Is Advance Selling Desirable with Competition?

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Abstract

It has been shown that a monopolist can use advance selling to increase profits. This paper documents that this may not hold when firms face competition. With advance selling a firm offers its service in an advance period, before consumers know their valuations for the firms' services, or later on in a spot period, when consumers know their valuations. We identify two ways in which competition limits the effectiveness of advance selling. First, while a monopolist can sell to homogeneous consumers at a high price, if consumers are equally uncertain as to which firm they prefer, their choice is based primarily on which firm has the lower price, and the resulting price competition lowers advance prices and profits. Second, competition in the spot period is likely to lower spot period prices, thereby forcing the firms to lower their advance period prices, which again is not favorable to profits. However, the firms may nevertheless find themselves in an equilibrium with advance selling. In this sense, advance selling is better described as a competitive necessity than as an advantageous tool to raise profits.

1 Introduction

Sometimes consumers purchase a good or service well before they actually consume it. This can involve some risk - the consumers may have an expectation that they will want or need the service, but they can be uncertain in what its actual value will be. There are different ways in which valuation uncertainty may arise when purchasing consumer goods or services. For example, customers may be uncertain about their value of a concert or a musical several weeks before the show or about their desire to eat at a popular restaurant in the future. Customer value may also be uncertain for a new or innovative product (e.g., an Apple iWatch, Jawbone's UP3 activity tracker, Sony's VR headset, or Windows 10), an experience item (such as movies, books, or video games), or if their need for an item is uncertain (e.g., a dress for a friend's wedding they may or may not be able to attend). Common to all these examples is that customers gain a better sense of their value for the product over time. Therefore, consumers can wait to resolve that uncertainty by delaying their purchasing decision to shortly before consumption.

Firms who are aware of consumers' value uncertainty may try to exploit it by selling in advance. We say a firm *advance sells* when the firm allows consumers to buy well in advance of their consumption. And

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we say that the firm *spot sells* when the firm sells to consumers at the time of consumption, such as selling movie tickets just before the movie begins. It has been shown that monopolies can earn more revenue by advance-selling than by selling exclusively on the spot (e.g., Xie and Shugan (2001)). When consumers purchase in advance, they are uninformed and are willing to pay at most their expected value for the service. In contrast, when consumers purchase on the spot, they know their exact values for the product. These values are generally different across consumers–some customers value the product more than others and no consumer is willing to pay more than his realized value. That is, in the spot period, consumers are less homogeneous relative to the advance period. Monopolistic firms can take advantage of consumers' *ex-ante* homogeneity: A firm can earn higher revenue by selling in advance to *all* consumers (but, at a lower price) than by selling on the spot to only a portion of consumers (those who discover they have a high value for the product).

Building on the advance selling results for a monopoly, in this paper we seek to understand whether advance selling is also desirable in a competitive environment. Competition complicates the consumers' purchasing options in that under competition individual consumers decide not only on *when* and *whether* to purchase the product, but also *which* firm to buy from. This paper demonstrates that while a monopolist benefits from selling in advance, advance selling does not necessarily help competing firms. We show that even though advance selling may occur in equilibrium, competitive firms would in most cases benefit if they could commit to sell only on the spot. There are two reasons for our finding, which we illustrate using two related models. First, as consumers are more homogeneous in the advance period, they are more sensitive to the firms' advance period prices. This intensifies the competition between the firms in the advance period, driving down prices in the advance period, making advance selling less desirable. Second, competition in the spot period lowers spot period prices. As consumers only buy in advance if they get a sufficiently good discount relative to the spot period price (to compensate them for the fact that they face valuation uncertainty if they buy in advance), spot period competition forces the firms to lower their advance period prices even if they do not compete in the advance period. Again, this is undesirable for advance selling.

2 Related Literature

Our work belongs in the intertemporal pricing literature. These papers consider pricing over multiple periods with forward looking consumers who make dynamic choices. In the durable goods literature consumers time their purchase in anticipation of markdowns: e.g., Coase (1972), Stokey (1981), Besanko and Winston (1990). In all of these papers, consumers never consider purchasing when their valuation for the service is uncertain, so purchasing early (i.e., in "advance") does not involve the risk of purchasing something that is later on not desired. In these papers, the only reason to purchase early at a high price is the time value of money (i.e., discounting), which we ignore here. Other reasons for consumers to time their purchase that have been discussed in the literature include product availability (e.g., Su (2007), Elmaghraby et al. (2008), Aviv and Pazgal (2008) and Cachon and Swinney (2009)) and product innovation (e.g., Dhebar (1994) and Kornish (2001)). These papers differ from ours. We do not model inventory decisions and assume that there are no capacity constraints and that the firms offer the same product in both periods.

Within that literature, a number of papers focus specifically on advance selling strategies. Gale and Holmes (1993), and Degraba (1995) each consider a monopoly firm with capacity constraints. Gale and Holmes (1993) show that advance-purchase discounts allow a firm to price discriminate between consumers who are reasonably certain of their future utility for a service and those who are more unsure. Degraba (1995) finds that a firm may benefit from intentional scarcity strategies that induce consumers to buy in advance to avoid a rationing risk. In our models there are no capacity constraints, so customers' incentive to purchase in advance is due to an advance price discount rather than limited availability. Xie and Shugan (2001) extend the work of Shugan and Xie (2000) and show that a firm can be better off by selling in advance even when there are no capacity constraints as long as the unit marginal cost is not too high. For simplicity, we assume the unit marginal cost is zero. Nasiry and Popescu (2012) consider advance selling to consumers who experience regret, whereas we work in an expected revenue maximization framework. Cachon and Feldman (2011) study subscription pricing, which can be a firm of advance selling applied to repeat purchases, whereas in our model consumers make a single purchase.

Several papers consider selling strategies that help mitigate consumer risks from advance purchases. Among those, in some studies, firms offer advance sales which are at least partially refundable: e.g., Xie and Gerstner (2007), Guo (2009), and Gallego and Sahin (2010). These papers focus on how much refund, if any, the firm should offer customers. In these papers advance selling may be beneficial because of the ability to sell the same unit of capacity twice. We do not consider partial refunds, as the benefits of partial refunds have been shown only in environments with limited capacity. Other papers consider strategies to help customers mitigate other types of risk. When there is limited capacity and customers are risk-averse, Png (1989) shows that offering reservations before consumers learn their valuations insures them against the possibility of being rationed. In our model customers are risk-neutral, but the firm needs to provide a sufficient discount to convince forward-looking consumers to buy in advance when their value is uncertain. Firms may also offer price guarantees which give customers a refund if they find the product at a lower price elsewhere: e.g., Png and Hirshleifer (1987) and Jain and Srivastava (2000). In our paper there is no price uncertainty. We assume that customers are capable of correctly anticipating firms' spot prices.

Although most of the advance selling literature assumes a market with a single firm, there are some that consider competition. Dana (1998) demonstrates that price-taking firms may offer an advance purchase discount in a market with capacity constraints and rationing. However, in his setting firms earn zero profit whether firms sell in advance or not, as the market is perfectly competitive. Hence, firms neither benefit nor are harmed by advance selling. Shugan and Xie (2005) find that "competition does not diminish the advantage of advance selling". In their models the firms sell either in advance or on the spot but not in both periods. Hence, consumers do not trade off buying in advance versus on the spot, i.e., consumers are not forward looking. In our model, we allow firms to sell in both periods and we assume that consumers are forward looking. Therefore, competitiveness in the spot period will reduce spot period prices and influence firms' advance sales. Finally, Guo (2009) considers advance selling in an oligopolistic market with capacity constraints, but as already mentioned, he focuses on the impact of including partial refunds or not.

3 Model Setup

To motivate the models, consider the following anecdotes:

- A married couple considers buying tickets to one of two plays to celebrate their anniversary. They have been to each playhouse before, but have not seen the plays themselves. They may learn which play they prefer to attend by reading expert critiques online as the date approaches.

- Travelers are interested in going on a vacation in the Mediterranean in July. They have not been on a cruise before, and debate on which cruise line to book for the trip. They may book their vacation now, or wait until June, when their friends return from a similar cruise and can share their experience.

- Both Apple and Samsung are designing innovative smart watches. Customers loyal to Samsung have no interest in Apple's iWatch. They may pre-order the watch now, uncertain on their need for it, or wait until it becomes available to the general public, at which time non-loyal consumers may also be interested in buying it.

- A's fans are considering whether to purchase baseball tickets early, fearing that they may have other commitments when game day arrives. Though both teams are located in the Bay Area, they have no interest in going to a Giant's game. Travelers to the Bay Area who are interested in watching a baseball game, but do not particularly care which ballpark to go to, may purchase tickets closer to the date, once they arrive to the area.

The first two examples describe situations where customers are initially uncertain regarding the firm they prefer and firms therefore engage in advance period competition. The uncertainty is resolved later so each firm is a monopoly in the spot period. The last two anecdotes are examples in which firms are monopolies in advance serving only their loyal customers, but compete on the spot for additional customers. Using two different models that fit these examples, we demonstrate that no matter the type of competition, advance selling is rarely desirable for competing firms.

In this section we describe the modeling framework common to both models and later describe the characteristics specific to each model. Two firms compete in a duopoly market. Each firm offers a single product for sale in two selling periods–the advance period and the spot period. The firms simultaneously announce advance period prices and then spot period prices to maximize expected revenues. A market of forward looking consumers decides whether to buy in the advance period or to wait for the spot. These consumers are initially uncertain about their product valuation, but this uncertainty is resolved in the spot period and customers' valuations are independent. The particular form of this uncertainty differs across the two models, as is described in detail later.

Firms' expected revenues are calculated based on the prices they charge and the number of consumers who decide to buy at that price. We assume that both the firms and the consumers are risk neutral and therefore all calculations and decisions are based on expectations (expected revenues for the firms and expected surplus for consumers) and that firms have sufficient capacity to sell to all consumers. Adding capacity constraints softens the competition between the firms and in the extreme the firms act as separate monopolists. Hence, we choose to ignore capacity constraints so as to emphasize the impact of competition.

For each model, we seek to characterize the subgame perfect Nash equilibria of the game in pure strategies. An equilibrium consists of the optimal actions chosen by the firms and the consumers given their beliefs about the actions taken by the other players. Moreover, the beliefs of all players are consistent with the equilibrium outcome.

4 Model I: Advance Period Competition

A market of consumers, which can be normalized to size 1 without loss of generality, decides which firm to buy from and in which period. In the advance period, consumers are uncertain about their consumption utility in the spot period. They face two types of uncertainty. First, they are unsure as to which firm they will prefer. Second, they are unsure as to how strongly they will value their preferred firm. To be specific, consumers know in the advance period that they will receive value V from their most preferred firm, where V can take on one of two values with equal probability, $V \in \{v_l, v_h\}$, $v_h \geq v_l > 0$. Consumers also know that they receive zero value from the other, non-preferred, firm.[1] Finally, consumers know in the advance period

[1]The results continue to hold qualitatively under more complicated value distributions, though the analysis becomes significantly more cumbersome.

that they will learn in the spot period which firm they prefer and how strong that preference is. Mayzlin (2006) and Guo (2009) use similar models of consumer uncertainty under competition.

The expected value of the preferred product is $\mathbb{E}[V] = (v_l + v_h)/2$. We define $\beta = v_l/v_h$. That is, β measures the value of v_l relative to v_h. As β increases, v_l approaches v_h. We assume $\beta \in [0, 1/2]$, which implies that $v_h \geq v_l$. Consequently, a firm's optimal spot price is v_h: a firm prefers to charge a high spot price (v_h) and sell to half of the consumers than to sell to all consumers at a low price. Thus, conditional on not owning a unit, only customers that have a high realized value for the product will purchase the unit in the spot period. We do not include cases in which $v_h < 2v_l$ because these are not interesting–in those cases there is no advantage to advance selling even for a monopolist: The optimal spot price is v_l, so forward-looking consumers would not pay more than v_l in the advance period, meaning that in all cases the monopolist sells to all consumers at v_l.

Even though consumers do not know which firm they prefer in the advance period, they are also not entirely clueless. Consumers have some information about their future spot period preference. This information can be obtained, for example, from consumers' previous experience with the firms or by reading expert reviews or blogs about the products. We model the degree of information each consumer has explicitly, by defining α as the probability that a customer attaches for preferring firm 1 in the advance period after processing the information. $(1 - \alpha)$ is therefore the probability that the same consumer attaches for preferring firm 2. We allow this information to differ among consumers by assuming that $\boldsymbol{\alpha} \sim U\left[\frac{1-\delta}{2}, \frac{1+\delta}{2}\right]$. Hence, there is a continuum of consumer types and a consumer's type is the probability that she will prefer firm 1. Some consumers attach a higher probability to preferring firm 1 than others.[2] When $\delta = 0$, consumers have no information with respect to which firm they will prefer and all consumers attach the same probabilities for preferring each firm (50/50). As δ increases, consumers become more heterogeneous in their knowledge. Some consumers are very informed about their firm preference, whereas others are not. This information helps consumers to better evaluate their eventual preference, before making their advance period purchasing decisions. The introduction of δ allows us to examine the effect that the heterogeneity in information has on firms' revenues and consumers' purchasing decisions in equilibrium, which we discuss in subsection 4.3.

Gale and Holmes (1993) also have consumers who are heterogeneous in the advance period. In their

[2] It is possible to derive these heterogeneous beliefs by assuming that customers receive a private signal on the firm they prefer. Assume that all customers share the same prior belief (1/2,1/2) for preferring each firm and let ω_i be the state of nature indicating that firm i is preferred, $i \in \{1, 2\}$. At the start of the advance period, each customer observes a private signal on his preferred firm. In particular, consider the case where the signal $s = \{s_1, s_2\}$ is binary with accuracy α. That is, the conditional probability distribution is $\mathbb{P}\{s_i|\omega_i\} = \alpha$ and $\mathbb{P}\{s_i|\omega_j\} = 1 - \alpha$, if $i \neq j$ with $i, j \in \{1, 2\}$. Customers are heterogeneous in the accuracy of their signal with $\alpha \sim U\left[\frac{1}{2}, \frac{1+\delta}{2}\right]$. The probability that a signal s_i is received by a customer with signal accuracy α is $\mathbb{P}\{s_i|\alpha\} = 1/2\mathbb{P}\{s_i|\omega_1\} + 1/2\mathbb{P}\{s_i|\omega_2\} = 1/2$. Let $\gamma_i(\alpha)$ be the posterior belief that a customer with signal accuracy α prefers firm 1 after receiving signal s_i. Bayes' rule yields $\gamma_1(\alpha) = \alpha$ and $\gamma_2(\alpha) = 1 - \alpha$. Therefore, the posterior probabilities of a customer who observed signal s_i are γ_i for firm 1 and $1 - \gamma_i$ for firm 2. Upon observation of the realization s_i (which happens with probability 1/2), a customer updates the belief from 1/2 to γ_i, which results in the continuum of customer types described above.

