ESSAYS ON OCCUPATIONAL CHOICE AND ENTREPRENEURIAL VENTURES

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ABSTRACT

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This dissertation consists of three independent articles on the subjects of entrepreneurial ventures and occupational choice. The first article examines the flow of information between entrepreneurs and investors in a theoretical venture capital market, analyzing how informational asymmetries and evaluative bias affect equilibrium fundraising, investment, and implementation strategies. The second article investigates the gender income differential using a model of occupational choice, maternity choice, and the ability to stochastically wage climb. The third article theoretically examines the location choice and endogenous growth dynamics of television productions facing labor supplies that are heterogeneous in skill level. Location choice is influenced by tax considerations, as well as the regional accessibility of talent; the model is used to explain why studios and producers have begun shifting television production outside of Hollywood despite the high concentration of production talent in Los Angeles.

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CHAPTER 1 : Venture Capital Search with Signalling and Asymmetric Uncertainty

Abstract. This paper examines the flow of information between entrepreneurs and investors in a theoretical venture capital market: How do informational asymmetries and evaluative bias affect equilibrium fundraising, investment, and implementation strategies? We present a search model of the VC market in which entrepreneurs seek to undertake risky projects and venture capitalists seek to fund projects. Both sides of the market have private information in each period of a three-stage game. The persistence of private information gives rise to a dynamic system with nonlinear feedback, resulting in a rich system of integral equations. Our analysis demonstrates how bias and informational asymmetries lead to inefficient capital allocation (and acceptance) strategies, while repeated interactions between entrepreneurs and VCs mitigate informational differences over time.

1.1. Introduction

Markets characterized by asymmetric information between participants often rely upon signalling before transaction. The flow of accurate information may be constrained by accessibility, communicability, and moral hazard. Informational differences are particularly complex in markets for financial capital. Venture capitalists and private equity firms see hundreds to thousands of business plans each year: They observe trends, conduct market research, hear pitches, meet with founders, and employ industry specialists with expertise and access to broad information networks—all means of collecting information by which to evaluate ventures. Entrepreneurs have intimate knowledge of their own character, commitment, and productivity, and may also have deeper understanding of the product or technology pending development.

Using their available resources, both the entrepreneur and the lender form impressions of a project's quality. These signals approximate the underlying true potential, with precision determined by experience, evaluative ability, and informational access.

The relationship between entrepreneur and venture capitalist conforms to the framework of global games introduced by Carlsson and van Damme (1993), wherein players receive noisy signals of the true state of the world. Global games have been applied to the study of financial markets frequently, particularly to crises including speculative attacks, bank runs, and bubbles, as in work done by Morris and Shin (1998), Morris and Shin (2001), Angeletos and Werning (2006), and Hellwig et al. (2006).

Observable actions of the entrepreneur and investor can convey information to be reflected

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in respective updated valuations of a project. Leland and Pyle (1977) show that an entrepreneur's inclination to invest in his own project, measured by shares retained, can act as a positive signal of quality to the lender.

The ability of entrepreneurs to signal the value of their firms to potential investors has been studied empirically. Keasey et al. (1992) examine firms seeking a listing on the U.K. Unlisted Securities Market (USM). The authors consider a number of signals, including the percentage of equity retained by entrepreneurs, and find firm value to be positively related to shares retained by the entrepreneur. Audretsch et al. (2009) consider patenting and prototyping activities as signals; data from over 900 nascent entrepreneurs suggest patenting and prototyping in early stages increase the likelihood of external financing. A study by Nofsinger and Wang (2011) examines the methods by which institutional vs. informal investors receive signals, finding institutional investors rely more on their own abilities and quality of investor protection, while informal investors tend to use product type, level of an entrepreneur's experience, and personal relationships to reduce information asymmetry.

This paper presents a search model wherein an entrepreneur carries ideas of stochastic quality to fruition in three phases: idea search (brainstorm), funding search, and implementation. The entrepreneur and investors receive noisy signals as to an idea's true quality, with varied relative precisions. Investor optimality is determined through a zero profit condition. The model is used as a tool to study the flow of information between investors and entrepreneurs; it helps quantify the impact of informational asymmetry and transfer on search behavior and equity contract formation, as well as the losses associated with the inability to transfer information accurately or from one side being systematically misguided. In one application, evaluative bias by entrepreneurs (overconfidence) is modeled as a (positive) linear shift in the distribution of signals received by entrepreneurs, so that their posterior beliefs are no longer centered at an idea's true quality. The framework presented serves as a basis for analysis of the welfare cost of overconfidence, which can induce suboptimal investment, fundraising, and implementation behavior by entrepreneurs. The model can be used to study the effects of optimism on expected search time, and to explain the existence of financial experts who receive more precise signals. It can also be used to compare the cost of overconfidence in different investment environments, i.e., at venture capital firms where investors' signals are relatively precise, versus government institutions where investors' signals are imprecise. The model can also be used to measure the value of learning by doing, in the spirit of Fraser and Green (2006), who find that overconfidence bias tends to diminish with experience.

This paper further validates that result, finding that an entrepreneur's willingness to shop a project can serve as a positive signal to venture capitalists, whose equity requirement in turn signals the entrepreneur before he accepts or rejects a contract.

Our contribution to the literature is threefold. First, our model speaks to the relative significance of adverse versus advantageous selection in search and matching process. Beginning with the seminal Akerlof (1970) and Rothschild and Stiglitz (1976), adverse selection has been shown to induce nonexistence of equilibria in many models. Efforts to overcome this problem have ranged from restricting models via capacity constraints and size of the economy, as in Inderst and Wambach (2001), to contract rationing, as in Prescott and Townsend (1984), Gale (1992), and Dubey (2002). This literature abstracts away from multiplicity of equilibria by restricting the size of the economy and the potential behaviors of their agents. Significant discussion has been devoted to how the properties of the learning mechanisms of the agents interacts with these restrictions. However, the bulk of this literature agrees that equilibria in models with adverse selection are Pareto inefficient, and this gives rise to a new body of literature discussing regulatory mechanisms for resolving this Pareto inefficiency. Voucher markets in health insurance markets have been discussed by Cutler and Reber (1998). Cohen and Dehejia (2004) examined the role that mandatory insurance plays in the prevalence of traffic fatalities. Chiappori and Salanie (2000) study the French motor insurance market and the role of regulation. Guerrieri, Shimer, and Wright (2010) more recently study a search theoretic setting, showing that adverse selection plays a significant role in frictional search markets.

The advantageous selection literature poses a paradigm in which these problems are not as severe as the prevailing literature suggests. Beginning with the seminal De Meza and Webb (2001), economists have asked what implications may hold when selection is based on risk aversion in addition to ex ante quality. In particular, selection is advantageous when the more risk averse participate more fully in a market (say, by buying more insurance), and in addition the more risk averse are actually less risky. The reason is that conditioning on risk aversion then induces positive correlation between coverage and ex post risk that an agent presents. This result is shown to have empirical significance in the health insurance market by Fang, Keane, and Silverman (2006). Davidoff and Welke (2007) find a similar result for reverse mortgages. In all cases the result is driven by the fact that agents heterogenous in their types are also heterogenous in their preferences, and that types and preferences are linked. In the above example, this manifests as risk-averse individuals also engaging in less risky behavior. Our paper will claim that the result holds for funding search markets as well, since agents of lower quality prefer not to reach an implementation stage. The model is stylized in such a way that "implementation" may be interpreted broadly, from starting a business to a generic firm-worker matchup. This allows us to conjecture that the result will carry through to general labor market search settings. We thus consider our paper part of an ongoing modern conversation on the theory of adverse versus advantageous selection.

The second contribution is to the role that overconfidence plays in entrepreneurial markets. The prevalence of overconfidence has been well established. One of the first related studies, by Cooper et al. (1988), examines data from 2,994 members of the National Federation of Independent Business (NFIB) and finds significant optimism in entrepreneurs' perception of their odds for success. Furthermore, entrepreneurs tend to rank their own odds for success higher than other business owners with similar ideas; evaluations also tend to have little relationship with objective predictors. Work by Cassar and Friedman (2007) links individual overconfidence with more aggressive entrepreneurial startup decisions, finding a positive association between overconfidence and propensity for startup activity. Using a variety of overconfidence measures, Dai and Ivanov (2010) find no evidence to suggest banks curtail lending to overconfident entrepreneurs; rather, banks are more likely to lend to the optimistic, and without charging an interest premium or requiring more collateral. One might a priori suspect that interactions between agents and markets could fix overconfidence, since prices are quite revealing as economic objects, and would therefore be surprised by the prevailing literature that this is not the case. One might also suspect that since information in an economy is widely dispersed, and overcoming overconfidence requires effectively aggregating private information, that markets may be inefficient at performing this task. Our model shows that both cases are possible, and which prevails is indeed highly contingent on the precise information structure of the economy.

Our third contribution is to the applied computational literature on dynamic systems with feedback. Systems of integral equations in the mathematics literature are described as Volterra integral equations. The computational economics literature has been aware that agents interacting and learning from the behavior of other agents induces a system of Volterra integral equations. Our paper presents a stylized model with nontrivial dynamics that through the use of the integral equation literature remains numerically tractable.

The paper is organized as follows. Section 1.2 presents the capital search model, characterizes optimal strategies for the investor and entrepreneur, and describes an effective reparametrization of the value functions. Section 1.3 examines three baseline cases, when both or either the entrepreneur and/or investor observes true quality directly. Section 1.4 introduces and discusses the qualitative effects of an overconfidence parameter. Section 1.5 analyzes results of the numerical solution. A timeline of the model and the feedback cycle, a programming method for a numerical solution, plots, relevant derivations, and a notation guide can be found in the Appendices.

1.2. The Model

An entrepreneur living in infinite discrete time may start a business by moving through three phases: brainstorming, funding, and implementation. During the brainstorming phase, exactly one idea arrives each period, at a cost c_B, with stochastic quality $q^* \sim N(\mu, \sigma_N^2)$ drawn by nature and which the entrepreneur views with a noisy signal $q^E \sim N(q^*, \sigma_E^2)$.

Aware of the prior on q^* and the likelihood of his own signal, the entrepreneur deduces a posterior $p(q^*|q^E)$ using Bayes' rule. Using this posterior, he decides whether to spend another period brainstorming or to begin the funding search. If he continues brainstorming, his current q^* is discarded, and he receives a new idea according to the same process.

During a period of fundraising, the entrepreneur pays a cost c_F and taps into a pool of risk-neutral, homogeneous, competitively-behaving venture capitalist investors. This pool of investors collectively receives its own signal as to the project's true quality. Investors within the pool, all seeing the same private investor signal, then each bid for the opportunity to enter a funding contract with the entrepreneur. A bid is a proposed share offer s, meaning the share of future revenue R the investor wishes to keep. Since all investors receive the same signal, they are participating in a common value auction with identical information. The auction is comparable to Bertrand competition in which profit is driven out. We henceforth regard the "pool" of investors as a single, representative investor bound by a zero-profit condition.

After receiving an offer from this representative investor, the entrepreneur has three options: accept the equity contract and move to the implementation phase, reject the offer and continue the funding search, or reject the offer and drop the idea altogether for further brainstorming. Both the entrepreneur and investors discount at β.

During the implementation phase, an idea yields a risky return R drawn from a truncated normal distribution centered at q^* with variance σ_R^2 and support $[\frac{K}{\beta}, \infty]$. The lower bound was selected to ensure nonnegative present discounted profits for any project.

$$p(R|q^*) = \begin{cases} \dfrac{\dfrac{1}{\sigma_R}\phi\left(\dfrac{R-q^*}{\sigma_R}\right)}{1-\Phi\left(\dfrac{\frac{K}{\beta}-q^*}{\sigma_R}\right)}, & R \geq \frac{K}{\beta} \\ 0, & otherwise \end{cases} \tag{1.2.1}$$

This lower bound is selected without significant loss of generality, since the goal of the entrepreneur is to select a single project for implementation. Thus fruitful analysis is still driven by opportunity costs for the entrepreneur. Moreover, this assumption yields significant tractability by allowing us to abstract away from the notion of default in this environment. Default penalties and walkaway options in the venture capital industry are complex legal objects we do not wish to consider.

The payoff structure is consistent with the idea that $q^{*'} > q^*$ implies $p(R|q^{*'})$ first order stochastic dominates $p(R|q^*)$, capturing the notion that higher quality ideas yield greater expected profit. The expectation of R conditional on q^* is

$$\mathbb{E}[R|q^*] = q^* + \sigma_R \frac{\phi\left(\frac{\frac{K}{\beta}-q^*}{\sigma_R}\right)}{1-\Phi\left(\frac{\frac{K}{\beta}-q^*}{\sigma_R}\right)} = q^* + \Psi(K, \sigma_R, \beta, q^*) \tag{1.2.2}$$

The entrepreneur has CRRA utility

$$U(c) = \frac{c^{1-\rho}}{1-\rho}$$

For the majority of analysis, we consider the risk-neutral case with $\rho = 0$.

A project requires fixed cost K to implement. This fixed cost may be considered to include technology, business connections, and other non-pecuniary costs that only investors can provide, hence the necessity of the fundraising stage. An investor's signal of true quality is governed by a different constant variance: $q^I \sim N(q^*, \sigma_I^2)$. $\sigma_I < \sigma_E$ implies investors better assess revenue potential than entrepreneurs (and vice versa). A private venture capital firm would most likely be represented by a small σ_I, while a government agency may have a larger σ_I due to less investment experience and a lack of profit incentive.

Investors exhibit no memory or evaluative spillover for ideas. That is, if an entrepreneur rejects an offer and continues the funding search next period with the same idea, investors treat the idea as new: They draw a new q^I and do not condition their posterior for q^* on previous assessments by themselves or other VC firms. Equivalently, during each period of fundraising, the entrepreneur is pitching the idea to a new risk-neutral, competitively-behaving pool of investors that has not seen the idea or communicated with other pools, and which draws its own noisy signal governed by the constant investor variance.

We make the following simplifying assumption of *investor naivety*: Each investor always believes himself to be the first who has been pitched the idea; that is, he always believes n equals 1, where n is the number of times a specific idea has been pitched. Each investor is aware that other investors are similarly naive, but does not question the validity of his own naive assumption. Entrepreneurs are aware of investor naivety. This simplification, which does not qualitatively affect our result, is made for tractability and will be defended in the next section.

For a given project, the entrepreneur retains all information of his previous offers, and with each new offer updates his posterior on q^*. Let $\vec{s}$ be the vector of all offers the entrepreneur has received during his funding search. The entrepreneur's posterior is then $p(q^*|q^E, \vec{s})$. Once an investment offer has been declined, it may not be revisited. Denote by s_{imp} the offer finally implemented by the entrepreneur. This will be the last element of $\vec{s}$.

The value functions $V, W,$ and X of the brainstorming, funding, and implementation phases are given respectively by:

$$V(q^E) = -c_B + \beta \max\{\mathbb{E}^{q^E} V(q^E), \mathbb{E}^s W(q^E, s)\} \tag{1.2.3}$$

$$W(q^E, \vec{s}) = -c_F + \beta \max\{\mathbb{E}^{q^E} V(q^E), \mathbb{E}^{s'} W(q^E, (\vec{s}, s')), X(q^E, \vec{s})\} \tag{1.2.4}$$

$$X(q^E, \vec{s}) = \mathbb{E}^{q^*} U((1 - s_{imp})(R|q^E, \vec{s})) \tag{1.2.5}$$

In the brainstorm phase, there will be a reservation signal of quality q_1^R such that the entrepreneur will proceed to fundraising with any signal q^E that is greater than q_1^R, return to brainstorming with any signal less than q_1^R, and be indifferent when the values are equal.

The timeline of the entrepreneur's venture capital search process and feedback cycle is illustrated in extensive form in Appendix 1.A.

1.2.1. Investor Behavior

Upon receiving a signal q^I, the investor must select a share offer s to make to the entrepreneur. An individual investor thus selects a function $s(q^I)$. Since entrepreneurs face a pool of investors, there also exists a market share offer function $s_M(q^I)$. We consider the construction where s_M is invertible. Entrepreneurs, observing a share offer s, invert the function s_M in order to learn q^I. Investors are aware of the function s_M and of how entrepreneurs respond to share offers s.

An investor considering a project formulates his posterior belief $p(q^*|q^I, q^E \geq q_1^R)$. Since the investor's side of this economy is determined competitively, the equilibrium strategy for investors is determined by a zero profit condition. However, the investor's posterior upon receiving a signal is not enough to determine zero profit in expectation; the investor is in fact concerned with the posterior probability distribution on q^* conditional on implementation being reached. Recalling the simplifying assumption of investor naivety—that the investor believes his share offer to be the first one received by the entrepreneur—we turn to equation 1.2.4. Let $Q_E(s)$ be the set of signals q^E that the entrepreneur could be holding such that given a share offer s (and the corresponding q^I fully revealed by that s), he prefers implementing to fundraising or is at least indifferent:

$$Q_E(s) = \{q^E : \mathbb{E}^{s'} W(q^E, (s, s')) \leq X(q^E, s)\}$$

It is worth pointing out at this stage that the set $Q_E(s)$ is not necessarily a half-open interval of the form $[\underline{Q}(s), \infty)$. There are nontrivial dynamics relating the relative returns to fundraising and implementation, so that $Q_E(s)$ for many values of s may become a disconnected collection of closed intervals. We will return to this point in due course. The full zero profit condition for the investor is then:

$$p(q^E \in Q_E(s)|q^I, q^E \geq q_1^R)\mathbb{E}^{q^*}(\beta s R - K|q^I, q^E \geq q_1^R, q^E \in Q_E(s)) + \\ (1 - p(q^E \in Q_E(s)|q^I, q^E \geq q_1^R)) \times 0 = 0 \quad (1.2.6)$$

After simplification:

$$\beta s \int_{-\infty}^{\infty} \int_{\frac{K}{\beta}}^{\infty} R p(R|q^*) p(q^*|q^I, q^E \geq q_1^R, q^E \in Q_E(s)) dR dq^* - K = 0 \quad (1.2.7)$$

Solving this equation for $s(q^I)$ yields:

$$s(q^I) = \frac{K}{\beta \int_{-\infty}^{\infty} (q^* + \Psi(K, \sigma_R, \beta, q^*)) p(q^*|q^I, q^E \geq q_1^R, q^E \in Q_E(s_M(q^I))) dq^*} \quad (1.2.8)$$

Now we can see how a given market share function s_M induces an optimal choice of $s(q^I)$ for an individual investor. The market share function satisfies $s(q^I) = s_M(q^I)$ for all q^I in monotonic equilibrium (see Definition 1.2.2). Thus, considering equation 1.2.8 as an operator mapping s_M to s, we see that our task is to find a fixed point of this operator.

Since the posterior mean of q^* is increasing in q^I, $p(q^*|q^I, q^E \geq q_1^R)$ first order stochastically dominates $p(q^*|q^{I'}, q^E \geq q_1^R)$ for $q^I \geq q^{I'}$, yielding downward monotonicity of $s(\cdot)$ in q^I over the range of q^I such that there is a positive probability the entrepreneur will accept. The interpretation is that the more promising the representative investor finds the idea, the smaller the share he is willing to accept to break even in expectation, and vice versa.

The entrepreneur knows $s_M(q^I)$, which is one-to-one. Hence after receiving an offer s the entrepreneur can take the function's inverse to deduce q^I. The entrepreneur updates his posterior, now given by $p(q^*|q^E, \vec{q}^I)$—where q^I is the latest in a vector of fully–revealed investors' signals—before ultimately making his decision to implement, search for better funding, or return to the chalkboard for brainstorming.

Without the assumption of investor naivety, $Q_E(s)$ would become difficult to calculate. An information structure would have to be assumed: Either the investor would know n exactly, in which case n would appear as an additional parameter in $Q_E(s)$ and therefore $s()$; or he would not know n, and an expectation would have to be taken over all possible $n \geq 1$. In either case, an arbitrarily large or infinite number of integrals would have to be considered to account for all possible prior signal paths of length n.

Imposing no more than a zero-profit condition on investor behavior opens the potential for a multiplicity of equilibria, since at any time the investor has the ability either to (1) make an offer that, if accepted, earns him zero expected profit ex ante, or to (2) make any offer he is certain will be rejected by the entrepreneur. Section 1.2.4 presents two definitions of equilibrium—one in the most generalized form, and another that restricts investors to share offer functions that are monotonic and hence fully revealing. According to the latter, more useful monotonic equilibrium notion, $s(q^I)$ is shaped like a logistic function ranging on $(0, 1)$. For the remainder of the paper we primarily consider this equilibrium notion and construction of $s(q^I)$.

1.2.2. Parametrizing Value Functions by the Entrepreneur's Posterior

In the brainstorming phase, the entrepreneur's only state variable is his own signal. In the funding phase, the state variables are his own signal and a finite vector of investor signals, the length of which is given by the number of periods he has pitched the idea to investors. For convenience, since the latter may grow in size without bound, we can instead describe the state by only three variables: the mean and variance governing the entrepreneur's current posterior on q^*, and the latest offer s received.

Since the zero-profit strategy employed by investors in equilibrium is injective, a share offer by the investor fully reveals the investor's signal q^I to the the entrepreneur. Seeing a sequence of share offers is as informative to the entrepreneur as seeing a sequence of investors' signals directly. Since the prior on q^* is normal and all signals are drawn from normal distributions, the posterior for the agent after his own signal of q^E and a history of n signals $\vec{q}^I$ is given by

$$q^*|q^E, \vec{q}^I \sim N(m, \gamma^2(n))$$

$$\gamma^2(n) = \left(\frac{1}{\sigma_E^2} + \frac{n}{\sigma_I^2} + \frac{1}{\sigma_N^2}\right)^{-1}$$

$$m = \gamma^2(n)\left(\frac{q^E}{\sigma_E^2} + \frac{\sum q_t^I}{\sigma_I^2} + \frac{\mu}{\sigma_N^2}\right)$$

The entrepreneur's posterior distribution on q^* can always be characterized entirely by the pair (m, n) after any history of signals. The posterior variance $\gamma^2(n)$ does not depend on the quality of the $\vec{q}^I$ signals received; it depends only on the quantity n. The posterior mean m depends on both the quality and quantity of signals $\vec{q}^I$. During brainstorm, as soon as q^E is drawn, since no offers have been made for the idea at hand, $n = 0$. Therefore posterior variance is always constant during the brainstorm, and the value function at that stage can be parametrized by a single variable: the posterior mean of $q^*|q^E$, given by the above equations at $n = 0$ and $q_t^I = 0$. Call this term m^E. Further define $\tilde{s}(m', n, m)$ as a function that returns the latest offer that must be received to generate a new posterior mean of m' after $n+1$ offers, given a previous posterior mean of m on the nth offer.

The value functions can now be reparametrized:

$$V(m^E) = -c_B + \beta \max\{\mathbb{E}^{m^E} V(m^E), \mathbb{E}^{m'} W(m', 1, \tilde{s}(m', 0, m^E))\} \tag{1.2.9}$$

$$W(m, n, s) = -c_F + \beta \max\{\mathbb{E}^{m^E} V(m^E), \mathbb{E}^{m'} W(m', n+1, \tilde{s}(m', n, m)), X(m, n, s)\} \tag{1.2.10}$$

$$X(m, n, s) = \mathbb{E}^{q^*}[U((1-s)R)|m, n] \tag{1.2.11}$$

To compute expectations in the first two value functions, we require an expression $p(m'|m,n)$ to denote the probability that, given a current posterior mean of m after n share offers, the posterior mean after observing the next share offer will be m'. This expression and an expression for $\tilde{s}(m',n,m)$ are derived in Appendix 1.D. A reparametrization of $s(\cdot)$ in terms of posterior means is included in this same appendix, as it is useful for the numerical solution.

For a risk-neutral entrepreneur, the value to implementation given a share offer and a posterior mean is simply:

$$X(m,n,s) = \mathbb{E}^R((1-s)R|m,n) = (1-s)\mathbb{E}^{q^*}[\mathbb{E}^R(R|q^*)|m,n]$$

$$= (1-s)\left(m + \int_{-\infty}^{\infty} \Psi(K,\sigma_R,\beta,q^*)p(q^*|m,n)dq^*\right) \tag{1.2.12}$$

The agent's expected earnings in the implementation phase are increasing in K, since the distribution of returns is truncated to guarantee nonnegative discounted profits. K represents the overall magnitude of the venture; it could span from an artisan jewelry store to a pharma company. A larger K should yield large expected revenue for the agent in the implementation phase: The agent is undertaking a more ambitious endeavor. In this paper we treat K as fixed.

The value functions and associated expressions may now appear more complex, but by taking advantage of conjugate priors, the new parametrization constrains dimensionality that might have otherwise grown without bound. The new method will make a numerical approach to solving the value functions more manageable.

1.2.3. Entrepreneur Optimality

A strategy for the entrepreneur is a pair (S_1, S_2) where $S_1 : \mathbb{R} \mapsto \{brainstorm, funding\}$ is a mapping during the brainstorm phase from the entrepreneur's signal to his current action space; and $S_2 : \mathbb{R} \times \mathbb{Z} \times (0,1) \mapsto \{brainstorm, funding, implement\}$ is a mapping during the funding phase from a parametrization of his posterior and his latest offer to his new action space.

