# Curved Path Walking 

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#### Abstract

Research on biped locomotion has focused on sagittal plane walking in which the stepping path is a straight line. Because a walking path is often curved in a three dimensional environment, a 3D locomotion subsystem is required to provide general walking animation. In building a 3D locomotion subsystem, we tried to utilize pre-existing straight path (2D) systems. The movement of the center of the body is important in determining the amount of banking and turning. The center site is defined to be the midpoint between the two hip joints. An algorithm to obtain the center site trajectory that realizes the given curved walking path is presented. From the position and orientation of the center site, we compute stance and swing leg configurations as well as the upper body configuration, based on the underlying 2D system.


## 1 Introduction

The workspace of a human figure would be quite restricted if the lower body were fixed. Locomotion provides a tremendous extension of the workspace by moving the body to places where other activities may be accomplished.

A locomotion system should provide the configuration of the figure at each time of the walk along a path specified by the input. There have been many efforts to make this process more realistic and automatic.

If the initial, final, and several intermediate keyframe configurations are given, a simple way of animating a jointed figure is interpolating the joint angles. When there are any constraints to be satisfied during the motion, this keyframe method may have some difficulties. This was pointed out by Girard and Maciejewski [1]. Even though the keyframe configurations themselves satisfy the constraints, the simple interpolation may make the inbetweens violate them.

Bruderlin and Calvert solved this problem by building a keyframeless system to simulate human locomotion [3, 4]. In doing animation they generated every single frame based on a dynamics and kinematics computation. Their system could generate a wide gamut of walking by changing the three primary parameters: step length, step frequency, and speed.

There have been many other studies on locomotion in robotics, biomechanics, and computer graphics $[5,6,7,8,9,10,11,12,13,14,15,16]$. But most of them concentrated on $2 D$ locomotion, in which walking is restricted to the sagittal plane. In a 3D animation system, one of the most important roles of the locomotion subsystem is to send the human figure from any position and orientation to any other arbitrary position and orientation whenever upper body adjustment alone cannot achieve such a posture. A straight locomotion path is very rare. In this point of view, 2D locomotion lacks the usefulness as a subsystem of general animation system, and the development of 3D locomotion subsystem seems inevitable to simulate general human behaviors. Unfortunately, simply treating a complex walking path as sequence of straight-line paths does not work. The problems of turning and coordinating the limb motions at the turns is frequently neglected and the rigid appearance of the resulting abrupt mid-air turns is clearly unacceptable animation.

In his paper [2], Girard discussed general concepts (time, position, velocity control, etc.) encountered in animating legged animal 3D motion. He also
mentioned turning. He interprets stepping (liftoff) as a way to give impulse to the human body in running. Each impulse contributes some acceleration to the whole body movement. He computed the impulse that is required to drive the center of the body along a given curve. By the impulse at the liftoff (this impulse includes rotational torque as well as upward force), the body gets a rotational torque and the whole body rotates in the air. So turning is basically done while the body stays in the air.

In walking, however, everything that turning entails is done while at least one of the feet is on the ground. So there are more constraints to be satisfied than in running. The whole body doesn't turn by the same angle: the body is rather twisted during the turning motion. And the ankle and hip joints have an important role in generating a natural turning motion.

In building a 3D locomotion system, we tried to utilize pre-existing 2D systems, that is, a 2D locomotion system is used as a subsystem to our 3D system. For every 3D step, we will consider its underlying 2D step, and the 2D system will provide some needed information.

Our generalization algorithm from 2D to 3D was based on the intuition that there should be a smooth transition between linear and curved locomotion. If the curvature is not large, the 3D walk generated by our system should be close to the 2 D walk given by the underlying 2 D system. In particular, if the given curve is actually a straight line, the walk by the 3D system should be exactly the same as the walk produced by the underlying 2D system. No assumptions were made about the underlying $2 D$ system, therefore most 2D locomotion systems can be generalized into 3D ones by applying our algorithm. Clearly the underlying 2D system will determine the stylistics of curved path walking. In this paper, a 2D locomotion system will be assumed to exist already and so its details will not be discussed.