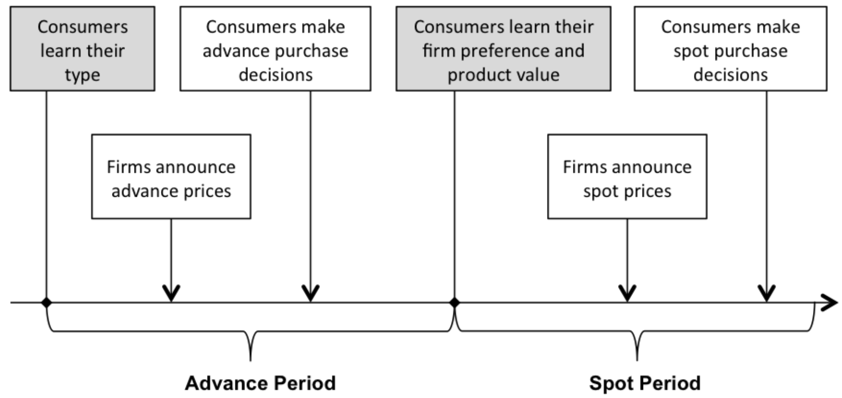

Figure 1. Sequence of events in Model I

model a single firm offers two services. Consumers differ in the values they assign to these services but they are equally knowledgable about which service they prefer. In contrast, in our model consumers have the same value distribution across the available services/firms but differ in their ability to identify which firm they will prefer, i.e., some may be reasonably sure which they will prefer whereas others are not. Dana (1998) allows consumers to vary in the value distribution for a service, but since all firms offer an equivalent service, consumers do not have firm preferences.

Without observing consumers' types, but knowing the distributions of V and $\boldsymbol{\alpha}$, the firms simultaneously announce advance prices p_1 and p_2. Consumers learn their types α, and decide whether to purchase in advance (and if so, from which firm) or to wait for the spot period. Being forward-looking, customers can correctly anticipate that the spot period prices are equal to v_h. We assume that consumers purchase only one unit, even if they realize that they end up preferring the other unit.[3] We believe that assuming that customers purchase only once is more realistic for the type of products that are usually sold in advance: Customers that hold tickets for a concert do not usually purchase other tickets, even if they realized that they do not value the tickets as much as they expected. This assumption is common to the literature on advance selling and competition (Shugan and Xie (2005); Guo (2009)). Figure 1 demonstrates the sequence of events. Note, the firms are monopolists in the spot period whereas they face competition in the advance period. The reverse holds in our second model.

In this model, we restrict attention to symmetric subgame perfect Nash equilibria, where both firms choose the same prices in equilibrium. To identify the equilibrium, in subsections 4.1 and 4.2 we analyze

[3]We provide an analysis of a variant of the model, where consumers can purchase two units in advance, one from each firm, in the online appendix. If consumers consider purchasing more than one unit, competition is dampened, which increases the benefit of advance selling. In particular, consumers will purchase from both firms in advance if the equilibrium prices are low enough, which is the case when the degree of heterogeneity is low. In this case, advance selling to consumers at a low price may be desirable, because it eliminates the competition between firms–firms do not have an incentive to undercut each other's price if they can sell to the entire market. Nevertheless, as in the model we present below, spot selling is still preferred to advance selling in the majority of cases.

consumers' behavior, given their beliefs with respect to the spot prices, and the firms' advance and spot period prices, given their beliefs about customers' purchasing decisions and the competitor's prices.

4.1 Consumer Behavior

After receiving the signals and observing the advance prices, p_1 and p_2, announced by both firms, consumers decide whether to purchase in advance and, if so, from which firm. According to the equilibrium concept, all customers share the same beliefs about the firms' spot prices and the behavior of other consumers. Moreover, because customers are forward-looking, they can correctly anticipate that each firm will charge v_h in the spot period and that they will obtain no surplus from purchasing on the spot. The next result follows. (The proofs of this and all subsequent results are given in the appendix.)

Lemma 1. *If firm 1 advance sells in equilibrium then there exists a unique threshold, $\bar{\alpha}$, such that only consumers with $\alpha \geq \bar{\alpha}$ buy from it in advance. Similarly, if firm 2 advance sells in equilibrium then there exists a unique threshold, $\underline{\alpha}$, such that only customers with $\alpha < \underline{\alpha}$ buy from it in advance.*

Lemma 1 enables us to simplify the search for consumer actions in equilibrium. Instead of analyzing each consumer's optimal purchasing decision, we can restrict attention to finding the equilibrium information thresholds that are induced by the firms' advance and spot prices. Let $\boldsymbol{p}$ be a vector of advance and spot prices charged by both firms. The thresholds $\bar{\alpha}(\boldsymbol{p})$ and $\underline{\alpha}(\boldsymbol{p})$ are the consumers' best responses to the firms' prices. In what follows, we analyze these best responses for a given set of prices, $\boldsymbol{p}$. Because in any equilibrium the firms' spot prices are v_h, we can restrict attention to best responses to the advance period prices p_1 and p_2 (and spot prices v_h).

All consumers are expected utility maximizers and therefore choose the strategy that maximizes their total expected surplus, i.e., the expected surplus of advance and spot purchases. In this model we focus on pure strategies and assume that in case of indifference between purchasing in the two periods, customers purchase in advance. Thus, a consumer that attaches a probability α for preferring firm 1, evaluates the expected utility of three different strategies:

1. Buy in advance from firm 1, which yields an expected utility of $\alpha\mathbb{E}[V] - p_1$
2. Buy in advance from firm 2, which yields an expected utility of $(1-\alpha)\mathbb{E}[V] - p_2$
3. Wait for the spot and then, if $V = v_h$, buy from the preferred firm, which yields an expected utility of zero.

We refer to a customer who obtains the same surplus by choosing two different strategies as an indifferent consumer. To buy in advance from firm 1, a customer who attaches a probability α for firm 1, must prefer to purchase in advance from firm 1 over firm 2, which happens if and only if $\alpha\mathbb{E}[V]-p_1 \geq (1-\alpha)\mathbb{E}[V]-p_2$; at the same time, this customer must prefer to purchase in advance from firm 1 rather than wait for the spot period to make her purchasing decision, which is the case if and only if $\alpha\mathbb{E}[V]-p_1 \geq 0$. Thus, for a consumer to buy unit 1 in advance, the attached probability, α, for preferring firm 1 should satisfy:

$$\alpha \geq \max\left\{\frac{p_1}{\mathbb{E}[V]}, \frac{1}{2}\left(1+\frac{p_1-p_2}{\mathbb{E}[V]}\right)\right\}. \tag{1}$$

Similarly, to buy in advance from firm 2, α should satisfy:

$$\alpha \leq \min\left\{1-\frac{p_2}{\mathbb{E}[V]}, \frac{1}{2}\left(1+\frac{p_1-p_2}{\mathbb{E}[V]}\right)\right\}. \tag{2}$$

Combining conditions (1) and (2), we get that if $p_1+p_2 \leq \mathbb{E}[V]$, all customers purchase in advance from either firm 1 or firm 2. Denote by $\hat{\alpha}$ the probability of the consumer indifferent between the firms. This probability is given by:

$$\hat{\alpha}(p_1,p_2) = \frac{1}{2}\left(1+\frac{p_1-p_2}{\mathbb{E}[V]}\right).$$

Thus, all customers with $\alpha \geq \hat{\alpha}$ purchase from firm 1 in advance and all customers with $\alpha \leq \hat{\alpha}$ purchase from firm 2 in advance. If, however, $p_1+p_2 > \mathbb{E}[V]$, some consumers buy in advance, while others wait for the spot to make their purchasing decision. Let $\bar{\alpha}$ be the preference probability of the consumer who is indifferent between buying in advance from firm 1 and waiting for the spot and $\underline{\alpha}$ be the preference probability of the consumer who is indifferent between buying in advance from firm 2 and waiting for the spot, where

$$\bar{\alpha}(p_1) = \frac{p_1}{\mathbb{E}[V]}$$

and

$$\underline{\alpha}(p_2) = 1-\frac{p_2}{\mathbb{E}[V]}.$$

(Observe that $\underline{\alpha} < \bar{\alpha}$, if $p_1+p_2 > \mathbb{E}[V]$.) Then, all consumers with $\alpha \geq \bar{\alpha}$ purchase in advance from firm 1, all consumers with $\alpha \leq \underline{\alpha}$ purchase in advance from firm 2, and all consumers with $\alpha \in (\underline{\alpha}, \bar{\alpha})$ wait for the spot.

Note that the advance period is analogous to a Hotelling line model of competition. Each firm is located at the endpoints of a segment of unit length and consumers are located uniformly along the interior segment $[(1-\delta)/2, (1+\delta)/2]$. Each customer receives a base value of $\mathbb{E}[V]$ from getting the unit and incurs a marginal travel cost of 1. Of course, the classic Hotelling line model is not concerned with dynamically pricing products across periods and the attractiveness of advance selling strategies, which is the focus of this

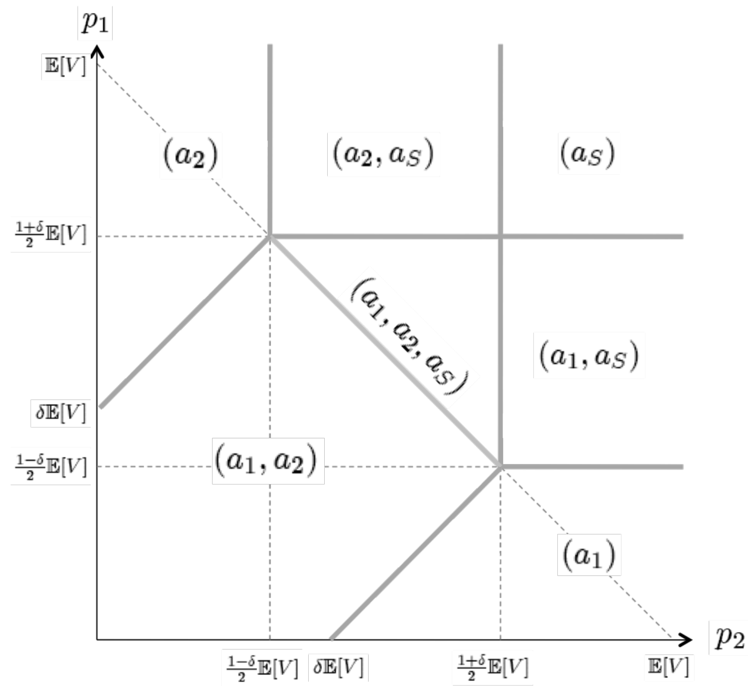

Figure 2. Customers' behavior in a (p_1, p_2) price space for $\delta > 1/3$. An a_i for a particular (p_1, p_2), denotes that there are consumers that follow purchasing strategy i, for that price tuple, $i \in \{1, 2, S\}$.

paper.

Figure 2 demonstrates the consumer equilibrium behavior on the advance-price space. The possible purchasing decisions in equilibrium are represented by $a_i,\ i \in \{1, 2, S\}$, where 1 denotes buying in advance from firm 1, 2 denotes buying in advance from firm 2, and S denotes waiting for the spot. A particular customer's purchasing decision depends on her α in the manner explained above. For example, the area represented by (a_1, a_S) implies that consumers with $\alpha \geq \bar{\alpha}$ buy in advance from firm 1, consumers with $\alpha < \bar{\alpha}$ wait for the spot to make their purchasing decision and no customer buys from firm 2 in advance. Figure 2 represents the equilibrium behavior for $\delta > 1/3$. With $\delta < 1/3$ the same seven regions exist and the only difference is that $(1 - \delta)\mathbb{E}[V]/2 > \delta\mathbb{E}[V]$.

4.2 Firms' Revenue Functions

From the analysis in the previous section, the firms can rationally conclude how a tuple of advance prices (p_1, p_2) affects customers' purchasing decisions. That is, given a set of advance period prices firms can correctly predict their expected demand in each period and consequently their expected revenues. If $p_1 + p_2 \leq \mathbb{E}[V]$, advance period demand is $\bar{F}(\hat{\alpha}(p_1, p_2))$ from firm 1 and $F(\hat{\alpha}(p_1, p_2))$ from firm 2, where $F(\cdot)$ is the cdf of a uniform random variable on $[0, 1]$ and $\bar{F}(\cdot) = 1 - F(\cdot)$. In this case all customers purchase in advance, so demand on the spot is 0. If, on the other hand, $p_1 + p_2 > \mathbb{E}[V]$, advance period demand is $\bar{F}(\bar{\alpha}(p_1))$ from

Table 1. Firms' total expected revenues as a function of customers' purchasing behavior and prices.

	$\Pi_1(p_1;p_2)$	$\Pi_2(p_2;p_1)$
(a_1,a_2)	$\frac{p_1}{2\delta}\left(\delta-\frac{p_1-p_2}{\mathbb{E}[V]}\right)$	$\frac{p_2}{2\delta}\left(\delta-\frac{p_2-p_1}{\mathbb{E}[V]}\right)$
(a_1,a_2,a_S)	$\frac{p_1}{\delta}\left(\frac{1+\delta}{2}-\frac{p_1}{\mathbb{E}[V]}\right)+\frac{2\mathbb{E}[V]}{\beta+1}D_1^S$	$\frac{p_2}{\delta}\left(\frac{1+\delta}{2}-\frac{p_2}{\mathbb{E}[V]}\right)+\frac{2\mathbb{E}[V]}{\beta+1}D_2^S$
(a_1,a_S)	$\frac{p_1}{\delta}\left(\frac{1+\delta}{2}-\frac{p_1}{\mathbb{E}[V]}\right)+\frac{\mathbb{E}[V]}{\delta(\beta+1)}\int_{\frac{1-\delta}{2}}^{\bar{\alpha}}\alpha d\alpha$	$\frac{\mathbb{E}[V]}{\delta(\beta+1)}\int_{\frac{1-\delta}{2}}^{\bar{\alpha}}(1-\alpha)\,d\alpha$
(a_2,a_S)	$\frac{\mathbb{E}[V]}{\delta(\beta+1)}\int_{\underline{\alpha}}^{\frac{1-\delta}{2}}\alpha d\alpha$	$\frac{p_2}{\delta}\left(\frac{1+\delta}{2}-\frac{p_2}{\mathbb{E}[V]}\right)+\frac{\mathbb{E}[V]}{\delta(\beta+1)}\int_{\underline{\alpha}}^{\frac{1-\delta}{2}}(1-\alpha)\,d\alpha$
(a_1)	p_1	0
(a_2)	0	p_2
(a_S)	$\frac{\mathbb{E}[V]}{\delta(\beta+1)}\int_{\frac{1-\delta}{2}}^{\frac{1+\delta}{2}}\alpha d\alpha$	$\frac{\mathbb{E}[V]}{\delta(\beta+1)}\int_{\frac{1-\delta}{2}}^{\frac{1+\delta}{2}}(1-\alpha)\,d\alpha$

firm 1 and $F(\underline{\alpha}(p_2))$ from firm 2. The expected spot period demand in this case is composed of customers who did not purchase in the advance period and whose realized spot value is $V=v_h$, which occurs with probability $1/2$. Therefore, the expected spot period demand from firm 1 is given by:

$$D_1^S=\int_{\underline{\alpha}}^{\bar{\alpha}}\frac{\alpha}{2}dF\alpha. \tag{3}$$

Similarly,

$$D_2^S=\int_{\underline{\alpha}}^{\bar{\alpha}}\frac{1-\alpha}{2}dF\alpha \tag{4}$$

is the expected spot period demand from firm 2.

