.9\omega_{p2}$. The cloak is designed to reduce the scattering from a dielectric particle of permittivity $\varepsilon = 5\varepsilon_0$.

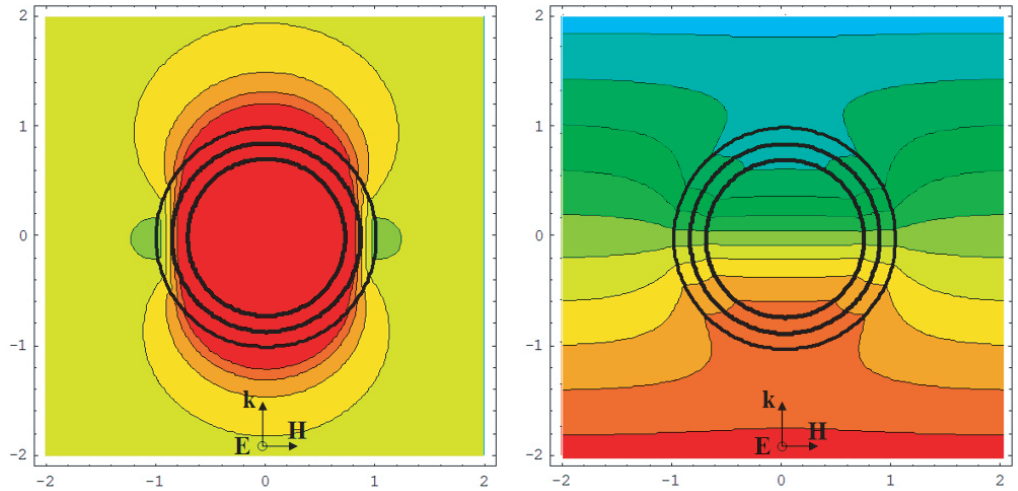


Figure 9. Total electric field distribution in the H plane at frequency ω_{c1} for the cloaked sphere of figure 8. Brighter colors correspond to larger field values.

with $\eta_{c1} = 1.2$, $\eta_{c2} = 1.4$ and $a = \lambda_0/5$ at the plasma frequency ω_{p2} of the outer layer and $\omega_{p1} = 0.9\omega_{p2}$ of the inner layer. The cloak is designed to reduce the scattering from a dielectric particle of permittivity $\varepsilon = 5\varepsilon_0$. The Drude model for the two cloak materials considered here is $\varepsilon_{ci} = \varepsilon_0(1 - \frac{\omega_{pi}^2}{\omega(\omega + i\gamma)})$, where the damping frequency γ takes into account realistic losses.

Figure 8 reports the scattering ‘gain’ caused by the presence of this two-layered cover, compared with the uncloaked scenario. Using equations (4) and (6), it is predicted that three cloaking frequencies may be simultaneously achieved at $\omega_{c1} = 0.34\omega_{p2}$, $\omega_{c2} = 0.95\omega_{p2}$ and

