Self-Protection and Insurance with Interdependencies

Alexander Muermann
Institute of Risk Management and Insurance
Vienna University of Economics and
Business Administration
alexander.muermann@wu-wien.ac.at

Howard Kunreuther
The Wharton School
of the University of Pennsylvania
kunreuth@wharton.upenn.edu

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Risk Management and Decision Processes Center
The Wharton School, University of Pennsylvania
3730 Walnut Street, Jon Huntsman Hall, Suite 500
Philadelphia, PA, 19104
USA
Phone: 215-898-4589
Fax: 215-573-2130
http://opim.wharton.upenn.edu/risk/

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Self-Protection and Insurance with Interdependencies*

Alexander Muermann† Howard Kunreuther‡

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Abstract

We study optimal investment in self-protection of insured individuals when they face interdependencies in the form of potential contamination from others. If individuals cannot coordinate their actions, then the positive externality of investing in self-protection implies that, in equilibrium, individuals underinvest in self-protection. Limiting insurance coverage through deductibles or selling "at-fault" insurance can partially internalize this externality and thereby improve individual and social welfare.

Key Words externality, mitigation, insurance

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†Muermann: Institute of Risk Management and Insurance, Vienna University of Economics and Business Administration, Nordbergstraße 15, A-1090 Wien, AUSTRIA, email: alexander.muermann@wu-wien.ac.at

‡Kunreuther: The Wharton School of the University of Pennsylvania, 3730 Walnut Street, 563 Jon M. Huntsman Hall, Philadelphia, PA, 19104, USA, email: kunreuth@wharton.upenn.edu.

1 Introduction

This paper is concerned with the question as to how much a consumer or firm who has purchased insurance should invest in loss reduction measures when they can be contaminated by others due to interdependencies. To motivate the analysis consider the following two examples:

Example 1: Ms. A is an owner of an apartment in a multi-unit building. Ms. A, who is required to purchase insurance as a condition for her mortgage, needs to determine how much she should invest in protective measures (e.g. a sprinkler system) to reduce the likelihood of a fire occurring in her apartment knowing that there is some chance that one of her unprotected neighbors could experience a fire that could spread to her apartment and cause damage even if she invests in these measures.

Example 2: An electric power company that has insured itself against customer law suits from business interruption due to power failures has to determine how much to invest in risk-reducing measures given the knowledge that other utilities in the power grid have not taken similar protective action. More specifically, one weak link in the system can wreak havoc over a much wider area than the customers served by the utility that failed. In the case of the August 2003, power failures in the northeastern US and Canada, the initiating event occurred in Ohio, but the worst consequences were felt hundreds of miles away. Experts believe these could have been greatly reduced if joint investments would have been undertaken to assure better system stability and decoupling properties across the boundaries of the several transmission and distribution companies that own and operate assets that make up the interconnected electric power grid in the northeastern US.[1]

More generally, our interest is in examining the equilibrium levels of investment in protective measures when there are interdependencies such as those illustrated by the above examples and when insurance rates are risk-based. We show that without coordination between those at risk, individuals and firms will, in equilibrium, underinvest in protection relative to the socially optimal decision due to the possibility of being contaminated by others. Restricting the amount of coverage an individual can take by requiring a deductible on insurance policies can encourage investment in protective measures and often improves both individual and social welfare.

To our knowledge no one has investigated optimal behavior by insureds when they have the op-

[1] See Kleindorfer (2004) for more details on interdependency of the grid and regulatory problems leading to underinvestment.

portunity to invest in protective measures and face interdependent risks. Ehrlich and Becker (1972) study the interaction between insurance and self-protection when there are no interdependencies. Schlesinger and Venezian (1986) focus on the joint production of insurance and self-protection in various market settings without interdependencies between insureds. The problem of optimal protection when there are interdependencies between agents has been recently studied by Kunreuther and Heal (2002) and Heal and Kunreuther (2005) when there is no insurance. They developed a game theoretic model for these interdependent security problems where there are two choices facing an agent: don't invest in protection at all or invest in full protection. For the case where there are negative externalities due to the possibility of contagion from others, they show that there can be two Nash equilibria—either everyone invests in protection or no-one invests. The key point is that the incentive that any agent has to invest in risk-reduction measures depends on how she expects the others to behave in this respect. If she thinks that they will **not** invest in protection, then this reduces the incentive for her to do so. On the other hand should she believe that others will invest in risk reducing measures, then it may be best for her to also do so. So there may be an equilibrium where no-one invests in protection, even though all would be better off if they had incurred this cost.

At the core of this problem is a stochastic negative externality—the possibility of being adversely affected by others in the system who have not invested in protection and hence pose a threat to others. To the extent that there is a reluctance to invest in mitigation measures because others have not taken similar measures, there is the potential for catastrophic losses. More specifically insured losses from natural disasters have increased significantly in recent years in part because many homeowners in hazard-prone areas have not invested in mitigation measures. One of the reasons that these measures may not have been adopted is because other homes in the area had not taken similar protective measures and could cause damage to the property even if it was mitigated. For example, an earthquake could cause a home that was not reinforced to collapse and severely damage a neighboring structure that was protected and/or a water heater that was not appropriately strapped could topple, causing a fire that could spread to other homes in the area (Levenson 1992).

On a related note, suppose a family in New Orleans was considering whether to elevate its house to reduce future water losses from hurricane damage. The family may opt not to invest in this mitigation measure if it had purchased flood insurance and this was the only structure raised on stilts. The house would look like an oddity in a sea of homes at ground level. Should the family choose to move, it would be concerned that the resale value of their home would be lower because the house was different from all the others. Behaviorally, there is a tendency not to think about a disaster until after it happens. The family may thus reason that it would be difficult to convince potential buyers that elevating their house actually increases its property value (Kunreuther 2006).

The interdependency problem we are studying raises the question as to the benefits of coordinating individuals' and firms' protective decisions so that one can reduce the externalities due to contamination and hence improve both individual and social welfare. In this sense it is related to the study by Shavell (1991) who investigated the optimal decision by individuals to protect their property against theft, acting alone or collectively, when precautions are observable (e.g. iron bars on a window) or unobservable (e.g. use of a safe for storing valuables). Ayres and Levitt (1998) have demonstrated the social benefits of protection when individuals invest in unobservable precautionary measures. They focus on the Lojack car retrieval system that criminals cannot detect. This generates positive externalities that lead to a sub-optimal level of private investment.

The paper is organized as follows. We first consider the case of two identical individuals (or firms) where there is no possibility of contamination from one individual to another and each individual has an opportunity to invest in mitigation to reduce its losses with premium reductions reflecting the reduced level of risk. We label this base case the **No Contamination** case. We compare this base case with a situation where there can be contamination between the two parties and where the two parties coordinate their actions. This case is labeled **Contamination — First Best**. It will be compared with a situation where the two parties cannot coordinate their actions and thus each party makes a decision independent of the other. This case is labeled **Contamination — Second Best**. We then turn to regulatory mechanisms that provide stronger incentives for agents to invest in self-protection and thereby internalize the externality if the two parties face the possibility of contamination and cannot coordinate their actions. We show that welfare can be improved by either a required deductible on each insurance policy or by making agents liable for losses to others and providing "at-fault" rather than no-fault insurance coverage. The concluding section discusses the policy implications of these findings by highlighting the importance of coordination between agents either voluntarily or through external involvement such as building codes. We also suggest directions for future research.

2 Model

There are two identical agents, i and j, who maximize expected utility with respect to an increasing, concave utility function $u(\cdot)$.[2] Each policyholder has initial wealth w_0 and is exposed to a loss of size L with probability p_0. There is a market for self-protection and a market for insurance. Investing in self-protection reduces the loss probability and investing in insurance transfers wealth from the no-loss to the loss state. The cost of reducing the loss probability to $p_i \leq p_0$ is given by a cost function $\gamma(\Delta p) = \gamma(p_0 - p_i)$ where $\gamma(0) = 0$, $\gamma' > 0$, and $\gamma'' > 0$. The policyholder can purchase insurance coverage I for an actuarially fair premium P. We assume that there is no moral hazard problem, i.e. the agents' investments in protection is verifiable and contractible by the insurer.

Optimal Insurance Coverage. As insurance is actuarially fair, it is optimal for the risk-averse agent to purchase full insurance, i.e. $I^* = L$, for any level of investment in self-protection. Hence one can investigate the decision on how much self-protection to purchase under conditions of no contamination and contamination independent of the insurance decision. Furthermore since individuals are fully protected by insurance they do not face any risk. They will thus determine their optimal amount of self-protection by maximizing their level of final wealth which, in this case, is equivalent to maximizing their expected utility of wealth. This equivalence does not hold if insurance coverage is restricted and individuals therefore face risk. The optimal level of self-protection is then derived under the maximization of expected utility of final wealth (see Section 3).