Table 1 lists firm 1's and 2's revenue functions, $\Pi_1(p_1;p_2)$ and $\Pi_2(p_2;p_1)$, for each case of customers' purchasing decisions, where D_1^S and D_2^S are the spot period expected demands in (3) and (4). Observe that for each customer behavior case, firm i's revenue function is concave in p_i. However, while the profit function is locally concave, it is not, in general, globally concave. To see this, fix p_2. As p_1 increases, customers' behavior changes and the profit function switches from one region to another. For example, for $p_2\in(\delta\mathbb{E}[V],(1+\delta)\mathbb{E}[V]/2)$, customers' behavior changes from $(a_1)\to(a_1,a_2)\to(a_1,a_2,a_S)\to(a_2,a_S)$ with an increase of p_1 (see figure 2). The revenue function is composed of the $\Pi_1(p_1;p_2)$s that correspond to each region (taken from Table 1). Thus, for a given p_2, firm 1's revenue function is continuous and piece-wise concave and uniqueness of equilibrium cannot be guaranteed.

4.3 Equilibrium Analysis

Before finding the subgame perfect Nash equilibrium of the game, we discuss the equilibrium of a special case in which one of the firms is restricted to sell only on the spot and the other firm decides whether or not to sell in advance, where it is essentially a monopoly. Should it sell in advance? The literature on advance selling for a monopoly confirms that it should when $\delta=0$. We find that this holds for all $\delta\in[0,1]$: the monopolist always chooses to sell in advance. Lemma 2 characterizes the equilibrium for a general value of δ.

Lemma 2. *Let $\tilde{\delta} = \beta/(1+3\beta)$. If firm 2 sells only on the spot (by charging a high advance price, p_2), then: (i) if $\delta < \tilde{\delta}$, the unique equilibrium is one in which firm 1 charges $p_1^{all} = (1-\delta)\,\mathbb{E}[V]/2$ in the advance period and sells to all consumers in advance; (ii) otherwise, the unique equilibrium is one in which firm i charges p_1^{part} in advance and advance sells only to consumers with $\alpha > p_1^{part}/\mathbb{E}[V]$, where*

$$p_1^{part} = \frac{(1+\delta)(1+\beta)\,\mathbb{E}[V]}{2(1+2\beta)}. \tag{5}$$

Lemma 2 suggests that there exists a $\tilde{\delta}$ so that the firm sells to all consumers in advance if $\delta < \tilde{\delta}$. Otherwise, the firm charges a relatively high advance price and sells to the more informed consumers, those with high values of α, in advance and to the less informed consumers on the spot. Regardless, at least in some capacity, advance selling is always beneficial to the monopolist.

Next, we turn to the construction that both firms can sell in advance, but assume that consumers have no information regarding their preferred firm in the advance period (i.e., $\delta = 0$). The equilibrium outcome is summarized in the following lemma.

Lemma 3. *When $\delta = 0$, the unique set of advance period prices is $p_1^* = p_2^* = 0$ and all consumers purchase in advance from one of the two firms. Any division of market demand between the two firms at these prices is an equilibrium.*

Lemma 3 demonstrates that advance selling always occurs in equilibrium when two firms are competing and consumers have no information in the advance period with respect to their eventual firm preference. However, while a monopolistic firm profits from the time separation of purchase from consumption, which allows the monopolist to sell to a homogenous group of consumers, a price setting competitive firm does not. When the firms are identical, the only price equilibrium is one in which both firms charge an advance price of zero, all consumers purchase in advance, and both firms obtain zero profits. Because the firms are identical, each firm has an incentive to undercut each other's advance price and obtain the entire market demand. As consumers are only interested in obtaining one unit, each firm knows that if it does not sell in advance, it gets no demand in the spot period. This leads to an intense Bertrand competition in the advance period, which eventually results in no revenues to the firms.[4] Thus, the benefit that advance selling had for

[4] The no profit result is closely related to the assumption that consumers only buy once. While allowing consumers to buy on the spot if they realize that they favor the other firm, will not result in zero profit, it still eliminates the benefit of advance selling. In this case, firms undercut each other's advance price until they are indifferent between selling in advance or only on the spot. Thus, the revenue obtained by advance selling is not higher than under spot selling. Allowing consumers to buy two units in advance eliminates the competition in the advance period when consumers are homogeneous, so that advance selling

a single firm is completely eliminated in this competitive case. If the firms were able to commit to sell only on the spot, they would obtain strictly positive profits. Thus, the possibility to advance sell makes both firms worse off. Limited capacity would mitigate the severity of this result, i.e., the equilibrium price would not fall to zero, but limited capacity does not change the fact that competition on homogeneous customers is intense and generally not beneficial to firms.

Next, we analyze the equilibrium of the general game. In this case, both firms are allowed to sell in advance and consumers are heterogeneous in the advance period, with degree of heterogeneity δ. We find that there are two possible symmetric price equilibria depending on the parameter conditions. Theorem 1 describes the equilibrium advance period prices and the corresponding consumer behavior.

Theorem 1. *Two symmetric price equilibria are possible:*

1. *For every β, there exists a $\delta_1(\beta)$, such that for every β and $\forall\delta \leq \delta_1(\beta)$, the firms charge $p_1^l = p_2^l = \delta\mathbb{E}[V]$ in advance. All consumers purchase in advance and $\hat{\alpha} = 1/2$. Consumers with $\alpha > \hat{\alpha}$ buy in advance from firm 1 and consumers with $\alpha < \hat{\alpha}$ buy in advance from firm 2.*

2. *For every β, there exists a $\delta_2(\beta)$, such that for every β and $\forall\delta \geq \delta_2(\beta)$, the firms charge*

$$p_1^h = p_2^h = \frac{(1+\delta)(1+\beta)\,\mathbb{E}[V]}{2(1+2\beta)}.$$

   Furthermore,

$$\bar{\alpha} = 1 - \underline{\alpha} = \frac{(1+\delta)(1+\beta)}{2(1+2\beta)} < 1.$$

   Customers with $\alpha > \bar{\alpha}$ purchase in advance from firm 1, those with $\alpha < \underline{\alpha}$ purchase from firm 2, and the rest wait for the spot.

Theorem 1 demonstrates that there are two types of symmetric equilibria in this game. The first type of equilibrium (case 1 of Theorem 1) is one in which both firms charge a relatively low price that makes all consumers purchase in advance. Half of the consumers–those with high values of α–purchase in advance from firm 1 and the other half purchases in advance from firm 2. In the second type of equilibrium (case 2 of Theorem 1), the firms charge a higher price, which makes only some consumers, those who are more informed, buy in advance. The other, relatively uninformed consumers, wait for the spot period to make their purchasing decisions.

In the low price equilibrium both firms charge $p_1^l = p_2^l = \delta\mathbb{E}[V]$. Each firm sells in advance to half of the consumers. Consumers with $\alpha > 1/2$ buy from firm 1 and consumers with $\alpha < 1/2$ buy from firm 2.

is preferable to spot selling. As we show in the online appendix, however, even when customers are allowed to buy from both firms in advance, advance selling is still inferior to spot selling in most cases with $\delta > 0$.

This equilibrium occurs when customers are more a-priori homogeneous (δ is low) and when the relative advantage of spot selling is low (β is high). In this case, in the advance period the firms engage in intense price competition as consumers are very price sensitive–they are relatively indifferent between the two firms, so price is low, because it is the key decider as to which firm to purchase from. Consequently, the firms capture the entire market in the advance period and their revenue is $\Pi_1 = \Pi_2 = \delta\mathbb{E}[V]/2$.

In the high price equilibrium both firms charge $p_1^h = p_2^h = \frac{(1+\delta)(1+\beta)}{2(1+2\beta)}\mathbb{E}[V]$ and sell to only a fraction of the consumers, the well-informed ones, in advance, while the relatively uninformed consumers wait for the spot period. This equilibrium occurs when δ is high and when β is low. When δ is high, consumers are more heterogeneous in the advance period. This decreases the firms' need to compete in advance to get demand–well-informed consumers will not purchase in advance from the firm for which they attach a low preference probability–and therefore the equilibrium prices are higher. When β is low, firms have more incentive to sell on the spot and, in fact, in this equilibrium the firms sell to some consumers on the spot. In this equilibrium, there does not exist a consumer who is indifferent between purchasing from either firm in the advance period and the advance period prices are equal to the monopolist's price (see equation (5)). However, this should not be taken to mean that this equilibrium outcome is not due to competition: Given that one firm sets a high price, p_j^h, to sell to informed consumers in advance, the other firm benefits from doing the same, even though, as is later shown, in the majority of cases, both firms prefer that they both only sell on the spot.

Both the optimal advance prices and the resulting total expected profits increase in δ and decrease with β. Greater consumer heterogeneity in advance implies that some consumers become more informed, so the firms can charge a higher advance period price. As β decreases, firms have an incentive to sell more on the spot so they increase the advance price. This results in fewer consumers buying in the advance period, but at a higher price, and in more consumers who wait and purchase at v_h.

Note that since the firms' revenue functions are not quasi-concave, uniqueness of equilibrium cannot be guaranteed. In fact, the next theorem shows that under some parameter values, both equilibria exist.

Theorem 2. $\delta_1(\beta) > \delta_2(\beta)\ \forall\beta$. *Thus, a symmetric equilibrium always exists, but it is not necessarily unique:* $\forall\delta \in [\delta_2(\beta), \delta_1(\beta)]$, *both symmetric equilibria of Theorem 1 exist. Further, in that range, the equilibrium where only part of the consumers purchase in advance (case 2 of Theorem 1) Pareto dominates the equilibrium in which all consumers purchase in advance (case 1 of Theorem 1).*

Figure 3 illustrates the ranges for which each of the two equilibria occurs on the (β, δ) parameter space. As shown in Theorem 2, a symmetric equilibrium always exists, but it is not necessarily unique. When δ is low (the bottom area) the unique equilibrium is such that the firms charge a low advance period price

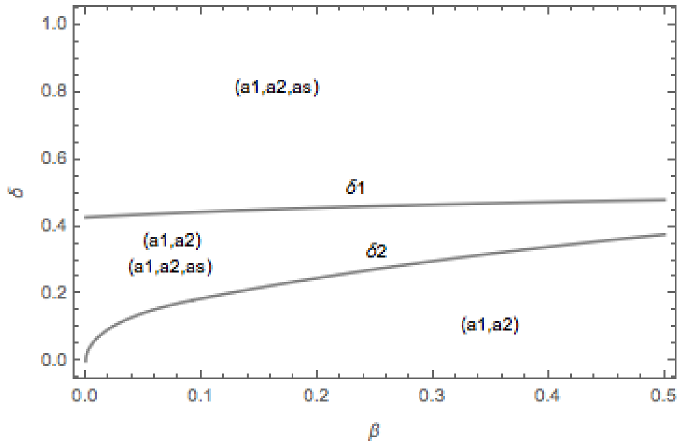

Figure 3. Equilibrium types on a (β, δ)parameter space. The tuple (a_1, a_2) corresponds to the equilibrium in which all customers purchase in advance (case 1 of Theorem 1) and (a_1, a_2, a_s) corresponds to the equilibrium in which some customers purchase in advance and others wait for the spot (case 1 of Theorem 1).

and all consumers purchase in advance (case 1 of Theorem 1). When δ is high (the upper area) the unique equilibrium has the firms charge a higher advance price and sell to some consumers in advance whereas some consumers wait for the spot (case 2 of Theorem 1). For mid-values of δ (the middle area between the two curves), both symmetric equilibria exist. Theorem 2 demonstrates that in this range, the higher price equilibrium (case 2) Pareto dominates the low price equilibrium (case 1).

Figure 3 suggests that competing firms prefer a market with a higher δ - with a high δ there are more consumers who are relatively certain of their preferences, which dampens competition between the firms, whereas with a low δ, consumers are uncertain of their preferences and competition is intense. The opposite holds for a monopolist, as confirmed by the next corollary.

Corollary 1. *As δ increases, the equilibrium revenue of the monopolist decreases, but the revenues of firms under competition increase.*

Hence, while a monopolist can use advance selling to profit from consumer homogeneity resulting from the time separation of purchase and consumption, this same homogeneity works against competing firms as it increases the intensity of price competition in the advance period.

4.4 The Choice to Sell in Advance

We know that a monopolist willingly chooses to advance sell. Does the same hold for competing firms? We find that under most parameter values, it does not. Recall from Theorem 1 that some sort of advance selling always occurs in equilibrium. However, even though advance selling is always an equilibrium, in most cases, the firms would be better off if they could commit not to sell in advance.

Corollary 2. *Firms' revenues from selling only on the spot are given by*

$$\Pi_1 = \Pi_2 = \frac{v_h}{4} = \frac{\mathbb{E}[V]}{2\,(1+\beta)}.$$

These revenues are strictly higher than the revenues obtained in the low price equilibrium range (case 1 of Theorem 1) and are strictly higher than the high price equilibrium revenues (case 2 of Theorem 1) if

$$\delta\,(\beta) < \frac{1+2\beta-\beta^2}{(1+\beta)^2}. \tag{6}$$

Corollary 6 demonstrates that advance selling is inferior in this model for most parameter values. If δ is low and consumers are rather homogeneous in advance, fierce competition in the advance period results in setting low prices and selling to all consumers, which clearly hurts profits. Even when customers are rather heterogeneous in advance and in equilibrium firms charge the monopoly price and sell to only the informed consumers in advance, advance selling may result in lower revenues compared to selling only on the spot. This is because under competition, when firms sell in advance to uninformed consumers, they lose the opportunity to sell to customers who purchased in advance from the other firm, but would have otherwise bought on the spot. Such a situation does not occur when one of the firm is restricted to sell on the spot, but happens under competition. Only when δ is high, so that there is little competition in advance (specifically, if condition (6) fails), advance selling results in higher profits compared to selling on the spot.