It was previously stated that the optimal S_1 can be characterized by a reservation signal q_1^R, such that the entrepreneur will attempt funding for any project with signal $q^E \geq q_1^R$

and continue brainstorming given any signal $q^E < q_1^R$. Analogously, there is a reservation posterior mean on q^*, denoted by m_1^R and corresponding to the reservation signal q_1^R, such that he will attempt funding for any project whose posterior mean satisfies $m^E \geq m_1^R$ and continue brainstorming for any $m^E < m_1^R$. To understand that during brainstorming the entrepreneur's optimal strategy will be a reservation strategy, observe the following: The value of returning to brainstorming is independent of the current posterior mean, while the value of proceeding to fundraising is increasing in the posterior mean. Therefore, if it is optimal for the entrepreneur to proceed with some m, it is optimal to proceed for all $m' > m$. We call the reservation strategy during the brainstorming phase the *first* reservation strategy, and it solves the indifference equation:

$$\int V(m^E)p(m^E)dm^E = \int W(m', 1, \tilde{s}(m', 0, m_1^R))p(m'|m_1^R, n = 0)dm' \tag{1.2.13}$$

After the entrepreneur has received an equity proposal during fundraising, one might assume his strategy S_2 could be characterized by two more reservation posterior means, $m_2^R(n)$ and $m_3^R(n, s)$, the former representing his indifference between brainstorm and funding, and the latter representing his indifference between funding and implementation. (Notationally, these reservation posterior means incorporate the latest fully revealing share offer, s.) Numerical results show us the decision to return to brainstorming can indeed be described by a reservation strategy, $m_2^R(n)$, and it is not too difficult to verify this analytically since the ex-ante value to brainstorming is constant. We call this the *second* reservation strategy, and it solves the following indifference equation for all n:

$$\int V(m^E)p(m^E)dm^E = \int W(m', n+1, \tilde{s}(m', n, m_2^R(n)))p(m'|m_2^R(n), n)dm' \tag{1.2.14}$$

In what is a key finding of this paper, the same numerical results tell us the rules governing a decision to implement or further fundraise cannot be expressed as a single reservation mean for a given (n, s). Instead they would have to be described by a correspondence that maps (n, s) into a set of posterior means. Alternatively, they could be described by a reservation share offer, as a function of the current posterior distribution that is given by (m, n). We call this function $s_3^R(m, n)$, and it is interpreted as "the share offer that, given a posterior described by (m, n) that has been conditioned on said share offer, makes the entrepreneur indifferent between implementation and further fundraising." For a given (m, n), this *third* reservation strategy function solves the following indifference equation:

$$\int W(m', n+1, \tilde{s}(m', n, m))p(m'|m, n)dm' = X(m, n, s_3^R(m, n)) \tag{1.2.15}$$

The system of indifference equations 1.2.13–1.2.15, together with 1.2.9, 1.2.10, and 1.2.11, fully describes the entrepreneur's optimal strategies and value functions. The model is solved numerically using value function iteration described in Appendix 1.B. The third reservation strategy function is later shown graphically, and economic intuition is provided.

1.2.4. Equilibrium Definitions and Discussion

Now that the optimal behavior of investors and entrepreneurs has been discussed, we define two types of equilibrium in this economy. The first is an equilibrium notion in its most generalized form, and the second a slightly more restrictive notion. Once again, let $S_1 : \mathbb{R} \mapsto \{brainstorm, funding\}$ map the entrepreneur's private signal to his action space during the brainstorming phase, and let $S_2 : \mathbb{R} \times \mathbb{Z} \times (0,1) \mapsto \{brainstorm, funding, implement\}$ map his parametrized posterior and his latest equity offer to his action space during the funding phase.

Definition 1.2.1. *Generalized equilibrium* in this market for ideas and venture capital consists of a strategy for the entrepreneur given by (S_1, S_2), a market share offer function $s_M(q^I)$, and a share offer strategy for the investor $s : \mathbb{R} \mapsto (0,1)$ such that

(1) the entrepreneur's strategy solves the reparametrized value functions 1.2.9–1.2.11.

(2) the investor's share offer strategy yields zero profit to the investor.

(3) $s(q^I) = s_M(q^I)$ for all q^I such that the investor is not certain $s_M(q^I)$ will be rejected.

(4) $s_M(q^I)$ is monotonic.

Definition 1.2.2. *Monotonic equilibrium* in this market for ideas and venture capital consists of a strategy for the entrepreneur given by (S_1, S_2), a market share offer function $s_M(q^I)$, and a share offer strategy for the investor $s : \mathbb{R} \mapsto (0,1)$ such that

(1) the entrepreneur's strategy solves the reparametrized value functions 1.2.9–1.2.11.

(2) the investor's share offer strategy yields zero profit to the investor.

(3) $s(q^I) = s_M(q^I)$ for all $q^I \in \mathbb{R}$.

(4) $s_M(q^I)$ and $s(q^I)$ are monotonic.

In monotonic equilibrium only, S_1 can be characterized by the formerly defined reservation strategy m_1^R, and S_2 by the formerly defined $m_2^R(n)$ and $s_3^R(m,n)$.

Observe that in generalized equilibrium, it is possible for the investor to make an offer he knows will be rejected, and which also misinforms the entrepreneur as to the q^I that was observed. This can never happen in monotonic equilibrium, where share offers are always correctly and fully revealing. For this reason, we are primarily interested in studying monotonic equilibrium; for the remainder of the paper it may be assumed we are in this framework.

Proving the existence of equilibria in this economy is a difficult mathematical task, and insofar as possible we abstract away from doing so. Heuristically, equilibrium in this economy is the outcome of a large fixed point problem, represented by equation 1.2.8 together with equations 1.2.9, 1.2.10, and 1.2.11. The key difficulty is showing that there exists a market share function s_M such that equation 1.2.8 is solved by $s = s_M$. Suppose such a market share function did not exist. Then no matter how the market is behaving, a competing investor pool will not converge to market behavior even as profits disappear. In our numerical exploration this is never the case; the operator that represents equation 1.2.8 always converges. We leave the mathematical question of when equilibrium might not exist in this economy to future research. The programming method used to compute a numerical solution, and the corresponding plots, are found respectively in Appendices 1.B and 1.C.

The entrepreneur wishes to proceed with ideas he believes to be better, since better ideas are more likely to yield higher rewards. However, waiting before implementation—through brainstorming, in attempt to draw a good idea, and through fundraising, in attempt to draw a favorable (and informative) share offer—is costly. A more patient entrepreneur is more willing to wait for a better idea as he brainstorms, and to wait for a more favorable share offer as he raises funds. The tradeoff between costly waiting, and the desire to implement good ideas with favorable share offers, is what governs the entrepreneur's strategies.

The entrepreneur's value to participating in the game (his expected value of brainstorming) decreases in σ_E; the more precise his information, the better his ability to determine whether an idea merits pursuing, as well as avoid costly fundraising on suboptimal ideas. If the entrepreneur's signal had infinite variance, his prior on q^* would be unaffected by q^E; his own signal would give him no new information. In this infinite variance case, the entrepreneur would not exercise a reservation strategy in the brainstorming phase, as further brainstorming would be strictly dominated, the value to brainstorming independent of his private signal. He would always proceed from brainstorming to fundraising, regardless of the signal.

The entrepreneur's reservation strategies also interact with the investor's zero-profit condition. When pitched an idea, the investor knows the entrepreneur must have seen a certain minimum signal. The investor's share offer is conditioned on the entrepreneur's having achieved that minimum signal. The entrepreneur is in turn aware of that conditioning, and such awareness impacts his strategies, and so on.

We now consider the relative role that selection plays in this economy. Since the investor cannot observe the quality of the entrepreneur, we might suspect that the pooling that occurs in the above equilibrium is indicative of adverse selection. Indeed, if entrepreneurs were only able to fundraise a single idea and could only fundraise once, then this would be a greater concern; in that model, all entrepreneurs would desperately proceed to implementation. Here, entrepreneurs have different preferences depending on their "type". Entrepreneurs with high-quality ideas want to proceed to implementation with a generous share offer, while entrepreneurs with lower-quality ideas may not want to proceed to implementation at all, unless the share offer were exceedingly generous. This induces the following curve that defines the entrepreneur's decision whether or not to implement:

$$S_R(m,n) = 1 - \left(\frac{\mathbb{E}^{m'} W(m', n+1, \tilde{s}(m', n, m))}{\mathbb{E} X(m,n)} \right)$$

This is the function that yields the nth share offer at which an entrepreneur with posterior mean m (after seeing s) is indifferent between proceeding to implementation and not. We might ask, as investors make increasingly generous share offers, which entrepreneurs are they attracting? If it is solely the left tail of the distribution, then this would be a negative result. It would indicate that increased participation in the market, in the form of issuing share offers that will attract a wider consumer base, can only be detrimental to the investor. A more economically positive result would be that it always attracts entrepreneurs at both tails of the distribution. Surprisingly, we find an intermediate result, shown in Figure 2.

There is an interval $s_1 < s < s_2$ such that increasingly generous share offers attract entrepreneurs at both tails of the distribution. The intuition is that over this interval, the higher quality entrepreneurs are pickier. Since their expected revenues are so large, the return on waiting for a better share offer outweighs the search costs c_F and depreciation rate β. These are precisely the agents that investors are seeking to attract when they make a more generous offer, and they are successful in doing so over this interval. For $s < s_1$, the classic concern dominates.

1.3. Baseline Cases

As a baseline measure, consider the following three cases: (1) the entrepreneur and the investor both observe q^* directly, (2) only the entrepreneur does, and (3) only the investor does. The first case is tantamount to solving the social planner's problem, and the remaining two cases can be used to measure the cost of uncertainty. We will compare the definition of $\mathbb{E}V(m)$ across the baseline cases and the full model, as it represents the ex ante welfare of the entrepreneur.

Case 1

Consider $\sigma_E = \sigma_I = 0$. The entrepreneur directly observes q^* in brainstorming, and knows the investor will also observe q^*. The entrepreneur correctly predicts all forthcoming share offers, and will not proceed to fundraising with an idea unless he knows a corresponding share offer will be acceptable. All ideas carried into fundraising will be implemented immediately after the first share offer; both sides know the probability of fundraising more than once is zero. This makes the value function in brainstorming after a posterior mean of $m^E = q^E = q^*$ equal to

$$V(m^E) = V(q^*) = -c_B + \beta \max\{\mathbb{E}^{q^*} V(q^*), -c_F + \beta X(q^*, s(q^*))\}$$

where

$$s(m) = s(q^*) = \frac{K}{\beta\left(q^* + \Psi(K, \sigma_R, \beta, q^*)\right)}$$

Combining these two equations with the definition of X, we have

$$V(q^*) = -c_B + \beta \max\{\mathbb{E}^{q^*} V(q^*), -c_F + \beta(1 - s(q^*))\left(q^* + \Psi(K, \sigma_R, \beta, q^*)\right)\}$$

Here the first element in the *max* operator is constant, while the second slot is increasing in q^*: There is a single reservation strategy, which we denote here by r, and use to express the expectation of the value function:

$$\mathbb{E}^{q^*}V(q^*) = -c_B + \beta[p(q^* < r)\mathbb{E}^{q^*}V(q^*)$$
$$+ p(q^* > r)\mathbb{E}^{q^*}[(-c_F + \beta(1 - s(q^*))\,(q^* + \Psi(K, \sigma_R, \beta, q^*))\,|q^* > r)]] \quad (1.3.1)$$

Rearrange to solve for the expectation of the value function, and plug in the appropriate normal CDFs:

$$\mathbb{E}^{q^*}V(q^*) = \frac{-c_B + \beta(1 - \Phi(\frac{r-\mu}{\sigma_N}))\mathbb{E}^{q^*}[(-c_F + \beta(1 - s(q^*))\,(q^* + \Psi(K, \sigma_R, \beta, q^*))\,|q^* > r)]}{1 - \beta\Phi(\frac{r-\mu}{\sigma_N})}$$
$$= -c_F + \beta(1 - s(r))\,(r + \Psi(K, \sigma_R, \beta, r)) \quad (1.3.2)$$

The second equality, required by the definition of the reservation signal r, is enough to solve for our one unknown r. With r in hand, we are able to define $\mathbb{E}V(q^*)$ via equation 1.3.1. With that in hand, $V(q^*)$ is defined piecewise above.

Case 2

Consider $\sigma_E = 0, \sigma_I > 0$. Now when only the entrepreneur observes the signal directly, there is the possibility of fundraising multiple times: Though the entrepreneur knows the true quality of his idea, if the investor receives a poor signal, the share offer may still be lower than desirable to the entrepreneur. However, it is convenient that once the entrepreneur proceeds to fundraising, he will never revert back to brainstorming, since no new information is revealed in the fundraising stage; no share offer could arrive that would make the entrepreneur want to abandon his current idea. The entrepreneur will proceed to implementation when the share offer he receives is sufficiently desirable.

Denote the reservation signal in brainstorming as r_1. The reservation strategy in fundraising can be expressed as a signal as a function of s. For every idea of accurately perceived quality q^*, the entrepreneur implements with any share offer less than a maximal s, or equivalently, whenever the investor's signal $q^I > r_2(q^*)$ since the share offer function is monotonic and fully revealing. Since from the entrepreneur's standpoint, there is no longer any uncertainty as to a project's quality interacting with the desire for a good share offer, the entrepreneur does follow reservation strategy in the fundraising phase governing his decision to implement or continue fundraising.

The value of being in brainstorming with a posterior mean of $m^E = q^E = q^*$ is given by $V(q^*)$, while the value of being in fundraising with the same mean and a share offer s is given by $W(q^*, s)$. These functions are defined by the equations

$$V(q^*) = -c_B + \beta \max\{\mathbb{E}^{q^*} V(q^*), \mathbb{E}^{q^I} W(q^*, s(q^I))\}$$
$$W(q^*, s) = -c_F + \beta \max\{\mathbb{E}^{q^I} W(q^*, s(q^I)), X(q^*, s)\}$$

Bound by the zero expected profit condition, the investor follows

$$s(q^I) = \frac{K}{\beta \int_{-\infty}^{\infty} (q^* + \Psi(K, \sigma_R, \beta, q^*))\, p(q^* | q^I, q^* \geq r_1, q^I \geq r_2(q^*)) dR dq^*}$$

The probability of the entrepreneur's acceptance is not shown in this equation since it cancels during rearrangement of the zero-profit condition. With this share offer function in mind, the expected value of returning to fundraising is

$$\begin{aligned}\mathbb{E}^{q^I} W(q^*, s(q^I)) = -c_F + \beta[&p(q^I < r_2(q^*))\mathbb{E}^{q^I} W(q^*, s(q^I)) \\ &+ p(q^I \geq r_2(q^*))\mathbb{E}^{q^I}[X(q^*, s(q^I)) | q^I \geq r_2(q^*)]]\end{aligned} \tag{1.3.3}$$

Rearranging, this yields

$$\mathbb{E}^{q^I} W(q^*, s(q^I)) = \frac{-c_F + \beta p(q^I \geq r_2(q^*))\mathbb{E}^{q^I}[X(q^*, s(q^I)) | q^I \geq r_2(q^*)]}{1 - \beta p(q^I < r_2(q^*))} \tag{1.3.4}$$

The reservation function $r_2(q^*)$ is now pinned down by the condition

$$\mathbb{E}^{q^I} W(q^*, s(q^I)) = X(q^*, s(r_2(q^*)))$$

With this reservation function in hand, $\mathbb{E}^{q^I} W(q^*, s(q^I))$ is now well-defined for all q^*. The brainstorm reservation signal r_1 is now found through the condition

$$\mathbb{E}^{q^I} W(r_1, s(q^I)) = \mathbb{E}^{q^*} V(q^*)$$

With this reservation signal in hand, $V(q^*)$ is now defined. As desired we can compute $\mathbb{E}^{q^*} V(q^*)$.

Case 3

Finally, consider $\sigma_E > 0, \sigma_I = 0$. Now only the investor observes the signal directly. The share offer $s(\cdot)$ is a direct function of $m = q^I = q^*$, and so it may be given as

$$s(q^*) = \frac{K}{\beta\left(q^* + \Psi(K, \sigma_R, \beta, q^*)\right)}$$

The entrepreneur will have a posterior mean m in the brainstorming phase, constructed after viewing his own noisy signal q^E. In the fundraising phase, however, the entrepreneur will accurately deduce the true q^* from a single share offer. The investor knows the entrepreneur will make this deduction; and so, being aware of the entrepreneur's strategy, the investor knows before making his share offer whether the offer will be accepted or rejected. It is meaningless to include any acceptance/rejection probabilities in the share offer function. It is now easier to comprehend the concept of the multiplicity of equilibria. When the investor knows an offer he makes will be rejected, he can also make any other offer he knows will similarly be rejected still earn zero profit. Above a certain threshold share offering, the investor can choose any number of odd mappings—not necessarily monotonic—and mixing strategies, simply because he knows any s above the threshold will be rejected anyway.

The value functions in brainstorming and fundraising are

$$V(m^E) = -c_B + \beta \max\{\mathbb{E}^{m^E} V(m^E), \mathbb{E}^{q^*}[W(q^*, s(q^*))|m^E]\}$$
$$W(q^*, s) = -c_F + \beta \max\{\mathbb{E}^{m^E} V(m^E), X(q^*, s)\}$$

As in Case 2, we have a reservation mean in the brainstorming phase which we will denote by r_1. There is also a reservation share offer in the fundraising phase which we will denote by $r_2(q^*)$. [1] The function $r_2(q^*)$ satisfies

$$\mathbb{E}^{m^E} V(m^E) = X(q^*, r_2(q^*))$$

Interestingly, the value of implementing at the reservation share offer is now independent of the quality of the idea. This is the case in any version of this model with a terminal condition (maximum number of fundraising periods), since the reservation share offer makes the entrepreneur indifferent between implementation and brainstorming anew, and the expected value of brainstorming is independent of quality.

Meanwhile, in brainstorming, the reservation mean r_1 is satisfies

$$\mathbb{E}^{m^E} V(m^E) = \mathbb{E}^{q^*}[W(q^*, s(q^*))|r_1]$$

We can write the expected value of returning to brainstorming as

$$\begin{aligned}\mathbb{E}^{m^E} V(m^E) = -c_B &+ \beta p(m^E < r_1)\mathbb{E}^{m^E} V(m^E) \\ &+ \beta p(m^E \geq r_1)\mathbb{E}^{m^E}[\mathbb{E}^{q^*}[W(q^*, s(q^*))|m^E]|m^E > r_1] \end{aligned} \quad (1.3.5)$$

While the expected value of continuing to fundraising from brainstorming is

$$\begin{aligned}\mathbb{E}^{q^*}[W(q^*, s(q^*))|m^E] = -c_F &+ \beta p(s(q^*) > r_2(q^*)|m^E)\mathbb{E}^{m^E} V(m^E) \\ &+ \beta p(s(q^*) \leq r_2(q^*)|m^E)\mathbb{E}^{q^*}[X(q^*, s(q^*))|s(q^*) \leq r_2(q^*), m^E] \end{aligned} \quad (1.3.6)$$

Combining equations 1.3.5 and 1.3.6, we arrive at

[1] In Case 2, $r_2(q^*)$ was a reservation investor signal; here we parametrize the object as a share offer but use the same notation. Though the object's type may differ, r is used repeatedly to denote "reservation strategy" to avoid a proliferation of notation.

$$\mathbb{E}^{m^E}V(m^E) = \frac{1}{1-\beta p(m^E < r_1) - \beta p(m^E \geq r_1)\mathbb{E}^{q^*}[p(s(q^*) > r_2(q^*))]} \times$$
$$(-c_B - \beta p(m^E \geq r_1)c_F + \beta \mathbb{E}^{m^E}[p(s(q^*) \leq r_2(q^*)|m^E)\times$$
$$\mathbb{E}^{q^*}[X(q^*, s(q^*))|s(q^*) < r_2(q^*), m^E]|m^E \geq r_1] \quad (1.3.7)$$

This pins down $\mathbb{E}^{m^E}V(m^E)$ in terms of r_1 and $r_2(q^*)$. Plugging this definition into 1.3.6 then yields $\mathbb{E}^{q^*}[W(q^*, s(q^*))|m^E]$ in terms of reservation strategies as well. The equations

$$\mathbb{E}^{m^E}V(m^E) = X(q^*, r_2(q^*))$$
$$\mathbb{E}^{m^E}V(m^E) = \mathbb{E}^{q^*}[W(q^*, s(q^*))|r_1]$$

then define r_1 and $r_2(q^*)$ precisely. As in the preceding cases, these equations can be solved numerically. With reservation strategies pinned down, we may evaluate $\mathbb{E}^{m^E}V(m^E)$ and compare with cases 1 and 2, as desired.

1.4. Overconfidence

We extend the model to explore the welfare effects of overconfidence on an economy. Given the setup of the model, this may be accomplished by adding a bias term of ϕ to the entrepreneur's signal distribution, so now $q_\phi^E \sim N(q^* + \phi, \sigma_E^2)$. The entrepreneur believes $\phi = 0$, but in reality $\phi > 0$.

Consider the entrepreneur after a period of brainstorming. He has a prior mean of μ, and has drawn a signal q_ϕ^E. Comparing the accurate posterior mean on q^*, denoted as before by m^E, with the overconfident posterior mean which we denote by m_ϕ^E, we have

$$m_\phi^E = \gamma^2(0)\left(\frac{\mu}{\sigma_N^2} + \frac{q_\phi^E}{\sigma_E^2}\right) = \gamma^2(0)\left(\frac{\mu}{\sigma_N^2} + \frac{q_\phi^E + \phi - \phi}{\sigma_E^2}\right) = m^E + \gamma^2(0)\frac{\phi}{\sigma_E^2} = m^E + \phi\frac{\sigma_N^2}{\sigma_N^2 + \sigma_E^2}$$

The posterior mean shifts up linearly in ϕ. The brainstorming reservation mean (marked by indifference between fundraising and continued brainstorming, denoted by m_1^R and corresponding to signal q_1^R) is now reached more easily, though possibly artificially. The true posterior mean for which the entrepreneur is willing to leave brainstorming has shifted linearly downward, meaning the overconfident entrepreneur will seek funding for some ideas of lower quality than he would otherwise require.

In the fundraising phase, two information structures can be considered: The investor either knows or does not know the entrepreneur is overconfident. In the latter case, the $s(\cdot)$ function remains unchanged. We consider the former, in which $s(\cdot)$ becomes

$$s_\phi(q^I) = \frac{K}{\beta \int_{-\infty}^{\infty} \int_{\frac{K}{\beta}}^{\infty} Rp(R|q^*)p(q^*|q^I, q^E + \phi \geq q_1^R, q^E + \phi \in Q_E(s_M(q^I)))dRdq^*}$$

Now $\phi > 0$ implies $s_\phi(q^I) > s(q^I)$ for all q^I. The investor, realizing entrepreneurs are pursuing lower quality ideas on average, demands higher compensation to meet the zero-profit condition. But the entrepreneur, believing $\phi = 0$, incorrectly deduces the signal to have been $q_\phi^I = s^{-1}(s) < s_\phi^{-1}(s) = q^I$; his perception of the investor's signal is artificially low. After seeking fundraising for n periods, the entrepreneur's posterior mean is given by

$$m_\phi(n) = \gamma^2(n)\left(\frac{\mu}{\sigma_N^2} + \frac{q_\phi^E}{\sigma_E^2} + \frac{\sum q_\phi^I}{\sigma_I^2}\right) = m(n) + \gamma^2(n)\left(\frac{\phi}{\sigma_E^2} + \frac{\sum(q_\phi^I - q^I)}{\sigma_I^2}\right)$$

Since the summation term is negative, the two terms in the large parentheses work in conflict, and are weighted by the entrepreneur by the relative signal precisions. This expression demonstrates that repeated interactions with VC firms mitigate potential problems of overconfidence.

Studying such a problem only explores the effect of an individual's overconfidence on himself or herself. Another compelling question is the effect of overconfident individuals on an entire entrepreneurial economy. We explore this issue by considering the existence of a continuum of entrepreneurs with some distribution of overconfidence levels, where an entrepreneur's type is unobservable to the investor. A fraction λ of the population are unbiased, and a fraction $1 - \lambda$ have $\phi > 0$. Now the investor's share offer function is

$$s_\phi(q^I) = \frac{K}{D}$$

where

$$D = \beta \int \int_{\frac{K}{\beta}}^{\infty} Rp(R|q^*)(\lambda p(q^*|q^I, q^E \geq q_1^R, q^E \in Q_E(s_M(q^I)))$$
$$+ (1-\lambda)p(q^*|q^I, q^E + \phi \geq q_1^R, q^E + \phi \in Q_E(s_M(q^I))))dRdq^* \quad (1.4.1)$$

Once again, $\phi > 0$ and $\lambda \in [0, 1)$ implies $s_\phi(q^I) > s(q^I)$ for all q^I: The investor, realizing some entrepreneurs are pursuing lower quality ideas but unable to observe who, demands higher compensation.

One of two information structures must be assumed: Unbiased entrepreneurs either know $s_\phi(q^I)$ or still falsely believe $s(q^I)$. This choice in information structure has implications for Q_E; given a share offer, the set of private signals for which the entrepreneur implements would be different between the two. In either information structure, both types of entrepreneur are negatively impacted by the overconfident fraction: Overconfident entrepreneurs implement lower-quality ideas than they would with correct beliefs, and unbiased entrepreneurs face lower share offers from the investor who cannot distinguish types. Additionally, discouraged unbiased entrepreneurs who are unaware of the overconfident fraction might receive lower signals than what they might otherwise accept with knowledge of the overconfident population—hence a form of adverse selection caused by the existence of an overconfident contingent.

Furthermore, as the new share offer function reflects, if in each period an investor is to receive a single pitch uniformly selected from the population of entrepreneurs proceeding to fundraising (and recalling the investor believes $n = 1$), an even greater fraction than $1 - \lambda$ of the pitches the investor receives will be from overconfident entrepreneurs—more than the population average, since overconfident entrepreneurs are more likely to draw private signals that exceed their reservation strategies. We observe adverse selection by entrepreneurs during both brainstorming and fundraising stages.

1.5. Numerical Results

The model's dynamic feedback characteristic induces a system of Volterra integral equations that is numerically tractable through value function iteration. A numerical solution to the case in which both signal variances are positive is found in MATLAB. The algorithm is described in Appendix 1.B; this section analyzes the results.

The plots in Figure 1 show (1) the value to being in the brainstorming phase, across posterior means m, (2) the value to being in the fundraising phase, across posterior means m for a preselected (n, s), (3) the evolution of the first reservation strategy across iterations, and (4) the expected value of going into fundraising from the brainstorm stage, across posterior means m. In the first plot, the kink occurs at the first reservation strategy. To the left of the kink, the value function is linear, since for all m below a minimum threshold, the entrepreneur finds it optimal to reject the current idea and brainstorm a new one. To the right of the kink, the value function is strictly increasing, since higher posterior means suggest a greater share of higher returns. The second and fourth plots exhibit a similar shape. The kinks in the third plot suggest a non-monotonic convergence pattern for the first reservation strategy.

Figure 2 plots the reservation share offer as a non-monotonic function of the posterior mean. This plot is relevant during the fundraising stage, and what it demonstrates is that there exists an s^* such that: For all $s < s^*$, the reservation mean follows a simple cutoff rule. For all $s < s^*$, the entrepreneur chooses to implement on a closed interval of reservation means, the intuition being that for high investor signals, the entrepreneur would rather search again to secure a more favorable share offer.