2.1 No Contamination

We first review the situation in which one individual cannot be contaminated by the other. As noted above, the optimal amount of self-protection and therefore the optimal loss probability p^* is determined by maximizing the value of final wealth

$$\max_p W(p) = w_0 - \gamma(p_0 - p) - pL$$

[2] One obtains the same qualitative results when considering n rather than two individuals.

where $P = pL$ is the actuarially fair premium for full coverage. The first and second derivatives with respect to p is $W'(p) = \gamma'(p_0 - p) - L$ and $W''(p) = -\gamma''(p_0 - p) < 0$. The objective function is thus globally concave which implies that we either have a corner solution $p^* = p_0$ if $\gamma'(0) \geq L$ or otherwise the optimal loss probability $p^* < p_0$ is determined by the first order condition

$$\gamma'(p_0 - p^*) = L. \tag{1}$$

The individual thus equates the marginal cost of the loss reduction, $\gamma'(\Delta p)$, with the marginal benefit in premium reduction, L. We now assume that $\gamma'(0) < L < \gamma'(p_0)$ which implies an inner solution $0 < p^* < p_0$. Note that if $\gamma'(p_0) < L$ then $p^* = 0$ because the marginal cost of eliminating the probability of a loss is sufficiently small relative to the magnitude of the loss itself that it is worth investing so there is no exposure to this risk. Similarly if $\gamma'(0) > L$ then the marginal cost of investing in any protection is so high relative to the benefits in reducing the expected loss that it is optimal not to commit any funds to mitigation.

2.2 Contamination

In this section, we introduce the possibility that one agent can be contaminated by the other agent. Denote by $q(p_j)$ the likelihood that agent i is contaminated by the other agent, j, as a function of the other agent's loss probability p_j. Contamination thus introduces an externality between the two agents in the sense that the decision of one policyholder to invest in protection affects the decision of the other policyholder. We assume that contamination is "perfect" in the sense that if a loss is incurred by one policyholder it spreads with probability one to the other policyholder, i.e. $q(p_i) = p_i$ and $q(p_j) = p_j$.[3] The loss and final wealth distribution faced by policyholder i is

event	prob	final wealth
loss	$p_i + (1 - p_i) p_j$	$w_0 - \gamma(p_0 - p_i) - P - L + I$
no loss	$(1 - p_i)(1 - p_j)$	$w_0 - \gamma(p_0 - p_i) - P$

where the actuarially fair premium is given by $P = (p_i + (1 - p_i) p_j) I$.

[3] In Appendix 7.1 we treat the general case by allowing for less-than-perfect contamination. We show that the same qualitative results obtain under the conditions $q(0) = 0$, $q(p) \leq p$, $0 \leq q' \leq 1$, and $q'' \geq 0$.

As above, given that insurance coverage is actuarially fair, it is optimal for the policyholder to purchase full insurance, $I^* = L$, independent of the amount invested in self-protection. Under full coverage, policyholder i's level of final wealth is given by

$$W_i = W(p_i, p_j) = w_0 - \gamma(p_0 - p_i) - (p_i + (1 - p_i)p_j)L.$$

In the following two subsections, we consider the optimal investment in self-protection under the first-best and second-best scenarios in which policyholders can and cannot, respectively, contract on the level of investment in protection.

First-Best. If policyholders can contract on the externalities, i.e. they jointly determine and implement p_i and p_j, the Coase theorem applies and the optimal solution is given by the socially optimal level that maximizes the aggregate level of final wealth

$$W_i + W_j = 2w_0 - \gamma(p_0 - p_i) - \gamma(p_0 - p_j) - 2(p_i + (1 - p_i)p_j)L.$$

The first and second derivative of the aggregate level of wealth with respect to p_i is given by

$$\begin{aligned} \frac{\partial W_i + W_j}{\partial p_i} &= \gamma'(p_0 - p_i) - 2(1 - p_j)L \\ \frac{\partial^2 W_i + W_j}{\partial p_i^2} &= -\gamma''(p_0 - p_i) < 0. \end{aligned}$$

The aggregate level of wealth is thus globally concave which implies a unique solution $p_i^*(p_j)$ for each p_j. As the maximization problem is symmetric in i and j, let p_{FB}^* denote the optimal solution which is determined by $p_{FB}^* = p_i^*(p_{FB}^*) = p_j^*(p_{FB}^*)$. If $\gamma'(0) \geq 2(1 - p_0)L$, then it is optimal not to invest in protection, i.e. $p_{FB}^* = p_0$. Note that $2(1 - p_0)L$ represents the expected joint loss to individuals i and j if neither party invests in protection. In this situation the marginal cost of investing even a penny in protection is greater than the marginal benefit of the joint reduction in losses to individuals i and j from incurring this cost. Note that the smaller p_0 is, the more likely one invests in protection for any given value of $\gamma'(0)$ because the marginal benefits to each individual of the other investing in mitigation is $(1 - p_0)L$ which increases as p_0 decreases. Otherwise, the optimal solution is determined by the first-order condition

$$\gamma'\left(p_0 - p_{FB}^*\right) = 2\left(1 - p_{FB}^*\right) L. \tag{2}$$

We can interpret this condition by rearranging it into

$$\gamma'\left(p_0 - p_{FB}^*\right) + p_{FB}^* L = L + \left(1 - p_{FB}^*\right) L. \tag{3}$$

The left hand side of (3) is the marginal cost of investing in protection which is the sum of the marginal dollar cost, $\gamma'\left(p_0 - p_{FB}^*\right)$, and the marginal increase in the premium, $p_{FB}^* L$, due to indirectly increasing the likelihood of being contaminated by the other agent. The right hand side of (3) is the marginal benefit of investing in protection which is decomposed into the marginal reduction in premium, L, due to the reduced likelihood of a direct loss and the marginal reduction in premium, $\left(1 - p_{FB}^*\right) L$, due to the reduction in the likelihood of contaminating the other agent. The latter marginal benefit represents the benefit from internalizing the positive externality.

Second-Best. In this section, we examine the setting in which the two policyholders cannot contract on the level of investment in self-protection and determine the pure-strategy Nash-equilibria. Policyholder i's best response function $p_i^*\left(p_j\right)$ is given by

$$p_i^*\left(p_j\right) \in \arg\max_{p_i} W_i\left(p_i, p_j\right) = w_0 - \gamma\left(p_0 - p_i\right) - \left(p_i + \left(1 - p_i\right) p_j\right) L.$$

It therefore satisfies the first-order condition

$$\gamma'\left(p_0 - p_i^*\left(p_j\right)\right) - \left(1 - p_j\right) L = 0.$$

Differentiating with respect to p_j yields

$$-p_i^{*\prime}\left(p_j\right) \gamma''\left(p_0 - p_i^*\left(p_j\right)\right) + L = 0$$

i.e.

$$p_i^{*\prime}\left(p_j\right) = \frac{L}{\gamma''\left(p_0 - p_i^*\left(p_j\right)\right)} > 0. \tag{4}$$

Policyholder i's strategy is thus a strategic complement to policyholder j's strategy which implies that there are only symmetric pure-strategy Nash-equilibria.

If policyholder j reduces the loss probability to zero, i.e. $p_j = 0$, then there is no contamination to policyholder i and thus $p_i^*(0) = p^*$ which is implicitly determined by (1). Under the assumption $\gamma'(0) < L < \gamma'(p_0)$ we have an inner solution $0 < p_i^*(0) = p^* < p_0$. If policyholder j does not invest in self-protection, i.e. $p_j = p_0$, then policyholder i's best response is determined by

$$\gamma'(p_0 - p_i^*(p_0)) = (1 - p_0) L.$$

If $\gamma'(0) \geq (1 - p_0) L$ then policyholder i's best response is also to not invest in self-protection, i.e. $p_i^*(p_0) = p_0$. Otherwise, if $\gamma'(0) < (1 - p_0) L$ then $p_i^*(p_0) < p_0$.

Since $0 < p_i^*(0) = p_j^*(0) < p_0$ and since the best-response functions are increasing, they can only cross the 45 degree line an odd number of times. We thus conclude that if $\gamma'(0) < (1 - p_0) L$ then there exists an odd number of pure-strategy Nash-equilibria, $p_{SB}^* = p_i^*(p_{SB}^*) = p_j^*(p_{SB}^*)$, which are all inner solutions and determined by the condition

$$\gamma'(p_0 - p_{SB}^*) = (1 - p_{SB}^*) L. \tag{5}$$

If $\gamma'(0) \geq (1 - p_0) L$, then there also exists an odd number of pure-strategy Nash-equilibria with the only difference that the largest equilibrium is at the corner $p_{SB}^* = p_0$, i.e. there is no investment in self-protection in this equilibrium.

In both cases, the smallest and the largest equilibrium are stable with respect to a myopic adjustment process and the other equilibria alternate in terms of stability and instability. The stability condition is characterized by $p_i^{*\prime}(p_{SB}^*) < 1$ which, by equation (4) is equivalent to $\gamma''(p_0 - p_{SB}^*) > L$. If the best-response functions are concave, then there exists a unique pure strategy Nash-equilibrium which is stable with respect to a myopic adjustment process. Figure 1 shows a situation in which there are three Nash-equilibria.