## 2 Our Model

At a certain moment, if a leg is between its own heelstrike (beginning) and the other leg's heelstrike (ending), it is called the stance leg. If a leg is between the other leg's heelstrike(beginning) and its own heelstrike (ending), it is called the swing leg. For example, in Figure 1, left leg is the stance leg during the interval 1 , and right leg is the stance leg during the interval 2. Thus at each moment we can refer to a specific leg by either stance or swing

## LEFT LEG



RIGHT LEG


## HS=HEELSTRIKE

 TO=TOE OFF DS=DOUBLE STANCEFigure 1: The Phase Diagram of Human Walk


Figure 2: The Hypothetical Structure of Human Body
leg with no ambiguity. The joints and segments in a leg will be referred to using prefixes swing or stance. For example, swing ankle is the ankle in the swing leg.

The structure of the body used in our locomotion is shown in Figure 2. There are 14 joints $J_{i},(i=1, \ldots, 14)$, and joint angles $\theta_{i},(i=1, \ldots, 14)$ at each joint. Even numbered joints $J_{2}, J_{4}, J_{6}$, and $J_{8}$ are the joints in the stance leg, and corresponding odd numbered joints are in the swing leg. $\theta_{13}$ is the angle of the torso with respect to the vertical line. $\theta_{7}, \theta_{8}, \theta_{11}, \theta_{12}$, and $\theta_{14}$ are measured with respect to the torso. Let the center site be the center of the two hip joints, $J_{7}$ and $J_{8}$. In general, $\theta_{i}$ can be a scalar or a vector according to the degree of freedom at that joint. For example, $\theta_{7}$ is a three dimensional vector, which is interpreted as the Euler angles at the swing hip joint. In human locomotion, the rotations at the joints after all produce translational movement of the whole body. To compute the translational movement, we need a reference point somewhere in the body, which we will call the root site. To specify the configuration of the body at a moment, we should have the position $\theta_{15}$ and orientation $\theta_{16}$ of the root site, as well as all the 14 joint angles. Let $\Theta=\left[\theta_{1}, \ldots, \theta_{16}\right]$.

On the other hand, consider what affects the value of $\Theta$. The step length $s l$ and the step frequency $s f$ are considered to be the most important factors that determine the style of the locomotion. In 3D walking, however, the walking depends also on the relative direction of the next foot. And the definition of step length is not clear any more. Instead of $s l$, we will use the position and orientation of each foot. Here, the position of the foot is the position of the heel of the foot, and the orientation of the foot is the direction of the foot when the foot is put flat on the ground. So the orientation of the foot is always parallel to the ground. Let $\vec{h}_{i}$ and $\vec{d}_{i}$ be the position and orientation of the $i$ th foot, respectively. Let $s f_{i}$ be the step frequency of the $i$ th step. ( $s f_{0}$ does not have any meaning.) Let $\Sigma$ be the sequence of $\left(\vec{h}_{i}, \vec{d}_{i}, s f_{i}\right.$, lorr $\left._{i}\right), i=0,1, \ldots, n$, where lorr ${ }_{i}$ is 0 when the $i$ th foot is left foot and 1 otherwise.

The shape and the mass distribution of the body also have something to do with the type of walking. Let $l_{1}, \ldots, l_{14}, l_{15}$ be the description of the links of the model. $l_{1}$ is the swing toe, $l_{2}$ is the stance toe, $\ldots, l_{14}$ is neck and head, etc. $l_{15}$ represents the pelvis (not shown in the Figure) and $l_{7}, l_{8}$, and $l_{13}$ are connected to $l_{15}$. Let $\Lambda=\left[l_{1}, \ldots, l_{15}\right]$.