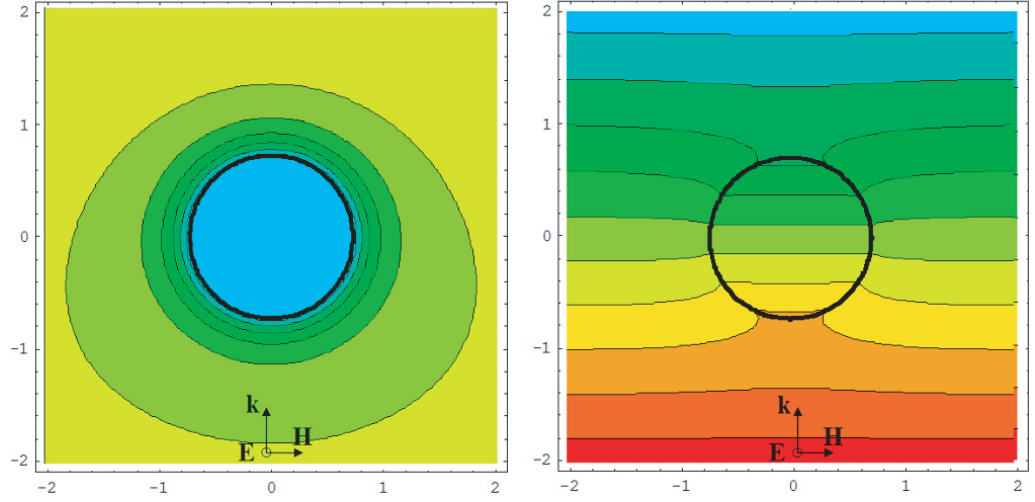


Figure 10. Total electric field distribution in the H plane at frequency ω_{c1} for the bare sphere of figure 8. The plots have the same absolute color scale as in figure 9, for the sake of comparison.

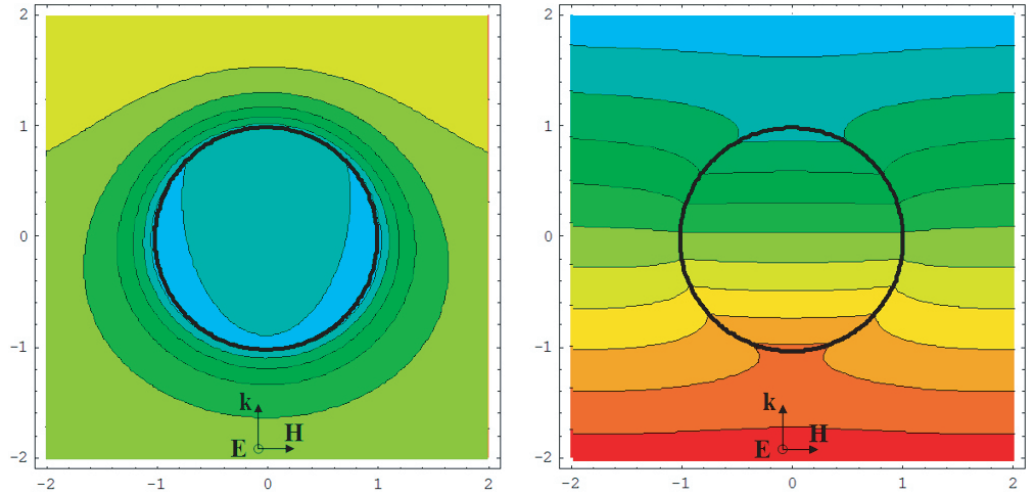


Figure 11. Total electric field distribution in the H plane at frequency ω_{c1} for the larger dielectric sphere of figure 8. The plots have the same absolute color scale as in figure 9, for sake of comparison.

$\omega_{c3} = 1.15\omega_{p2}$, respectively, interleaved by three frequencies for which resonant scattering may be expected $\omega_{r1} = 0.27\omega_{p2}$, $\omega_{c2} = 0.71\omega_{p2}$ and $\omega_{c3} = 0.96\omega_{p2}$. It is noticed that the cloaking frequencies always arise in the three distinct frequency regions $\omega_{c1} < \min(\omega_{p1}, \omega_{p2})$ (both permittivities of the layered cloak being negative), $\min(\omega_{p1}, \omega_{p2}) < \omega_{c2} < \max(\omega_{p1}, \omega_{p2})$ (one permittivity being negative, the other being positive) and $\omega_{c3} > \max(\omega_{p1}, \omega_{p2})$ (both permittivities being low positive). This ensures that moving from one cloaking frequency to the next, the intersections in the equivalent of figure 4 pass from one region to another. Similar considerations may be drawn for the resonant frequencies.