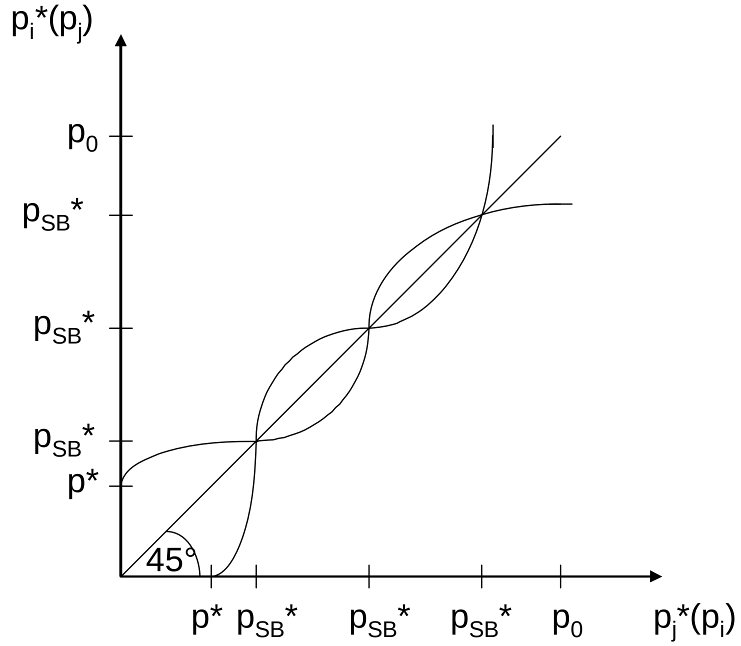

Figure 1

To interpret condition (5), we rearrange it into

$$\gamma' \left(p_0 - p_{SB}^*\right) + p_{SB}^* L = L. \tag{6}$$

The left hand side of (6) is the same as under the first-best scenario (3) , i.e. the sum of the marginal dollar cost, $\gamma' \left(p_0 - p_{SB}^*\right)$, and the marginal increase in the premium, $p_{SB}^* L$, due to indirectly increasing the likelihood of being contaminated by the other agent. The right hand side of (6), however, differs from the first-best scenario (3). The only marginal benefit of investing in protection is the marginal reduction in premium, L, due to the reduced likelihood of a direct loss. As policyholders cannot contract on the level of investment in self-protection, it is not possible for a policyholder to benefit from the positive externality that his investment poses on the other policyholder as shown in equation (3) for the joint solution.

2.3 Comparison

In the following subsection, we compare the level of investment in any Nash equilibrium with both the one in the first-best scenario and the one if policyholders do not face contamination.

Comparing Second-Best with First-Best. In this section, we compare the optimal level of investment in self-protection in the first-best with the one in the second-best scenario. Suppose it is optimal to not invest in self-protection in the first-best world, i.e. $\gamma'(0) \geq 2(1-p_0)L$. Then it is also not optimal to invest in self-protection in the second-best world, as $\gamma'(0) \geq 2(1-p_0)L > (1-p_0)L$, since an individual does not take into account the positive externalities provided the others when making an investment decision. Now suppose it is optimal to invest in self-protection in the first-best world, i.e. $\gamma'(0) < 2(1-p_0)L$. The optimal solution is then determined by

$$\gamma'(p_0 - p_{FB}^*) = 2(1 - p_{FB}^*)L.$$

This implies

$$\gamma'(p_0 - p_{FB}^*) > (1 - p_{FB}^*)L$$

and condition (5) yields $p_{SB}^* > p_{FB}^*$. In any pure-strategy Nash-equilibrium the level of investment in self-protection is thus lower compared to the first-best scenario. The intuition behind this result can be derived from comparing the first-order condition (3) in the first-best scenario under contamination

$$\gamma'(p_0 - p) + pL = L + (1-p)L$$

with the first-order condition (5) in the second-best scenario under contamination

$$\gamma'(p_0 - p) + pL = L.$$

We note that in the second-best scenario it is not possible to internalize the marginal benefit of the policyholder's effect on the other policyholder, $(1-p)L$, and he therefore underinvests in self-protection compared to the first-best scenario.

Comparing First-Best with No-Contamination. Let us compare the optimal level of investment without contamination with the one in the first-best scenario with contamination. The first-order condition (1) under no contamination is

$$\gamma'(p_0 - p) = L$$

and the first-order condition (3) in the first-best scenario under contamination is

$$\gamma'(p_0 - p) + pL = L + (1-p)L.$$

By comparing the marginal costs and benefits of the two scenarios, we see that contamination adds both a marginal cost and a marginal benefit of investing in self-protection. The additional marginal cost, pL, is due to the indirect increase in the likelihood of being contaminated by the other agent while the marginal benefit, $(1-p)L$, is due to the internalized positive of effect on the other agent. This implies that investment in self-protection under contamination can be larger or smaller than under no contamination depending on whether the additional marginal benefit is larger or smaller than the additional marginal cost.

Under the condition $p_0 < 1/2$—which seems most relevant for insurance events—the additional marginal benefit is larger than the additional marginal cost of investing in self-protection. It is thus optimal to invest more in self-protection under the first-best scenario with contamination compared to the scenario in which agents cannot be contaminated which yields $p^*_{FB} < p^*$.

To be more precise, suppose it is optimal to not invest in self-protection if there is no contamination, i.e. $\gamma'(0) \geq L$. Then it may still be optimal to invest in self-protection in the first-best world with contamination since the condition for not investing is $\gamma'(0) \geq 2(1-p_0)L$ and $2(1-p_0)L > L$ for all $p_0 < 1/2$. Now suppose that it is optimal to invest in self-protection without contamination. Then policyholders invest more in self-protection in the first-best world with contamination as the first-order condition $\gamma'(p_0 - p^*) = L$ under no contamination implies

$$\gamma'(p_0 - p^*) < 2(1-p^*)L$$

for all $p_0 < 1/2$ and condition (3) yields $p^*_{FB} < p^*$.

Comparing Second-Best with No-Contamination. Let us now compare the optimal level of investment without contamination with the one in the second-best scenario with contamination. Suppose it is optimal to not invest in self-protection if there is no contamination, i.e. $\gamma'(0) \geq L$. Then it is also optimal to not invest in self-protection in the second-best world as $\gamma'(0) \geq L$ implies $\gamma'(0) > (1-p_0)L$. Now suppose that it is optimal to invest in self-protection without contamination. Then policyholders invest less in self-protection in the second-best world with contamination as the first-order condition $\gamma'(p_0 - p^*) = L$ implies

$$\gamma'(p_0 - p^*) > (1 - p^*) L$$

and condition (5) yields $p^*_{SB} > p^*$. In any pure-strategy Nash-equilibrium the level of investment in self-protection is thus lower compared to the scenario in which policyholders do not face possible contamination. The intuition behind this result can again be derived by comparing the first-order conditions (6) in the second-best scenario under contamination

$$\gamma'(p_0 - p) + pL = L$$

with the first-order condition (1) under no contamination is

$$\gamma'(p_0 - p) = L.$$

In the second-best scenario policyholders, by investing in self-protection, face the additional marginal cost of implicitly increasing the likelihood of being contaminated by the other policyholder, pL. The marginal cost equates the marginal benefit of investing in self-protection thus at a lower level of investment.

We summarize all findings of this section in Table 1 below under the assumption $p_0 < 1/2$.

$\gamma'(0) \in ...$	$]0, (1-p_0) L[$	$[(1-p_0) L, L[$	$[L, 2(1-p_0) L[$	$[2(1-p_0) L, \infty[$
No Contamination p^*	$p^* < p_0$	$p^* < p_0$	$p^* = p_0$	$p^* = p_0$
First-Best p^*_{FB}	$p^*_{FB} < p_0$	$p^*_{FB} < p_0$	$p^*_{FB} < p_0$	$p^*_{FB} = p_0$
Second-Best NE p^*_{SB}	$p^*_{SB} < p_0$ holds in all NE	$p^*_{SB} = p_0$ is a NE	$p^*_{SB} = p_0$ is unique NE	$p^*_{SB} = p_0$ is unique NE
Comparison	$p^*_{FB} < p^* < p^*_{SB}$	$p^*_{FB} < p^* < p^*_{SB}$	$p^*_{FB} < p^* = p^*_{SB}$	$p^*_{FB} = p^* = p^*_{SB}$

Table 1: Equilibrium levels of loss probabilities and comparison if $p_0 < 1/2$.

3 Improving Welfare

In the section above, we have shown that individuals inefficiently underinvest in self-protection if they cannot coordinate their activities. This raises the question about regulatory mechanisms that provide stronger incentives for agents to invest in self-protection and thereby improve both individual and social welfare. We show that this can be achieved by either restricting insurance coverage on each policy or if agents are liable for losses to others and at-fault insurance is provided as opposed to no-fault insurance.