Generally, the locomotion problem in computer graphics is to find the function $f$ that relates $\Lambda, \Sigma$ with $\Theta$ at each time $t$.

$$
\begin{equation*}
\Theta=f(\Lambda, \Sigma, t) \tag{1}
\end{equation*}
$$

Usually the function $f$ is not simple, and we try to devise a set of algorithms that computes the value of $\Theta$ for the given value of $(\Lambda, \Sigma, t)$, depending on the case.

At every joint, there are 2 underlying local coordinates, both of which can be represented by $3 \times 3$ matrices,

$$
\left[\begin{array}{c}
\overrightarrow{v_{x}} \\
\overrightarrow{v_{y}} \\
\overrightarrow{v_{z}}
\end{array}\right]
$$

where $\overrightarrow{v_{x}}, \overrightarrow{v_{y}}$, and $\overrightarrow{v_{z}}$ are unit vectors along $x, y$, and $z$ directions in the local coordinates, respectively. Let the upper local coordinate of $J_{i}$ be $T_{i}^{U}$, and the lower one be $T_{i}^{L}$. For example, at $J_{6}, T_{6}^{U}$ is the local coordinates at the lower end of the stance thigh, and $T_{6}^{L}$ is the upper end of the stance shin. Let $R_{i}^{U}$ and $R_{i}^{L}$ be the global rotation matrices (orthogonal) that are given by resolving the local coordinates $T_{i}^{U}$ and $T_{i}^{L}$ in the global coordinates, respectively.

## 3 The Specification of Walk

The specification of a walk in 2D can be done by giving a sequence of $\left(s l_{i}, s f_{i}\right), i=1, \ldots, n$. Each $\left(s l_{i}, s f_{i}\right)$ affects the type of current step, starting from the current heelstrike to the next one. In 3D, however, even defining $s l$ is not easy, and we need another way of specifying the steps.

As discussed in the previous section, the direct input to our locomotion subsystem is a sequence $\Sigma$ of 4 -tuples,

$$
\begin{equation*}
\sigma_{i}=\left(\vec{h}_{i}, \vec{d}_{i}, s f_{i}, \text { lorr }_{i}\right), i=0, \ldots, n \tag{2}
\end{equation*}
$$

which will be called the step sequence. Each tuple $\sigma_{i}$ is called the $i$ th foot description. The pair of adjacent two foot descriptions $\left(\sigma_{i-1}, \sigma_{i}\right)$ is called the ith step description or simply the ith step. Even though we can get maximum control of locomotion by using the step sequence, generating such a sequence


Figure 3: The Two Ways of Input Specification
might be a tedious job. Most of the users may not be interested in each step it takes, but just want to send the human figure to a goal position and orientation along a curved path, to avoid any possible objects in the way. To provide this kind of userfriendliness, we have 2 types of input, curves and step sequences (Figure 3).

If a curve is given as input, it is automatically transformed to a step sequence by the algorithm below (Figure 4 ): $\lambda$ is a constant whose value is the distance from the center site to either of the hip joints. $\mu$ is a variable, and its value is given by the user. At every point on the curve whose linear distance is $\mu$, say $P$, the derivative is computed and the normalized vector of it is assigned to $d_{i}$. The point whose distance from $P$ is $\lambda$ is given as the value of $\overrightarrow{h_{i}}$. Whether $\overrightarrow{h_{i}}$ should be in the left or right side of $P$ is determined by tracking from the first step by alternating the feet. The value of $\operatorname{lorr}_{i}$ is also given from this consideration.