Hence, in most cases, the possibility of advance selling ends up hurting firms under competition. Firms would be better off if they were both able to commit to sell only on the spot. In these cases, the firms are in a Prisoner's Dilemma situation–even though the firms are better off selling only on the spot, they both are forced to sell in advance (given that the other firm sells in advance). Corollary 6 also demonstrates that for high values of β and δ, both firms obtain higher revenues from advance selling, rather than spot selling. That is, under these parameter values, selling in advance (to highly informed consumers) is, in fact, a good pricing strategy for firms operating in a competitive environment, essentially because there is very little advance period competition when δ is large.

Figure 4 illustrates the range for which advance selling yields higher revenues for both firms (the area above the solid line) and the range where, if possible, the firms would benefit from committing to sell on the spot (the areas below the solid line). The latter decreases with a decrease in β. When β decreases, the firms' benefit from spot selling increases and therefore firms increase the advance period price to make more consumers purchase in the spot period. In the limit, when $\beta = 0$, the price charged in advance is so high, that none of the consumers purchase in advance.

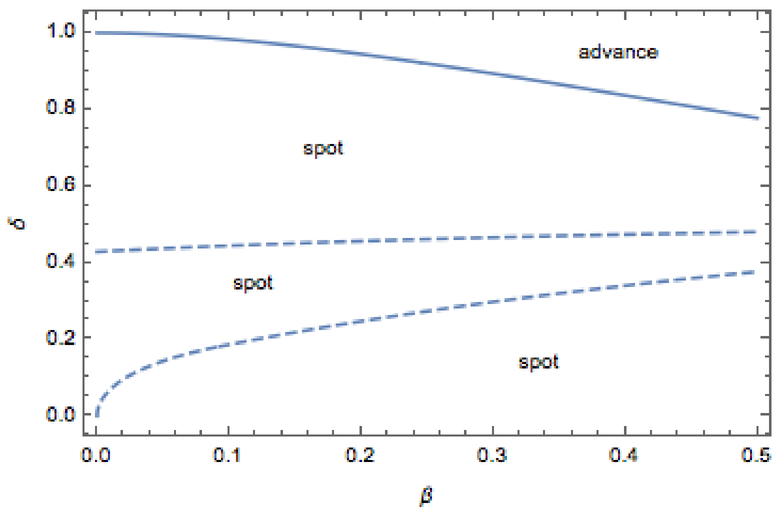

Figure 4. Areas in the (β, δ) parameter space for which the revenue obtained by selling only on the spot is preferred to the revenue obtained in the advance selling equilibrium.

In sum, as long as advance-period heterogeneity is not too high, the possibility to advance sell hurts firms–firms would be better off if they could commit to sell only on the spot. Competitive firms can benefit from advance selling only if consumers are heterogeneous enough in the advance period, because in this case there is limited competition in the advance period–firms charge the monopoly price in the advance period, but are still able to sell to a large fraction of consumers on the spot at a high price.

5 Model II: Spot Period Competition

In the previous section we showed that advance selling is in most cases an undesirable strategy when firms compete in the advance period. This occurs because firms who compete on a homogeneous market of consumers are pushed to decrease their prices, which makes selling to such consumers unprofitable. Thus, for the same reason that advance selling is attractive for a monopoly, it is a problem for competitive firms. In this model, we eliminate advance period competition and investigate whether the benefits of advance selling survive spot period competition.

Two firms are located on two ends of a Hotelling line of length v and are sequentially setting advance period and then spot period prices to maximize revenues. The market consists of two customer types, which taken together can be normalized to 1 without loss of generality. The first type of consumers of size $\phi \in (0, 1)$ arrives in the advance period. These consumers are loyal to one of the two firms and never consider buying from the other firm. Half of the consumers are loyal to firm 1 and the other half is loyal to firm 2. Loyal consumers are forward-looking and consider whether to purchase in advance or to postpone their purchasing decision to the spot period. As in model I, these consumers are uncertain about their spot period value of

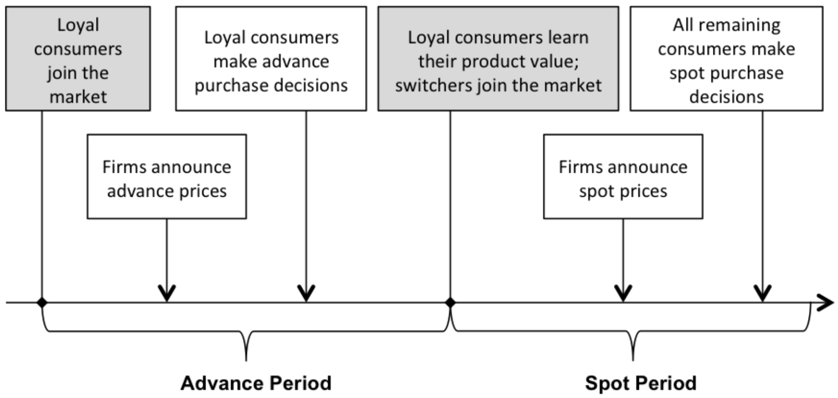

Figure 5. Sequence of events in Model II

the product. We assume that their value V is uniformly distributed between 0 and v and that their exact value is realized in the spot period.

The second type of consumers, of size $1-\phi$, arrive to the market only in the spot period knowing their value for each firm. These consumers do not have an a-priori preferred firm and buy from the firm that provides them with a higher surplus. These consumers are located uniformly along the line, so that even though they are homogeneous in their value for the product, they are heterogenous in location. A consumer's location represents their ideal point, but they are willing to consider purchasing from either firm. That is, these consumers are switchers and are willing to purchase from either firm depending on prices and location. To be specific, a customer located at $x \in [0, v]$ on the line incurs a cost $t \cdot x$ to buy from firm 1 and a cost $t(v-x)$ to buy from firm 2, where $t \in [0, 1]$ is the travel cost per unit of distance. Moreover, if p_1 and p_2 are the prices charged by firm 1 and firm 2, respectively, a switcher located at x prefers buying from firm 1 (firm 2) if $v - p_1 - tx \geq (\leq)\, v - p_2 - t(v-x)$.

Note that in this model competition arises only in the spot period: firms act as monopolies when serving loyal consumers, but compete on prices to serve switchers and the loyals who waited for the spot period. We assume that the firms cannot price discriminate between switchers and loyal consumers. That is, each firm sets a single spot price at which both switchers and loyal customers can purchase. We look for a subgame perfect Nash equilibrium in pure strategies where firms set advance period prices and then spot period prices, loyal consumers choose whether to purchase in advance or wait for the spot, and switchers arriving on the spot decide whether to purchase and from which firm.[5] Figure 5 demonstrates the sequence of events for Model II.

[5]As loyal consumers are homogeneous in the advance period, in a pure strategy equilibrium either all loyal consumers of a particular firm buy in advance or they all wait. The analysis can be extended to allow loyal consumers to adopt a mixed strategy, which is a probability that a loyal consumer of firm i purchases in advance from firm i. The analysis becomes more cumbersome, but the intuition and results remain. (A complete analysis of the mixed strategy equilibrium is available from the authors.)

5.1 Consumer Behavior

Loyal consumers decide whether to purchase in advance or wait for the spot, but will only consider purchasing from their preferred firm. These customers are forward-looking: given the advance prices the firms announce, they anticipate the equilibrium prices on the spot and will therefore only purchase in advance if they obtain a higher utility from doing so. We assume that loyal consumers purchase in advance if they are indifferent between doing so or waiting and potentially purchasing in the spot period. Specifically, consumers who are loyal to firm i, are indifferent between purchasing in the two periods if: $\mathbb{E}[V] - p_i^a = (\mathbb{E}[V|V \geq p_i^*] - p_i^*) \mathbb{P}\{V \geq p_i^*\}$, where the left-hand side of the equality is the utility from purchasing in advance, the right-hand side of the equation is the expected utility from waiting for the spot, p_i^* is the spot price charged by firm i in equilibrium, and p_i^a is the advance price set by firm i. Of course, the equilibrium spot period price depends on the composition of consumers on the spot. As loyal consumers are a-priori homogeneous, in equilibrium either all purchase in advance or all wait. To make all loyal consumers buy in advance, firm i needs to set the advance price, p_i^a, such that

$$\begin{aligned} p_i^a &\leq \mathbb{E}[V] - (\mathbb{E}[V|V \geq p_i^*] - p_i^*) \mathbb{P}\{V \geq p_i^*\} \\ &= \mathbb{E}[V] - \frac{(v - p_i^*)^2}{2v}. \end{aligned} \tag{7}$$

Switchers arrive in the spot period, observe spot period prices and their location on the line, x, $X \sim U[0, v]$. A switcher with realized location x that observes spot prices p_1 and p_2 then decides whether to buy in the spot period and from which firm. A switcher may:

1. Buy on the spot from firm 1 and get utility $v - p_1 - tx$
2. Buy on the spot from firm 2 and get utility $v - p_2 - t(v - x)$
3. Do not buy, which yields a utility of 0.

Combining the purchasing decisions and using an argument similar to Lemma 1, we get that all switchers with

$$x \leq \min\left\{\frac{v - p_1}{t}, \frac{p_2 - p_1 + vt}{2t}\right\} \tag{8}$$

buy from firm 1 and all switchers with

$$x \geq \max\left\{\frac{v - p_2}{t}, \frac{p_2 - p_1 + vt}{2t}\right\} \tag{9}$$

buy from firm 2. If $p_1 + p_2 \leq (2 - t)v$, all switchers buy. Switchers with $x > \hat{x}$ buy from firm 1 and those

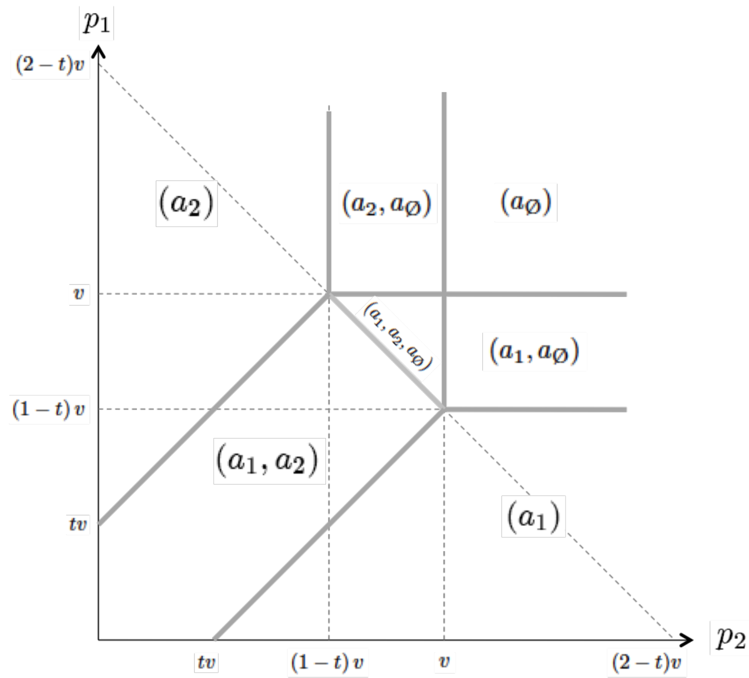

Figure 6. Switchers' behavior in a (p_1, p_2) spot price space and $t \leq 1/2$. An a_i for a particular (p_1, p_2), denotes that there are switchers that follow purchasing strategy i, for that price tuple, $i \in \{1, 2, \emptyset\}$.

with $x < \hat{x}$, buy from firm 2, where

$$\hat{x} = \frac{p_2 - p_1 + vt}{2t}.$$

However, if $p_1 + p_2 > (2-t)\,v$, then switchers with $x \in (\underline{x}, \bar{x})$ do not buy from either firm, where

$$\underline{x} = \frac{v - p_1}{t}$$

and

$$\bar{x} = \frac{p_2 - v\,(1-t)}{t}.$$

Figure 6 demonstrates the switchers' behavior on the spot-price space for $t \leq 1/2$. The possible purchasing decisions are represented by $a_i,\ i \in \{1, 2, \emptyset\}$, where 1 denotes buying on the spot from firm 1, 2 denotes buying on the spot from firm 2, and $\emptyset$ denotes not buying. The same seven regions emerge when $t > 1/2$, and the only difference is that $v\,(1-t) < tv$.

5.2 Firm's Revenue Functions

Each firm can control whether their loyal consumers purchase in advance or wait by setting the advance price, $p_i^a\,(p_i^*)$, according to condition (7). Further, for each set of spot prices (p_1, p_2), firms can predict their

Table 2. Firm 1's spot period revenue as a function of customers' purchasing behavior and prices. The revenues in column 1 assume all loyals buy in advance. The revenues in column 2 assume all loyals wait for the spot period.

	$\Pi_1(p_1;p_2)$	$\Pi_1(p_1;p_2)$
(a_1,a_2)	$p_1(1-\phi)\frac{1}{2}\left(1-\frac{p_1-p_2}{tv}\right)$	$p_1\left(\frac{\phi}{2}\frac{v-p_1}{v}+(1-\phi)\frac{1}{2}\left(1-\frac{p_1-p_2}{tv}\right)\right)$
$(a_1,a_2,a_\varnothing)$	$p_1(1-\phi)\frac{v-p_1}{tv}$	$p_1\left(\frac{\phi}{2}\frac{v-p_1}{v}+(1-\phi)\frac{v-p_1}{tv}\right)$
$(a_1,a_\varnothing)$	$p_1(1-\phi)\frac{v-p_1}{tv}$	$p_1\left(\frac{\phi}{2}\frac{v-p_1}{v}+(1-\phi)\frac{v-p_1}{tv}\right)$
$(a_2,a_\varnothing)$	0	0
(a_1)	$p_1(1-\phi)$	$p_1\left(\frac{\phi}{2}\frac{v-p_1}{v}+1-\phi\right)$
(a_2)	0	$p_1\frac{\phi}{2}\left(\frac{v-p_1}{v}\right)^+$
$(a_\varnothing)$	0	0

demand from loyals, in case they decided to wait for the spot, and from switchers. In particular, given that loyals wait for the spot, the fraction of loyals who purchase from firm i, is

$$D_i^l=\begin{cases}\frac{\phi}{2}\frac{v-p_i}{v} & 0\le p_i\le v\\ 0 & p_i>v.\end{cases}$$

Demand from switchers depends on (p_1,p_2) and can be inferred from switchers' behavior in Figure 6 and inequalities (8) and (9). If $p_1+p_2\le(2-t)v$, all switchers purchase on the spot, firm 1's demand from switchers is $D_1^s=(1-\phi)F(\hat{x})$ and firm 2's demand is $D_2^s=(1-\phi)\bar{F}(\hat{x})$. If, however, $p_1+p_2>(2-t)v$, then only a fraction of switchers purchase on the spot, and demand from the firms is $D_1^s=(1-\phi)\underline{x}/v$ and $D_2^s=(1-\phi)(1-\bar{x}/v)$. Table 2 lists firm 1's revenue function when loyals buy in advance (column 1) and wait for the spot (column 2). Firm 2's revenue functions are symmetric. The revenue function is continuous and piece-wise concave in the firm's own price.