3.1 Restricting Insurance Coverage

In this section, we show that restricting insurance coverage, e.g. by requiring a deductible in the insurance policy, can improve individual and social welfare in a second best world with contamination. With a deductible, each individual has to bear part of their own loss and is likely to have more of an incentive to invest in self-protection than if he had full insurance coverage. The additional investment in self-protection creates an extra marginal benefit, $(1-p)\,L$, through the positive externality that exists between individuals. In the following Proposition, we specify conditions under which this benefit outweighs the cost of bearing part of the loss and implies that partial insurance is optimal. It is important, however, that the deductible is enforced by some regulatory entity. In an unregulated environment, an insurer always will deviate by offering full coverage to attract all customers.[4]

Proposition 1 *Suppose that the stability condition $\gamma''(p_0 - p^*_{SB}) > L$ holds where p^*_{SB} is the loss probability in a Nash-equilibrium under full insurance coverage, implicitly defined by* (5)*. Then the optimally enforced deductible is strictly positive if and only if*

$$(1-p^*_{SB})^2\,L > \left(1-(1-p^*_{SB})^2\right)\gamma''(p_0 - p^*_{SB})\,. \tag{7}$$

Proof. See Appendix 7.2. ■

Imposing a strictly positive deductible and thereby forcing agents to invest more in self-protection can only be welfare-improving if the marginal benefit of internalizing the externality, i.e. $(1-p^*_{SB})\,L$,

[4] Since individuals face risk under restricted insurance coverage, the proof requires the maximization of expected utility of final wealth.

is relatively large compared to marginal cost of investing in self-protection. By substituting $\gamma'(p_0 - p^*_{SB}) = (1 - p^*_{SB}) L$ into the necessary and sufficient condition (7) we obtain

$$(1 - p^*_{SB}) L > \left(1 - (1 - p^*_{SB})^2\right) L \cdot \frac{\gamma''(p_0 - p^*_{SB})}{\gamma'(p_0 - p^*_{SB})}.$$

The left-hand side of this inequality reflects the marginal benefit of internalizing the externality whereas the right-hand side includes the marginal cost of investing in self-protection, as measured by the degree of convexity $\gamma''(p_0 - p^*_{SB}) / \gamma'(p_0 - p^*_{SB})$ of the cost function. We conclude that the optimally enforced deductible is more likely to be strictly positive the higher the marginal benefit of internalizing the externality $(1 - p^*_{SB}) L$ and the lower the convexity of the cost function $\gamma''(p_0 - p^*_{SB}) / \gamma'(p_0 - p^*_{SB})$.

This result should be contrasted with the case of terrorism insurance considered by Lakdawalla and Zanjani (2005) where protection by one target leads the terrorist to attack a less protected target. Protection thus creates a negative externality and an inefficient overinvestment in self-protection. A governmental subsidy of terrorism insurance can improve social welfare by discouraging investment in protection. In our case, there is a positive externality associated with investment in protection. Social welfare is now improved by limiting insurance through a deductible, thereby encouraging investment in protection.

Remark 2 *Note that inequality* (7) *does not depend on the specific form of the utility function. As shown in the proof of Proposition 1, any risk-averse agent will want to invest more in self-protection under a deductible policy than under full coverage if and only if* $p^*_{SB} < 1 - \sqrt{1/2}$ *which is implied by the necessary and sufficient condition* (7)*. This improves both individual and social welfare. The optimal deductible level depends on the specific form of the utility function, i.e. on the degree risk aversion.*

Put differently, if the probability of a loss under full insurance coverage is relatively large, i.e. $p^*_{SB} > 1 - \sqrt{1/2}$, limiting insurance by enforcing a deductible discourages investment in protection. This is related to the finding of Ehrlich and Becker (1972) who show that the absence of market insurance can discourage the investment in self-protection if the probability of a loss is relatively large.

3.2 At-Fault Insurance

Consider the case where an apartment owner is liable in case the fire in his apartment spreads over to the other apartment. At-fault insurance would then include coverage of losses to others.[5] For owner i, the overall premium is therefore $P = [p_i + p_i (1 - p_j)] L$. The best-response function of owner i is given by

$$p_i^* (p_j) \in \arg\max_{p_i} W_i (p_i, p_j) = w_0 - \gamma (p_0 - p_i) - p_i (2 - p_j) L.$$

All features of the best-response function and NE are equivalent to the second-best scenario in Section 2. The one important difference is that the equilibrium level(s) of loss probability, p_{AF}^*, is determined by the following condition

$$\gamma' (p_0 - p_{AF}^*) = (2 - p_{AF}^*) L. \tag{8}$$

Comparing equation (8) with the first-order condition in the First-Best case, that is equation (2),

$$\gamma' (p_0 - p_{FB}^*) = 2 (1 - p_{FB}^*) L < (2 - p_{FB}^*) L$$

implies $p_{AF}^* < p_{FB}^*$. In any pure-strategy Nash-equilibrium under at-fault insurance, agents inefficiently overinvest in self-protection. The intuition behind this result can be derived from comparing the first-order condition (3) in the first-best scenario under contamination

$$\gamma' (p_0 - p) + pL = L + (1 - p) L \tag{9}$$

with the first-order condition (8) in the at-fault scenario under contamination

$$\gamma' (p_0 - p) = L + (1 - p) L. \tag{10}$$

[5] This is not how current insurance practice operates. An insurer who provides protection to individual i is responsible for losses incurred by that agent no matter who caused the losses. One reason for this contractual arrangement between insurer and insured is the difficulty in assigning causality for a particular event. With respect to fire damage a classic case is H.R. Moch Co., Inc. v Rensselaer Water Co. 247N.Y.160, 159 N.E. 896 which ruled that "A wrongdoer who by negligence sets fire to a building is liable in damages to the owner where the fire has its origin, but not to other owners who are injured when it spreads". We are indebted to Victor Goldberg who provided us with this case.

Since the agent is not liable for being contaminated by the other agent under the at-fault scenario, the marginal cost of investing in self-protection is lower than in the first-best scenario scenario. Under at-fault, investing in self-protection does not indirectly increase the likelihood of being contaminated by the other agent and thereby increase the premium marginally by pL. The reduction in the marginal cost implies an inefficient level of overinvestment under at-fault insurance.

Welfare Comparison. When the efficient level of investment in self-protection is relatively high, i.e. p^*_{FB} is relatively low, the marginal cost and benefit under at-fault insurance, see equation (10) approximate the marginal cost and benefit in the first-best scenario, see equation (9). Under this condition, welfare can therefore be improved by at-fault insurance or alternatively by a required deductible, see condition (7) and $p^*_{SB} < p^*_{FB}$. If, however, the efficient level of investment in self-protection is relatively low, i.e. p^*_{FB} is relatively high, then the second-best scenario under no-fault will dominate at-fault insurance coverage.

4 Implications for Policy

The bundling of protection and insurance has a long history dating back to the factory mutuals founded in the early 19th century in New England (Bainbridge, 1952). These mutual companies offered factories an opportunity to pay a small premium in exchange for protection against potentially large losses from fire while at the same time requiring inspections of the factory both prior to issuing a policy and after one was in force. Poor risks had their policies canceled; premium reductions were given to factories that instituted loss prevention measures. For example, the Boston Manufacturers worked with lantern manufacturers to encourage them to develop safer designs and then advised all policyholders that they had to purchase lanterns from those companies whose products met their specifications. In many cases, insurance would only be provided to companies that adopted specific loss prevention methods. For example one company, the Spinners Mutual, only insured risks where automatic sprinkler systems were installed. The Manufacturers Mutual in Providence, Rhode Island developed specifications for fire hoses and advised mills to buy only from companies that met these standards.

Private insurers today should consider requiring protective measures as a condition for insurance with respect to standard homeowners coverage to reduce the negative externalities due to contagion. However, all insurers would have to find it in their financial interest to follow this strategy because of the contractual arrangements with respect to claims payments. As pointed out above any insurer who provides protection to individual i is responsible for losses incurred by this policyholder no matter who caused the damage. Without protection requirements by other insurers, a competitive insurer will have to charge premiums that reflect the actions of policyholders who are independently deciding how much to invest in protection.

One way of coordinating protective decisions of individuals is through a monopolistic insurer who can require the adoption of such measures or provide premium incentives for those at risk to adopt them to internalize the externalities due to interdependencies. A competitive insurer may not be able to do this as easily if others in the industry do not take similar actions. von Ungern-Sternberg (1996) provides an empirical study of the pricing and performance of insurance markets in Switzerland and compares the performance of competitive insurers in seven cantons of the country with local state monopolies in the 19 other cantons. The study finds that for very similar products the monopolies charge premiums that are 70 percent lower than for the competitive insurance, they spend substantially more on fire prevention and have much lower damage rates.