For every step description $\left(\sigma_{i-1}, \sigma_{i}\right)$ in 3D, we consider its underlying $2 D$ step. The step frequency $s f_{2 D}$ of this 2D step is given by $s f_{i}$ of $\sigma_{i}$. We can draw 2 lines $\alpha_{1}, \alpha_{2}$ on the horizontal plane as shown in the Figure 6. $\alpha_{1}$ is in the direction of $\vec{d}_{i-1}$ displaced by $\lambda$ from $\vec{h}_{i-1} . \alpha_{2}$ is in the direction of $\vec{d}_{i}$ displaced by $\lambda$ from $\overrightarrow{h_{i}}$. Let $\delta$ be the arc length of the spline curve from $E_{i-1}$ to $E_{i}$, where $E_{i-1}$ and $E_{i}$ are the projections of the heel positions to the lines $\alpha_{1}$, and $\alpha_{2}$, respectively. (The derivatives at the end points of this spline curve are given by $\vec{d}_{i-1}$ and $\vec{d}_{i}$.) The step length $s l_{2 D}$ of the underlying


Figure 4: Getting a Step Sequence from a Curve

2D step is given by this $\delta$.
The overall structure of our algorithm is depicted in Figure 5. The underlying 2D step of the current step is sent to the 2D locomotion system. The 2D system provides the 2D step information to the 3 D system. In our 3D system, we first compute the center site trajectory. ¿From the center site location (position and orientation), we obtain the locations of both hips. The locations of the feet are computed based on the 2D step information and our assumptions. Because we have the locations of hip and foot of both legs, the configurations of the stance leg and the swing can be computed. The banking angle is computed, and the upper body is adjusted to move the center of mass until such a banking is achieved. The parameters $\Theta$ that determine the configuration of the whole body is now sent to the graphics system to draw the human figure on the screen.

The details of the algorithm will be given in the following two sections. In Section 4, higher level concepts are discussed such as center site trajectory, stance and swing hip locations, banking, and turning, etc. Section 5 deals with lower level ideas, including the computation of the joint angles in the legs, the computation of banking angles and the subsequent upper body adjustment, etc.


Figure 5: The Overview of the Algorithm


Figure 6: The Step Length of the Underlying 2D Step


Figure 7: The Trajectory of the Center Site (Top View)

## 4 Path of the Center Site

In our model, the center site is assumed to move along a straight line from the heelstrike moment of the stance leg (HSM) to the toe off moment of the swing leg (TOM), as shown in the Figure 7. For the time being we will restrict ourselves to the planar movement (top view) of the center site. The height component will be discussed later in this section. The trajectory of the center site during this double stance phase (DS) is given by the underlying 2D step, and it is a straight line at top view. The direction of this straight line is determined from the direction of the current stance foot direction $\vec{d}_{i-1}$.

From the TOM to the next HSM, the center site moves along a spline interpolation(Figure 7). At the both ends of the spline, the derivative of the curve is the same with that of the adjacent line segments, to maintain the


Figure 8: The Position of Center Site at Heelstrike Moment
first order continuity. Through the whole locomotion, the pelvis is assumed to face the derivative direction of the center site path, and be vertical to the ground. The torso can be bent in any direction, a part of which is given by the underlying 2 D algorithm, and another part is given from the banking computation which will be discussed later in this section.

To derive the spline curve, we need the position $\vec{C}_{N H S M}$ and derivative $\dot{\vec{C}}_{N H S M}$ of the center site at the next HSM, as well as $\vec{C}_{T O M}, \dot{\vec{C}}_{T O M}$ at TOM which are provided by the underlying 2 D system (Figure 8 ). The assumptions above imply that $\dot{\vec{C}}_{N H S M}=\vec{d}_{i}$, and $\vec{C}_{N H S M}$ should be put somewhere on the line $\alpha_{2}$. Let $X$ be a point on $\alpha_{2}$.