Figure 3 illustrates the effect of overconfidence on an entrepreneur's beliefs, and how overconfidence can lead to adverse selection. $s(q^I)$ is the share offer function in an economy without overconfidence. $s_\phi(q^I)$ is the share offer function in an economy exhibiting the simplest form of entrepreneurial overconfidence: All entrepreneurs are overconfident, and investors are aware of this fact. Observe that for all q^I, $s_\phi(q^I) > s(q^I)$. This is because the entrepreneur is more likely to observe a signal satisfying his first reservation strategy, so the investor's prior is less favorable; as a result, the investor must demand a greater share to break even. The overconfident entrepreneur is not aware of $s_\phi(q^I)$, and believes $s(q^I)$ to be the investor's share offer function. Using $s(q^I)$, the entrepreneur deduces a false investor signal, q^I_ϕ, which is necessarily lower than the true q^I. Misinformation in the brainstorming and fundraising stages induces adverse entrepreneurial search decisions.

Figure 4 compares the shapes of the $m(q^I)$ function for two entrepreneurial signal variances, $\sigma_E = .5$ and 2. Recall that $m(q^I)$, derived in Appendix 1.D, is the function that returns the investor's posterior mean on true idea quality given a signal. Why should this function depend on the entrepreneur's signal variance? The answer lies in the fact that the investor conditions his posterior on the knowledge that the entrepreneur's signal exceeded a minimum threshold during brainstorming. This information is more meaningful when the entrepreneur's signal is more precise.

A real-world example can be considered as follows. Suppose an investor gets a signal of poor quality on a venture, but sees that the team pitching the idea has a track record of successful startups, and is therefore qualified to make a sound evaluation. The investor would take this experienced team's enthusiasm into consideration more so than than he would an inexperienced team's. On the right side of the $m(q^I)$ plots, where q^I is relatively high, the two functions are close in value, because the investor and entrepreneur "agree" that the venture is worthwhile. On the left side, the two plots diverge with greater "disagreement". In the $\sigma_E = .5$ case, the entrepreneur is more precise, and so the investor is more willing to consider the entrepreneur's enthusiasm than in the $\sigma_E = 2$ case, even when the investor himself is pessimistic in both cases.

1.6. Conclusion

This model was originally motivated by a stark stylized fact of the venture capital industry: entrepreneurs are overconfident. Since venture capital firms as well as entrepreneurs hold private information and communicate with each other in repeated, structured environments, we were a priori surprised by this fact, as it indicates that there are frictions in the venture capital industry, and that the mechanisms of transmitting information between each side of the economy are imperfect.

We analyze the information transfer problem from the perspective of a search model, in which entrepreneurs seek both good ideas and favorable equity contracts, and firms seek entrepreneurs with good ideas. Because the model was initially motivated by studying the flow of information to entrepreneurs, we restrict ourselves to the case of actuarially fair share offers by venture capital firms, and are still able to generate nontrivial and wholly unexpected dynamics. Though the model was constructed with the idea of search frictions in mind, one of the primary results finds advantageous selection to be a significant factor in equilibrium. A classical concern in venture capital economies is that since firms cannot perfectly distinguish entrepreneurs with good ideas from those with bad ideas, equilibrium induced by actuarially fair prices will be characterized by a proliferation of the latter. We find the opposite holds. As investors make increasingly generous equity contracts, entrepreneurs are added to both tails of the distribution of those willing to proceed to implementation. We argue this is because entrepreneur preferences align with the traditional investor's incentives; both prefer implementing high-quality projects. We recognize this result may break down if intrinsic, static types were to be added to the entrepreneurial pool—if some entrepreneurs systematically generated worse ideas than others, different results may be obtained. However, in the baseline model, since incentives are aligned and all available information is transmitted actionably and perfectly to the entrepreneur, we argue that a socially optimal outcome is achieved.

Pursuing the initial stylized fact of the economy, we then introduce overconfidence. This is both in the form of analyzing the effect of overconfidence on a single individual, and the effect of persistent overconfidence in a fixed proportion of the economy. In the case where all entrepreneurs are overconfident, we show that the investor pool can help mitigate the effect of such bias through repeated communication. However, in the case where only a fixed proportion of the investor pool is overconfident, this ability dwindles. We thus provide a stylized explanation for how overconfidence can persist in an economy despite regular communication with rational firms holding accurate beliefs, the key mechanism being that while an individual may be aware of the existence of overconfidence, it is difficult to construct an information transmission mechanism that will allow a firm to convince a particular individual of his own overconfidence. This is a problem: Though the incentives between entrepreneur and investor are still aligned, the beliefs over what is considered optimal startup behavior become misaligned. We demonstrate that this misalignment has negative welfare effects.

Extensions

The model can be used to study other aspects of venture capital economies. To encourage further research within this environment, we suggest a few potentially promising veins.

Relaxing the assumption of investor naivety will enable researchers to learn how histories of declined equity contracts can affect future share offers. To keep the solution tractable, researchers may wish to approach the problem by structurally imposing a maximum length of fundraising time, x; that is, after x periods of fundraising an entrepreneur must choose between brainstorming and implementing. Instead of believing $n = 1$ in all cases, investors may either view n perfectly or they take an expectation of n over $\{1, ...x\}$.

Individual profit-seeking investors can be considered instead of investor "pools" bidding for the right to enter a contract. A venture capitalist wishing to maximize profits faces an interesting tradeoff. By demanding a greater share of realized returns, he increases his utility from each contract implemented—but at the same time, he raises the probability that the entrepreneur will decline his offer and approach a different investor next period. In this environment competition still exists within the venture capital market; now the competition is intertemporal, and the investor's bargaining power is limited by the entrepreneur's ability to continue seeking funding elsewhere. The dynamics of informational asymmetry and transfer become more complex in such a model, and the investor's beliefs or knowledge about n would continue to affect the optimal share offer function. An environment with individual profit-seeking investors will likely give rise to new and informative equilibrium concepts.

The baseline environment lends itself well to the study of learning by doing, and how the capacity for an entrepreneur to improve his ideas over time, or his discerning ability, can affect how investors propose equity contracts and how entrepreneurs search. Learning by doing would affect the way investors interact with individuals who have advanced learning, or with everyone, depending on how well investors can distinguish types. Researchers will have to decide how the learning factor will emerge: deterministically or probabilistically, after each period of brainstorming or only after downward movement to brainstorming from fundraising, and as an increase in μ or as a reduction in σ_E.

Another direction we propose would examine the "caste system" of VC firms—that is, the heterogeneity of firms with respect to reputation, profitability, deal flow, and consequently, behavior. Researchers might consider an environment that would permit strategic behavior by VCs who are heterogeneous in reputation, which would affect the likelihood of receiving scarce proposals by entrepreneurs. Construction would be consistent with the data in Hsu (2004), which finds high-reputation firms to be three times more likely to have their offers accepted, and at a (10-14%) discount. Due to the increased deal flow, such firms can be more patient with the ideas they fund. Having a top tier reputation allows breathing room if the VC fund underperforms, but prolonged underperformance threatens a demotion to a lower tier. This type of paper could preserve the environment of search and informational asymmetry, and shift the strategic focus to the VC side, examining the effect of reputation on behavior.

Finally, we propose an analysis of the economy using Monte Carlo simulation, which will enable precise estimation of several interesting objects, including: a distribution of quality of implemented projects; a distribution of search time, in total and broken down by stages; and the average difference between the "overconfident" and unbiased posterior means as a function of the number of share offers. Such an analysis should study how these objects change in σ_E and σ_I.

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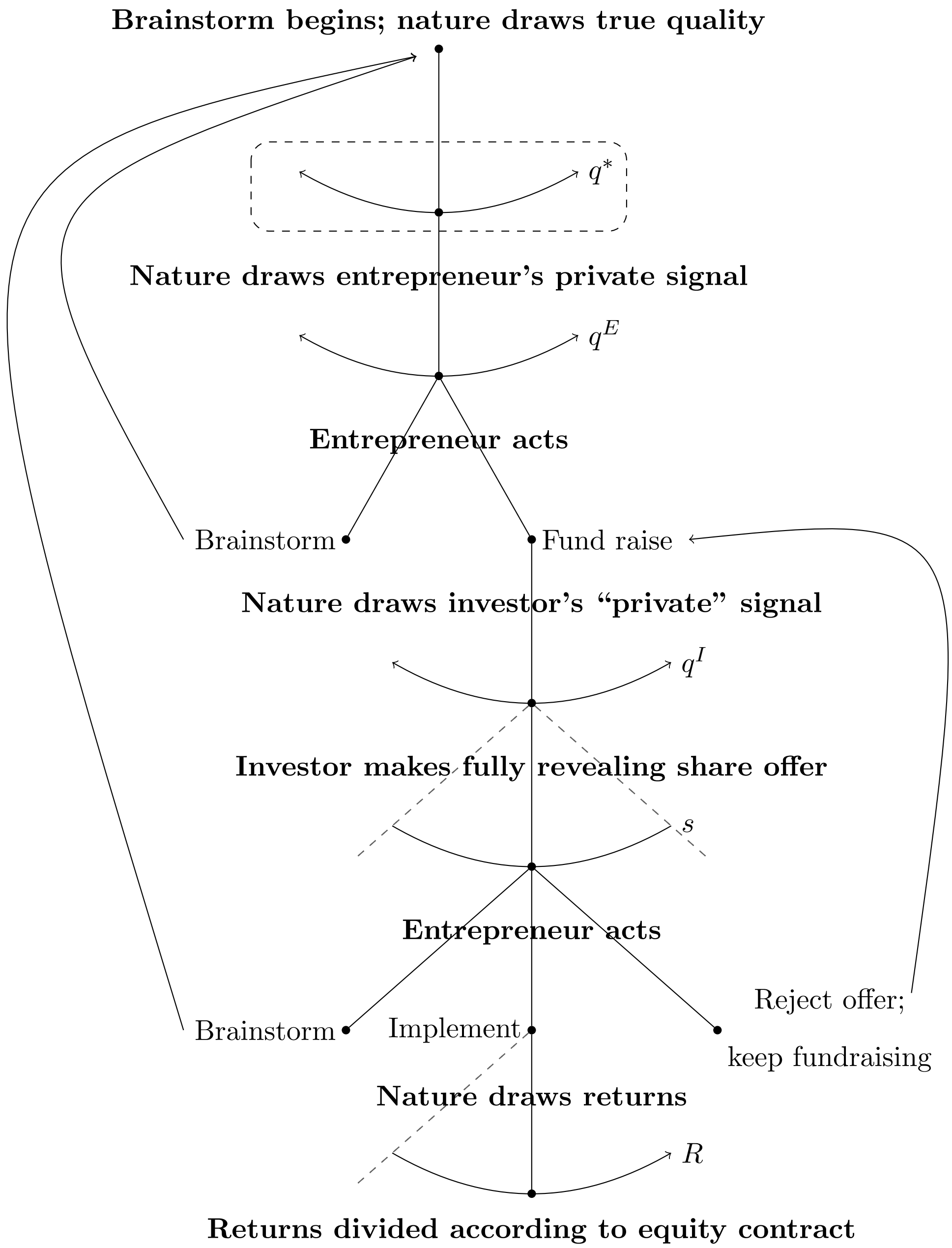
Appendix 1.A.

Timeline and Feedback Cycle



Appendix 1.B.

Programming Method for Numerical Solution

A numerical solution is found using value function iteration in MATLAB. We begin by choosing baseline parameters as follows:

Parameter	**Value**
β	0.95
μ	2
σ_N	5
σ_E	.5
σ_I	1
σ_R	1
c_B	.1
c_F	.1
K	1

In various experiments, these parameters are changed. We now construct discrete grids for the posterior mean and share offer. Though the posterior mean can theoretically take any value on the real line, we constrain the grid of *mlength* points to within five standard deviations σ_N around μ. The share offer grid remains on $(0, 1)$. Each venture requires a maximum number of fundraising periods; this is chosen to be $nlength = 5$.

A number $expV$ representing the entrepreneur's expected value when entering the brainstorming phase is initialized with a best guess of zeros. A $[mlength, nlength]$ matrix called $expW$ is also initialized with zeros; this contains the expected value when entering the fundraising phase given any combination of posterior mean m and signals drawn n. We similarly make an initial guess for the first reservation strategy. A vector v, representing the current value at brainstorming for any posterior mean, is initialized, and populated using a reasonable piecewise function with linear segments. A $[mlength, nlength, slength]$ array called w is initialized and populated according to a similar initial guess; w contains the values at fundraising given any posterior mean m, signals drawn n, and share offer s. Another vector is initialized to track the brainstorming reservation strategy as it converges.

Functions are coded for $p(m'|m, n)$, $s(m)$, $\tilde{s}(m', n, m)$, $q^I(m)$, $m(q^I)$, and $X(m, n, s)$. Since some of these functions are heavy on integrals and slow to compute, in some cases, values are presolved and stored in an array; interpolated approximations are also used.

The value function iteration algorithm works as follows. For brevity in, numbers in parentheses following a vector, matrix, or array correspond to values instead of indices. On each iteration, the number $expV$ is updated by taking the expectation across values in vector v using the appropriate distribution. Each entry in $expW$ is updated in a similar fashion. $res1$ is updated to be the value m for which $abs(expV - expW(m, 1))$ is minimized. Finally, v and w are updated by plugging in the latest $expV$ and $expW$ into

$$v(m) = -c_B + \beta max\{expV, expW(m, 1)\}, \forall m$$

$$w(m, n, s) = -c_F + \beta max\{expV, expW(m, n + 1), X(m, n, s)\}, \forall m, n, s$$

Iteration continues to until the value functions converge. Resulting graphs are shown in Figure 1.

Appendix 1.C.

Plots

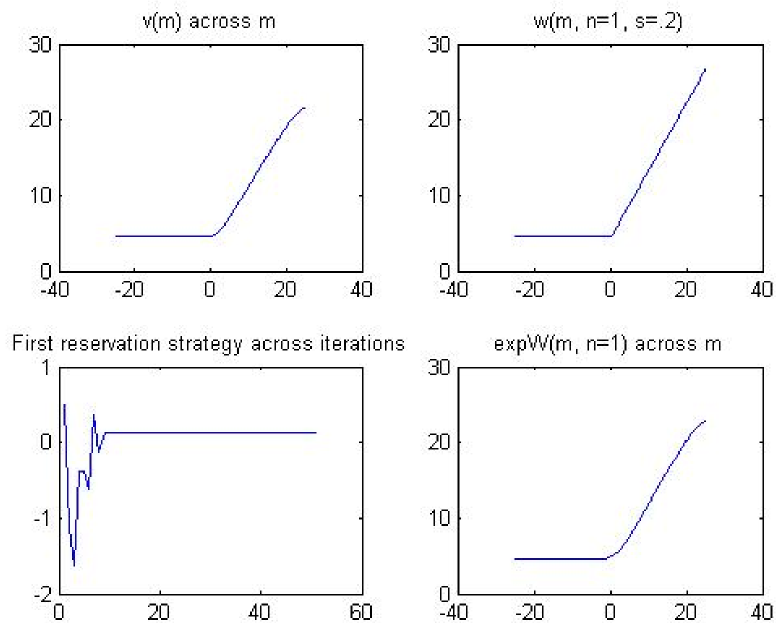


Figure 1: Value function plots and reservation strategy evolution.

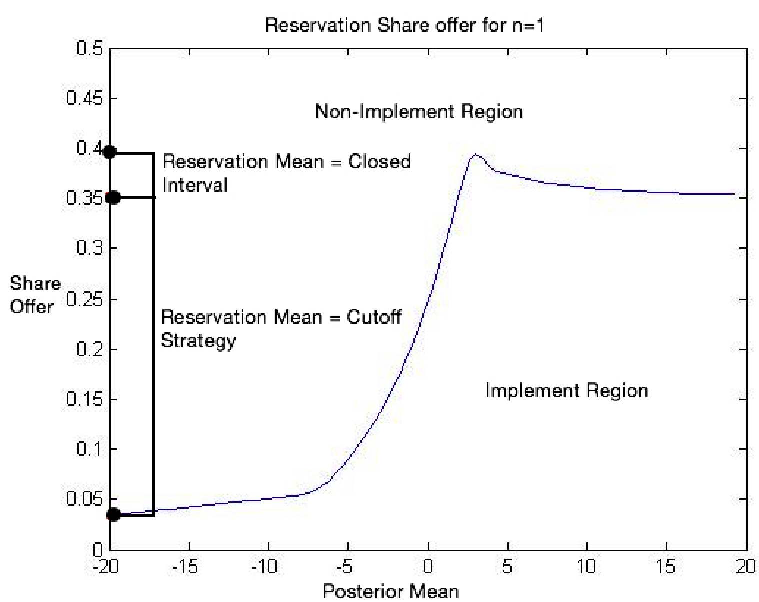


Figure 2: Reservation share offer as a non-monotonic function of posterior mean.

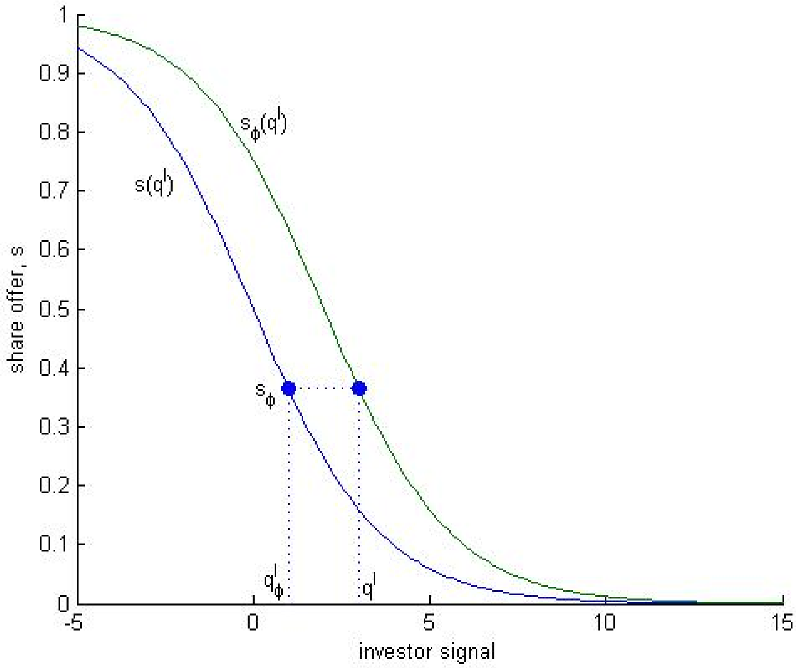


Figure 3: Effects of overconfidence: $s_\phi(\cdot) > s(\cdot)$ and $q^I_\phi < q^I$

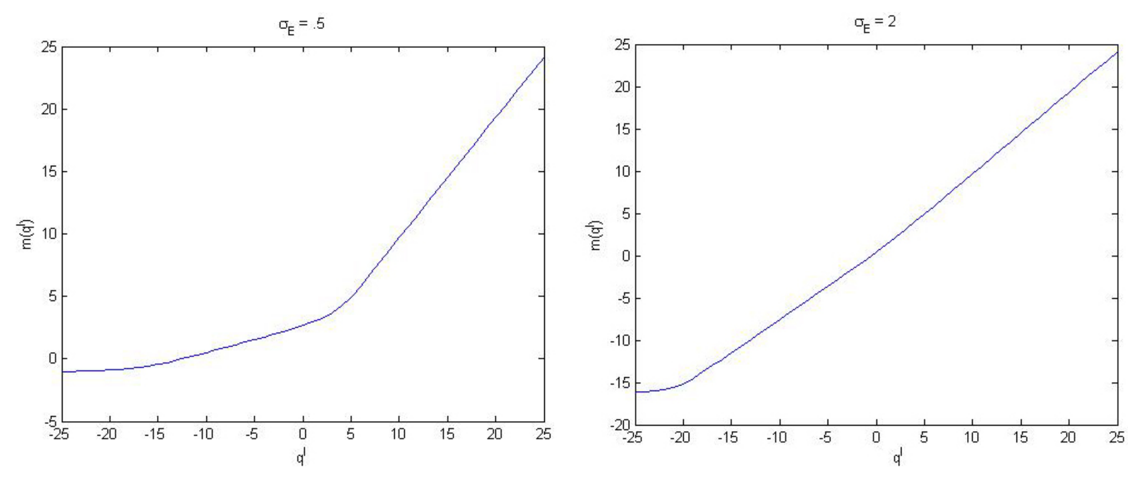


Figure 4: A comparison of the $m(q^I)$ function for $\sigma_E = .5$ and $\sigma_E = 2$.

Appendix 1.D.

Derivations of $p(m'|m,n)$, $\tilde{s}(m',n,m)$, and reparametrized $s(\cdot)$

A mathematical exercise which occurs frequently in this analysis is the following. Suppose $\theta \sim N(\mu, \sigma_N^2)$. Further suppose that $x|\theta \sim N(\theta, \sigma_I^2)$, and that θ is unobserved. What then is the total probability distribution $p(x)$? Consider the random variable $Z = N(0, \sigma_I^2)$. Then we have

$$x = \theta + Z$$

Thus, by standard formulas for the sum of normal distributions, we have that the unconditional distribution of x is given by $x \sim N(\mu, \sigma_N^2 + \sigma_I^2)$.

We now employ this result to find $p(m'|m,n)$. The expression for the current posterior mean m is

$$m = \gamma^2(n)\left(\frac{q^E}{\sigma_E^2} + \frac{\sum_{t=1}^n q_t^I}{\sigma_I^2} + \frac{\mu}{\sigma_N^2}\right)$$

While next period's posterior mean m' is given by

$$m' = \gamma^2(n+1)\left(\frac{q^E}{\sigma_E^2} + \frac{\sum_{t=1}^n q_t^I}{\sigma_I^2} + \frac{q_{n+1}^{I'}}{\sigma_I^2} + \frac{\mu}{\sigma_N^2}\right)$$

Since m' follows from m, combining these two equations yields

$$m' = \gamma^2(n+1)\left(\frac{q^{I'}}{\sigma_I^2} + \frac{m}{\gamma^2(n)}\right) = \frac{\gamma^2(n+1)}{\sigma_I^2}q^{I'} + \frac{\gamma^2(n+1)}{\gamma^2(n)}m \qquad (1.D.1)$$

We now apply the above fact to determine the entrepreneur's beliefs regarding the distribution of $q^{I'}$:

$$q^*|m,n \sim N(m, \gamma^2(n))$$

$$q^{I'}|m, n, q^* \sim N(q^*, \sigma_I^2)$$
$$\Rightarrow q^{I'}|m, n \sim N(m, \gamma^2(n) + \sigma_I^2)$$

It follows that

$$m'|m, n \sim N\left(m, \frac{\gamma^2(n)\gamma^2(n+1)}{\sigma_I^2}\right)$$

The function $\tilde{s}(m', n, m)$ conveniently follows from equation 1.D.1 above, using the s function from 1.2.8:

$$\tilde{s}(m', n, m) = s\left(\frac{m'\sigma_I^2}{\gamma^2(n+1)} - \frac{m\sigma_I^2}{\gamma^2(n)}\right) \tag{1.D.2}$$

We will now parametrize $s(\cdot)$ as function not of the investor's signal, but as a function of the investor's beliefs about q^*. This is useful for the numerical solution. Begin by defining $m(q^I)$, a function which delivers the investor's posterior mean m^I given a signal q^I, conditional on the knowledge that q^E exceeded the entrepreneur's first reservation signal:

$$m^I = m(q^I) = \int q^* p(q^*|q^I, q^E \geq q_1^R) dq^* = \frac{\int \int_{q_1^R} q^* p(q^I|q^*) p(q^E|q^*) p(q^*) dq^E dq^*}{\int \int_{q_1^R} p(q^I|q^*) p(q^E|q^*) p(q^*) dq^E dq^*} \tag{1.D.3}$$

Evaluating the integral in the denominator of 1.2.8 yields:

$$\int (q^* + \Psi(K, \sigma_N, \beta, q^*)) p(q^*|q^I, q^E \geq q_1^R) dq^*$$
$$= m(q^I) + \frac{\int \int_{q_1^R} \Psi(K, \sigma_N, \beta, q^*) p(q^I|q^*) p(q^E|q^*) p(q^*) dq^E dq^*}{\int \int_{q_1^R} p(q^I|q^*) p(q^E|q^*) p(q^*) dq^E dq^*}$$

Since we prefer $s(\cdot)$ to be a function of posterior means, not signals, we change variables and normalize, defining $q(m^I)$ to be the functional inverse of $m(q^I)$:

$$s(m^I) = \frac{K}{\beta\left(m^I + \frac{\int\int_{q_1^R} \Psi(K, \sigma_N, \beta, q^*) p(q(m^I)|q^*) p(q^E|q^*) p(q^*) dq^E dq^*}{\int\int_{q_1^R} p(q(m^I)|q^*) p(q^E|q^*) p(q^*) dq^E dq^*}\right)} \tag{1.D.4}$$

Appendix 1.E.

Notation Guide

Variable	**Meaning**
q^*	true idea quality
q^E	entrepreneur's signal
q^I	investor's signal
μ	prior mean of q^*
σ_N^2	prior variance of q^*
σ_E^2	entrepreneur's signal variance
σ_I^2	investor's signal variance
σ_R^2	variance of returns
β	discount factor
K	non-pecuniary capital requirement
R	return
c_B	flow cost of brainstorming
c_F	flow cost of fundraising
ρ	risk aversion
m^E	posterior mean on q^* at brainstorming
m	posterior mean on q^*
n	number of investor signals received
$V(\cdot)$	value to entrepreneur in brainstorming
$W(\cdot)$	value to entrepreneur in fundraising
$X(\cdot)$	value to entrepreneur at implementation
$s(\cdot)$	investor's share offer function
$\tilde{s}(m', n, m)$	share offer required to move from (m, n) to m'
$\Psi()$	mean bias function due to normal truncation

$p(m'\|m,n)$	probability of going from (m,n) to m'
$\gamma^2(n)$	posterior variance on q^* after n investor signals
q_1^R	reservation signal in brainstorming
m_1^R	reservation mean in brainstorming
$m_2^R(n)$	first reservation mean in fundraising
$m_3^R(n,s)$	second reservation mean in fundraising
$s_3^R(n,m)$	second reservation share offer in fundraising
ϕ	overconfidence bias
q_ϕ^E	overconfidence-biased signal in brainstorming
q_ϕ^I	entrepreneur's bias-tainted perception of investor's signal after offer s_ϕ
m_ϕ^E	overconfidence-biased posterior mean on q^* in brainstorming
λ	fraction of unbiased entrepreneurs
$s_\phi(\cdot)$	investor's share offer function, corrected for overconfidence

CHAPTER 2 : A Theory of Gender Income Differentials, Occupation Choice, and Childcare Regulation

Abstract. Though the gender earnings differential attributed to workplace discrimination and social pressures has diminished over time, the gap due to unexplained factors has remained constant. This paper attempts to shed light on the gender earnings differential using a model of occupational choice, maternity choice, and the ability to stochastically "wage climb." The model is solved analytically and numerically, yielding a result for the man and woman's normalized income that is consistent with the empirical gender wage gap. The result is driven by the fact that a woman's optimal occupational choice is a shorter-term investment made to accommodate the potential for time out of the labor force following maternity. The model is then extended to include a child day care services component. Results are consistent with empirical findings that regulation, and hence prices, in childcare services reduce demand and discourage female labor participation. A relationship is established between childcare services regulation and the gender wage gap.