Some type of coordinative mechanism may also improve both individual and social welfare without having to rely on the power of the monopolist insurer. One option is for a well-enforced standard or regulation, such as a building code, that requires individuals and firms to adopt cost-effective protective mechanisms when they would not do so voluntarily.[6] One could also turn to the private sector to coordinate decisions through an industry association that stipulates that any member has to follow certain rules and regulations. For example, an apartment owners association could require that all residents in the building adopt certain fire protective measures such as installing a smoke alarm and/or a sprinkler system. The association could then arrange to purchase insurance for all units in the building where the premiums would reflect the required protection that would reduce the chances of a fire occurring. Today some apartment associations install smoke alarm and/or a sprinkler system in all their units as a way of detecting and extinguishing a fire at any early stage before it causes damage to the unit and spreads to surrounding ones.

[6] Some states require smoke alarm and/or sprinkler systems to be installed.

5 Future Research

There are a number of questions that could be studied both theoretically and empirically with respect to the linkages between undertaking investments in mitigation and purchasing insurance when there are interdependencies and hence negative stochastic externalities.

On the behavioral side there is considerable evidence that individuals misperceive the risk with respect to a disaster occurring. To the extent that individuals overestimate the probability, they will want to purchase full insurance even when the premiums are higher than the actuarially fair rate to the extent that $p'L > P(1 + \lambda)$ where p' is the perceived probability of the loss, P is the actuarially fair premium for full coverage and λ is the loading factor. It would be interesting to determine empirically what people's perception of the risk are for different hazards and how it compares to estimates by experts.

To date there is limited empirical data on how buyers and sellers consider the nature of the interdependencies in their decision making process. In what ways do homeowners and firms take into account the possibility of being adversely affected by others and/or the presence of weak links in the system when they make their decisions on how much insurance to purchase and what risk-reducing measures to adopt? Do insurers consider the nature of the interdependencies facing them when they market policies and set premiums for coverage against property damage and business interruption risk? We are not aware of any empirical evidence indicating that the impact of negative externalities on future losses are factored into insurers' coverage and pricing decisions. Yet we do know that the insurance industry is more concerned today than ever before of the importance of encouraging those at risk to adopt mitigation measures to reduce losses and is supporting research by organizations such as the Institute for Business and Home Safety to evaluate the cost-effectiveness of alternative protective measures (Wharton Risk Center Report, 2007).

On the theoretical side, it would be interesting to determine how the equilibrium solution with and without coordination is affected when agents are heterogenous so that some create more negative externalities than others. Can one induce tipping and cascading by inducing one agent to increase their investment in loss reduction measures through either subsidies or fines. Answers to these questions will provide more insight into the linkages between insurance and mitigation in a world where there are intedependencies.

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7 Appendix

7.1 The Case of General Contamination

In this section, we introduce the possibility that one agent can be contaminated by the other agent. Denote by $q(p_j)$ the likelihood that agent i is contaminated by the other agent, j, as a function of the other agent's loss probability p_j. We assume $q(0) = 0$, $q(p) \leq p$, $0 \leq q' \leq 1$, and $q'' \geq 0$. If $q(p) = 0$ for all p then there is no contamination. If $q(p) = p$ for all p, then contamination is "perfect" in the sense that if a loss is incurred by one policyholder it spreads with probability one to the other policyholder. Contamination thus introduces an externality between the two agents in the sense that the decision of one policyholder to invest in protection affects the decision of the other policyholder. The loss and final wealth distribution faced by policyholder i is

event	prob	final wealth
loss	$p_i + (1-p_i)\,q(p_j)$	$w_0 - \gamma(p_0 - p_i) - P - L + I$
no loss	$(1-p_i)(1-q(p_j))$	$w_0 - \gamma(p_0 - p_i) - P$

where the actuarially fair premium is given by $P = (p_i + (1-p_i)\,q(p_j))\,I$.

As above, given that insurance coverage is actuarially fair, it is optimal for the policyholder to purchase full insurance, $I^* = L$, independent of the amount invested in self-protection. Under full coverage, policyholder i's level of final wealth is given by

$$W_i = W(p_i, p_j) = w_0 - \gamma(p_0 - p_i) - (p_i + (1-p_i)\,q(p_j))\,L.$$

In the following two subsections, we consider the optimal investment in self-protection under the first-best and second-best scenarios in which policyholders can and cannot, respectively, contract on the level of investment in protection.

First-Best. If policyholders can contract on the externalities, i.e. they jointly determine and implement p_i and p_j, the Coase theorem applies and the optimal solution is given by the socially optimal level that maximizes the aggregate level of final wealth

$$W_i + W_j = 2w_0 - \gamma(p_0 - p_i) - \gamma(p_0 - p_j) - (p_i + (1-p_i)\,q(p_j))\,L - (p_j + (1-p_j)\,q(p_i))\,L.$$

The first and second derivative of the aggregate level of wealth with respect to p_i is given by

$$\begin{aligned}
\frac{\partial W_i + W_j}{\partial p_i} &= \gamma'(p_0 - p_i) - (1 - q(p_j) + (1-p_j)\,q'(p_i))\,L \\
\frac{\partial^2 W_i + W_j}{\partial p_i^2} &= -\gamma''(p_0 - p_i) - (1-p_j)\,q''(p_i)\,L < 0.
\end{aligned}$$

The aggregate level of wealth is thus globally concave which implies a unique solution $p_i^*(p_j)$ for each p_j. As the maximization problem is symmetric in i and j, let p_{FB}^* denote the optimal solution which is determined by $p_{FB}^* = p_i^*(p_{FB}^*) = p_j^*(p_{FB}^*)$. If $\gamma'(0) \geq (1 - q(p_0) + (1-p_0)\,q'(p_0))\,L$, then it is optimal not to invest in protection, i.e. $p_{FB}^* = p_0$. Note that $(1 - q(p_0) + (1-p_0)\,q'(p_0))\,L$ represents the expected joint loss to individuals i and j if neither party invests in protection. In this situation the marginal cost of investing even a penny in protection is greater than the marginal benefit of the joint reduction in losses to individuals i and j from incurring this cost. Otherwise, the optimal solution is determined by the first-order condition

$$\gamma'(p_0 - p_{FB}^*) = (1 - q(p_{FB}^*) + (1-p_{FB}^*)\,q'(p_{FB}^*))\,L. \tag{11}$$

We can interpret this condition by rearranging it into

$$\gamma'(p_0 - p_{FB}^*) + q(p_{FB}^*)\,L = L + (1-p_{FB}^*)\,q'(p_{FB}^*)\,L. \tag{12}$$

The left hand side of (12) is the marginal cost of investing in protection which is the sum of the marginal dollar cost, $\gamma'(p_0 - p_{FB}^*)$, and the marginal increase in the premium, $q(p_{FB}^*)\,L$, due to indirectly increasing the likelihood of being contaminated by the other agent. The right hand side of (12) is the marginal benefit of investing in protection which is decomposed into the marginal reduction in premium, L, due to the reduced likelihood of a direct loss and the marginal reduction in premium, $(1-p^*_{FB})\,q'(p^*_{FB})\,L$, due to the reduction in the likelihood of contaminating the other agent. The latter marginal benefit represents the benefit from internalizing the positive externality.

Second-Best. In this section, we examine the setting in which the two policyholders cannot contract on the level of investment in self-protection and determine the pure-strategy Nash-equilibria. Policyholder i's best response function $p^*_i(p_j)$ is given by

$$p^*_i(p_j) \in \arg\max_{p_i} W_i(p_i, p_j) = w_0 - \gamma(p_0 - p_i) - (p_i + (1-p_i)\,q(p_j))\,L.$$

It therefore satisfies the first-order condition

$$\gamma'(p_0 - p^*_i(p_j)) - (1 - q(p_j))\,L = 0.$$

Differentiating with respect to p_j yields

$$-p^{*\prime}_i(p_j)\,\gamma''(p_0 - p^*_i(p_j)) + q'(p_j)\,L = 0$$

i.e.

$$p^{*\prime}_i(p_j) = \frac{q'(p_j)\,L}{\gamma''(p_0 - p^*_i(p_j))} > 0. \tag{13}$$

Policyholder i's strategy is thus a strategic complement to policyholder j's strategy which implies that there are only symmetric pure-strategy Nash-equilibria.

If policyholder j reduces the loss probability to zero, i.e. $p_j = 0$, then there is no contamination to policyholder i and thus $p^*_i(0) = p^*$ which is implicitly determined by (1). Under the assumption $\gamma'(0) < L < \gamma'(p_0)$ we have an inner solution $0 < p^*_i(0) = p^* < p_0$. If policyholder j does not invest in self-protection, i.e. $p_j = p_0$, then policyholder i's best response is determined by

$$\gamma'(p_0 - p^*_i(p_0)) = (1 - q(p_0))\,L.$$

If $\gamma'(0) \geq (1 - q(p_0))\,L$ then policyholder i's best response is also to not invest in self-protection, i.e. $p^*_i(p_0) = p_0$. Otherwise, if $\gamma'(0) < (1 - q(p_0))\,L$ then $p^*_i(p_0) < p_0$.