5.3 Equilibrium Analysis

For comparison, we first briefly discuss the equilibrium of the monopoly case. Suppose that there is a single firm serving both loyal and non-loyal consumers. As in the general case, there are $\phi/2$ of consumers who are loyal and arrive in the advance period and $(1-\phi)$ consumers who are non-loyal and arrive in the spot period. The equilibrium is summarized in the following lemma.

Lemma 4. *Let p_s be the spot period price, Π_s be the revenue of a monopolist who sells only on the spot and Π_a be the revenue of a monopolist who sells to loyals in advance and non-loyals on the spot. Then, the monopolist sets $p_s=\max\{v/2,v(1-t)\}$ on the spot and prefers to sell in advance: $\Pi_a\ge\Pi_s$, with $\Pi_a=\Pi_s$ if and only if $t=1$.*

Since the monopolist does not strictly benefit from selling in advance when $t = 1$, in the rest of the analysis we assume that $t < 1$.

Next consider competition. Each group of loyal consumers is homogeneous in advance and therefore makes the same purchasing decision in that period: if a consumer loyal to firm i decides to purchase in advance, all consumers loyal to firm i purchase in advance as well. Alternatively, if that consumer decides to wait for the spot, all consumers loyal to firm i wait. Therefore, there are three possible spot period subgames to consider: (1) only switchers remain on the spot (all loyals purchased in advance); (2) both loyals and switchers remain on the spot (all loyals decided to wait); and (3) consumers loyal to firm i and switchers remain on the spot (firm j's loyals purchased in advance). Define γ_i as:

$$\gamma_i = \begin{cases} 1, & \text{loyals to firm } i \text{buy in advance} \\ 0, & \text{otherwise} \end{cases}$$

and $\Gamma = (\gamma_1, \gamma_2)$. The next lemma establishes the equilibrium of the three spot period subgames.

Lemma 5. *The price equilibria for the spot period subgame depend on the composition of consumers that remain in that period.*

(i) Suppose $\Gamma = (1, 1)$*: The unique spot period price equilibrium is given by* $p_s^1 = p_s^2 = tv$*.*

(ii) Suppose $\Gamma = (0, 0)$*: Let*

$$t' = \frac{1}{2\phi}\left(\sqrt{\frac{1-\phi}{1+\phi}} - (1-\phi)\right).$$

If $t \geq t'$*, the unique spot period price equilibrium is given by*

$$p_s^1 = p_s^2 = \frac{tv}{1-\phi+2\phi t}.$$

Otherwise, there does not exist a spot period price equilibrium in pure strategies.

(iii) Suppose $\Gamma = (0, 1)$*: Let*

$$t'' = \frac{1}{6\phi(3+\phi)}\left((3-\phi)\sqrt{9-6\phi-2\phi^2} + 5\phi^2 + 6\phi - 9\right).$$

If $t \geq t''$*, the unique spot period price equilibrium is given by*

$$p_s^1 = \frac{tv(3-\phi)}{3(1-\phi)+4\phi t} \quad p_s^2 = \frac{tv(3-\phi) - \phi tv(1-2t)}{3(1-\phi)+4\phi t}.$$

Otherwise, there does not exist a spot period price equilibrium in pure strategies. The case where $\Gamma = (1, 0)$

is symmetric.

Lemma 5 shows that assuming that a spot period equilibrium exists, in all spot period subgames the equilibrium prices are such that all switchers buy on the spot. The exact spot period price tuple depends on the composition of consumers in the spot period and the existence of equilibrium in cases (ii) and (iii) depends on the values of t and ϕ. If $t > 1/2$, competition on switchers is low and the inclusion of loyal consumers in the spot ($\Gamma = (0,0)$) actually reduces the equilibrium spot period prices compared to the case where firms only sell to switchers ($\Gamma = (1,1)$). This implies that in this range, advance selling must be desirable for firms. For the rest of the analysis we therefore concentrate on the $t \leq 1/2$ range.

By setting an appropriate advance period price, firms can influence whether loyals purchase in advance or wait. Loyal customers are forward-looking and after observing the advance period prices charged by the firms, they can anticipate purchasing behavior of fellow customers as well as spot prices. Given their expectation of their firm's spot period price, loyal consumers' expected utility from waiting for the spot is:

$$\begin{aligned}\mathbb{E}\left[U^s\left(p_i^s\right)\right] &= \mathbb{P}\left\{V \geq p_i^s\right\}\left(\mathbb{E}\left[V | V \geq p_i^s\right] - p_i^s\right) \\ &= \frac{\left(v - p_i^s\right)^2}{2v},\end{aligned}$$

where p_i^s is shorthand notation for $p_i^s\left(\Gamma; v, \phi, t\right)$. Clearly, the lower the spot price, the more likely it is that a loyal consumer will wait for the spot period. To make $\gamma_i = 1$, firm i must charge an advance price that is sufficiently low and one that decreases if the spot price decreases:

$$\begin{aligned}p_i^a\left(\Gamma; v, \phi, t\right) &= \mathbb{E}[V] - \mathbb{E}\left[U^s\left(p_i^s\left(\Gamma; v_h, v_l, \phi\right)\right)\right] \\ &= = \frac{v}{2} - \frac{\left(v - p_i^s\right)^2}{2v}.\end{aligned} \tag{10}$$

The price in (10) is the best advance price firm i can charge to make loyal consumers buy in advance. Setting a higher advance price will make all loyals wait for the spot, while setting a lower advance price will still result in all loyals purchasing in advance, but at a lower price, hence reducing revenues. The total revenue function including both the advance and the spot period for firm i is given by:

$$\Pi_i = \frac{\phi}{2}\gamma_i p_i^a\left(\Gamma\right) + \Pi_i^s\left(p_i^s; p_j^s, \Gamma\right).$$

Corollary 3 summarizes the total revenue functions obtained for each combination of purchasing decisions made by loyal consumers, Γ, which result from an appropriate selection of advance period prices.

Corollary 3. *Let* $\underline{p}^a = (1 - t/2)\, tv$ *and*

$$\bar{p}^a = \frac{v}{2}\left(1 - \left(\frac{3\,(1-t) - \phi\,(3 - 6t + 2t^2)}{3 - \phi\,(3 - 4t)}\right)^2\right)$$

be two advance prices. The total revenues obtained by each firm are as follows:

(i) If $p_i^a = \underline{p}^a\ \forall i$*, then* $\Gamma = (1,1)$ *and* $\Pi_1 = \Pi_2 = (2 - \phi t)\, tv/4$.

(ii) If $p_i^a > \bar{p}^a\ \forall i$*, then* $\Gamma = (0,0)$ *and*

$$\Pi_1 = \Pi_2 = \frac{1 - \phi\,(1-t)}{2\,(1 - \phi\,(1 - 2t))^2} tv.$$

(iii) If $p_1^a = \bar{p}^a$ *and* $p_2^a > \bar{p}^a$*, then* $\Gamma = (0,1)$ *and*

$$\Pi_1 = \frac{(3-\phi)^2\,(1 - \phi\,(1-t))}{2\,(3\,(1-\phi) + 4\phi t)^2} tv;\ \Pi_2 = \frac{(3 - 2\phi\,(1-t))\left(6 - \phi\,(4-t) - 2\phi^2\left(1 - 3t + t^2\right)\right)}{4\,(3\,(1-\phi) + 4\phi t)^2} tv.$$

The revenues obtained when $\Gamma = (1,0)$ *are defined symmetrically.*

Finally, we analyze the game in which firms, by choosing an appropriate advance price, first decide whether they want to offer their product in advance or not. Comparing the revenue functions in Corollary 3, Theorem 3 specifies the equilibrium outcomes of the full game.

Theorem 3. *The equilibria of the full game depend on the values of* t *and* ϕ*. There exist* t_1 *and* t_2*,* $t_2 > t_1$*, such that*

(i) if $t \geq t_2$*, both firms set advance prices* $p_i^a = \underline{p}^a\ \forall i$ *and spot prices* $p_i^s = tv\ \forall i$ *and sell to loyal consumers in advance;*

(ii) if $t'' \leq t \leq t_1$*, both firms set advance prices* $p_i^s = \frac{tv}{1-\phi+2\phi t}\ \forall i$ *and sell only on the spot; and*

(iii) otherwise, there exist two equilibria: one of the firm sets advance price $\bar{p}^a$ *and sells to loyals in advance and the other firm sets a higher advance price and sells only on the spot.*

Figure 7 illustrates the types of equilibria resulting in the game. The dark area represents the range for which there does not exist a spot period equilibrium ($t < t''$). Observe that when firms offer products in both periods and engage in price competition in the spot period, advance selling does not always occur in equilibrium. Competition in the spot period may be sufficiently fierce, yielding sufficiently low spot period prices so that firms cannot offer a profitable advance period price that attracts customers. In contrast, a monopolist always sells in advance.

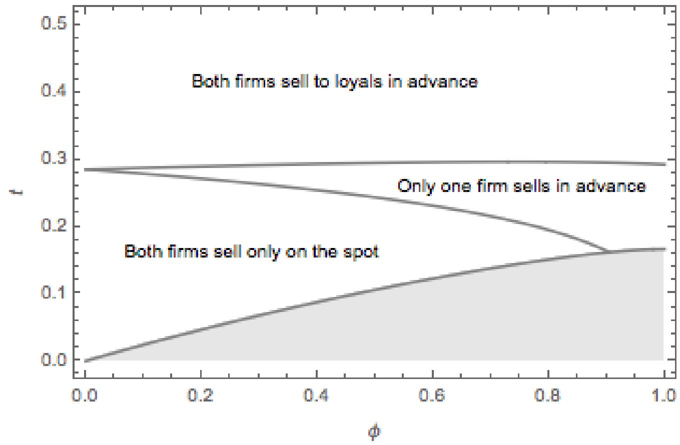

Figure 7. The regions for the different types of equilibria on the (ϕ, t) parameter space.

5.4 The Choice to Sell in Advance

In this section we compare the equilibrium revenues under advance selling with the equilibrium revenues of firms who sell only on the spot. The equilibrium prices and revenues when restricting both firms to sell only on the spot follow from Lemma 5 and Corollary 3 for $\Gamma = (0, 0)$. Naturally, the conditions for existence of equilibrium when limiting both firms to sell only on the spot are less restrictive when compared to the general game, i.e., $t' < t''$. As $t \leq 1/2$, an increase in the number of loyal consumers, i.e., the fraction ϕ increases, increases the spot period price. Put another way, as more loyal consumers purchase in advance, the spot period becomes less competitive.

Next, we compare the revenues obtained from selling on the spot and the revenues obtained from offering the product in both the advance and the spot period. The comparison is focused on the parameter values for which an existence of equilibrium is guaranteed, i.e., the area above the dark area and is illustrated in Figure 8. The (ϕ, t) space is divided into four areas. When t is low, the equilibrium is such that even when firms offer the product in the advance period, all consumers purchase in the spot. Thus, in this case, both games result in the same equilibrium in which firms sell only on the spot. When t is high, there is little competition in the spot period, so selling in advance yields a higher revenue than the spot only equilibrium. Notice also that as the fraction of loyal consumers increases, firms benefit from selling in advance for a wider range of travel costs–in the limit, when all customers are loyal, the case is equivalent to having two monopolies who decide whether to sell in advance. Finally, for medium values of t, the spot period competition is not high enough to discourage all loyal consumers from purchasing in advance, so in equilibrium either one or both firms sell in advance. However, the firms are hurt by this: spot selling dominates advance selling for both firms when they both sell in advance and their combined revenue is lower when only one sells in advance. Even if firms sell in advance in equilibrium, they might both prefer that they did not. Thus, overall, in most

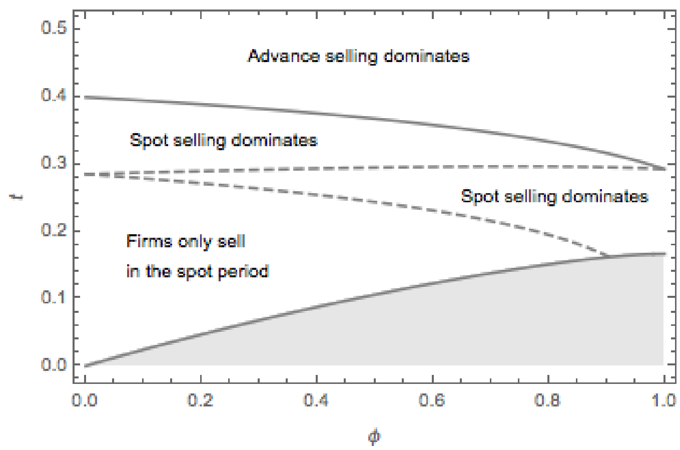

Figure 8. Regions for which the possibility to sell in both periods is (i) equivalent to selling only on the spot, (ii) superior to selling only on the spot, and (iii) dominates selling only on the spot.

cases, advance selling is undesirable under spot period competition: in some cases, when t is low the level of competition is so high that advance selling does not even occur in equilibrium; in other cases, advance selling occurs in equilibrium, but firms are hurt by selling in advance, because doing so lowers t he spot price. Only if t is so high that the level of spot period competition is low, firms may actually benefit from selling in advance. These cases describe situations where there is little competition between the firms.

6 The Commitment to Sell on the Spot

Both models illustrate that in many cases firms end up selling in advance at a discount, but are harmed by it and would benefit if they could commit to charge a higher price and only sell on the spot. In this section we discuss the credibility of such commitments and how likely it is for both firms to sell on the spot, even when the equilibrium involves advance selling. Firms can generally achieve credibility through repeated interaction (e.g., Fudenberg and Levine (1989)), which we do not model in this paper. Although in both our models we analyze settings where firms compete over a single season, with an advance and a spot period, realistically, firms interact over multiple such seasons. For example, competing baseball teams sell tickets for many games and different cruise lines offer vacation packages for many future trips. Therefore, it is reasonable that firms should be able to establish a long-run reputation for how they conduct business. This is especially true in the modern economy, where most firms offer their products and services online, so competitors' prices are easier to verify.

Although both models demonstrate that in most cases the commitment to sell only on the spot benefits firms assuming that both firms adhere to their commitment, in any such one-shot seasons a firm may have an incentive to deviate by charging a lower price and selling in advance. Therefore, if firms only interacted

Table 3. Parameter Values Used in the Numerical Study.