2.1. Introduction

2.1.1. The Gender Wage Gap

It is a well-known fact that in almost every country, the earnings of men exceed those of women. Gender income differentials have been attributed to a variety of causes, including differences in industry choice, choices in the types of positions held within an industry, discrimination due to sexism and/or variations in labor mobility as explained by monopsony theory, work experience, marginal productivities, and cultural pressures and other sociological factors. Such income gaps have also been identified using a variety of measures. The gap is generally given by a statistic reflecting female earnings as a percentage of male earnings; "earnings" might be hourly wages, weekly earnings, or annual salaries, and may be reflected by a median or an average. Comparisons differ in their usefulness, as certain facts may bias estimates. For instance, men work more hours on average, and tend to select into fields with average higher pay. 2008 OECD estimates of median earnings of full-time employees reflect an average differential of 18% for OECD countries. For the U.S., the differential was 19%.

The seminal book by Becker (1957) on prejudice and the economics of discrimination sparked an eventual proliferation of microdata-driven research into the study of gender wage differentials. For instance, the application of the decomposition technique as developed by Blinder (1973) and Oaxaca (1973) to microdata enabled the comparison of wages across

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genders while controlling for productivity levels.

Surveys of the ample literature on the gender wage differential—which span a wide range of countries, motivations, and classes of data—have struggled to condense results in a way that facilitates meaningful meta-analysis. Successful meta-analysis has been conducted by Stanley and Jarrell (1998), who identify causes and sources of variation in the U.S. wage gap, and Jarrell and Stanley (2004), who expand their former dataset. Weichselbaumer and Winter-Ebmer (2005) extend the analysis of these papers to a worldwide level, studying more than 260 published adjusted pay gap studies for over sixty countries. Their quantitative review of the vast amount of empirical literature on gender earnings differentials concerns differences in methodology, data, time periods, and countries. The study finds that over time, raw wage differentials have fallen globally, mostly due to a better labor market endowment of females, but the "unexplained component" of the gap remains unchanged. The constancy of the unexplained component is further substantiated by the U.S. Congress Joint Economic Committee, which finds that as explained inequities decrease, the unexplained pay gap remains unchanged (see Figure 5). Furthermore, Blau and Kahn (2007) find that despite evidence suggesting a decline in discrimination against working women, the convergence between wages of men and women has declined in recent years, and "the possibility arises that the narrowing of the gender pay gap will not continue into the future."

The current paper proposes a model that attempts to shed light on the "unexplained pay gap" that persists despite observable decreases in workplace discrimination, cultural pressures, and other factors to which gender differentials have been attributed. The mechanism hinges on the assumption that the mere possibility of a future maternity decision affects the optimal vocational choice for women. In the model, men and women begin working life by choosing an occupation. As time progresses, workers of either gender attempt to *climb* the job by seeking higher paying positions, which arrive stochastically as a function of the requested wage increase. Occupations are heterogeneous in their growth potential, a parameter that governs the probability of earning a promotion conditional on a worker's current wage. Example "high ladder" industries might be investment banking or corporate law, which promise ample opportunity to climb in rank and pay grade over time, contrasted with "low ladder" occupations like librarianship or restaurant services, which do not. The theory finds that, faced with a potential maternity choice, female workers find it optimal to choose jobs associated with a lower ladder than those of males.

2.1.2. Relating Maternity Choice to Income and Occupation

The results of the current model are consistent with the inverse relationship between socioeconomic status and fertility generally seen in the data. Using 1968-1988 data from the

National Longitudinal Survey of Young Women, Waldfogel (1997) finds that women undergo a 4% wage penalty for having one child and a 12% wage penalty for multiple children. For other similar examples, see Budig and England (2001), Rindfuss and Brewster (1996), and Smith et al. (2001).

Abma and Martinez (2006) use data from the National Survey of Family Growth to study women aged 35-44 who are voluntarily, temporarily, and involuntarily childless. Here voluntarily childless women are those who are fecund but choose not to have children or are contraceptively sterile; temporarily childless are those who expect to have children in the future; and involuntarily childless women are biologically incapable of having children. The authors find that 11% of voluntarily childless women hold "executive, administrative, and managerial" or "professional specialty occupations", compared to 6% among women with children, and 9% among those who are temporarily childless.

The negative relationship between motherhood and socioeconomic status may be due in part to educational differences between childless and childbearing women; see Figure 6, adapted from Livingston and Cohn (2010). However, education gaps between women with and without children are not as pronounced as the socioeconomic gap, suggesting the reasons for the latter are deeper, and may be caused an anticipation of incompatibility between the woman's role as worker and mother.

2.1.3. Childcare Regulation, Prices, and Demand

The past few decades have seen swift growth in the childcare industry. The U.S. Census Bureau estimates the existence of 262,511 childcare facilities in 1987, and 766,401 in 2007. Increased demand has been driven by higher female labor participation rates, as suggested by Blau (2001), resulting from changes in family structure, a desire to provide young children with educational opportunities, and other factors. Financial statement analysis by Sageworks indicates sales among private childcare experienced average annual growth of 6-7% between 2008 and 2012, while other companies' sales declined 5% on average in 2009.

Recent growth in the childcare market has been accompanied by political and philosophical debate as to the extent of private versus public responsibility in child rearing, but movement toward federal involvement in childcare began as early as 1933, when the Federal Economic Recovery Act and the Work Projects Administration specified funding for the expansion of childcare. With the Community Facilities Act of 1941, the Children's Bureau under the Office of Education recommended standards in staff training, staff-child ratios, nutrition, age ranges, and group sizes in subsidized day care facilities. Though the act was discontinued after the war and standards were never enforced, stringent procedures and regulations would soon follow. Legislation in 1962 would require state licenses for childcare facilities seeking federal funding grants. Ongoing government debates would converge toward the 1968 Federal Interagency Day Care Requirements (FIDCR), a mandate on a set of regulations to govern childcare provision, including staff-children ratios by age group and by staff status (volunteer, clerical, etc), space, meals, ventilation. Administrative bodies were specified. In 1969, President Nixon submitted his Family Assistance Plan (FAP), which included an estimated 400 new childcare programs to be established in five years. In consideration of this plan, the 1968 FIDCR would be revised to introduce and enable more enforceable standards in a 1972 FIDCR, whose authority would extend to in-home as well as facility-based childcare. Over subsequent decades, national day care studies would abound, administrative authorities would frequently shift, and regulatory control of day care services at federal and state levels would grow enormously. A history of regulation in the childcare industry is found in Phillips and Zigler (1973), and an overview of the structure of U.S. childcare regulations is found in Heeb and Kilburn (2004).

Complex childcare regulations, on top of other enterprise regulations including zoning laws, pose serious startup and operational challenges for day care providers today. The Pennsylvania code, for example, specifies a list of general requirements, staff suitability and ratio requirements, as well as rules for the physical site, fire safety, equipment, programing/activities, admission procedures, child health, adult health, nutrition, transportation, child records, adult records, special exceptions, and school-age programs. Rules for the physical site alonee space, measurement and use of play space, unsafe areas in outdoor space, outside walkways, protective electrical covers, toxics, sanitation, smoking, water, indoor temperature, hot water pipes and sources of heat, ventilation, telephone availability/usage, emergency phone number posting, first-aid kit availability, building surface requirements, paint, lighting, firearms, glass, and toilet areas.

The effects of state regulations on childcare prices and choices have been studied by Heeb and Kilburn (2004) using regulations from state archives and 1990 National Childcare Survey data. The authors explicitly account for the way regulations increase labor costs to specify a relationship between staff-to-child ratios and day care prices. Using simultaneous estimation of prices and wages, and including price and quality-assurance effects of regulations, they also empirically assess the effect of regulations and quality-assurance on household decisions for childcare. They find an economically significant effect of regulations on the price of childcare, which in turn reduces childcare services demand and discourages mothers from participating in the labor force. No quality assurance effect of regulation on demand is found.

Using mixed logit and universal logit models on Canadian data, Powell (2002) finds a similar result that childcare prices reduce the probability of labor force participation and opting for childcare services. Her work, along with that of Michalopoulos and Robins (2000), Kimmel (1998), and Ribar (1992), also uses childcare regulations to identify prices, finding a strong correlation between strictness and complexity of regulations and the price of day care services.

The current paper's baseline model attempts to shed light on the persistent "unexplained" component of the gender wage gap, the mechanism being that women—in anticipation of taking time out of the workforce for some period of maternity and childcare—find lower-ladder occupations optimal relative to men. The baseline model is also extended to include an option of day care services for each family. Results of the extended model are consistent with the empirical findings that regulation (hence prices) in childcare services reduce demand and discourage labor participation among women. Since women expecting to leave the labor force for an extended time period choose lower-ladder occupations, a connection between childcare regulation and gender income inequality is established, even when comparing normalized incomes that only examine wages during working periods.

2.1.4. Organization

In what follows, Section 2.2 presents the baseline model. Section 2.3 describes the solution to the baseline model. Section 2.4 provides numerical analysis. Section 2.5 describes and solves an extended model that includes an option for day care service, enabling women to remain to remain in the workforce after childbearing. Section 2.6 provides numerical analysis for this extended model. Section 2.7 concludes and offers possible extensions to the baseline model.

2.2. Baseline Model

Consider a model wherein workers live for n periods in discrete time, with impatience given by β. Workers face a continuum of occupations indexed by a parameter $\phi \in [0, 1]$, which governs the evolution of the potential for wage growth. The minimum wage across all occupations is $\underline{w}$. Each occupation has a set of employment positions offering wages on the support $[\underline{w}, \infty)$. A worker begins his or her life by choosing an industry, ϕ. A worker may enter an occupation by applying to its lowest paying job, which the worker receives with certainty. Once employed in an occupation, a worker may in each period apply for a raise of $\varepsilon \in [0, 1]$. If received, the then higher-paying job offers a wage of $w + \varepsilon$ where w was the worker's previous wage. We will refer to this process as "climbing" an industry. If the raise is denied, the worker keeps his or her current wage. After choosing ε, the worker receives the promotion with probability:

$$p(w, \varepsilon, \phi) = (1-\varepsilon)^{\frac{1}{2}}(1 - \frac{1}{1 + e^{(1-\phi)\frac{(-w+\mu)}{\sigma}}}) \tag{2.2.1}$$

Here μ and σ are exogenous parameters that specify the shape of the logistic portion of $p()$; they control the underlying stochastic rates at which industries can be climbed over time. The intuition behind this probability function is as follows. The first term implies that the probability of a promotion being accepted is decreasing in the size of the promotion requested; and the marginal decrease in acceptance probability becomes infinite as the requested raise approaches one. (E.g., it is more difficult for an analyst to directly become a company's vice president than to become a senior analyst first.) The second term, a logistic function, implies that occupations with high ϕ are more difficult to climb up front, but over time show relatively greater promise for career growth. Occupations with low ϕ are easier to climb up front, but the opportunity for career growth quickly drops. Hence occupations with higher ϕ are more suitable for workers seeking a longer term career investment, and vice versa. For the purposes of this paper, occupations with high and low ϕ will respectively be termed "high ladder" and "low ladder" occupations.

The process by which promotions are requested and accepted is a simplifying generalization of the many contexts in which wage climbing occurs in reality. Real-world examples might include employees submitting applications to be considered for higher-paying roles, speaking with bosses about taking on greater responsibility and being compensated accordingly, renegotiation of salary, or simply working harder to signal high marginal productivity in the hope of a raise.

There are both male and female workers, about whom some simplifiying assumptions are made. No worker is ever fired. Moreover, in the baseline model, a worker in a particular occupation may not switch except after having given birth to a child. Hence male workers commit to an industry at the beginning of their lives, and never switch.

For the first $m < n$ periods, the worker's utility in each period is given by his or her current wage, w. In period m (denoting "marriage" or "match"), the worker is exogenously partnered with a member of the opposite gender. At this time, the couple draws a random non-pecuniary benefit to having a child given by $B \sim U[\underline{w}, \underline{w} + \Delta]$. If the couple decides to have a child, they will enjoy this benefit each period after the child's birth for the remainder of their lives. Both members of the household now face the same period utility function, given as the sum of household income plus the benefit to having a child:

$$U(w_M, w_W, B) = w_M + w_W + \mathbb{1}(\text{child born})B \tag{2.2.2}$$

Wages are subscripted by man and woman. Assume for simplicity the couple can have at most one child, who can be born in period m only if the couple chooses. For a child to be born, the woman must exit the workforce for $a < n - m$ periods of birthing and childcare, after which she may reenter the workforce in period $m + a + 1$, once again choosing an occupation to enter at the minimum wage. Mandatory time out of the work force after a birth is a reasonable aspect of the model as suggested by Figure 7, which shows the cumulative percents of women working by number of months after a first birth, by time cohort.

Requiring the woman to reenter the workforce at a lower (minimum) wage after exiting is a simplification based in reality. One might ask whether a child's consumption would actually require the woman to work harder and thereby have the opposite effect. Much empirical work has been done comparing women's wages before and after maternity; see Waldfogel (1997) for one of the most cited examples. The fact that mothers tend to enter an occupation on lower ranks than non-mothers is established by Kunze (2014) using Norwegian panel data. This research also finds mothers to be 1.6 percentage points less likely to receive promotions than non-mothers (the "family gap"). Using register data on West Germany, Ejrnæs and Kunze (2011) find that on reentering the workforce after giving birth, mothers experience a decrease in wages of 3-5.7% per year of maternity leave, and a negative selection back to full-time employment. Regardless, the current paper obtains the same essential qualitative results whether or not mothers are allowed to return to previous wage levels after birth. Additionally, the mother's need to worker harder after a child is born can be offset, in the case of married couples, by the empirically observed rise in male wages after fatherhood; see Hodges and Budig (2010).

The goal is now to solve for optimal behavior by both men and women, and compare the average or expected per-period income among all working individuals who are behaving optimally. Such a comparison must only examine working individuals and normalize away the possible a periods of maternity time. The result is that women's normalized income is lower than men's when both genders behave optimally, suggesting a component of income inequality arising from natural biological differences, unrelated to employment discrimination.

2.3. Solution to Baseline Model

In the following section, optimal behavior is determined for the man and woman with respect to the baseline model. First, a framework is established for solving the man's problem. The framework is then extended in the more complex woman's problem. Expected lifetime incomes for the man and woman are calculated. A normalized income statistic, "average income per period worked," is found for both genders.

2.3.1. Optimality for Man

In the first period of the man's life, he chooses an occupation. In periods $t = 1...n-1$, he must choose a raise ε_t to request. The decision to have a child can be viewed as exogenous to the man, since he can take no action in birthing the child; any birthing action will be taken by his spouse, whose utility perfectly aligns with his own. Each receives the same non-pecuniary benefit B, and the remaining utility is given by the sum of the household's wages. Thus the man's only concern is to maximize his discounted lifetime income.

The man's strategy can be expressed as $(\phi_M, \{\Omega_t\}_{t=1}^{n-1})$ where $\phi_M \in [0,1]$ is his occupational choice at the start of life, and $\Omega_t : [\underline{w}, \underline{w}+t-1) \times [0,1] \mapsto [0,1]$ is a mapping from all possible current wages and occupations at time t to a wage request $\varepsilon_t()$.

To solve for the optimal strategy, we first seek to establish an occupation-specific value function for the man, $V_M(t, w, \phi)$, where t, w, and ϕ are his current period, wage, and occupation. This value function should be interpreted as the value to the man derived exclusively from working in that occupation, not accounting for additional benefits received via his wife's income, or from having a child. We know his value in the final period of life:

$$V_M(n, w, \phi) = w \tag{2.3.1}$$

In general, his value function is given by:

$$V_M(t, w, \phi) = \max_{0 \le \varepsilon \le 1} \{w + \beta(p(w, \varepsilon, \phi)V_M(t+1, w+\varepsilon, \phi) + (1 - p(w, \varepsilon, \phi))V_M(t+1, w, \phi))\} \tag{2.3.2}$$

We proceed by backward induction. Using substitution to find his value in period $n-1$, and by plugging in the raise probabilities, we see that $V_M(n-1, w, \phi) =$

$$\max_{0\leq\varepsilon\leq 1}\{w+\beta((1-\varepsilon)^{\frac{1}{2}}(1-\frac{1}{1+e^{(1-\phi)\frac{(-w+\mu)}{\sigma}}})(w+\varepsilon)+(1-(1-\varepsilon)^{\frac{1}{2}}(1-\frac{1}{1+e^{(1-\phi)\frac{(-w+\mu)}{\sigma}}}))w\} \tag{2.3.3}$$

Taking the FOC with respect to ε, we get an interior solution of $\varepsilon^*_{n-1}=\frac{2}{3}$, independent of the occupation or current wage. Plugging this number back into 2.3.3, we get

$$V_M(n-1,w,\phi)=w(1+\beta)+\frac{\beta 2\sqrt{3}}{9}(1-\frac{1}{1+e^{(1-\phi)\frac{(-w+\mu)}{\sigma}}}) \tag{2.3.4}$$

To determine $\varepsilon^*_{n-2}()$, we plug expression 2.3.4 into 2.3.2 and once again take the FOC with respect to ε. The process iterates in this manner.

By numerically approximating the solution, it is found that $\varepsilon^*_t\approx\frac{2}{3}$ for $t=1...n-2$; in every period of the man's life, the optimal raise request is approximately $\frac{2}{3}$ with a high degree of precision, regardless of the occupation or current wage. For a high computation speed approximation to the value function that is still quite accurate, it is possible to construct the value function at time $t<n$ that treats the raise request as exogenous:

$$V_M(t,w,\phi)=w+\beta(p(w,\tfrac{2}{3},\phi)V_M(t+1,w+\tfrac{2}{3},\phi)+(1-p(w,\tfrac{2}{3},\phi))V_M(t+1,w,\phi)) \tag{2.3.5}$$

Nonetheless, 2.3.2 should be used for greatest accuracy. All explicit values can now be found by backward induction from 2.3.1. We now know the man's (and later, by the same logic, the woman's) optimal raise request paths within an occupation. Finally, the man chooses an occupation at the beginning of his life by solving:

$$\phi^*_M=\arg\max_{0\leq\phi\leq 1}V_M(1,\underline{w},\phi) \tag{2.3.6}$$

2.3.2. Man's Lifetime Expected Income

The man's undiscounted expected lifetime income can be approximated to a high degree of accuracy in two ways (undiscounted because here we are interested in quantity, not utility value.) The first is to Monte Carlo simulate a large number of lifetime wage paths under the man's optimal strategy. The second is to take advantage of the fact that $\varepsilon_t^* \approx \frac{2}{3}$ for $t = 1...n-2$, for all $w \in [\underline{w}, \underline{w}+t-1]$, and for all $\phi \in [0,1]$, and solve for the lifetime expected income analytically using value functions that treat ε_t^* as exogenous.

The second method is implemented as follows. Since the man will always request a raise of $\frac{2}{3}$, starting at time 1 by entering an occupation at wage $\underline{w}$, he proceeds to stochastically climb a ladder of possible wages $\{\underline{w} + \frac{2}{3}\vec{X}\}$, where the elements of $\vec{X}$ comprise the set of "wage rungs" or "number of promotions received." These elements are $\vec{X} = 0, 1, ...n-1$. Let $j \in \vec{X}$ be a variable describing the number of promotions received. Since the male worker has chosen his optimal occupation at time 1, each jump occurs with probability $p(w, \frac{2}{3}, \phi_M^*)$. For example, if he always receives the raise, an event which happens with probability $\prod_{j=0}^{n-1} p(\underline{w} + \frac{2}{3}j, \frac{2}{3}, \phi_M^*)$, his wage in the last period of life will be $\underline{w} + \frac{2}{3}(n-1)$. To find the man's lifetime expected income at ϕ_M^*, we require a probability-weighted sum of all possible income paths along this "wage ladder". This expression is difficult to simplify, since at each rung the probability of the next raise changes, even if the requested increments are the same. We can condense an expression for the man's lifetime expected wages as follows:

$$\mathbb{E}(\sum_{t=1}^{n} w_t) = n\underline{w} + \begin{pmatrix} 1 & 2 & ... & n \end{pmatrix} \begin{pmatrix} p_{11} & ... & p_{1n} \\ ... & ... & ... \\ p_{n1} & ... & p_{nn} \end{pmatrix} (\tfrac{2}{3}\vec{X}^T) \tag{2.3.7}$$

In the n-by-n matrix above, which we will call $\mathbf{P}$, element p_{ij} is the probability of spending exactly i periods of one's life working at wage rung j. The vector to the left of $\mathbf{P}$ scales by the number of periods worked at the wages $\frac{2}{3}\vec{X}^T$. We can expand p_{ij} as follows:

$$p_{ij} = p(\text{earn wage } j \text{ for exactly } i \text{ periods}) =$$
$$\sum_{k=j+1}^{n-i+1} \underbrace{p(\text{wage } j \text{ first attained in } k)}_{\text{denote } q_{jk}} \underbrace{p(\text{stay } j \text{ for } i \text{ periods})}_{= (1-p(\underline{w}+\frac{2}{3}j, \frac{2}{3}, \phi_M^*))^i} \quad \underbrace{p(\text{leave } j}_{= p(\underline{w}+\frac{2}{3}j, \frac{2}{3}, \phi_M^*)} \quad)^{\mathbb{1}(k<n-i+1)} \tag{2.3.8}$$

The lower bound on the sum in 2.3.8 is the first period at which wage level j could possibly be reached. (Do not be confused by the indexing; wage rung $j = 0$ implies zero promotions have been received, which is by construction possible at time period $j + 1 = 1$.) The upper bound is the last period at which a worker can enter rung j and remain there for i periods without dying first. An indicator function is placed as an exponent on $p(\text{leave } j)$ because if a worker enters rung j in period $k = n - i + 1$ and remains there for i total periods, he has reached the end of his life and cannot "leave".

There are $\binom{k-1}{j}$ possible paths of getting to (k, j) while climbing the wage ladder. q_{jk} is calculated by taking the probability-weighted sum of all paths to $(k - 1, j - 1)$, and then multiplying by the probability of making an immediate wage jump, $p(\underline{w}+\frac{2}{3}(j-1), \frac{2}{3}, \phi_M^*)$. q_{jk} is trivially equal to 1 at $(j = 0, k = 1)$, and trivially 0 for the impossible cases $(j = 0, k > 1)$ and $j > k$. After incorporating these probability-weighted path sums into equation 2.3.8 and filling in our matrix $\mathbf{P}$, 2.3.7 gives us a man's lifetime expected earnings.

A statistic later useful in comparing income streams across gender is "average earnings per period worked," or $\mathbb{E}(\frac{\text{lifetime income}}{\text{working periods}})$. For men, denote this statistic by I_M and calculate by dividing 2.3.7 by n, since the number of periods worked by the man is constant at n.

2.3.3. Optimality for Woman

The woman's problem has an added complexity: She can give birth to a child in period m. She does not know at the beginning of her life how much she will want a child, since the non-pecuniary benefit is drawn in period m. The possibility of wanting a child may affect her career choice pre-maternity; the act of having a child may affect her career choice post-maternity. Moreover, the act of giving birth affects her ability to work for a periods.

The woman's strategy can be expressed as $(\phi_{W,1}, \{\Omega_t\}_{t=1}^{n-1}, M, \phi_{W,2})$. Here $\phi_{W,1} \in [0, 1]$ is her occupational choice at the start of life. $\Omega_t : [\underline{w}, \underline{w} + t - 1) \times [0, 1] \mapsto [0, 1]$ maps all possible current wages and occupations at time t to a wage request $\varepsilon_t()$. M is a maternity decision mapping from $[\underline{w}, \underline{w} + \Delta] \times [0, 1] \times [\underline{w}, \underline{w} + m - 1] \mapsto \{\textit{child}, \textit{no child}\}$; that is, from all possible benefits to having a child, occupations, and current wages to a discrete motherhood choice. Finally, $\phi_{W,2} \in [0, 1]$ is her post-maternity occupational choice.

Once again, we solve by backward induction. Suppose a woman gave birth to a child in period m and has just completed the a periods of maternity. The period is $m + a + 1$. She will choose to enter an occupation by maximizing a value function analogous to the man's:

$$\phi^*_{W,2} = \arg\max_{0\leq\phi\leq 1} V_W(m+a+1, \underline{w}, \phi) \tag{2.3.9}$$

A subscript of 2 was placed on ϕ^*_W to denote the post-maternity occupation choice, as opposed to a subscript of 1 which will later denote the pre-maternity occupation choice made at time 1. (Once again, the woman will ask for raises $\varepsilon_t()$ close to $\frac{2}{3}$ in all states.)