Since $0 < p^*_i(0) = p^*_j(0) < p_0$ and since the best-response functions are increasing, they can only cross the 45 degree line an odd number of times. We thus conclude that if $\gamma'(0) < (1 - q(p_0))\,L$ then there exists an odd number of pure-strategy Nash-equilibria, $p^*_{SB} = p^*_i(p^*_{SB}) = p^*_j(p^*_{SB})$, which are all inner solutions and determined by the condition

$$\gamma'(p_0 - p^*_{SB}) = (1 - q(p^*_{SB}))\,L. \tag{14}$$

If $\gamma'(0) \geq (1 - q(p_0))\,L$, then there also exists an odd number of pure-strategy Nash-equilibria with the only difference that the largest equilibrium is at the corner $p^*_{SB} = p_0$, i.e. there is no investment in self-protection in this equilibrium.

In both cases, the smallest and the largest equilibrium are stable with respect to a myopic adjustment process and the other equilibria alternate in terms of stability and instability. The stability condition is characterized by $p^{*\prime}_i(p^*_{SB}) < 1$ which, by equation (4) is equivalent to $\gamma''(p_0 - p^*_{SB}) > q'(p^*_{SB})\,L$. If the best-response functions are concave, then there exists a unique pure strategy Nash-equilibrium which is stable with respect to a myopic adjustment process.

To interpret condition (14), we rearrange it into

$$\gamma'(p_0 - p^*_{SB}) + q(p^*_{SB})\,L = L. \tag{15}$$

The left hand side of (15) is the same as under the first-best scenario (12) , i.e. the sum of the marginal dollar cost, $\gamma'(p_0 - p^*_{SB})$, and the marginal increase in the premium,$q(p^*_{SB})\,L$, due to indirectly increasing the likelihood of being contaminated by the other agent. The right hand side of (15), however, differs from the first-best scenario (12). The only marginal benefit of investing in protection is the marginal reduction in premium, L, due to the reduced likelihood of a direct loss. As policyholders cannot contract on the level of investment in self-protection, it is not possible for a policyholder to benefit from the positive externality that his investment poses on the other policyholder as shown in equation (12) for the joint solution.

In the following subsection, we compare the level of investment in any Nash equilibrium with both the one in the first-best scenario and the one if policyholders do not face contamination.

Comparing Second-Best with First-Best. In this section, we compare the optimal level of investment in self-protection in the first-best with the one in the second-best scenario. Suppose it is optimal to not invest in self-protection in the first-best world, i.e. $\gamma'(0) \geq (1 - q(p_0) + (1 - p_0) q'(p_0)) L$. Then it is also not optimal to invest in self-protection in the second-best world, as $\gamma'(0) \geq (1 - q(p_0) + (1 - p_0) q'(p_0)) L > (1 - q(p_0)) L$ since an individual does not take into account the positive externalities provided the others when making an investment decision. Now suppose it is optimal to invest in self-protection in the first-best world, i.e. $\gamma'(0) < (1 - q(p_0) + (1 - p_0) q'(p_0)) L$. The optimal solution is then determined by

$$\gamma'(p_0 - p^*_{FB}) = (1 - q(p^*_{FB}) + (1 - p^*_{FB}) q'(p^*_{FB})) L.$$

This implies

$$\gamma'(p_0 - p^*_{FB}) > (1 - q(p^*_{FB})) L$$

and condition (14) yields $p^*_{SB} > p^*_{FB}$. In any pure-strategy Nash-equilibrium the level of investment in self-protection is thus lower compared to the first-best scenario. The intuition behind this result can be derived from comparing the first-order condition (12) in the first-best scenario under contamination

$$\gamma'(p_0 - p) + q(p) L = L + (1 - p) q'(p) L$$

with the first-order condition (14) in the second-best scenario under contamination

$$\gamma'(p_0 - p) + q(p) L = L.$$

We note that in the second-best scenario it is not possible to internalize the marginal benefit of the policyholder's effect on the other policyholder, $(1 - p) q'(p) L$, and he therefore underinvests in self-protection compared to the first-best scenario.

Comparing First-Best with No-Contamination. Let us compare the optimal level of investment without contamination with the one in the first-best scenario with contamination. The first-order condition (1) under no contamination is

$$\gamma'(p_0 - p) = L$$

and the first-order condition (12) in the first-best scenario under contamination is

$$\gamma'(p_0 - p) + q(p) L = L + (1 - p) q'(p) L.$$

By comparing the marginal costs and benefits of the two scenarios, we see that contamination adds both a marginal cost and a marginal benefit of investing in self-protection. The additional marginal cost, $q(p) L$, is due to the indirect increase in the likelihood of being contaminated by the other agent while the marginal benefit, $(1 - p) q'(p) L$, is due to the internalized positive of effect on the other agent. This implies that investment in self-protection under contamination can be larger or smaller than under no contamination depending on whether the additional marginal benefit is larger or smaller than the additional marginal cost.

Under the condition $p_0 < 1/2$—which seems most relevant for insurance events—the additional marginal benefit is larger than the additional marginal cost of investing in self-protection. It is thus optimal to invest more in self-protection under the first-best scenario with contamination compared to the scenario in which agents cannot be contaminated which yields $p^*_{FB} < p^*$.

To be more precise, suppose it is optimal to not invest in self-protection if there is no contamination, i.e. $\gamma'(0) \geq L$. Then it may still be optimal to invest in self-protection in the first-best world with contamination since the condition for not investing is $\gamma'(0) \geq (1 - q(p_0) + (1 - p_0) q'(p_0)) L$ and

$(1 - q(p_0) + (1 - p_0) q'(p_0)) L > L$ for all $p_0 < 1/2$. Now suppose that it is optimal to invest in self-protection without contamination. Then policyholders invest more in self-protection in the first-best world with contamination as the first-order condition $\gamma'(p_0 - p^*) = L$ under no contamination implies

$$\gamma'(p_0 - p^*) < (1 - q(p^*) + (1 - p^*) q'(p^*)) L$$

for all $p_0 < 1/2$ and condition (12) yields $p^*_{FB} < p^*$.

Comparing Second-Best with No-Contamination. Let us now compare the optimal level of investment without contamination with the one in the second-best scenario with contamination. Suppose it is optimal to not invest in self-protection if there is no contamination, i.e. $\gamma'(0) \geq L$. Then it is also optimal to not invest in self-protection in the second-best world as $\gamma'(0) \geq L$ implies $\gamma'(0) > (1 - q(p_0)) L$. Now suppose that it is optimal to invest in self-protection without contamination. Then policyholders invest less in self-protection in the second-best world with contamination as the first-order condition $\gamma'(p_0 - p^*) = L$ implies

$$\gamma'(p_0 - p^*) > (1 - q(p^*)) L$$

and condition (14) yields $p^*_{SB} > p^*$. In any pure-strategy Nash-equilibrium the level of investment in self-protection is thus lower compared to the scenario in which policyholders do not face possible contamination. The intuition behind this result can again be derived by comparing the first-order conditions (15) in the second-best scenario under contamination

$$\gamma'(p_0 - p) + q(p) L = L$$

with the first-order condition (1) under no contamination is

$$\gamma'(p_0 - p) = L.$$

In the second-best scenario policyholders, by investing in self-protection, face the additional marginal cost of implicitly increasing the likelihood of being contaminated by the other policyholder, $q(p) L$. The marginal cost equates the marginal benefit of investing in self-protection thus at a lower level of investment.

We summarize all findings of this section under the assumption $p_0 < 1/2$ in Table 2 below.

$\gamma'(0) \in ...$	$]0, (1 - q(p_0)) L[$	$[(1 - q(p_0)) L, L[$	$[L, \bar{p}L[$	$[\bar{p}L, \infty[$
No Contamination p^*	$p^* < p_0$	$p^* < p_0$	$p^* = p_0$	$p^* = p_0$
First-Best p^*_{FB}	$p^*_{FB} < p_0$	$p^*_{FB} < p_0$	$p^*_{FB} < p_0$	$p^*_{FB} = p_0$
Second-Best NE p^*_{SB}	$p^*_{SB} < p_0$ holds in all NE	$p^*_{SB} = p_0$ is a NE	$p^*_{SB} = p_0$ is unique NE	$p^*_{SB} = p_0$ is unique NE
Comparison	$p^*_{FB} < p^* < p^*_{SB}$	$p^*_{FB} < p^* < p^*_{SB}$	$p^*_{FB} < p^* = p^*_{SB}$	$p^*_{FB} = p^* = p^*_{SB}$

Table 2: Equilibrium levels of loss probabilities and comparison under $p_0 < 1/2$ where $\bar{p} = (1 - q(p_0) + (1 - p_0) q'(p_0))$.