Let $\eta_{2 D}$ and $\tau_{2 D}$ be the length of the center site trajectory from HSM to TOM, and from TOM to the next HSM, respectively, during the underlying 2 D step. Let $\eta_{3 D}$ of corresponding 3D step be similarly defined as $\eta_{2 D}$. Let $\tau_{3 D}(X)$ be the arc length (top view) of the spline from $\vec{C}_{T O M}$ to $X$ in the Figure 8 , which is a function of $X$. Now the position of the center site $\vec{C}_{N H S M}$ at the next HSM is set to the point $X$ on the line $\alpha_{2}$ such that

$$
\begin{equation*}
\frac{\eta_{2 D}}{\tau_{2 D}}=\frac{\eta_{3 D}}{\tau_{3 D}(X)} \tag{3}
\end{equation*}
$$

This definition of $\vec{C}_{N H S M}$ is based on the smooth transition assumption from the 2D locomotions to the 3D ones. By a mapping which preserves arc length


Figure 9: Banking of the Center Site
ratio $[2,17,19]$, we can find the correspondence between the 3 D trajectory of the center site and underlying 2D one. Note that this definition also makes the degenerate case of 3D walk exactly same with the corresponding 2 D walk. Bisection method can be used in computing such $X$.

The displacement of the curved path from the underlying linear path is produced by banking as shown in Figure 9. In the figure, $C_{3 D}$ is the position of the displaced center site in 3D step, and $H_{s w}$ is the position of the swing hip. This banking mostly results from the ankle joint adjustment. The computation of the ankle angle is presented in the next section. Even though the center site is put on the spline curve by the ankle angle, the upper body has not bent yet to generate the overall correct banking of the whole body. The banking should be considered in terms of the center of mass of the body. The overall banking is given by

$$
\begin{equation*}
\phi=\arctan \left(\frac{\kappa v^{2}}{g}\right) \tag{4}
\end{equation*}
$$

where $v$ is the velocity, $g$ is the gravity, and $\kappa$ is the curvature of the curve [2]. Here we use the spline curve of the center site as an approximation to get the curvature. (The center site is not far away from the center of mass, specially when it is seen from the top.) The upper body should be bent so that the center of mass (which is in the upper body) may make the angle $\phi$ around the stance ankle with respect to the ground. Iterative methods can be used to compute the current center of mass and reduce the difference from the current one and the desired one.

The amount of turning is measured with respect to the direction of the underlying 2D locomotion, as shown in Figure 10. Turning is produced mostly at the stance hip joint. More details of it will be explained in the next section.


Figure 10: Turning of the Center Site


Figure 11: Dynamic Leg Length $\omega$

The dynamic leg length $\omega$ at a certain moment is defined to be a hypothetical length of the stance leg from the ankle to the hip, and here the position of ankle comes from the foot which was flat on the ground, as shown in Figure 11 [3, 4].

We assume that the dynamic leg length $\omega$ in 3D locomotion at a moment $t$, is same with $\omega$ at the corresponding moment in the underlying 2D step. So the displaced center site $C_{3 D}$ will be lower than the corresponding 2D center site $C_{2 D}$. The position ( $x_{1}, y_{1}, z_{1}$ ) of the hypothetical ankle is available from the old foot location. Let $\left(x_{2}, y_{2}, z_{2}\right)$ be the position of the swing hip $H_{s w}$ in Figure 9. The horizontal components $x_{2}$ and $z_{2}$ can be computed from the derivative at $C_{3 D}$ and $\lambda$ : note that the horizontal components of $C_{3 D}$ are
already available at this point by the discussion above. In Figure $9, \lambda$ is the distance between $H_{s w}$ and $C_{3 D}$, and this distance is along the perpendicular direction of the derivative.

Because we assumed that $\omega$ is the same in 2D and 3D locomotions, we have

$$
\begin{equation*}
\left|\left(x_{1}, y_{1}, z_{1}\right)-\left(x_{2}, y_{2}, z_{2}\right)\right|=\omega \tag{5}
\end{equation*}
$$

where w is given by the underlying 2D system. The value of $y_{2}$ that satisfies the above equation is the height of both $H_{s w}$ and $C_{3 D}$. Because the pelvis is assumed to be straight up all through the steps, the height of the stance hip $H$ is also $y_{2}$.