In period m once B is realized, the discounted lifetime benefit to having a child is given by the sum of a geometric series:

$$\sum_{i=m+1}^{n} \beta^{i-m} B = B\frac{\beta - \beta^{n-m+1}}{1-\beta} \tag{2.3.10}$$

We can then calculate the lifetime discounted utility for a woman who births a child at time m, knowing her child value B, and who resumes work in period $m+a+1$. This is given by the LHS of equation 2.3.11. Given state (m, w, ϕ) and a realized benefit B, the woman will choose to birth a child if and only if

$$\beta^{a+1} V_W(m+a+1, \underline{w}, \phi^*_{W,2}) + B\frac{\beta - \beta^{n-m+1}}{1-\beta} \geq V_W(m, w, \phi) - w \tag{2.3.11}$$

The RHS of the equation is the present value to remaining employed in the current occupation. Rearranging 2.3.11, the maternity condition can be rewritten

$$B \geq (\frac{1-\beta}{\beta - \beta^{n-m+1}})[V_W(m, w, \phi) - w - \beta^{a+1} V_W(m+a+1, \underline{w}, \phi^*_{W,2})] \equiv \underline{B}(m, w, \phi; \beta, a) \tag{2.3.12}$$

Given some wage and occupation state in period m, $\underline{B}()$ is the minimum non-pecuniary benefit the woman must receive to become a mother; using the uniform distribution of B, the probability of becoming a mother before seeing B, having just entered state (m, w, ϕ), is therefore

$$p(B \geq \underline{B}(m,w,\phi;\beta,a)) = \begin{cases} \dfrac{\underline{w}+\Delta-\underline{B}(m,w,\phi;\beta,a)}{\Delta}, & \underline{B} \in [\underline{w},\underline{w}+\Delta] \\ 0, & \underline{B} > \underline{w}+\Delta \\ 1, & \underline{B} < \underline{w} \end{cases} \tag{2.3.13}$$

The value to a woman at state (m, w, ϕ) with a realized B is the maximum of the two sides of 2.3.11. Let $\hat{V}_W(t, w, \phi)$ denote the value to the woman before B is realized. We can write $\hat{V}_W$ in period m as follows:

$$\hat{V}_W(m,w,\phi) = w + \mathbb{E}\max\{\beta^{a+1}V_W(m+a+1,\underline{w},\phi^*_{W,2}) + B\frac{\beta-\beta^{n-m+1}}{1-\beta}, V_W(m,w,\phi)-w\}$$

$$\begin{aligned} = w + p(B \geq \underline{B}(m,w,\phi;\beta,a))(\beta^{a+1}V_W(m+a+1,\underline{w},\phi^*_{W,2}) \\ + (\frac{\beta-\beta^{n-m+1}}{1-\beta})\mathbb{E}(B|B > \underline{B}(m,w,\phi;\beta,a))) \\ + (1-p(B \geq \underline{B}(m,w,\phi;\beta,a)))(V_W(m,w,\phi)-w) \end{aligned} \tag{2.3.14}$$

The woman's optimal pre-maternity wage-climbing strategy and career choice are solved using backward induction from $\hat{V}_W(m, w, \phi)$ in the spirit of 2.3.5. With an expression for the value function in the first period, we can finally solve for the woman's initial occupation choice in a fashion analogous to 2.3.6:

$$\phi^*_{W,1} = \underset{0\leq\phi\leq 1}{\arg\max}\, \hat{V}_W(1,\underline{w},\phi) \tag{2.3.15}$$

We have now solved for the woman's optimal strategy, which includes an initial occupation choice, a raise-request for each period of her working life, a maternity decision in state m after the realization of B, and a post-maternity occupation choice.

A natural question to ask is, if the a periods away from the labor force encompass both birthing and childcare, why can't parents share responsibility for the latter aspect? Why must the entirety of the a periods be spent by the female? The model can be viewed in a way that eliminates this restriction but induces the same behavior. Suppose a were composed of two sequential components: giving birth, followed by early childcare. The woman must leave the workforce to birth a child. Now the family must decide whether the mother will continue at home for the remainder of a, or whether the father and mother will swap places. Since a mother has already paid the wage costs associated with motherhood, the family finds it optimal for her to continue at home; it would be inefficient for the man to take over for her, as his wages would then suffer, too. Hence the mother will always choose her first occupation keeping in mind the possibility of spending a whole periods out of the workforce as a mother, and the man will choose his first occupation anticipating no time out of the workforce.

2.3.4. Woman's Lifetime Expected Income

Here again, the expected lifetime income can be approximated with a high degree of accuracy in two ways. The first is by Monte Carlo simulating a large number of wage growth paths according to the woman's optimal strategy. The second is by assuming $\varepsilon \approx \frac{2}{3}$ in all states and solving analytically, as follows.

Let there be a vector $\vec{Y}$ of length m, whose i^{th} component is $\underline{w} + \frac{2}{3}(i-1)$ for $i = 1...m$, describing the set of possible wages in period m. Wages are subscripted by time period. The woman's lifetime expected income is the following probability-weighted sum of all possible income paths:

$$\begin{aligned}\mathbb{E}(\sum_{t=1}^{n} w_t) = \mathbb{E}(\sum_{t=1}^{m} w_t | w_1 = \underline{w}, \phi = \phi^*_{W,1}) + \sum_{i=1}^{m-1} p(w_m = Y_i)[... \\ p(B \geq \underline{B}(m, w_i, \phi^*_{W,1}))\mathbb{E}(\sum_{t=m+a}^{n} w_t | w_{m+a} = \underline{w}, \phi = \phi^*_{W,2}) + \\ (1 - p(B \geq \underline{B}(m, w_i, \phi^*_{W,1} f)))\mathbb{E}(\sum_{t=m+1}^{n} w_t | w_m = w_i, \phi = \phi^*_{W,1}))] \quad (2.3.16)\end{aligned}$$

The maternity probability is given by 2.3.13, while the expected wage sums are calculated in a fashion analogous to the process used in 2.3.7.

Since the woman sometimes spends a periods out of occupation, it is necessary to use a normalized statistic for comparing income across genders. Let us use $\mathbb{E}(\frac{\text{lifetime income}}{\text{working periods}})$. Since the man spends every period working, the denominator for him is constant, and thus his normalized "expected period income" was simply his lifetime expected income divided by n. For the woman, the expression is more complicated. Begin by calculating the ex ante (period 1) probability of giving birth as:

$$p(motherhood) = \sum_{i=1}^{m-1} p(w_m = Y_i)p(B \geq \underline{B}(m, w_i, \phi^*_{W,1})) \tag{2.3.17}$$

Remaining lifetime expected income after motherhood, denoted I_2^{child}, is given by:

$$I_2^{\text{child}} = \mathbb{E}(\sum_{t=m+a+1}^{n} w_t | w_{m+a+1} = \underline{w}, \phi = \phi^*_{W,2}) \tag{2.3.18}$$

The ex ante (viewed from period 1) expected total income earned after m and after the decision not to have a child is denoted $I_2^{\text{no child}}$ and given by

$$I_2^{\text{no child}} = \sum_{i=1}^{m-1} p(w_m = Y_i)\mathbb{E}(\sum_{t=m+1}^{n} w_t | w_m = w_i, \phi = \phi^*_{W,1}) \tag{2.3.19}$$

Expected total income from periods 1...m is denoted I_1 and given by

$$I_1 = \mathbb{E}(\sum_{t=1}^{m} w_t | w_1 = \underline{w}, \phi = \phi^*_{W,1}) \tag{2.3.20}$$

Finally, we can compute the woman's normalized income, $\mathbb{E}(\frac{\text{lifetime income}}{\text{working periods}})$, denoted by I_W:

$$I_W = p(motherhood)\left(\frac{I_1 + I_2^{\text{child}}}{n-a}\right) + (1 - p(motherhood))\left(\frac{I_1 + I_2^{\text{no child}}}{n}\right) \tag{2.3.21}$$

We are now ready to compare the normalized incomes of women and men for any set of parameters $(n, m, a, \beta, \mu, \sigma, \underline{w}, \Delta)$.

2.4. Baseline Numerical Analysis

A numerical analysis was performed using the following baseline parameter values:

Parameter	**Value**	**Parameter**	**Value**
β	0.99	μ	3
n	45	σ	2
m	15	Δ	5
a	5	$\underline{w}$	0

The fact that $\varepsilon^* \approx \frac{2}{3}$ in all states (regardless of the current period, wage, or occupation) was numerically verified using backward iteration from period n.

It is helpful to see how ϕ affects a worker's ability to stochastically wage climb. Figure 8 shows the evolution of the probability of receiving a promotion of ε^* in two occupations, with high and low ϕ. The high-ϕ occupation exhibits relatively slow advancement at the beginning, but offers more room to grow. For example, we could compare the wage evolution of an investment banker (high ϕ) who begins as an analyst with low wages but who eventually makes vice president, versus a restaurant employee (low ϕ) who quickly becomes manager, but who sees little opportunity for advancement beyond his managerial wage.

$V_M(t, w, \phi)$ was solved using backward iteration as described in Section 2.3.1. ϕ_M^* was calculated by finding the argmax over $V_M(1, \underline{w}, \phi)$ with respect to ϕ. Figure 9 shows how the man's period-one value peaks at $\phi_M^* = 0.569$. The intuition is that the man considers any occupation $\phi > 0.569$ to be too difficult to climb during his finite lifespan. An occupation with $\phi < .569$ does not show enough potential for long-term advancement.

The woman's optimal occupation strategies were solved by similar means of backward induction with numerical maximization, with a result that $\phi_{W,2}^* = 0$ and $\phi_{W,1}^* = 0.318$.

The longer a worker knows he/she will be committed to working, i.e., the longer the lifespan, the more the worker will value long-term career advancement. Figure 10 demonstrates this effect, showing that higher n implies a higher optimal occupation choice, ϕ^*. This result also explains the inequality $\phi_{W,2}^* < \phi_M^*$: When a woman reenters the workforce after motherhood, her remaining number of working periods is $n - m - a < n$, and thus she favors an occupation with the opportunity for more rapid advancement.

There is also intuition for the fact $\phi_{W,1}^* < \phi_M^*$. In period m, if the woman does not become a mother, in retrospect she would like to have chosen the same occupation as the man, since ϕ_M^* is the optimal occupation choice for someone knowing she will work for n straight periods. But if she does become a mother, since she is now exiting the workforce for a time, she would like to have chosen an occupation she had been able to climb relatively rapidly. So the woman faces a tradeoff, owing to the fact that at time 1, she does not yet know whether she will later choose motherhood. It is the potential for motherhood in this tradeoff that induces $\phi^*_{W,1} < \phi^*_M$.

Due to the possibility of motherhood, women optimally choose lower ladder occupations. This drives the fact that the woman's normalized average per-period income (expectation of total income over periods worked) is smaller than the man's. These normalized incomes were Monte Carlo estimated with trial sizes of $50,000$. Random wage-promotion paths for the man were stochastically drawn following his occupation decision, according to the evolution of his promotion probabilities. A similar process was followed for the woman, with an additional random draw B in period m and an optimal decision regarding motherhood. The woman's normalized earnings were found to be 3.014, or 29.03% less than the man's, which were 4.247.

A woman's initial occupational choice, and hence her expected earnings, are a function of Δ, which sets an upper bound on the non-pecuniary benefit to motherhood. As Δ approaches 0, the woman's optimal initial occupation choice approaches the man's, and her earnings approach the man's; this is because she approaches the certainty of having no child. As Δ grows, so does the probability of having a child, and hence her $\phi^*_{W,1}$ approaches zero, and her earnings approach a lower bound less than the man's. These relationships are captured in Figures 11 and 12, the latter having been smoothed with a moving average filter. The irregularities in the first plot arise from a high variance Monte Carlo simulation, using $3,000$ trials per data point. The result here is consistent with the fact that non-mothers earn higher wages and select into occupations associated with higher prestige; women with lower values of Δ know they are less likely to become mothers.

Figure 13 plots comparative statics for a, showing how the woman's normalized earnings change as the number of mandatory periods out of the work force changes. The result is that an increase in a leads to an increase in the female's income. Why? As a increases, the opportunity cost to having a child increases, making it less likely the woman will give birth; this incents the woman to choose a higher-ladder industry at the start of life, which raises her normalized average income.

2.5. Extended Model with Childcare

The model is now extended to include a day care services option. In period m, the woman is no longer obligated to exit the workforce for a periods. Her family has the alternative of paying a day care cost c for each of the a periods, so she can remain in her current occupation without wage demotion and continue to climb her wage ladder. During that time, a mother opting for day care services receives non-pecuniary benefit γB to having a child, where $\gamma \in (0,1)$ is a utility-reducing factor due to time away from the child. (The father can experience this, too, but the problem remains unchanged.)

2.5.1. Optimality for Man

The man's problem remains unchanged. He chooses an occupation at the beginning of life according to 2.3.6, and his strategy encompasses a raise request choice for all possible states of time, occupation, and current wage.

2.5.2. Optimality for Woman

We begin solving the woman's problem by considering her decision at time m, after she and her husband have jointly drawn B. The woman now has three options: (1) Do not birth a child. (2) Birth a child, and exit the workforce for a periods to care for the child. (3) Birth a child, and hire a day care service for a periods while remaining in the workforce.

Suppose in period m the woman's current wage and occupation are (w, ϕ_1). The current value of choosing the first option is $V_1 = V_W(m, w, \phi_1) - w$, where the value function is the same as before. The current value of choosing the second option is determined in the same manner as in section 2.3.3, and equals:

$$V_2 = \beta^{a+1} V_W(m + a + 1, \underline{w}, \phi^*_{W,2}) + B \frac{\beta - \beta^{n-m+1}}{1 - \beta} \tag{2.5.1}$$

The value to choosing the third option is:

$$V_3 = V_W(m, w, \phi_1) - w + (\gamma B - c) \frac{\beta - \beta^{a+1}}{1 - \beta} + B \frac{\beta^{a+1} - \beta^{n-m+1}}{1 - \beta} \tag{2.5.2}$$

The woman's choice in period m is determined by $\max\{V_1, V_2, V_3\}$ as derived in Appendix 2.B. The woman's optimal pre-maternity wage-climbing strategy and career choice are solved using backward induction from $\hat{V}_W(m, w, \phi)$ in the spirit of 2.3.5. With an expression for the value function in the first period, we solve for the woman's initial occupation choice in a fashion analogous to 2.3.6:

$$\phi^*_{W,1} = \arg\max_{0 \leq \phi \leq 1} \hat{V}_W(1, \underline{w}, \phi) \tag{2.5.3}$$

We have now solved for the woman's optimal strategy in a model permitting families to opt for day care services. The strategy includes an initial occupation choice, a raise-request for each period of her working life, a maternity and day care decision in state m after the realization of B, and a post-maternity occupational choice.

2.6. Analysis of Extended Childcare Model

The natural question to ask is how the woman's normalized expected income behaves in relation to the cost of day care. Figure 14 plots this relationship using Monte Carlo draws to compute the expected income at each data point, and Figure 15 shows the smoothed values after a moving average filter is applied. Income levels fall in c, approaching a lower bound on the right. As the cost of day care is reduced, the woman's income level rises toward that of the man's.

When the woman knows c to be low, day care is more attractive for her relative to dropping out of the workforce. The opportunity cost to having a child becomes lower as the day care option becomes more attractive. Hence at the start of life, the woman believes herself more likely to remain in the workforce continuously, and so she chooses a higher ladder occupation. By contrast, when c is high, day care is a less attractive option relative to dropping out of the workforce during the a required periods. At the start of life, the woman believes herself less likely to work for n periods straight, and so her optimal first-period occupation choice is of a lower ladder.

It is via this mechanism that a relationship between the regulation of day care markets and the gender income gap is proposed in theory. The empirical work of Heeb and Kilburn (2004), Powell (2002), Michalopoulos and Robins (2000), Kimmel (1998), and Ribar (1992) establishes the positive effect of state regulations on prices of day care services, as well as a negative effect on childcare services demand and labor participation rates among women. Additionally, Kabátek (2014) develops a structural model of labor supply, fertility, and childcare of Dutch women using administrative panel data spanning 2001-2009, finding a positive change in the maternal labor supply induced by childcare subsidies. Consistent with the data, the current model finds a negative relationship between day care costs and female labor participation, and explains an additional relationship between day care costs and gender income differentials.

2.7. Conclusion

This paper proposes a theory for explaining the persistent "unexplained" component of the gender wage gap. The theory is consistent with the empirical literature relating regulations of the childcare industry to higher prices and reduced female labor participation.

It furthermore establishes a relationship between such regulations and this “unexplained” component of the gender wage gap. The theory’s central workhorse is a model of optimal occupational choice for men and women, involving the ability to stochastically wage climb. Men and women have aligned preferences and seek to maximize the discounted lifetime income stream of the household plus a randomly drawn non-pecuniary benefit to having a child.

Numerical results find that increasing the expected benefit to having a child, and hence the likelihood of maternity, induces women to optimally choose lower ladder industries in expectation of exiting the workforce. Chosing occupations with reduced wage growth potentials causes women to earn less per working period on average, hence an amplification of the gender wage differential. As the probability of maternity—as predicted upon entering the workforce—approaches zero, women choose higher ladder occupations, and the gender wage gap narrows.

The gender wage gap can be mitigated with the introduction of childcare services. Depending on the price of such services, women may opt to remain in the workforce after giving birth. In anticipation of this option, a woman entering the workforce at the start of life puts a new weight on the probability of working continuously, and so her optimal occupational choice differs. The lower the cost of childcare services, the more likely the woman will remain in the labor force post motherhood, and hence the higher the ladder of her chosen industry. As the woman choses a higher ladder at the start of life, the gender income differential is reduced.

2.7.1. Possible Extensions to Model

Future researchers may wish to consider extend the model with the purpose of exploring new uses or more closely approximating reality. The following considerations can be made. (1) Letting the woman decide to give birth to her child in any period $\geq m$ adds interesting complexities to the initial occupational choice. (2) Figures 10 and 8 suggest that workers’ occupational preferences change over time. For example, an unlucky worker who has received few promotions after many working periods may wish to switch to a lower ladder job mid-life, thereby taking better advantage of his remaining income-generating periods. A more dynamic model should be considered wherein workers may switch occupations at any time. (3) A simple tweak that should be considered: Allow women to reenter the workforce, post motherhood, at an income level between minimum wage and the former wage before childbirth. (4) The characteristic that women draw B in period m, and before m have no idea how much they will value a child, can also be adjusted for greater realism and interest. More likely, women decide at different ages whether they will want a child in the future, and this preference can evolve. A modified model can be considered wherein females receive noisy signals as to their true B, in periods leading to m—and during this process, they are also allowed to switch occupations. We would expect to observe women with high true B's converging toward lower ladder occupations, and vice versa. A simpler variation of this modification would entail considering an economy of $m-1$ types of females, where a female of type m realizes B in period m; all other aspects of the model would remain unchanged, but now the normalized earnings of females would have to be considered in aggregate.

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Appendix 2.A.

Plots

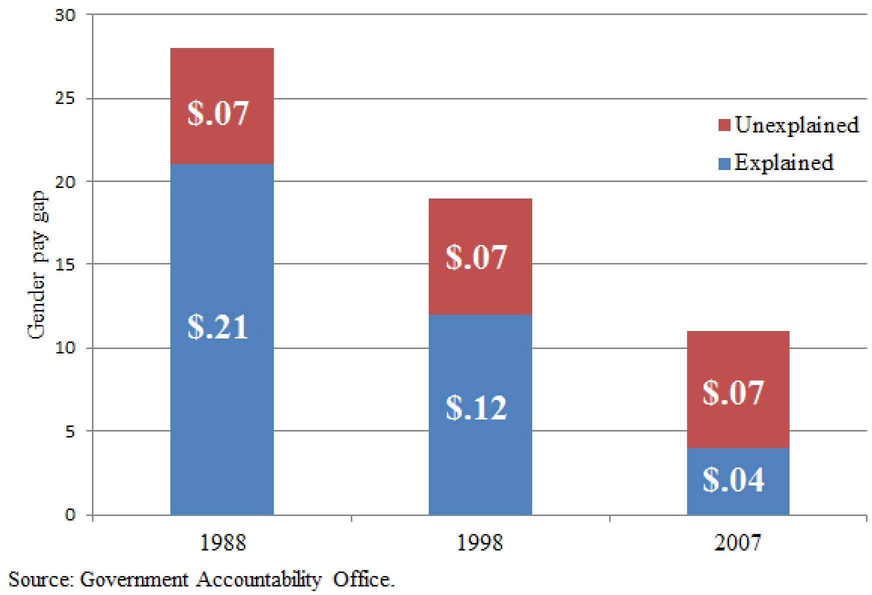

Figure 5: Persistence of the unexplained gender wage differential.

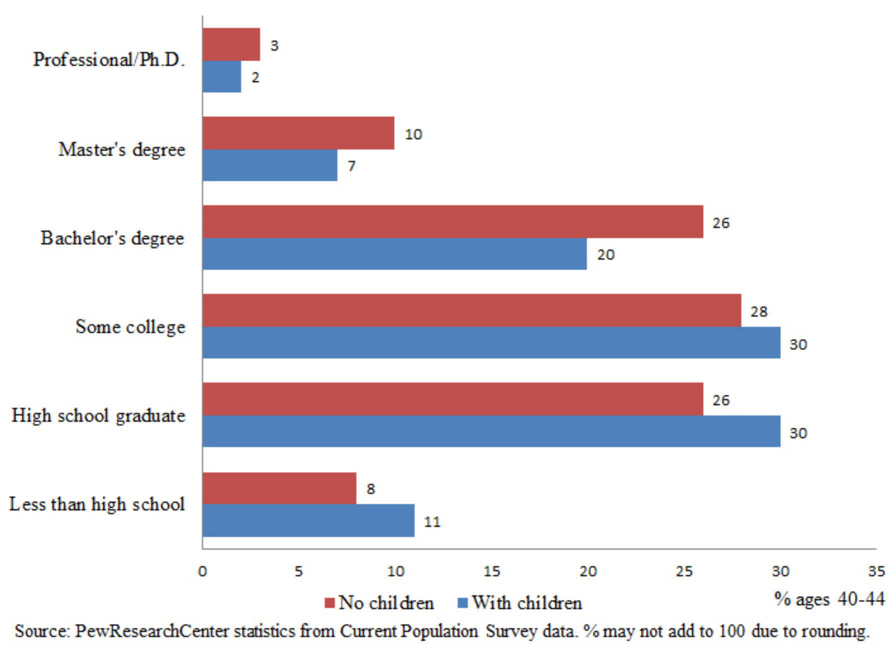

Figure 6: Education levels of mothers and non-mothers, 2006-2008, adapted from Livingston and Cohn (2010).

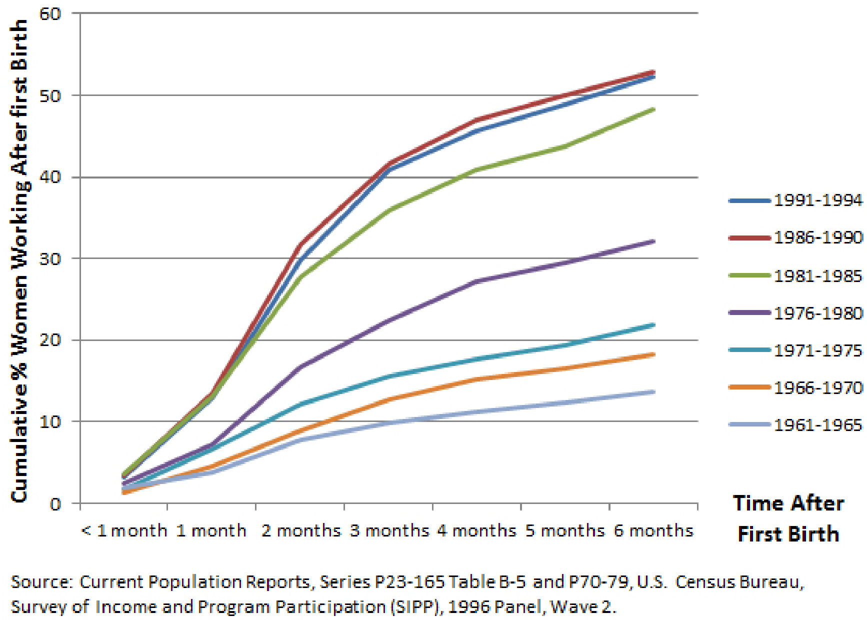

Figure 7: Cumulative percents of women working after first birth, by time cohort.

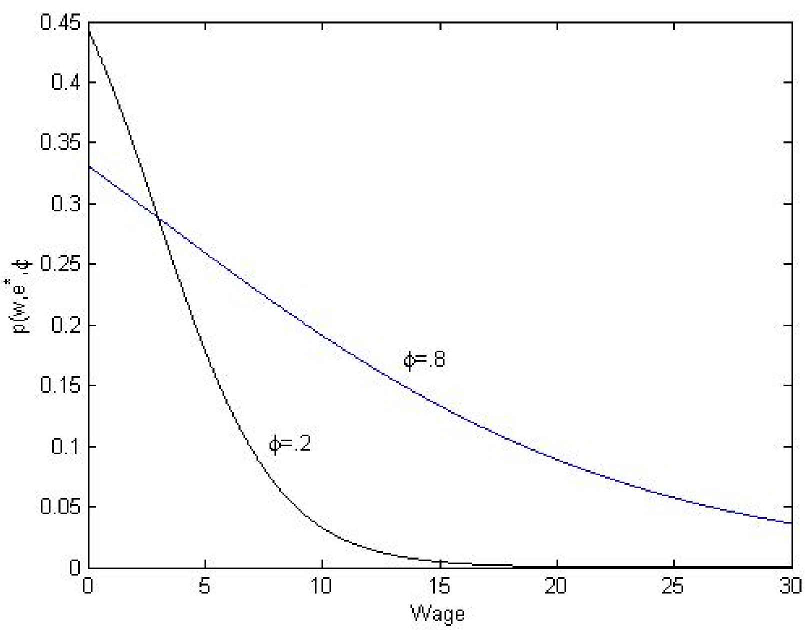

Figure 8: Comparison of wage-climbing opportunity in two occupations. y-axis shows probability that a raise of $\varepsilon^* = \frac{2}{3}$ is accepted.

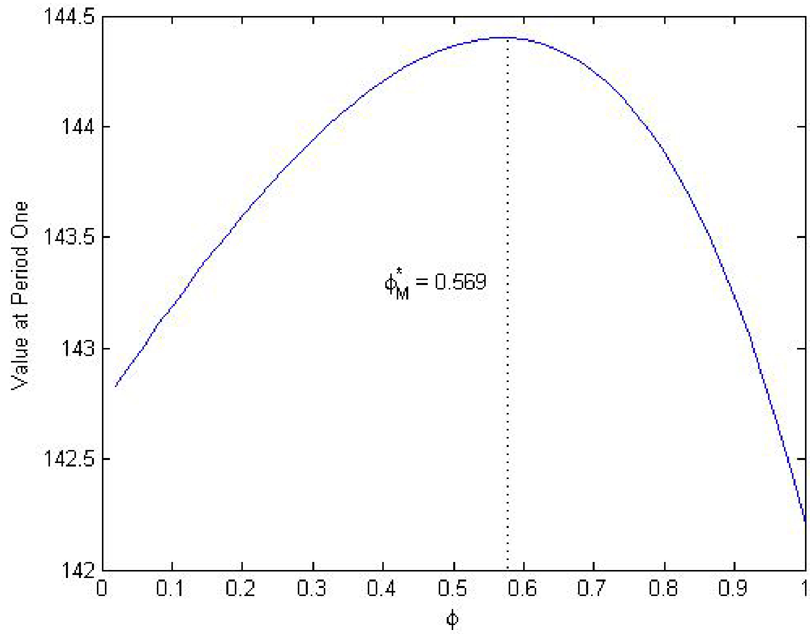

Figure 9: Period-one value to man by occupation, showing optimal ϕ_M^* choice.