7.2 Proof of Proposition 1

The economic environment is as above in the second-best scenario in which individuals cannot contract on their investment in self-protection. The only difference is that the insurance policy includes a deductible D, i.e., the insurer pays $L-D$ in case of a loss. The loss and final wealth distribution faced by policyholder i is

event	prob	final wealth
loss	$p_i + (1-p_i)\, p_j$	$w_0 - \gamma(p_0 - p_i) - P(D) - D$
no loss	$(1-p_i)(1-p_j)$	$w_0 - \gamma(p_0 - p_i) - P(D)$

where the actuarially fair premium is given by $P(D) = (p_i + (1-p_i)\, p_j)(L-D)$. Policyholder 1's expected utility of final wealth is given by

$$EU_i(p_i, p_j, D) = (1-p_i)(1-p_j)\, u(w_0 - \gamma(p_0 - p_i) - P(D)) + (p_i + (1-p_i)\, p_j)\, u(w_0 - \gamma(p_0 - p_i) - P(D) - D).$$

Policyholder i's best response function $p_i^*(p_j, D)$ is given by

$$p_i^*(p_j, D) \in \arg\max_{p_i} EU_i(p_i, p_j).$$

Let $p_{SB}^*(D)$ denote the symmetric Nash-equilibrium, i.e. $p_{SB}^*(D) = p_i^*(p_{SB}^*(D), D) = p_i^*(p_{SB}^*(D), D)$, which satisfies the first-order condition

$$\begin{aligned}
& (1-p_{SB}^*(D))\,(u(w_0 - \gamma(p_0 - p_{SB}^*(D)) - P(D) - D) - u(w_0 - \gamma(p_0 - p_{SB}^*(D)) - P(D))) \\
& + (1-p_{SB}^*(D))^2\,(\gamma'(p_0 - p_{SB}^*(D)) - (1-p_{SB}^*(D))(L-D))\, u'(w_0 - \gamma(p_0 - p_{SB}^*(D)) - P(D)) \\
& + p_{SB}^*(D)(2 - p_{SB}^*(D))(\gamma'(p_0 - p_{SB}^*(D)) - (1-p_{SB}^*(D))(L-D))\, u'(w_0 - \gamma(p_0 - p_{SB}^*(D)) - P(D) - D) \\
= \; & 0.
\end{aligned} \tag{16}$$

At $D=0$ we get the condition

$$\gamma'(p_0 - p_{SB}^*(0)) - (1-p_{SB}^*(0))\, L = 0, \tag{17}$$

where $p_{SB}^*(0) = p_{SB}^*$.

Note that

$$\begin{aligned}
P(D) &= p_{SB}^*(D)(2 - p_{SB}^*(D))(L-D) \\
P'(D) &= 2(1-p_{SB}^*(D))\, p_{SB}^{*\prime}(D)(L-D) - p_{SB}^*(D)(2 - p_{SB}^*(D)) \\
P'(0) &= 2p_{SB}^{*\prime}(0)\, \gamma'(p_0 - p_{SB}^*(0)) - p_{SB}^*(0)(2 - p_{SB}^*(0)) \\
P''(D) &= -2p_{SB}^{*\prime 2}(D)(L-D) + 2(1-p_{SB}^*(D))\, p_{SB}^{*\prime\prime}(D)(L-D) - 2(1-p_{SB}^*(D))\, p_{SB}^{*\prime}(D) - 2p_{SB}^{*\prime}(D)(1-p_{SB}^*(D))
\end{aligned}$$

For a given deductible D, the level of expected utility is given by

$$EU_i(p_{SB}^*(D), p_{SB}^*(D), D) = (1-p_{SB}^*(D))^2\, u(w_0 - \gamma(p_0 - p_{SB}^*(D)) - P(D)) + (2 - p_{SB}^*(D))\, p_{SB}^*(D)\, u(w_0 - \gamma(p_0 - p_{SB}^*(D)) - P(D) - D). \tag{18}$$

Differentiating expected utility with respect to the deductible level yields

$$\begin{aligned}\frac{\partial EU_i\left(p_{SB}^*\left(D\right),p_{SB}^*\left(D\right),D\right)}{\partial D} &= -2\left(1-p_{SB}^*\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(u\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)\right)-u\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)-D\right)\right)\\ &\quad+\left(1-p_{SB}^*\left(D\right)\right)^2\left(p_{SB}^{*\prime}\left(D\right)\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-P'\left(D\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)\right)\\ &\quad+\left(2-p_{SB}^*\left(D\right)\right)p_{SB}^*\left(D\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-P'\left(D\right)-1\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)-D\right)\end{aligned}$$

Evaluating the first derivative at $D=0$ and substituting $\gamma'\left(p_0-p_{SB}^*\left(0\right)\right)-\left(1-p_{SB}^*\left(0\right)\right)L=0$ and $P'\left(0\right)=2p_{SB}^{*\prime}\left(0\right)\gamma'\left(p_0-p_{SB}^*\left(0\right)\right)-p_{SB}^*\left(0\right)\left(2-p_{SB}^*\left(0\right)\right)$ implies

$$\frac{\partial EU_i\left(p_{SB}^*\left(D\right),p_{SB}^*\left(D\right),D\right)}{\partial D}|_{D=0}=-p_{SB}^{*\prime}\left(0\right)\left(1-p_{SB}^*\left(0\right)\right)Lu'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right). \tag{19}$$

To determine the sign of the first derivative we implicitly differentiate the first-order condition (16) with respect to D which yields the identity

$$\begin{aligned}&-p_{SB}^{*\prime}\left(D\right)\left(u\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)-D\right)-u\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)\right)\right)\\ &+\left(1-p_{SB}^*\left(D\right)\right)\begin{pmatrix}\left(p_{SB}^{*\prime}\left(D\right)\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-P'\left(D\right)-1\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)-D\right)\\ -\left(p_{SB}^{*\prime}\left(D\right)\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-P'\left(D\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)\right)\end{pmatrix}\\ &-2\left(1-p_{SB}^*\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-\left(1-p_{SB}^*\left(D\right)\right)\left(L-D\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)\right)\\ &+\left(1-p_{SB}^*\left(D\right)\right)^2\left(-p_{SB}^{*\prime}\left(D\right)\gamma''\left(p_0-p_{SB}^*\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^*\left(D\right)\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)\right)\\ &+\left(1-p_{SB}^*\left(D\right)\right)^2\left(\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-\left(1-p_{SB}^*\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-P'\left(D\right)\right)u''\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)\right)\\ &+2\left(1-p_{SB}^*\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-\left(1-p_{SB}^*\left(D\right)\right)\left(L-D\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)-D\right)\\ &+p_{SB}^*\left(D\right)\left(2-p_{SB}^*\left(D\right)\right)\left(-p_{SB}^{*\prime}\left(D\right)\gamma''\left(p_0-p_{SB}^*\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^*\left(D\right)\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)-D\right)\\ &+p_{SB}^*\left(D\right)\left(2-p_{SB}^*\left(D\right)\right)\left(\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-\left(1-p_{SB}^*\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma'\left(p_0-p_{SB}^*\left(D\right)\right)-P'\left(D\right)-1\right)u''\left(w_0-\gamma\left(p_0-p_{SB}^*\left(D\right)\right)-P\left(D\right)-D\right)\\ =&\ 0.\end{aligned} \tag{20}$$

Evaluating at $D=0$ and substituting $\gamma'\left(p_0-p_{SB}^*\left(0\right)\right)-\left(1-p_{SB}^*\left(0\right)\right)L$ we derive

$$p_{SB}^{*\prime}\left(0\right)\left(L-\gamma''\left(p_0-p_{SB}^*\left(0\right)\right)\right)=0.$$

The stability condition $\gamma''\left(p_0-p_{SB}^*\left(0\right)\right)>L$ then implies $p_{SB}^{*\prime}\left(0\right)=0$ and (19) yields

$$\frac{\partial EU_i\left(p_{SB}^*\left(D\right),p_{SB}^*\left(D\right),D\right)}{\partial D}|_{D=0}=0.$$

To determine whether expected utility increases or decreases in the deductible level for small deductible levels, we have to determine the sign of the second derivative of expected utility evaluated at $D = 0$. Differentiating (18) twice with respect to D yields

$$
\begin{aligned}
& \frac{\partial^2 EU_i \left(p_{SB}^{*}(D), p_{SB}^{*}(D), D\right)}{\partial D^2} \\
= \; & -2\left(p_{SB}^{*\prime\prime}(D)\left(1-p_{SB}^{*}(D)\right) - p_{SB}^{*\prime 2}(D)\right)\left(u\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) - u\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right)\right) \\
& -2\left(1-p_{SB}^{*}(D)\right)p_{SB}^{*\prime}(D)\begin{pmatrix} \left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \\ -\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D) - 1\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) \end{pmatrix} \\
& -2\left(1-p_{SB}^{*}(D)\right)p_{SB}^{*\prime}(D)\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \\
& +\left(1-p_{SB}^{*}(D)\right)^2\left(p_{SB}^{*\prime\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - p_{SB}^{*\prime 2}(D)\gamma''\left(p_0 - p_{SB}^{*}(D)\right) - P''(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \\
& +\left(1-p_{SB}^{*}(D)\right)^2\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D)\right)^2 u''\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \\
& +2\left(1-p_{SB}^{*}(D)\right)p_{SB}^{*\prime}(D)\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D) - 1\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) \\
& +\left(2-p_{SB}^{*}(D)\right)p_{SB}^{*}(D)\left(p_{SB}^{*\prime\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - p_{SB}^{*\prime 2}(D)\gamma''\left(p_0 - p_{SB}^{*}(D)\right) - P''(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) \\
& +\left(2-p_{SB}^{*}(D)\right)p_{SB}^{*}(D)\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D) - 1\right)^2 u''\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right)
\end{aligned}
$$