As a summary of this section, note that if the locomotion path is actually a straight line, the path of the center site will be the same as that of the underlying 2D locomotion.

## 5 Lower Level Details

### 5.1 Stance Leg

The center site movement of 3D locomotion during DS is the same as that of the 2 D one, including the height component, so the stance leg configurations are given by the underlying 2D system. During the single stance phase, we still use 2D system to get the joint angle at the ball of foot. But because the center site begins to deviate, the other joint angles should be computed.

Meaningful twist is defined to be a rotation at a joint (not in the swing leg), which can kinematically affect the position of the swing leg, and therefore affects the next stance foot. Thus a meaningful twist can change the planar direction of the locomotion. For example, the twist at the spine (rooted at the stance foot) doesn't affect the position of the swing leg. But the rotation of pelvis around the stance hip affects the later walk because the swing leg will be displaced accordingly. There are 3 possible joints in the stance leg that can generate meaningful twist of the body during the turning motion. They are the ball of foot, ankle, and hip. (There is also the possibility of sliding in the case of tough curvature. But it will not be considered in this paper.)

From the locomotion point of view, the worst discrepancy between the real body and our model lies at the ball of the foot. In a real body, the toe
group is flexibly attached to the hind foot, and the flexion axis tends to vary according to the direction of lifting during the heel off. That is how it can contribute to turning. But that kind of joint is difficult to model, because the foot is usually modeled by 2 rigid segments connected by a revolute joint. So we rule out the ball of foot. Thus in our model, the rotation of the pelvis relative to the stance foot comes from the ankle and hip joints. Even though each of them contributes to the turning, rotation around ankle is much more restricted than around the hip. So the potential redundancy can be eliminated by considering the ankle as a joint of 2 degrees of freedom. Actually, in most of the human anatomy books, ankle is regarded as a joint of 2 degrees of freedom, flexion-extension and inversion-eversion [18].

In the stance leg, after the foot is put flat on the ground, the toetip is regarded as the root (fixed point) because that point is not moved until the next toe off. Because the joint angle at the ball of foot is provided by the 2 D algorithm, we have the position $A$ and global rotation matrix $R_{4}^{L}$ of the ankle. From the distance between the stance hip $H$ and stance ankle $A$, and the lengths of thigh and shin, we get the stance knee angle $\theta_{6}$, and the opposite angle $\theta_{\text {thigh }}$ of the thigh, by applying the cosine law to the triangle formed by $A, H$, and the knee. Let $\vec{\rho}$ be the vector from $H$ to $A$. The local coordinates at the joints in the stance leg are shown in Figure 12. We use the convention that $x$ axis is forward, $y$ axis is to the right side, and $z$ axis is downward when the body is straight up. In this section, components of every vector are resolved in the global coordinates. At each joint, let $\vec{x}_{i}{ }^{L}$, $\vec{y}_{i}^{L}$, and $\vec{z}_{i}^{L}$ be the unit vectors along $x, y, z$ axis of the local coordinates corresponding to the global rotation matrix $R_{i}^{L}$, respectively, that is, $\vec{x}_{i}{ }^{L}$ is the first row of $R_{i}^{L}$, and so on. The unit vectors $\vec{x}_{i}^{U}, \vec{y}_{i}^{U}$, and $\vec{z}_{i}^{U}$ associated with $R_{i}^{U}$ are similarly defined. Let

$$
\begin{equation*}
\vec{u}_{z}=\frac{\vec{\rho}}{|\vec{\rho}|} \tag{6}
\end{equation*}
$$

Let $\vec{u}_{x}$ be a unit vector that is normal to both $\vec{y}_{4}^{L}$ and $\vec{u}_{z}$, whose direction is given by

$$
\begin{equation*}
\vec{u}_{x}=\frac{\vec{y}_{4}^{L} \times \vec{u}_{z}}{\left|\vec{y}_{4}^{L} \times \vec{u}_{z}\right|} \tag{7}
\end{equation*}
$$