Parameter	Values
Model I	
v_h	1
v_l	$\{0.01, 0.05, 0.1, 0.15, ..., 0.45, 0.49\}$
δ	$\{0.01, 0.05, 0.1, 0.15, ..., 0.95, 0.99\}$
Model II	
v	1
ϕ	$\{0.01, 0.05, 0.1, 0.15, ..., 0.95, 0.99\}$
t	$\{0.13, 0.14, 0.15, ..., 0.39\}$

over a single advance-spot season, then the commitment to sell only on the spot would not be credible. However, as firms are likely to interact over multiple seasons, the commitment to sell only on the spot could be credible if firms implement a trigger strategy that penalizes deviations. With a trigger strategy, firms charge prices so that they sell only on the spot until they detect a deviation by the other firm, at which point they switch to sell in advance in all subsequent selling seasons. If the penalty from detection is large relative to the gain from deviation, a commitment to sell on the spot is likely credible.

To be specific, let $\Pi_1^{x,y}$ be firm 1's profit when both firms charge advance period prices such that firm 1 implements strategy x and firm 2 implements strategy y, where $(x, y) = (\{a, s\} \times \{a, s\})$. Consider a possible deviation by firm 1. Firm 1's gain from a deviation to sell in advance is $\Pi_1^{a,s} - \Pi_1^{s,s}$. If a deviation is detected, firm 1's profit loss in each season is $\Pi_1^{s,s} - \Pi_1^{a,a}$. To sense whether a commitment to sell only on the spot is likely in situations where spot selling by both firms is not an equilibrium, we evaluate the ratio of the short-term gain from deviation to a single season loss from detection:

$$\frac{\Pi_1^{a,s} - \Pi_1^{s,s}}{\Pi_1^{s,s} - \Pi_1^{a,a}}.$$

We conduct a numerical analysis to evaluate the magnitude of this ratio. Table 3 lists the parameter values we use in each model. In model I, it is sufficient to manipulate $v_l \in (0, v_h)$ and $\delta \in (0, 1)$, without loss of generality. Hence, we fix the value of v_h to 1. In model II, it is sufficient to manipulate $\phi \in (0, 1)$ and $t \in (0, 1/2)$. We fix the value of v to 1, without loss of generality. We then select values that cover the range of the parameters uniformly, according to Table 3.

There are 231 parameter combinations for model I. Of those, we disregard 27 combinations which correspond to the range in which advance selling is preferable, so 204 relevant combinations remain. 46 combinations fall in the range in which both high and low advance period prices may result in equilibrium. We take a worst-case scenario approach and use the high-price equilibrium profit when calculating the ratio. As the high-price equilibrium Pareto dominates the low-price equilibrium in this range, the penalty from detection would be greater if a low-price equilibrium emerges, which would strengthen the credibility of commitment. The average ratio in this sample is 1.15 and in 80.9% of the scenarios the ratio is 1.0 or lower. That is, on average, the gain in a single season deviation from spot selling is 115% of the loss that is incurred in each season after the trigger is activated and in 81% of the scenarios it is lower than the loss in a single season. For model II, there are 567 parameter combinations. Of those, we disregard the 302 combinations which result in either no equilibrium or spot-selling equilibrium ($t < \max\{t'', t_1\}$) and the ones for which advance selling dominates (high values of t). This results in a total of 265 relevant parameter combinations. In this sample, the ratio is, on average, 2.19, in 64.9% of the scenarios the ratio is 1.0 or lower and in 79.2% of the scenarios it is 2.0 or lower.

In both models, as long as the firms care about future profits, the the loss of future profit is unlikely to be covered by the short-term gain from deviation, suggesting that a commitment not to sell in advance can be credible in most scenarios. In some cases, however, the ratio is rather high, suggesting that a commitment to sell on the spot is unlikely to be credible. This occurs in settings where there is not much competition between the two firms – high values of δ and t, so the profit from advance selling approaches that of spot selling. In such cases it is reasonable to assume that firms would sell in advance, even though they would benefit from selling on the spot.

Of course, the credibility of a commitment to sell only the spot hinges not only on the benefit/loss ratio, but also on the ability to detect a deviation. As the required length of detection increases, the deviating firm may gain $\Pi_1^{a,s} - \Pi_1^{s,s} \geq 0$ over multiple seasons. However, in cases where the benefit/loss ratio is low, credibility can be achieved if firms do not heavily discount future profit. (A similar analysis to justify the credibility of commitment in a different setting is performed in Cachon and Feldman (2015).)

7 Conclusion

It has been shown that a monopolist can benefit from advance selling because consumers are more homogeneous in the advance period than in the spot period. The monopolist must give an advance period discount, but because the monopolist is expected to charge a high spot period price, consumers choose to purchase in advance. There are two reasons why this logic does not carry over well into a competitive setting. For one, competition in the spot period is likely to reduce the spot period price (model 2), which means that the firm must further discount the advance period price - consumers will not purchase in advance, unsure of their valuation, if they can anticipate a low price in the spot period, when they know they will learn their valuation. Second, as consumers are more homogeneous in the advance period, they may choose which firm to purchase from in advance primarily based on price (model 1). The resulting price competition lowers the advance period prices and therefore the attractiveness of selling in advance. Both of our models demonstrate that in most cases advance selling occurs in equilibrium (at least to some degree), but the possibility of advance selling hurts competitive firms. In most of these cases, firms would be better off if they were able to commit not to advance sell. We demonstrate that in many cases a commitment not to sell in advance is likely to be credible. Overall, we conclude that competition may either eliminate advance selling or advance selling may be better described as a competitive necessity.

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A Proofs

Proof of Lemma 1. Let p_1 and p_2 be the firms' advance period prices. In equilibrium, v_h is the spot price charged by the two firms. The surplus of a customer with α who purchases in advance from firm 1 is $\alpha\mathbb{E}[V]-p_1$. If the same customer purchases in advance from firm 2, her expected surplus is $(1-\alpha)\mathbb{E}[V]-p_2$. Finally, if the customer waits for the spot, her expected surplus is 0. A customer purchases in advance from firm 1 if and only if

$$\alpha\mathbb{E}[V]-p_1 \geq \max\{(1-\alpha)\mathbb{E}[V]-p_2, 0\}. \tag{11}$$

Suppose there exists an $\bar{\alpha}$ such that $\bar{\alpha}\mathbb{E}[V]-p_1 = \max\{(1-\bar{\alpha})\mathbb{E}[V]-p_2, 0\}$. Since the left-hand-side increases with α and the right-hand-side decreases with α, such $\bar{\alpha}$ is unique. Further, condition (11) holds $\forall\alpha \geq \bar{\alpha}$ and fails otherwise. An analogous argument follows for purchasing in advance from firm 2. □

Proof of Lemma 2. Without loss of generality, let firm 2 be the firm restricted to selling only on the spot. (That is, firm 2 charges a high enough advance price, $p_2 > (1+\delta)\mathbb{E}[V]/2$, so that consumers do not purchase from it in advance.) Given this p_2, firm 1's revenue function is:

$$\Pi_1(p_1;p_2) = \begin{cases} p_1 & p_1 \in [0, (1-\delta)\mathbb{E}[V]/2] \\ \frac{p_1}{\delta}\left(\frac{1+\delta}{2}-\bar{\alpha}\right) + \frac{\mathbb{E}[V]}{\delta(1+\beta)}\int_{(1-\delta)/2}^{\bar{\alpha}} \alpha d\alpha & p_1 \in ((1-\delta)\mathbb{E}[V]/2, (1+\delta)\mathbb{E}[V]/2] \\ \frac{\mathbb{E}[V]}{\delta(1+\beta)}\int_{(1-\delta)/2}^{(1+\delta)/2} \alpha d\alpha = \frac{v_h}{4} & p_1 > (1+\delta)\mathbb{E}[V]/2 \end{cases}$$

Observe that $\Pi_1(p_1;p_2)$ is continuous and is composed of three parts: (1) a linearly increasing function with slope 1; (2) a strictly concave function ($d\Pi_1^2/d^2p_1 = \left(\frac{1}{1+\beta}-2\right)/(\delta\mathbb{E}[V]) < 0$); and (3) a constant function. Thus, the optimal price, p_1^*, must fall in the range $\left[\frac{1-\delta}{2}\mathbb{E}[V], \frac{1+\delta}{2}\mathbb{E}[V]\right]$. If $p_1^* = \frac{1+\delta}{2}\mathbb{E}[V]$, then there are infinitely many prices that result in the same optimal revenue. Otherwise, in that range both existence (from the maximum theorem) and uniqueness ($\Pi_1(p_1;p_2)$ is strictly concave in this range) is guaranteed. Differentiating the second segment of$\Pi_1(p_1;p_2)$, we get

$$\frac{d\Pi_1(p_1;p_2)}{dp_1} = \frac{p_1}{\delta\mathbb{E}[V]}\left(\frac{1}{1+\beta}-2\right) + \frac{1+\delta}{2\delta}.$$

Equating to zero and rearranging, we get:

$$p_1^* = \frac{(1+\delta)(1+\beta)\,\mathbb{E}[V]}{2(1+2\beta)}. \tag{12}$$

(12) is the optimal advance price if it belongs to $\left[\frac{1-\delta}{2}\mathbb{E}[V], \frac{1+\delta}{2}\mathbb{E}[V]\right]$. Otherwise, the optimal price is a corner solution. To see that $p_1^* \neq \frac{1+\delta}{2}\mathbb{E}[V]$, note that

$$\left.\frac{d\Pi_1(p_1;p_2)}{dp_1}\right|_{p_1=\frac{1+\delta}{2}\mathbb{E}[V]} = -\frac{1+\delta}{2\delta}\left(\frac{\beta}{1+\beta}\right) < 0.$$

Applying some algebra to compare (12) to $\frac{1-\delta}{2}\mathbb{E}[V]$, the result follows. □

Proof of Lemma 3. Let $p_k > 0$ be the advance period price charged by firm k, $k \in \{1, 2\}$. Because customers only buy once and the firms are a-priori identical, if firm i undercuts firm j's advance price by charging $p_i = p_j - \epsilon$, it gets all market demand and firm j gets no demand. The proof of this Lemma is therefore analogous to the well-known proof of the Bertrand equilibrium for a zero marginal cost and will not be repeated. Refer to Kreps (1990), pp. 330–335 for a complete proof of the Bertrand result. □

Proof of Theorem 1. It follows from the profit functions in Table 1 that there are two candidates for a symmetric price equilibrium. The first candidate is a set of prices which results in all customers buying in advance (falls in the (a_1, a_2) range). The second candidate is a set of prices which results in only a fraction of consumers buying in advance (falls in the (a_1, a_2, a_s) range). We check under which conditions these prices are sustainable in equilibrium. In checking for profitable deviations, we focus on firm 1. The behavior of firm 2 is identical due to the symmetry of the game.

(i) The (a_1, a_2) range: Given p_2, in this range, $\Pi_1(p_1;p_2) = \frac{p_1}{2\delta}\left(\delta - \frac{p_1-p_2}{\mathbb{E}[V]}\right)$, which is strictly concave and maximized at $p_1(p_2) = (\delta\mathbb{E}[V] + p_2)/2$. Symmetry and the concavity of the profit functions in this range imply that the only interior candidate in this range is

$$p_1^l = p_2^l = \delta\mathbb{E}[V].$$

To show that it is an equilibrium, we check for which parameter values these prices fall in the (a_1, a_2) range and whether there are no profitable price deviations. If $\delta > 1/2$, the prices $\left(p_1^l, p_2^l\right)$ fall outside the (a_1, a_2) range and therefore cannot be an equilibrium. Otherwise, $\left(p_1^l, p_2^l\right)$ is in the range. If $\delta \leq 1/3$, firm 1 has no profitable deviations and thus p_1^l is the optimal price in the (a_1, a_2) range. To see this, note that choosing any price outside this range, i.e., setting $p_1 > 2\delta\mathbb{E}[V]$ results in all customers buying in advance from firm 2 (range(a_2)), which is clearly not profitable. Finally, if $1/3 < \delta \leq 1/2$, a price increase may result in some customers waiting for the spot (the (a_1, a_2, a_s) range). Increasing the price further results in no customers buying in advance from firm 1 (the (a_2, a_s) range). Can an increase in p_1 be profitable? The profit function in the (a_1, a_2, a_s) range is given by:

$$\Pi_1 (p_1; p_2) = \frac{p_1}{\delta} \left(\frac{1+\delta}{2} - \frac{p_1}{\mathbb{E}[V]} \right) + \frac{2\mathbb{E}[V]}{\beta + 1} D_1^S. \tag{13}$$

Taking the first order conditions, we get that the function is maximized at p_1^h. If $p_1^h < \mathbb{E}[V] - p_2^l = (1-\delta)\,\mathbb{E}[V]$, then from continuity and the fact that the function is piece-wise concave, it follows that $\left(p_1^l, p_2^l\right)$ is an equilibrium. If, however, $p_1^h \in ((1-\delta)\,\mathbb{E}[V], (1+\delta)\,\mathbb{E}[V]/2]$, i.e., it falls in the (a_1, a_2, a_s) range, then comparing between the two profit functions, we get that deviating to p_1^h is profitable if $\delta_1 < \delta \leq 1/2$, where

$$\delta_1 = \frac{5 + 10\beta + \beta^2 - 2\,(1-\beta)\,\sqrt{(1+\beta)\,(1+2\beta)}}{7 + 18\beta + 7\beta^2}.$$

Finally, $p_1^h \not> (1+\delta)\,\mathbb{E}[V]/2$, so deviating to the (a_2, a_s) range cannot be profitable. Combining the conditions for deviation, we find that $\left(p_1^l, p_2^l\right)$ is an equilibrium if and only if $\delta \leq \delta_1$.