Evaluating at $D = 0$ and substituting $p_{SB}^{*\prime}(0) = 0$, $\gamma'\left(p_0 - p_{SB}^{*}(0)\right) - \left(1-p_{SB}^{*}(0)\right)L$, $P'(0) = -p_{SB}^{*}(0)\left(2-p_{SB}^{*}(0)\right)$, and $P''(0) = 2\left(1-p_{SB}^{*}(0)\right)p_{SB}^{*\prime\prime}(0)L$ yields

$$
\begin{aligned}
& \frac{\partial^2 EU_i \left(p_{SB}^{*}(D), p_{SB}^{*}(D), D\right)}{\partial D^2}\Big|_{D=0} \qquad (21) \\
= \; & -p_{SB}^{*\prime\prime}(0)\left(1-p_{SB}^{*}(0)\right)Lu'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(0)\right) - P(0)\right) + \left(1-p_{SB}^{*}(0)\right)^2 p_{SB}^{*}(0)\left(2-p_{SB}^{*}(0)\right)u''\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(0)\right) - P(0)\right).
\end{aligned}
$$

Differentiating the identity (20) with respect to D yields

$$
\begin{aligned}
& -p_{SB}^{*\prime\prime}(D)\left(u\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) - u\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right)\right) \\
& -p_{SB}^{*\prime}(D)\begin{pmatrix} \left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D) - 1\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) \\ -\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \end{pmatrix} \\
& -p_{SB}^{*\prime}(D)\begin{pmatrix} \left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D) - 1\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) \\ -\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \end{pmatrix} \\
& +\left(1-p_{SB}^{*}(D)\right)\begin{pmatrix} \left(p_{SB}^{*\prime\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - p_{SB}^{*\prime 2}(D)\gamma''\left(p_0 - p_{SB}^{*}(D)\right) - P''(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) \\ -\left(p_{SB}^{*\prime\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - p_{SB}^{*\prime 2}(D)\gamma''\left(p_0 - p_{SB}^{*}(D)\right) - P''(D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \end{pmatrix} \\
& +\left(1-p_{SB}^{*}(D)\right)\begin{pmatrix} \left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D) - 1\right)^2 u''\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D) - D\right) \\ -\left(p_{SB}^{*\prime}(D)\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - P'(D)\right)^2 u''\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right) \end{pmatrix} \\
& +2p_{SB}^{*\prime}(D)p_{SB}^{*\prime}(D)\left(\gamma'\left(p_0 - p_{SB}^{*}(D)\right) - \left(1-p_{SB}^{*}(D)\right)(L-D)\right)u'\left(w_0 - \gamma\left(p_0 - p_{SB}^{*}(D)\right) - P(D)\right)
\end{aligned}
$$

$$
\begin{aligned}
& -2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime\prime}\left(D\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& -2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& -2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)\right)u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& -2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& +\left(1-p_{SB}^{*}\left(D\right)\right)^{2}\left(-p_{SB}^{*\prime\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime 2}\left(D\right)\gamma^{\prime\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime\prime}\left(D\right)\left(L-D\right)-p_{SB}^{*\prime}\left(D\right)-p_{SB}^{*\prime}\left(D\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& +\left(1-p_{SB}^{*}\left(D\right)\right)^{2}\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)\right)u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& -2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)\right)u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& +\left(1-p_{SB}^{*}\left(D\right)\right)^{2}\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)\right)u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& +\left(1-p_{SB}^{*}\left(D\right)\right)^{2}\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-p_{SB}^{*\prime\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime\prime}\left(D\right)\right) \\
& \cdot u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& +\left(1-p_{SB}^{*}\left(D\right)\right)^{2}\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)\right) \\
& \cdot u^{\prime\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)\right) \\
& -2p_{SB}^{*\prime}\left(D\right)p_{SB}^{*\prime}\left(D\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime\prime}\left(D\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)-1\right)u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +2p_{SB}^{*\prime}\left(D\right)\left(1-p_{SB}^{*}\left(D\right)\right)\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +p_{SB}^{*}\left(D\right)\left(2-p_{SB}^{*}\left(D\right)\right)\left(-p_{SB}^{*\prime\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime 2}\left(D\right)\gamma^{\prime\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime\prime}\left(D\right)\left(L-D\right)-p_{SB}^{*\prime}\left(D\right)-p_{SB}^{*\prime}\left(D\right)\right) \\
& \cdot u^{\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +p_{SB}^{*}\left(D\right)\left(2-p_{SB}^{*}\left(D\right)\right)\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)-1\right) \\
& \cdot u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +2\left(1-p_{SB}^{*}\left(D\right)\right)p_{SB}^{*\prime}\left(D\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)-1\right)u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +p_{SB}^{*}\left(D\right)\left(2-p_{SB}^{*}\left(D\right)\right)\left(-p_{SB}^{*\prime}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)+p_{SB}^{*\prime}\left(D\right)\left(L-D\right)+\left(1-p_{SB}^{*}\left(D\right)\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)-1\right) \\
& \cdot u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +p_{SB}^{*}\left(D\right)\left(2-p_{SB}^{*}\left(D\right)\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-p_{SB}^{*\prime 2}\left(D\right)\gamma^{\prime\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime\prime}\left(D\right)\right) \\
& \cdot u^{\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
& +p_{SB}^{*}\left(D\right)\left(2-p_{SB}^{*}\left(D\right)\right)\left(\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-\left(1-p_{SB}^{*}\left(D\right)\right)\left(L-D\right)\right)\left(p_{SB}^{*\prime}\left(D\right)\gamma^{\prime}\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P^{\prime}\left(D\right)-1\right)^{2}u^{\prime\prime\prime}\left(w_{0}-\gamma\left(p_{0}-p_{SB}^{*}\left(D\right)\right)-P\left(D\right)-D\right) \\
= \; & 0.
\end{aligned}
$$

Evaluating at $D = 0$ and substituting $p^{*\prime}_{SB}(0) = 0$, $\gamma'(p_0 - p^*_{SB}(0)) - (1 - p^*_{SB}(0))L$, $P'(0) = -p^*_{SB}(0)(2 - p^*_{SB}(0))$, and $P''(0) = 2(1 - p^*_{SB}(0))\,p^{*\prime\prime}_{SB}(0)\,L$ yields

$$-(1 - p^*_{SB}(0))\left(1 - 2(1 - p^*_{SB}(0))^2\right)u''(w_0 - \gamma(p_0 - p^*_{SB}(0)) - P(0)) + p^{*\prime\prime}_{SB}(0)(L - \gamma''(p_0 - p^*_{SB}(0)))\,u'(w_0 - \gamma(p_0 - p^*_{SB}(0)) - P(0)) = 0.$$

i.e.

$$p^{*\prime\prime}_{SB}(0) = \frac{(1 - p^*_{SB}(0))\left(1 - 2(1 - p^*_{SB}(0))^2\right)}{(L - \gamma''(p_0 - p^*_{SB}(0)))} \cdot \frac{u''(w_0 - \gamma(p_0 - p^*_{SB}(0)) - P(0))}{u'(w_0 - \gamma(p_0 - p^*_{SB}(0)) - P(0))}. \tag{22}$$

If $p^*_{SB}(0) < 1 - \sqrt{1/2}$ then $p^{*\prime\prime}_{SB}(0) < 0$ and small deductible levels thus increase the investment in self-protection. Note that this condition is implied by the necessary and sufficient condition. Substitution of (22) into the second derivative of expected utility (21) implies

$$\frac{\partial^2 EU_i(p^*_{SB}(D), p^*_{SB}(D), D)}{\partial D^2}|_{D=0} = \left(1 - (1 - p^*_{SB}(0))^2 - \frac{\left(1 - 2(1 - p^*_{SB}(0))^2\right)L}{(L - \gamma''(p_0 - p^*_{SB}(0)))}\right)(1 - p^*_{SB}(0))^2\,u''(w_0 - \gamma(p_0 - p^*_{SB}(0)) - P(0)) \tag{23}$$

A strictly positive deductible is optimal if and only if expected utility is convex at $D = 0$, i.e. if $\frac{\partial^2 EU_i(p^*_{SB}(D), p^*_{SB}(D), D)}{\partial D^2}|_{D=0} > 0$. Equation (23) implies the condition

$$(1 - p^*_{SB}(0))^2 L > \left(1 - (1 - p^*_{SB}(0))^2\right)\gamma''(p_0 - p^*_{SB}(0)).$$