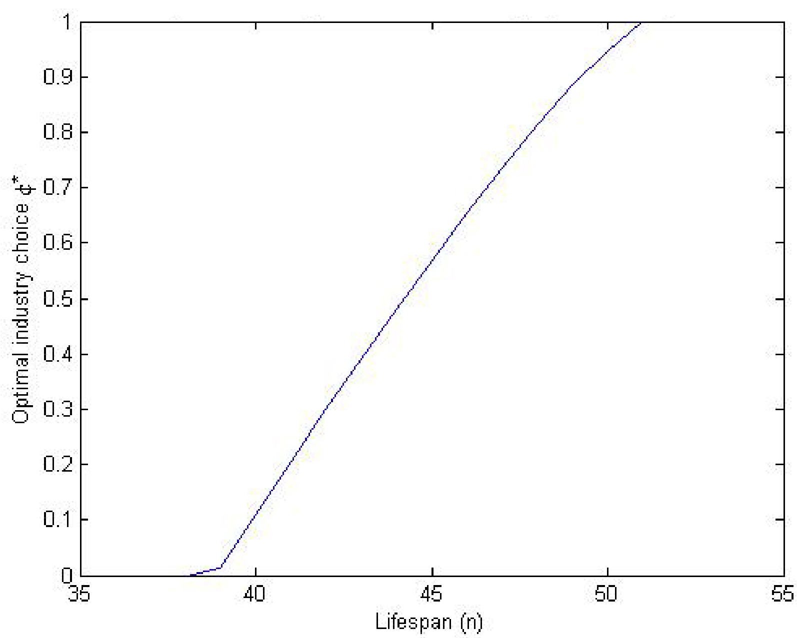

Figure 10: Man's optimal occupation choice varied along lifespans.

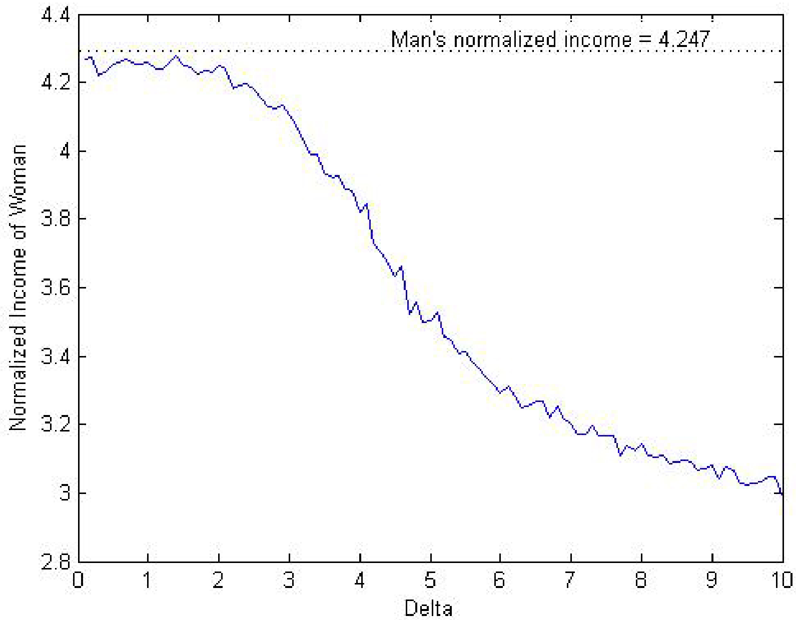

Figure 11: Woman's normalized earnings as Monte Carlo draws, versus Δ.

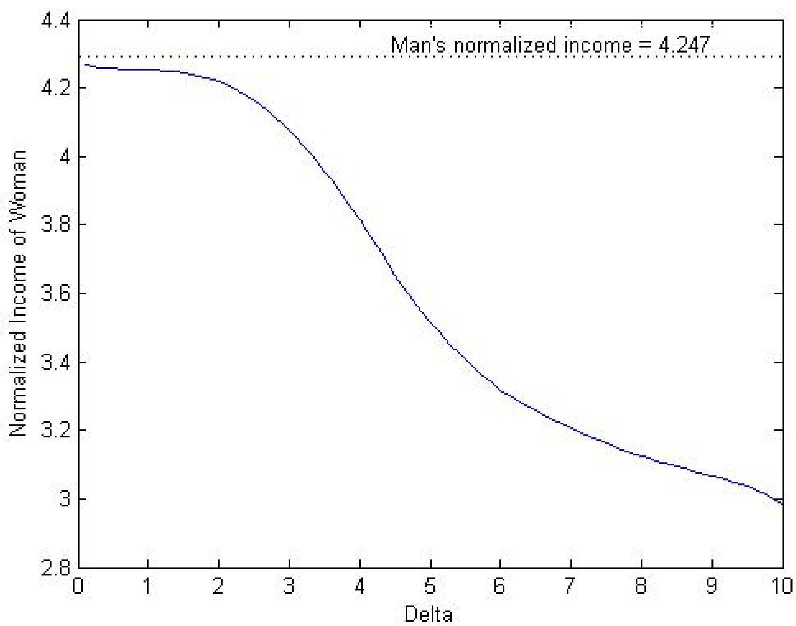

Figure 12: Woman's normalized earnings varying with Δ, smoothed.

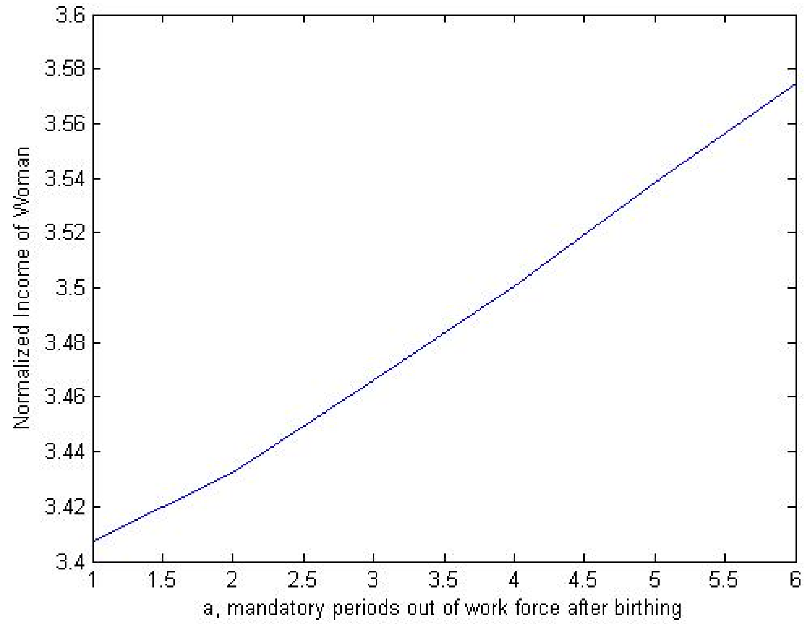

Figure 13: Woman's normalized earnings varying with a, for high Δ.

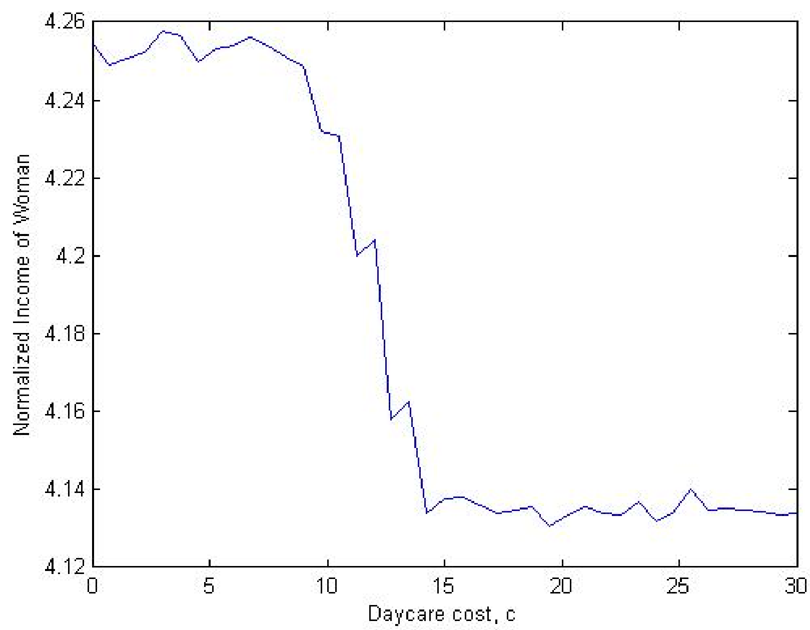

Figure 14: Woman's normalized earnings as Monte Carlo draws, varying with c.

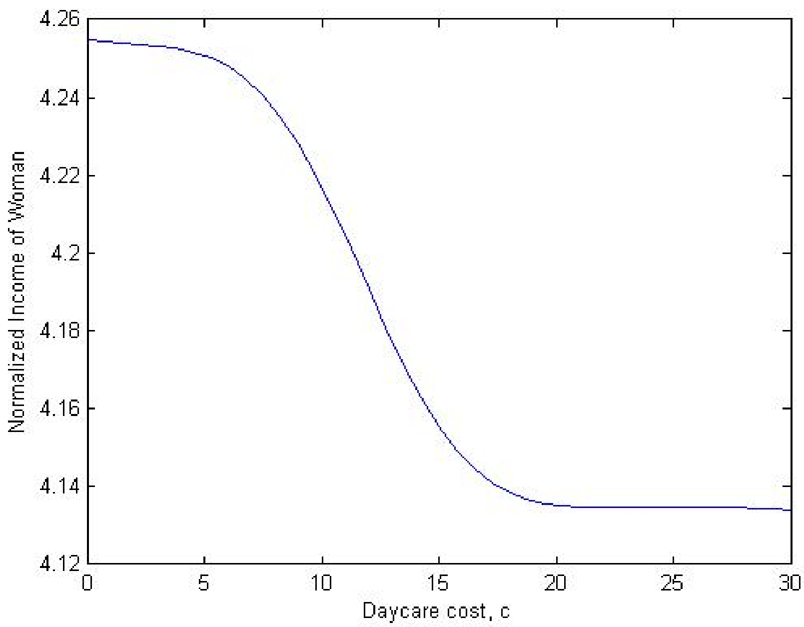

Figure 15: Woman's normalized earnings as MC draws varying with c, smoothed.

Appendix 2.B.

Derivation of Optimal Maternity/Daycare Choice

The woman's maternity and day care choice is determined by $\max\{V_1, V_2, V_3\}$, where:

$$V_1 = V_W(m, w, \phi_1) - w \tag{2.B.1}$$

$$V_2 = \beta^{a+1} V_W(m + a + 1, \underline{w}, \phi^*_{W,2}) + B\frac{\beta - \beta^{n-m+1}}{1 - \beta} \tag{2.B.2}$$

$$V_3 = V_W(m, w, \phi_1) - w + (\gamma B - c)\frac{\beta - \beta^{a+1}}{1 - \beta} + B\frac{\beta^{a+1} - \beta^{n-m+1}}{1 - \beta} \tag{2.B.3}$$

These are the respective values in period m to choosing (1) no child, (2) a child and no day care service, or (3) a child and a day care service. For simplicity while solving, constants are grouped so the expressions can be rewritten:

$$V_1 = J$$

$$V_2 = K + BL$$

$$V_3 = J + (\gamma B - c)M + BN$$

The expected value of the maximum in period m, before drawing $B \sim U[\underline{w}, \underline{w} + \Delta]$, is:

$$\begin{aligned}\mathbb{E}(\max\{V_1, V_2, V_3\}) = p(V_1 > \max\{V_2, V_3\})\mathbb{E}(V_1|V_1 > \max\{V_2, V_3\}) \\ + p(V_2 > \max\{V_1, V_3\})\mathbb{E}(V_2|V_2 > \max\{V_1, V_3\}) \\ + p(V_3 > \max\{V_1, V_2\})\mathbb{E}(V_3|V_3 > \max\{V_1, V_2\}) \quad (2.B.4)\end{aligned}$$

The first term is derived as follows:

$$p(V_1 > V_2) = p(B < \frac{J-K}{L})$$

$$p(V_1 > V_3) = p(B < \frac{cM}{N+\gamma M})$$

$$\begin{aligned}\Rightarrow p(V_1 > \max\{V_2, V3\} = p(B < \min\{\frac{J-K}{L}, \frac{cM}{N+\gamma M}\}) \\ = \frac{\max\{\min\{\frac{J-K}{L}, \frac{cM}{N+\gamma M}, \underline{w} + \Delta\}, \underline{w}\} - \underline{w}}{\Delta} \quad (2.B.5)\end{aligned}$$

Composed solely of constants, $\mathbb{E}(V_1|V_1 > \max\{V_2, V_3\})$ is trivially equal to J. The second term is derived as follows:

$$p(V_2 > V_1) = p(B > \frac{J-K}{L})$$

$$p(V_2 > V_3) = p(B < \frac{K + cM - J}{N + \gamma M - L})$$

$$\begin{aligned} \Rightarrow p(V_2 > \max\{V_1, V3\}) &= p(\frac{J-K}{L} < B < \frac{K + cM - J}{N + \gamma M - L}) \\ &= \frac{\max\{\min\{\frac{K+cM-J}{N+\gamma M-L}, \underline{w} + \Delta\}, \underline{w}\} - \min\{\max\{\frac{J-K}{L}, \underline{w}\}, \underline{w} + \Delta\}}{\Delta} \end{aligned} \quad (2.B.6)$$

and

$$\mathbb{E}(V_2 | V_2 > \max\{V_1, V_3\}) = K + L\mathbb{E}(B | V_2 > \max\{V_1, V_3\}) \quad (2.B.7)$$

where

$$\mathbb{E}(B | V_2 > \max\{V_1, V_3\}) = \frac{\max\{\min\{\frac{K+cM-J}{N+\gamma M-L}, \underline{w} + \Delta\}, \underline{w}\} + \min\{\max\{\frac{J-K}{L}, \underline{w}\}, \underline{w} + \Delta\}}{2}$$

The third term is derived as follows:

$$p(V_3 > V_1) = p(B > \frac{cM}{N + \gamma M})$$

$$p(V_3 > V_2) = p(B > \frac{K + cM - J}{N + \gamma M - L})$$

$$\Rightarrow p(V_3 > \max\{V_1, V2\} = p(B > \max\{\frac{cM}{N+\gamma M}, \frac{K+cM-J}{N+\gamma M-L}\})$$
$$= \frac{\underline{w} + \Delta - \min\{\max\{\frac{cM}{N+\gamma M}, \frac{K+cM-J}{N+\gamma M-L}, \underline{w}\}, \underline{w}+\Delta\}}{\Delta} \quad (2.B.8)$$

and

$$\mathbb{E}(V_3|V_3 > \max\{V_1, V_2\})$$
$$= J + (\gamma\mathbb{E}(B|V_3 > \max\{V_2, V_3\}) - c)M + \mathbb{E}(B|V_3 > \max\{V_2, V_3\})N \quad (2.B.9)$$

where

$$\mathbb{E}(B|V_3 > \max\{V_2, V_3\}) = \frac{\underline{w} + \Delta + \min\{\max\{\frac{cM}{N+\gamma M}, \frac{K+cM-J}{N+\gamma M-L}, \underline{w}\}, \underline{w}+\Delta\}}{2}$$

Subsituting the terms into 2.B.4 yields the mother's expected maximum value of the three choices before the benefit to having a child is known.

CHAPTER 3 : The Talent-Tax Trade-off in Locational Decisions of Television Productions

Abstract. The past decade has seen a flight from Hollywood television production despite the comparative advantages Los Angeles has developed for over a century. We theoretically examine the location choice and endogenous growth dynamics of television productions facing labor supplies that are heterogeneous in skill level. The location choice is influenced by tax considerations, as well as the regional accessibility of talent. An endogenously dynamic Cobb-Douglas production function is developed to capture changing productivities of inputs over the life cycle of a TV production. Equilibrium is found numerically. The model is used to explain why studios and producers have begun shifting television production outside of Hollywood despite the high concentration of production talent in Los Angeles.

3.1. Introduction

The location choice of firms is a popular subject of economic research. Why does the United States have regional hotbeds of entrepreneurial activity? The upper East Coast attracts biotech companies; Silicon Valley has long been popular for internet and information technology startups; Colorado is becoming competitive in its ability to attract young, vibrant entrepreneurs seeking startup capital; and Virginia, Florida, and Texas are competing to accomplish the same, as documented by the Startup America Partnership and Small Business Administration. Determining how and why up-and-coming entrepreneurial economies can become hotbeds of innovation—or lose their attractiveness over time—is a question of significant interest.

The same questions apply in the context of the entrepreneurial entertainment industry, where every production is operated like an independent business with a set of products that may extend beyond the production itself to include merchandising, books, spinoff series, games, etc. Movie "ventures" are even formed as individual LLCs. Every film or television series can be regarded as a unique enterprise facing its own locational decisions. In fact, the development of Los Angeles as the major film/TV hub can be attributed to locational incentives dating back to the early 1900s, when filmmakers moved west to avoid lawsuits by Thomas Edison's Motion Picture Patents Company in New Jersey; Edison's company, which owned most motion picture patents, had been aggressively suing filmmakers and putting a halt to productions. As a result, and owing in part to the wide variety of sets/geographies available in California, Hollywood became the capital of the film and television industries and has since maintained its status as the premiere nexus for production.

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Key words and phrases. Heterogeneous labor, growth

Nonetheless, production in the past few years has seen a flight from Hollywood. Producers have reduced costs by locating in production-friendly states, namely New York, North Carolina, Louisiana, and Georgia. In 2012, fewer than 10% of the new fall and midseason hour-long dramas were shot in Los Angeles County—dropped from 50% in 2010 and close to 80% in 2005, as shown in Figure 16. A report by Klowden et al. (2014) at the Milken Institute shows California lost 11% of its film/TV-related jobs from 2004 to 2012 (a total of 16,137) while New York state gained 25% (a total of 10,675), following New York's expanded film and television tax credit program allocating $420 million annually to attract film/television productions (four times that of California's credit program). Figure 17 shows the relative change in absolute employment numbers; Table 4 contains data from other leading states, as well. The entertainment jobs are also high paying, with an average of nearly $98,500 per person, and linked with ancillary jobs including those at prop houses, equipment rental houses, catering firms, etc.

The shift away from Los Angeles is made even more surprising by the fact that Hollywood has had over a century to develop its comparative advantages. Entertainment talent—including directors, writers, actors, crew, production assistants, cinematographers, editors, sound/music-related talent, and other categories—is heavily concentrated in the Hollywood area. Not only that, but there exists a well-developed infrastructure in Los Angeles to assist in matching between talent and productions; agencies and management companies like Creative Artists Agency and Anonymous Content make a business of connecting talent with the right projects. This infrastructure is not as well-developed in other states, nor do other states have complex infrastructures for set creation and other aspects of TV and filmmaking.

The current paper addresses two conflicting broad categories of variables relevant to a television venture's location choice decision. The first is the availability of labor in the area, coupled with the relative availability of talented (or "skilled") and unskilled labor. The second is the role government plays in the economy of the area: tax rates/credits, ease of navigating bureaucratic snares, costs of obeying regulations. Qualitatively, the relationship between these two variables is simple. A producer prefers a location with a large pool of available labor in a favorable skilled/unskilled ratio, desires tax credits, and objects to burdensome government intervention. Quantitatively, it is unclear to what extent one aspect of the tradeoff can outweigh the other, or how the question can be approached from a theoretical standpoint. In areas where skilled labor is so plentiful that it becomes relatively inexpensive, does the government hold some semblance of market power, so that it can charge high taxes and not see a flight of television productions to other regions? In areas with scarce labor and scarce talent, can governments attract entertainment ventures by being friendly to production? We wish to develop a theory that will enable the study of these and related questions.

Of further interest is the conjecture that these answers may depend on the age and development of a venture in a given region. This paper restricts the focus to *television* so that this subject may also be studied. Movies have a “one-shot” structure: The movie is filmed, produced, and then brought to market. Television has a continuing structure: When a show is brought to market, if it performs well, further investment is made for continued episodes/seasons. By investing more heavily in a certain television production, the producer (typically with the help of a major studio) can improve the quality and value of the product: More labor-hours can go into filming/editing episodes, prominent cast members can be hired, new locations can be afforded, and so on. This higher quality product can in turn attract more viewers, increasing a network’s willingness to pay for more episodes and making the show more profitable to the producer.

Young and long-standing television productions may face different location-based tradeoffs. Young series have less labor-based attachment to their surroundings, but may also not have the capital necessary to move even a small infrastructure. More developed shows may find this transition more affordable, but have a more complex infrastructure and greater attachment to their environment—settings, crew, etc. In light of this possibility, we find it crucial to address locational decisions in the context of a TV series life cycle. Continuing in this vein, there may be reflexivity between these variables. Government selection of regional parameters may be influenced by life cycle patterns of TV productions, but the life cycle patters of TV productions may themselves be a function of regional parameters. Thus it is important to operate in the context of an endogenous firm life cycle. One of the innovations developed in this paper is an endogenously dynamic Cobb-Douglas production function that captures changing marginal productivities of skilled and unskilled labor over the course of a production’s life cycle.

We approach this issue from multiple perspectives. As a foundation we begin with a basic search and matching problem with heterogeneous TV productions and heterogeneous labor. We extend this baseline model to include government and regional parameters that may exogenously vary between locations. This reveals the object of interest: the locationial choice of TV productions. In what follows, Section 3.2 provides a literature review and motivation. Section 3.3 presents the model that is used for quantitative analysis. Section 3.4 describes the numerical method used to solve and analyze the baseline model. Section 3.5 presents an alternative formulation that is more analytically tractable, but whose computational difficulties necessitate the model of Section 3.3. Section 3.6 concludes.

3.2. Literature Review

The areas of economics and mathematics that motivate this question and our approach span multiple fields. We restrict our network theoretical toolkit by considering only two regions. The economic tools and concepts we find useful are firm and worker heterogeneity (especially the former), dynamic search and matching for firms, and regional location decisions for firms. The following literature review discusses relevant research in firm theory in general; for our purposes, television productions will assume the role of "firms" and operate as such.

The study of firm heterogeneity has a long and successful history. It is commonly accepted that firms are homogeneous in their preferences, unlike households; heterogeneity is generally discussed in the context of productive capabilities. However, for much of early labor/firm theory, decisions made by firms in the context of heterogeneity were taken to be quite simple: Firms seek to profit maximize. The focus of the literature was on the outcomes of labor and firm choices in the face of heterogeneity. For a good example of this, see Hopenhayn (1992), which develops a dynamic model for determining entry, exit, and growth of firms' output and employment in a competitive industry. Stepping back further, a complex theory of firm heterogeneity arguably began with Dixit and Stigitz (1977) in their discussion of optimal product diversity. The history of this theory certainly extends beyond these two works, but they provide a picture of the distinction between historical and modern firm theory.

A starting point for modern theory of firm heterogeneity, though with a different application than ours, is found in Melitz (2003) in which industry reallocation decisions are studied. The key components of firm heterogeneity therein are differences in marginal productivity of labor and in overall productivity, the latter being randomly drawn from a distribution. Firms do not know what their productivity will be prior to the production phase. A similar approach is taken in the first chapter of Melitz and Redding (2014). The key feature distinguishing this work from Melitz (2003) is its focus on analyzing the decision problem of firms who differ by productivity, and the dynamics that result from entry and exit. Felbermayr, Prat, and Schmerer (2011) introduce this new aspect of the theory into a search setting. Our approach has parallels to that of Felbermayr et al., not surprisingly: Embedding search frictions into a complicated decision problem for heterogeneous firms is a young enough practice that a canonical approach is still useful for a variety of settings.

We distinguish our work from the results of Melitz and Felbermayr in two meaningful ways. For one, we consider ex post heterogeneity rather than ex ante, as we are more interested in the life cycle of firms than past work, and we seek to understand how firms make decisions at different points in their lifetimes. Our theory begins with identical firms ex ante, and heterogeneity is introduced via the birth, growth, and death of firms and the outcomes of their investments. This will also create a dynamic economy. Firms invest in their own productivity, and thus grow (or die) as time passes. Our second distinct contribution is the incorporation of a locational decision undertaken by firms ex ante. There exist multiple locations where firms may choose to operate, and each location is parametrized by tax rates, and an availability of skilled and unskilled labor. The location decision by firms, and the incentives corresponding to such a choice, constitute the second primary focus of this paper.

Research in the area of regional location decisions has also proven fruitful. With multiple regions, this problem can have deep mathematical and topological undertones; see Jackson (2010). However, even in the case of deciding between two regions, the incentives and dynamics that come into consideration are complex. Devereux et al. (2007) explore this problem from an industrial organization perspective, examining how agglomeration of industries affects the choices for new entrants. Bhat, Paleti, and Singh (2014) address this question from a purely empirical standpoint using sophisticated mathematical and topological methods. The empirics used in their case are also motivated by elements of industrial organization, and include the role that the government can play in manipulating IO variables—anti-trust laws, discretionary grants to specific industries, etc.

A growing body of research continues to address location choice from a labor-oriented perspective. Modern work providing a foundation for study includes Moretti (2010), in which a GE framework of regional labor markets is fitted with a spatial equilibrium. Building from this work, Lutgen et al. (2013) and many others develop models wherein homogenous workers are faced with a regional location decision. Lutgen et al. help to empirically motivate our approach, insofar as they find that matching effectiveness is a significant determinant of unemployment. This indicates that though location decisions at first pass may seem to fall under the scope of industrial organization, search theory has something to teach us about actual outcomes in the data. In particular, Lutgen et al. find that former notions of efficiency no longer hold in regional settings; the Hosios condition is not sufficient to ensure efficiency. Moreover, there are even cases in which increasing matching effectiveness in a region increases unemployment rates. The intuition is that if increasing the matching effectiveness has too strong of an effect in terms of attracting new workers, the total rate of unemployment could still rise in response.