(ii) The (a_1, a_2, a_s) range: Given p_2, in this range, the profit function of firm 1 is given by equation (13), which is strictly concave and maximized at p_1^h. This price is independent of p_2. For $\left(p_1^h, p_2^h\right)$ to be an equilibrium, it must fall in the (a_1, a_2, a_s) range, i.e., we must have $p_1^h \in \left[\mathbb{E}[V] - p_2^h, (1+\delta)\,\mathbb{E}[V]/2\right]$, which happens $\forall \delta \geq \beta/(1+\beta)$. Therefore, $\left(p_1^h, p_2^h\right)$ is not an equilibrium if $\delta < \beta/(1+\beta)$. Next, we check whether it is worth while to deviate from this price. Charging a price $p > p_1^h$ is surely not profitable, because the profit function is constant at range (a_2, a_s). So it remains to check whether lowering the price and deviating to ranges (a_1, a_2) or (a_1) is profitable. Deviation to the (a_1, a_2) range: the profit function in the (a_1, a_2) range is maximized at $p_1(p_2) = (\delta\mathbb{E}[V] + p_2)/2$. A deviation will be profitable, if $p_1\left(p_2 = p_2^h\right) \in \left(\max\left\{0, p_2^h - \delta\mathbb{E}[V]\right\}, \mathbb{E}[V] - p_2^h\right)$ and the profit from deviating is higher. Deviation to the (a_1) range is profitable if $p_1\left(p_2 = p_2^h\right) < \max\left\{0, p_2^h - \delta\mathbb{E}[V]\right\}$ and the profit from deviating to $p_1 = p_2^h - \delta\mathbb{E}[V]$ is higher. Combining the conditions, we get that $\left(p_1^h, p_2^h\right)$ is an equilibrium if $\delta \geq \delta_2(\beta)$, where

$$\delta_2(\beta) = \begin{cases} \delta_3 & \beta > \bar{\beta} \\ \delta_4 & otherwise, \end{cases}$$

where $\bar{\beta}$ is the unique solution to the cubic equation $1 - 8\beta - 31\beta^2 + 14\beta^3 = 0$ in the range $\beta \in [0, 1/2]$ and is approximately equal to $\bar{\beta} \approx 0.093$,

$$\delta_3 = \frac{5 + 16\beta + 3\beta^2 - 4(1-\beta)(1+2\beta)\sqrt{\frac{1+2\beta}{1+\beta}}}{9 + 22\beta + 17\beta^2}$$

and

$$\delta_4 = \frac{\beta^2 + (1+2\beta)\sqrt{\frac{\beta(2+7\beta-3\beta^2)}{1+\beta}}}{2 + 11\beta + 13\beta^2}.$$

Finally, it remains to show that the prices $p_1 = p_2 = \mathbb{E}[V]/2$ and $p_1 = p_2 = (1+\delta)\mathbb{E}[V]/2$, i.e., prices in the boundaries, cannot be an equilibrium. For $p_1 = p_2 = \mathbb{E}[V]/2$ to be an equilibrium, we must have that $\delta > 1/2$ and that $p_1^h < \mathbb{E}[V]/2$. These two conditions cannot be satisfied together. For $p_1 = p_2 = (1+\delta)\mathbb{E}[V]/2$ to be an equilibrium, $p_1^h > (1+\delta)\mathbb{E}[V]/2$. This never holds, because $\beta \geq 0$. $\square$

Proof of Theorem 2. Multiplicity of equilibria: it is sufficient to show that $\delta_1(\beta) > \delta_2(\beta)\ \forall\beta$. Since $\delta_3 \geq \delta_4\ \forall\beta \in [0, 1/2]$, it is sufficient to show that $\delta_1(\beta) > \delta_3(\beta)\ \forall\beta$. Differentiating δ_k with respect to β, we get that $\delta_k'(\beta) > 0\ \forall k \in \{1, 3\}$ for $\beta \in [0, 1/2]$. Next, $\delta_1(\beta = 0) = 3/7$ and $\delta_3(\beta = 1/2) < 3/7$. This implies that an equilibrium always exists, but it is not necessarily unique: $\forall\delta : \delta \in [\delta_2, \delta_1]$, both (p_1^l, p_2^l) and (p_1^h, p_2^h) are sustainable.

Pareto dominance: an equilibrium is Pareto dominant if it is Pareto superior to all other equilibria in the game. To show that the price pair (p_1^h, p_2^h) Pareto dominates the price pair (p_1^l, p_2^l), we need to show that $\Pi_1^*(p_1^h; p_2^h) > \Pi_1^*(p_1^l; p_2^l)$ in the range $\delta \in [\delta_2(\beta), \delta_1(\beta)]$, where

$$\begin{aligned}
\Pi_1^*(p_1^h; p_2^h) &= \frac{p_1^h}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1^h}{\mathbb{E}[V]}\right) + \frac{\mathbb{E}[V]}{\beta+1}\int_{1-\frac{p_2^h}{\mathbb{E}[V]}}^{\frac{p_1^h}{\mathbb{E}[V]}} \alpha d\alpha \\
&= \frac{\mathbb{E}[V]}{2\delta}\left(\frac{(1+\delta)^2(1+\beta)\beta}{2(1+2\beta)^2} + \frac{1+\delta}{1+2\delta} - \frac{1}{1+\beta}\right)
\end{aligned}$$

and $\Pi_1^*(p_1^l; p_2^l) = \delta\mathbb{E}[V]/2$. Comparing the profit functions, we get that $\Pi_1^*(p_1^h; p_2^h) > \Pi_1^*(p_1^l; p_2^l)\ \forall\delta(\beta) \in (\delta^-(\beta), \delta^+(\beta))$, where

$$\delta^{\pm}(\beta) = \frac{1 + 3\beta + \beta^2 \pm (1+2\beta)\sqrt{1 - \frac{2\beta(1-\beta)}{1+\beta}}}{2 + 7\beta + 7\beta^2}.$$

To prove Pareto dominance, we need to show that $\delta^-(\beta) \leq \delta_4(\beta)$ and that $\delta_1(\beta) < \delta^+(\beta)\ \forall\beta$: We have $\delta^{-'}(\beta) > 0$ and $\delta_4'(\beta) > 0\ \forall\beta \in [0, 1/2]$. Further, let $\hat{\beta}$ be the solution to the equation $\delta^-(\hat{\beta}) - \delta_4(\hat{\beta}) = 0$. Algebra reveals that $\hat{\beta} = 0$ is the unique solution to the equation. Thus, to show that $\delta^-(\beta) \leq \delta_4(\beta)$, it is enough to find one $\beta \neq 0$, which satisfies the inequality. Take $\beta = 1/2$. $\delta^-(\beta = 1/2) < \delta_4(\beta = 1/2)$

and the result follows. Further, since $\delta^{+'}(\beta) < 0$ and $\delta_1'(\beta) > 0$, it is enough to show that $\delta_1(\beta = 1/2) < \delta^+(\beta = 1/2)$. Plugging $\beta = 1/2$ in, we get the desired result. (Symmetry implies that $\Pi_2^*\left(p_1^h; p_2^h\right) > \Pi_2^*\left(p_1^l; p_2^l\right)$ as well.) $\square$

Proof of Lemma 4. Suppose only switchers remain on the spot. Then, the firm's revenue function is:

$$\Pi_s = p_s (1-\phi)\left(\frac{v - p_s}{tv}\right),$$

which is concave and maximized at $\hat{p} = v/2$. Thus, if $t > 1/2$, the equilibrium spot price is $p^* = \hat{p}$ and the firm sells to a fraction of switchers. Otherwise, $p^* = v(1-t)$ and the firm sells to all consumers. In advance, loyals anticipate the spot period price, p^*. Their expected utility from waiting for the spot is: $(\mathbb{E}[V|V \geq p^*] - p^*)\bar{F}(p^*)$. Therefore, the best advance period price to charge to make all loyals purchase in advance is: $p_a = \mathbb{E}[V] - (\mathbb{E}[V|V \geq p^*] - p^*)\bar{F}(p^*)$ and the total revenue is $\phi p_a/2 + \Pi_s$. If $t > 1/2$, $p_a = \mathbb{E}[V] - \frac{v}{8}$ and if $t \leq 1/2$, $p_a = vt^2/2$ and $\Pi_s = v(1-\phi)/(4t)$. Now suppose the monopolist sets a high advance price to sell to all consumers on the spot. The firm's revenue function is:

$$\Pi_{all} = p_s\left(\frac{\phi}{2}\frac{v - p_s}{v} + (1-\phi)\frac{v - p_s}{tv}\right),$$

which is concave and maximized at $\hat{p} = v/2$. Thus, if $t > 1/2$, the equilibrium spot price is $p^* = \hat{p}$ and the firm sells to a fraction of of loyal consumers and switchers. Otherwise, $p^* = v(1-t)$ and the firm sells to all consumers. Comparing with the revenue function when selling to loyals in advance, the result follows. $\square$

Proof of Lemma 5. We prove the equilibria of each subgame separately.

(i) Suppose all loyals bought in advance and only switchers remain on the spot. The spot period revenue function is continuous and piece-wise concave in p_i for every price p_j. We write the best response function $p_i(p_j)$ based on the analysis of the revenue functions given in column 1 of Table 2. If $t \geq 1/2$, $p_i(p_j) = (p_j + tv)/2 \; \forall t \Rightarrow p_s^1 = p_s^2 = tv$. If $1/3 < t < 1/2$,

$$p_i(p_j) = \begin{cases} (p_j + tv)/2 & p_j < 4v/3 - tv \\ (2-t)v - p_j & otherwise. \end{cases} \tag{14}$$

The best response functions are continuous and composed of two parts: a linearly increasing function with slope $1/2$, followed by a linearly decreasing function with slope (-1). If the best response functions intersect in the first part, the spot period equilibrium satisfies $p_1 = p_2 = tv$. As $tv < 4v/3 - tv$ for all values of t in this range, this is the unique equilibrium. Finally, if $t \leq 1/3$,

$$p_i(p_j) = \begin{cases} (p_j + tv)/2 & p_j < 3tv \\ p_j - tv & otherwise. \end{cases}$$

The best response functions are continuous and composed of two parts: a linearly increasing function with slope $1/2$, followed by a linearly increasing function with slope 1. If the best response functions intersect in the first part, the spot period equilibrium satisfies $p_1 = p_2 = tv$. As $tv < 3tv$, this is the unique equilibrium. Hence, $p_1 = p_2 = tv$ is the unique equilibrium $\forall t \in [0, 1]$.

(ii) Suppose all consumers remain on the spot. Similarly to the analysis in (i), we can write the best response function based on the revenue functions given in column 2 of Table 2. If $t \geq 1/2$,

$$p_i(p_j) = \begin{cases} \frac{p_j(1-\phi)+tv}{2(1-\phi+\phi t)} & p_j \leq p_j' \\ (2-t)v - p_j & p_j' < p_j \leq 3v/2 - t \\ v/2 & p_j > 3v/2 - t, \end{cases}$$

where

$$p_j' = \frac{4v(1-\phi) - 3tv(1-2\phi) - 2\phi t^2 v}{3(1-\phi) + 2\phi t}.$$

The best response functions are continuous and composed of three parts: a linearly increasing function with slope $\frac{1-\phi}{2(1-\phi+\phi t)}$, followed by a linearly decreasing function with slope (-1), followed by a constant function. If the best response functions intersect in the first part, the spot period equilibrium satisfies

$$p_1 = p_2 = \frac{tv}{1-\phi+2\phi t}. \tag{15}$$

As $p_j' > (2-t)v - p_j'$, this is the unique equilibrium. Next, we analyze the case $t < 1/2$. The best response function is composed of at most 5 parts:

$$p_i(p_j) = \begin{cases} v/2 & \\ \frac{p_j(1-\phi)+tv}{2(1-\phi+\phi t)} & \\ (2-t)v - p_j & 1/3 < t < 1/2, \phi < \frac{1-3t}{1-4t+2t^2}, p_j > p_j' \\ p_j - tv & \\ \frac{(2-\phi)v}{2\phi} & \phi > 2/(3-2t), p_j > \frac{v(2-\phi+2\phi t)}{2\phi} \end{cases} \tag{16}$$

The exact conditions for each part are complex and depend on the values of the parameters v, ϕ and t. The best response function is almost always continuous except for a potential discontinuity between the first and second part. To show the equilibrium is unique and is given by (15), it's sufficient to argue that it is an equilibrium and that other equilibria candidates are not. $\left(\frac{(2-\phi)v}{2\phi}, \frac{(2-\phi)v}{2\phi}\right)$ can't be an equilibrium, because $\frac{v(2-\phi+2\phi t)}{2\phi} > \frac{v(2-\phi)}{2\phi}$. $p_i = p_j - tv$ cannot be an equilibrium because there do not exist $p_i = p_j$ that satisfies the equation. $p_1 = p_2 = (2-t)\,v/2$ cannot be an equilibrium, because $2p_j' > (2-t)\,v$. This condition also implies that (15) must be the unique equilibrium, if it exists. Existence is not guaranteed, however, because of the possibility that the best response equals $v/2$. Therefore, we conclude by checking for which parameter values (15) is not an equilibrium. Prices (15) are not an equilibrium, if there exists profitable deviations from it. Suppose $p_s^1 = p_s^2 = tv/\left(1-\phi+2\phi t\right)$. Is it optimal for firm 1 to deviate by increasing the price to $p_s^{dev} = v/2$? Such an increase may only be profitable if $v/2$ falls in the $\left(a_2, a_{\varnothing}\right)$ range when $p_2 = p_s^2$. That is, if $v/2 > vt + p_s^2$. Otherwise, a deviation to $v/2$ is surely not profitable. $v/2$ falls in range if $t < \frac{\sqrt{1-\phi}-(1-\phi)}{2\phi}$. Then, the revenue from deviating to $v/2$ is $\phi v/8$. The revenue obtained if $p_s^1 = p_s^2 = tv/\left(1-\phi+2\phi t\right)$ is

$$\Pi_s = \frac{1-\phi\left(1-t\right)}{2\left(1-\phi\left(1-2t\right)\right)^2} tv.$$

Comparing the revenues we get that (15) is an equilibrium if and only if $t > t'$. Otherwise, there does not exist a spot period price equilibrium.