Let

$$
\begin{equation*}
\vec{u}_{y}=\vec{u}_{z} \times \vec{u}_{x} \tag{8}
\end{equation*}
$$



Figure 12: Local Coordinates at Ankle and Hip Joints

We construct $R_{4}^{U}$ by

$$
R_{4}^{U}=\left[\begin{array}{c}
\vec{u}_{x}  \tag{9}\\
\vec{u}_{y} \\
\vec{u}_{z}
\end{array}\right]
$$

Then the joint angle(s) $\theta_{4}=\left(\theta_{4}^{x}, \theta_{4}^{y}, \theta_{4}^{z}\right)$ at the ankle is given by computing the Euler angles associated with the rotation matrix $R_{4}^{L}\left(R_{4}^{U}\right)^{-1}$, where $\theta_{4}^{x}, \theta_{4}^{y}, \theta_{4}^{z}$ are the Euler angles around the local $x, y, z$ axis, respectively.
$\theta_{4}^{y}$ should be incremented by $\theta_{\text {thigh }}$, because $\vec{\rho}$ is not in the same direction with shin. The definition of $R_{4}^{U}$ above always ensures that $\theta_{4}^{z}$ is 0 . In this way, the redundancy along $\vec{\rho}$ is eliminated.

By the computations above, we can determine the configuration of the stance leg from the toe group to the thigh. So the global rotation matrix $R_{8}^{L}$ at stance hip is now available. By the assumption that the pelvis always faces the derivative direction and is vertical to the ground, we already have the global rotation matrix $R_{8}^{U}$ of the pelvis: if we let $\vec{\gamma}$ be the unit vector
along the derivative direction, we have

$$
R_{8}^{U}=\left[\begin{array}{c}
\vec{\gamma}  \tag{10}\\
(0,-1,0) \times \vec{\gamma} \\
(0,-1,0)
\end{array}\right]
$$

From these 2 matrices we compute Euler angles to get the joint angle(s) $\theta_{8}$ between the pelvis and the hip. Thus we now have all the joint angles in the stance leg.

### 5.2 Swing Leg at the Double Stance Phase

Because a revolute joint is assumed at the ball of foot, if we exclude the possibility of sliding, the toe group of the swing foot should stay fixed on the ground during the DS. Because there are 3 links between the swing hip $J_{7}$ and the ball of foot $J_{1}$, we should resolve the redundancy in a reasonable way. If we use the joint angle in 2D algorithm at $J_{1}$, this redundancy goes away. This approximation works well in most of the cases. But when both the direction change and the step length (the distance between the adjacent steps) are extremely large, the distance $\left|\vec{\rho}_{s w}\right|$ from $H_{s w}$ to $A_{s w}$ becomes too long to be connected by the lengths of thigh and shin. $\left(\vec{\rho}_{s w}, H_{s w}\right.$, and $A_{s w}$ are similarly defined in the swing leg as in the stance leg.) This problem was solved by increasing the angle at the ball of foot until $\left|\vec{\rho}_{s w}\right|$ becomes less than the sum of thigh and shin. Once this condition is satisfied, we can use $\vec{\rho}_{s w}$ to get the joint angles at ankle, knee, and hip, similarly as in the stance leg.

### 5.3 Swing Leg at the Single Stance Phase

The trajectory (top view) followed by the swing ankle is approximated by a second degree Casteljau curve [19]. The 3 control points are given by the position $D_{1}$ of the current swing ankle at TOM, $D_{2}$ which is symmetric point of the stance ankle with respect to the line $\alpha$, and the ankle position $D_{3}$ at the next heel strike point, as shown in Figure 13.