3.3. Baseline Model

Terminology

Since the current model uses a firm theoretical toolkit in the context of television production, for consistency with the economic literature, certain terms must be defined so the analogies are clear in what follows. Television productions will sometimes be referred to as "TV firms" or simply "firms"; these always refer to a single TV-series production and are not to be confused with production houses as a whole. Workers of talent (directors, cinematographers, etc.) will be called "skilled" and others are "unskilled."

Television producers are paid by networks for the right to air or stream a show (and large studios often provide capital for the production of a show). Since a network's willingness to pay increases with viewership, the popularity of a show drives its revenue-earning potential for producers. A TV firm's "productivity" in the context of this paper is not merely the ability to create episodes, but the ability to earn high profits from networks.

A TV firm is considered to have a level of "technological investment" or a "productivity technology" which governs its ability to earn profit from networks. In reality television producers typically are not creating new technologies, but rather investing in the sum of elements that create revenue-generating production value, including high-reputation actors, film sets, equipment, etc. A TV firm's productive technology/capability should be viewed in this way, and the "technological frontier" should be viewed as the sum of elements that create the highest possible production value.

Model

We consider an economy wherein a TV firm living in infinite discrete time begins its life by choosing to permanently locate in one of two regions indexed by $j \in \{1, 2\}$. Each region has a labor force comprised of skilled (type t) and unskilled (type l) workers; a worker's skill type s is contained in $\{t, l\}$. Each region has an infinite measure of immobile workers of each type. Firms have measure unity. In a given period, a measure δ of firms is born, and each existing firm dies with probability δ, ensuring a constant measure of firms in the economy. (In the real-world analogy, the continuation or death of a TV production is determined by networks, whose decision to renew or cancel a show is influenced by myriad unpredictable factors.) Firms are heterogeneous in productivity, λ, which strictly increases in a firm's total level of technological investment, denoted by I, according to a logistic function with parameters μ and σ:

$$\lambda(I) = \frac{1}{1 + e^{-\frac{I-\mu}{\sigma}}}$$

A basic property of logistic functions implies $\lambda(I) \in (0, 1)$. This s-shape of the logistic function is commonly used to describe the growth pattern of emerging technology, companies, and whole industries. Figure 18 plots the proximity to the technological frontier as a function of total investment. Let t and l denote the number of respective skilled and unskilled workers employed by a firm. Per period revenue, denoted by R, is given by a Cobb-Douglas function whose dynamic exponents are governed by the current total internal investment (technological capability). TV firms are taxed regionally at the marginal rate τ_j. Let the technological frontier be given by a constant A, so as $\lambda(I)$ increases, a firm's coefficient on revenue approaches the industry maximum. Revenue in a period is given by:

$$R(I, t, l, j) = \lambda(I) A (t + l)^{\lambda(I)} t^{1-\lambda(I)} (1 - \tau_j)$$

A nascent or undeveloped production (with low I) strongly prefers skilled workers, whereas a large, developed production (with high I) sees a smaller marginal benefit to hiring talent and cares more about the size of the work force. The dynamic exponents on the Cobb-Douglas revenue function capture the notion that developed firms have a higher marginal need for routinized or low-skill tasks (e.g., grips, assistants) relative to the marginal need for high-skill tasks, than do less-developed firms who require powerhouse teams to establish an early success. Nonetheless, even among highly developed and productive firms, a skilled worker is always preferred to an unskilled one; this is captured by the inclusion of t in both terms of the Cobb-Douglas revenue function. The parameters μ and σ determine the shape of the company's technological growth function, i.e., how quickly internal investment yields higher productivity, and in turn how quickly the relative talent premium diminishes.

A TV firm enters a given period with a level of current investment, and a work force, given by state variables (I, t, l). The firm then posts vacancies for skilled and unskilled work, v_{jt} and v_{jl}, with marginal costs of posting equal to ϕ_t and ϕ_l. TV firms also simultaneously invest some incremental amount i into their productivity-enhancing technology, with total adjustment costs equal to ηi^2.

Workers and firms are matched according to an exogenous matching function $m(W_{js}, \theta_{js})$, where W_{js} is the current measure of workers of type s employed in region j, and θ_{js} is a regional friction parameter specific to the labor type s. Search frictions imply that $m(W_{js}, \theta_{js}) \in (0, 1]$. The higher the current level of employment, the more difficult it is for a match to occur; the probability of a match approaches zero as the measure of employment approaches infinity. The matching function used is:

$$m(W_{js}, \theta_{js}) = e^{-\theta_{js} W_{js}} \tag{3.3.1}$$

A smaller friction parameter θ_{js} means skill of type s is more readily available; that is, for a smaller fixed θ_{js}, $m(W_{js}, \theta_{js})$ approaches zero in W_{js} less quickly.

Workers receive an outside option b in unemployment insurance and/or benefits while unemployed. Let b be small enough to guarantee that workers always find job search profitable while unemployed. Workers' wages w_{jt} and w_{jl} are determined via a Nash bargaining process to be explained later.

The Firm's Problem

The firm faces a dynamic optimization problem which we present here in recursive form. Assume the firm discounts future value at rate r. A firm in region j with total investment I, skilled labor t, and unskilled labor l has a value function given by:

$$V(I, t, l, j) = \max_{v_t, v_l, i} [log(R(I, t, l, j) - w_{jt} t - w_{jl} l - \phi_t v_t - \phi_l v_l - \eta i^2) + \frac{1-\delta}{1+r} V(I', t', l', j)] \tag{3.3.2}$$

such that

$$R(I, t, l, j) \geq w_{jt} t + w_{jl} l + \phi_t v_t + \phi_l v_l + \eta i^2 \tag{3.3.3}$$

$$I' = I + i \tag{3.3.4}$$

$$t' = t + m(W_{jt}, \theta_{jt}) \tag{3.3.5}$$

$$l' = l + m(W_{jl}, \theta_{jl}) \tag{3.3.6}$$

TV firms are born with some initial money, k, which they use in the first period of existence to invest and hire. At birth, firms choose a region according to:

$$j = \arg\max_{j \in \{1,2\}} \{ \max_{v_t, v_l, i} [log(k - \phi_t v_t - \phi_l v_l - \eta i^2) + \frac{1-\delta}{1+r} V(i, m(W_{jt}, \theta_{jt}) v_t, m(W_{jt}, \theta_{jl}) v_l, j)] \} \tag{3.3.7}$$

where the inner objective function is subject to the budget constraint

$$k \geq \phi_t v_t + \phi_l v_l + \eta i^2 \tag{3.3.8}$$

Suppose in equilibrium that new TV firms are indifferent between regions 1 and 2, and mix their geographical entry choice with probabilities $(\omega, 1-\omega)$. The size of next period's labor force, by type, is a deterministic function of the number of vacancies posted. All firms with a given state (I, t, l) will have the same optimal behavior. After the economy has reached an equilibrium steady state by existing for an infinite number of periods, the firms' side of the economy will consist of a sequence of generations $n \in \{1, 2, ...\infty\}$, each of which has measure $(1-\delta)^n$. There will be an optimal growth path in equilibrium given by the sequences $(\{I_{nj}, t_{nj}, l_{nj}\}_{n=1}^{\infty})_{j \in \{1,2\}}$ and corresponding vacancy postings $(\{v_{njt}, v_{njl}\}_{n=1}^{\infty})_{j \in \{1,2\}}$, which can be used to calculate market frictions. The measure of employed workers of type s in region j is given by:

$$W_{js} = \omega^{\mathbb{1}\{j=1\}} (1-\omega)^{\mathbb{1}\{j=2\}} m(W_{js}, \theta_{js}) \sum_{n=1}^{\infty} \sum_{a=1}^{n} v_{ajs} (1-\delta)^{n+1} \tag{3.3.9}$$

Wage Determination

In the first period of life, firms Nash bargain with each skill type of workers using the equation:

$$(1-\chi)[E(w_{js}) - U(w_{js})] = \chi \frac{dV(0,0,0,j)}{ds'} \tag{3.3.10}$$

Let us drop the regional and skill-type subscripts for a moment. Here χ denotes the relative bargaining power between workers and TV firms. s' is the measure of workers of type s

being brought into the first revenue-generating period by newborn firms. $E(w)$ and $U(w)$ are the respective benefits to being employed and unemployed in this economy where the wage for the worker's type is w; the former is given by:

$$E(w) = w + \frac{1}{1+r}[(1-\delta)E(w) + \delta U(w)] \tag{3.3.11}$$

Let the probability of an unemployed worker receiving a job offer equal the total measure of new hires over the current measure of unemployed. Since the denominator in either region is always infinite, this probability will be zero. Hence the value to being unemployed is given by:

$$U(w) = b + \frac{1}{1+r}U(w) \Rightarrow U = \frac{b(1+r)}{r} \tag{3.3.12}$$

After nascent firms Nash bargain, they remain committed to hiring at those wages for the remainder of life. In equilibrium, there will thus exist a single, constant pair of wages (w_{jt}, w_{jl}) in each region attained via Nash bargaining between workers and nascent firms. By plugging equations 3.3.12 and 3.3.11 into 3.3.10, the wage determination equation attained is:

$$w_{js} = b + \left(\frac{r+\delta}{1+r}\right)\left(\frac{\chi}{1-\chi}\right)\frac{dV(0,0,0,j)}{ds'} \tag{3.3.13}$$

This amounts to a differential equation, since the value function $V()$ is also a function of wages; a numerical solution will later be computed. We now proceed to a notion of equilibrium.

Definition 3.3.1. *Competitive equilibrium* in this economy consists of (A) regional growth paths and hires by skill type, given by sequences $(\{I_{nj}, t_{nj}, l_{nj}\}_{n=1}^{\infty})_{j\in\{1,2\}}$, (B) corresponding vacancy postings $(\{v_{njt}, v_{njl}\}_{n=1}^{\infty})_{j\in\{1,2\}}$, (C) wages $\{w_{jt}, w_{jl}\}_{j\in\{1,2\}}$, (D) a mixing strategy $(\omega, 1-\omega)$ over regions 1 and 2 exercised by newborn TV firms, and (E) aggregate employment levels $\{W_{jt}, W_{jl}\}_{j\in\{1,2\}}$ given by equation 3.3.9, such that:

(1) Newborn TV firms are indifferent between regions; the mixing strategy is optimal for newborn firms.

(2) Given wages and aggregate employment levels, a TV firm of generation $n \geq 2$ chooses $\{I_{nj}, v_{njt}, v_{njl}\}$ that maximize the objective function given by 3.3.2 subject to the budget constraint in 3.3.3 and transition equations 3.3.4–3.3.6.

(3) Wages $\{w_{jt}, w_{jl}\}_{j\in\{1,2\}}$ are determined via Nash bargaining between nascent TV firms and workers in each region as given by 3.3.13.

(4) Given aggregate employment levels and a region choice, newborn firms choose $\{I_{1j}, v_{1jt}, v_{1jl}\}$, and hence make a simultaneous choice of the endogenous wages in 3.3.13, that together maximize their objective function given inside 3.3.7 subject to the budget constraint in 3.3.8.

3.4. Numerical Method

A numerical equilibrium solution is found using standard value function iteration absorbed under three further levels of iteration, each of which endogenizes another variable or variable pair. The lowest level of iteration treats matching probabilities and wages as exogenous, and solves for the optimal investment and vacancy choices by iteratively searching across a three-dimensional discretized control space for each (I, t, l) state combination. We call this basic VFI the "level 1" loop.

In a "level 2" algorithm, aggregate employment levels and matching probabilities are endogenized as follows. Based on some initial guess for the matching probabilities, ordinary VFI yields a policy function. From this policy function, we calculate aggregate employment levels by summing hires across the growth path and weighting each generation of hires by $(1-\delta)^n$; new aggregate employments yield new matching probabilities. Using the old policy function and new matching probabilities, aggregate employment is recalculated again, and iteration continues to converged matching probabilities. These converged matching probabilities are then used in level 1 VFI to recalculate the policy function. The new policy function is now used to calculate a new set of converged matching probabilities (and aggregate employment levels). The process continues until both the policies, and the matching probabilities (and aggregate employment levels), have converged simultaneously.

These processes are further subsumed under a "level 3" loop, wherein wages are endogenized. For a set of endogenous value/policy functions and matching probabilities (with aggregate employment levels), new wages are further calculated using a discrete approximation to the Nash bargaining equation 3.3.10. Level 2 then repeats, yielding new values, policies, and matching probabilities. The process continues to convergence in wages.

Finally, wage convergence and all lower-level loops are subsumed under "level 4" iteration, which endogenizes the mixing strategy using a bisection method on the support $(0,1)$. The upper (lower) bound on the support is iteratively reduced (increased) whenever locating in region 1 is less (more) attractive than region 2. The level 4 iteration can be computed across various tax plans and matching frictions, to see how changing regional characteristics of appeal affects the mixing strategy. It is this type of analysis that enables us to examine the nature of the talent-tax trade-off in equilibrium.

Global parameters are calibrated as follows. The death rate δ solves $(1-\delta)^5 = 0.25$, in keeping with the finding of Cooper et al. (1988) that 25% of businesses exist after five years. Alternatively, one could calibrate δ by using data on the percentage of television shows that are cancelled each year, but this is highly variable and also conditional on the number of seasons of a show to date. The discount rate is the commonly used $r = 0.05$. The remaining parameters were chosen to ensure an equilibrium growth path from which intuition could be gleaned from plots and visualizations.

Table 1: Global Parameters

Parameter	*Value*	*Parameter*	*Value*
r	0.05	ϕ_t	0.5
δ	0.242	ϕ_l	0.5
μ	20	k	5
σ	10	b	0.1
A	30	χ	0.1
η	1	-	-

The baseline regional parameters (taxes and hiring frictions) are likewise calibrated for visual intuition in this theoretical framework, as follows, in two cases to allow for comparison.

Table 2: Regional Parameters, Two Cases

Parameter	*Value*
τ_1	0.1
τ_2	0
θ_{1t}	0.04
θ_{2t}	0.05
θ_{1l}	0.07
θ_{2l}	0.07

Parameter	*Value*
τ_1	0.05
τ_2	0
θ_{1t}	0.03
θ_{2t}	0.05
θ_{1l}	0.07
θ_{2l}	0.07

Some parameters for region 1 vary between the two cases. In the first case, taxes and frictions are higher than in the second case. The mechanism by which frictions and taxes affect the equilibrium mixing strategy works as follows. Suppose equilibrium has been found at ω^*, and suddenly more talented workers are made available in region 1 via a reduction in the corresponding friction parameter. This raises the matching probability and makes region 1 more attractive, causing more firms to choose this region. As more firms locate to region 1, aggregate employment levels rise, reducing the matching probability, until a new equilibrium is reached with some $\omega^{*'} > \omega^*$. Taxes affect the system in a similar way. In increase in region 1 taxes discourages entry, causing more firms to locate to region 2. This reduces aggregate employment in region 1, raising the matching probabilities. There will be a new equilibrium $\omega^{*'} < \omega^*$. It is through this mechanism that taxation and the availability of talent constitute opposing forces in the location decision of firms.

3.4.1. Solution in One Region

Levels 1–3 of the algorithm are solved in the first region, treating the measure of entering TV firms as exogenous. Matching probabilities for skilled and unskilled workers are found to be $(0.43, 0.59)$ respectively, despite the relative availability of talent in region 1, reflecting the greater demand for talented workers.

Both low-tech and high-tech firms exhibit value functions characterized by a steep concavity in skilled workers. However, technologically undeveloped firms do not benefit as much from unskilled workers, whereas high-tech firms do, requiring more routinized tasks. For high-tech firms, the value function is more steeply concave in unskilled labor, showing greater returns to that input. A side-by-side comparison between low-tech and high-tech firms is shown in Figure 19, illustrating firms' changing labor productivities as a function of total investment.[1] The changing productivities are caused by the dynamic exponents in the Cobb-Douglas function. Figure 20 illustrates the value function of a mid-tech firm. Once again, value varies across current labor force compositions. Skilled workers are still more productive, though unskilled labor is worth more to a high-tech firm than it is to a low-tech firm. This is observed in the steepening concavity along the unskilled axis.

Figure 21 shows a mid-tech firm's incremental investment policy for a given labor force composition. The function is concave and increasing in skilled and unskilled labor, but less so in unskilled labor, making the shape similar to the value function. The jaggedness of the policy function arises from insufficient density of the grid, though improving density comes

[1] In Figures 19–23, the labor force domains begin at a small positive integer rather than at zero. This is because when both inputs are close to zero, consumption is negative in firms' log-utility.

at a cost of computational burden given the number of variables for which we are solving. The policy for skilled hires is shown in Figure 22, and appears to be linear and independent of the number of unskilled hires. Finally, Figure 23 shows the policy for hiring unskilled labor. This variable does depend on both states of the labor composition. Unskilled labor hires are increasing in the current size of the skilled labor force, and decreasing in that of the unskilled labor force. Once again, the kinks in the surface are due to sparsity in the control space.

3.4.2. Solution in Two Regions

Now the mixing strategy is endogenized using the fourth level of iteration, consisting of a bisection algorithm on $(0,1)$ to find the ω for which the difference in values between the two regions is minimized. The value to locating in a region decreases in the fraction of firms choosing that region; a higher measure of firms in a region causes the aggregate employment levels to rise, and this reduces the matching probabilities. Reducing the hiring frictions in a region increases the value to locating there; a region with lower hiring frictions exhibits a slower convergence of the matching probability to zero as aggregate employment climbs. Finally, the value to locating in a region decreases in the local tax rate. Equilibrium strikes a balance in these various factors of appeal between the two regions.

We compare equilibrium under the two sets of regional parameters given in Table 2, and find the following results:

Table 3: Equilibria in Two Cases

Parameter	*Region 1*	*Region 2*
$\omega, 1-\omega$	0.47	0.53
m_t	0.5204	1.0000
m_l	0.6489	1.0000
w_t	0.1255	0.1119
w_l	0.1026	0.1016

Parameter	*Region 1*	*Region 2*
$\omega, 1-\omega$	0.57	0.43
m_t	0.7256	0.6844
m_l	0.6895	0.7788
w_t	0.1112	0.1231
w_l	0.1017	0.1021

The equilibrium on the right hand side of Table 3 corresponds to a scenario in which region 1 exhibits lower taxes and hiring frictions for talented workers. As an equilibrium result, a greater percentage of newborn TV firms choose region 1. Case two shows an increase in the matching probability of skilled workers in region 1; this is a consequence of the exogenous decrease in the corresponding frictions. The region 1 matching probability of unskilled workers increases, most likely due to a fall in demand for unskilled workers; skilled workers have become a more viable substitute. Region 2 sees a reduction in both matching probabilities due to a reduced presence of firms, and hence lower employment levels. The effects on wages is minimal. In region 1, wages of both types fall; the reverse is true of region 2.

Figures 24 and 25 show the value to locating in a region as a function of the mixing strategy. Equilibrium is found at the intersections. The first plot corresponds to the first case, and the second plot shows how the equilibrium ω increases with a reduction in taxes and frictions for talented hires; the region 1 line shifts upward. The plots should theoretically exhibit monotonicity; the results show noisy kinks that would likely disappear with a reduction in tolerances and an increase in the density of the grids.

3.5. Alternative Formulation

We now consider a more canonical formulation with a non-exponential matching function and a more robust Nash bargaining condition. This construction will be more analytically tractable, but induces some numerical complexities; here we present the main analytical results. For simplicity, only the 1-region case will be considered. As before, there are two types of labor: skilled and unskilled (or talented and untalented). The primitives of the model are the same as in the baseline case, save for the matching function and wage determination, and utility is made to be linear.

Search Frictions

Firms recruit both talented and untalented laborers by posting vacancies v_t and v_l, to which workers apply. Vacancy posting costs are ϕ_t and ϕ_l. To capture search frictions, an exogenous function $M(u, v)$ governs the matching process, outputting a number of matches for given inputs u (unemployed) and v (vacancies). We will let $M(u, v)$ exhibit CRS, so we can focus on functions of the market tightness of a particular labor sector: $\theta = u/v$. We will thus let $m(\theta) \equiv M(u, v)/v$ and $p(\theta) \equiv m(\theta)/\theta$. The first represents matches per vacancy, which is relevant for firms posting vacancies. The second is matches per unemployed laborer, which is relevant to laborers seeking a job; it is their probability of finding one. Thus a firm that pays ϕ_t in order to post a single vacancy for talent receives not 1 but instead $m(\theta_t)$ talented workers the next period:

$$t' = t + m(\theta_t)v_t$$

$$l' = l + m(\theta_l)v_l$$

Firm's Problem

The firm faces a dynamic optimization problem in which it discounts future value at rate r and also faces hazard rate δ. It will be easier to analyze the problem recursively. If a firm currently has running total investment I, talent t, and untalented labor l, it faces the following problem:

$$
\begin{aligned}
V(I,t,l) = \max_{v_t,v_l,i} &[R(I,t,l) - w_t t - w_l l - \phi_t v_t - \phi_l v_l - \eta i^2 + \frac{1-\delta}{1+r} V(I',t',l')] \\
I' &= I + i \\
t' &= t + m(\theta_t) v_t \\
l' &= l + m(\theta_l) v_l
\end{aligned}
$$

Here we must redefine $R(I,t,l)$ and restrict attention to revenue functions that satisfy

$$
\frac{\partial R}{\partial t} = (1-\alpha_t)\frac{R}{t}
$$

$$
\frac{\partial R}{\partial l} = (1-\alpha_l)\frac{R}{l}
$$

We do not necessarily need revenue to be constant returns to scale in labor (see above, where revenue is in fact increasing returns to scale). Thus we do not need $\alpha_t + \alpha_l = 1$. However, as we will see in the main theorem of this section, equilibrium in this economy will only obey clean properties if the marginal product of each labor sector is a constant fraction of average output attributed to that sector.

Laborer's Problem

Here we will briefly provide a deeper understanding of the objects $E(w,\theta)$ and $U(w,\theta)$, the respective values to laborers of being employed and unemployed. Employed workers receive wages as long as their employing companies continue to survive (firms face a hazard rate δ). Unemployed workers receive unemployment benefits b. Laborers discount at the same rate r as firms. Assume for a moment that wages offered are constant across all firms. We thus arrive at the following result for laborers.

For each worker type $s = \{t, l\}$, the value of being employed is given by the following recursive equation:

$$E_s = w_s + \frac{1}{1+r}(\delta U_s + (1-\delta)E_s)$$

That is, an employed laborer receives a wage today. Then tomorrow the laborer is unemployed with probability δ and employed with probability $1-\delta$. Rearranging yields the first equation above. The value of being unemployed is given by the following recursive equation:

$$U_s = b + \frac{1}{1+r}(p(\theta_s)(1-\delta)E_s + p(\theta_s)\delta U_s + (1-p(\theta_s))U_s)$$

An unemployed laborer receives benefits b. Then tomorrow there are three outcomes. Either (1) no job offer arrives, (2) a job offer arrives but the firm offering that job succumbs to its hazard rate, or (3) a job offer arrives and the firm survives until next period. In (1) and (2) the laborer remains unemployed tomorrow, while in (3) the laborer becomes employed tomorrow.

Rearranging these two equations, we arrive at:

$$U_s = \frac{b(1+r)(r+\delta) + p(\theta_s)(1-\delta)w_s(1+r)}{r(r+\delta) - p(\theta_s)(1-\delta)(r+2\delta)} \tag{3.5.1}$$

$$E_s = \frac{w_s(1+r) + \delta U_s}{r+\delta} \tag{3.5.2}$$

With a system of two equations and two unknowns, we have been able to define the value functions for laborers as a function of the equilibrium wage and market tightness ratio. From this point forward we will suppress the arguments of each object, and only be interested in values in equilibrium. We need to pin down equations for the equilibrium wage and market tightness ratio, and turn to the problem of the firm to accomplish this.

Wage Determination

Firms that post vacancies receive applications from workers, and then hire workers. After matching occurs, the worker and firm Nash bargain to determine how to split the surplus that results from the matching. This is contrary to the baseline formulation, wherein Nash bargaining is conducted only between laborers and nascent firms. Now there is a

Nash bargaining stage between an individual firm and an individual worker upon every single hire. This formulation is numerically less tractable, but gives rise to an interesting analytical result. For $s = \{t, l\}$, the Nash bargaining condition is given by:

$$(1 - \chi)[E(w_s) - U(w_s)] = \chi \frac{dV(I', t', l')}{ds'} \tag{3.5.3}$$

For simplicity, the continuation probability $(1 - \delta)$ is omitted by definition from the RHS. Here $E(w)$ and $U(w)$ denote the value to a laborer of being employed and unemployed, respectively, when the wage is w. Since (I, t, l) appears on the RHS of this equation, we might suspect that the wage paid to a laborer by a firm with resources (I, t, l) should be a function of I, t, and l, but we will show this not to be the case: Rather, there is a single wage paid to skilled labors, and a single wage paid to unskilled labor. Intuitively this follows from the fact that under the optimal hiring policy for a firm, $\frac{dV(I', t', l')}{ds'}$ is constant across I, t, l in equilibrium, but it will be proven below. Observe that our formulation preserves the notion of a Hosios condition. Since our matching function is canonical, there remains the standard definition of matching share of workers, given by

$$S(\theta_s) = \frac{u_s}{M(u_s, v_s)} \frac{\partial M(u_s, v_s)}{\partial u_s} = \frac{\theta m'(\theta)}{m(\theta)}$$

Thus the standard Hosios condition in each sector is given by

$$\chi = S(\theta_s)$$

First, observe that since it is highly unlikely for different sectors (with different supplies and demands for labor) to converge to the same market tightness ratio, unless χ is different in each sector, then the Hosios condition is unlikely to hold. Second, since firms earn profits in this economy, the Hosios condition is insufficient for guaranteeing efficiency.

Proceeding in our study of wage determination in this environment, let us begin solving the firm's problem. The first order condition for each labor sector is given by

$$\frac{\phi_s}{m(\theta_s)} = \frac{1 - \delta}{1 + r} \frac{\partial V(I', t', l')}{\partial s'}$$

Since θ_s is constant in equilibrium, we observe from this FOC that $\frac{\partial V(I,t,l)}{\partial s} = \frac{\partial V(I',t',l')}{\partial s'}$. Thus the marginal value of an additional laborer to the firm must be constant in equilibrium. The intuition is that as firms grow, they hire until the last worker hired has value

$$\frac{\phi_s(1+r)}{m(\theta_s)(1-\delta)}$$

Since this is constant across time, the value of the last laborer hired must be constant across time.