(iii) Suppose firm i's loyals waited for the spot and firm j's loyals purchased in advance. The best response function of firm i is as in (16) and for firm j is as in (14) (with i replacing j). Similarly to the procedure in parts (i) and (ii) of the proof, one can verify that the unique equilibrium is the solution to the system of equations:

$$\begin{cases} p_i = \frac{p_j(1-\phi)+tv}{2(1-\phi(1-t))} \\ p_j = \frac{p_i+tv}{2}, \end{cases}$$

or

$$p_s^1 = \frac{tv\left(3-\phi\right)}{3\left(1-\phi\right)+4\phi t}\; p_s^2 = \frac{tv\left(3-\phi\right)-\phi tv\left(1-2t\right)}{3\left(1-\phi\right)+4\phi t}. \tag{17}$$

For (17) to be an equilibrium, neither firm should find it profitable to deviate to $v/2$. Following the same argument as in part (ii) of the proof, we get that it is an equilibrium if and only if $t > t''$. □

Proof of Theorem 3. To prove the ranges for the equilibria, we need to show that there do not exist any profitable deviations. For advance selling to be an equilibrium for both firms, we need the revenues in (i) to be higher than Π_1 in (iii). Comparing the revenue functions we get that this is the case if $t > t_2$, such that t_2 is the second root of the cubic equation: $6-4\phi-2\phi^2+\left(11\phi^2+18\phi-21\right)t-\left(8\phi+24\phi^2\right)t^2+16\phi^2t^3 = 0$. For spot selling to be an equilibrium, we need the revenues in (ii) to be higher than Π_2 in (iii). Comparing the revenue functions we get that this is the case if $t < t_1$, such that t_1 is the second root of the quintic equation: $a + bt + ct^2 + dt^3 + et^4 + ft^5 = 0$, where $a = 6 - 14\phi + 6\phi^2 + 6\phi^3 - 4\phi^4$, $b = -21 + 58\phi - 21\phi^2 - 48\phi^3 + 32\phi^4$, $c = -48\phi + 20\phi^2 + 124\phi^3 - 96\phi^4$, $d = -8\phi^2 - 120\phi^3 + 132\phi^4$, $e = 32\phi^3 - 80\phi^4$ and $f = 16\phi^4$. For firm 1 to sell only on the spot and firm 2 to sell to loyals in advance, we need Π_1 in (iii) to be higher than the revenue in (i) and Π_2 in (iii) to be higher than the revenue in (ii), which occurs for all other values of t. Finally, no spot period equilibrium occurs if $t < t''$ and therefore, in this range, it is impossible to predict the equilibrium of the full game. Finally, we argue that $\underline{p}^a$ and $\bar{p}^a$ are the only two prices that can be part of an advance period equilibrium. Since loyals are homogeneous in advance, in any pure strategy equilibrium they will either all buy in advance or all wait. For each such decision, there is a range of advance period prices that result in the same purchasing decision. The lower price, $\underline{p}^a$, is an advance price such that it and any lower price will result in all customers purchasing in advance given that all loyals purchased in advance from the other firm. Since demand is inelastic in this range, charging any price below the threshold is suboptimal and will never be part of an equilibrium. Similarly, the higher price, $\bar{p}^a$, is the advance price such that it and prices below it, but above $\underline{p}^a$, result in all loyals buying in advance given that loyals of the other firm wait. Charging an advance price in $\left(\underline{p}^a, \bar{p}^a\right)$ is suboptimal. Finally, setting any price above $\bar{p}^a$ will lead all loyals to wait and result in the same outcome and revenue. Therefore, to find the equilibrium of the current model, it is sufficient to consider three advance prices for each firm: $\underline{p}^a$, $\bar{p}^a$, and an arbitrarily higher than $\bar{p}^a$. $\square$

Online Appendix: "Is Advance Selling Desirable with Competition?"

Gérard P. Cachon and Pnina Feldman

In this document, we analyze a version of Model I in which we allow consumers to purchase from both firms in advance. Allowing the purchase of two units decreases the competition between the two firms in advance–if firms charge a low enough price customers are going to buy from both of them. This happens when consumers are relatively homogeneous. Therefore, it is possible that advance selling dominates spot selling not only when customers are very heterogeneous (large δ) as in Model 1, but also when they are homogeneous or the degree of heterogeneity is low (small δ).

A Model I: Consumers may purchase two units in advance

Allowing consumers to purchase two units in advance expands their choice set. A consumer that attaches a probability α for preferring firm 1, now evaluates the expected utility of four different strategies:

1. Buy in advance from both firms, which yields an expected utility of $\mathbb{E}[V] - p_1 - p_2$
2. Buy in advance from firm 1, which yields an expected utility of $\alpha \mathbb{E}[V] - p_1$
3. Buy in advance from firm 2, which yields an expected utility of $(1-\alpha)\mathbb{E}[V] - p_2$
4. Wait for the spot and then, if $V = v_h$, buy from the preferred firm, which yields an expected utility of zero.

Comparing the different strategies, we get that consumers purchase from both firms in advance if $p_1 + p_2 \leq \mathbb{E}[V]$ and $\alpha \in (p_1/\mathbb{E}[V], 1 - p_2/\mathbb{E}[V])$. Consumers whose $\alpha > 1 - p_2/\mathbb{E}[V]$, purchase only from firm 1 in advance and those with $\alpha < p_1/\mathbb{E}[V]$ purchase only from firm 2 in advance. That is, compared to the model where customers are restricted to purchase only once, consumer behavior differs when $p_1 + p_2 \leq \mathbb{E}[V]$. It is the same when $p_1 + p_2 > \mathbb{E}[V]$.

Suppose first that customers are homogeneous in advance ($\delta = 0$). The next theorem describes the equilibrium in this case.

Theorem 1. *When $\delta = 0$, the unique set of advance period price equilibrium is $p_1^* = p_2^* = \mathbb{E}[V]/2$ and all consumers buy from both firms in advance. The corresponding revenues, $\Pi_1^* = \Pi_2^* = \mathbb{E}[V]/2$, are greater than the revenues from selling only on the spot.*

Proof. Customers are going to buy from both firms if $p_i \leq \mathbb{E}[V]/2\ \forall i = 1,2$, buy only from firm i if $p_i \leq \mathbb{E}[V]/2$ and $p_j > \mathbb{E}[V]/2$ and wait if $p_i > \mathbb{E}[V]/2\ \forall i$. Therefore, in equilibrium each firm charges $\mathbb{E}[V]/2$ and sells to all consumers: Charging a lower price will decrease revenues per sale without increasing the total sales. Increasing the price will eliminate sales altogether. The revenue obtained from selling only on the spot is $\Pi_S^* = v_h/4$. Therefore, selling in advance is superior. $\square$

If consumers are completely homogeneous in advance and may purchase from both firms, there is no competition between firms in advance. Each firm is able to charge the monopolist advance period price and sell to all consumers. Therefore, the advance selling result holds in this case and advance selling dominates spot selling.

Next, assume that $\delta > 0$. The revenues listed in Table 1 of the main text remain the same, aside for the revenues in range (a_1, a_2), which, when allowing consumers to purchase from both firms in advance, become $\Pi_i = \frac{p_i}{\delta}\left(\frac{1+\delta}{2} - \frac{p_i}{\mathbb{E}[V]}\right)$ $\forall i$. The next theorem describes the equilibria of the game.

Theorem 2. *Assume $\delta > 0$. Let $\delta_1(\beta) = 4\sqrt{2(1+\beta)(1+2\beta)} - 5 - 8\beta$ and $\delta_2(\beta) = 3 + 8\beta - 2\sqrt{\frac{2(1+2\beta)^3}{1+\beta}}$. Two symmetric price equilibria are possible:*

1. *If $\delta \leq \delta_1(\beta)$, the firms charge $p_1^l = p_2^l = (1+\delta)\mathbb{E}[V]/4$ in advance and all customers purchase in advance. If $\delta < 1/3$, all customers purchase from both firms. Otherwise, customers with $\alpha > (3-\delta)/4$ purchase only from firm one, customers with $\alpha < (1+\delta)/4$ purchase only from firm 2, and the rest purchase from both firms in advance.*

2. *If $\delta \geq \delta_2(\beta)$, the firms charge*
$$p_1^h = p_2^h = \frac{(1+\delta)(1+\beta)\mathbb{E}[V]}{2(1+2\beta)}.$$
Furthermore,
$$\bar{\alpha} = 1 - \bar{\alpha} = \frac{(1+\delta)(1+\beta)}{2(1+2\beta)} < 1.$$
Customers with $\alpha > \bar{\alpha}$ buy from in advance from firm 1, those with $\alpha < \underline{\alpha}$ buy in advance from firm 2 and the rest wait for the spot.

Proof. The steps for the proof are similar to the proof of Theorem 1 of the main text. There are four candidates for a symmetric price equilibrium. The first candidate is a set of prices which results in all customers buying in advance, some from both firms. The second candidate is a set of prices which results in only a fraction of consumers buying in advance from one of the two firms and the rest wait for the spot. The third candidate falls in the boundary: half of the customers buys in advance from firm 1 and the other half buys from firm 2. None buys from both firms and none waits for the spot. The fourth candidate is the spot selling equilibrium where both firms charge a high enough advance price, so that all customers wait. We check under which conditions these prices are sustainable in equilibrium. In checking for profitable deviations, we will focus on firm 1. The behavior of firm 2 is identical due to the symmetry of the game.

(i) The (a_1, a_2) range: in this range, $\Pi_1(p_1) = \frac{p_1}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1}{\mathbb{E}[V]}\right)$, which is strictly concave and is maximized at p_1^l. Thus, the only interior equilibrium candidate in this range is (p_1^l, p_2^l). The equilibrium profits in this range are: $\Pi_i^*(p_i^l) = (1+\delta)^2\mathbb{E}[V]/(16\delta)$. To show that it is an equilibrium, it remains to check for which parameter values these prices fall in the range and whether there are no profitable deviations. If $\delta \leq 1/3$, p_1^h fall below $(1-\delta)\mathbb{E}[V]/2$. Given p_2^l, the only possible deviation is to increase the price so that only firm 2 sells, which is definitely not profitable. Therefore, (p_1^l, p_2^l) is an equilibrium in this range. If $\delta > 1/3$, p_1^l falls in $((1-\delta)\mathbb{E}[V]/2, (1+\delta)\mathbb{E}[V]/2)$. The only possible profitable deviation is to increase the price to p_1^h, if it falls in (a_1, a_2, a_s) range and earns higher profits. p_1^h falls in $\left(\mathbb{E}[V] - p_2^l, (1+\delta)\mathbb{E}[V]/2\right)$ if $\delta > (v_h + 4v_l)/(3v_h/4v_l)$. In this case, firm 1's revenue from deviating is:

$$\Pi_1\left(p_1^h; p_2^l\right) = \frac{p_1^h}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1^h}{\mathbb{E}[V]}\right) + \frac{v_h}{2\delta}\int_{1-\frac{p_2^l}{\mathbb{E}[V]}}^{\frac{p_1^h}{\mathbb{E}[V]}} \alpha d\alpha.$$

It dominates $\Pi_1^*\left(p_1^l\right)$ if $\delta > \delta'(\beta)$. Therefore,(p_1^l, p_2^l) is an equilibrium otherwise.

(ii) The (a_1, a_2, a_s) range: As in the base model, in this range, the revenue function is maximized at p_1^h. For the high price to be an equilibrium it must fall in $\left(\mathbb{E}[V] - p_2^h, (1+\delta)\,\mathbb{E}[V]/2\right)$. It does $\forall \delta > v_l/\left(v_l + v_h\right)$. In addition, for the high price to be an equilibrium, there shouldn't be any profitable deviation. The only possible profitable deviation may be to p_1^l. This is only possible if the low price falls in (a_1, a_2), i.e., if $p_1^l < \mathbb{E}[V] - p_2^h$. This happens if $\delta \in \left(\frac{v_l}{v_l+v_h}, \frac{v_h+4v_l}{3v_h+4v_l}\right)$. Furthermore, for the deviation to be profitable, the revenue obtained from deviating should be higher. This revenue is given by:

$$\Pi_1\left(p_1^l\right) = \frac{(1+\delta)^2\,\mathbb{E}[V]}{16\delta},$$

which is higher than the high-price equilibrium profit if $\delta \in \left(\frac{v_l}{v_l+v_h}, \delta_2(\beta)\right)$. Therefore, $\left(p_1^h, p_2^h\right)$ is an equilibrium if $\delta > \delta_2(\beta)$.

(iii) $p_1 = p_2 = \mathbb{E}[V]/2$: The revenue in this case is $\mathbb{E}[V]/4$. For this to be an equilibrium, we must have $(1+\delta)\,\mathbb{E}^2[V]/\left(v_h + 2v_l\right) < \mathbb{E}[V]/2 < (1+\delta)\,\mathbb{E}[V]/4$, which never holds. Therefore $p_1 = p_2 = \mathbb{E}[V]/2$ is never an equilibrium.

(iv) Spot selling: For both firms to sell in the spot the candidate symmetric equilibrium is every pair of prices that satisfy $p_1 = p_2 > (1+\delta)\,\mathbb{E}[V]/2$. This yields revenue of $v_h/4$ for each firm. This, however is not an equilibrium. Given that sets a high advance price, the other benefits from deviating by setting a lower price, p', such that:

$$p' = \max\left\{\frac{1+\delta}{2}\mathbb{E}[V], \frac{1+\delta}{\left(v_h + 2v_l\right)}\mathbb{E}^2[V]\right\}.$$

□

Similarly to the original model, here too there are two types of symmetric equilibria. One in which the firms sell to all consumers in advance with some consumers purchasing from both firms and the other in which some consumers purchase in advance from either firm 1 or firm 2 and others wait for the spot period. The latter equilibrium is the same as the one in the original model.

As in the original model, $\delta_1(\beta) > \delta_2(\beta)$, implying that an equilibrium always exists, but is not necessarily unique. Comparing the profit functions in the range $\delta \in (\delta_2(\beta), \delta_1(\beta))$, we find that equilibrium (ii) Pareto dominates equilibrium (i) in that range. Finally, Corollary 1 shows that firms, in most cases, would benefit if they could commit not to sell in advance.

Corollary 1. *Firms' revenues from selling only on the spot, $v_h/4$, are strictly higher than the revenue obtained from selling at least partly in advance if $\delta'(\beta) < \delta < \delta''(\beta)$, where*

$$\delta' = \frac{3 - \beta - 2\sqrt{2\,(1-\beta)}}{1+\beta}$$

and

$$\delta'' = \frac{1 + 2\beta - \beta^2}{(1+\beta)^2}$$

Corollary 1 demonstrates that in most cases the possibility of advance selling ends up hurting firms under competition, even when consumers may purchase from both firms in advance. Figure 1 illustrates the ranges where advance selling / spot selling dominates. The range $\delta > \delta''$ is similar to the range in the original model, where because of the high level of heterogeneity among customers, advance selling is better, because there is essentially no competition between firms. The range $\delta < \delta'$ is new. Here, advance selling is better

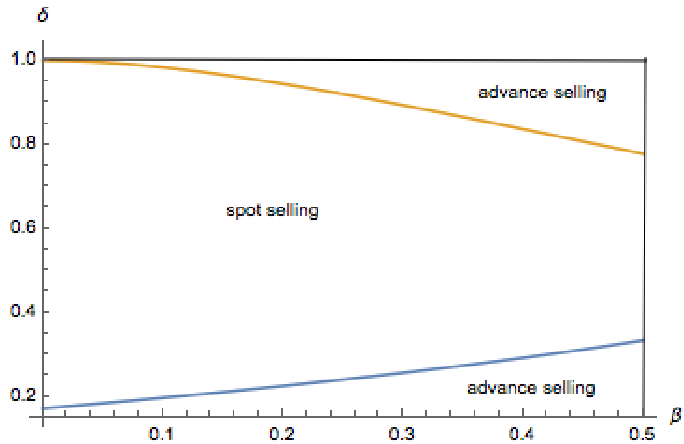

Figure 1. Areas in the (β, δ) parameter space for which selling only on the spot is preferred to the revenue obtained in the advance selling equilibrium.

because consumers end up buying from both firms in advance, again, limiting the level of competition. Still, in most cases, $\delta' < \delta < \delta''$ implying that firms would benefit if they were able to commit to sell only on the spot, even though spot selling alone is never an equilibrium.