The height component of the ankle is determined by the underlying 2 D algorithm. So now we have $A_{s w}$ and $H_{s w}$, and should determine $\theta_{3}^{x}, \theta_{3}^{y}, \theta_{5}$, $\theta_{7}^{x}, \theta_{7}^{y}$, and $\theta_{7}^{z}$. We use the underlying 2 D locomotion system to get $\theta_{3}^{y}$. $\theta_{5}$


Figure 13: The Path of the Swing Foot
is given by the triangle formed by $H_{s w}, A_{s w}$, and the swing knee. Note that $R_{7}^{U}$ is given by the computation in the section 5.1 , because $R_{7}^{U}=R_{8}^{U}$. Let the vector $\vec{\rho}_{s w}$ be from $H_{s w}$ to $A_{s w}$. ¿From the relationship between $R_{7}^{U}$ and $\vec{\rho}_{s w}$ we can compute $\theta_{7}^{x}$ and $\theta_{7}^{y}$ in similar way as in the section 5.1. Now only $\theta_{3}^{x}, \theta_{7}^{z}$ remain to be computed.

At the moment of heelstrike, the swing leg should have been prepared for the next step. Because linear walking is assumed from HSM and TOM, the hip and ankle angles in swing leg should become 0 except for the bending direction (around the $y$ axis). That is, the values of $\theta_{7}^{x}, \theta_{7}^{z}$, and $\theta_{3}^{x}$ should get to 0 at the heelstrike moment. So we should somehow adjust the swing leg and foot from the chaos configuration (at TOM) to the ordered configuration (at the next HSM). By the stance leg movement and the ankle trajectory given above, $\theta_{7}^{x}$ becomes 0 automatically at the heelstrike moment. At toe off, the displaced (rotated around the $x$ axis) foot gets to the normal position very quickly and it is approximated by an exponential function that decreases rapidly to 0 . That is, for some positive constant $G$, we let $\theta_{3}^{x}$ at time $t$ be

$$
\begin{equation*}
\theta_{3}^{x}(t)=\theta_{3}^{x}(0) \cdot \exp ^{\frac{-G t}{t_{s w}}} \tag{11}
\end{equation*}
$$

where $t$ is the elapsed time after TOM, and $t_{s w}$ is the duration between the TOM and the next HSM. The $z$ component of the hip joint angle is approximated by a parabola given by

$$
\begin{equation*}
\theta_{7}^{z}(t)=\theta_{7}^{z}(0) \cdot\left(\frac{t_{s w}-t}{t_{s w}}\right)^{2} \tag{12}
\end{equation*}
$$

As a summary of this section, note that if the 3 D locomotion path is actually a straight line, the walk generated by the computations above is exactly the same as the walk given by the underlying 2D system. For example, in the consideration of swing leg motion during the single stance phase, $D_{1}, D_{2}$, and $D_{3}$ will be collinear and parallel to the walking path. Also in the equations (8) and (9), both $\theta_{3}^{x}(0)$ and $\theta_{7}^{z}(0)$ will be 0 and the trajectory of the swing leg will be exactly the same as that of 2 D walking.

## 6 Results and Conclusion

Figure 14 shows the foot steps generated by the interactive step editor developed for our walking animation. Steps can be added, deleted, moved, and rotated interactively on the screen, and finally we can get the step sequence.

Figure 15 shows the walking path generated by the interactive path editor. A curve can be edited by adding (at the ends), inserting, deleting, moving the control points. If $\mu$ values are given by the user, the foot steps are generated according to the algorithm in section 3 .

The curved path walking algorithm is implemented in $J a c k{ }^{\text {TM }}$ [20]. Figure 16 shows the snapshots during a turning step. The torso of the human figure in Jack is modeled by 17 segments. It allows the torso to bend in the forward/backward, lateral, and axial directions [21]. During walking, we could bend the torso to avoid simple obstacles. At the end of a walk, walking motion could be connected to a stepping motion [20], which is useful in generating local locomotion.

This method is a robust and effective way to produce curved path locomotion.


Figure 14: Steps Generated by the Step Editor


Figure 15: Steps Generated by the Path Editor


Figure 16: Snapshots during a Turning Step

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