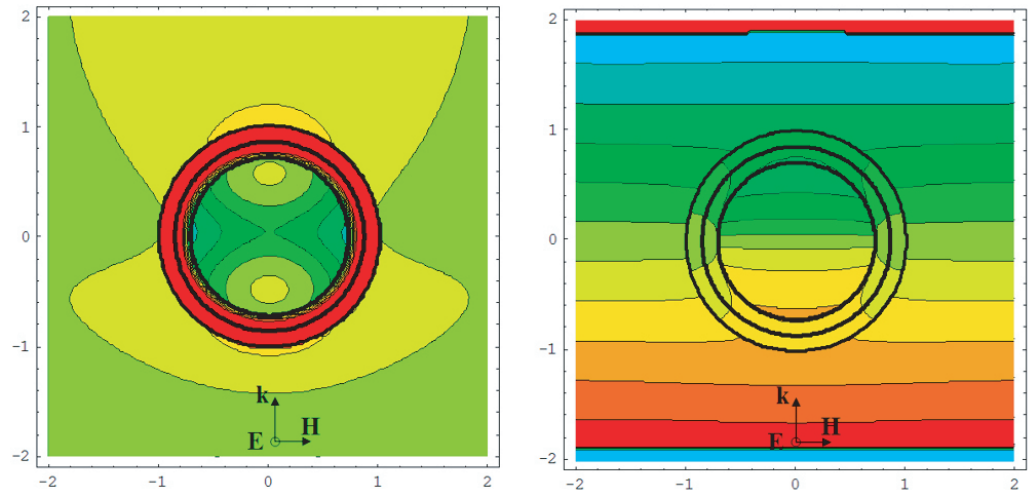


Figure 12. Similar to figure 9, but for frequency ω_{c2} : total electric field distribution in the H plane at frequency ω_{c2} for the cloaked sphere of figure 8.

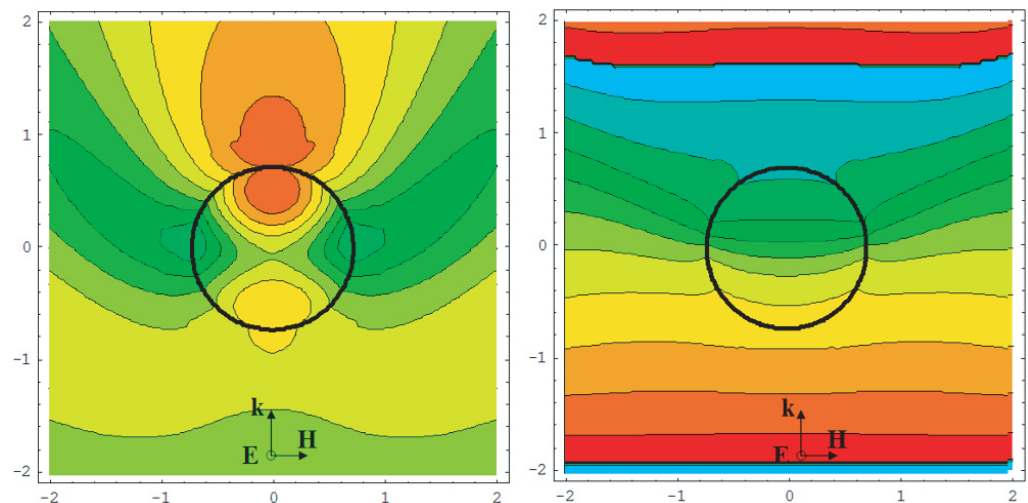


Figure 13. Similar to figure 10, but for frequency ω_{c2} . The plots have the same absolute color scale as in figure 12, for sake of comparison.

It can be seen in figure 8 that the cloak may indeed reduce the scattering cross section in the neighborhood of the three cloaking frequencies. The first cloaking frequency, at relatively low frequencies, reduces the total scattering of the cloaked system by about -6.3 dB. This is a fairly good scattering reduction, but is not as good as can be achieved with thinner cloaks. The reason resides in the fact that at these low frequencies the scattering from the uncloaked object is mainly dominated by its electric dipole contribution, which may be totally canceled by a thin plasmonic layer. Having two rather thick layers, as in this case, creates unwanted spurious scattering from the cloak layer that is not effectively being cloaked, not approaching a total scattering suppression. At the higher frequency ω_{c2} , the scattering reduction is more pronounced, due to the lower negative permittivity of the cloak materials. The third cloaking frequency is characterized by a smaller scattering reduction, due to the presence of higher-order scattering orders. It is evident that there is an arguably relatively large frequency range