The envelope theorem result for this economy is given by

$$\frac{\partial V}{\partial s} = \frac{1+r}{r+\delta}\left[\frac{\partial R}{\partial s} - w_s - \frac{\partial w_s}{\partial s}s\right] \tag{3.5.4}$$

Why the differential terms for the wage? This follows from the fact that firms are individually Nash bargaining with workers. Every time they hire a new worker, the marginal value of the last worker falls and thus the wage falls. But firms pay the same wage to all of their employees. Thus the wage falls for their entire stock of labor in a given sector, s.

Recall the revenue function is constructed to satisfy:

$$\frac{\partial R}{\partial s} = (1-\alpha_s)\frac{R}{s}$$

Combining this equation with the envelope theorem and the FOC yields the pricing function pinning down wages as a function of current revenue:

$$(1-\alpha_s)R = sw_s + \frac{\partial w_s}{\partial s}s^2 + \frac{s\phi_s}{m(\theta_s)}\frac{r+\delta}{1-\delta}$$

We now turn to the Nash bargaining equation. Combining the Nash bargaining equation with the envelope result above yields a differential equation in w_s:

$$w_s = \chi\frac{(1-\alpha_s)R}{s} + \frac{1-\chi}{1+r}rU_s - \chi\frac{\partial w_s}{\partial s}s$$

The solution to this differential equation must take into account the fact that $R()$ is a function of s, and U_s is a function of w_s which is a function of s. We have now arrived at the conditions on E_s, U_s, w_s, θ_s, which are jointly determined in equilibrium.

3.6. Conclusion

Despite its many comparative advantages in the film and television industry, California has seen a flight from Hollywood over the past decade as productions have been locating to other states with more favorable tax incentives. California established the 2009 California Film and Television Production Tax Credit in response to a series of domestic production incentives that were introduced in other states in 2005. Still, California remains at a competitive disadvantage relative to other states, namely its biggest rival, New York, which has a $420 million annual cap on tax credits and returns close to 30% to productions. California's 2009 program offered a tax credit of 20%, or 25% for indie films and relocating TV series, with an annual cap of $100 million—less than a quarter of New York's cap.

The result has been an erosion of California's local entertainment workforce. The Milken Institute finds that between 2004 and 2012, California's filmed production employment levels dropped from 152,525 to 136,388 while New York added 10,675 jobs. The economic impact of this runaway production is significant, as these are high-paying jobs that support a variety of other local markets.

Los Angeles has spent over a century developing the world's largest production infrastructure. The fact that productions are being lured away speaks to the importance of tax incentives to producers. It also raises questions about how tradeoffs between tax incentives and production infrastructure can influence the locational decisions of productions in theory. The current paper examines this question in the context of television productions, which have the characteristic of growing and changing over the course of a life-cycle. The category of "production infrastructure" is narrowed to encompass labor, specifically the regional availability of talent.

We have formulated a dynamic search and matching problem with heterogeneous TV firms and heterogeneous labor supplies and shown how it can help us analyze the regional location decision faced by television productions. Two models were considered: one with analytical tractability, and one with numerical tractability. Endogenously dynamic marginal products of labor were introduced through a novel Cobb-Douglas revenue function whose exponents vary with a firm's total level of investment.

The numerical analysis, subject to a baseline calibration, showcases the following properties. Increasing matching efficiency increases employment within the region and exerts downward pressure on employment in the other region. This is contrary to certain calibrations in Lutgen et al., but the structure of our numerical calibration means the results cannot be directly compared. In particular, rising employment in one region is not synonymous with falling unemployment rates, because laborers are not mobile. The remaining numerical results are consistent with intuition. The value of entering a region monotonically decreases with the fraction of TV firms entering that region, precisely because a higher measure of firms makes the region more competitive. As desired, in the case when two regions are identical, TV firms are indifferent between the two regions when half the firms are in each region. As a region becomes more attractive (lowering taxes, increasing matching efficiency), entrants require more firms to be in that region before becoming indifferent.

In the analytically tractable formulation, we show that the results of Felbermayr et al. by and large hold in a dynamic, ex post heterogeneous setting, which in itself is a success. It is shown that the results will not hold in a multi-region, mobile-firm setting.

There are a number of promising directions for continued work. (1) Foremost is bringing the analytically tractable formulation to a numerical setting, for a more natural fit with the traditional literature. (2) An interesting extension to be considered is to allow for labor mobility in addition to firm mobility, as we were motivated by creating a theory that could inform government policy with the purposes of attracting entertainment jobs and innovation. (3) In the context of the existing model, there is an interesting optimal policy problem. What is the optimal tax credit that should be offered as a function of labor availability and relative scarcity of talent? How does optimal policy change allowing for *temporary* tax breaks? This second question taps into an interesting component of the regional selection problem, and remains relatively unstudied in the heterogeneous firm literature. When trying to attract new businesses or workers, governments are often trying to change structural parameters of their economy. Businesses and workers may be savvy to the fact that such changes are potentially temporary, or to the fact that governments face a time inconsistency problem. In light of these issues, we propose the optimal government policy problem as a future vein of study.

Acknowledgements

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Appendix 3.A.
Plots

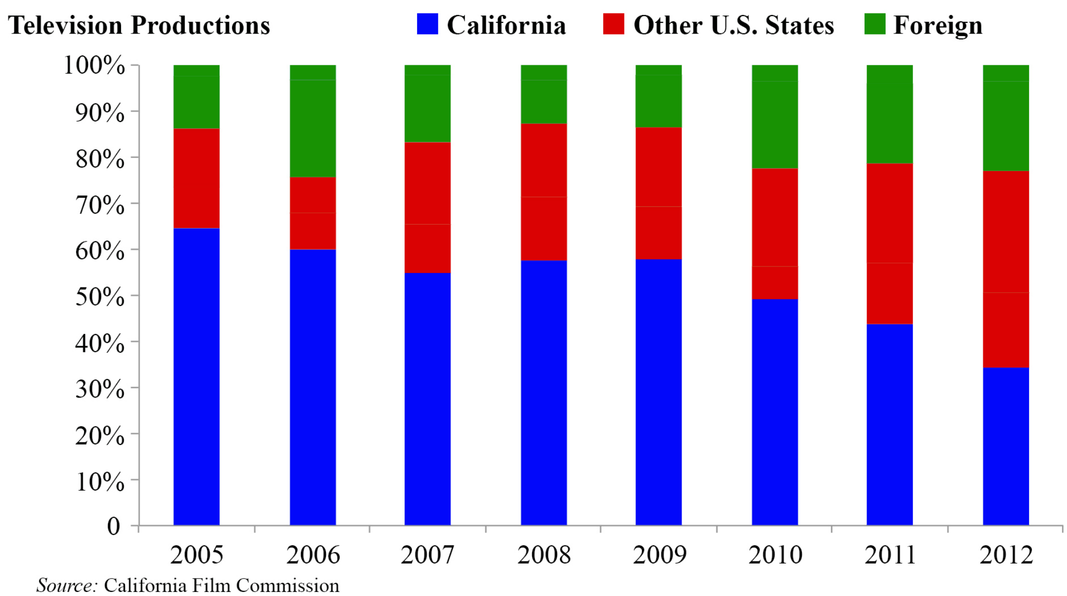

Figure 16: Decline in hour-long dramas filmed in California; adapted from Klowden et al. (2014).

State	2004	2005	2006	2007	2008	2009	2010	2011	2012
California	152525	146678	143448	146061	150069	137817	140902	139902	136388
New York	43085	45899	46901	46934	49477	50310	51388	52356	53760
Texas	14168	13970	14590	14294	14586	15068	14780	15239	16250
Florida	15369	15392	14531	14287	12615	11711	10975	10988	11037
Georgia	6306	6169	6059	6233	6253	6855	6285	6205	6519
Tennessee	6714	6656	6704	6858	7110	6473	6604	5914	6032
Louisiana	3768	4081	3499	4345	5155	3090	4174	3777	5378
North Carolina	3392	3923	3803	3635	3406	3545	3265	4018	4054
New Mexico	1445	1978	2721	3209	3335	2897	3103	2536	3176
Nevada	2013	1943	2296	2759	2612	2244	2268	2377	2525

Table 4: Absolute employment numbers in filmed production. Adapted from Klowden et al. (2014). Sources: Moody's, Bureau of Labor Statistics.

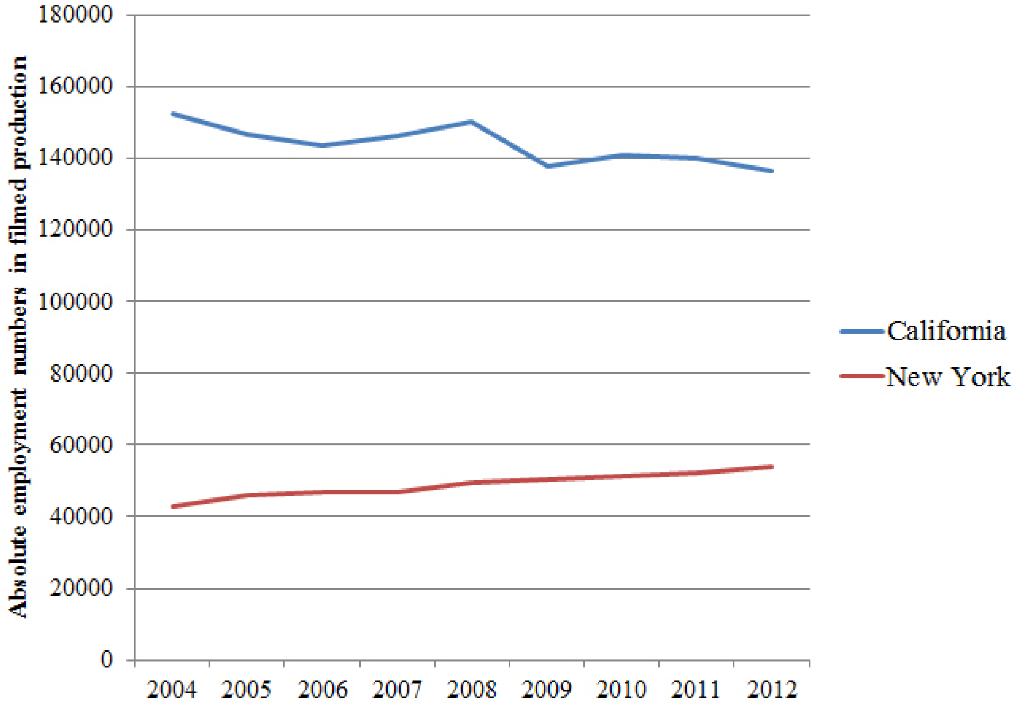

Figure 17: Absolute employment numbers in filmed production, CA and NY.

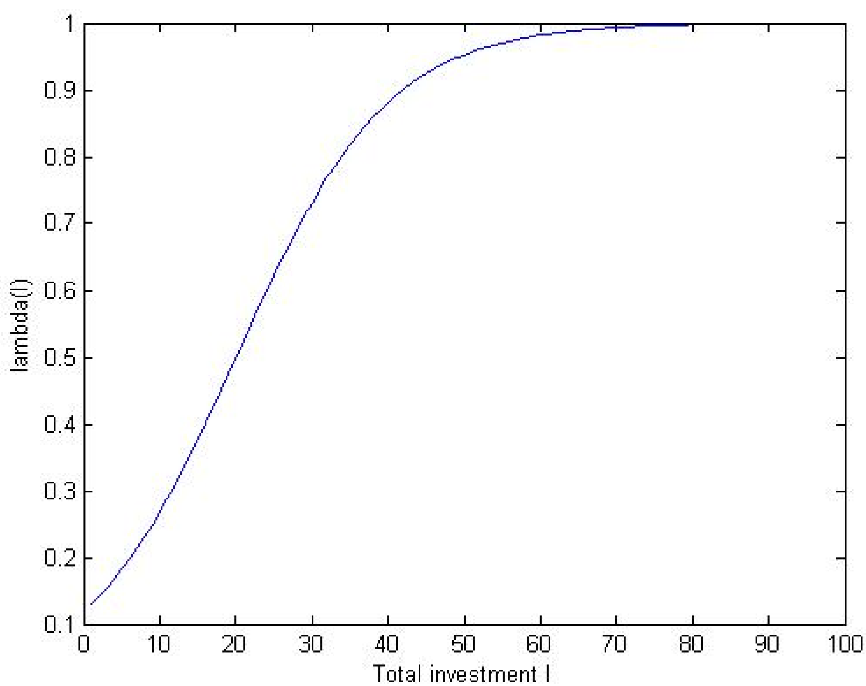

Figure 18: Technological capability as a function of total investment.

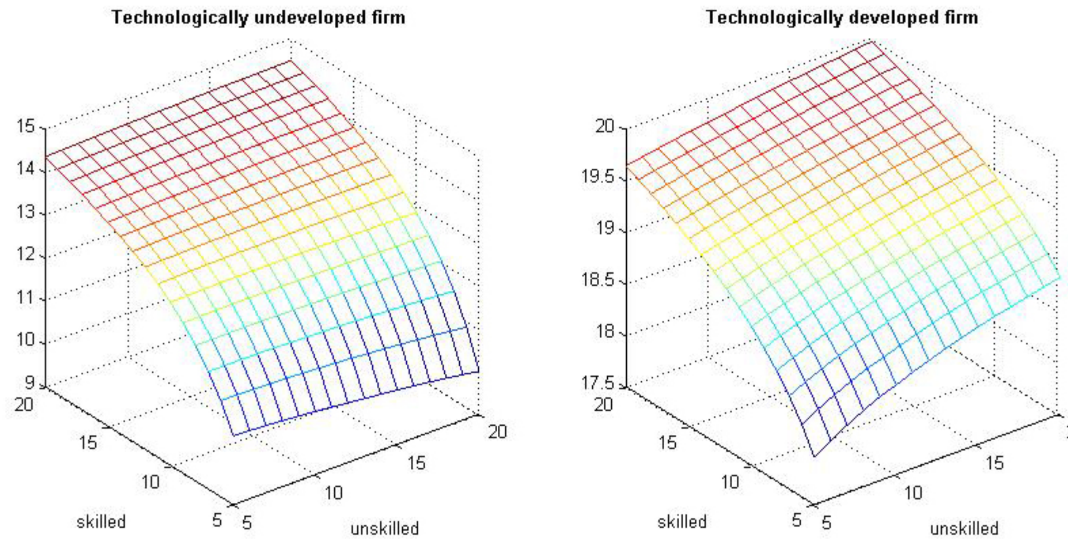

Figure 19: Comparing value function convexities between low-tech and high-tech TV firms.

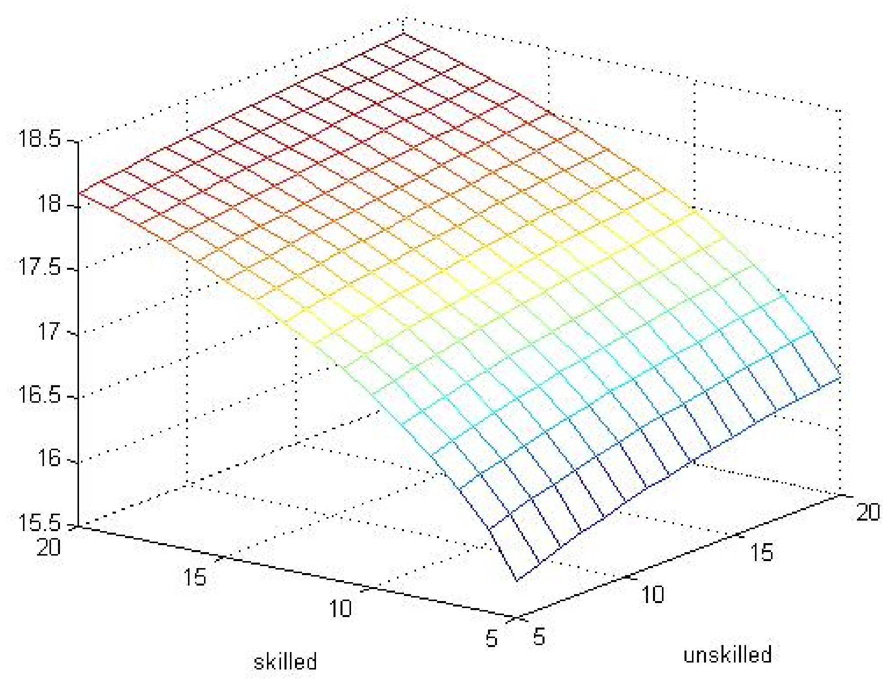

Figure 20: Value function of mid-developed TV firm by labor force composition.

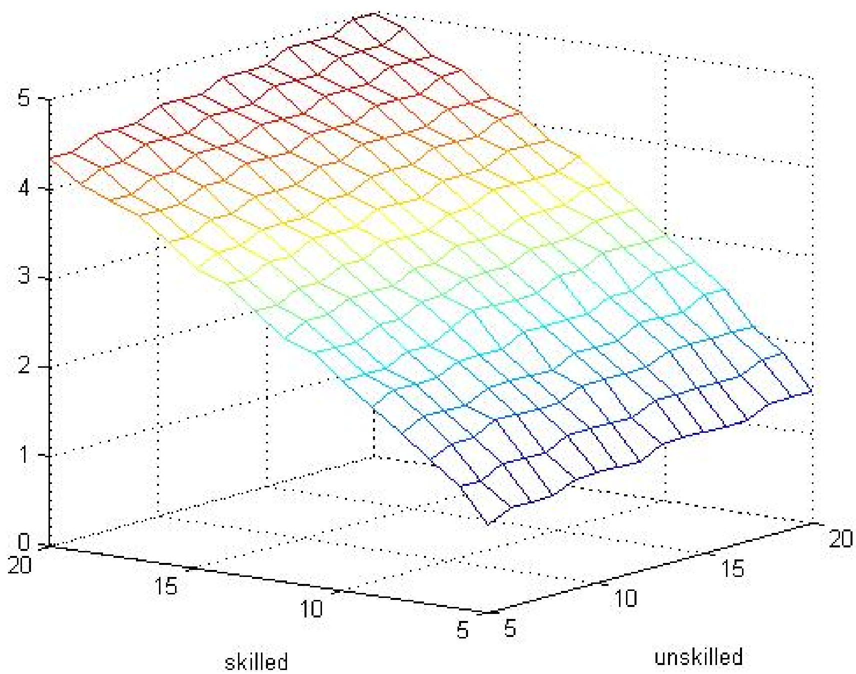

Figure 21: Optimal investment policy of mid-developed firm by labor force composition.

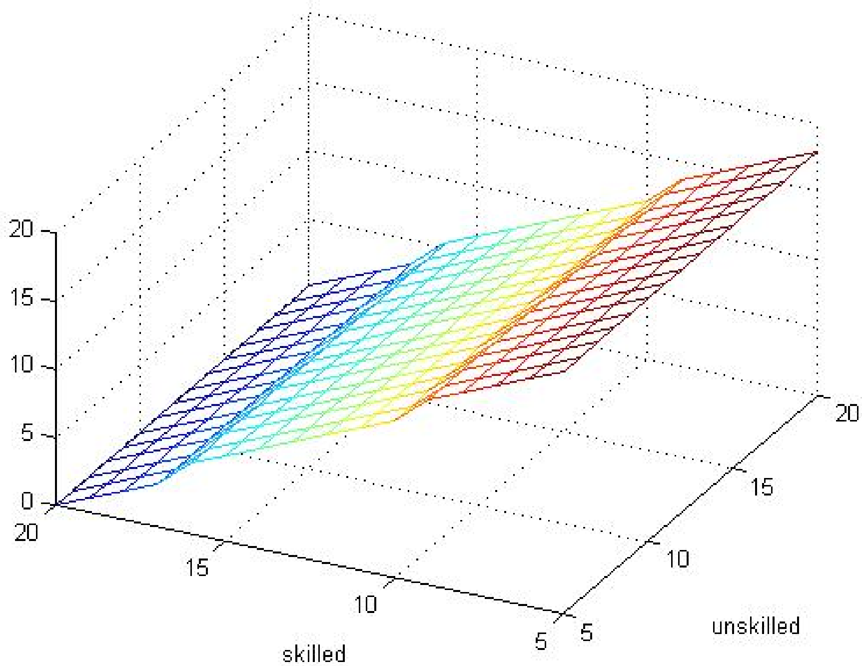

Figure 22: Optimal skilled hiring policy of mid-developed TV firm by labor force composition.

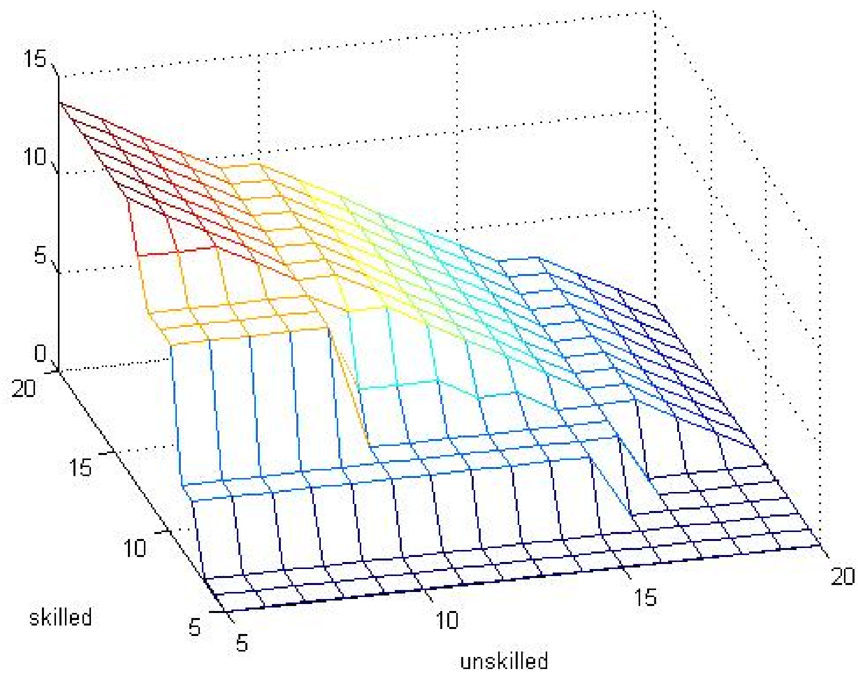

Figure 23: Optimal unskilled hiring of mid-developed TV firm by labor force composition.

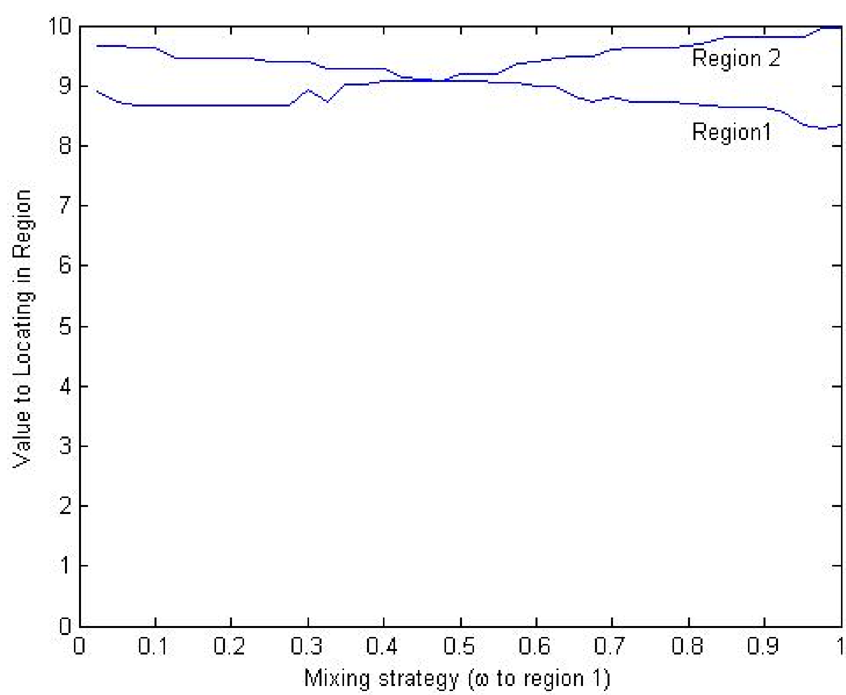

Figure 24: Value to locating in each region as function of mixing strategy (case 1).

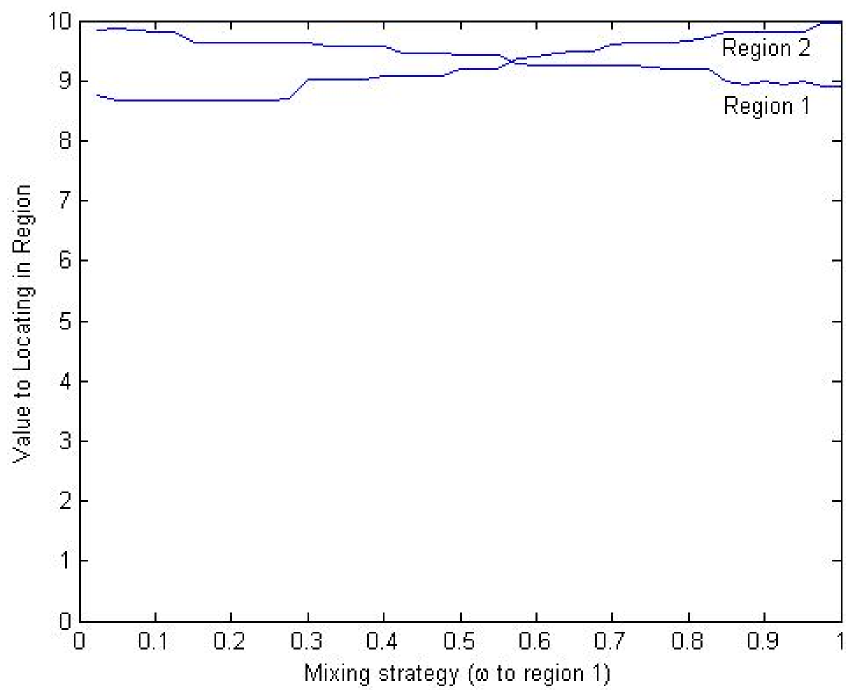

Figure 25: Value to locating in each region as function of mixing strategy (case 2).