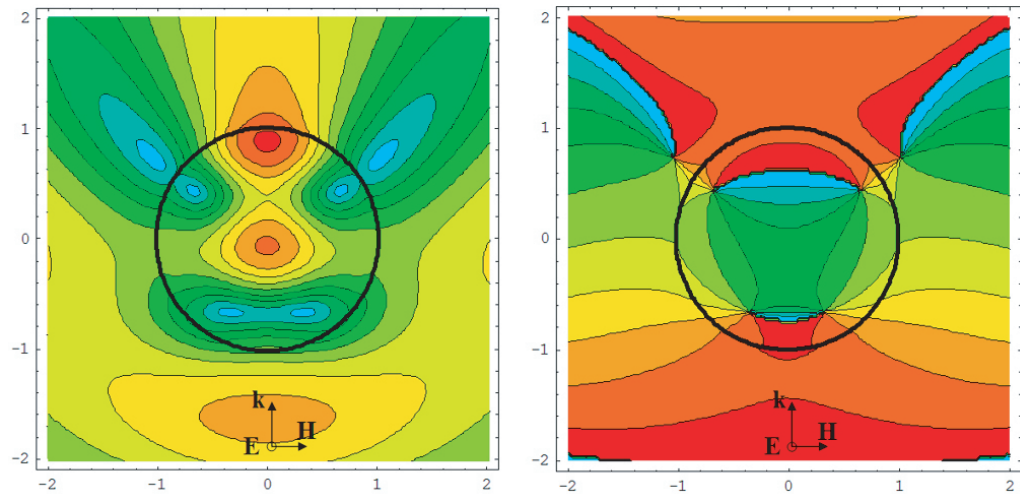


Figure 14. Similar to figure 11, but for frequency ω_{c2} . The plots have the same absolute color scale as in figure 12, for sake of comparison.

over which the scattering gain may be negative, when compared with the uncloaked scenario. This is obtained simply with a two-layered cloak composed of homogeneous and isotropic materials. However, the resonant scattering does exist at frequencies in between these cloaking frequencies.

Figure 8 also takes into account the presence of realistic losses. It is evident how the cloaking is not sensibly affected by reasonable losses (up to $\gamma = 0.01 \omega_p$), whereas the resonant scattering peaks are sensibly reduced as soon as losses are considered. This is in agreement with our previous findings for homogeneous cloaks [2].

In order to provide an example of the potentials of this technique, figure 9 reports the amplitude and phase of the electric field distribution on the H plane of polarization for this geometry at frequency ω_{c1} , under plane wave incidence traveling from bottom to top of the panels. The figures show almost uniform amplitude and planar phase fronts, thanks to the cloaking effect of the layered cover. For sake of comparison, we have reported analogous plots for the uncloaked dielectric particle (figure 10) and for the same particle covered by two analogous layers with the same dielectric permittivity (figure 11). We notice that at this lower frequency, the cloaking effect is less drastic than the cloaking at higher frequencies of interest, as discussed above; however, even at this frequency, there is a 6 dB scattering reduction in the cloaked system, when compared with the bare sphere.

The scattering reduction and operation of the cloak are even more evident in figures 12–14, where the same set of plots has been reported for the higher frequency of operation ω_{c2} .

It may be clearly seen from these plots that the peculiar functionality of this cloaking mechanism, with wave penetration from layer to layer inside the cloak, allows the multifrequency operation of this cloak.

To conclude this section, we point out that these concepts may be applied, with proper limitations, to relatively larger particles with overall size comparable to or larger than the wavelength of operation at the expense of having some residual nonzero scattering at the cloaking frequency. For a cloak composed of a single, homogeneous layer this cloaking mechanism and its size dependence and limitations have been thoroughly discussed and reported in [18].

5. Conclusions

Here, we have reported on the theory and possibility of using multi-layered plasmonic shells in order to drastically reduce the total scattering of a dielectric or conducting particle at different frequencies simultaneously. Inherent limitations dictated by the passivity of the materials have been outlined, together with the interesting potentials of the present method. Applications of these concepts may be envisioned in different scientific fields and different frequency regimes, spanning camouflaging and noninvasive probing in optics or obstacle and noise reduction at radio frequencies.

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